1. Charge and current

Two possible types of charge: positive and negative

Opposites attract, likes repel

Definition: current (*I*) is moving charge (vector)

Units: current: Ampere [A], charge: Coulomb [C]

$$I = \frac{\partial Q}{\partial t} \rightarrow Q = \int_t I(t)dt$$

Point charge and charge densities:

Point charge: mathematic concept to simplify problems

Charge Q located on a point without spatial extension

Line charge density λ: used when only one coordinate is important (wires etc)

Surface charge density σ: used when two coordinates are important (plane, surface of sphere, cylinder etc)

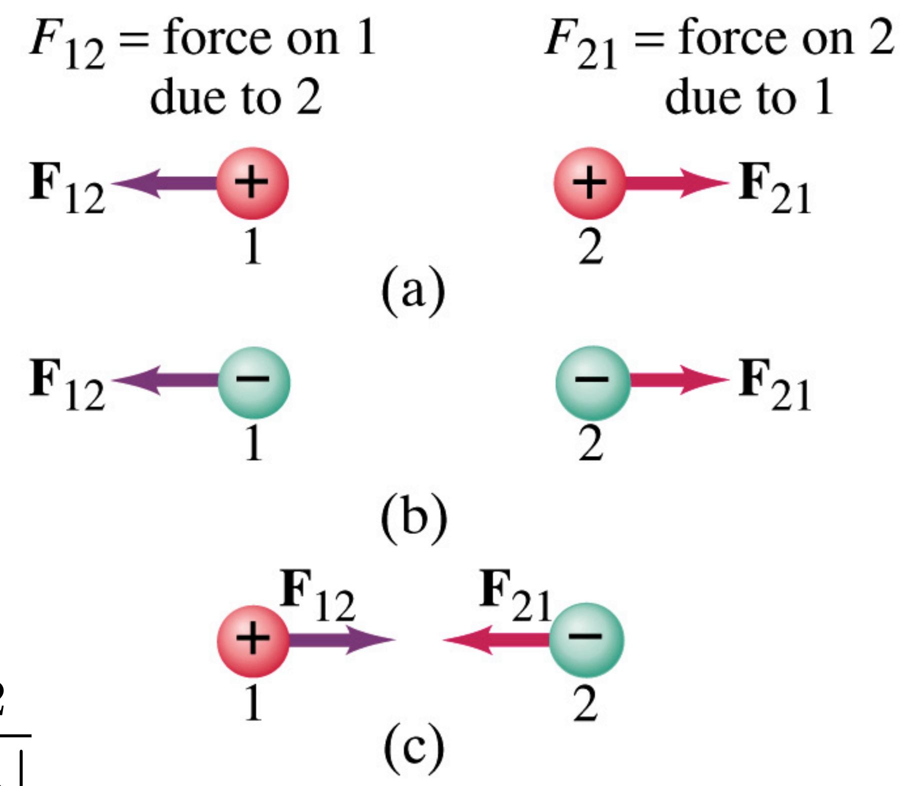
Volume charge density ρ: most realistic (depends on three coordinates), used for charged volumes and when no details are specified (general laws)

For currents mostly only the total current (*I*) and current density (*j*) are used

2.1 Coulomb's law

Describes the force between charges; magnitude and direction

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{12}^2}\hat{r}_{12}$$



$$\frac{1}{4\pi\epsilon_0} \approx 9\times 10^9 Nm^2C^{-2}[mF^{-1}]$$

$$\epsilon_0 \approx 8.8\times 10^{-12}Fm^{-1}$$

Electric constant or permittivity of free space

unit vector between Q_1 and Q_2 $\quad \hat{r}_{12} = \frac{\vec{r_{12}}}{|r_{12}|}$

Central force, just like gravity, but much larger and can be attractive and repulsive

Superposition principle

Total force can be obtained by summing all individual contributions (vector sum!) or by integrating if a charge density is given

$$\vec{F} = Q\sum_i \frac{Q_i\hat{r}_i}{4\pi\epsilon_0 r_i^2}$$

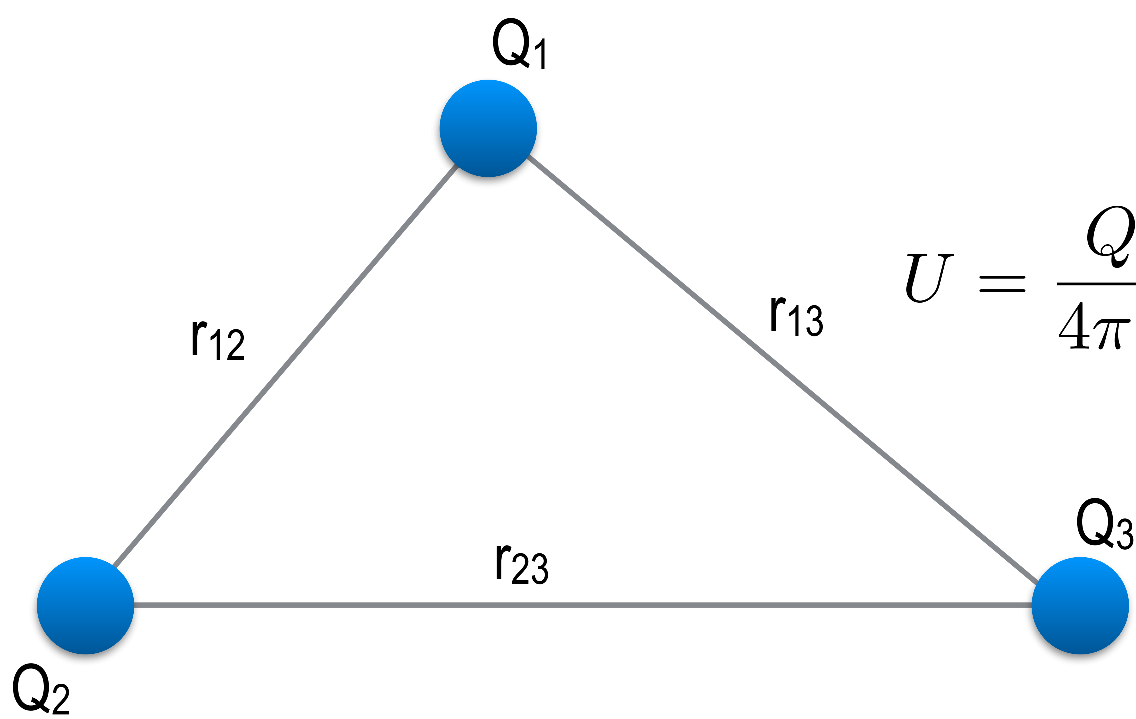
$$\vec{F} = \int d\vec{F} = Q\int_\tau \frac{\rho\hat{r}d\tau}{4\pi\epsilon_0 r^2}$$

2.2 Electric potential energy

Can be directly compared to other energies

Definition: Change in potential energy ΔU_{12} is work done by (external) force solely in changing from configuration 1→2

For two charges at distance *r* : $U = 0 \text{ for } r' = \infty \;\rightarrow\; U = \dfrac{Q_1 Q_2}{4\pi\epsilon_0 r}$



For a collection of charges:

$$U = \frac{Q_1 Q_2}{4\pi\epsilon_0 r_{12}} + \frac{Q_2 Q_3}{4\pi\epsilon_0 r_{23}} + \frac{Q_1 Q_3}{4\pi\epsilon_0 r_{13}}$$

Can be used to determine force: $\vec{F} = -\nabla U$

3.1. Electric field

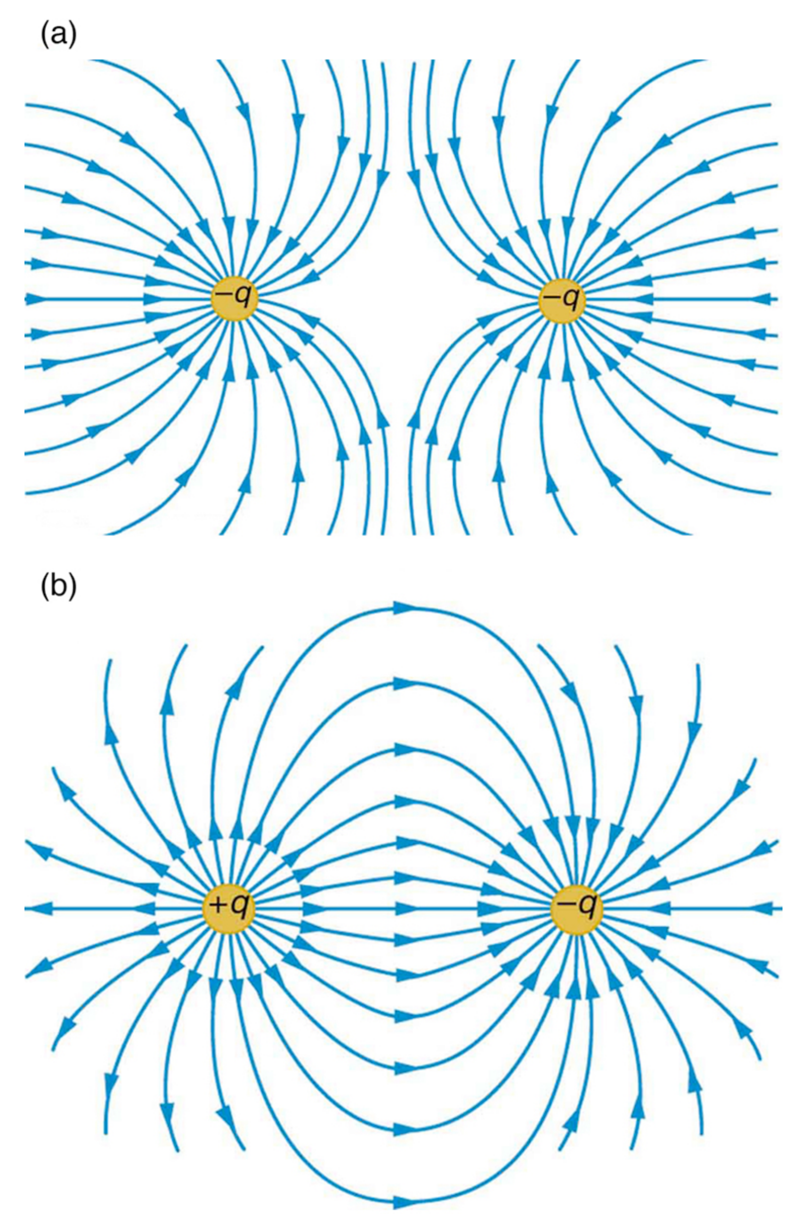
E-field: Force that would be exerted on a (test) charge if placed at any point relative to other charges

E-field is property of source charge, Coulomb force is between two (or more) charges

Definition: $\vec{E} = \dfrac{\vec{F}}{Q_t}$ Force per unit positive charge. Units: NC^{-1}, more commonly Vm^{-1} Q_t : "test charge"

Vector field

Field lines for two charges



<u>E-field due to charges and charge distributions:</u>

Single point charge: $$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \left(\approx \frac{\vec{F}(Q_t)}{Q_t}\right)$$

Collection of point charges: $$\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{Q_i \hat{r}_i}{r_i^2}$$

Charge distribution: $$\vec{E} = \int d\vec{E} = \int_\tau \frac{\rho \hat{r} d\tau}{4\pi\epsilon_0 r^2}$$

Example: "infinite plane": $$E = \frac{\sigma}{2\epsilon_0}$$

3.2. Electric potential

Potential energy a charge would have if placed relative to other charges: scalar field

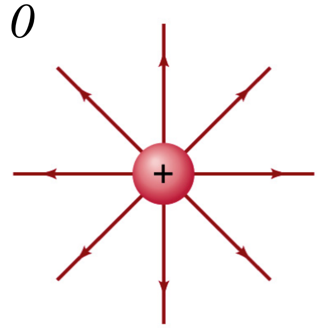
Potential is property of source charge, potential energy (U=QV) depends on both charges

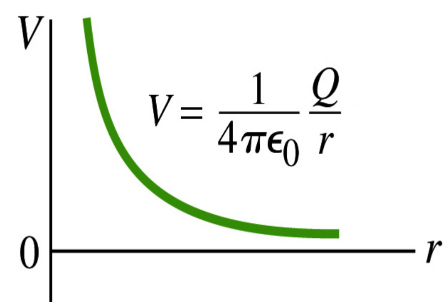
Definition: work done to bring (test) charge from A to B $V_{AB} = \dfrac{W_{AB}}{Q_t}$ Unit: JC^{-1}, more common V (Volt)

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{L} = \int_A^B -Q_t \vec{E} \cdot d\vec{L} \Rightarrow \boxed{V_B = \int_A^B -\vec{E} \cdot d\vec{L}} \quad \text{(zero at A)}$$

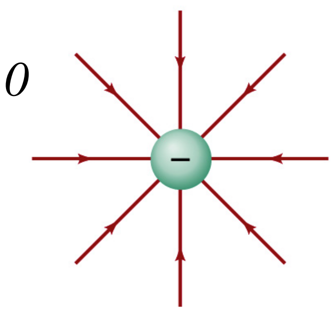
Point charge:

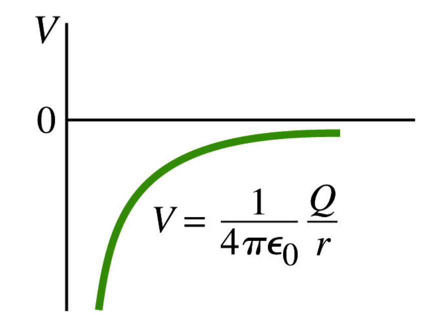
$Q > 0$





$Q < 0$



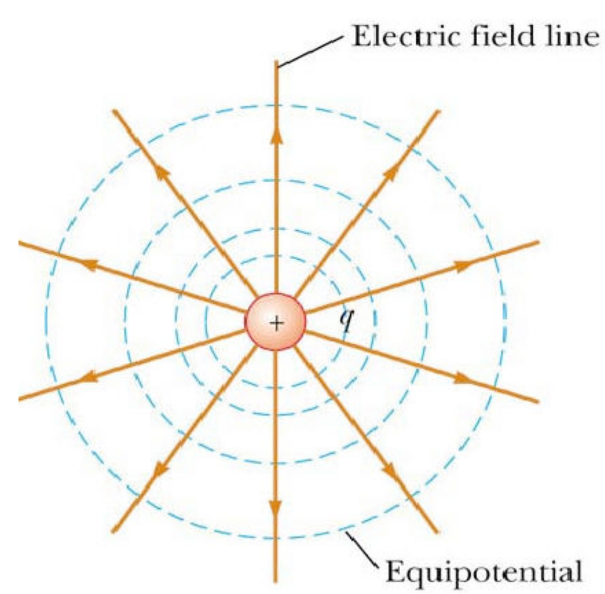


Uniform E-field $$V = \int -E dx = C - Ex$$

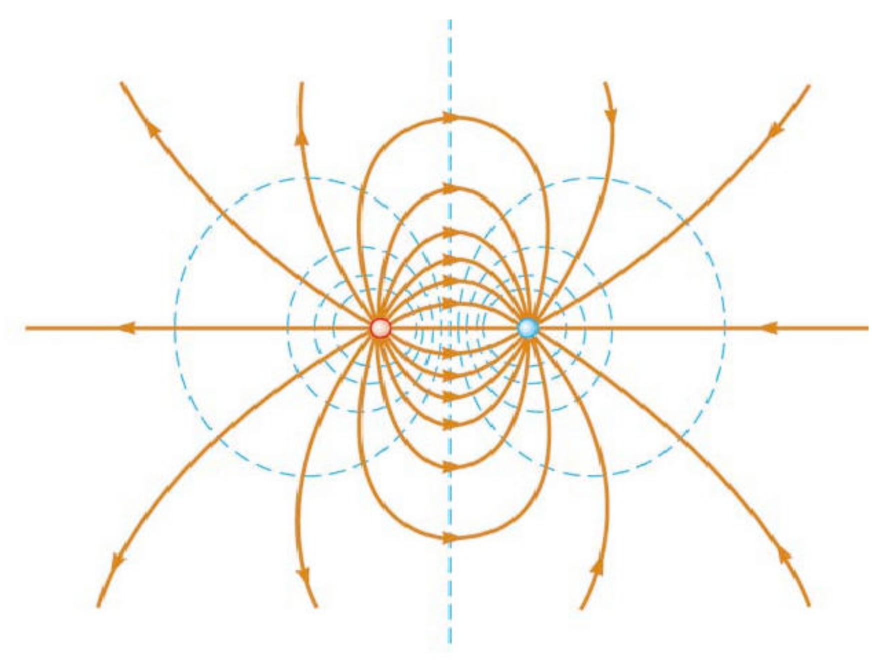
<u>Obtaining the E-field from a given potential</u>

$$\vec{E}(\vec{r}) = -\nabla V(x, y, z)$$

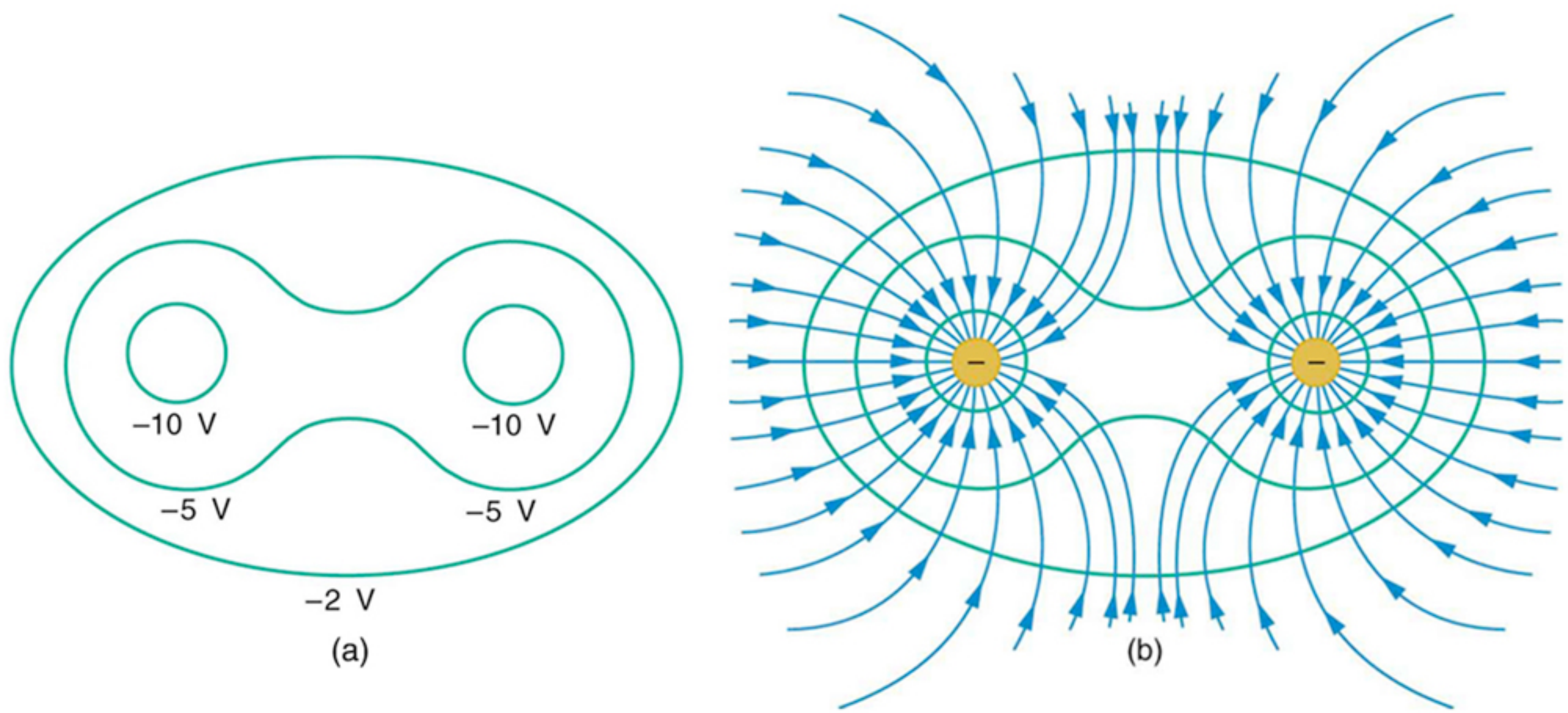
E-field lines are **always** normal to equipotential lines



(a)


(b)

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\boxed{\begin{array}{c}\text{Coulomb force}\\ \vec{F}\end{array}} \arrow[r, shift left, "\frac{1}{Q_t}"] \arrow[d, shift right, "\int \vec{F}\cdot d\vec{L}"'] & \boxed{\begin{array}{c}\text{Electric field}\\ \vec{E}\end{array}} \arrow[l, shift left, "\times Q"] \arrow[d, shift left, "\int \vec{E}\cdot d\vec{L}"] \\
\boxed{\begin{array}{c}\text{Potential energy}\\ U_E\end{array}} \arrow[u, shift right, "-\nabla"'] \arrow[r, shift left, "\frac{1}{Q_t}"] & \boxed{\begin{array}{c}\text{Electric potential}\\ V\end{array}} \arrow[u, shift left, "-\nabla"] \arrow[l, shift left, "\times Q"]
\end{tikzcd}
$$



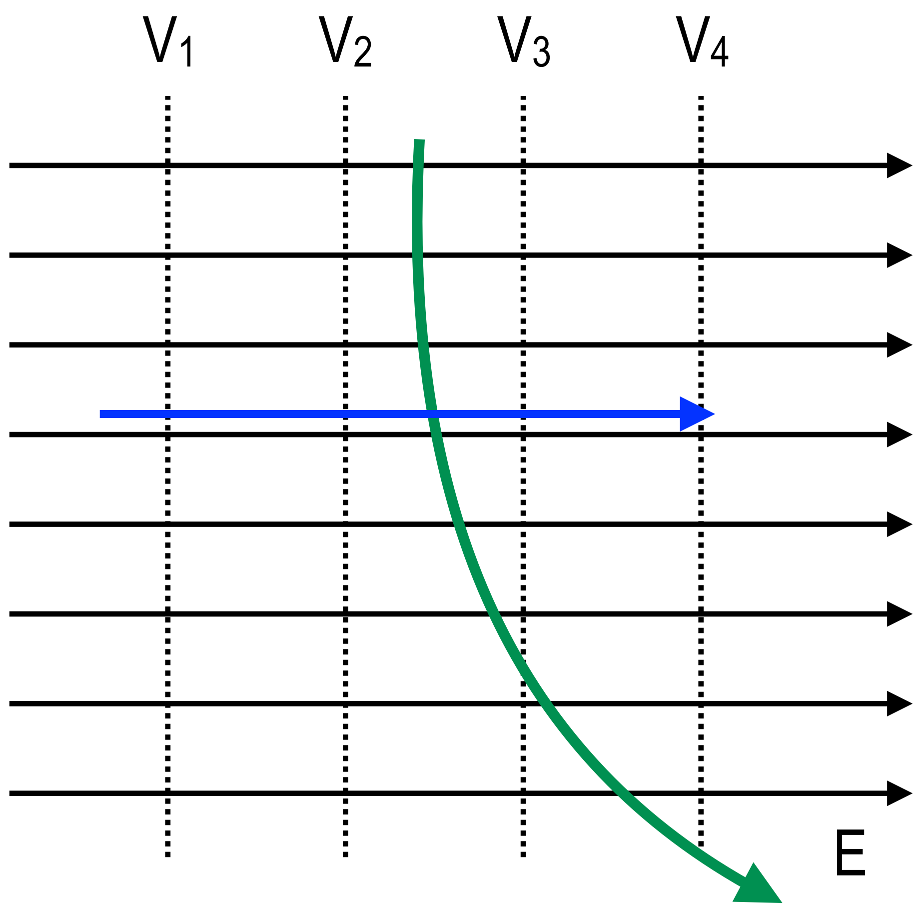
3.3. Charged particles in E-field

$$\vec{F} = Q\vec{E} = m\vec{a}$$

Charges experience a force and thus acceleration along the field lines

Electronvolt: energy of e^- passing 1V potential difference (1 eV = 1.6×10^{-19} J)

Trajectories of charges in E-field



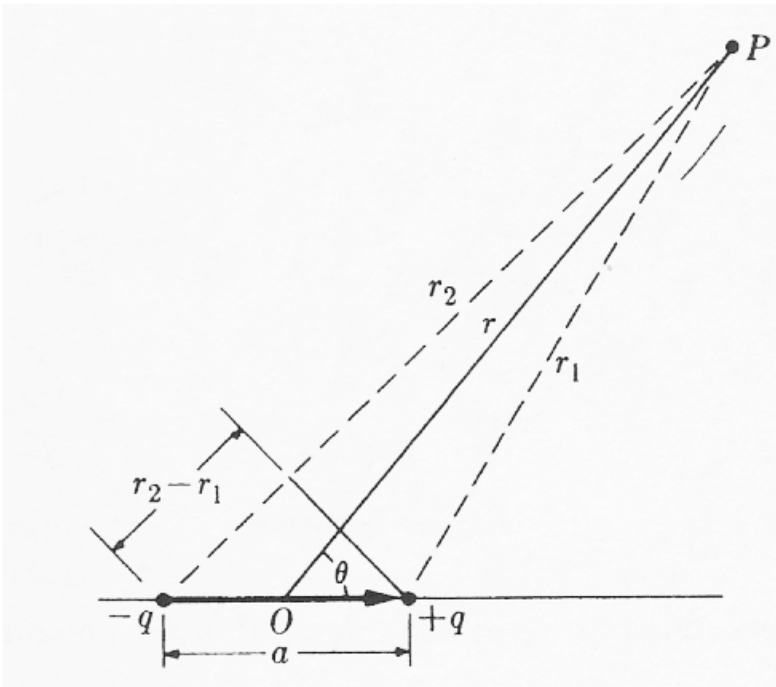
Distinguish cases by initial velocity:

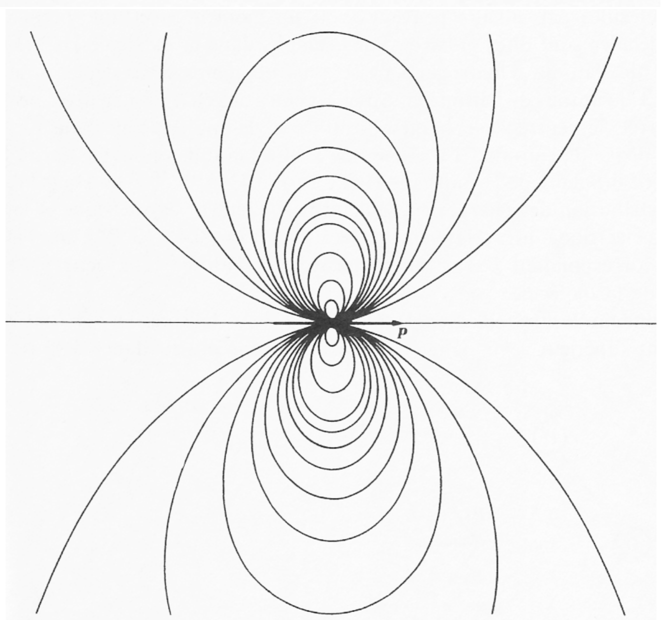
$v_i = 0$ *Particle is accelerated along E-field*

$\vec{v}_i || \vec{E}$ *Particle is accelerated along initial direction*

$\vec{v}_i \perp \vec{E}$ *Particle is deflected*

3.4. Electric dipole





$$\sum Q = 0$$ total charge is zero

-*q* and +*q* connected by vector $\vec{a}$ (from - to +)

Ideal dipole: a is small compared to other distances

Dipole moment: $\vec{p} = q\vec{a}$

Potential due to a dipole $V_P = \dfrac{p\cos\theta}{4\pi\epsilon_0 r^2} = \dfrac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2}$

E-field due to a dipole

$$V = \frac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2} \rightarrow E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{p\sin\theta}{4\pi\epsilon_0 r^3}$$

$$V = \frac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2} \rightarrow E_r = -\frac{\partial V}{\partial r} = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3}$$

<u>When placed in E-field a dipole will experience a force and torque</u>

Uniform E-field $\sum \vec{F} = 0$ (non-uniform fields are only as info)

