Hibernating during the pandemic. A quantitative assessment of business failures and risk of zombification of French firms

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The COVID-19 shock resulted in an economic crisis of an unprecedented scale and nature. Even if the GDP loss was massive, business failures decreased by almost 40 % in 2020 in France. The massive government interventions let firms with liquidity and solvency problems to hibernate during the pandemic. Cros, Epaulard, and Martin (2020) highlight this kind of behavior. By delaying failures, a 'time bomb' may explode once government support fades out. In order to assess the share of firms concerned by this risk, a structural model of industry equilibrium with firm-level heterogeneity, based on the canonical Melitz (2003) model, is developed and calibrated. In the central calibration, 3% of firms have a negative value after the COVID-19 crisis and 6.9-9.4 % of firms hibernate during the pandemic. The model can be extended to quantify the share of firms with risk of zombification. It cannot be excluded that, in the post-COVID equilibrium, a mass of firms may be sufficiently efficient to remain profitable but unable to pay the burden of the higher debt inherited from the COVID-19 crisis.
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The COVID-19 epidemic and the measures implemented to stem the spread of the virus resulted in an economic crisis of an unprecedented scale and nature. Entire sections of the economy shut down to ensure social distancing. Many employees were declared as non-essential and had to stay at home and were excluded from production process. Some firms suffered from disruptions in supply chains. It is possible to add a productivity shock linked to the massive unanticipated recourse to remote-work. Finally, demand from private agents felt sharply - either through the inability to make certain purchases or through purchase deferral to limit the risk of infection. Lastly, the macroeconomic savings rate skyrocketed in a context of high uncertainty. This unusual combination of supply and demand-side shocks produced massive GDP losses in all economies, advanced or emerging. The impact of an unanticipated lockdown was huge. In April 2020, in France the monthly-GDP decreased by an unprecedented rate of 31 % and if we restrict to the business sector the fall was even more dramatic close of 50 %. In 2020, French

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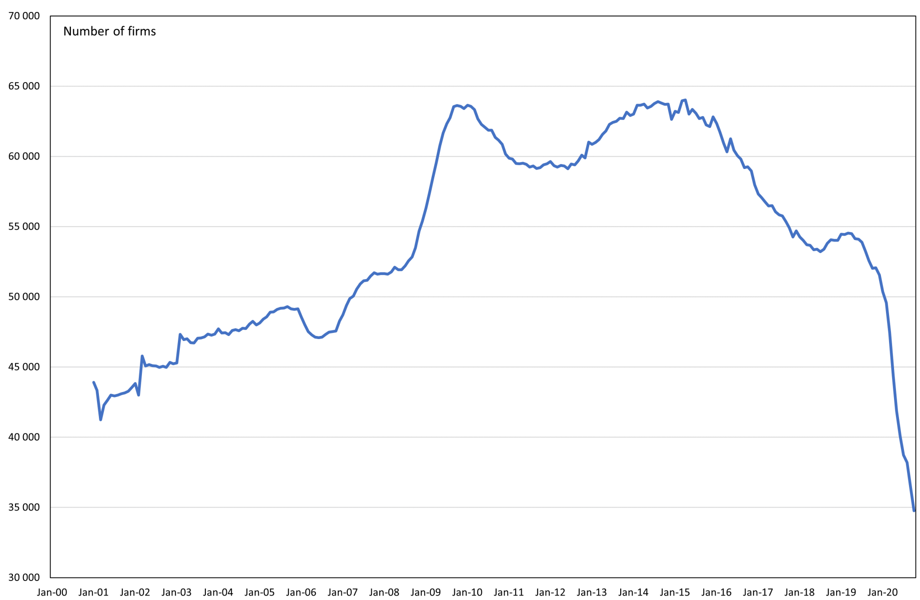
GDP fell by 9 %, the biggest recession since WW2.

Surprisingly, the COVID-19 crisis has not lead to a surge in business failures. According to the data published by the Banque de France, from January to October 2020 business failures decreased by 39 % with respect to the same period of 2019 (Figure 1). Cros, Epaulard, and Martin (2020) show a similar pattern using judiciary data. This paradox can be explained by the generous policy response that gave direct subsidies to firms (*fonds de solidarité*), the large extension of partial unemployment schemes (*activité partielle*) that allowed firms to lower the cost of employees not involved in the production process during the pandemic. Moreover, deferral of taxes and social contributions and public guarantees granted by the French government supported firm's treasury. According to Dauvin et al. (2020) the COVID-19 cut off business sector profits by 2.45 points of GDP, an amount covered by state guarantees and loans. So far, unprofitable firms are hibernating as stated by Cros, Epaulard, and Martin (2020) but this creates the risk of creating a fringe of zombie firms in the forthcoming years.

Figure 1. Cumulative business failures over the last 12 months



Note: Source: Banque de France.

The objective of this paper is to assess the share of firms that hibernate during the pandemics and the share of firms that face a risk of *zombification* after the pandemic. This assessment relies on a structural model of firm behavior. Gourinchas et al. (2021) measure the 'time bomb' of SME failures that can be expected in 2021 if fiscal policy becomes less generous. Our setting builds on a partial-equilibrium of heterogeneous firms facing simultaneous supply and demand shocks. The basis of the model relies on Melitz (2003) model, well known in the international trade literature, which is extended to include the idiosyncrasies of the COVID-19 shock. The model is calibrated to quantify the share of firms affected by solvency and liquidity problems during the pandemic.

Obviously, this is paper is not the first to model the idiosyncrasies of the COVID-19 crisis. A quick search shows that on February 3, 2021, 579 working papers published by the NBER contain the word "COVID" in their title. Economists proved inspired (and inspiring) to contribute to the public debate using new tools to answer new questions (or forgotten ones). It is impossible to cite and value appropriately all the relevant literature linked with this paper. In this mass of articles, it is necessary to highlight the seminal work of Baqaee and Farhi (2020a), Baqaee and Farhi (2020b), Baqaee and Farhi (2021) and Guerrieri et al. (2020). These paper take into account the simultaneous effect of supply and demand shocks heterogeneous among industries. The model built for this article is much closer of Gourinchas et al. (2020) which is more suitable for an analysis without strategic interactions among firms or complementarities in demand or production process.

This article seeks to identify the binding shocks that modify firm's behavior. In Dauvin and Sampognaro (2021) we show that when each industry has one representative firm, only the harsher shock restricts the production of the industry. With multiple and heterogeneous firms, firms constrained by supply and by demand may coexist within an industry, depending on the relative size of both shocks and some sectoral structural parameters. Once calibrated, the structural model is easily tractable, especially if the heterogeneity between firms follows a Pareto distribution. It is possible to assess the share of firms that risk failure and the number of firms that need an open access to liquidity in order to avoid the consequences of a negative cash flow.

It is also important to be clear about what this paper is not about. Buera et al. (2021) develop a model of firm-behavior during the pandemics when labor market and financial frictions matter. Here, firms cannot adjust to their optimal labor demand (even if *activité partielle* covers the impact of unproductive employees on its profitability) but financial frictions do not play any major role. Moreover, there is not heterogeneity in the preference for risk among firm-owners that can explain some heterogeneity in the balance sheet structure of firms before the emergence of the pandemic which can explain a heterogeneous exposure to risks between firms depending on their efficiency.

The heterogeneity among firms within an industry is only related to their relative productive efficiency. Firms that are more productive set lower prices, reach a higher demand and make bigger profits. Automatically, all the liquidity and solvency problems generated by the COVID-19 shock will be concentrated among smaller and less productive firms. This model is unable to generate an interesting pattern shown by . The authors show that 4 %[1] of big firms could face solvency problems due to COVID-19, linked with their higher leverage before the pandemics. This result cannot emerge from our model.

In the preferred calibration, our model predicts that 2% of firms have a negative value after the COVID-19 crisis, which should lead to failure. Moreover, 10 % of firms hibernate during the pandemics. In line with very heterogeneous sectoral shocks, the hibernation behavior is heterogeneous among industries. More than 20 % of firms in accommodation and food service activities and more of 10 % of firms in wholesale and retail trade, transportation and storage, construction and manufacture of transport equipment follow this kind of behavior. The sectoral heterogeneity of failure risk is smaller, as no permanent shock of the pandemics is assumed neither on demand nor on supply.

The outline of the paper is as follows. In Section 1, the model is exposed. In Section 2, the concepts of liquidity and solvency that can be tracked within the theoretical model is presented. In Section 3, some quantitative evaluations are performed in order to assess the share of firms concerned by liquidity and solvency problems during the COVID-19 shock. In Section 4, the model is extended to allow for the emergence of zombie firms after the pandemics (TO COME). Lastly, I conclude.

