explain these topics one by one try to cover hidden aspects of them
Basic Probability
Counting principles, permutations, combinations
Sample spaces & events
Basic probability rules (union, intersection, complement)

Great — let's make **Basic Probability** *crystal clear* with intuition, diagrams-in-words, hidden details, and the mathematical structure that ties everything together.
I'll go topic-by-topic exactly in your order:

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1. Counting Principles (Fundamental of all Probability)

Counting tells you **how many outcomes are possible**. Probability is always:

$$P(\text{event}) = \frac{\#(\text{favorable outcomes})}{\#(\text{total outcomes})}$$

So we must master counting first.

1.1 Fundamental Counting Principle (FCP)

If an action has:

- **a** ways to happen for Step 1
- **b** ways for Step 2
- **c** ways for Step 3

Then total outcomes =

$$a \times b \times c$$

Hidden Aspect 1 — FCP works only when choices are independent

Example:
Choose **a shirt** (3 choices) and **pants** (4 choices) → 3×4.

But if pants choice depends on shirt color (e.g., you cannot pick red pants for red shirt), then FCP **cannot be applied blindly**.

1.2 Permutations (Order matters)

Permutations count the number of arrangements **when order matters**.

Formula

Arranging **n** items taken **r** at a time:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Example: 5 people in a line but only 3 positions.

$$P(5,3) = 5 \times 4 \times 3$$

Hidden Aspect 2 — Why permutations divide by $(n-r)!$?

Imagine arranging all 5 people → $5!$.
But you only care about the **first 3 positions**, so you ignore how the remaining 2 are ordered → divide by $2!$.

1.3 Combinations (Order does NOT matter)

Choosing **r people from n**, but order doesn't matter.

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

Hidden Aspect 3 — Why divide by $r!$?

If selecting {A, B, C}, the permutations:

ABC, ACB, BAC, BCA, CAB, CBA
are *all the same team*, so combinations remove duplicates.

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2. Sample Spaces & Events

2.1 Sample Space (S)

The set of *all possible outcomes*.

Examples:

- Toss a coin: $S = \{H, T\}$
- Roll a die: $S = \{1, 2, 3, 4, 5, 6\}$

- Pick 2 cards from 52 (unordered):
  Sample space size = $C(52, 2)$

Hidden Aspect 4 — Sample space must be defined before probability

Different sample spaces change probability:

Example:
Picking 2 cards **ordered** → sample space size $= 52 \times 51$
Picking 2 cards **unordered** → sample space size $= C(52, 2)$

Probability changes because the *definition of outcome* changes.

2.2 Events

An event is any **subset** of the sample space.

Examples:

- Event A = "roll an even number" → {2,4,6}
- Event B = "sum of dice is 7"
- Event C = "at least one head in two tosses"

Hidden Aspect 5 — Events can overlap

For two dice:

Event A = “sum is even”
Event B = “sum > 6”

Many outcomes belong to **both events** → intersection.

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3. Basic Probability Rules

Let outcomes be equally likely.

$$P(A) = \frac{|A|}{|S|}$$

3.1 Complement Rule

Event A + not A = whole sample space.

$$P(A^c) = 1 - P(A)$$

Example:

Probability of **not** getting at least one head in 2 tosses:

$A =$ "at least one head"

$A^c =$ "no heads" = {TT}

$$P(A) = 1 - \frac{1}{4} = \frac{3}{4}$$

Hidden Aspect 6 — Complement is easiest in many problems

Probability of:

- "At least one success"
- "At least one person shares your birthday"
- "At least one defective item"

Are usually easier by complement.

3.2 Union Rule (A OR B)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Subtract intersection because it's counted twice.

Example

Rolling a die:

A = even = {2,4,6}

B = >3 = {4,5,6}

Overlap = {4,6}

$$P(A \cup B) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6}$$

Hidden Aspect 7 — Union formula generalizes

For 3 events:

$$P(A \cup B \cup C) = \sum P(\text{single}) - \sum P(\text{pairwise}) + P(\text{triple intersection})$$

3.3 Intersection Rule (A AND B)

If A and B are independent:

$$P(A \cap B) = P(A)P(B)$$

Example:

Coin toss + die roll.

But if not independent:

$$P(A \cap B) = P(A)P(B|A)$$

Hidden Aspect 8 — Most real-life events are NOT independent

Examples:

- “Rain today” and “Cloudy today”
- “Card is Ace” and “Card is red”
- “Student passed math” and “Student passed physics”

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4. Hidden Links (How everything connects)

Counting → Sample Space → Probability Rules

1. Use counting (FCP, permutation, combination) to determine sample space size.
2. Represent events as subsets.
3. Apply rules (union, intersection, complement).
4. Solve probability.

Example Problem (Complete Flow):

Pick 2 cards from 52. What's the probability BOTH are aces?

Step 1: Total sample space

Unordered → $C(52, 2)$

Step 2: Favorable outcomes = choose 2 from 4 aces

$$C(4, 2)$$

Step 3: Probability

$$\frac{C(4, 2)}{C(52, 2)}$$

This merges:

- Combinations
- Sample space
- Events as subsets
- Probability definition

✔️ Your Next Step

If you want, I can now go **topic-by-topic with examples**, or give you:

- Practice problems
- Visual explanations
- Hidden intuition for each rule
- Advanced probability (conditional probability, Bayes, distributions)

Which one do you want next?