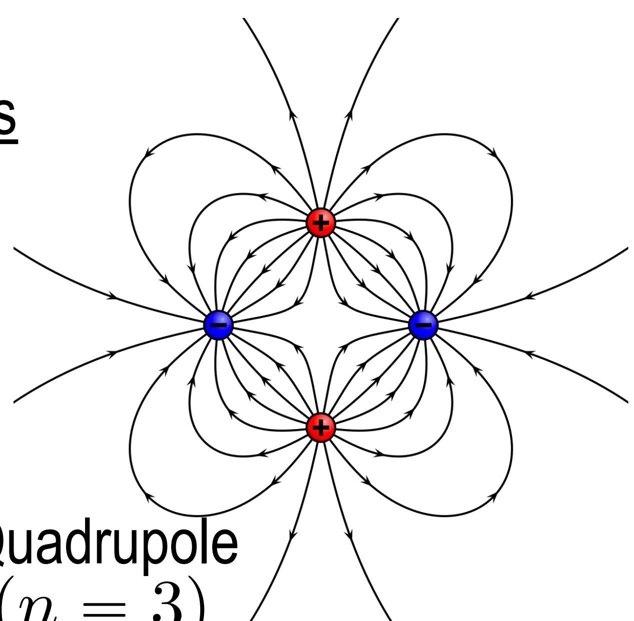
Torque: $\vec{\tau} = \vec{p}\times\vec{E}$ Torque will align dipole to E-field

Potential energy of dipole in E-field: $U = -\vec{p}\cdot\vec{E}$

<u>Multipoles</u>

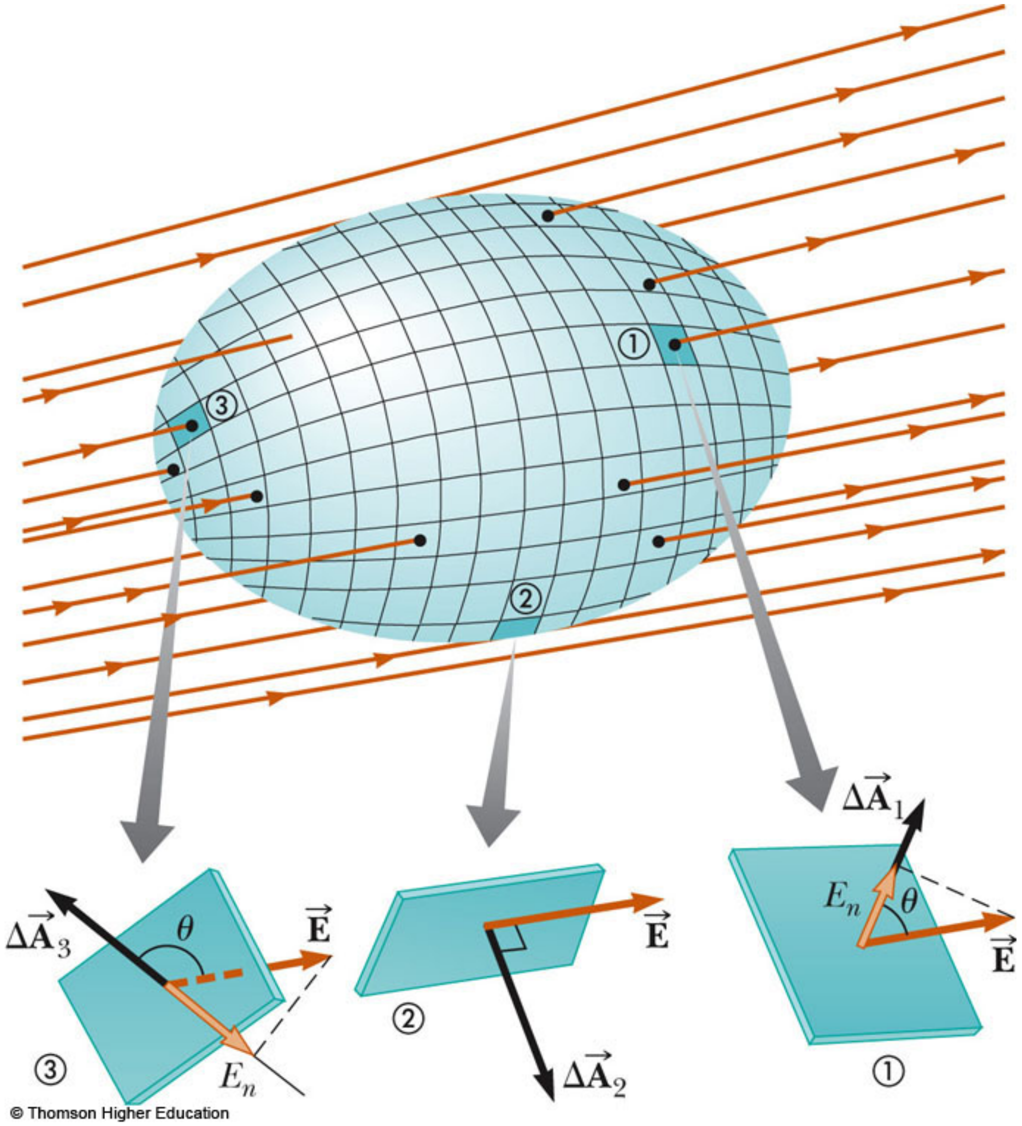
$$V \propto \frac{1}{r^n}$$

$$E \propto \frac{1}{r^{n+1}}$$



Quadrupole
$(n = 3)$

4.1. Gauss's law



Flux $\Phi_{dA} = \vec{E} \cdot d\vec{A}$

Flux of E-field over any closed surface is "equal" to enclosed charge

$$\oint\!\!\oint_S \vec{E} \cdot d\vec{S} = \sum_i \frac{Q_i}{\epsilon_0}$$

Local, or differential form based on charge density

$$\left(\frac{\partial E}{\partial x} + \frac{\partial E}{\partial y} + \frac{\partial E}{\partial z} \right) = \frac{\rho}{\epsilon_0}$$

This can be expressed using the nabla vector

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

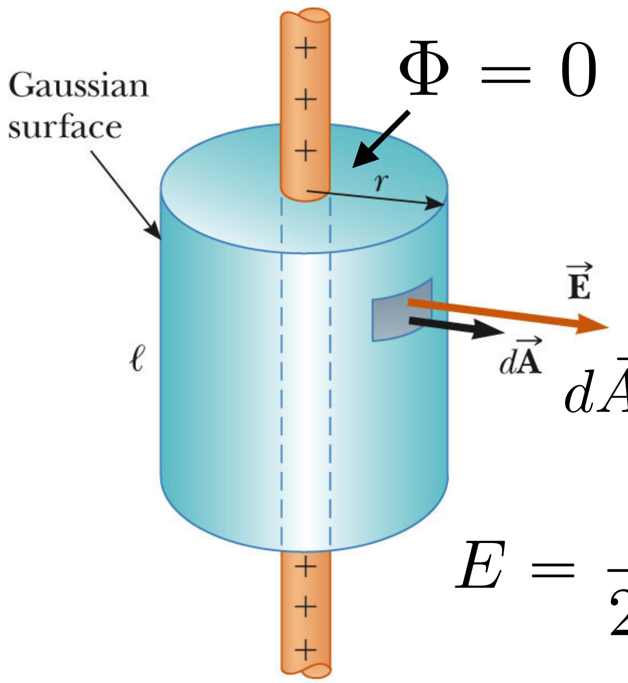
Using Gauss's law to find E-fields:

1. **Choose a smart and easy closed surface based on symmetry of system**
2. Write down expression or solve integral for flux through surface and charge contained inside
3. Simplify to obtain E=…
4. Integrate to obtain potential

Examples from lecture

Uniformly charged cylinder (wire or e⁻ beam)

line charge density λ



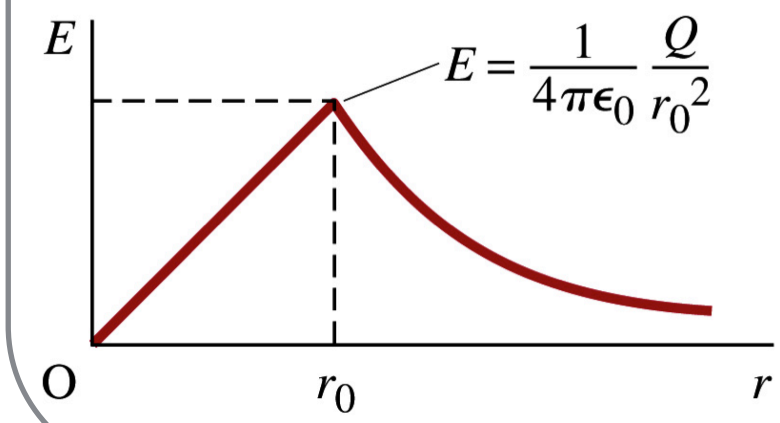
$$\Phi = 0$$

$$d\vec{A} \parallel \vec{E}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_0}{r}$$

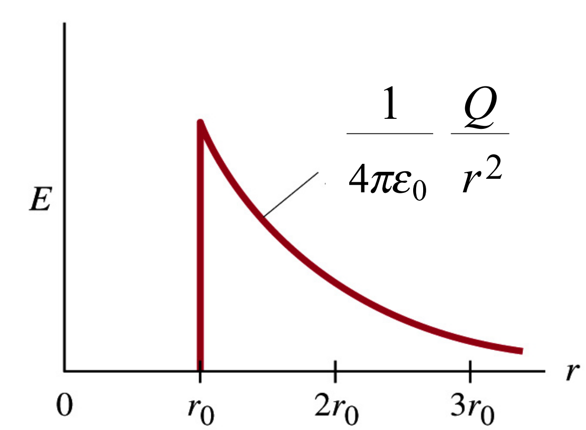
Uniformly charged sphere (not conductor)

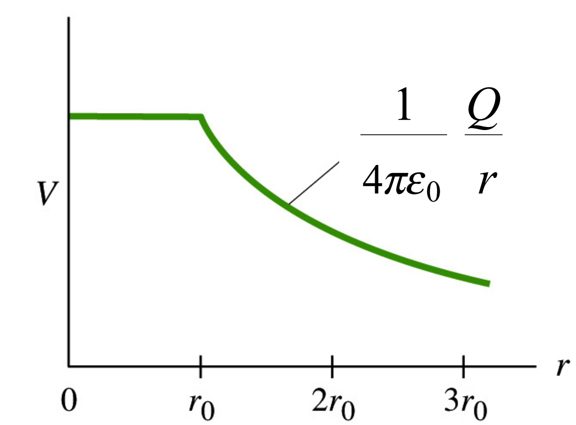


$$r \geqslant r_0$$

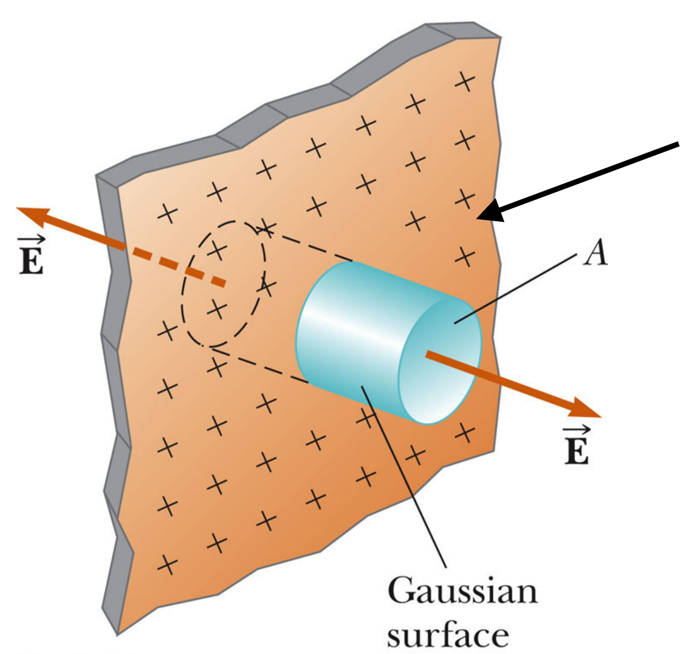
$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Conducting sphere or shell (charge on surface)





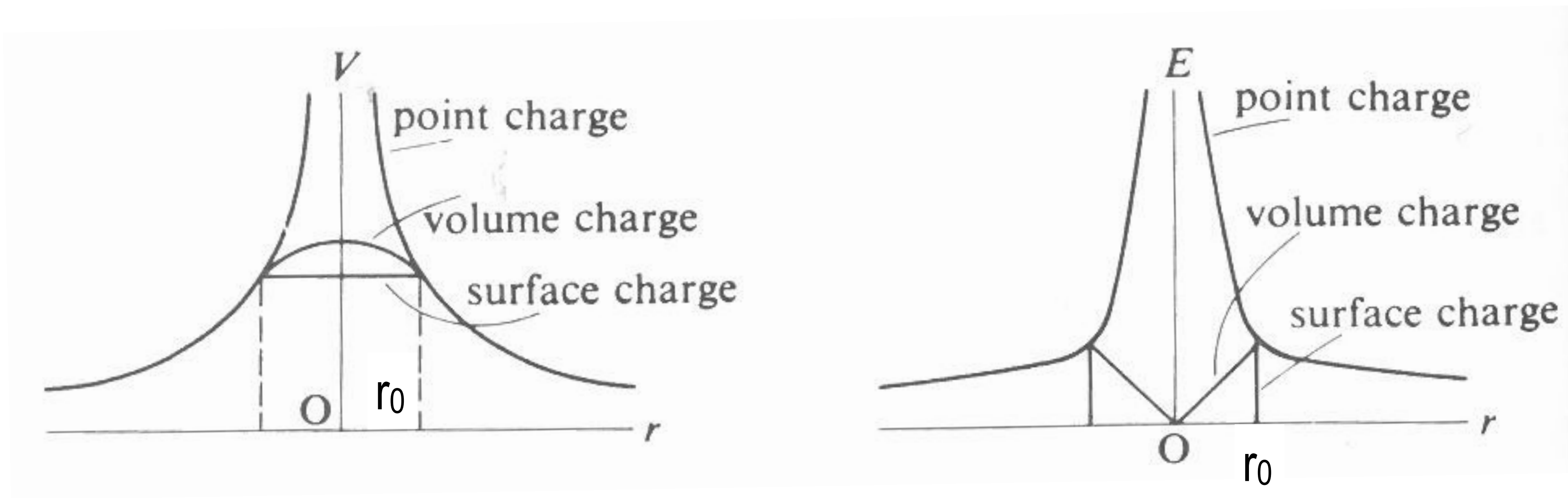
Plane conductor with surface charge σ



$$E = \frac{\sigma}{2\epsilon_0}$$

From symmetry: $\vec{E} \perp$ surface

Comparison of spherical systems



Point effect and Corona discharge

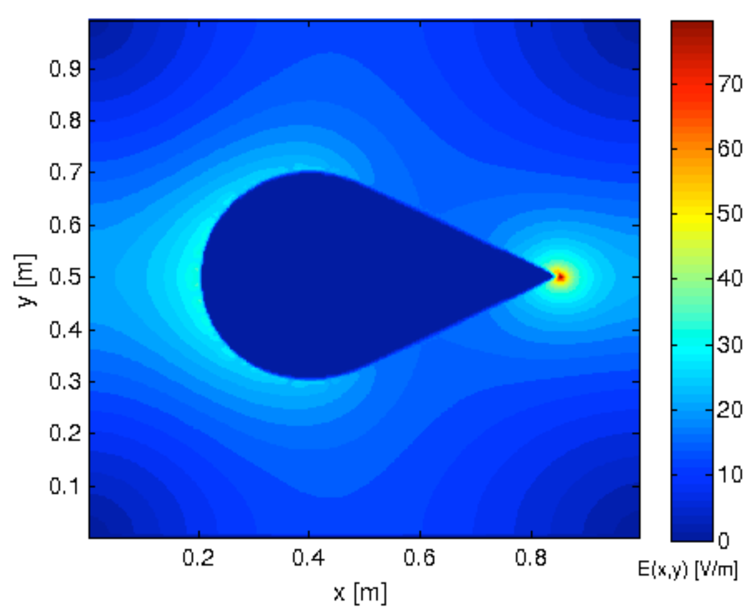
$$V \propto \frac{1}{r_0}$$

Locally:

$$\sigma = \frac{\epsilon_0 V}{r_0}$$

$$E = \frac{V}{r_0}$$

For same potential



Corona discharge due to high field at sharp points

4.2. Circuital law

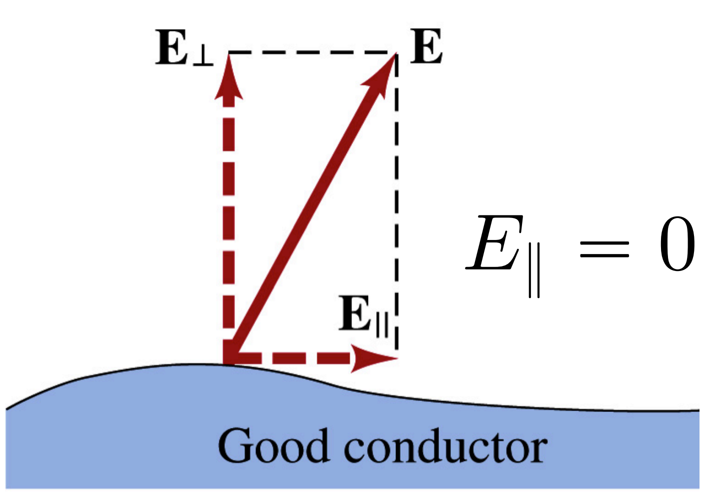
There are no closed E-field lines for electrostatic fields.

Follows from path independence of potential difference (central force)

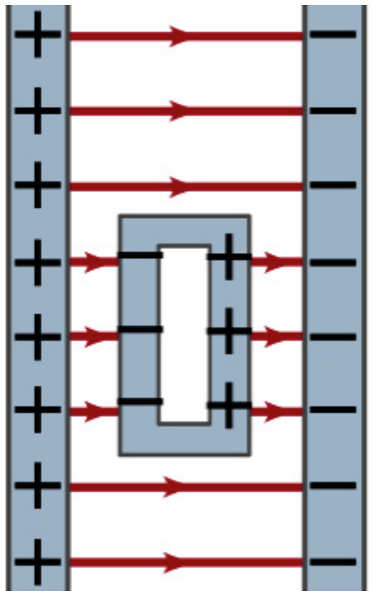
$$\oint \vec{E} \cdot d\vec{L} = 0 \quad \text{or} \quad \nabla \times \vec{E} = 0$$

4.3. General consequences of Gauss and circuital law

- **No E-field inside conductor:** All charge is located on surface of conductor (Potential is the same throughout conductor)
- Charge inside hollow conductor is **screened**
- **External E-fields blocked inside a hollow conductor:** Electrostatic shielding or screening (Faraday cage)
- E-field lines are **always perpendicular** to the surface of a conductor



$$E_{\parallel} = 0$$



4.4. Poisson's and Laplace's equation

Poisson's equation: $$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

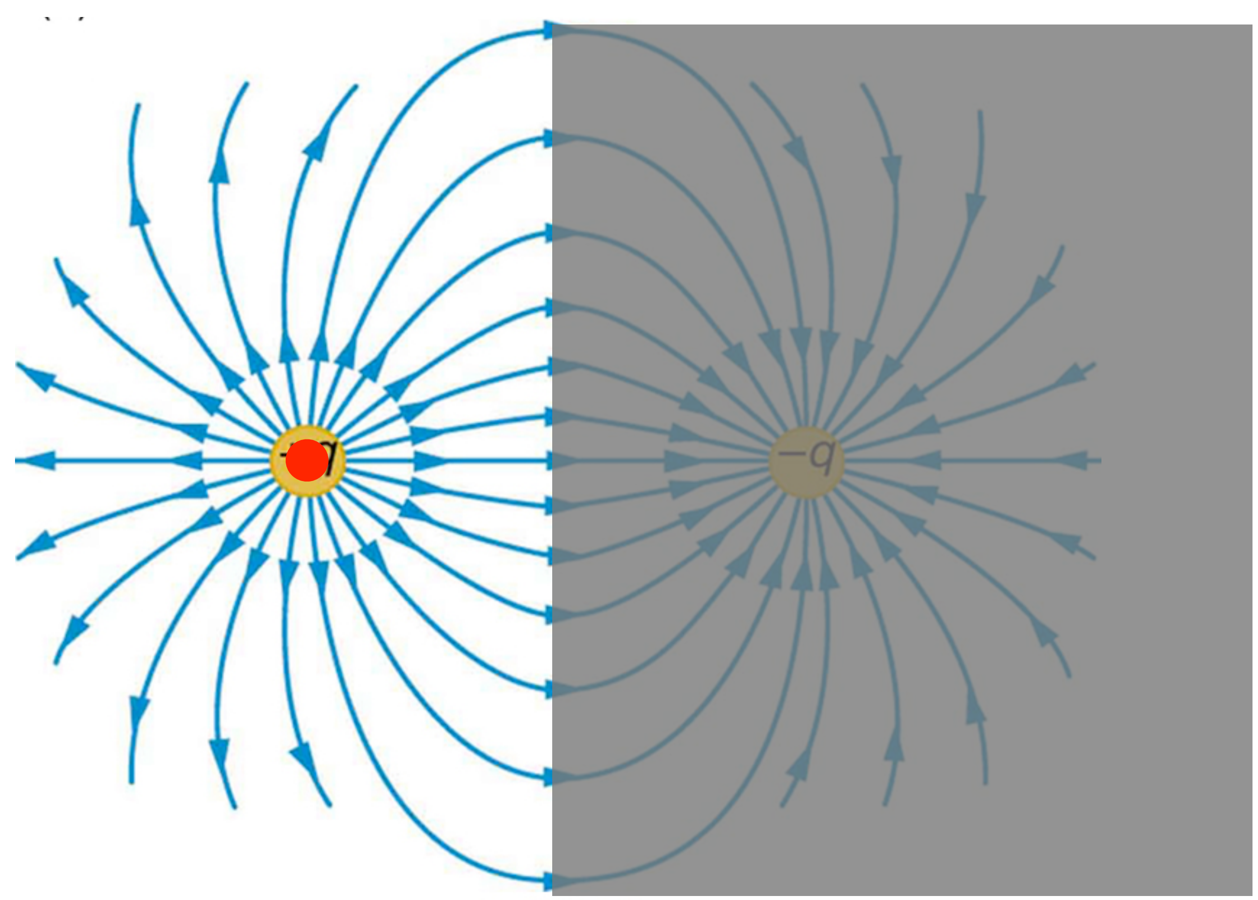
with $$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

Laplace's equation (no charge): $$\nabla^2 V = 0$$

Can be used to find the potential in any situation if boundary conditions are given

Most important consequence is **uniqueness theorem**:
"If a solution can be found then it is the only correct one"

<u>Image charge solution to problems:</u>



V=0

Example: charge in front of conductor

5.1. Capacitance

Capacitance of a conductor: Amount of charge a conductor can hold for a given potential

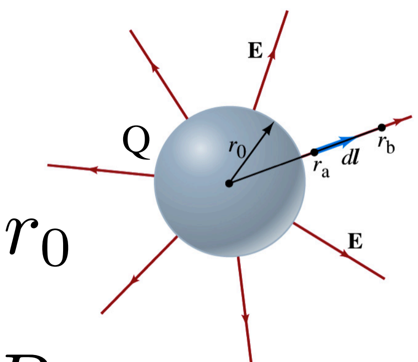
"capacity for charge"

Definition: $$C = \frac{Q}{V}$$ units: CV^{-1} or F (Farad)

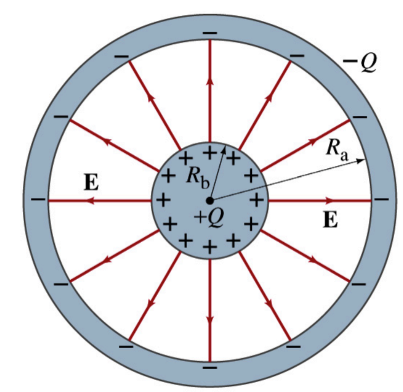
C only depends on geometry!

<u>Some examples from lecture 5</u>

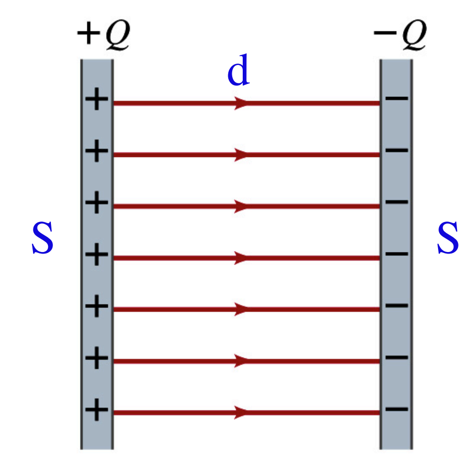
Lone conducting sphere: $$C = \frac{Q}{V} = 4\pi\epsilon_0 r_0$$



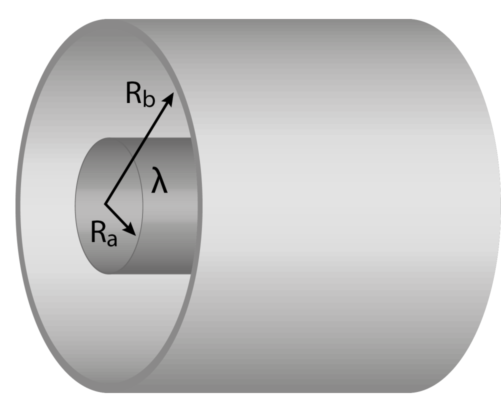
Spherical capacitor: $$C = 4\pi\epsilon_0 \frac{R_a R_b}{(R_a - R_b)}$$

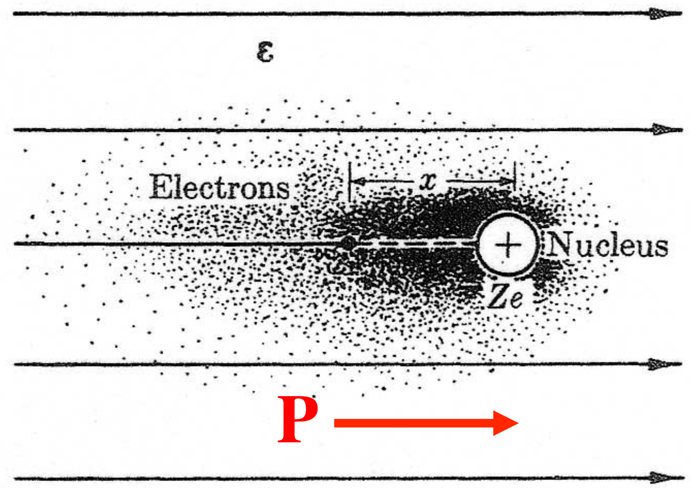


Parallel plate capacitor: $$C = \epsilon_0 \frac{S}{d}$$



Cylindrical capacitor: $$C = \frac{2\pi\epsilon_0 l}{\ln \frac{R_b}{R_a}}$$





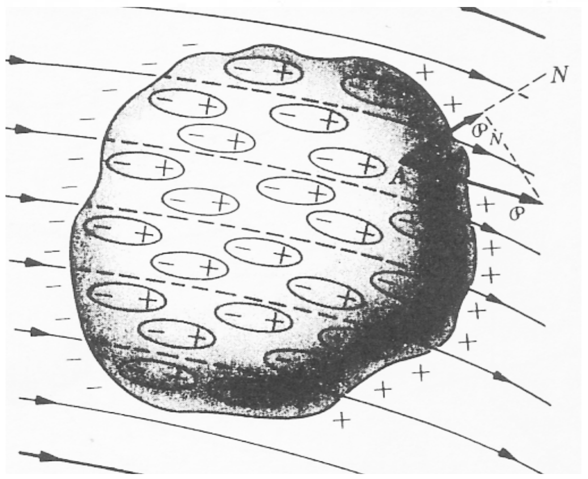
5.2. Dielectrics

No true insulator exists: matter can be polarized

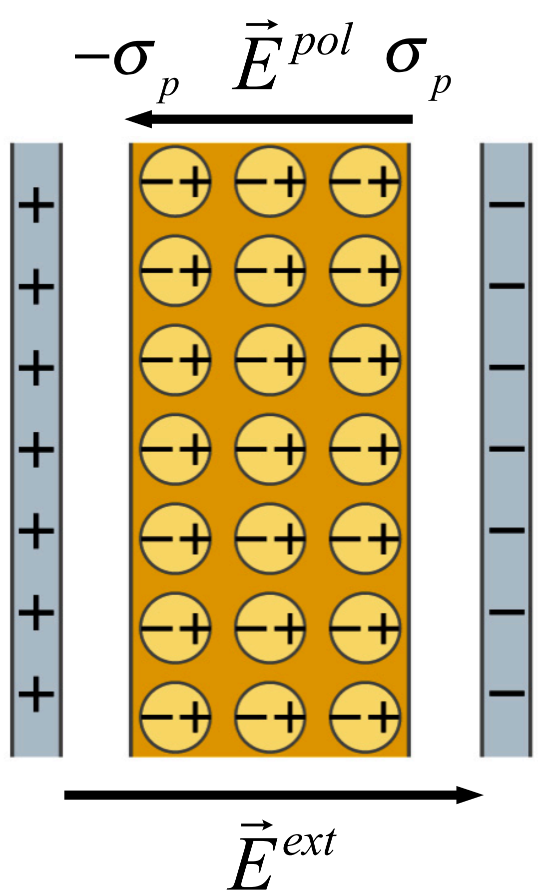
E-field is partially screened, but +/- charge always bound

