EE 102: Signal Processing and Linear Systems

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Homework #2: Properties of Signals

Name: ______________ **Submission Date:** ______________

Problem 1 [Adapted from Lathi 1.1-11] Consider a signal that is a sum of complex exponentials given by

$$x(t) = \sum_{k=m}^{n} D_k \, e^{j\omega_k t}$$

where $D_k \in \mathbb{C}$ and $\{\omega_k\}$ are pairwise distinct, that is, $\omega_i \neq \omega_j$ for all $i \neq j$.

(a) [10 points] Show that the time-averaged power of $x(t)$ is

$$P_\infty(x) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \; = \; \sum_{k=m}^{n} |D_k|^2.$$

Hint: expand $|x(t)|^2$ and use that $\frac{1}{2T}\int_{-T}^{T} e^{j(\omega_k - \omega_\ell)t} \, dt \to 0$ when $\omega_k \neq \omega_\ell$.

(b) [5 points] Determine $E_\infty(x) = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$. State clearly whether it is finite or infinite for this $x(t)$ and justify.

(c) [5 points] Let

$$\tilde{x}(t) \triangleq \overline{x(t)} = \sum_{k=m}^{n} \overline{D_k} \, e^{-j\omega_k t}.$$

What is the power for this conjugate signal?

(d) [5 points] Is $x(t)$ even, odd, or neither? If it is not necessarily even/odd, state the sufficient conditions on $\{D_k, \omega_k\}$ under which $x(t)$ becomes even or odd. For a challenge, you can attempt to show both necessary and sufficient conditions.

[use more pages if needed]

Problem 2 Use the unit step $u(t)$ and unit impulse $\delta(t)$ function definitions discussed in class to answer the following questions.

(a) [5 points] Show that

$$\int_{-\infty}^{t} u(\tau)\, d\tau = t\, u(t) \quad \text{(the ramp } r(t)\text{).}$$

(b) [5 points] Show that

$$\int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t).$$

(c) [10 points] Prove in the sense of distributions that

$$\frac{d}{dt}\, u(t) = \delta(t), \qquad \frac{d}{dt}\, [t\, u(t)] = u(t) + t\, \delta(t).$$

(d) [10 points] For the finite pulse

$$p(t) = A\, \mathrm{rect}\left(\frac{t - t_0}{T}\right),$$

compute $E_\infty(p)$ and $P_\infty(p)$ and state when each is finite/nonzero.

[use more pages if needed]

Problem 3 For each signal below, state whether it is periodic. If periodic, find the fundamental period (T_0 for continuous time, N_0 for discrete time). If it is not periodic, justify your answer.

(a) [5 points] $x(t) = 5\sin(10t - 0.5) + \cos(5t)$

(b) [5 points] $x(t) = j\,e^{j10t}$

(c) [5 points] $x(t) = e^{\,(-0.5+j)\,(t+0.5)}$

(d) [5 points] $x[n] = e^{j\,12\pi n}$

(e) [5 points] $x[n] = 1 + e^{j\,\frac{4\pi}{7}n} - e^{j\,\frac{2\pi}{3}n}$

[use more pages if needed]

Problem 4 [Adapted from Vierinen Ch.5 P7] Practice with real audio signals. The file `amplifier.ipynb` (on GitHub) implements the linear amplifier system. The amplified signal is

$$y(t) = \alpha\, x(t).$$

The code reads `guitar_clean.wav` (file on GitHub), plots original vs. amplified, normalizes the output to 0.9 peak, and writes `guitar_amp.wav`.

(a) [2.5 points] Run the script, and produce a figure showing the original $x(t)$ and $\alpha x(t)$ on the same axes. Experiment with different amplification factors α (e.g., $1, 2, 5, 10$) and plot each amplified signal on a separate graph.

(b) [10 points] Using the discrete-time samples $x[n]$ from the WAV file, estimate $P_\infty(x)$ and $P_\infty(\alpha x)$ via

$$\widehat{P} = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2.$$

Verify the relationship between the powers of the original and the amplified signal.

(c) [5 points] Modify the script to *hard-clip* the amplified signal to $[-1, 1]$ before saving (no renormalization). Experiment with at least three different values of β and present your results by plotting the waveforms (slice your arrays so that the signal clipping is clearly visible). Discuss how clipping affects the spectrum qualitatively.

(d) [2.5 points] Is $y(t) = \alpha x(t)$ linear? time-invariant? Is the *clipper* system that we discussed during lecture linear? time-invariant? Briefly justify each of the four properties of the systems.

[use more pages if needed]