

Maths - 3.5_Visual demonstration rotation

demonstrate this thing using visuals $(\cos(\theta + \pi/2), \sin(\theta + \pi/2)) = (-\sin \theta, \cos \theta)$

Here is a **clear visual demonstration** of the identity:

$$\left(\cos \left(\theta + \frac{\pi}{2} \right), \sin \left(\theta + \frac{\pi}{2} \right) \right) = \left(-\sin \theta, \cos \theta \right)$$

$$(\cos(\theta+2\pi), \sin(\theta+2\pi)) = (-\sin\theta, \cos\theta)$$

We'll see it **geometrically on the unit circle**.

1. Visual: Point Rotated by 90° on the Unit Circle

Below you see the original point at angle θ , and another point rotated by **+90° ($\pi/2$)** from it.

2. Key Idea

A **rotation by +90° ($\pi/2$)** sends the vector:

$$(\cos \theta, \sin \theta)$$

$$(\cos\theta, \sin\theta)$$

to:

$$(-\sin \theta, \cos \theta)$$

$$(-\sin\theta, \cos\theta)$$

This corresponds exactly to the rotation matrix:

$$R_{\pi/2} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

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Apply it:

$$R_{\pi/2} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} 0 \cdot \cos \theta - 1 \cdot \sin \theta \\ 1 \cdot \cos \theta + 0 \cdot \sin \theta \end{pmatrix} = \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix}$$

$$R_{\pi/2}(\cos\theta, \sin\theta) = (0 \cdot \cos\theta - 1 \cdot \sin\theta, 1 \cdot \cos\theta + 0 \cdot \sin\theta) = (-\sin\theta, \cos\theta)$$

This matches the trigonometric identity.

3. Visual Explanation (Step-by-Step)

Step 1 — Original angle θ

The point on the unit circle is:

$$P_{\theta} = (\cos \theta , \sin \theta)$$

$$P_{\theta} = (\cos\theta, \sin\theta)$$

Step 2 — Rotate by $+90^\circ$

Rotating 90° CCW moves the point to the position shown in the diagrams above.

Step 3 — New coordinates

The rotated point has coordinates:

$$P_{\theta + \pi/2} = (\cos (\theta + \frac{\pi}{2}) , \sin (\theta + \frac{\pi}{2}))$$

$$P_{\theta + \pi/2} = (\cos(\theta + 2\pi), \sin(\theta + 2\pi))$$

Step 4 — Using angle-shift identities

$$\cos (\theta + \frac{\pi}{2}) = - \sin \theta$$

$$\cos(\theta + 2\pi) = -\sin\theta$$

$$\sin (\theta + \frac{\pi}{2}) = \cos \theta$$

$$\sin(\theta + 2\pi) = \cos\theta$$

Therefore:

$$P_{\theta + \pi/2} = (- \sin \theta , \cos \theta)$$

$$P_{\theta + \pi/2} = (-\sin\theta, \cos\theta)$$

4. Visual Intuition in Plain Words

- Moving a point by **90° anticlockwise** swaps the x–y values.
- The x-coordinate becomes negative because the point jumps into the **left** half-plane.
- The y-coordinate becomes the old x-value.

So the transformation:

→ **rotate vector by 90°**

→ **swap components**

→ **negate the first one**

Exactly gives:

$$(\cos (\theta + \frac{\pi}{2}) , \sin (\theta + \frac{\pi}{2})) = (- \sin \theta , \cos \theta)$$

$$(\cos(\theta+2\pi), \sin(\theta+2\pi)) = (-\sin\theta, \cos\theta)$$

If you want, I can also show:

- ✓ animations
- ✓ vector rotation matrices for other angles
- ✓ 3D geometric interpretation

Just tell me!

animation

 Generated image

Generated image

 Generated image

is above concept is demonstrating the same above thing or different from below things
0 to $90^\circ \rightarrow$ both projections positive $\rightarrow \sin +, \cos +$
 90 to $180^\circ \rightarrow$ x negative, y positive $\rightarrow \cos -, \sin +$
 180 to $270^\circ \rightarrow$ both negative $\rightarrow \sin -, \cos -$
 270 to $360^\circ \rightarrow$ x positive, y negative $\rightarrow \cos +, \sin -$

