EE 102 Week 2, Lecture 1 (Fall 2025)

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1 Goals

- Review: time reversal and combined operations
- Review: energy and power — metrics to quantify signals
- Properties of signals: even, odd, and periodic.
- The fundamental period of a signal
- Complex exponential signals
- Next class: The unit impulse and step functions

2 Review: signal operations

For a signal $x(t)$, common time operations include:

1. Reversal: $x(-t)$
2. Compression: $x(2t)$
3. Expansion: $x\left(\frac{t}{2}\right)$
4. Delay: $x(t-6)$
5. Advance: $x(t+6)$

How to sketch: keep the vertical axis unchanged; apply horizontal changes only. For $x(at)$, compress if $|a| > 1$ and expand if $0 < |a| < 1$; for $x(t \pm T)$, shift right by T for $x(t-T)$ and left by T for $x(t+T)$; for $x(-t)$, reflect across the vertical axis.

3 Example #1: a clipping amplifier

A simple hard-clipping (overdrive) model:

$$y_d(t) = \begin{cases} -\beta, & \alpha\, x(t) < -\beta, \\ \alpha\, x(t), & |\alpha\, x(t)| \le \beta, \\ \beta, & \alpha\, x(t) > \beta, \end{cases} \qquad \alpha > 0,\ \beta > 0.$$

This is a *memoryless* nonlinearity: at each t, $y_d(t)$ depends only on $x(t)$. It amplifies small inputs by α and saturates at $\pm\beta$ for large inputs.

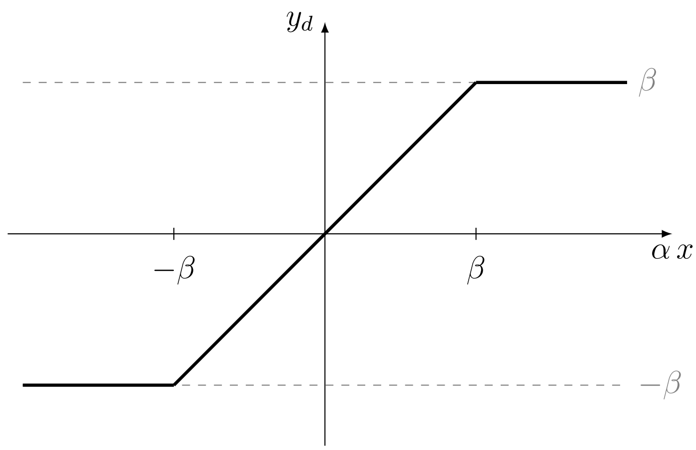

Figure 1: Hard-clipping nonlinearity: linear region $|\alpha x| \le \beta$, saturation outside.

Task: Draw all time operations discussed above for $y_d(t)$.

3.1 Optional: Audio tone signal example and time operations

In the supplementary notes, you will find a Python notebook that creates a guitar-like audio tone. You can use computer programming to compute various time-transformed versions $x(-t)$, $x(2t)$, $x(t/2)$, and $x(t+6)$. The transformed signals are shown in Figures 2 and 3.

