EE102 Week 1, Lecture 1 (Fall 2025)

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1 Goals

- Logistics, grading, extensions, expectations
- Motivation to study signal processing

2 Pre-requisite #1: Vectors

When studying problems with many entities/observations, we structure our variables into vectors.

An n-dimensional vector $\mathbf{x}$ can be written as

$$\mathbf{x} = [\, x_1,\ x_2,\ \ldots,\ x_n \,], \qquad \mathbf{x} \in \mathbb{R}^n.$$

Matrices are transformations

If you transform a vector $\mathbf{x}$ to a new vector $\mathbf{y}$ such that all elements in $\mathbf{y}$ are linear combinations of elements in $\mathbf{x}$, then the transformation is called a matrix.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \longmapsto \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

such that

$$y_1 = \sum_{i=1}^{n} \alpha_{1i} x_i, \qquad y_2 = \sum_{i=1}^{n} \alpha_{2i} x_i, \quad \ldots, \quad y_m = \sum_{i=1}^{n} \alpha_{mi} x_i.$$

Then $A\mathbf{x} = \mathbf{y}$, where

$$A \in \mathbb{R}^{m\times n}, \qquad A = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{bmatrix}.$$

We write

$$A: \; X \to Y,$$

where X is the vector space in $\mathbb{R}^n$ where $\mathbf{x}$ lies and Y is the vector space in $\mathbb{R}^m$ where $\mathbf{y}$ lies.

Recall

- Diagonal matrix
- Identity matrix
- Symmetric matrix
- Zero matrix
- Matrix transpose
- Matrix algebra ($+$, $-$, $\times$, inverse)

3 Pre-requisite #2: Complex numbers

Although the usual way we learn about the complex unit "j" is as a convenient notation for a solution of

$$x^2 + 1 = 0 \quad \Rightarrow \quad x = \sqrt{-1} := \mathrm{j},$$

it is useful to recognize other places where this convenience is beneficial. In signal processing we are often looking for easy ways to analyze physical signals, not only to solve algebraic equations.

From vectors to a complex scalar. Given a vector

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

define the (complex) scalar

$$z_{\mathbf{x}} = x_1 + \mathrm{j}x_2.$$

The entries x_1 and x_2 are not "added" in $\mathbb{R}$; they are bound only because they are components of the same vector. Writing $z_{\mathbf{x}}$ lets us treat the vector like a single scalar living in $\mathbb{C}$.

Inner product via complex numbers. For $\mathbf{x} = [x_1, x_2]^\top$ and $\mathbf{y} = [y_1, y_2]^\top$,

$$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^\top \mathbf{y} = x_1 y_1 + x_2 y_2.$$

With the complex representations

$$z_{\mathbf{x}} = x_1 + \mathrm{j}x_2, \qquad z_{\mathbf{y}} = y_1 + \mathrm{j}y_2,$$

their product with conjugation is

$$\begin{aligned} z_{\mathbf{x}}\,\overline{z_{\mathbf{y}}} &= (x_1 + \mathrm{j}x_2)(y_1 - \mathrm{j}y_2) \\ &= (x_1 y_1 + x_2 y_2) \;+\; \mathrm{j}\,(-x_1 y_2 + x_2 y_1). \end{aligned}$$

Taking the real part gives the vector inner product:

$$\Re(z_{\mathbf{x}}\,\overline{z_{\mathbf{y}}}) = x_1 y_1 + x_2 y_2 = \langle \mathbf{x}, \mathbf{y} \rangle.$$

Polar form viewpoint. Write $\mathbf{x}$ in polar coordinates with $r_x = \|\mathbf{x}\|$ and angle θ_x:

$$x_1 = r_x \cos\theta_x, \qquad x_2 = r_x \sin\theta_x,$$

so

$$z_{\mathbf{x}} = r_x(\cos\theta_x + \mathrm{j}\sin\theta_x) = r_x e^{\mathrm{j}\theta_x}.$$

Similarly $z_{\mathbf{y}} = r_y e^{\mathrm{j}\theta_y}$. Then

$$\begin{aligned} z_{\mathbf{x}}\,\overline{z_{\mathbf{y}}} &= r_x e^{\mathrm{j}\theta_x}\; r_y e^{-\mathrm{j}\theta_y} = r_x r_y e^{\mathrm{j}(\theta_x - \theta_y)} \\ &= r_x r_y \Big[\cos(\theta_x - \theta_y) + \mathrm{j}\sin(\theta_x - \theta_y)\Big], \end{aligned}$$

hence

$$\Re(z_{\mathbf{x}}\,\overline{z_{\mathbf{y}}}) = r_x r_y \cos(\theta_x - \theta_y) = \langle \mathbf{x}, \mathbf{y} \rangle.$$

You can make intuitive sense of the inner product in polar form by understanding that the angle between the two vectors is $\theta_x - \theta_y$ (plot the two vectors in a 2D plane). Therefore, the scalar projection (see notes below) of $\mathbf{x}$ onto $\mathbf{y}$ is $r_x \cos(\theta_x - \theta_y)$, which when multiplied by the absolute value of y gives the inner product (that is, $r_x r_y \cos(\theta_x - \theta_y)$ in complex polar form).

Scalar projection. Let $\hat{\mathbf{y}} = \mathbf{y}/\|\mathbf{y}\|$ and let ϕ be the angle between $\mathbf{x}$ and $\mathbf{y}$. Then

$$\mathrm{proj}_{\mathbf{y}}(\mathbf{x}) = (\mathbf{x} \cdot \hat{\mathbf{y}})\, \hat{\mathbf{y}}, \qquad \|\mathrm{proj}_{\mathbf{y}}(\mathbf{x})\| = \mathbf{x} \cdot \hat{\mathbf{y}} = \|\mathbf{x}\| \cos\phi,$$

and

$$\mathbf{x} \cdot \mathbf{y} = \mathbf{x} \cdot \big(\|\mathbf{y}\|\hat{\mathbf{y}}\big) = \|\mathbf{y}\|\, (\mathbf{x} \cdot \hat{\mathbf{y}}).$$