I. A model of firm behavior during the pandemic

In a no-COVID-19 world, the model of sectoral equilibrium is exactly an autarky version of the Melitz (2003), presented in detail in Melitz and Redding (2014). The economy consists in a set of S industries and the analysis is in partial equilibrium.

A. The model set-up: Melitz model

Households. There is a population of households with identical preferences, and each household owns one unit of only one of the industry-specific factor[2] . Consumers allocate their budget according to the following nested-CES preferences:

[1]According to the classification of the Insee, a firm belongs to the category *Grande entreprise* if it employs more of 5 000 employees and if its sales are bigger of 1.5 billion euros or its balance sheet is bigger than 2.0 billion of euros.

[2]To simplify each factor is specific to each industry in this version of the paper. Allowing for mobility of factors between industries would be necessary to analyze a general-equilibrium version of the model.

$$C = \left[\sum_{j \in S} \phi_j C_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

Where ϕ_j is a demand shifter and η is the constant elasticity of substitution among goods of different industries[3], C represents the aggregate consumption in volume and C_j the aggregate consumption of goods of industry j. Macroeconomic variables, like C, are not modeled and are considered as given.

Then, the demand for variety $\omega \in \Omega_j$ is given by a CES aggregator:

$$C_j = \left[\int_{\omega \in \Omega} c_j(\omega)^{\frac{\sigma_j - 1}{\sigma_j}} d\omega \right]^{\frac{\sigma_j}{\sigma_j - 1}}$$

with $\sigma_j > 1$, the elasticity of substitution between varieties produced by firms of the same industry.

P_j and P are the usual CES-price index of varieties produced by sector j and the price of aggregate consumption:

$$P_j = \left[\int_{\omega \in \Omega_j} p_j^{1-\sigma_j} d\omega \right]^{\frac{1}{1-\sigma_j}} \text{ and } P = \left[\sum_{i=1}^{J} \phi^{\eta} P_j^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

The demand in volume (c_j) for a variety of a firm with productivity φ is:

$$c_j(\omega) = \frac{\phi_j PC}{P_j} \left(\frac{p_j \omega}{P_j} \right)^{-\sigma_j}$$

The demand depends on the total expenses of households allocated to goods from industry j ($\frac{\phi_j PC}{P_j}$) and the relative price of the firm relatively to the price index of available varieties.

Producers. Each firm can differentiate horizontally and has a local monopoly power over its variety. The production of a variety only requires using the specific factor. All firms of the industry remunerate the specific-factor at the same wage rate w_j. If a firm of productivity φ wants to produce $q_j(\varphi)$ units $l_j(\varphi)$ units of the specific factors are employed:

$$l_j(\varphi) = f_j + \frac{q_j(\varphi)}{\varphi}$$

The firm has a constant marginal cost ($\frac{w_j}{\varphi}$) and the presence of the fixed cost (f_j) ensures that the production function presents increasing returns to scale. Firms set prices equal to their marginal cost augmented by a constant mark-up

[3]To simplify notation we will stick to Cobb-Douglas case to allocate the consumer budget among industries ($\eta = 1$)

(decreasing on σ_j).

At the equilibrium, the value of sales of a firm of productivity φ is:

$$r_j(\varphi) = \phi_j PCP_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{1-\sigma_j} w_j^{1-\sigma_j}\varphi^{\sigma_j-1}$$

A firm will be active in the market if it can make positive profits (π_j):

$$\pi_j(\varphi) = \frac{r(\varphi)}{\sigma_j} - f_j w_j$$

Defining a productivity threshold (φ^*) for which all firms with $\varphi \geq \varphi^*$ make non-negative profits:

$$\varphi^* \Leftrightarrow \pi_j(\varphi^*) = 0 \Leftrightarrow \frac{r(\varphi^*)}{\sigma_j} = f_j w_j$$

B. Introducing the COVID-19 shock to the Melitz Model

From now on, the hat-notation $\widehat{x} = \frac{x'}{x}$ is used to represent the change of variable x between the pre-COVID period (x) and its value during the pandemic (x').

Demand shock. The demand of consumers is affected by the spread of the virus. Households cut their aggregate expenses in order to increase their precautionary savings ($\widehat{C} < 1$). Moreover, households reallocate their budget in order to avoid the purchase of products requiring social interactions. This is modeled through a change in the demand shifter ($\widehat{\phi_j}$). In the Cobb-Douglas case is very easy to see that ($\widehat{P_jC_j} = \widehat{PC}\widehat{\phi_j}$). Under these circumstances and supposing that firms cannot adjust their prices during the pandemic, the demand addressed to a firm of productivity φ:

$$\widehat{c_j(\varphi)} = \widehat{PC}\widehat{\phi_j}\widehat{P_j}^{\sigma_j-1}$$

This demand depends on the industry-level shock $(\widehat{PC})\widehat{\phi_j}$) and the evolution of the sectoral price index ($\widehat{P_j}^{\sigma_j-1}$). As individual prices are unchanged, this price index only evolves if the composition of active firms changes during the pandemic. Measured by the hat-algebra, the change in demand does not depend on the efficiency of the firm. This is a very useful feature of the model.

Supply shock. Firms are constrained by the absenteeism of non-essential employees, of sick employees or employees unable to go to their workplaces as schools may be closed during the lockdown. Only a fraction $\nu_j \in [0, 1]$ of employees is able to produce during the pandemic. This share is the same for all firms within the industry. We can also assume that firms suffer from a productivity losses to the diffusion of remote-work. However, as there is no clear evidence that the massive and unanticipated recourse to remote-work had an impact on productivity, this kind of effect is neglected[4].

Decision timing. By assumption, firms set their demand for specific factors $l_j(\varphi)$ and their price $p_j(\varphi)$ at the start of the year according to the expected macroeconomic and sectoral context (P_j,P,C,w_j,ϕ_j). Those decisions depend on the pre-COVID equilibrium. After the unexpected irruption of the pandemics, only $\nu_j l_j(\varphi)$ employees are able to produce[5]. Firms cannot hire new employees and only incumbent firms can produce during the pandemic.

Potential labor demand during COVID-19. In this simple framework, the only margin of adjustment at the disposal of the firm is the number of units of labor effectively productive. Unproductive factors do not weigh on firm profitability as *activité partielle* is supposed to finance the wage bill of all the unproductive employees. The firm should choose its COVID-labor demand under the constraint that $l'_j(\varphi) \leq \nu_j l_j(\varphi)$. If profits are insufficient to cover the fixed costs, a firm chooses to remain inactive during the pandemic.

Before the COVID-19 shock, the optimal factor demand for a firm of productivity φ is:

$$l_j(\varphi) = f_j + PC\phi_j P_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{-\sigma_j} w_j^{-\sigma_j}\varphi^{\sigma_j-1}$$

After the COVID-19 shock, the optimal labor demand ($l_j(\varphi)'$) is:

$$l_j(\varphi)' = f_j + \widehat{PC}\widehat{\phi_j}\widehat{P_j}^{\sigma_j-1}\phi_j PC P_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{-\sigma_j} w_j^{-\sigma_j}\varphi^{\sigma_j-1}$$

However, some firms are unable to reach their optimal size. In that case, the maximum productive workers of the firm is:

$$\nu_j l_j(\varphi) = \nu_j f_j + \nu_j PC\phi_j P_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{-\sigma_j} w_j^{-\sigma_j}\varphi^{\sigma_j-1}$$

This constraint defines a new threshold φ^{**}, separating firms unable to adjust its labor force to its optimal-level (see Appendix A.A1):

$$\varphi^{**} = \begin{cases} \frac{1-\nu_j}{\nu_j-\widehat{PC}\widehat{\phi_j}\widehat{P_j}^{\sigma_j-1}} \times (\sigma_j-1)\times(\varphi^*)^{\sigma_j-1} & \text{if } \nu_j - \widehat{PC}\widehat{\phi_j}\widehat{P_j}^{\sigma_j-1} \geq 0 \\ +\infty & \text{if } \nu_j - \widehat{PC}\widehat{\phi_j}\widehat{P_j}^{\sigma_j-1} < 0 \end{cases}$$

When $\varphi \geq \varphi^{**}$ the supply shock is not binding and firms adjust to their optimal

[4]In the absence of productivity losses and without changes in the price of factors, the assumption that prices of varieties remain fixed during the pandemics is unnecessary.

