EE 102: Signal Processing and Linear Systems

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Homework #1: Introduction to Signals

Name: ______________ **Submission Date:** ______________

Problem 1

[Adapted from Problem 3 in Vierinen and Jensen] A time scaling system adjusts the scaling of the independent variable: $y(t) = x(\alpha t)$, when $x(t)$ is the signal fed into the system and $y(t)$ is the output. Answer the following questions for this system:

1. [**5 points**] Draw a block diagram for the system. Label the input, output, and the system.
2. [**5 points**] What is the effect on the signal when $0 < \alpha < 1$. What about $\alpha > 1$?
3. [**5 points**] Prove that the system is linear.
4. [**5 points**] Give an example of a real-world application where studying this system can be useful. Then, for this example, propose a nonlinear modification to the system, which captures a real situation.

[use more pages if needed]

Problem 2 Consider the signal $x(t) = a^{-tu(t)}$, where $u(t)$ is the unit step function.

1. [**5 points**] Sketch $x(t)$ for time $-2 < t < 2$ for $a > 0$. You are not allowed to use computer programs to do this.
2. [**5 points**] Sketch $y(t)$ for time $-2 < t < 2$ for $a > 0$, where $y(t) = 2x(5 - 0.5t)$
3. [**5 points**] Find out whether the signal $y(t)$ is time-invariant.
4. [**5 points**] Find out whether the signal $y(t)$ converges to 0 as $t \to \infty$ for $a > 0$.
5. [**5 points**] Find the value of a such that $y(1) = 0.1$.
6. [**5 points**] Think of a signal that you can relate $x(t)$ with. Analyze the properties of $x(t)$ to find out a real-world signal that shares similar characteristics.

[use more pages if needed]

Problem 3 For each of the signals below, you have four tasks:

1. **[4 points, per signal]** Sketch the signal (clearly label the amplitude and axes)
2. **[8 points, per signal]** Compute E_∞ and P_∞ using the definitions provided in the lecture notes.
3. **[3 points, per signal]** Using Python (or MATLAB), plot the signal over an appropriate interval and confirm your findings.
4. **[5 points total]** Briefly discuss a relevant example where the properties of the signal can be important.

You must do the three parts above for each of the three signals below

(a) $x_1[n] = \left(\frac{1}{3}\right)^n u[n]$

(b) $x_2(t) = \mathrm{e}^{\mathrm{j}\,(3t+\pi/7)}$

(c) $x_3[n] = \mathrm{e}^{\mathrm{j}\,(\frac{\pi}{3}n+\pi/10)}$

Hint: To sketch a signal, compute its values for multiple values of time to understand the pattern. Remember that $u(n)$ is the continuous-time unit step function and $u[n]$ is the discrete-time unit step function. For signals in complex polar form, you can sketch the real part and the imaginary part on separate axes.