EE 102 Week 5, Lecture 2 (Fall 2025)

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1 Goals

Apply convolution to an image processing example. Visualize convolution using graphs. Motivate the frequency domain using the general complex exponential signal.

2 Example: A discrete-time echo system

An audio receiver system produces an echo. When excited by a unit impulse, it responds with an echo of magnitude 1 at $n = 0$ that decays exponentially as α^n for $\alpha \in (0, 1)$ until $n = 5$ (that is, for six seconds in total). You may assume that $\alpha = \frac{1}{2}$ for numerical parts. Answer the following:

(A) Sketch the impulse response $h[n]$ and label $h[0], h[1], \ldots, h[5]$.

(B) We want to understand the kind of echo that will be produced when the audio receiver system is excited by a pulse input of unit amplitude lasting three seconds, starting at $n = 0$ and staying at unit amplitude until $n = 3$. Find $y[n]$ for this input using convolution and show your steps.

The impulse response of the system is

$$h[n] = \begin{cases} \alpha^n, & 0 \leq n \leq 5, \\ 0, & \text{otherwise,} \end{cases}$$

The input is a unit amplitude tone that starts at $n = 0$ and lasts three seconds. So, we can write the pulse signal for the input $x[n]$ as

$$x[n] = u[n] - u[n-3] = \begin{cases} 1, & n = 0, 1, 2, \\ 0, & \text{otherwise.} \end{cases}$$

Now, we can compute the output $y[n]$ using convolution:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k],$$

and give $y[n]$ explicitly for all n where it is nonzero. It is important that we are careful about all values of n for which $y[n]$ is nonzero. Echos can last longer than the original sound!

We will solve this by using illustrations of convolution. For each index $n = 0, 1, \ldots,$ draw three plots in a row for each n:

$$x[k], \qquad h[n-k] \text{ (as a function of } k\text{)}, \qquad \text{and the resulting single sample } y[n],$$

so that the overlap of $x[k]$ and $h[n-k]$ and the accumulation giving $y[n]$ are visually clear.

Let's start by drawing $h[n]$ for $\alpha = \frac{1}{2}$:

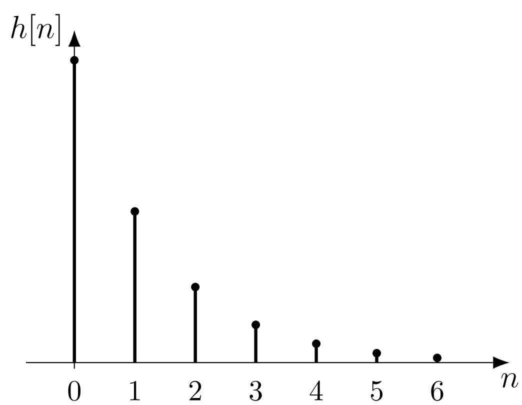
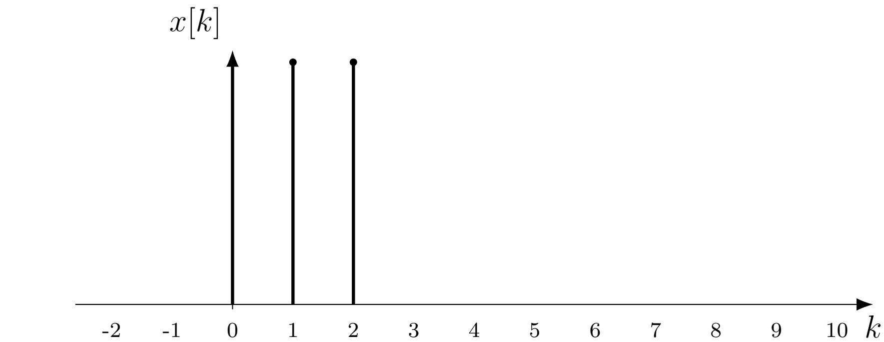
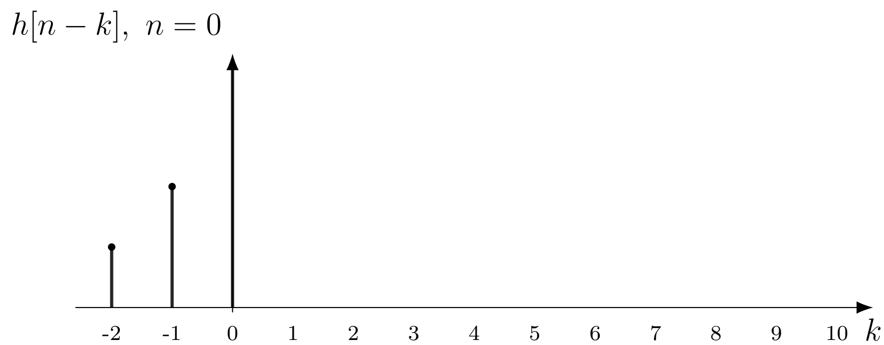
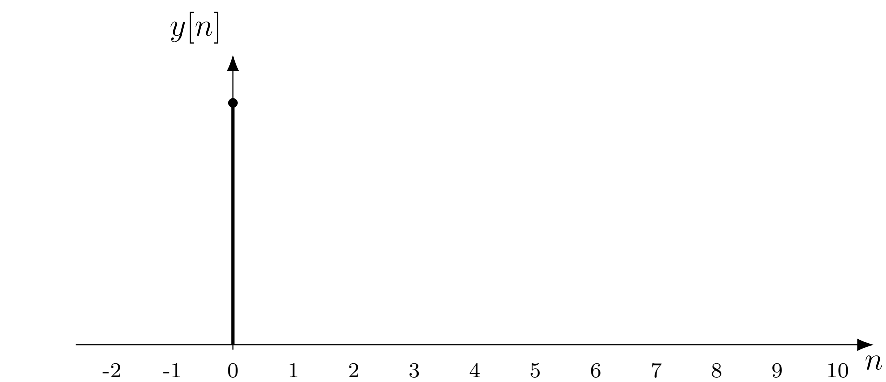