You're asking whether the **quadrant–sign rules**:

- $0^\circ - 90^\circ \rightarrow (\cos +, \sin +)$

- $90^\circ\text{--}180^\circ \rightarrow (\cos -, \sin +)$
- $180^\circ\text{--}270^\circ \rightarrow (\cos -, \sin -)$
- $270^\circ\text{--}360^\circ \rightarrow (\cos +, \sin -)$

are **the same concept** as the identity:

$$\left(\cos \left(\theta + \frac{\pi}{2} \right), \sin \left(\theta + \frac{\pi}{2} \right) \right) = \left(-\sin \theta, \cos \theta \right)$$

$$(\cos(\theta+2\pi), \sin(\theta+2\pi)) = (-\sin\theta, \cos\theta)$$

✓ Short Answer:

They are *related*, but **not the same**.

✓ Relationship:

- The **quadrant rule** tells you **the sign of sin and cos** depending on the angle's location in the coordinate plane.
- The **rotation identity** tells you **how a vector transforms** when rotated by 90° .

But they BOTH come from **the geometry of the unit circle**.

● How They Connect (with visuals)

1 Quadrant signs

These diagrams show how **cos = x-coordinate** and **sin = y-coordinate** change sign depending on where the angle lies.

Example:

In Quadrant II ($90^\circ\text{--}180^\circ$) $\rightarrow x < 0, y > 0 \rightarrow \cos -, \sin +$

2 Rotation identity

The second kind of diagram shows that rotating a point by **+90°** gives:

- $\text{new_x} = \text{old_y}$ *with sign change*
- $\text{new_y} = \text{old_x}$

This is a **transformation rule**, not a sign rule.

● Are they demonstrating the same thing?

✓ **Quadrant signs** → tell you *where* the point lies and what sign sin/cos have.

✓ **Rotation identity** → tells you *how* the coordinates change after a 90° rotation.

✗ They are NOT identical concepts.

✓ But they are deeply connected through the unit circle geometry.

● How these two concepts align

Take the identity:

$$\left(\cos \left(\theta + \frac{\pi}{2} \right), \sin \left(\theta + \frac{\pi}{2} \right) \right) = \left(-\sin \theta, \cos \theta \right)$$

$$(\cos(\theta+2\pi), \sin(\theta+2\pi)) = (-\sin\theta, \cos\theta)$$

Suppose θ is in Quadrant I → sin+, cos+.

After adding 90° → $\theta + 90^\circ$ is in Quadrant II → cos-, sin+.

Using the formula:

- new x = $-\sin(\theta)$ → negative
- new y = $\cos(\theta)$ → positive

These MATCH Quadrant II sign rules.

So the identity **predicts the correct quadrant signs automatically**.

● Final Summary

Concept	Purpose	Same or Different?
Rotation identity	mathematically rotates a vector by 90°	different idea
Quadrant sign rule	tells sign of sin/cos in each quadrant	different idea
Connection	both arise from the geometry of unit circle	related

If you want, I can create:

- ✓ animation of the rotation
- ✓ animation showing quadrant signs while θ moves
- ✓ combined animation showing both concepts working together

Just tell me which one you want!

what if we add pi in theta

When you **add π to θ** , you rotate the point on the unit circle by **180°** .

Let's analyze the transformation **visually and mathematically**.