Polarization of material $\vec{P}$: collection of dipoles

dipole moment/volume $\quad d\vec{p} = \vec{P} d\tau$



<u>Dielectric in external E-field</u>



$$E^{ext} = \text{constant}$$

$$E^{pol} \text{ due to } \sigma_p$$

$$\langle \vec{E} \rangle = \vec{E}^{ext} - \vec{E}^{pol}$$

$$\vec{P} = \epsilon_0 \chi_e \langle \vec{E} \rangle$$

χ_e Electric susceptibility (*how easy it is to polarize a material*)

5.3. The D-field

Gauss's law applied to a dielectric yields: $\oiint (\epsilon_0 \vec{E} + \vec{P}) \cdot d\vec{S} = \sum Q_c$

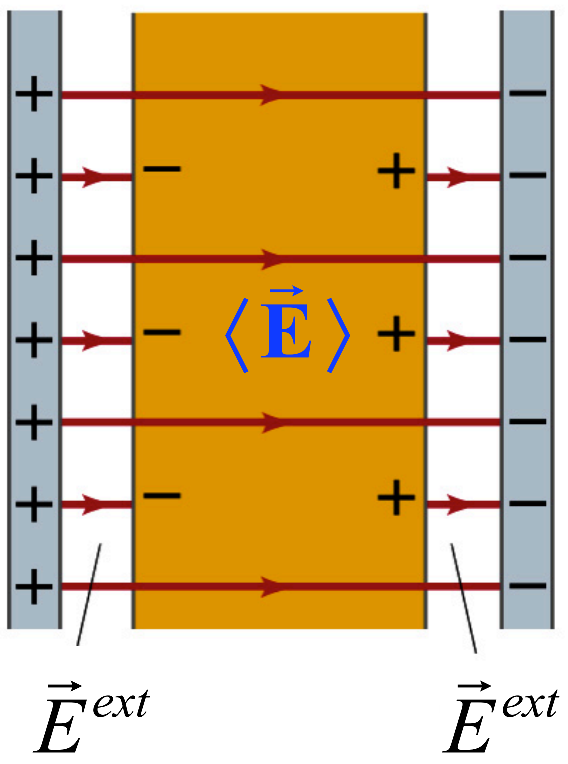
Where Qc are the conduction charges; the charges we considered before and not the polarization charges

Define D-field: $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ and thus $\oiint \vec{D} \cdot d\vec{S} = \sum Q_c$ and $\nabla \cdot \vec{D} = \rho_c$

Only conduction charges are source of D-field

$$\vec{D} = \epsilon_0 (1 + \chi_e) \vec{E}$$

Definition relative permittivity: $\epsilon_r = (1 + \chi_e)$ thus: $\boxed{\vec{D} = \epsilon_0 \epsilon_r \vec{E}}$

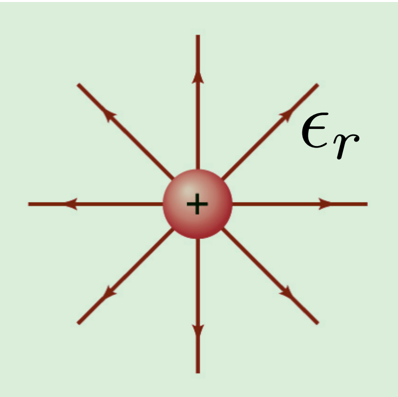


In dielectric E-field is reduced by factor ϵ_r

The magnitude of the D-field stays the same

In vacuum $\vec{D} = \epsilon_0 \vec{E}$

Point charge



$$\vec{E} = \frac{Q\hat{r}}{4\pi\epsilon_0\epsilon_r r^2}$$

$$\vec{D} = \frac{Q\hat{r}}{4\pi r^2}$$

5.4. Electric energy

Energy stored in capacitor or collection of charges

General case or capacitor: $$U_E = W = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Collection of charges: $$U_E = \sum_{i=1}^{N} \frac{1}{2} Q_i V_i$$

Electric energy in terms of E- or D-fields

$$U_E = \frac{1}{2}\epsilon_0 \int_{space} E^2 d\tau$$

In a dielectric this changes to:

$$U_E = \frac{1}{2}\epsilon_0 \int_\tau \epsilon_r E^2 d\tau$$

or more formally:

$$U_E = \frac{1}{2} \int_\tau \vec{D} \cdot \vec{E} d\tau$$

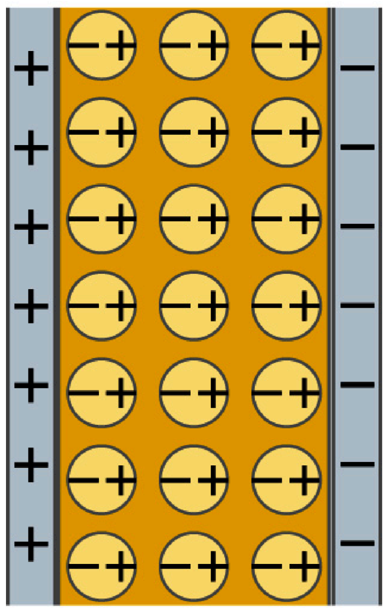
The force between conductors can be determined from the electric energy

$$\vec{F} = -(\nabla U_E)_Q$$ For constant Q

$$\vec{F} = +(\nabla U_E)_V$$ For constant V

5.5. Capacitor with dielectric

Result depends on whether capacitor is connected or isolated



$$C = \epsilon_r C_0$$

Isolated (Q constant) $\quad E = \dfrac{E_0}{\epsilon_r} \quad$ and $\quad V = \dfrac{V_0}{\epsilon_r}$

Connected (V constant) $\quad Q = Q_0\epsilon_r \quad \sigma = \sigma_0\epsilon_r$

$$E = \frac{\sigma_0\epsilon_r}{\epsilon_0\epsilon_r} = E_0$$

Electric energy of capacitor without dielectric $\quad U_E^0 = \dfrac{1}{2}C_0V_0^2 = \displaystyle\int_{cap} \frac{1}{2}\epsilon_0 E_0^2 d\tau$

<u>Electric energy of capacitor with dielectric</u>

Isolated (Q constant) $\quad U_E = \dfrac{1}{\epsilon_r}U_E^0$

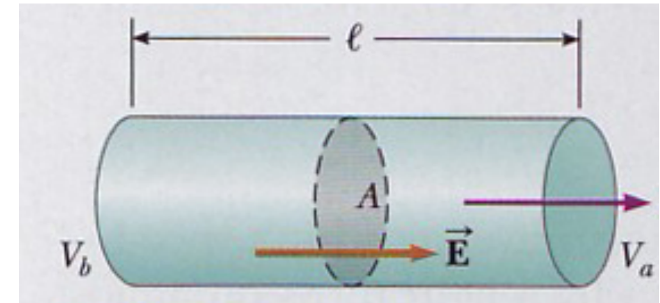
Connected (V constant) $\quad U_E = \epsilon_r U_E^0 \quad$ *Increase in energy provided by voltage source*

6.1. DC networks

Electromotive force: work done to provide useful current

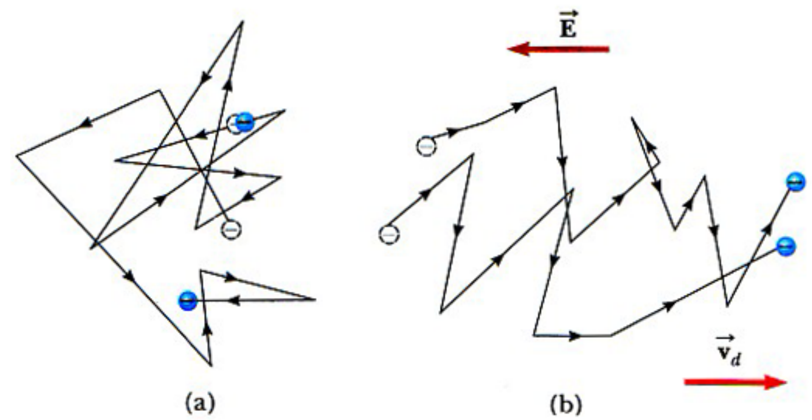
Current: $\vec{j} = \sigma \vec{E} \quad (\sigma : \text{conductivity})$

Resistivity: $\rho = \dfrac{1}{\sigma} = \dfrac{RA}{l}$ Resistance $R = \dfrac{\rho l}{A}$



Microscopic Drude model

conductivity: $\sigma = \dfrac{ne^2\lambda}{2mv}$



<u>Kirchhoff's laws</u>

Conservation of charge $\sum_i I_i = 0$ Path independence of potential difference $\sum_i^N \Delta V_i = 0$

<u>Combinations of resistors</u> In series: $R' = \sum_i R_i$ In parallel: $\dfrac{1}{R'} = \sum_i \dfrac{1}{R_i}$

<u>Combinations of capacitors</u> in series $\dfrac{1}{C'} = \sum_i \dfrac{1}{C_i}$ in parallel $C' = \sum_i C_i$

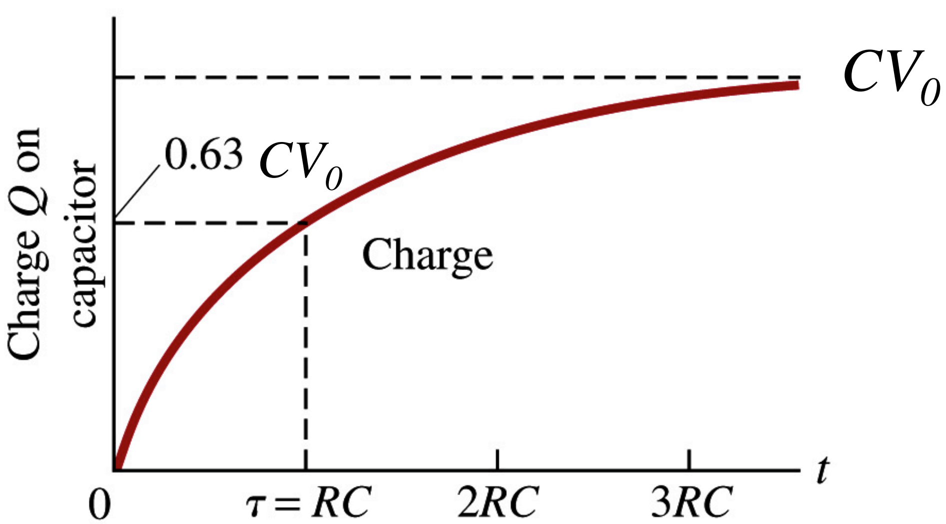
6.2. Combination of capacitor and resistor

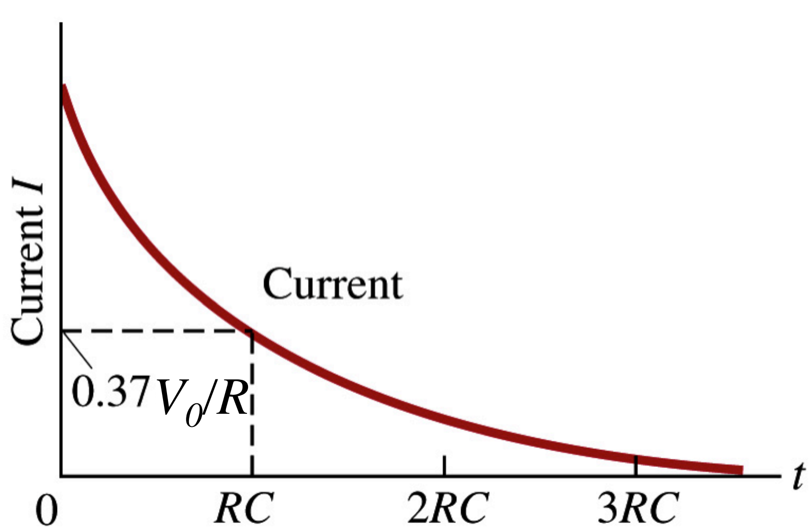
Charging a capacitor

$$V_C(t) = V_0\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$I(t) = C\frac{dV_C}{dt} = \frac{V_0}{R}e^{\frac{-t}{\tau}}$$

Characteristic time: $\tau = RC$





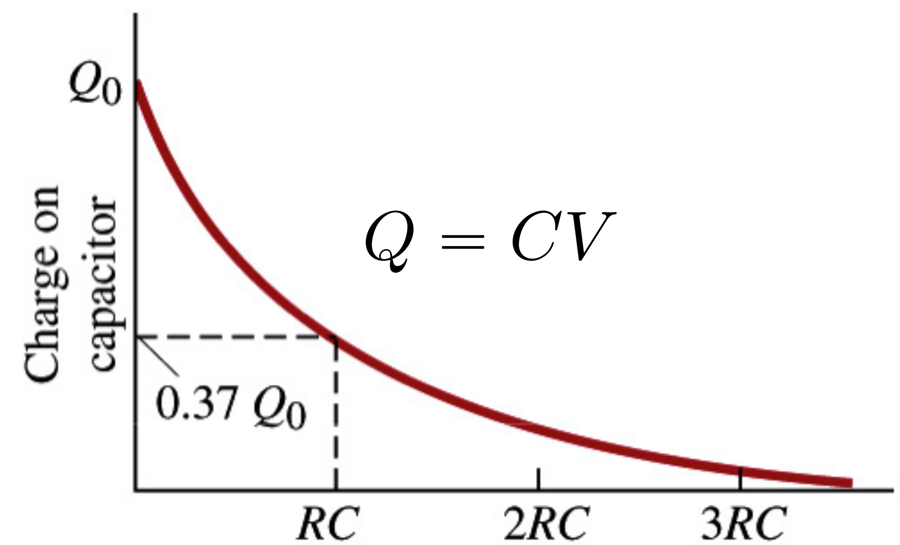
Discharging a capacitor

$$V_C(t) = V_0 e^{\frac{-t}{\tau}}$$

$$I(t) = \frac{V_0}{R}e^{\frac{-t}{\tau}}$$

$$\tau = RC$$

"How long discharge takes"



