

Maths - 3.5_Visual demonstration rotation

demonstrate this thing using visuals $(\cos(\theta + \pi/2), \sin(\theta + \pi/2)) = (-\sin \theta, \cos \theta)$,
 $\cos(\theta+2\pi),\sin(\theta+2\pi))=(-\sin\theta,\cos\theta)$

Here is a **clear visual demonstration** of the identity:

$$(\cos(\theta + \frac{\pi}{2}), \sin(\theta + \frac{\pi}{2})) = (-\sin \theta, \cos \theta)$$

$$(\cos(\theta+2\pi),\sin(\theta+2\pi))=(-\sin\theta,\cos\theta)$$

We'll see it **geometrically on the unit circle**.

1. Visual: Point Rotated by 90° on the Unit Circle

Below you see the original point at angle θ , and another point rotated by **+90° ($\pi/2$)** from it.

2. Key Idea

A **rotation by +90° ($\pi/2$)** sends the vector:

$$(\cos \theta, \sin \theta)$$

$$(\cos\theta,\sin\theta)$$

to:

$$(-\sin \theta, \cos \theta)$$

$$(-\sin\theta,\cos\theta)$$

This corresponds exactly to the rotation matrix:

$$R_{\pi/2} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$R\pi/2=(01-10)$$

Apply it:

$$R_{\pi/2} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} 0 \cdot \cos \theta - 1 \cdot \sin \theta \\ 1 \cdot \cos \theta + 0 \cdot \sin \theta \end{pmatrix} = \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix}$$

$$R\pi/2(\cos\theta\sin\theta)=(0\cdot\cos\theta-1\cdot\sin\theta1\cdot\cos\theta+0\cdot\sin\theta)=(-\sin\theta\cos\theta)$$

This matches the trigonometric identity.

3. Visual Explanation (Step-by-Step)

Step 1 — Original angle θ

The point on the unit circle is:

$$P_\theta = (\cos \theta, \sin \theta)$$

$$P\theta = (\cos\theta, \sin\theta)$$

Step 2 — Rotate by +90°

Rotating 90° CCW moves the point to the position shown in the diagrams above.

Step 3 — New coordinates

The rotated point has coordinates:

$$P_{\theta + \pi/2} = (\cos(\theta + \frac{\pi}{2}), \sin(\theta + \frac{\pi}{2}))$$

$$P\theta + \pi/2 = (\cos(\theta + 2\pi), \sin(\theta + 2\pi))$$

Step 4 — Using angle-shift identities

$$\cos(\theta + \frac{\pi}{2}) = -\sin \theta$$

$$\cos(\theta + 2\pi) = -\sin \theta$$

$$\sin(\theta + \frac{\pi}{2}) = \cos \theta$$

$$\sin(\theta + 2\pi) = \cos \theta$$

Therefore:

$$P_{\theta + \pi/2} = (-\sin \theta, \cos \theta)$$

$$P\theta + \pi/2 = (-\sin\theta, \cos\theta)$$

4. Visual Intuition in Plain Words

- Moving a point by **90° anticlockwise** swaps the x–y values.
- The x-coordinate becomes negative because the point jumps into the **left** half-plane.
- The y-coordinate becomes the old x-value.

So the transformation:

→ **rotate vector by 90°**

→ **swap components**

→ **negate the first one**

Exactly gives:

$$(\cos(\theta + \frac{\pi}{2}), \sin(\theta + \frac{\pi}{2})) = (-\sin \theta, \cos \theta)$$

$$(\cos(\theta+2\pi), \sin(\theta+2\pi)) = (-\sin\theta, \cos\theta)$$

If you want, I can also show:

- animations
- vector rotation matrices for other angles
- 3D geometric interpretation

Just tell me!

animation

 Generated image

 Generated image

is above concept is demonstrating the same above thing or different from below things
0 to 90° → both projections positive → $\sin+$, $\cos+$
 90 to 180° → x negative, y positive → $\cos-$, $\sin+$
 180 to 270° → both negative → $\sin-$, $\cos-$
 270 to 360° → x positive, y negative → $\cos+$, $\sin-$

You're asking whether the **quadrant-sign rules**:

- 0° – 90° → ($\cos +$, $\sin +$)

- $90^\circ - 180^\circ \rightarrow (\cos -, \sin +)$
- $180^\circ - 270^\circ \rightarrow (\cos -, \sin -)$
- $270^\circ - 360^\circ \rightarrow (\cos +, \sin -)$

are **the same concept** as the identity:

$$(\cos(\theta + \frac{\pi}{2}), \sin(\theta + \frac{\pi}{2})) = (-\sin\theta, \cos\theta)$$

$$(\cos(\theta+2\pi), \sin(\theta+2\pi)) = (-\sin\theta, \cos\theta)$$

Short Answer:

They are *related*, but **not the same**.