1. The Identity for Adding π

$$(\cos(\theta + \pi), \sin(\theta + \pi))$$

$$(\cos(\theta + \pi), \sin(\theta + \pi))$$

Using angle-shift formulas:

$$\cos(\theta + \pi) = -\cos \theta$$

$$\cos(\theta + \pi) = -\cos \theta$$

$$\sin(\theta + \pi) = -\sin \theta$$

$$\sin(\theta + \pi) = -\sin \theta$$

✓ **Final result:**

$$(\cos(\theta + \pi), \sin(\theta + \pi)) = (-\cos \theta, -\sin \theta)$$

$$(\cos(\theta + \pi), \sin(\theta + \pi)) = (-\cos \theta, -\sin \theta)$$

2. Visual Meaning — 180° Rotation

Here are visuals to help you see this transformation directly:

Interpretation:

- Adding π (**180°**) moves a point to the **diametrically opposite point** on the circle.
- The new point has the **same absolute values** but **opposite signs** for both x and y.

That is exactly:

$$(x, y) \rightarrow (-x, -y)$$

$$(x, y) \rightarrow (-x, -y)$$

3. Connection to Quadrant Signs

This transformation **switches the quadrant**:

Original Quadrant	θ	$\theta + \pi$ (180° later)
Q1 (+, +)	$(\cos+, \sin+)$	Q3 (-, -)

Original Quadrant	θ	$\theta + \pi$ (180° later)
Q2 (-,+)	(cos-, sin+)	Q4 (+,-)
Q3 (-,-)	(cos-, sin-)	Q1 (+,+)
Q4 (+,-)	(cos+, sin-)	Q2 (-,+)

So adding π **flips both signs**.

4. Matrix Form (Optional but powerful)

Rotation by π has matrix:

$$R_{\pi} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$R_{\pi}(-100-1)$

Apply it:

$$R_{\pi} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} -\cos \theta \\ -\sin \theta \end{pmatrix}$$

$R_{\pi}(\cos \theta \sin \theta) = (-\cos \theta -\sin \theta)$

Exactly the same result.

5. Intuition Summary

Adding π to θ :

- Rotates the point **half the circle**.
- Takes the point to the **opposite side**.
- Flips the **sign of both cosine and sine**.

So:

Adding $\pi \rightarrow$ Reflects the point across the origin

Adding $\pi \rightarrow$ Reflects the point across the origin

If you want, I can also show:

✓ animation of rotation by π

✓ comparison of adding $\pi/2$ vs. π vs. 2π

Just tell me!

and what if $\theta + 2\pi/3$

2/2

When you add $\frac{2\pi}{3}$ to θ , you are rotating the point on the unit circle by **120° counter-clockwise**.

This rotation is not as simple as adding $\frac{\pi}{2}$ (90°) or π (180°), but we can **express it exactly** using identities, geometry, and visuals.

Let's break it down clearly.

1. Compute the New Coordinates

We want:

$$\left(\cos \left(\theta + \frac{2\pi}{3} \right), \sin \left(\theta + \frac{2\pi}{3} \right) \right)$$

$$(\cos(\theta+32\pi), \sin(\theta+32\pi))$$

Use cosine and sine angle-addition formulas:

Cosine

$$\cos \left(\theta + \frac{2\pi}{3} \right) = \cos \theta \cos \frac{2\pi}{3} - \sin \theta \sin \frac{2\pi}{3}$$

$$\cos(\theta+32\pi) = \cos\theta \cos 32\pi - \sin\theta \sin 32\pi$$

Insert exact values:

$$\cos \frac{2\pi}{3} = -\frac{1}{2}, \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos 32\pi = 1, \sin 32\pi = 0$$

√

Therefore:

$$\cos \left(\theta + \frac{2\pi}{3} \right) = -\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$$

$$\cos(\theta+32\pi) = -\frac{1}{2} \cos\theta - \frac{\sqrt{3}}{2} \sin\theta$$

√

$$\sin\theta$$

Sine

$$\sin \left(\theta + \frac{2\pi}{3} \right) = \sin \theta \cos \frac{2\pi}{3} + \cos \theta \sin \frac{2\pi}{3}$$

$$\sin(\theta+32\pi) = \sin\theta \cos 32\pi + \cos\theta \sin 32\pi$$

Insert values:

$$= -\frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta$$

$$= -\frac{1}{2} \sin\theta + \frac{\sqrt{3}}{2} \cos\theta$$

√

$$\cos\theta$$

✓ Final transformation for adding $\frac{2\pi}{3}$ 32π :

$$\left(\cos \left(\theta + \frac{2\pi}{3} \right), \sin \left(\theta + \frac{2\pi}{3} \right) \right) = \left(-\frac{1}{2}\cos \theta - \frac{\sqrt{3}}{2}\sin \theta, \frac{\sqrt{3}}{2}\cos \theta - \frac{1}{2}\sin \theta \right)$$
$$(\cos(\theta+32\pi), \sin(\theta+32\pi)) = (-21\cos\theta - 23$$

$$\sqrt{\sin\theta, 23}$$

$$\sqrt{\cos\theta - 21\sin\theta}$$

This is the coordinate of the point rotated by **120° counterclockwise**.

● 2. Visual of 120° Rotation

A **120° rotation**:

- moves a point to a new location forming a **120° sector**,
- produces a vector that is a specific linear combination of $\cos \theta$ and $\sin \theta$.

● 3. Special Case Check ($\theta = 0$)

Let's verify the formula is correct.

If $\theta = 0$:

$$(\cos 0, \sin 0) = (1, 0)$$

$$(\cos 0, \sin 0) = (1, 0)$$

Applying the derived formula:

Cosine:

$$-\frac{1}{2} \cdot 1 - \frac{\sqrt{3}}{2} \cdot 0 = -\frac{1}{2}$$

$$-21 \cdot 1 - 23$$

$$\sqrt{\cdot 0 = -21}$$

Sine:

$$\frac{\sqrt{3}}{2} \cdot 1 - \frac{1}{2} \cdot 0 = \frac{\sqrt{3}}{2}$$

$$23$$

$$\sqrt{\cdot 1 - 21 \cdot 0 = 23}$$

$$\sqrt{\cdot}$$

These are exactly:

$$\left(\cos \frac{2\pi}{3}, \sin \frac{2\pi}{3} \right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$$

$$(\cos 32\pi, \sin 32\pi) = (-1, 0)$$

✓
)

Correct ✓

4. Matrix Form (Elegant Perspective)

Rotation by angle $\alpha = \frac{2\pi}{3}$ uses:

$$R_{\alpha} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$

$$R_{\alpha} = (\cos \alpha \sin \alpha - \sin \alpha \cos \alpha)$$

Insert values:

$$R_{\frac{2\pi}{3}} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

$$R_{32\pi} = (-1, 0)$$

✓
-23

✓
-21)

Then:

$$R_{\frac{2\pi}{3}} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}\cos \theta - \frac{\sqrt{3}}{2}\sin \theta \\ \frac{\sqrt{3}}{2}\cos \theta - \frac{1}{2}\sin \theta \end{pmatrix}$$

$$R_{32\pi}(\cos \theta \sin \theta) = (-1 \cos \theta - 0 \sin \theta)$$

✓
sin θ 23

✓
cos θ - 21 sin θ)

Same as our formula.

5. Summary (Easy to Remember)

Angle Added	Meaning	Transformation
$\pi/2$	90° rotation	swap + sign change
π	180° rotation	flip both signs
$2\pi/3$	120° rotation	linear combination with coefficients $-1/2, -\sqrt{3}/2$

The 120° rotation is more complex because **$\cos(120^\circ)$ and $\sin(120^\circ)$** are not 0 or ± 1 .

If you want, I can also generate:

- ✓ **animation** of rotation by $2\pi/3$
- ✓ **step-by-step geometric construction** of 120° rotation
- ✓ **comparison** of $\pi/2$, π , $2\pi/3$ rotations

Just tell me!

$\theta + 3\pi/2$

When you add $\frac{3\pi}{2}$ to θ , you are rotating the point on the unit circle by **270° counter-clockwise** (or equivalently **-90°**).

This gives a clean and beautiful transformation similar to the $\pi/2$ case.

