

demonstrate this thing using visuals

$$(\cos(\theta + \pi/2), \sin(\theta + \pi/2)) = (-\sin \theta, \cos \theta)$$



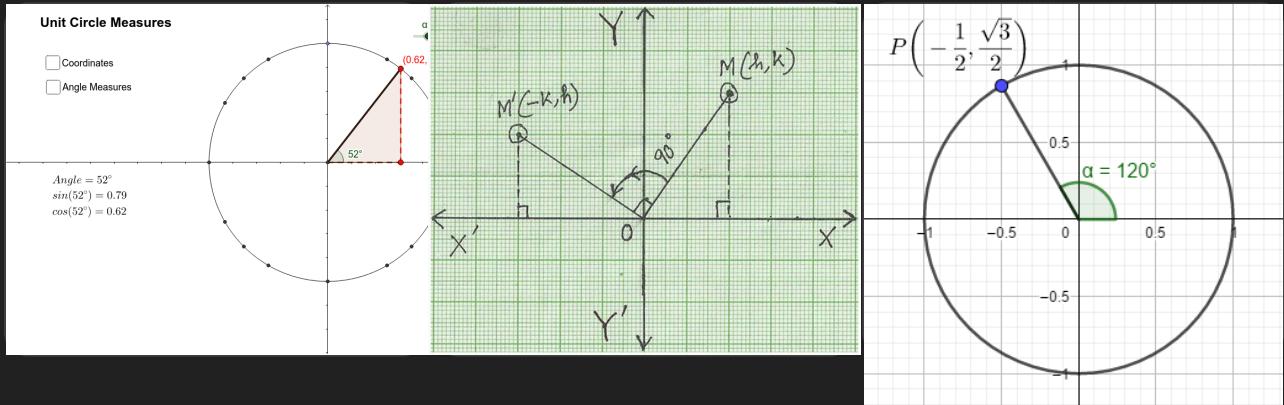
Here is a **clear visual demonstration** of the identity:

$$(\cos(\theta + \frac{\pi}{2}), \sin(\theta + \frac{\pi}{2})) = (-\sin \theta, \cos \theta)$$

We'll see it **geometrically on the unit circle**.

1. Visual: Point Rotated by 90° on the Unit Circle

Below you see the original point at angle θ , and another point rotated by $+90^\circ (\pi/2)$ from it.



2. Key Idea

A **rotation by $+90^\circ (\pi/2)$** sends the vector:

$$(\cos \theta, \sin \theta)$$

to:

$$(-\sin \theta, \cos \theta)$$

This corresponds exactly to the rotation matrix:

$$R_{\pi/2} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Apply it:

$$R_{\pi/2} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} 0 \cdot \cos \theta - 1 \cdot \sin \theta \\ 1 \cdot \cos \theta + 0 \cdot \sin \theta \end{pmatrix} = \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix}$$

This matches the trigonometric identity.

3. Visual Explanation (Step-by-Step)

Step 1 — Original angle θ

The point on the unit circle is:

$$P_\theta = (\cos \theta, \sin \theta)$$

Step 2 — Rotate by $+90^\circ$

Rotating 90° CCW moves the point to the position shown in the diagrams above.

Step 3 — New coordinates

The rotated point has coordinates:

$$P_{\theta+\pi/2} = \left(\cos\left(\theta + \frac{\pi}{2}\right), \sin\left(\theta + \frac{\pi}{2}\right) \right)$$

Step 4 — Using angle-shift identities

$$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin \theta$$

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos \theta$$

Therefore:

$$P_{\theta+\pi/2} = (-\sin \theta, \cos \theta)$$

4. Visual Intuition in Plain Words

- Moving a point by **90° anticlockwise** swaps the x-y values.
- The x-coordinate becomes negative because the point jumps into the **left** half-plane.
- The y-coordinate becomes the old x-value.

