EE 102: Signal Processing and Linear Systems

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Homework #4: Convolutions

Name: _____ **Submission Date:** _____

Problem 1 A system responds to an impulse input $\delta(t)$ in an exponentially decaying manner. So, the impulse response of the system is given by:

$$h(t) = e^{-2t}u(t)$$

where $u(t)$ is the unit step function.

(a) [10 points] What is the output $y(t)$ of the system when the input is $x(t) = k\delta(t)$, where k is a constant?

(b) [10 points] Prove that the system is linear and time-invariant.

Hint: For an impulse input $\delta(t)$, the output is $h(t)$. What is the output when the input is $\delta(t - t_0)$? Use this to prove time-invariance. For linearity, use the properties of the delta function.

(c) [15 points] What is the output $y(t)$ of the system when the input is $x(t) = u(t)$? You must find this by starting from the relationship between the step signal and the impulse signal (from HW 2):

$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$

Then, use linearity and the impulse response $h(t)$ to find the output $y(t)$.

(d) [15 points] Find the output $y(t)$ of the system when the input is a pulse signal of amplitude A and duration τ, $\quad x(t) = A[u(t) - u(t - \tau)]$.

(e) [10 points each] Find the output of the system to the following general input signals

1. $x(t) = e^{-3t}u(t)$
2. $x(t) = \cos(2t)u(t)$
3. $x(t) = \sin(4t)u(t)$
4. $x(t) = tu(t)$

Problem 2

Reflect on your understanding of the following:

(a) [4 points] What is the purpose of convolutions in signal processing?

(b) [4 points] Give an example of a system from one of your other classes (you must mention the name and number of the class) that can be modeled as an LTI system. Write about what would be the impulse response of that system and where you might use convolutions to analyze that system.

(c) [2 points] How long did this assignment take you to complete (this does not include the time spent in lectures or in labs, but it does include the time spent programming).