1. Identity for Adding $3\pi/2$

We want:

$$\left(\cos \left(\theta + \frac{3\pi}{2} \right), \sin \left(\theta + \frac{3\pi}{2} \right) \right)$$

$$(\cos(\theta + 23\pi), \sin(\theta + 23\pi))$$

Use known values:

$$\cos \frac{3\pi}{2} = 0, \sin \frac{3\pi}{2} = -1$$

$$\cos 23\pi = 0, \sin 23\pi = -1$$

Apply angle-sum identities:

Cosine term

$$\cos \left(\theta + \frac{3\pi}{2} \right) = \cos \theta \cdot 0 - \sin \theta \cdot (-1)$$

$$\cos(\theta + 23\pi) = \cos \theta \cdot 0 - \sin \theta \cdot (-1)$$

$$= \sin \theta$$

$$= \sin \theta$$

Sine term

$$\sin \left(\theta + \frac{3\pi}{2} \right) = \sin \theta \cdot 0 + \cos \theta \cdot (-1)$$

$$\sin(\theta + 23\pi) = \sin \theta \cdot 0 + \cos \theta \cdot (-1)$$

$$= -\cos \theta$$

$$= -\cos \theta$$

✓ **Final transformation:**

$$\left(\cos \left(\theta + \frac{3\pi}{2} \right), \sin \left(\theta + \frac{3\pi}{2} \right) \right) = (\sin \theta, -\cos \theta)$$

$$(\cos(\theta + 23\pi), \sin(\theta + 23\pi)) = (\sin \theta, -\cos \theta)$$

2. Visual Meaning — 270° Rotation

A **270° CCW rotation** is the same as a **90° clockwise rotation**.

Compare with 90° CCW ($\pi/2$):

$$\theta + \frac{\pi}{2} \rightarrow (-\sin \theta, \cos \theta) \quad (-\sin \theta, \cos \theta)$$

Now for 270° CCW:

$$\theta + \frac{3\pi}{2} \rightarrow (\sin \theta, -\cos \theta) \quad (\sin \theta, -\cos \theta)$$

You can see the pattern:

Rotation	New Point
+90° ($\pi/2$)	$(-\sin \theta, \cos \theta) \quad (-\sin \theta, \cos \theta)$
+180° (π)	$(-\cos \theta, -\sin \theta) \quad (-\cos \theta, -\sin \theta)$
+270° ($3\pi/2$)	$(\sin \theta, -\cos \theta) \quad (\sin \theta, -\cos \theta)$

This cycles neatly through quadrants.

3. Quadrant Behavior

Because 270° rotation moves a point **three quadrants ahead**, the signs change like this:

- x becomes **old y**
- y becomes **negative old x**

Exactly matching:

$$(\sin \theta, -\cos \theta)$$

4. Matrix Form (for completeness)

Rotation by $3\pi/2$ uses:

$$R_{\frac{3\pi}{2}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$R_{23\pi} = (0-110)$

Applying:

$$R_{\frac{3\pi}{2}} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} \sin \theta \\ -\cos \theta \end{pmatrix}$$

$R_{23\pi}(\cos\theta, \sin\theta) = (\sin\theta, -\cos\theta)$
Matches our identity perfectly.

● Final Summary

$\theta + \frac{3\pi}{2}$ rotates the point by 270° CCW → gives $(\sin \theta, -\cos \theta)$

$\theta + 23\pi$ rotates the point by 270° CCW → gives $(\sin\theta, -\cos\theta)$

This is the **third step** in the rotation cycle:

$(\cos\theta, \sin\theta) \rightarrow (-\sin\theta, \cos\theta) \rightarrow (-\cos\theta, -\sin\theta) \rightarrow (\sin\theta, -\cos\theta)$

After this, adding another $\frac{\pi}{2}2\pi$ returns you to the start.

If you want, I can create:

- ♦ animation of rotation by $3\pi/2$
- ♦ a comparison of all four rotations ($\pi/2, \pi, 3\pi/2, 2\pi$)
- ♦ a full table of all trig angle-shift identities

Just tell me!