> when MLE is maximum than MAP is also give biggest change in positive or negative side

Let's clarify this carefully — **MLE vs MAP** and what happens when **MLE is large** or **MAP changes more positively/negatively**.

✅ Key Insight

MLE and MAP are NOT directly proportional.

A *high MLE value does NOT guarantee that MAP will also move strongly positive or negative.*

MAP includes **prior**, and that prior can:

- **boost** MLE
- **shrink** MLE
- **flip the direction** of the change
- **limit the growth** even when MLE wants to increase

🔥 Formula Comparison

MLE

$$\hat{\theta}_{\mathrm{MLE}} = \arg\max_{\theta}\ p(D \mid \theta)$$

MAP

$$\hat{\theta}_{\mathrm{MAP}} = \arg\max_{\theta}\ p(D \mid \theta)p(\theta)$$

Equivalent in log-space:

⌐   ⟨⟩   ›   ChatGPT 5.2 ⌄   ✑   ⬆   ⋯

➡ **MLE maximizes likelihood only**

➡ **MAP maximizes (likelihood + prior)**

🎯 Why MAP may not follow MLE direction

Imagine likelihood wants θ to move **strong positive direction**.

But prior may say:

- "Stay near zero"
- "Stay small"
- "Stay negative"
- "Don't move too much"

So MAP responds depending on the prior strength.

📌 Case-by-Case Explanation

Case 1 — MLE increases; MAP also increases

This happens when the prior is **weak or flat**.

Example:

Gaussian prior with very large variance (almost uniform).

Likelihood dominates.

Then:

$$\theta_{MAP} \approx \theta_{MLE}$$

- ✔️ MAP follows MLE
- ✔️ Both changes have same sign
- ✔️ MAP change size ≈ MLE change size

Case 2 — MLE increases strongly; MAP increases but LESS

Prior pulls estimate back.

Example:

Prior says θ should be near 0 (Gaussian with small variance).

Then:

$$\theta_{MAP} = \frac{n}{n + \lambda} \theta_{MLE}$$

✔ MAP increases

✗ But smaller than MLE

✔ Change is *shrunk*

✔ Same direction, smaller magnitude

Case 3 — MLE increases; MAP stays SAME or barely changes

Strong prior dominates.

Example:

Laplacian prior → L1 regularization → leads to sparsity.

If prior prefers θ = 0, MAP may stay 0 until data becomes very strong.

✔ MAP direction = none

✗ MAP magnitude ≠ MLE magnitude

✔ Prior overrides likelihood

Case 4 — MLE increases; MAP decreases (opposite direction!)

Yes — this can happen.

If prior is strongly negative (prior mean < 0), then:

- MLE tries to push θ positive
- Prior pushes θ negative

MAP = compromise.

If prior is stronger, MAP may even go negative.

Case 5 — MLE unchanged; MAP changes

If data gives no signal but prior is informative.

⚡ Visual 3D Intuition

Imagine two surfaces:

1. Likelihood surface → pushes θ toward MLE

2. Prior surface → pushes θ toward prior mean

MAP = highest point of the **sum of these two hills**.

Even if the likelihood hill rises, the prior may:

- pull MAP backward
- flatten the slope
- completely dominate

🎯 Final Answer to Your Question

❓ **When MLE becomes maximum, does MAP also show largest positive/negative change?**

✅ **NO, not necessarily.**

MAP can:

- increase less
- increase same
- not increase
- even move in opposite direction

because **MAP = MLE + Prior** and **prior modifies both the sign and magnitude**.

If you want next, I can show:

- ✔️ **complete derivation of MLE vs MAP**

- ✔️ **geometric interpretation**

- ✔️ **numeric example with dataset**

- ✔️ **code simulation in Python**

- ✔️ **diagrams comparing MLE and MAP estimation surface**