[5]It is left for a future extension the impact of an *activité partielle* that does not fully compensate the cost of unproductive factors for the firm.

specific-factor demand. If the supply shock is harsher that the demand shock faced by the firm, all firms in the industry will be constrained by absenteeism.

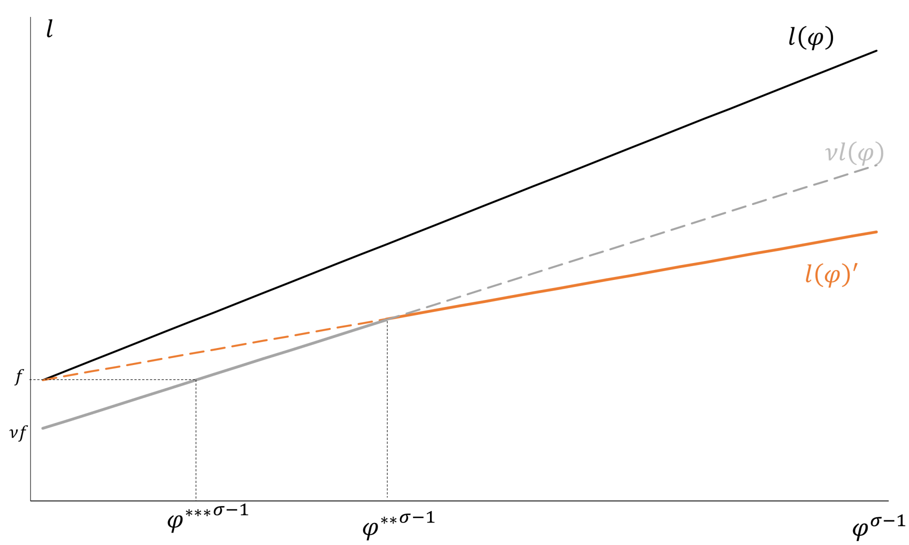
It is theoretically possible that some small firms can be technologically unable to produce due to the absenteeism shock. This may arise if the supply shock is big enough: $\nu_j l_j(\varphi) < f_j$. This defines a new technical threshold. Firms with productivity lower than φ^{***} are forced to stop their production (details in Appendix A.A2):

$$(\varphi^{***})^{\sigma_j-1} = \frac{1-\nu_j}{\nu_j} \times (\sigma_j - 1) \times (\varphi^*)^{\sigma_j-1}$$

It is evident that $\varphi^{***} < \varphi^{**}$ if $\widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1} > 0$, a very general condition as this means that a positive demand for products j exist during the pandemic. If there is no absenteeism $\nu_j = 1$), this constraint never binds ($\varphi^{***} = 0$) and if all employees are considered as non-essential all firms are inactive during the pandemic ($\varphi^{***} \to \infty$).

Figure 2 shows graphically the potential demand for labor of firms of different productivity when the demand shock is bigger (in absolute terms) than the supply shock. In this example, some firms are unable to produce because of the absenteeism shock ($\varphi < \varphi^{***}$), others are able to produce but could be constrained by the supply shock ($\varphi^{***} \leq \varphi < \varphi^{**}$), while the last group is limited by consumer expenses ($\varphi \geq \varphi^{**}$).

Figure 2. Labor demand when the demand shock is bigger than the supply shock



Firm profitability. The thresholds φ^{**} and φ^{***} determine the productivity threshold separating firms in function of supply and demand shocks. However, these thresholds do not exhaust all the reactions of firms. Lower sales means lower profits and less possibility to cover the fixed cost of production. Firms that are constrained by demand face a new profitability threshold $\varphi^{*'}$ (see Appendix A.A3):

$$\left(\varphi^{*'}\right)^{\sigma_j - 1} = \left[\frac{1}{\widehat{PC}\widehat{\phi}_j \widehat{P_j}^{\sigma_j - 1}}\right] (\varphi^*)^{\sigma_j - 1}$$

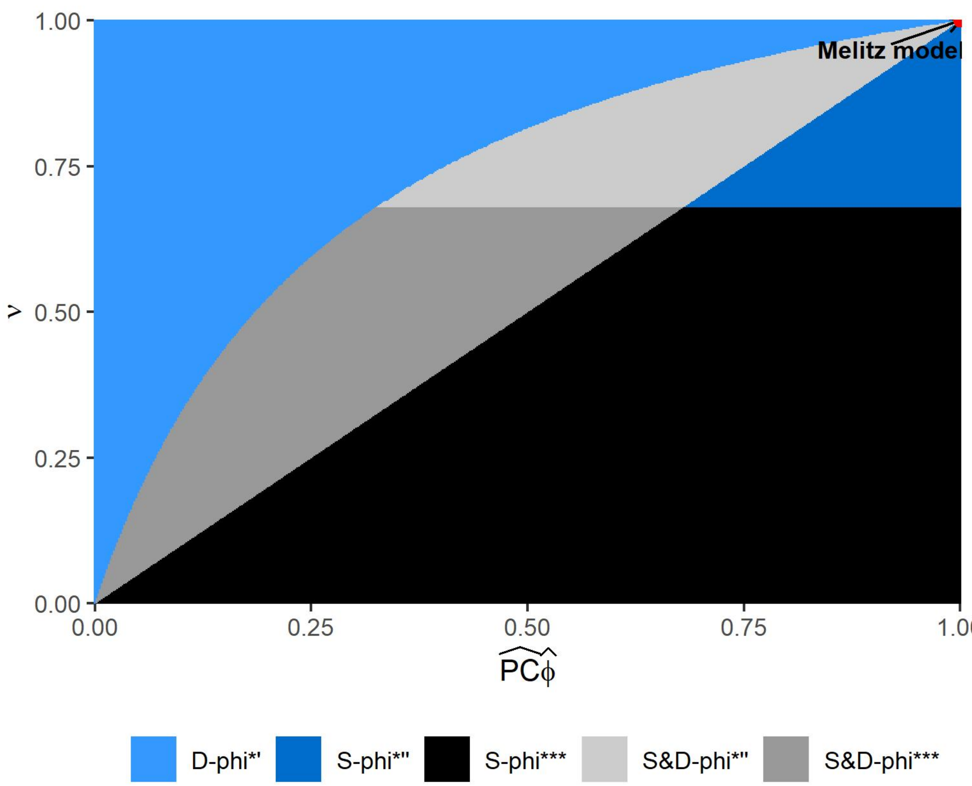
Otherwise, firms constrained by absenteeism face a the profitability threshold $\varphi^{*''}$ (see Appendix A.A4):

$$\left(\varphi^{*''}\right)^{\sigma_j - 1} = \left[\frac{1}{\nu_j} \times \frac{\sigma_j - \nu_j}{\sigma_j - 1}\right] \times (\varphi^*)^{\sigma_j - 1}$$

It is necessary to compare the values of φ^{**}, φ^{***}, $\varphi^{*'}$ and $\varphi^{*''}$ to know if the constraints identified on labor are effectively binding. While $\varphi^{*'}$ depends only on the demand shock, φ^{***} and $\varphi^{*''}$ depend only on the supply shock and φ^{**} depends on both supply and demand shocks.

Partial equilibrium. The formulas for $\varphi^{*'}$ and $\varphi^{*''}$ depend on the exogenous shocks ($\widehat{PC}\widehat{\phi}_j$ and ν_j) and the endogenous change of the industry-level price index ($\widehat{P_j}$). The last variable is endogenous as it depends on the set available varieties, which can change during the epidemic. In order to compute this price change it is necessary to make some additional assumptions about the distribution of the firm-level efficiency. Then it will be possible to solve for the equilibrium of ($\widehat{P_j}$ and compare which constraints are binding of firm-participation and its production. For the moment, let us consider $\widehat{P_j}$ as exogenous. Multiple configurations can emerge, depending on the relative size of demand and supply shocks. Some firms remain inactive during the pandemic if their productivity $\varphi \in \left[\varphi^*, \max(\varphi^{*'}, \varphi^{*''}, \varphi^{***})\right]$. The cause of inactivity may be linked either with the shortage of demand ($\varphi^{*'} > \max(\varphi^{*''}, \varphi^{***})$) or with the absenteeism shock. For firms remaining active, multiple cases may emerge to know how the COVID-19 crisis affects its sales. If the demand shock is more severe than the supply shock, some firms may nevertheless be constrained by absenteeism (if $\varphi^{**} > \varphi^{*'}$ while all remaining firms ($\varphi > \varphi^{**}$) adjust their labor demand to their optimal level, which is compatible with the fall in demand. If the supply shock is more severe than the demand shock, all the active firms are constrained by supply. All the possible configurations that can emerge in function of the size of the demand and supply shocks, for a given σ_j, are shown in Figure 3.