Relationship:

- The **quadrant rule** tells you **the sign of sin and cos** depending on the angle's location in the coordinate plane.
- The **rotation identity** tells you **how a vector transforms** when rotated by 90° .

But they BOTH come from **the geometry of the unit circle**.

How They Connect (with visuals)

Quadrant signs

These diagrams show how **cos = x-coordinate** and **sin = y-coordinate** change sign depending on where the angle lies.

Example:

In Quadrant II ($90^\circ - 180^\circ$) $\rightarrow x < 0, y > 0 \rightarrow \cos -, \sin +$

Rotation identity

The second kind of diagram shows that rotating a point by $+90^\circ$ gives:

- $\text{new_x} = \text{old_y}$ *with sign change*
- $\text{new_y} = \text{old_x}$

This is a **transformation rule**, not a sign rule.

● Are they demonstrating the same thing?

✓ Quadrant signs → tell you *where* the point lies and what sign sin/cos have.

✓ Rotation identity → tells you *how* the coordinates change after a 90° rotation.

✗ They are NOT identical concepts.

✓ But they are deeply connected through the unit circle geometry.

● How these two concepts align

Take the identity:

$$(\cos(\theta + \frac{\pi}{2}), \sin(\theta + \frac{\pi}{2})) = (-\sin\theta, \cos\theta)$$

$$(\cos(\theta+2\pi), \sin(\theta+2\pi)) = (-\sin\theta, \cos\theta)$$

Suppose θ is in Quadrant I → sin+, cos+.

After adding 90° → $\theta + 90^\circ$ is in Quadrant II → cos-, sin+.

Using the formula:

- new x = $-\sin(\theta)$ → negative
- new y = $\cos(\theta)$ → positive

These MATCH Quadrant II sign rules.

So the identity **predicts the correct quadrant signs automatically**.

● Final Summary

Concept	Purpose	Same or Different?
Rotation identity	mathematically rotates a vector by 90°	different idea
Quadrant sign rule	tells sign of sin/cos in each quadrant	different idea
Connection	both arise from the geometry of unit circle	related

If you want, I can create:

- ✓ animation of the rotation
- ✓ animation showing quadrant signs while θ moves
- ✓ combined animation showing both concepts working together

Just tell me which one you want!

what if we add pi in theta

When you **add π to θ** , you rotate the point on the unit circle by **180°** .

Let's analyze the transformation **visually and mathematically**.

1. The Identity for Adding π

$$(\cos(\theta + \pi), \sin(\theta + \pi))$$

$$(\cos(\theta + \pi), \sin(\theta + \pi))$$

Using angle-shift formulas:

$$\cos(\theta + \pi) = -\cos\theta$$

$$\cos(\theta + \pi) = -\cos\theta$$

$$\sin(\theta + \pi) = -\sin\theta$$

$$\sin(\theta + \pi) = -\sin\theta$$

Final result:

$$(\cos(\theta + \pi), \sin(\theta + \pi)) = (-\cos\theta, -\sin\theta)$$

$$(\cos(\theta + \pi), \sin(\theta + \pi)) = (-\cos\theta, -\sin\theta)$$

2. Visual Meaning — 180° Rotation

Here are visuals to help you see this transformation directly:

Interpretation:

- Adding **π (180°)** moves a point to the **diametrically opposite point** on the circle.
- The new point has the **same absolute values** but **opposite signs** for both x and y.

That is exactly:

$$(x, y) \rightarrow (-x, -y)$$

$$(x, y) \rightarrow (-x, -y)$$

3. Connection to Quadrant Signs

This transformation **switches the quadrant**:

Original Quadrant θ $\theta + \pi$ (180° later)

Q1 (+,+) $(\cos+, \sin+)$ Q3 (−,−)

Original Quadrant	θ	$\theta + \pi$ (180° later)
Q2 (-,+)	(cos-, sin+)	Q4 (+,-)
Q3 (-,-)	(cos-, sin-)	Q1 (+,+)
Q4 (+,-)	(cos+, sin-)	Q2 (-,+)

So adding π **flips both signs**.

4. Matrix Form (Optional but powerful)

Rotation by π has matrix:

$$R_\pi = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$R\pi = (-1 \ 0 \ 0 \ -1)$$

Apply it:

$$R_\pi \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} -\cos \theta \\ -\sin \theta \end{pmatrix}$$

$$R\pi(\cos \theta \sin \theta) = (-\cos \theta \ -\sin \theta)$$

Exactly the same result.

5. Intuition Summary

Adding π to θ :

- Rotates the point **half the circle**.
- Takes the point to the **opposite side**.
- Flips the **sign of both cosine and sine**.