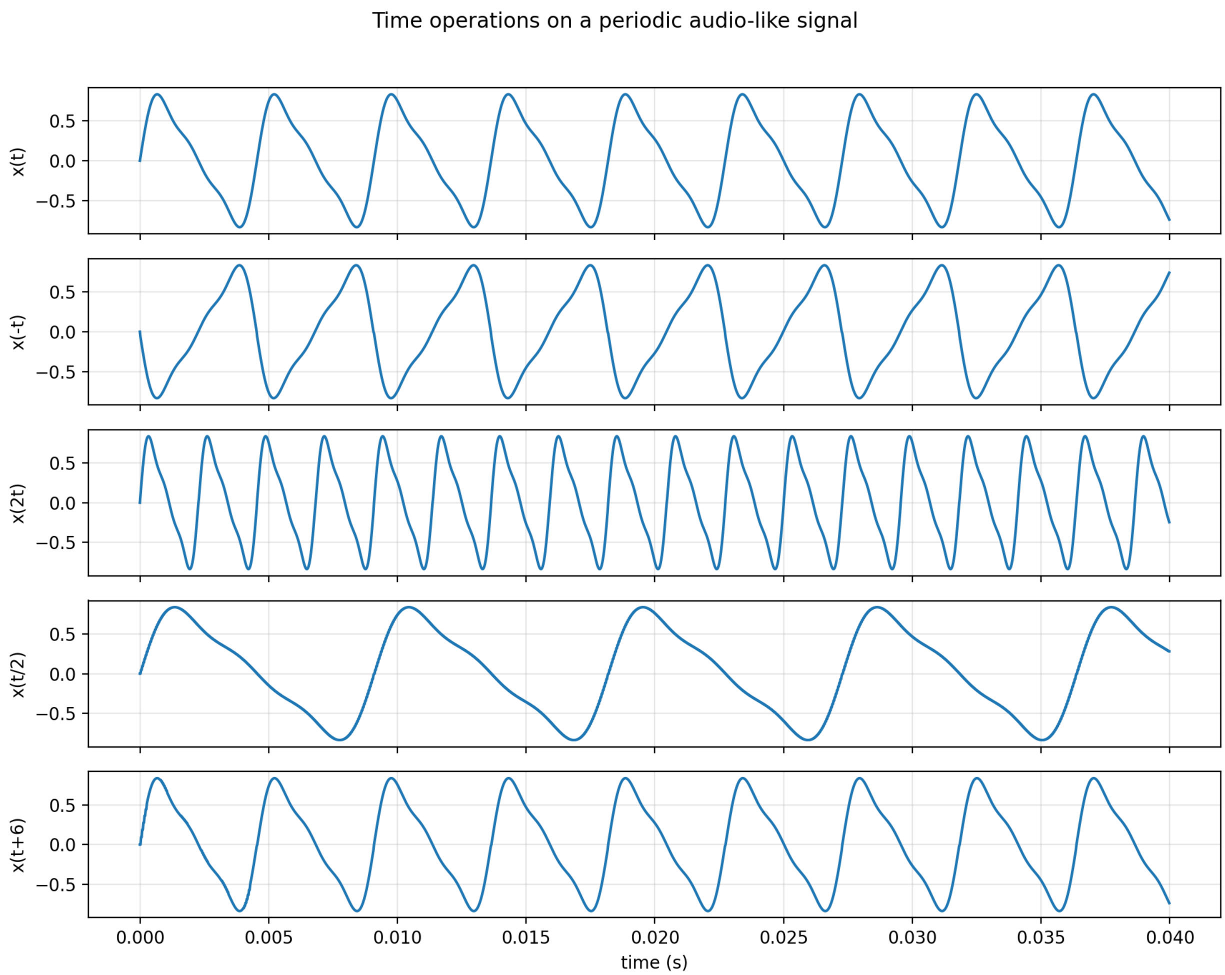

Figure 2: Time operations on a short audio-like signal $x(t)$.

4 Periodic signals and fundamental period

A signal $x(t)$ is periodic if $\exists\, T_0 > 0$ such that $x(t+T_0) = x(t)$ for all t. The smallest such T_0 is the *fundamental period*. For periodic x,

$$P_\infty(x) = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} |x(t)|^2 dt \quad \text{(independent of } t_0\text{)}, \qquad E_\infty(x) = \infty \text{ unless } x \equiv 0.$$

A useful trick: When a signal is defined on a finite interval (e.g., a single cycle), it is often useful to *periodically extend* it by repeating that interval end-to-end. This makes the time-average power well defined and makes symmetries/harmonics easier to see. Memoryless nonlinearities (like clipping) do not change T_0 but *do* add harmonics (distortion) while preserving periodicity.