FIGURE 3. FIVE CONFIGURATIONS OF BINDING CONSTRAINTS



II. Hibernating during the pandemic and business failures

The theoretical model developed in Section 1 allows to identify the binding constraints weighing on firm's static decisions. The impact of the COVID-19 related shocks on firm sales, and thus profits, depends on their productivity. The fall in their contemporaneous profits, influence the cash flow of the firm and its value. Some firms may be pushed to failure. In this section, the notions of liquidity and solvency coherent with the theoretical framework are defined.

A. *Defining liquidity and insolvency risk*

Liquidity. Strictly speaking, in the current setting a firm cannot have a negative cash flow. The wage bill of unproductive labor is fully paid by *activité partielle* and fixed cost are paid in terms of the specific-labor, eligible to the program. Therefore, a firm that stops its production makes no losses and has a null cash flow. Like Cros, Epaulard, and Martin (2020), these inactive firms hibernate during the COVID-19 crisis. This happens for firms with a productivity $\varphi \in \left[\varphi^*, \max(\varphi^{*'}, \varphi^{*''}, \varphi^{***})\right])$[6].

Insolvency. A firm is insolvent and exit the market if its value becomes negative. Classically, the value of a firm is defined as the present value of its current and future profits. Before COVID-19, the value of a firm of productivity φ is:

$$V_j(\varphi) = \sum_{t=0}^{T} \frac{\pi_j(\varphi)}{(1+r)^t} = \frac{r_j(\varphi)}{\sigma_j r} - \frac{w_j f_j}{r}$$

Firms do not hold equity capital. If a firm holds equity, its value would be risen by this amount and this would reduce the risk of failure. Implicitly, it assumed that firms distribute all the profits to shareholders. Chaney (2016) extends the Melitz model to incorporate reserves to this model. Extending the theoretical model would be easy. However, the main objective of this article is to quantify the liquidity and insolvency risk without data[7]. Guerini et al. (2020) include this element to their analysis.

Without COVID-19, $V_j(\varphi) > 0$ implies that $\varphi > \varphi^*$, the static zero profit condition of the Melitz model. Even if the COVID-19 is transitory, the value of firms close to the threshold may become negative and force some firms to exit. It is also possible that a firm hibernating keeps a positive value. In that case, the COVID-19 shock does not translate into a permanent exit.

[6]If we add some fixed costs paid in terms of other goods or services (i.e. rents) then an inactive firm has a negative cash flow. In that case, liquidity and insolvency risks emerge. **Extension to come**.

[7]This model was developed during the first lockdown, exclusively with access to public data. The main objective is to make fast evaluations of the liquidity and failure risk.

B. Insolvency thresholds

Assuming that the COVID-19 crisis lasts only one period, that fixed cost are eligible to *activité partielle* and that the post-COVID equilibrium is identical to the pre-COVID one:

$$V_j'(\varphi) = \left[\frac{r_j'(\varphi) - r_j(\varphi)}{\sigma_j}\right] + V_j(\varphi)$$

Therefore, it is evident that the evolution of the value of the firm depends exclusively on the impact of COVID-19 on its sales. As mentioned in Section 2, the change in sales during the pandemics depends on the binding shock for the firm.

Demand constrained firm. A firm constrained by the demand shock is insolvent if its productivity is lower to the threshold φ^V (see Appendix A.A5):

$$(\varphi^V)^{\sigma_j - 1} = \frac{(\varphi^*)^{\sigma_j-1}}{\left[1 - r\left(\widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1}\right)\right]}$$

For negative demand shocks $(\widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1} < 1)$ φ^V is higher than the pre-COVID solvability ratio φ^* but lower to the static profitability ratio $(\varphi^{*'})$.

Supply constrained firm. A firm constrained by the supply shock is insolvent if its productivity is lower to the threshold φ^{V2} (see Appendix A.A6):

$$\left(\varphi^{V2}\right)^{\sigma_j-1} = \frac{\sigma_j - 1 + r(1-\nu_j)}{(\sigma_j - 1)\times\left[1 - r\left(1-\nu_j\right)\right]} \times (\varphi^*)^{\sigma_j - 1}$$

Inactive firm. Finally, a hibernating firm is insolvent if its productivity is lower to the threshold φ^{V3} (see Appendix A.A7):

$$\left(\varphi^{V3}\right)^{\sigma_j-1} = \frac{(\varphi^*)^{\sigma_j-1}}{1-r}$$

It is interesting to notice that the probability of insolvency of an inactive firm during the pandemic only depends on the interest rate, which can be interpreted as the waiting cost for the shareholder[8].

III. Taking the model to the data: a quantitative assessment using French data

Adding demand and supply shocks to the Melitz model let us to compute the fundamental thresholds that influence static decisions of firms $(\varphi^{*'}, \varphi^{*''}, \varphi^{**}, \varphi^{***})$

[8] Again, this result will be modified once fixed costs are not covered by *activité partielle*. Extension to come.

and their dynamic decision to remain in the market or to stop permanently its activity (φ^V,φ^{V2},φ^{V3}). All this thresholds are function of two structural parameters (σ_j,r). The empirical literature is followed to calibrate those parameters. However, this is not enough. All the former thresholds depend on a single pre-COVID variable: the initial profitability – and solvency- threshold (φ^*). Moreover, to quantify the share of incumbent firms concerned by the different thresholds it is needed to define the shape of the distribution of φ. As is common in the literature, a Pareto distribution is chosen.

A. A Pareto distribution for φ

Advantages of using a Pareto distribution. The distribution of productivity is posed to be Pareto. This is a usual assumption for calibrating this kind of model, as the Pareto distribution has several important features. First, this distribution allows to compute the closed-form solution for several variables of interest (as sales, the number of active firms,. . .). Chaney (2008) used this property of the distribution to test some important implications of the Melitz model using the aggregate quantitative data available. Second, the Pareto distribution is a very good representation of the heterogeneity among firms (in terms of sales, employment, productivity). Gabaix (2009) surveys the literature using this kind of distribution and exposes the theoretical models that explain how this kind of distribution can emerge. Lastly, another interesting feature of Pareto distributions particularly useful in our setting is that a left-truncated Pareto distribution remains Pareto.

We assume that the cumulative distribution function of $G(\varphi)$ is:

$$G(\varphi) = 1 - \varphi^{-\theta_j} \text{ with } \varphi \geq 1 \text{ and } \theta_j > \sigma_j - 1 > 0$$

Moreover, to compute the share of incumbent firms ($\varphi \geq \varphi^*$) concerned by the new thresholds ($\varphi^{*'}$,$\varphi^{*''}$,φ^{**},φ^{***},φ^V,φ^{V2},φ^{V3}) can be computed using the following formula:

$$\Pr\left(\varphi < \text{threshold } \mid \varphi > \varphi^*\right) = 1 - \left(\frac{\text{threshold}}{\varphi^*}\right)^{-\theta_j}$$

It is important to notice that the share of firms concerned by a threshold depends exclusively of the ratio between the threshold and φ^*, precisely the pre-COVID ratio that is no longer needed to identify. To compute the share of incumbent firms that face liquidity or insolvency risk it useless to compute the pre-COVID equilibrium and to identify φ^*. Thus, the ratio of interest depends exclusively on the calibrated structural parameters and the measure of demand and supply shocks.

Partial equilibrium with a Pareto distribution. The level of sales of active firms, not constrained by absenteeism, depends on the evolution of the industry-level price index ($\widehat{P_j}$). Moreover, to determine $\varphi^{*'}$ and $\varphi^{*''}$ it is also necessary to solve for the price-index. It is important to remember that even if we suppose that firms do not change their individual price, the industry-level price index is modified exclusively by self-selection of firms that hibernate during the pandemic. In that case:

$$P_j^{1-\sigma_j} = \left(\frac{\sigma_j}{\sigma_j - 1}\right)\left(\frac{w_j\theta_j}{\theta_j + 1 - \sigma_j}\right)(\varphi^*)^{\sigma_j-\theta_j-1}$$

Thus,

$$(P_j)^{1-\sigma_j} = \left[\frac{\max\left(\varphi^{*\prime}, \varphi^{*\prime\prime}, \varphi^{***}\right)}{\varphi^*}\right]^{\sigma_j-\theta_j-1}$$

The participation threshold of firms constrained by demand depends on the industry-price level, which depends itself on the participation threshold. In the case, where all firms are constrained by demand: :

$$\varphi^{*\prime} = \left(\frac{1}{\widehat{PC}\widehat{\phi}_j}\right)^{\frac{1}{\theta_j}}$$

If some firms are constrained by absenteeism:

$$\widehat{P_j} = \left(\frac{1}{\nu_j}\right)^{\frac{\sigma_j-\theta_j-1}{1-\sigma_j}}\left(\frac{\sigma_j - \nu_j}{\sigma_j - 1}\right)^{\frac{1-\sigma_j+\theta_j}{(1-\sigma_j)^2}}$$

which can be used to compute de shift in the demand of firms constrained by demand.