So the transformation:

→ rotate vector by 90°

→ swap components

→ negate the first one

Exactly gives:

$$(\cos(\theta + \frac{\pi}{2}), \sin(\theta + \frac{\pi}{2})) = (-\sin \theta, \cos \theta)$$

If you want, I can also show:

- animations
- vector rotation matrices for other angles
- 3D geometric interpretation

Just tell me!

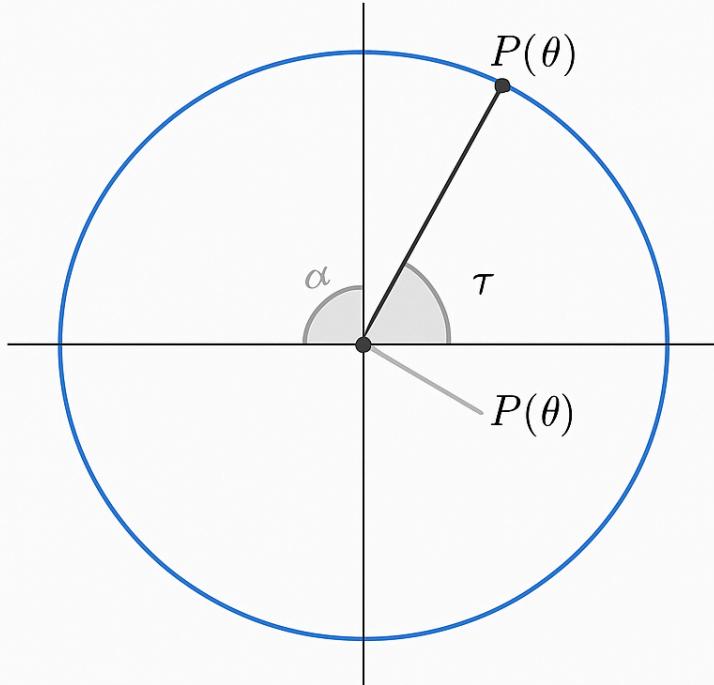


animation



Image created • Rotating vector on unit circle

$$(\cos(\theta + \frac{\pi}{2}), \sin(\theta + \frac{\pi}{2})) = (-\sin \theta, \cos \theta)$$



□ ⌂ ...

The concept above is demonstrating the same thing as the one below:
or different from the things below?

0 to 90° → both projections positive → sin+, cos+

90 to 180° → x negative, y positive → cos-, sin+

180 to 270° → both negative → sin-, cos-

270 to 360° → x positive, y negative → cos+, sin-

□ ⌂ ⌐

You're asking whether the **quadrant-sign rules**:

- 0°–90° → (cos +, sin +)
- 90°–180° → (cos -, sin +)
- 180°–270° → (cos -, sin -)
- 270°–360° → (cos +, sin -)

are the **same concept** as the identity:

$$(\cos(\theta + \frac{\pi}{2}), \sin(\theta + \frac{\pi}{2})) = (-\sin \theta, \cos \theta)$$

Short Answer:

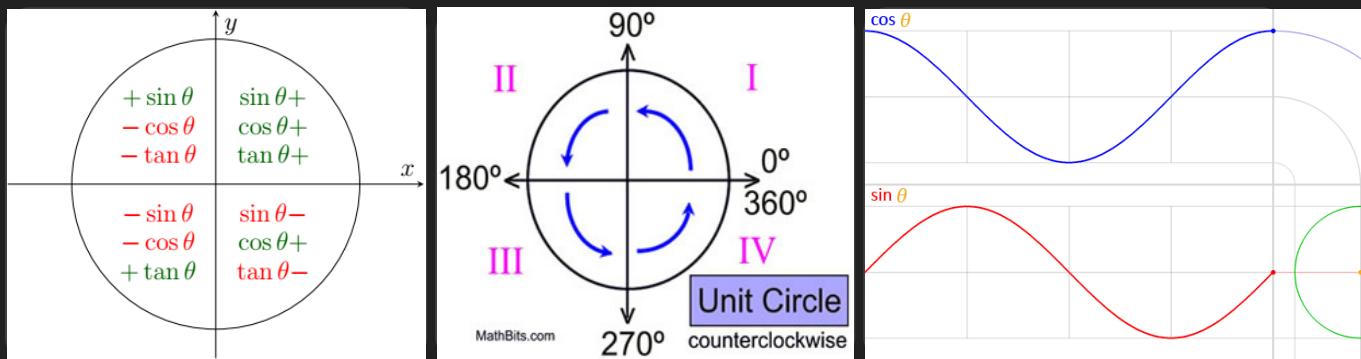
They are *related*, but **not the same**.