So:

Adding $\pi \rightarrow$ Reflects the point across the origin

Adding $\pi \rightarrow$ Reflects the point across the origin

If you want, I can also show:

- animation of rotation by π
- comparison of adding $\pi/2$ vs. π vs. 2π

Just tell me!

and what if theta + 2pi/3

2/2

When you add $\frac{2\pi}{3}$ to θ , you are rotating the point on the unit circle by **120° counter-clockwise**.

This rotation is not as simple as adding $\pi/2$ (90°) or π (180°), but we can **express it exactly** using identities, geometry, and visuals.

Let's break it down clearly.

1. Compute the New Coordinates

We want:

$$(\cos(\theta + \frac{2\pi}{3}), \sin(\theta + \frac{2\pi}{3}))$$

$$(\cos(\theta+32\pi), \sin(\theta+32\pi))$$

Use cosine and sine angle-addition formulas:

Cosine

$$\cos(\theta + \frac{2\pi}{3}) = \cos\theta \cos \frac{2\pi}{3} - \sin\theta \sin \frac{2\pi}{3}$$

$$\cos(\theta+32\pi) = \cos\theta \cos 32\pi - \sin\theta \sin 32\pi$$

Insert exact values:

$$\cos \frac{2\pi}{3} = -\frac{1}{2}, \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos 32\pi = -21, \sin 32\pi = 23$$

Therefore:

$$\cos(\theta + \frac{2\pi}{3}) = -\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$$

$$\cos(\theta+32\pi) = -21\cos\theta - 23$$

$$\sin\theta$$

Sine

$$\sin(\theta + \frac{2\pi}{3}) = \sin\theta \cos \frac{2\pi}{3} + \cos\theta \sin \frac{2\pi}{3}$$

$$\sin(\theta+32\pi) = \sin\theta \cos 32\pi + \cos\theta \sin 32\pi$$

Insert values:

$$= -\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$$

$$= -21\sin\theta + 23$$

$$\cos\theta$$

Final transformation for adding $\frac{2\pi}{3}$ 32π :

$$(\cos(\theta + \frac{2\pi}{3}), \sin(\theta + \frac{2\pi}{3})) = (-\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta, \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta)$$

$$(\cos(\theta+32\pi), \sin(\theta+32\pi)) = (-21\cos\theta - 23)$$

$$\sqrt{\sin\theta, 23}$$

$$\sqrt{\cos\theta - 21\sin\theta})$$

This is the coordinate of the point rotated by **120° counterclockwise**.

2. Visual of 120° Rotation

A **120° rotation**:

- moves a point to a new location forming a **120° sector**,
- produces a vector that is a specific linear combination of $\cos\theta\cos\theta$ and $\sin\theta\sin\theta$.

3. Special Case Check ($\theta = 0$)

Let's verify the formula is correct.

If $\theta = 0$:

$$(\cos 0, \sin 0) = (1, 0)$$

$$(\cos 0, \sin 0) = (1, 0)$$

Applying the derived formula:

Cosine:

$$-\frac{1}{2} \cdot 1 - \frac{\sqrt{3}}{2} \cdot 0 = -\frac{1}{2}$$

$$-21 \cdot 1 - 23$$

$$\sqrt{\cdot 0 = -21}$$

Sine:

$$\frac{\sqrt{3}}{2} \cdot 1 - \frac{1}{2} \cdot 0 = \frac{\sqrt{3}}{2}$$

$$23$$

$$\sqrt{\cdot 1 - 21 \cdot 0 = 23}$$

These are exactly:

$$(\cos \frac{2\pi}{3}, \sin \frac{2\pi}{3}) = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$$

$$(\cos 32\pi, \sin 32\pi) = (-21, 23)$$

✓
)
Correct ✓

4. Matrix Form (Elegant Perspective)

Rotation by angle $\alpha = \frac{2\pi}{3}$ uses:

$$R_\alpha = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$

$$R\alpha = (\cos \alpha \sin \alpha - \sin \alpha \cos \alpha)$$

Insert values:

$$R_{\frac{2\pi}{3}} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

$$R32\pi = (-21, 23)$$

✓
-23
✓
-21)

Then:

$$R_{\frac{2\pi}{3}} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}\cos \theta - \frac{\sqrt{3}}{2}\sin \theta \\ \frac{\sqrt{3}}{2}\cos \theta - \frac{1}{2}\sin \theta \end{pmatrix}$$

$$R32\pi (\cos \theta \sin \theta) = (-21 \cos \theta - 23 \sin \theta)$$

✓
sin 023
✓
cos 0 - 21 sin 0)

Same as our formula.

5. Summary (Easy to Remember)

Angle Added	Meaning	Transformation
$\pi/2$	90° rotation	swap + sign change
π	180° rotation	flip both signs
$2\pi/3$	120° rotation	linear combination with coefficients $-\frac{1}{2}, -\frac{\sqrt{3}}{2}, -\frac{1}{2}, -\frac{\sqrt{3}}{2}$

The 120° - 120° rotation is more complex because $\cos(120^\circ)$ and $\sin(120^\circ)$ are not 0 or ± 1 .