Finally, if φ^{***} is binding:

$$\widehat{P_j} = \left[\frac{1-\nu_j}{\nu_j} \times (\sigma_j - 1)\right]^{\frac{\sigma_j-\theta_j-1}{1-\sigma_j}}$$

B. *Calibration of σ_j and θ_j*

The central calibration for the elasticity of substitution (σ_j) and the shape parameter of the Pareto distribution of firm-level efficiency (θ_j) are taken form Cavallo, Feenstra, and Inklaar (2021). Following Simonovska and Waugh (2014), they preferred estimate for $\theta_j = 5.1$ across all sectors. For the elasticities of substitution, they considered the recent work of Redding and Weinstein (2020) , who estimate elasticities of substitution across bar code varieties using the Nielsen Homescan data. The median estimate for $\sigma_j = 6.5$. The problem with those separate estimates is that they do not fulfill the condition: $\theta_j > \sigma_j - 1$. In order to set this problem they use Eaton and Kortum (2002) using a very general theoretical framework where $\frac{\theta_j}{\sigma_j - 1} = 1.5$[9]. In this context, Cavallo, Feenstra, and Inklaar (2021) use two set of parameters in order to calibrate their model, inspired by Melitz (2003): $(\sigma_j, \theta_j) = (6.5, 8.25)$ and $(\sigma_j, \theta_j) = (4.4, 5.1)$. It important to notice that this central evaluation is used for services industries.

Other estimates are available in the literature. Ossa (2014) a mean value of 3.4 for σ_j. With a novel identification strategy, Fontagné, Martin, and Orefice (2018) suggest that the elasticity of substitution is higher and closer to 5.0, leading to a lower markup of 25%. Time-series data used by Ruhl (2008) suggest that the elasticity of substitution can be much lower (close to 2).

C. *COVID-19 shock*

The exogenous demand shock ($\widehat{PC}\widehat{\phi}_j$) can be recovered directly from observed data at the industry-level. The drop in aggregate sales by industry observed from national accounts is used to measure the demand shock. The supply shocks are calibrated following Dauvin et al. (2020) for the period going from April 2020 to June 2020 and following Dauvin and Sampognaro (2021) for November and December 2020 to take into account the effects of the second lockdown (Table 1).

Table 1—Demand and supply shocks

Industry	$\widehat{PC}\widehat{\phi}_j$	ν_j
AZ Agriculture, forestry and fishing	0.99	0.99
C1 Manufacture of food products, beverages and tobacco products	0.99	0.98
C3 Manufacture of electrical, computer and electronic equipment	0.91	0.99
C4 Manufacture of transport equipment	0.71	1.00
C5 Other manufacturing	0.90	0.99
FZ Construction	0.86	1.00
GZ Wholesale and retail trade ; repair of motor vehicles	0.96	0.97
HZ Transportation and storage	0.86	0.99
IZ Accommodation and food service activities	0.60	0.92
JZ Information and communication	1.03	1.00
KZ Financial and insurance activities	0.92	1.00
LZ Real estate activities	0.99	1.00
MN Business services	0.95	0.99
OQ Mainly non market services	1.01	0.95
RU Other services	0.92	0.94

Note: Source: National accounts, OFCE (2020) and Dauvin and Sampognaro (2021).

[9]Eaton, Kortum, and Kramarz (2011) using French micro-data find $\frac{\theta_j}{\sigma_j - 1} = 2.46$. This estimate implies a much higher value for $\theta_j = 5.8$. A higher shape parameter implies less heterogeneity of productivity among firms. The lower heterogeneity identified by the authors may arise from the fact that they use exclusively data from the manufacturing sector.

D. Results

On the basis of the preferred calibrated structural parameters and the COVID-19 shocks discussed in the previous sub-sections it is possible to assess the binding constraints identified by the model.

TABLE 2—BINDING CONSTRAINTS AND PARTICIPATION THRESHOLD

	$\sigma_j = 6.5$, θ_j=8.25		σ_j=4.4, θ_j=5.1	
Industry	Participation	Binding shocks	Participation	Binding shocks
AZ Agriculture, forestry and fishing	${\varphi^*}''$	ν_j	${\varphi^*}''$	ν_j
C1 Manufacture of food products, beverages and tobacco products	${\varphi^*}''$	ν_j	${\varphi^*}''$	ν_j
C3 Manufacture of electrical, computer and electronic equipment	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$
C4 Manufacture of transport equipment	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$
C5 Other manufacturing	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$
FZ Construction	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$
GZ Wholesale and retail trade ; repair of motor vehicles	${\varphi^*}''$	ν_j	${\varphi^*}''$	ν_j
HZ Transportation and storage	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$
IZ Accommodation and food service activities	${\varphi^*}''$	ν_j & $\widehat{PC}\widehat{\phi_j}$	${\varphi^*}'$	ν_j & $\widehat{PC}\widehat{\phi_j}$
JZ Information and communication	${\varphi^*}''$	ν_j	${\varphi^*}''$	ν_j
KZ Financial and insurance activities	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$
LZ Real estate activities	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$	${\varphi^*}'$	$\widehat{PC}\widehat{\phi_j}$
MN Business services	${\varphi^*}''$	ν_j & $\widehat{PC}\widehat{\phi_j}$	${\varphi^*}''$	ν_j & $\widehat{PC}\widehat{\phi_j}$
OQ Mainly non market services	${\varphi^*}''$	ν_j	${\varphi^*}''$	ν_j
RU Other services	${\varphi^*}''$	ν_j	${\varphi^*}''$	ν_j

All firms in the following industries are constrained by the absenteeism shock: *agriculture*; *manufacture of food products*; *wholesale and retail trade*; *information and communication*; *mainly non-market activities* and *other services*. On the contrary all the firms in *manufacture of equipment goods*; *manufacture of transport equipment*; *other manufacturing*; *construction*; *transportation and storage*; *financial activities*; *real estate activities* are constrained exclusively by demand factors. In all other industries co-exist firms constrained by supply and by demand (Table 2). The constraints identified by the model are robust to our two central calibrations: only the binding participation threshold is modified in the industry of *accommodation and food service activities*.

In our two central calibration, between 2.7 - 3.1% of firms face an insolvency risk and between 6.9% and 9.4% of firms hibernate during the pandemics (Table 3). This result is close to Guerini et al. (2020) and Bauer, Hadjibeyli, and Roulleau (2021) even if the methodologies used are different. There is low heterogeneity among industries for the insolvency risk. This reflects essentially the fact that the cost of hibernation is very low and that there is no heterogeneity of structural parameters in the current version of the article (σ_j, θ_j and r are identical among industries). On the contrary, there is massive heterogeneity in the hibernation decision, dependent on the heterogeneity of demand and supply shocks. Close of 30% of firms in *manufacture of transport equipment* face liquidity problems during the pandemic but 1% of firms in *information and communication*; *finan-*

cial activities; *retail state* activities or *professional and support services activities* follow this behavior.