Relationship:

- The **quadrant rule** tells you **the sign of sin and cos** depending on the angle's location in the coordinate plane.
- The **rotation identity** tells you **how a vector transforms** when rotated by 90° .

But they BOTH come from **the geometry of the unit circle**.

● How They Connect (with visuals)



1 Quadrant signs

These diagrams show how **cos = x-coordinate** and **sin = y-coordinate** change sign depending on where the angle lies.

Example:

- In Quadrant II (90° – 180°) → $x < 0, y > 0 \rightarrow \cos -, \sin +$
-

2 Rotation identity

The second kind of diagram shows that rotating a point by $+90^\circ$ gives:

- $\text{new_x} = \text{old_y}$ *with sign change*
- $\text{new_y} = \text{old_x}$

This is a **transformation rule**, not a sign rule.

● Are they demonstrating the same thing?

- ✓ Quadrant signs → tell you *where* the point lies and what sign sin/cos have.
 - ✓ Rotation identity → tells you *how* the coordinates change after a 90° rotation.
 - ✗ They are NOT identical concepts.
 - ✓ But they are deeply connected through the unit circle geometry.
-

● How these two concepts align

Take the identity:

$$(\cos(\theta + \frac{\pi}{2}), \sin(\theta + \frac{\pi}{2})) = (-\sin \theta, \cos \theta)$$

Suppose θ is in Quadrant I → sin+, cos+.

After adding 90° → $\theta + 90^\circ$ is in Quadrant II → cos-, sin+.

Using the formula:

- new x = $-\sin(\theta)$ → negative
- new y = $\cos(\theta)$ → positive

These MATCH Quadrant II sign rules.

So the identity **predicts the correct quadrant signs automatically**.

● Final Summary

Concept	Purpose	Same or Different?
Rotation identity	mathematically rotates a vector by 90°	different idea
Quadrant sign rule	tells sign of sin/cos in each quadrant	different idea
Connection	both arise from the geometry of unit circle	related

If you want, I can create:

- animation of the rotation
- animation showing quadrant signs while θ moves
- combined animation showing both concepts working together

Just tell me which one you want!



what if we add pi in theta



When you **add π to θ** , you rotate the point on the unit circle by **180°** .

Let's analyze the transformation **visually and mathematically**.

1. The Identity for Adding π

$$(\cos(\theta + \pi), \sin(\theta + \pi))$$

Using angle-shift formulas:

$$\cos(\theta + \pi) = -\cos \theta$$

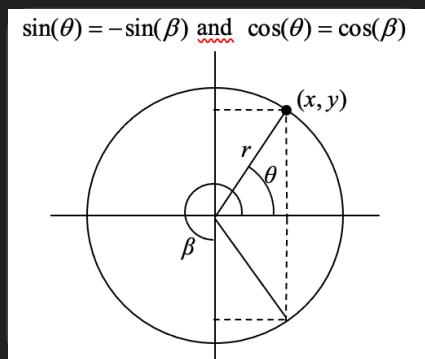
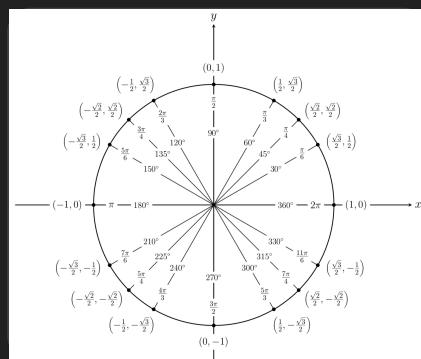
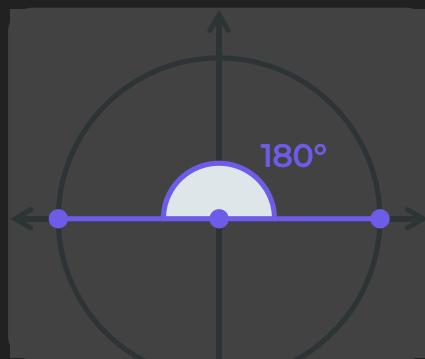
$$\sin(\theta + \pi) = -\sin \theta$$

