EE 102: Signal Processing and Linear Systems

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Homework #4: Convolutions

Name: _____   **Submission Date:** _____

Problem 1 A system responds to an impulse input $\delta(t)$ in an exponentially decaying manner. So, the impulse response of the system is given by:

$$h(t) = e^{-2t}u(t)$$

where $u(t)$ is the unit step function.

(a) [5 points] What is the output $y(t)$ of the system when the input is $x(t) = k\delta(t)$, where k is a constant?

(b) [5 points] Prove that the system is linear and time-invariant.

Hint: For an impulse input $\delta(t)$, the output is $h(t)$. What is the output when the input is $\delta(t - t_0)$? Use this to prove time-invariance. For linearity, use the properties of the delta function.

(c) [10 points] What is the output $y(t)$ of the system when the input is $x(t) = u(t)$? You must find this by starting from the relationship between the step signal and the impulse signal (from HW 2):

$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$

(d) [20 points] Find the output $y(t)$ of the system when the input is a pulse signal of amplitude A and duration τ:

$$x(t) = A[u(t) - u(t - \tau)]$$