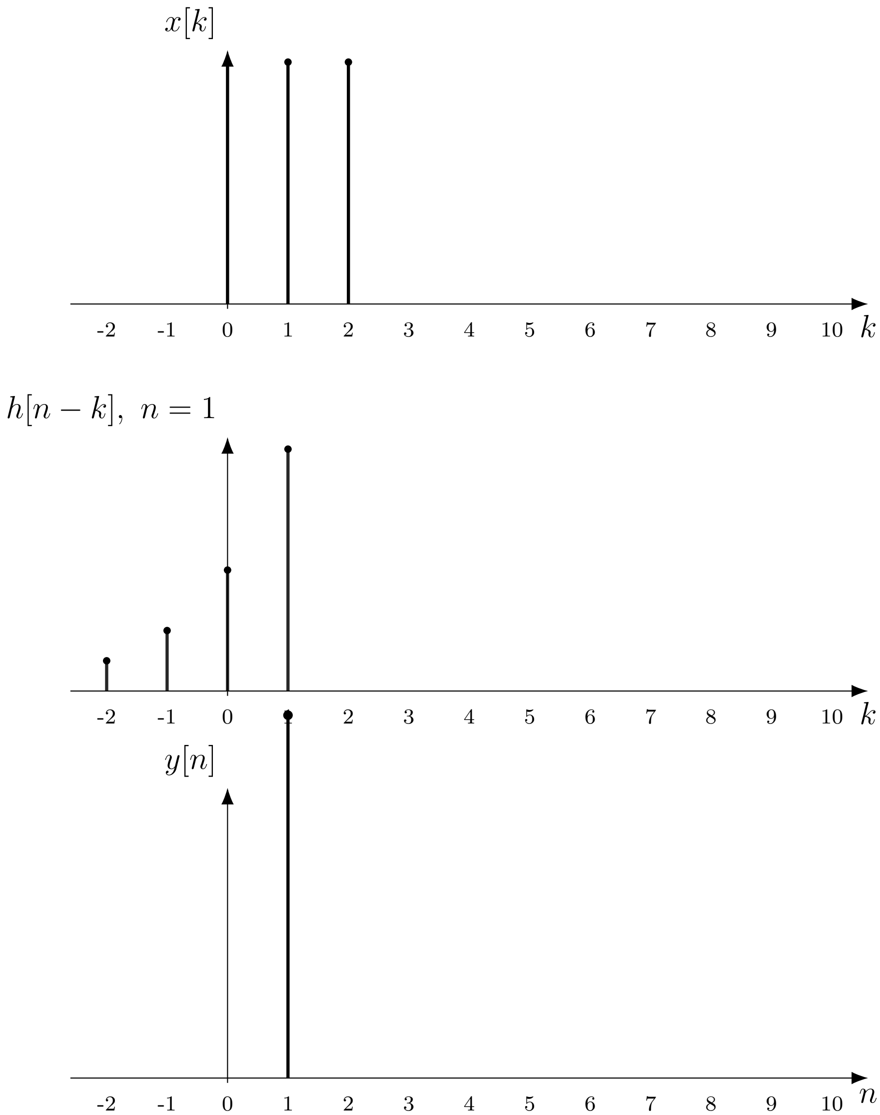
Figure 1: Impulse response $h[n]$ for $\alpha = \frac{1}{2}$.