Table 3—Share of firms concerned by the liquidity and insolvency risk

	$\sigma_j = 6.5$, θ_j=8.25		σ_j=4.4, θ_j=5.1	
Industry	Liquidity	Solvency	Liquidity	Solvency
AZ Agriculture, forestry and fishing	1.9	1.9	2.1	2.1
C1 Manufacture of food products, beverages and tobacco products	4.1	3.6	4.5	3.6
C3 Manufacture of electrical, computer and electronic equipment	9.0	3.6	9.0	3.6
C4 Manufacture of transport equipment	29.5	3.6	29.5	3.6
C5 Other manufacturing	9.7	3.6	9.7	3.6
FZ Construction	14.0	3.6	14.0	3.6
GZ Wholesale and retail trade ; repair of motor vehicles	5.9	3.6	6.4	.3
HZ Transportation and storage	14.5	3.6	14.5	3.6
IZ Accommodation and food service activities	13.4	3.6	40.5	3.6
JZ Information and communication	0.7	0.7	0.8	0.8
KZ Financial and insurance activities	8.0	3.6	8.0	3.6
LZ Real estate activities	0.4	0.4	0.5	0.5
MN Business services	1.4	1.4	4.7	3.6
OQ Mainly non market services	8.0	3.6	8.7	3.6
RU Other services	9.9	3.6	10.8	3.6
TOT Total Economy	6.9	2.7	9.4	3.1

Table 4 shows the sensitivity of the results to the chosen calibration. The results are particularly sensitive to the calibration of θ, that determines the heterogeneity of productivity within an industry. A higher θ, implies more homogeneity of productivity among competitors. When θ is higher, more firms are concentrated around the threshold of participation pre-COVID. Then, a higher θ (compatible with Eaton, Kortum, and Kramarz (2011)) implies more firms concerned by the risk of failure (4.4% in total) and most than 11% of firms *hibernate* during the pandemic. The elasticity of substitution also plays an important role. A higher σ means a lower markup, making the pre-COVID selection harsher. Incumbents are more productive and more resilient during the pandemic. Lastly, a change in the interest rate does not modify the hibernation behavior but modifies the cost of waiting the end of the pandemic. Lower interest rate leading to lower insolvency problems.

Table 4—Sensitivity analysis – Total Economy

Calibrated parameters	Liquidity	Solvency
$\boldsymbol{\sigma_j = 4.4 \; ; \; \theta_j = 5.1 \; ; \; r = 2.4\%}$	**9.4**	**3.1**
$\sigma_j = 8.8 \; ; \; \theta_j = 5.1 \; ; \; r = 2.4\%$	6.6	1.3
$\sigma_j = 4.4 \; ; \; \theta_j = 8.25 \; ; \; r = 2.4\%$	11.1	4.4
$\sigma_j = 4.4 \; ; \; \theta_j = 5.1 \; ; \; r = 1.2\%$	9.4	1.6

IV. Zombification after the pandemic

Forthcoming.

V. Conclusion

3% of firms are concerned by a failure risk while between 6.9 and 9.4% of firms *hibernate* during the pandemic according to the structural model developed. If this model lets to account for simultaneous demand and supply shocks when firms are heterogeneous, some extensions may be useful to make more realistic predictions. First, in the model, the *activité partielle* program compensates all the wage bill of unproductive employees and firms can finance their fixed costs through this instrument. Plane (2020) shows that significant fixed costs weighted on firm profitability in 2020. Second, if firms can modify their prices, some firms will be able to preserve their financial situation by increasing their prices. In particular, supply-constrained firms that are unable to satisfy all their potential demand. Lastly, the inclusion of fixed costs paid in terms of goods or services will lead to the inclusion of firms with negative cash flows and suffering losses during the pandemic and leading to higher leverage after the pandemic. If the post-COVID equilibrium is the same than the pre-COVID, profitable firms near the threshold may be unable to reimburse those debts, generating a fringe of *zombie* firms. This kind of effect will be incorporated to the model, as it will become a central question in 2021.

Even if this is not the main topic of the paper, it is hard not to discuss about the long-term effect of the COVID-19 crisis on the distribution of productivity of firms. The Melitz model predicts that firms with lower productivity will be the one concerned by failures. It is tempting to say that this cleansing effect will contribute to rise the mean productivity. However, this is incomplete. In a Melitz framework, if the pandemic does not modify the ratio of fixed costs of production (f_j) relative to the cost of creating a new project (f_E)[10], the productivity distribution will be the same at an unmodified equilibrium. It is important to notice that this remains true even if the crisis has a permanent impact on the level and the composition of demand. Only changes in the structure of fixed costs can have this kind of permanent effect on the productivity distribution. Assuming that fixed costs (f_j and f_E) remain unchanged in the post-COVID world means that there is no efficiency gain to expect from the failures that can arise in the forthcoming years. However, does this mean that all failures should be avoided? It is not clear and this paper cannot answer this question. If dM_j firms exit the market, $\frac{dM_j}{1-G(\varphi^*)}$ projects will be engaged at the equilibrium. Some will fail. This implies to mobilize $\frac{dM_j}{1-G(\varphi^*)} \times f_E$ specific factors, which constitutes a loss for the

[10] This parameter of the Melitz model was not presented as in this article, the behavior of entrants was neglected. f_E represents the number of specific-factor units needed to create a new firm. See Melitz and Redding (2014) for more details.

society linked with an excess turnover of firms. So, to answer the question using a Melitz model it is needed to compare the loss of value of incumbent firms that may exit with the social cost of launching new projects. Obviously, this raises several questions of how to identify the firms that face solvency problems exclusively linked with the COVID-19 and how to identify the cost of launching new projects.

VI. References

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Mathematical Appendix

A1. Computation of φ^{**}

We compute the threshold φ^{**} separating firms that are unable to reach their optimal labor demand during the COVID-19 outbreak:

$$\nu_j l_j(\varphi^{**}) = l_j(\varphi^{**})'$$

$$\nu_j f_j + \nu_j PC\phi_j P_j^{\sigma_j - 1}\left(\frac{\sigma_j}{\sigma_j - 1}\right)^{-\sigma_j} w_j^{-\sigma_j}(\varphi^{**})^{\sigma_j - 1} = f_j + \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j - 1}\phi_j PCP_j^{\sigma_j - 1}\left(\frac{\sigma_j}{\sigma_j - 1}\right)^{-\sigma_j} w_j^{-\sigma_j}(\varphi^{**})^{\sigma_j - 1}$$

$$f_j\left(1 - \nu_j\right) = \left[\nu_j - \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j - 1}\right] PC\phi_j P_j^{\sigma_j - 1}\left(\frac{\sigma_j}{\sigma_j - 1}\right)^{-\sigma_j} w_j^{-\sigma_j}(\varphi^{**})^{\sigma_j - 1}$$

$$(\varphi^{**})^{\sigma_j - 1} = \frac{(1 - \nu_j)}{\left[\nu_j - \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j - 1}\right]}\frac{f_j}{PC\phi_j P_j^{\sigma_j - 1}\left(\frac{\sigma_j}{\sigma_j - 1}\right)^{-\sigma_j} w_j^{-\sigma_j}}$$

$$(\varphi^{**})^{\sigma_j - 1} = \frac{(1 - \nu_j)}{\left[\nu_j - \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j - 1}\right]}\frac{f_j}{PC\phi_j P_j^{\sigma_j - 1}\left(\frac{\sigma_j}{\sigma_j - 1}\right)^{-\sigma_j} w_j^{-\sigma_j}}\frac{\sigma_j - 1}{\sigma_j - 1}$$

$$(\varphi^{**})^{\sigma_j - 1} = \frac{(1 - \nu_j)}{\left[\nu_j - \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j - 1}\right]}\frac{f_j}{PC\phi_j P_j^{\sigma_j - 1}\frac{\sigma_j^{-\sigma_j}}{(\sigma_j - 1)^{1 - \sigma_j}} w_j^{-\sigma_j}}(\sigma_j - 1)$$

Remember that in the classical Melitz (2003) model:

$$(\varphi^{*})^{\sigma_j - 1} = \frac{f_j}{PC\phi_j P_j^{\sigma_j - 1}\frac{\sigma_j^{-\sigma_j}}{(\sigma_j - 1)^{1 - \sigma_j}} w_j^{-\sigma_j}}$$

Hence:

$$(\varphi^{**})^{\sigma_j - 1} = \frac{(1 - \nu_j)}{\left[\nu_j - \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j - 1}\right]}(\sigma_j - 1)(\varphi^{*})^{\sigma_j - 1}$$

When $\nu_j - \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j - 1} \leq 0$, all firms are binded by the supply shock. Then, $\varphi^{**} \to \inf$:

$$\varphi^{**} = \begin{cases} \frac{1-\nu_j}{\nu_j - \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1}} \times (\sigma_j - 1) \times (\varphi^*)^{\sigma_j-1} & \text{if } \nu_j - \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1} \geq 0 \\ +\infty & \text{if } \nu_j - \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1} < 0 \end{cases}$$