7.1. Magnetic field

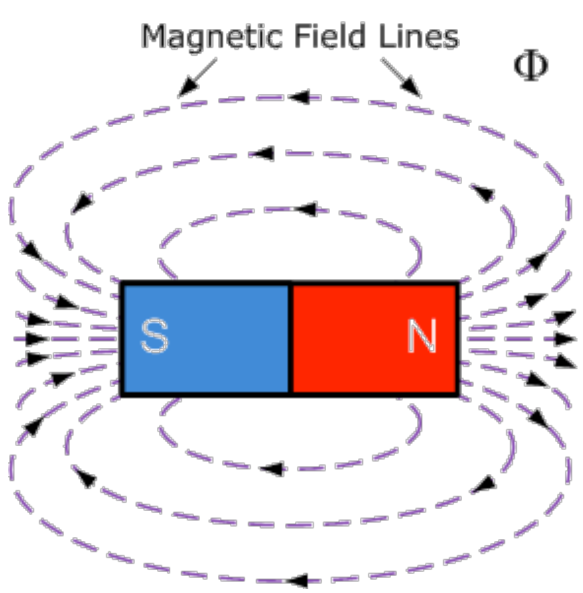
The magnetic field (B-field) is a vector field just like the E-field

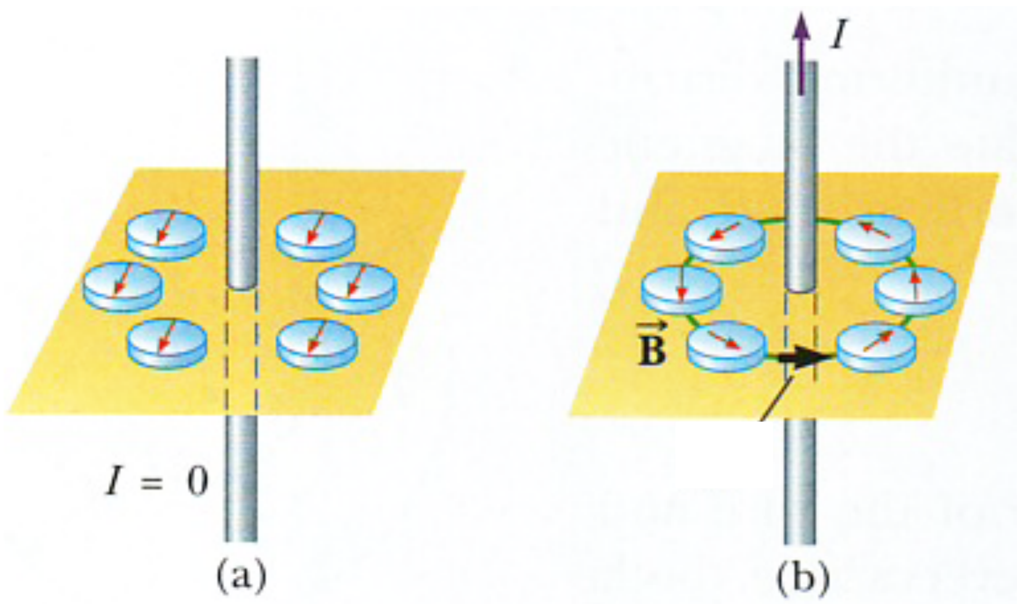
No magnetic monopole (equivalent to point charge) so derivation is less clear

Important differences to electrostatics:

- Force is perpendicular to B-field
- B-field is not central (and not conservative)
- No equivalent medium to conductor

Most straightforward is to look at different "objects" and determine the B-field they produce





7.1.1 Sources of B-fields

Permeability of free space

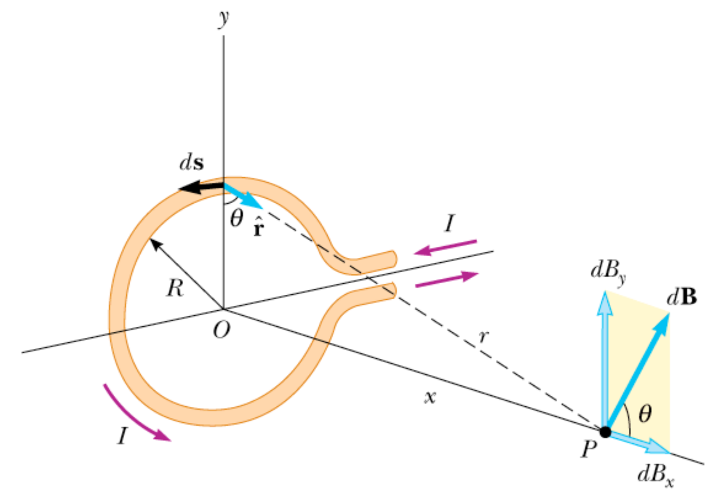
$$\mu_0 = 4\pi \times 10^{-7}$$

Current element

$$d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$$

Wire

$$\vec{B} = \frac{\mu_0 \vec{I} \times \hat{r}}{2\pi r}$$



Single loop

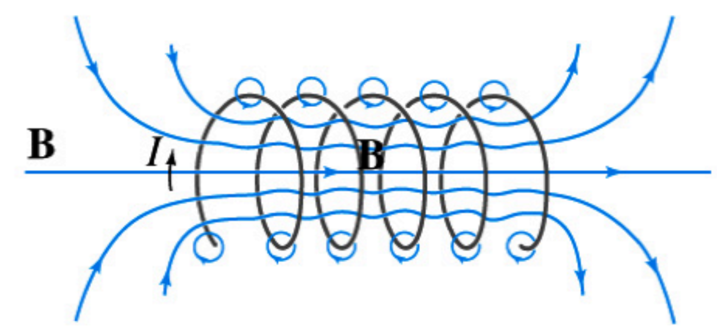
$$B_x = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{\frac{3}{2}}} \qquad B_y = B_z = 0$$

$$x = 0 \rightarrow B_x = \frac{\mu_0 I}{2R}$$

Solenoid

$$B = \mu_0 n I$$

n: number of windings per unit length



Moving charge

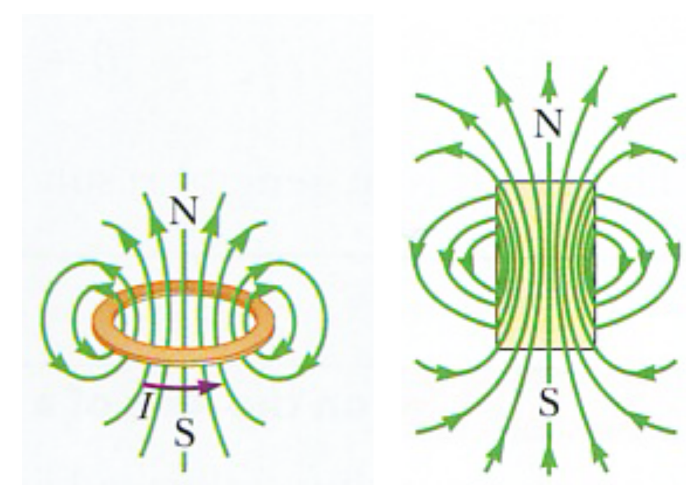
$$\vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2}$$

Magnetic dipole

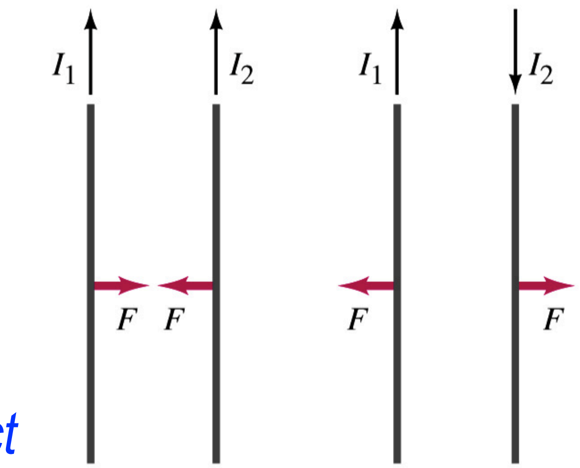
$$\boxed{B_r = \frac{2\mu_0 m \cos\theta}{4\pi r^3} \qquad B_\theta = \frac{\mu_0 m \sin\theta}{4\pi r^3}}$$

Dipole moment:

$$\vec{m} = I\vec{A} = IA\hat{n}$$



7.1.2 Force of B-field on different objects



Current element

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

Between two wires

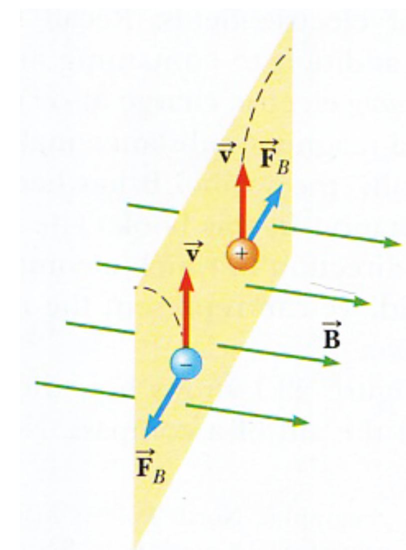
$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

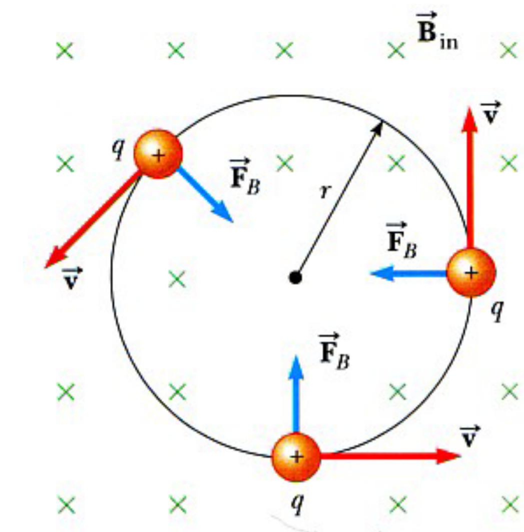
Opposites repel, likes attract

Moving charge

$$\vec{F} = q\vec{v} \times \vec{B}$$

Lorentz force





Magnetic dipole

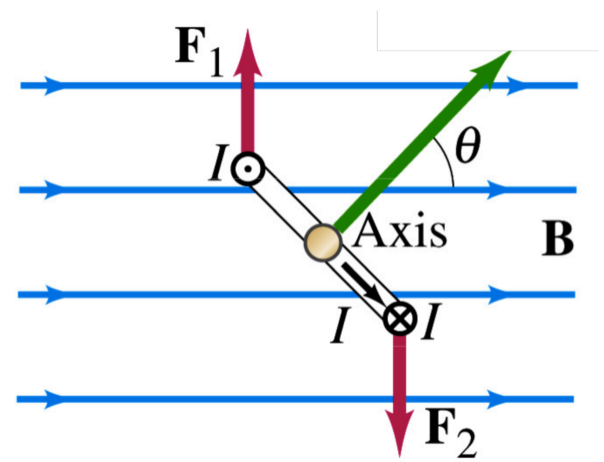
$$\vec{T} = \vec{m} \times \vec{B}$$

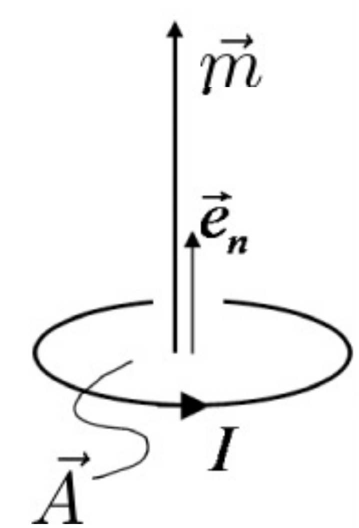
Dipole moment:

$$\vec{m} = I\vec{A} = IA\hat{n}$$

$$\sum \vec{F} = 0$$

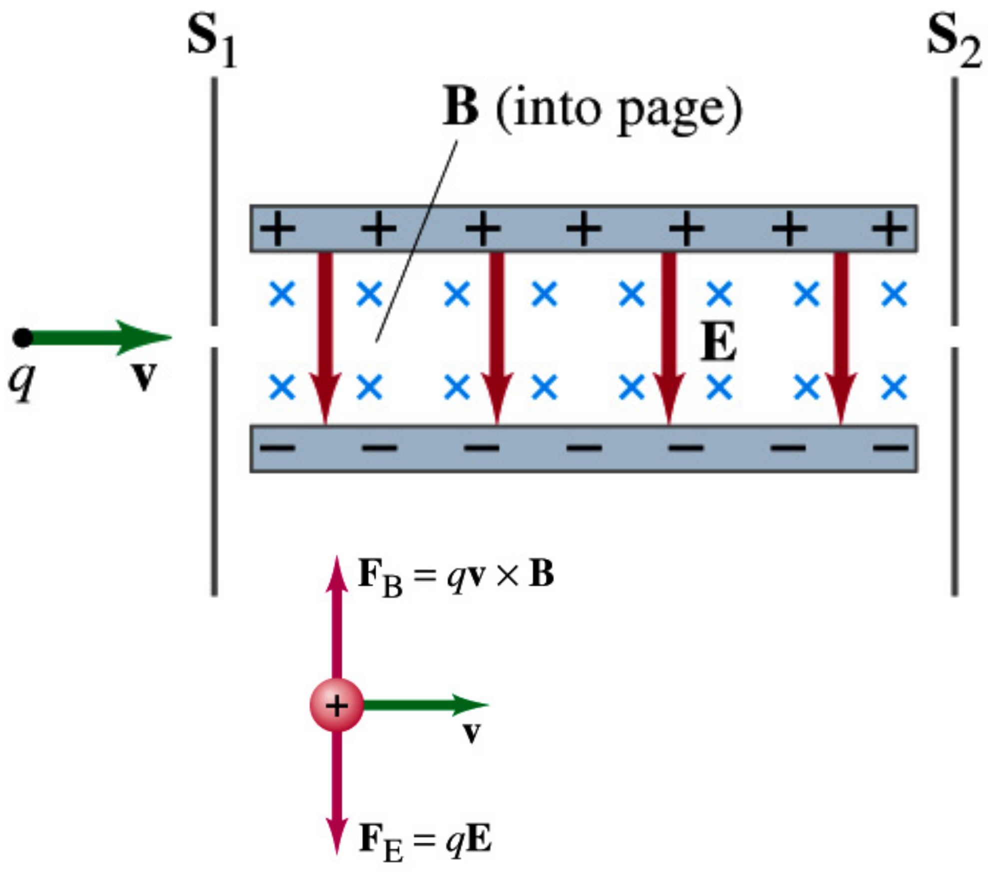
(in homogeneous field)