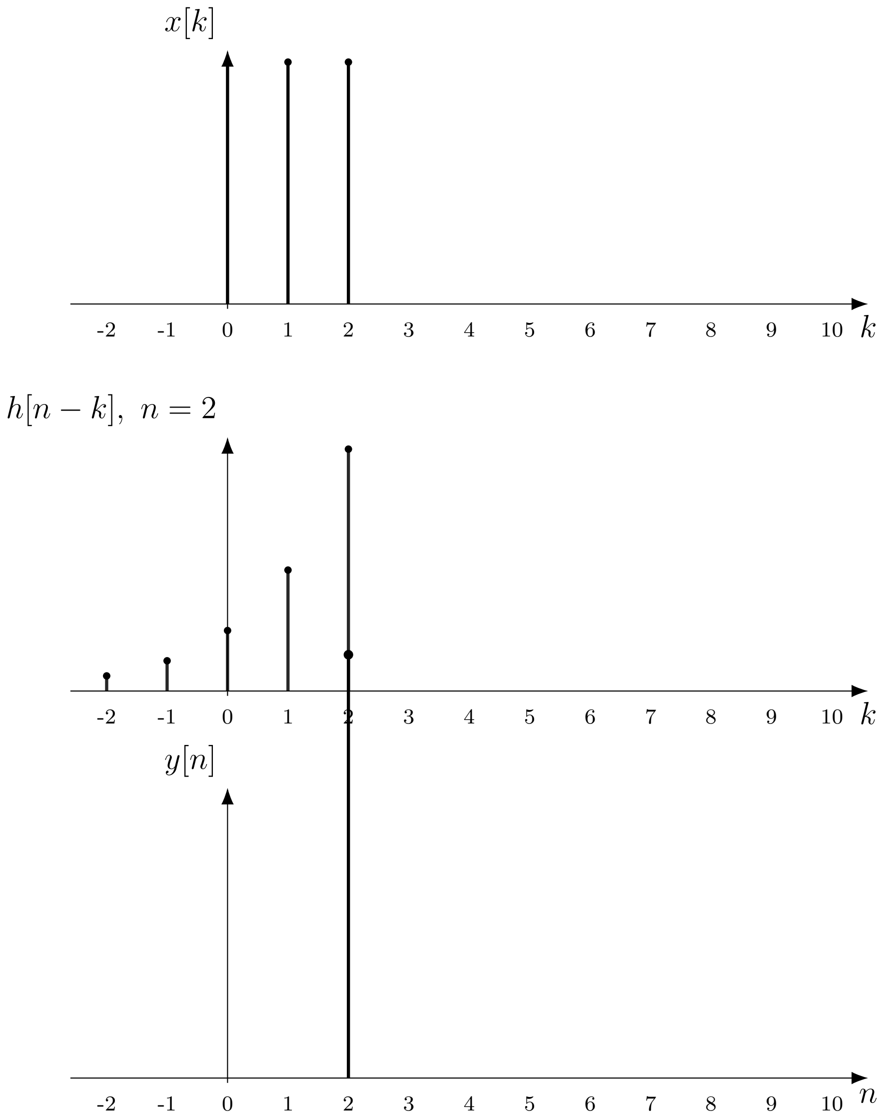
Sliding convolution panels


$x[k]$
-2
-1
0
1
2
3
4
5
6
7
8
9
10
k


$h[n-k],\ n=0$
-2
-1
0
1
2
3
4
5
6
7
8
9
10
k


$y[n]$
-2
-1
0
1
2
3
4
5
6
7
8
9
10
n


$x[k]$
-2 -1 0 1 2 3 4 5 6 7 8 9 10 k
$h[n-k],\ n=1$
-2 -1 0 1 2 3 4 5 6 7 8 9 10 k
$y[n]$
-2 -1 0 1 2 3 4 5 6 7 8 9 10 n


$x[k]$
-2 -1 0 1 2 3 4 5 6 7 8 9 10 k
$h[n-k],\ n=2$
-2 -1 0 1 2 3 4 5 6 7 8 9 10 k
$y[n]$
-2 -1 0 1 2 3 4 5 6 7 8 9 10 n

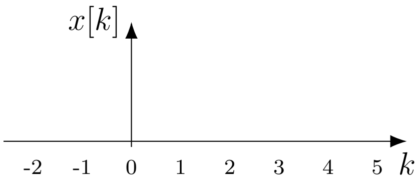
2.1 Convolution computation

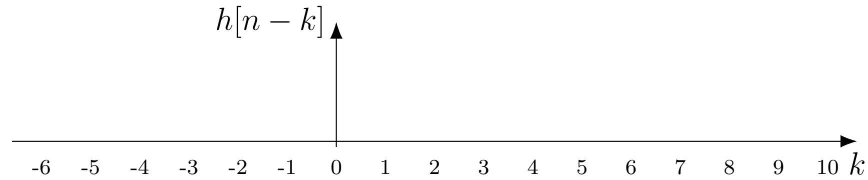
Let us compute the output for various values of n:

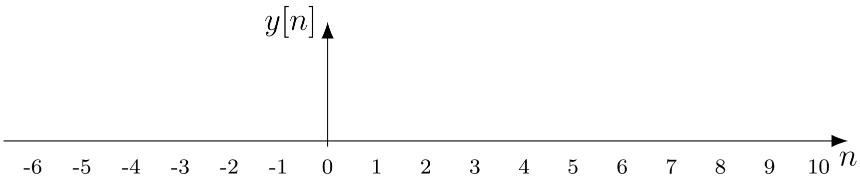
$$\begin{array}{r|l} n & y[n] \\ \hline -2: & 0 \\ -1: & 0 \\ 0: & 1 \\ 1: & 1+\alpha \\ 2: & 1+\alpha+\alpha^2 \\ 3: & \alpha+\alpha^2+\alpha^3 \\ 4: & \alpha^2+\alpha^3+\alpha^4 \\ 5: & \alpha^3+\alpha^4+\alpha^5 \\ 6: & \alpha^4+\alpha^5+\alpha^6 \\ 7: & \alpha^5+\alpha^6 \\ 8: & \alpha^6 \\ 9: & 0 \\ 10: & 0 \end{array} \quad \Rightarrow \quad \text{with } \alpha = \tfrac{1}{2}: \; y[0..8] = \left[1, \tfrac{3}{2}, \tfrac{7}{4}, \tfrac{7}{8}, \tfrac{7}{16}, \tfrac{7}{32}, \tfrac{7}{64}, \tfrac{3}{64}, \tfrac{1}{64}\right].$$