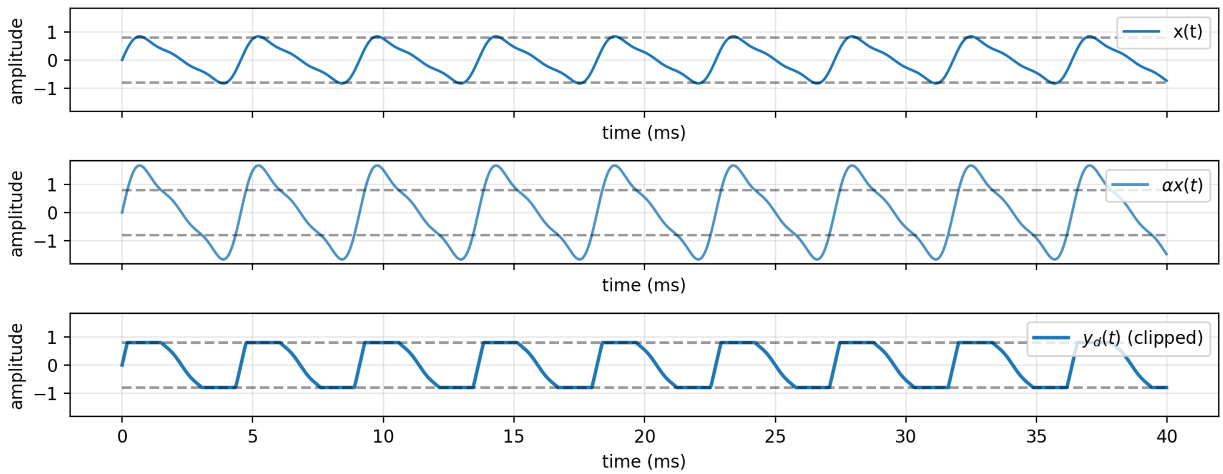

Figure 3: Clipping demo: input $x(t)$, scaled $\alpha x(t)$, and clipped output $y_d(t)$.

Energy and power for a periodic input

If $x(t)$ is periodic with fundamental period T_0, then $y_d(t)$ is also periodic with the *same* T_0 (memoryless mapping preserves period). Hence

$$E_\infty(y_d) = \int_{-\infty}^{\infty} |y_d(t)|^2 \, dt = \infty, \qquad P_\infty(y_d) = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} |y_d(t)|^2 \, dt \text{ (finite).}$$

For a finite-duration input, E_∞ is finite and $P_\infty = 0$ (time average over an unbounded window goes to zero).

5 Even and odd signals

A signal $x(t)$ can be decomposed uniquely as

$$x(t) = x_e(t) + x_o(t), \qquad x_e(t) = \frac{x(t) + x(-t)}{2}, \quad x_o(t) = \frac{x(t) - x(-t)}{2}.$$

Properties. x_e is even ($x_e(-t) = x_e(t)$), x_o is odd ($x_o(-t) = -x_o(t)$), and $x_e \perp x_o$ in the energy inner product.
Clipping intuition. The hard–clipper $f(u) = \text{clip}(u, -\beta, \beta)$ is an *odd* nonlinearity ($f(-u) = -f(u)$). Thus, if x is even, y_d is generally neither even nor odd; if x is odd, then y_d remains odd.

6 Complex exponential signals

Continuous time

$$x(t) = A\,e^{j(\omega_0 t+\phi)} = A\cos(\omega_0 t+\phi) + j\,A\sin(\omega_0 t+\phi).$$

Real and imaginary parts are orthogonal sinusoids. Fundamental period $T_0 = \frac{2\pi}{\omega_0}$.

Discrete time

$$x[n] = A\,e^{j(\Omega_0 n+\phi)}.$$

This is periodic iff $\dfrac{\Omega_0}{2\pi} = \dfrac{M}{N}$ with integers M, N coprime. Then the fundamental period is $N_0 = N$. Otherwise, it is *aperiodic* on $\mathbb{Z}$.

Geometric (phasor) picture

The complex exponential traces a circle of radius A in the complex plane at angular speed ω_0 (continuous) or advances by a fixed angle Ω_0 per sample (discrete). The real part is the projection on the horizontal axis; the imaginary part is the vertical projection.

Next class

The unit impulse $\delta(t)$ and step $u(t)$; convolution preview.