7.2. Moving charges in E- and B-fields

Velocity filter

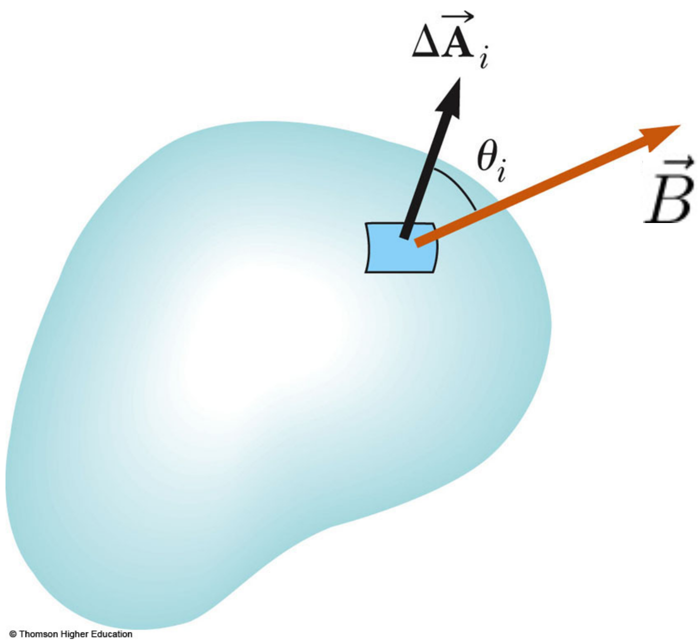


$$v = \frac{E}{B} \rightarrow \vec{F} = 0$$

E-field determines ***kinetic energy***

B-field determines ***momentum***

E- and B-field together determine ***velocity***



7.3.1 Magnetic flux

Similar to flux of E-field: $\Phi = \int_A \vec{B} \cdot d\vec{A}$

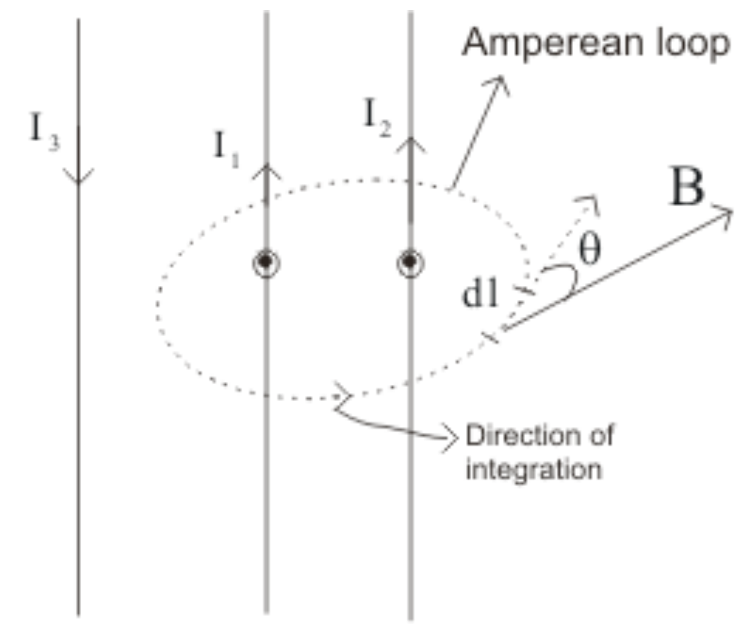
Crucial to understand **induction**

7.3.2 Ampère's circuital law

$$\oint_L \vec{B} \cdot d\vec{L} = \mu_0 I \quad \text{or} \quad \nabla \times \vec{B} = \mu_0 \vec{j}$$

Currents are the sources of magnetic fields

Can be used to find magnetic fields (see lecture 9 for examples)



7.3.3 Gauss's law for B-fields

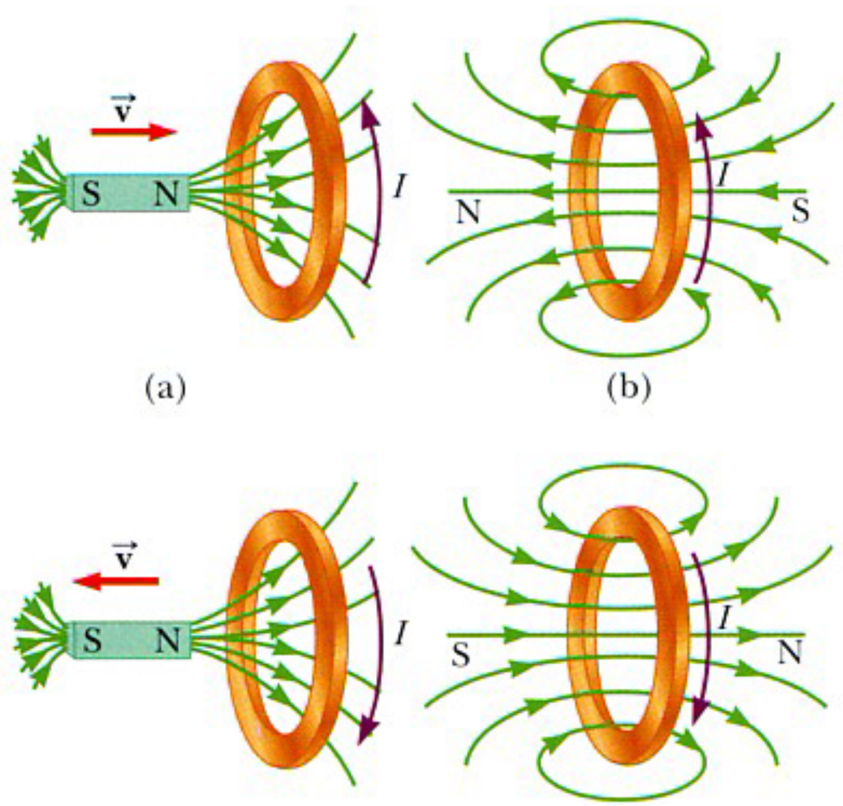
$$\oiint_S \vec{B} \cdot d\vec{S} = 0 \quad \text{or} \quad \nabla \cdot \vec{B} = 0$$

Absence of sources and sinks of magnetic field lines

8.1 Magnetic induction

A changing magnetic flux induces a voltage in any loop or circuit

$$V_{emf} = -\frac{d\Phi_m}{dt}$$



A voltage or electromotance (V_{emf}) is induced in

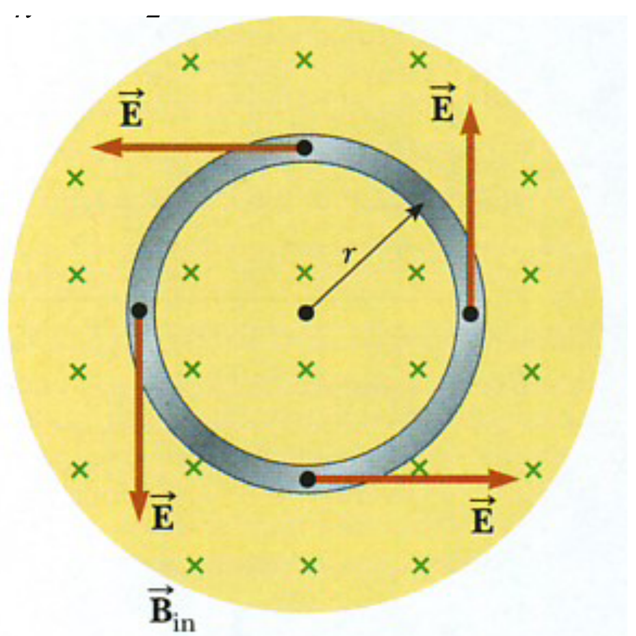
1. a rigid stationary circuit across which there is a varying magnetic field
2. a rigid circuit moving or rotating in a B-field such that the magnetic flux through it changes
3. a part of a circuit which moves and cuts magnetic flux

The direction of any magnetic induction effect is such as to oppose the cause of the effect

<u>The changing B-field induces an E-field with closed lines</u>

Work can be done

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$



The electric field is no longer conservative in presence of changing magnetic field!!!

8.2 Inductance

A current in any element will cause a magnetic flux in the same element

Definition of **self-inductance** *L*: $\Phi = LI$

A change of current will induce a change of flux and thus voltage drop

$$\Delta V = -L\frac{dI}{dt}$$

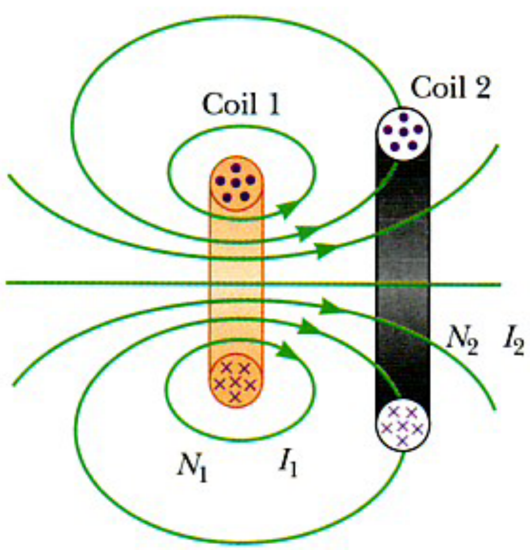
Internal resistance against changes in the current.
Can’t be avoided!

Solenoid:

$$L = \mu_0 n^2 l A$$

co-axial cable:

$$L = \frac{\mu_0 l}{2\pi}\ln(\frac{b}{a})$$



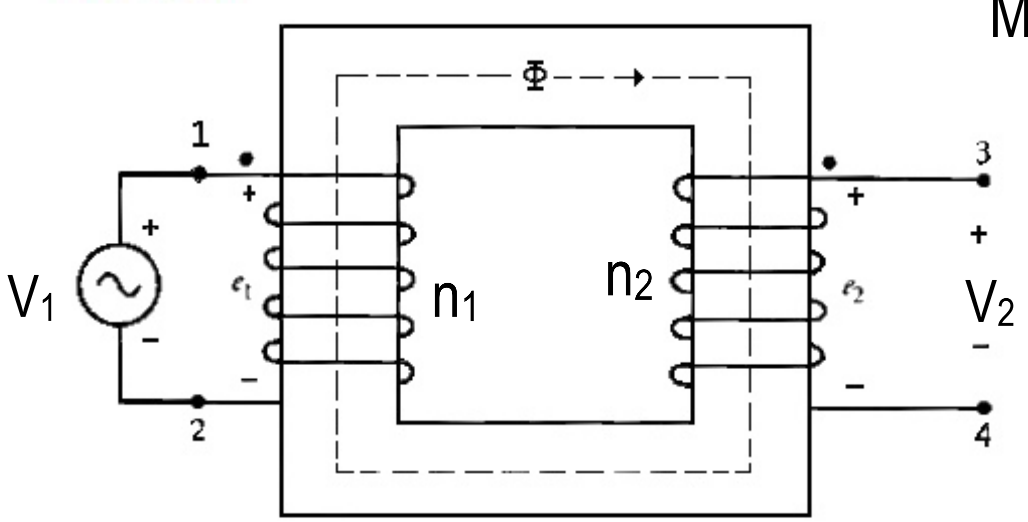
Mutual inductance

Flux through one element as function of current in another

Definition of **mutual-inductance** *M*: $\Phi_2 = MI_1$

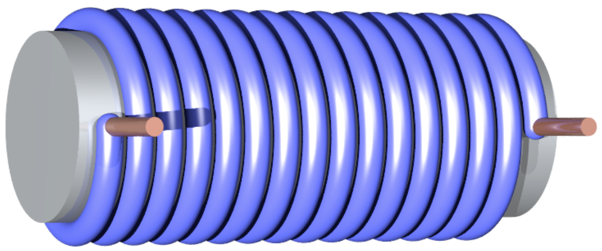
Voltage induced in 2 due to changing current in 1: $\Delta V_2 = -M\frac{dI_1}{dt}$

Main application: **Transformer**



$$\frac{V_2}{V_1} = -\frac{n_2}{n_1} \quad \text{and} \quad \frac{I_2}{I_1} = \frac{n_1}{n_2}$$

9. Magnetic materials



Inductance of coil is changed in presence of core:

$$\frac{L_m}{L_0} = \mu_r$$ Relative permeability μ_r

All equations for the B-field change by μ_r if material is present!

$$\frac{B_m}{B_0} = \mu_r$$

<u>Magnetisation of materials</u>

Magnetic dipole moment/volume $d\vec{m} = \vec{M} d\tau$ $\vec{M}$: Magnetisation

<u>The H-field</u>

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \quad \text{and} \quad \vec{B} = \mu_0(\vec{H} + \vec{M})$$

Magnetic susceptibility: $\vec{M} = \chi_m \vec{H}$ $\mu_r = (1 + \chi_m),\ \vec{B} = \mu_0 \mu_r \vec{H}$

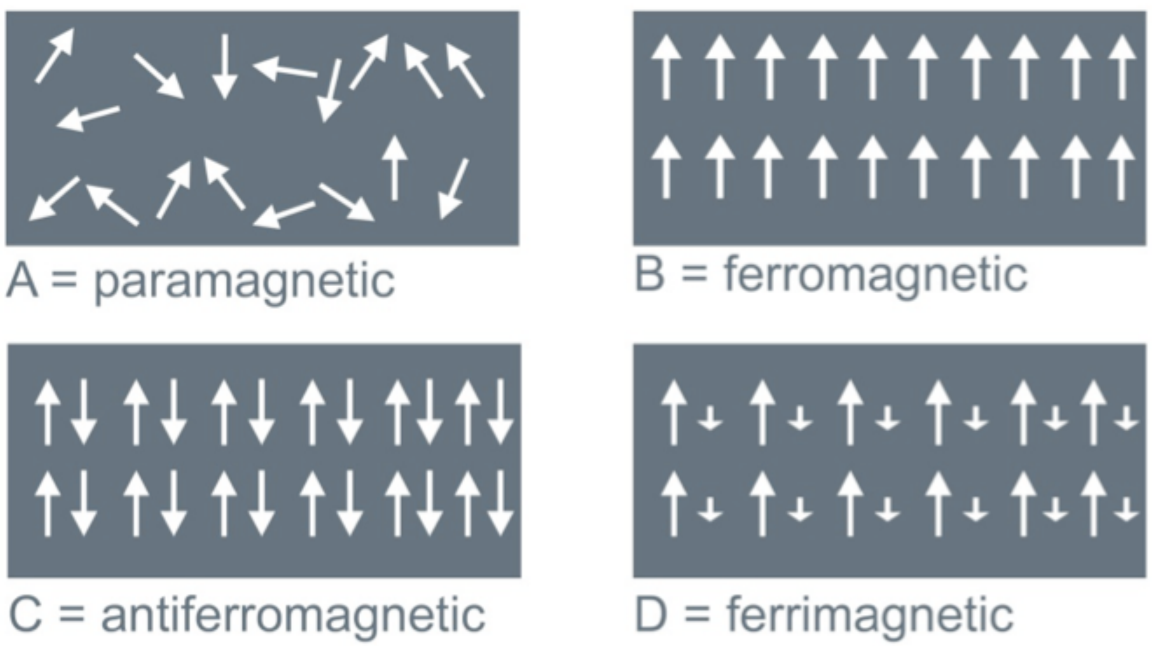
<u>At interfaces</u>

$$H_{\|2} = H_{\|1} \Rightarrow B_{\|1} = \frac{\mu_{r1}}{\mu_{r2}} B_{\|2} \qquad B_{\perp 1} = B_{\perp 2}$$