Visualize convolution

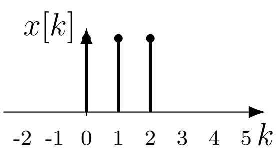
Graphically solve for $n = -1$

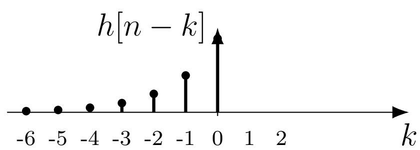




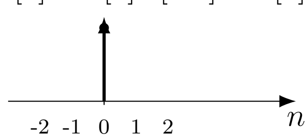


(Solved) Graphically show for $n = 0$

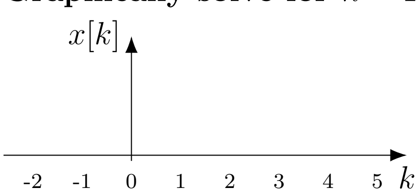


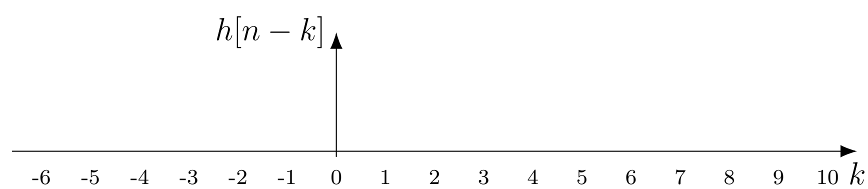


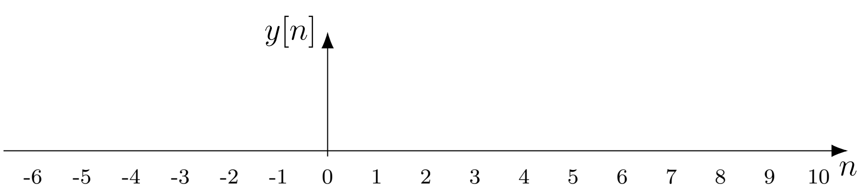
$$y[0] = \sum_k x[k]\,h[0-k] = x[0]\,h[0] + x[1]\,h[-1] + x[2]\,h[-2] = 1 + 0 + 0 = 1.$$