- Final result:

$$(\cos(\theta + \pi), \sin(\theta + \pi)) = (-\cos \theta, -\sin \theta)$$

2. Visual Meaning — 180° Rotation

Here are visuals to help you see this transformation directly:



Interpretation:

- Adding π (180°) moves a point to the **diametrically opposite point** on the circle.
- The new point has the **same absolute values** but **opposite signs** for both x and y .

That is exactly:

$$(x, y) \rightarrow (-x, -y)$$

3. Connection to Quadrant Signs

This transformation **switches the quadrant**:

Original Quadrant	θ	$\theta + \pi$ (180° later)
Q1 (+,+)	(cos+, sin+)	Q3 (−,−)
Q2 (−,+)	(cos−, sin+)	Q4 (+,−)
Q3 (−,−)	(cos−, sin−)	Q1 (+,+)
Q4 (+,−)	(cos+, sin−)	Q2 (−,+)

So adding π **flips both signs**.

4. Matrix Form (Optional but powerful)

Rotation by π has matrix:

$$R_\pi = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Apply it:

$$R_\pi \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} -\cos \theta \\ -\sin \theta \end{pmatrix}$$

Exactly the same result.

5. Intuition Summary

Adding π to θ :

- Rotates the point **half the circle**.
- Takes the point to the **opposite side**.
- Flips the **sign of both cosine and sine**.

So:

Adding $\pi \rightarrow$ Reflects the point across the origin

If you want, I can also show:

- animation of rotation by π
- comparison of adding $\pi/2$ vs. π vs. 2π

Just tell me!



and what if theta + 2pi/3



When you add $\frac{2\pi}{3}$ to θ , you are rotating the point on the unit circle by **120° counter-clockwise**.

This rotation is not as simple as adding $\pi/2$ (90°) or π (180°), but we can **express it exactly** using identities, geometry, and visuals.

Let's break it down clearly.

➊ 1. Compute the New Coordinates

We want:

$$(\cos(\theta + \frac{2\pi}{3}), \sin(\theta + \frac{2\pi}{3}))$$

Use cosine and sine angle-addition formulas:

Cosine

$$\cos(\theta + \frac{2\pi}{3}) = \cos \theta \cos \frac{2\pi}{3} - \sin \theta \sin \frac{2\pi}{3}$$

Insert exact values:

$$\cos \frac{2\pi}{3} = -\frac{1}{2}, \quad \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

Therefore:

$$\cos(\theta + \frac{2\pi}{3}) = -\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$$

Sine

$$\sin(\theta + \frac{2\pi}{3}) = \sin \theta \cos \frac{2\pi}{3} + \cos \theta \sin \frac{2\pi}{3}$$

Insert values:

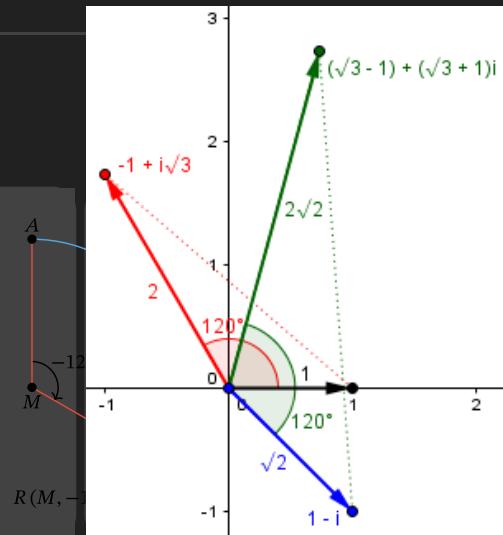
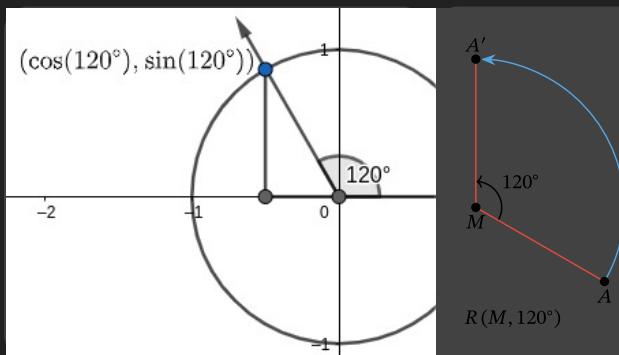
$$= -\frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta$$

Final transformation for adding $\frac{2\pi}{3}$:

$$(\cos(\theta + \frac{2\pi}{3}), \sin(\theta + \frac{2\pi}{3})) = \left(-\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta, \frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right)$$

This is the coordinate of the point rotated by 120° counterclockwise.

2. Visual of 120° Rotation



A 120° rotation:

- moves a point to a new location forming a 120° sector,
- produces a vector that is a specific linear combination of $\cos \theta$ and $\sin \theta$.

3. Special Case Check ($\theta = 0$)

Let's verify the formula is correct.

If $\theta = 0$:

$$(\cos 0, \sin 0) = (1, 0)$$

Applying the derived formula:

Cosine:

$$-\frac{1}{2} \cdot 1 - \frac{\sqrt{3}}{2} \cdot 0 = -\frac{1}{2}$$

Sine:

$$\frac{\sqrt{3}}{2} \cdot 1 - \frac{1}{2} \cdot 0 = \frac{\sqrt{3}}{2}$$

These are exactly:

$$(\cos \frac{2\pi}{3}, \sin \frac{2\pi}{3}) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

Correct ✓

4. Matrix Form (Elegant Perspective)

Rotation by angle $\alpha = \frac{2\pi}{3}$ uses:

$$R_\alpha = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$

Insert values:

$$R_{\frac{2\pi}{3}} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

Then:

$$R_{\frac{2\pi}{3}} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta \\ \frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \end{pmatrix}$$

Same as our formula.

5. Summary (Easy to Remember)

Angle Added	Meaning	Transformation
$\pi/2$	90° rotation	swap + sign change
π	180° rotation	flip both signs
$2\pi/3$	120° rotation	linear combination with coefficients $-1/2, -\sqrt{3}/2$

The 120° rotation is more complex because $\cos(120^\circ)$ and $\sin(120^\circ)$ are not 0 or ± 1 .

If you want, I can also generate:

- animation of rotation by $2\pi/3$
 - step-by-step geometric construction of 120° rotation
 - comparison of $\pi/2$, π , $2\pi/3$ rotations

Just tell me!



theta + 3pi/2



When you add $\frac{3\pi}{2}$ to θ , you are rotating the point on the unit circle by 270° counter-clockwise (or equivalently -90°).

This gives a clean and beautiful transformation similar to the $\pi/2$ case.

1. Identity for Adding $3\pi/2$

We want:

$$(\cos(\theta + \frac{3\pi}{2}), \sin(\theta + \frac{3\pi}{2}))$$

Use known values:

$$\cos \frac{3\pi}{2} = 0, \quad \sin \frac{3\pi}{2} = -1$$

Apply angle-sum identities:

Cosine term

$$\begin{aligned}\cos(\theta + \frac{3\pi}{2}) &= \cos \theta \cdot 0 - \sin \theta \cdot (-1) \\ &= \sin \theta\end{aligned}$$