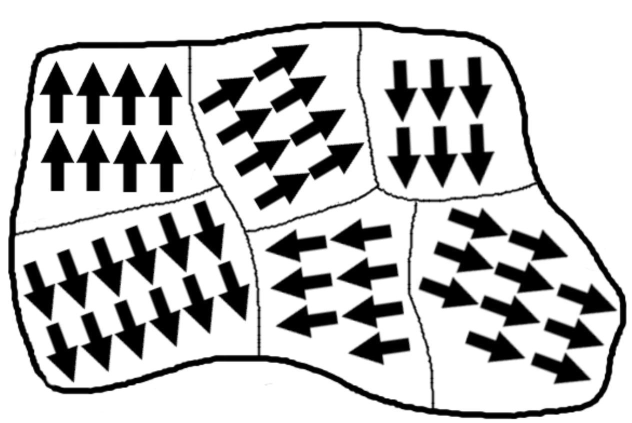
Magnetic screening

Magnetic response of different types of materials

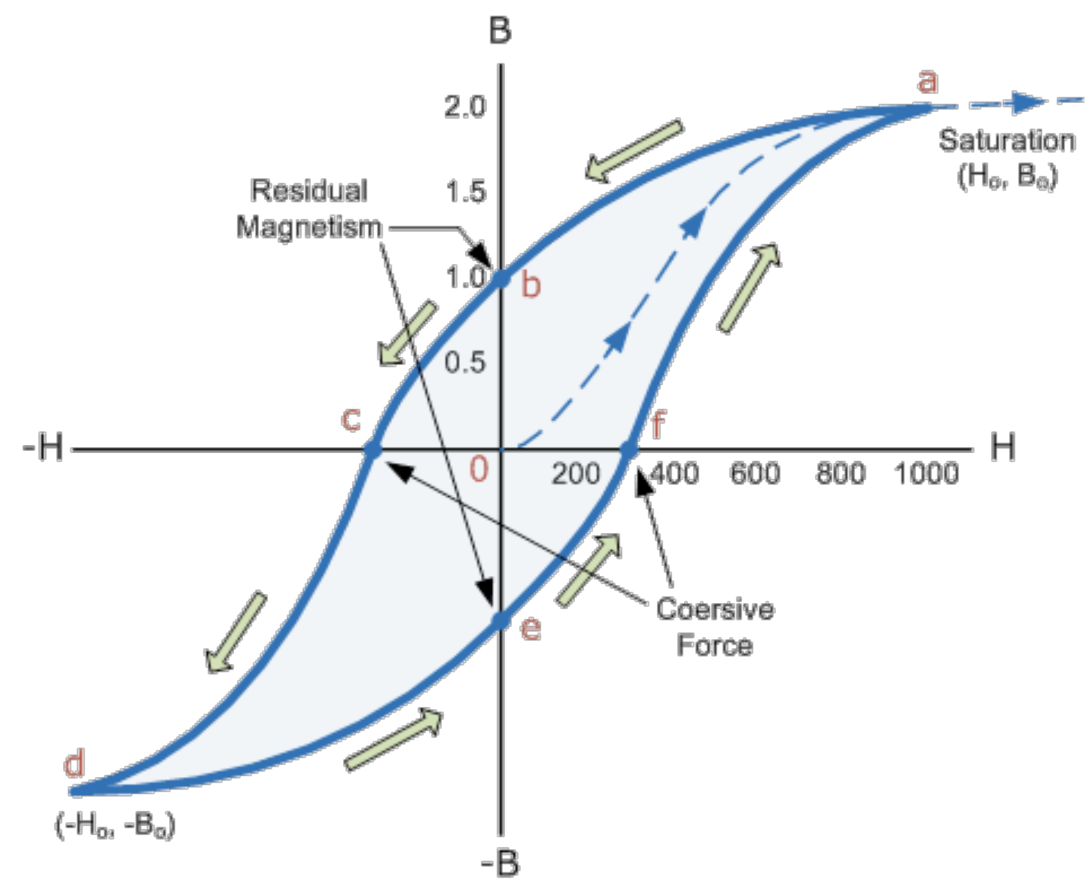
1. **Diamagnetic**: small negative χ_m, independent of H and temperature (T)
   Mechanism: small circular currents create opposing field.
2. **Paramagnetic**: small positive χ_m, independent of H, decreasing with increasing T
   Mechanism: unpaired spin, but not ordered, starts ordering in external field
3. **Ferromagnetic**: metallic, large positive χ_m, strong dependence on H and history (hysteresis), become paramagnetic above critical T (Curie temperature)
   Mechanism: ordered unpaired spin, also magnetisation without external field



Domains in ferromagnet



Hysteresis cycle in ferromagnets



Work done in one cycle: area of hysteresis curve

$$W = \oint B dH = \oint H dB$$

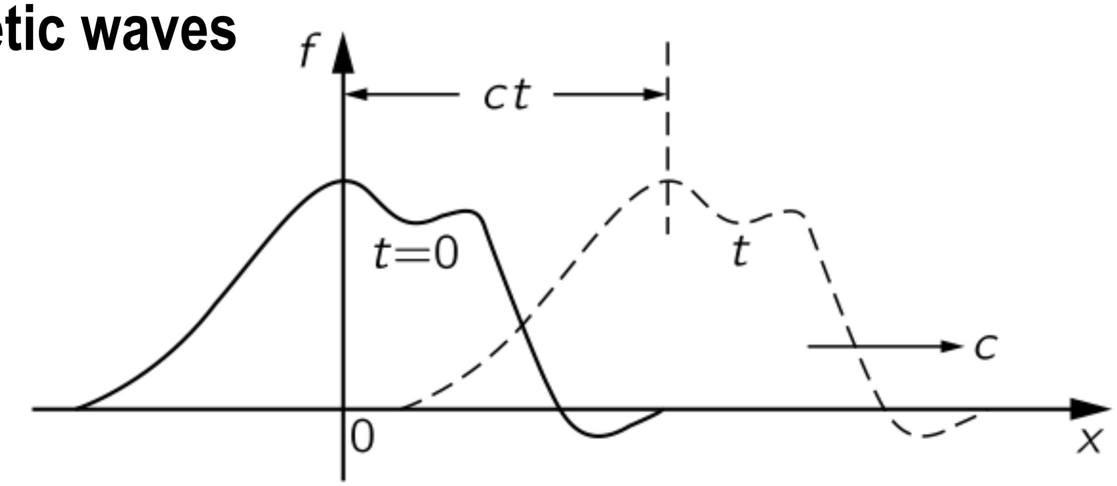
10. Electromagnetic waves

Wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

general solution:

$$u(x,t) = f(x - ct) + g(x + ct)$$



signal moving with velocity *c* along x-axis

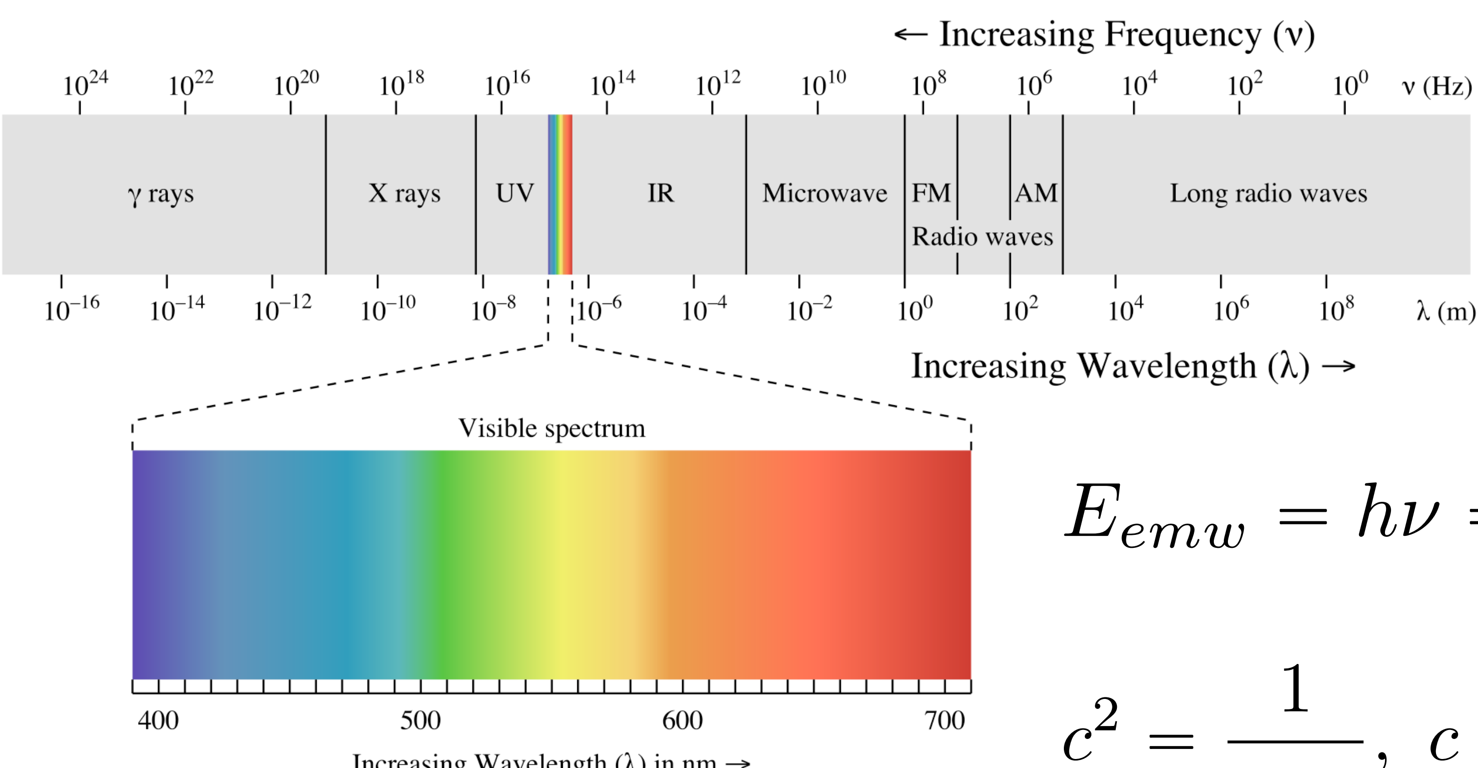
Extra term to Maxwell equations:

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Wave equations for E- and B-field

$$\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \nabla^2 \vec{E}$$

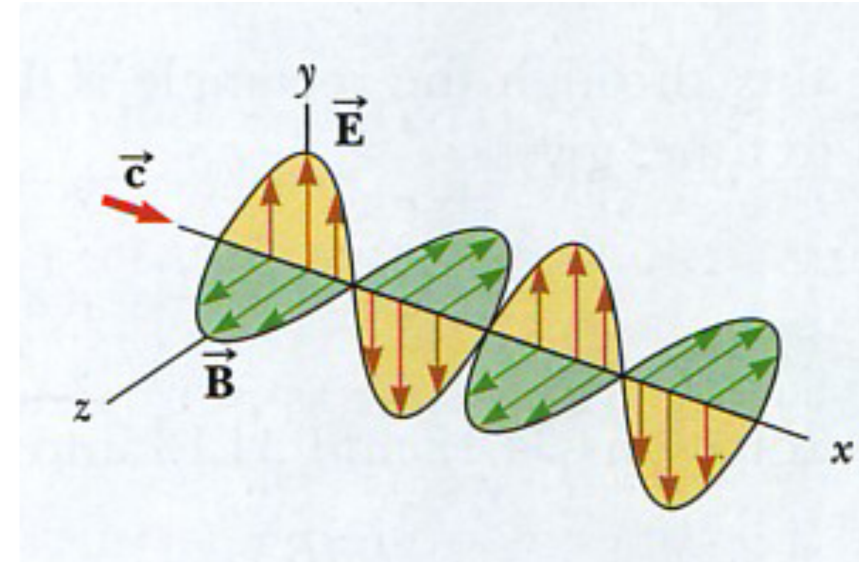
$$\frac{\partial^2 \vec{B}}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \nabla^2 \vec{B}$$



$$E_{emw} = h\nu = \frac{hc}{\lambda}$$

$$c^2 = \frac{1}{\epsilon_0 \mu_0}, \quad c \approx 3 \times 10^8 \ \mathrm{m/s}$$ (speed of light)

Properties of monochromatic plane waves



Relative orientation of E and B: $\vec{E} \perp \vec{B}$

Propagation direction (Poynting vector) along: $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$

Relative magnitude of E and B: $E_y = cB_z$ and in phase

Polarization: oscillation direction of E-field

E-M waves in a non-conducting medium

velocity in medium: $$c_m = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}} = \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

refractive index: $$n = \frac{c}{c_m} \approx \sqrt{\epsilon_r}$$

ϵ_r can be anisotropic and is function of wavelength

Generation of electromagnetic waves

accelerating charges!!

Accelerating charge: $$E_x(t) = \frac{q}{4\pi\epsilon_0 c^2 r} a_x(t - \frac{r}{c})$$

Oscillating dipole (antenna): $$E_{rad}(r,t) = \frac{\ddot{p}(t - \frac{r}{c})\sin\theta}{4\pi\epsilon_0 r c^2}$$

Maximum power: length = $\lambda/2$

Maxwell's equations for fields in vacuo

	Integral form	Differential form
Circuital law (Faraday)	$\oint_L \vec{E}\cdot d\vec{L} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Gauss's law:	$\oiint_S \vec{E}\cdot d\vec{S} = \frac{q}{\epsilon_0}$	$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
Ampère's circuital law including E-field flux:	$\oint_L \vec{B}\cdot d\vec{L} = \mu_0 I_c + \mu_0\epsilon_0 \frac{d}{dt}\iint_S \vec{E}\cdot d\vec{S}$	$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t}$
Gauss's law for B-fields:	$\oiint_S \vec{B}\cdot d\vec{S} = 0$	$\nabla \cdot \vec{B} = 0$

In presence of media: use circuital laws for H and E fields, use Gauss's law for B and D fields