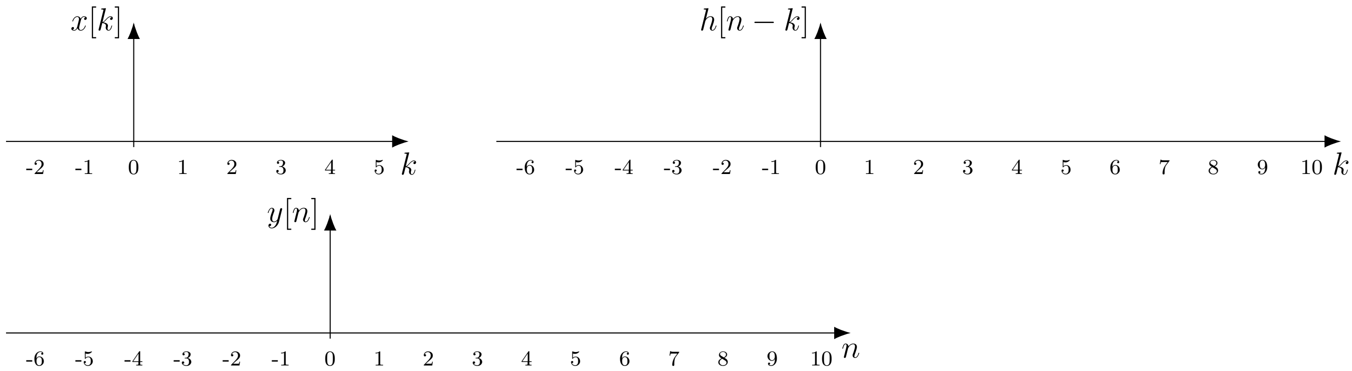
Graphically solve for $n = 1$



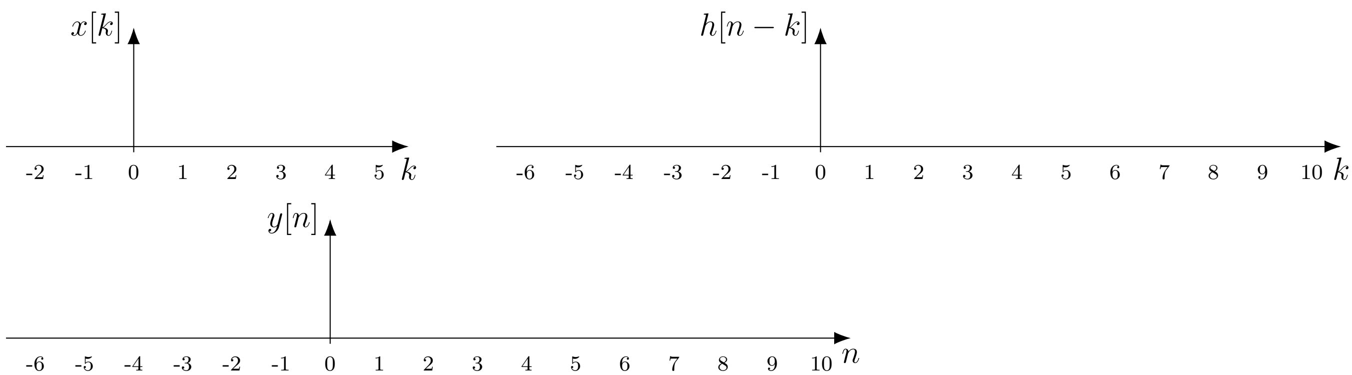




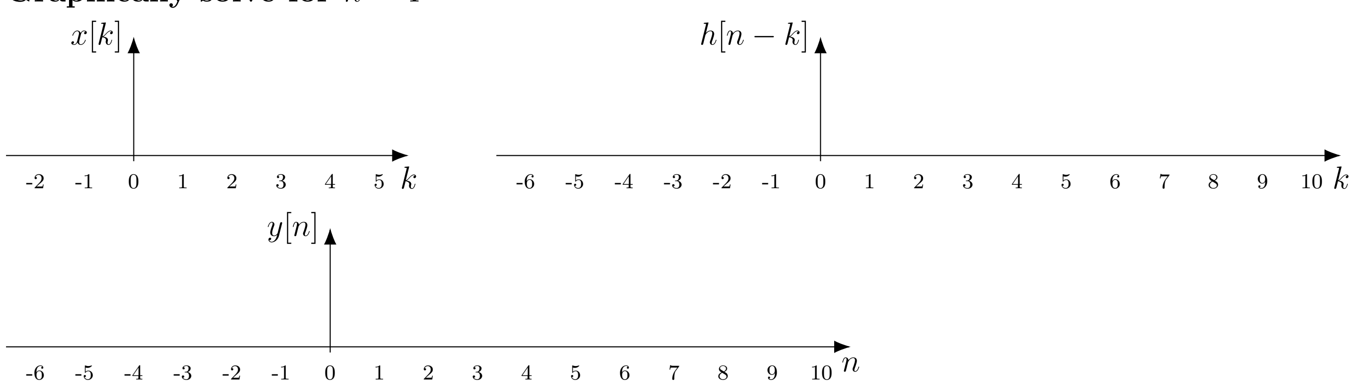
Graphically solve for $n = 2$



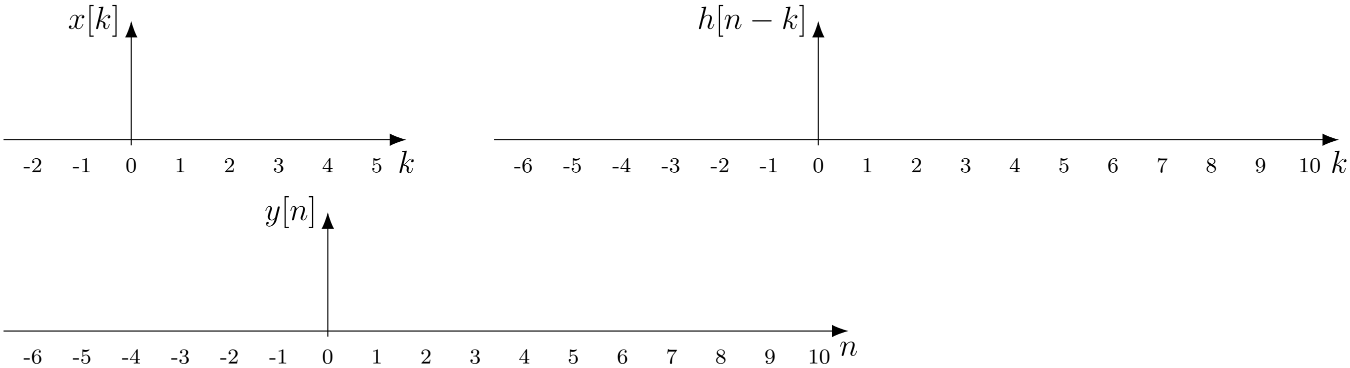
Graphically solve for $n = 3$