If you want, I can also generate:

- animation of rotation by $2\pi/3$
- step-by-step geometric construction of 120° rotation
- comparison of $\pi/2$, π , $2\pi/3$ rotations

Just tell me!

$$\theta + \frac{3\pi}{2}$$

When you add $\frac{3\pi}{2}$ to θ , you are rotating the point on the unit circle by **270° counter-clockwise** (or equivalently **-90°**).

This gives a clean and beautiful transformation similar to the $\pi/2$ case.

1. Identity for Adding $3\pi/2$

We want:

$$(\cos(\theta + \frac{3\pi}{2}), \sin(\theta + \frac{3\pi}{2}))$$
$$(\cos(\theta+23\pi), \sin(\theta+23\pi))$$

Use known values:

$$\cos \frac{3\pi}{2} = 0, \sin \frac{3\pi}{2} = -1$$

$$\cos 23\pi = 0, \sin 23\pi = -1$$

Apply angle-sum identities:

Cosine term

$$\begin{aligned}\cos(\theta + \frac{3\pi}{2}) &= \cos \theta \cdot 0 - \sin \theta \cdot (-1) \\ \cos(\theta+23\pi) &= \cos \theta \cdot 0 - \sin \theta \cdot (-1) \\ &= \sin \theta \\ &= \sin \theta\end{aligned}$$

Sine term

$$\begin{aligned}\sin(\theta + \frac{3\pi}{2}) &= \sin \theta \cdot 0 + \cos \theta \cdot (-1) \\ \sin(\theta+23\pi) &= \sin \theta \cdot 0 + \cos \theta \cdot (-1) \\ &= -\cos \theta \\ &= -\cos \theta\end{aligned}$$

Final transformation:

$$\begin{aligned}(\cos(\theta + \frac{3\pi}{2}), \sin(\theta + \frac{3\pi}{2})) &= (\sin \theta, -\cos \theta) \\ (\cos(\theta+23\pi), \sin(\theta+23\pi)) &= (\sin \theta, -\cos \theta)\end{aligned}$$

2. Visual Meaning — 270° Rotation

A 270° CCW rotation is the same as a 90° clockwise rotation.

Compare with 90° CCW ($\pi/2$):

$$\theta + \frac{\pi}{2} \rightarrow (-\sin \theta, \cos \theta) (-\sin \theta, \cos \theta)$$

Now for 270° CCW:

$$\theta + \frac{3\pi}{2} \rightarrow (\sin \theta, -\cos \theta) (\sin \theta, -\cos \theta)$$

You can see the pattern:

Rotation	New Point
+90° ($\pi/2$)	($-\sin \theta, \cos \theta$) ($-\sin \theta, \cos \theta$)
+180° (π)	($-\cos \theta, -\sin \theta$) ($-\cos \theta, -\sin \theta$)
+270° ($3\pi/2$)	($\sin \theta, -\cos \theta$) ($\sin \theta, -\cos \theta$)

This cycles neatly through quadrants.

3. Quadrant Behavior

Because 270° rotation moves a point **three quadrants ahead**, the signs change like this:

- x becomes **old y**
- y becomes **negative old x**

Exactly matching:

$$(\sin \theta, -\cos \theta)
(\sin \theta, -\cos \theta)$$

4. Matrix Form (for completeness)

Rotation by $3\pi/2$ uses:

$$R_{\frac{3\pi}{2}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

R $23\pi/2$ =(0-110)

Applying:

$$R_{\frac{3\pi}{2}} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} \sin \theta \\ -\cos \theta \end{pmatrix}$$

$$R23\pi(\cos\theta\sin\theta)=(\sin\theta-\cos\theta)$$

Matches our identity perfectly.

Final Summary

$\theta + \frac{3\pi}{2}$ rotates the point by 270° CCW → gives $(\sin \theta, -\cos \theta)$

$\theta+23\pi$ rotates the point by 270° CCW → gives $(\sin\theta, -\cos\theta)$

This is the **third step** in the rotation cycle:

$$(\cos\theta, \sin\theta) \rightarrow (-\sin\theta, \cos\theta) \rightarrow (-\cos\theta, -\sin\theta) \rightarrow (\sin\theta, -\cos\theta)$$

After this, adding another $\frac{\pi}{2}2\pi$ returns you to the start.

If you want, I can create:

- ◆ animation of rotation by $3\pi/2$
- ◆ a comparison of all four rotations ($\pi/2, \pi, 3\pi/2, 2\pi$)
- ◆ a full table of all trig angle-shift identities

Just tell me!