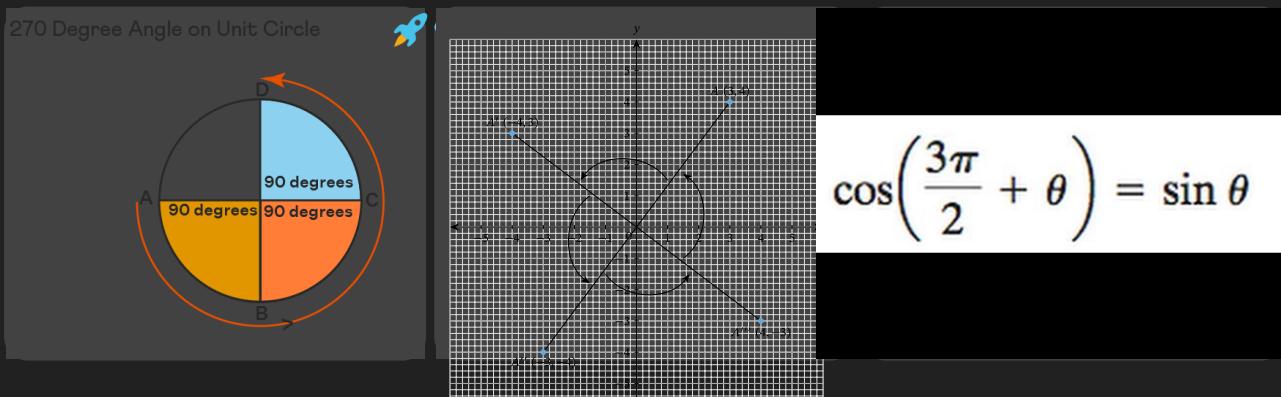
Sine term

$$\begin{aligned}\sin(\theta + \frac{3\pi}{2}) &= \sin \theta \cdot 0 + \cos \theta \cdot (-1) \\ &= -\cos \theta\end{aligned}$$

Final transformation:

$(\cos(\theta + \frac{3\pi}{2}), \sin(\theta + \frac{3\pi}{2})) = (\sin \theta, -\cos \theta)$

2. Visual Meaning — 270° Rotation



A 270° CCW rotation is the same as a 90° clockwise rotation.

Compare with 90° CCW ($\pi/2$):

- $\theta + \frac{\pi}{2} \rightarrow (-\sin \theta, \cos \theta)$

Now for 270° CCW:

- $\theta + \frac{3\pi}{2} \rightarrow (\sin \theta, -\cos \theta)$

You can see the pattern:

Rotation**New Point**

+90° ($\pi/2$) $(-\sin \theta, \cos \theta)$

+180° (π) $(-\cos \theta, -\sin \theta)$

+270° ($3\pi/2$) $(\sin \theta, -\cos \theta)$

This cycles neatly through quadrants.

3. Quadrant Behavior

Because 270° rotation moves a point **three quadrants ahead**, the signs change like this:

- x becomes **old y**
- y becomes **negative old x**

Exactly matching:

$$(\sin \theta, -\cos \theta)$$

4. Matrix Form (for completeness)

Rotation by $3\pi/2$ uses:

$$R_{\frac{3\pi}{2}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Applying:

$$R_{\frac{3\pi}{2}} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} \sin \theta \\ -\cos \theta \end{pmatrix}$$

Matches our identity perfectly.

Final Summary

$\theta + \frac{3\pi}{2}$ rotates the point by 270° CCW \rightarrow gives $(\sin \theta, -\cos \theta)$

This is the **third step** in the rotation cycle:

$(\cos\theta, \sin\theta) \rightarrow (-\sin\theta, \cos\theta) \rightarrow (-\cos\theta, -\sin\theta) \rightarrow (\sin\theta, -\cos\theta)$

After this, adding another $\frac{\pi}{2}$ returns you to the start.

If you want, I can create:

- ◆ animation of rotation by $3\pi/2$
- ◆ a comparison of all four rotations ($\pi/2, \pi, 3\pi/2, 2\pi$)
- ◆ a full table of all trig angle-shift identities

Just tell me!