A2. *Computation of φ^{***}*

A firm of productivity $\varphi_j \leq \varphi^{***}$ is unable to produce for technological reasons if $\nu_j l_j \varphi^{***} = f_j$. This is the case when:

$$\nu_j f_j + \nu_j PC\phi_j P_j^{\sigma_j-1} \left(\frac{\sigma_j}{\sigma_j - 1}\right)^{-\sigma_j} w_j^{-\sigma_j} (\varphi^{***})^{\sigma_j-1} = f_j$$

$$f_j (1-\nu_j) = \nu_j PC\phi_j P_j^{\sigma_j-1} \left(\frac{\sigma_j}{\sigma_j - 1}\right)^{-\sigma_j} w_j^{-\sigma_j} (\varphi^{***})^{\sigma_j-1}$$

$$(\varphi^{***})^{\sigma_j-1} = \frac{1-\nu_j}{\nu_j} \frac{f_j}{PC\phi_j P_j^{\sigma_j-1} \left(\frac{\sigma_j}{\sigma_j-1}\right)^{-\sigma_j} w_j^{-\sigma_j}}$$

$$(\varphi^{***})^{\sigma_j-1} = \frac{1-\nu_j}{\nu_j} \frac{f_j}{PC\phi_j P_j^{\sigma_j-1} \left(\frac{\sigma_j}{\sigma_j-1}\right)^{-\sigma_j} w_j^{-\sigma_j}} \frac{\sigma_j - 1}{\sigma_j - 1}$$

And using the formula of φ^* exposed in the Appendix A.A1, we get the final result:

$$(\varphi^{***})^{\sigma_j-1} = \frac{1-\nu_j}{\nu_j} \times (\sigma_j - 1) \times (\varphi^*)^{\sigma_j-1}$$

A3. *Computation of $\varphi^{*'}$*

If a firm can mobilize its optimal labor demand during the COVID-19 outbreak, then its sales and profits are exclusively determined by the demand shocks. Those firms behave as in the canonical Melitz model:

$$\frac{r(\varphi^{*'})}{\sigma_j} = f_j w_j$$

$$\frac{r(\varphi^{*'})}{\sigma_j} = \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1} \phi_j PC P_j^{\sigma_j-1} \left(\frac{\sigma_j^{-\sigma_j}}{(\sigma_j - 1)^{\sigma_j-1}}\right) w_j^{1-\sigma_j} (\varphi^{*'})^{\sigma_j-1} = w_j f_j$$

$$\widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1} \phi_j PC P_j^{\sigma_j-1} \left(\frac{\sigma_j^{-\sigma_j}}{(\sigma_j - 1)^{\sigma_j-1}}\right) w_j^{1-\sigma_j} (\varphi^{*'})^{\sigma_j-1} = w_j f_j$$

$$(\varphi^{*'})^{\sigma_j-1} = \frac{w_j f_j}{\widehat{PC}\widehat{\phi_j}\widehat{P_j}^{\sigma_j-1}\phi_j PCP_j^{\sigma_j-1}\left(\frac{\sigma_j^{-\sigma_j}}{(\sigma_j-1)^{\sigma_j-1}}\right)w_j^{1-\sigma_j}}$$

$$(\varphi^{*'})^{\sigma_j-1} = \frac{f_j}{\widehat{PC}\widehat{\phi_j}\widehat{P_j}^{\sigma_j-1}\phi_j PCP_j^{\sigma_j-1}\left(\frac{\sigma_j^{-\sigma_j}}{(\sigma_j-1)^{\sigma_j-1}}\right)w_j^{-\sigma_j}}$$

$$(\varphi^{*'})^{\sigma_j-1} = \frac{(\varphi^{*})^{\sigma_j-1}}{\widehat{PC}\widehat{\phi_j}\widehat{P_j}^{\sigma_j-1}}$$

A4. Computation of $\varphi^{*''}$

A firm constrained by the absenteeism shock can employ $\nu_j l_j(\varphi)$ employees. With this level of employment, the new output of a supply-constrained firm of productivity φ ($q_j'(\varphi)$) is given by:

$$\nu_j l_j(\varphi) = f_j + \frac{q_j'(\varphi)}{\varphi}$$

$$\nu_j f_j + \nu_j PC\phi_j P_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{-\sigma_j} w_j^{-\sigma_j}\varphi^{\sigma_j-1} = f_j + \frac{q_j'(\varphi)}{\varphi}$$

$$\varphi\left(\nu_j-1\right)f_j + \nu_j PC\phi_j P_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{-\sigma_j} w_j^{-\sigma_j}\varphi^{\sigma_j} = q_j'(\varphi)$$

Multiplying the output of the firm by the optimal price ($p_j(\varphi) = \frac{\sigma_j}{\sigma_j-1}\frac{w_j}{\varphi}$:

$$\frac{\sigma_j}{\sigma_j-1}\left(\nu_j-1\right)f_j w_j + \nu_j PC\phi_j P_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{1-\sigma_j} w_j^{1-\sigma_j}\varphi^{\sigma_j-1} = r_j'(\varphi)$$

Supply-constrained firms will be profitable if $\frac{r_j'(\varphi)}{\sigma_j} \geq w_j f_j$. The threshold $\varphi^{*''}$ is given for a firm with sales equal to $\sigma_j w_j f_j$:

$$\frac{\sigma_j}{\sigma_j-1}\left(\nu_j-1\right)f_j w_j + \nu_j PC\phi_j P_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{1-\sigma_j} w_j^{1-\sigma_j}(\varphi^{*''})^{\sigma_j-1} = w_j f_j \sigma_j$$

$$\frac{1}{\sigma_j-1}\left(\nu_j-1\right)f_j w_j + \frac{1}{\sigma_j}\nu_j PC\phi_j P_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{1-\sigma_j} w_j^{1-\sigma_j}(\varphi^{*''})^{\sigma_j-1} - w_j f_j = 0$$

$$\left(\frac{\nu_j-1}{\sigma_j-1}-1\right)f_j w_j + \frac{1}{\sigma_j}\nu_j PC\phi_j P_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{1-\sigma_j} w_j^{1-\sigma_j}(\varphi^{*''})^{\sigma_j-1} = 0$$

$$\left(\frac{\nu_j - \sigma_j}{\sigma_j - 1}\right) f_j w_j + \nu_j PC\phi_j P_j^{\sigma_j - 1} \frac{\sigma_j^{-\sigma_j}}{(\sigma_j - 1)^{\sigma_j - 1}} w_j^{1-\sigma_j} (\varphi^{*''})^{\sigma_j - 1} = 0$$

$$\nu_j PC\phi_j P_j^{\sigma_j - 1} \frac{\sigma_j^{-\sigma_j}}{(\sigma_j - 1)^{\sigma_j - 1}} w_j^{1-\sigma_j} (\varphi^{*''})^{\sigma_j - 1} = \left(\frac{\sigma_j - \nu_j}{\sigma_j - 1}\right) f_j w_j$$

$$\nu_j (\varphi^{*''})^{\sigma_j - 1} = \left(\frac{\sigma_j - \nu_j}{\sigma_j - 1}\right) \frac{f_j}{PC\phi_j P_j^{\sigma_j - 1} \frac{\sigma_j^{-\sigma_j}}{(\sigma_j - 1)^{\sigma_j - 1}} w_j^{-\sigma_j}}$$

Using the formula of φ^* (see Appendix A.A1), then we have:

$$\nu_j (\varphi^{*''})^{\sigma_j - 1} = \left(\frac{\sigma_j - \nu_j}{\sigma_j - 1}\right) (\varphi^*)^{\sigma_j - 1}$$

And finally:

$$\left(\varphi^{*''}\right)^{\sigma_j - 1} = \left[\frac{1}{\nu_j} \times \frac{\sigma_j - \nu_j}{\sigma_j - 1}\right] \times (\varphi^*)^{\sigma_j - 1}$$

A5. Computation of φ^V

It is straightforward to show that if all the fixed costs are eligible to *activité partielle*, then the evolution of the value of a firm is dependent on the drop in sales:

$$V_j'(\varphi) = \left[\frac{r_j'(\varphi) - r_j(\varphi)}{\sigma_j}\right] + V_j(\varphi)$$

Firms constrained by demand behave exactly like in the canonical Melitz (2003):

$$r_j(\varphi) = \phi_j PC P_j^{\sigma_j - 1} \left(\frac{\sigma_j}{\sigma_j - 1}\right)^{1-\sigma_j} w_j^{1-\sigma_j} \varphi^{\sigma_j - 1}$$

and,

$$r_j(\varphi)' = \widehat{PC}\widehat{\phi_j}\widehat{P_j}^{\sigma_j - 1} \phi_j PC P_j^{\sigma_j - 1} \left(\frac{\sigma_j}{\sigma_j - 1}\right)^{1-\sigma_j} w_j^{1-\sigma_j} \varphi^{\sigma_j - 1}$$

To simplify notations let's call $B_j = \phi_j PC P_j^{\sigma_j - 1} \left(\frac{\sigma_j}{\sigma_j - 1}\right)^{1-\sigma_j} w_j^{1-\sigma_j}$ and $B_j' = \widehat{PC}\widehat{\phi_j}\widehat{P_j}^{\sigma_j - 1} \phi_j PC P_j^{\sigma_j - 1} \left(\frac{\sigma_j}{\sigma_j - 1}\right)^{1-\sigma_j} w_j^{1-\sigma_j}$.