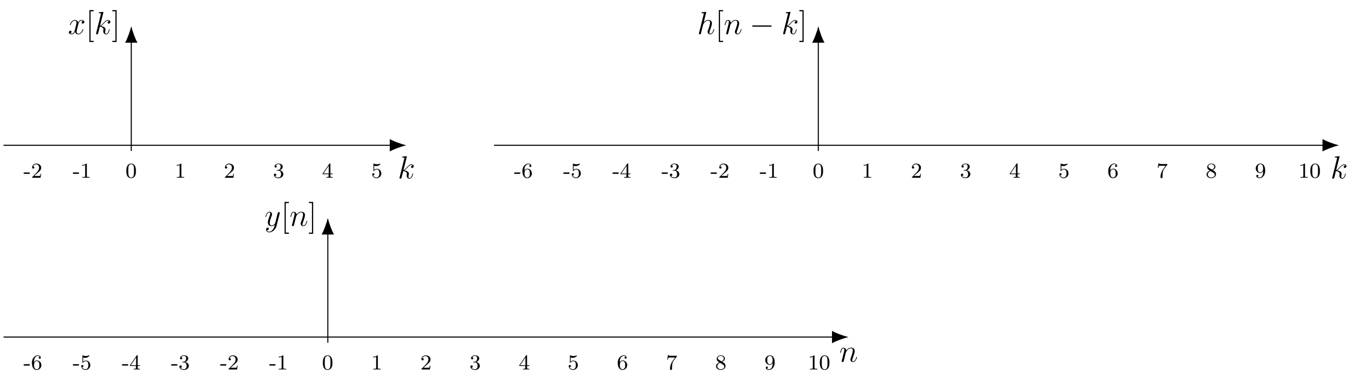
Graphically solve for $n = 4$



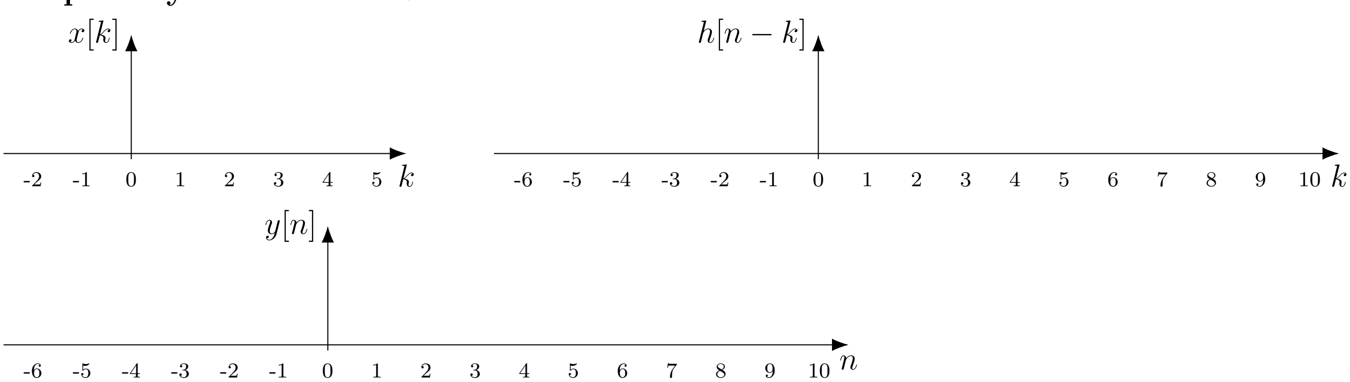
Graphically solve for $n = 5$



Graphically solve for $n = 6$



Graphically solve for $n = 7$



3 Convolution Example: Image Processing