Before the COVID-19 outbreak, the value of a firm of productivity φ active in industry j is:

$$V_j(\varphi) = \frac{B_j \varphi^{\sigma_j - 1}}{r\sigma_j} - \frac{w_j f_j}{r}$$

After the COVID-19 outbreak, the new solvability threshold for demand-constrained firms (φ^V) is defined by:

$$V_j'(\varphi^V) = \left[\frac{(B_j' - B_j)(\varphi^V)^{\sigma_j-1}}{\sigma_j}\right] + \frac{B_j(\varphi^V)^{\sigma_j-1}}{r\sigma_j} - \frac{w_j f_j}{r} = 0$$

$$\left[\frac{r(B_j' - B_j)(\varphi^V)^{\sigma_j-1} + B_j(\varphi^V)^{\sigma_j-1}}{r\sigma_j}\right] = \frac{w_j f_j}{r}$$

$$r(B_j' - B_j)(\varphi^V)^{\sigma_j-1} + B_j(\varphi^V)^{\sigma_j-1} = \sigma_j w_j f_j$$

$$(\varphi^V)^{\sigma_j-1}\left[r(B_j' - B_j) + B_j\right] = \sigma_j w_j f_j$$

With $B_j' = \widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1}B_j$:

$$(\varphi^V)^{\sigma_j-1}\left[r(\widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1}B_j - B_j) + B_j\right] = \sigma_j w_j f_j$$

$$(\varphi^V)^{\sigma_j-1}\left[B_j\left(r\left(\widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1} - 1\right) + 1\right)\right] = \sigma_j w_j f_j$$

$$(\varphi^V)^{\sigma_j-1} = \frac{\sigma_j w_j f_j}{B_j\left(r\left(\widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1} - 1\right) + 1\right)}$$

And with $\frac{\sigma_j w_j f_j}{B_j} = \varphi^*$:

$$(\varphi^V)^{\sigma_j-1} = \frac{(\varphi^*)^{\sigma_j-1}}{\left[1 - r\left(\widehat{PC}\widehat{\phi}_j\widehat{P_j}^{\sigma_j-1}\right)\right]}$$

A6. Computation of φ^{V2}

First, compute the level of sales of a firm of productivity φ binded by the absenteeism shock:

$$\nu_j l_j(\varphi) = \nu_j f_j + \nu_j PCP_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{-\sigma_j} w_j^{-\sigma_j}\varphi^{\sigma_j-1}$$

$$c_j(\varphi)' = \varphi\left(\nu_j - 1\right) f_j + \nu_j PCP_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{-\sigma_j} w_j^{-\sigma_j}\varphi^{\sigma_j}$$

$$r_j(\varphi)' = \frac{\sigma_j}{\sigma_j-1}\left(\nu_j - 1\right) w_j f_j + \nu_j PCP_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{1-\sigma_j} w_j^{1-\sigma_j}\varphi^{\sigma_j-1}$$

with $B_j = PCP_j^{\sigma_j-1}\left(\frac{\sigma_j}{\sigma_j-1}\right)^{1-\sigma_j} w_j^{1-\sigma_j}$:

$$r_j(\varphi)' - r_j(\varphi) = \frac{\sigma_j}{\sigma_j-1}\left(\nu_j - 1\right) w_j f_j + \left(\nu_j - 1\right) B_j\varphi^{\sigma_j-1}$$

Then, the new value of the firm is:

$$V_j(\varphi)' = \left[\frac{\sigma_j}{\sigma_j - 1}(\nu_j - 1) w_j f_j + (\nu_j - 1) B_j \varphi^{\sigma_j - 1}\right] \times \frac{1}{\sigma_j} + B_j \frac{\varphi^{\sigma_j - 1}}{r\sigma_j} - \frac{w_j f_j}{r}$$

$$V_j(\varphi)' = \frac{\nu_j - 1}{\sigma_j - 1} w_j f_j + \frac{(\nu_j - 1) B_j \varphi^{\sigma_j - 1}}{\sigma_j} + \frac{B_j \varphi^{\sigma_j - 1}}{r\sigma_j} - \frac{w_j f_j}{r}$$

$$V_j(\varphi)' = \frac{B_j \varphi^{\sigma_j - 1}}{\sigma_j}\left[(\nu_j - 1) + \frac{1}{r}\right] + \frac{\nu_j - 1}{\sigma_j - 1} w_j f_j - \frac{w_j f_j}{r}$$

$$V_j(\varphi)' = \frac{B_j \varphi^{\sigma_j - 1}}{\sigma_j}\left[\frac{1 - r(1 - \nu_j)}{r}\right] - \frac{\sigma_j - 1 + r(1 - \nu_j)}{(\sigma_j - 1)r} w_j f_j$$

And the insolvency threshold φ^{V2} is computed by:

$$V_j(\varphi^{V2})' = \frac{B_j (\varphi^{V2})^{\sigma_j - 1}}{\sigma_j}\left[\frac{1 - r(1 - \nu_j)}{r}\right] - \frac{\sigma_j - 1 + r(1 - \nu_j)}{(\sigma_j - 1)r} w_j f_j = 0$$

$$\frac{B_j (\varphi^{V2})^{\sigma_j - 1}}{\sigma_j}\left[\frac{1 - r(1 - \nu_j)}{r}\right] = \frac{\sigma_j - 1 + r(1 - \nu_j)}{(\sigma_j - 1)r} w_j f_j$$

$$(\varphi^{V2})^{\sigma_j - 1}\left[\frac{1 - r(1 - \nu_j)}{r}\right] = \frac{\sigma_j - 1 + r(1 - \nu_j)}{(\sigma_j - 1)r} \frac{\sigma_j w_j f_j}{B_j}$$

$$(\varphi^{V2})^{\sigma_j - 1}\left[1 - r(1 - \nu_j)\right] = \frac{\sigma_j - 1 + r(1 - \nu_j)}{(\sigma_j - 1)r} (\varphi^*)^{\sigma_j - 1}$$

And we get:

$$\left(\varphi^{V2}\right)^{\sigma_j - 1} = \frac{\sigma_j - 1 + r(1 - \nu_j)}{(\sigma_j - 1) \times \left[1 - r\left(1 - \nu_j\right)\right]} \times (\varphi^*)^{\sigma_j - 1}$$

A7. Computation of φ^{V3}

Sales of an hibernating firm are equal to 0. Then the value of an hibernating firm of productivity φ is:

$$V_j(\varphi)' = \left[0 - \frac{B_j \varphi^{\sigma_j - 1}}{\sigma_j}\right] + \frac{B_j \varphi^{\sigma_j - 1}}{r\sigma_j} - \frac{w_j f_j}{r}$$

$$V_j(\varphi^{V3})' = \frac{B_j (\varphi^{V3})^{\sigma_j - 1}}{r\sigma_j} - \frac{B_j (\varphi^{V3})^{\sigma_j - 1}}{\sigma_j} - \frac{w_j f_j}{r} = 0$$

$$\frac{B_j (\varphi^{V3})^{\sigma_j - 1}}{r\sigma_j} - \frac{B_j (\varphi^{V3})^{\sigma_j - 1}}{\sigma_j} = \frac{w_j f_j}{r}$$

$$\frac{B_j(\varphi^{V3})^{\sigma_j-1} - rB_j(\varphi^{V3})^{\sigma_j-1}}{r\sigma_j} = \frac{w_j f_j}{r}$$

$$\frac{B_j(\varphi^{V3})^{\sigma_j-1}(1-r)}{\sigma_j} = w_j f_j$$

And using the formula of φ^*:

$$\left(\varphi^{V3}\right)^{\sigma_j-1} = \frac{(\varphi^*)^{\sigma_j-1}}{1-r}$$