

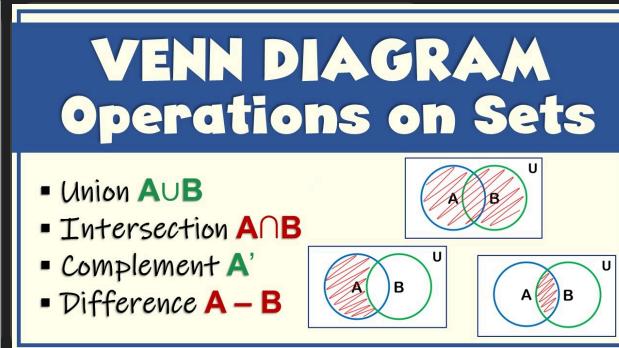
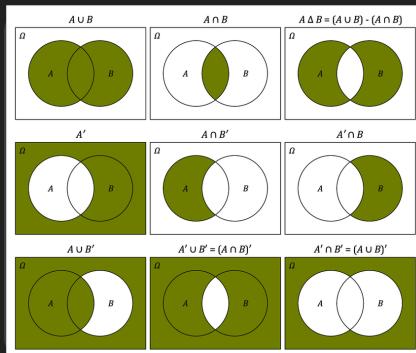
explain these topics one by one try to cover hidden aspects of them

Basic Set Theory & Logic
 Sets, unions, intersections
 Logical statements, truth tables
 Proof methods (direct, contradiction, induction)

Great — I'll explain **each topic one by one**, in your preferred style:
deep clarity, hidden aspects, diagrams-in-words, and links to how this math appears in ML, CS, and advanced math.

We will cover **Basic Set Theory → Logic → Proof Methods** in a clean learning pipeline.

1 SETS (FOUNDATIONS)



What is a set? (Hidden intuition)

A **set** is simply a *well-defined collection* of objects (called **elements**).

Hidden Aspect → *Well-defined* means: **you can always decide whether an element belongs or not**, no ambiguity.

Example:

- $\{1,2,3\}$ ✓ well-defined
- "All beautiful birds" ✗ not well-defined

Basic Notation

- $x \in A \rightarrow x$ is in A
- $x \notin A \rightarrow x$ is NOT in A

- $A \subseteq B \rightarrow A$ is a subset of B
- $A = B \rightarrow$ they have the *same* elements

Hidden Aspect \rightarrow Sets don't care about order or duplicates:

$$\{1,2,2,3\} = \{3,2,1\}$$

Key Operations

Union ($A \cup B$)

All elements in A or B or both.

Hidden Aspect \rightarrow union corresponds to **logical OR**.

Intersection ($A \cap B$)

Elements common to both A and B .

Hidden Aspect \rightarrow intersection corresponds to **logical AND**.

Complement (A^c)

Elements NOT in A (relative to some universal set U).

Hidden Aspect \rightarrow complement depends on **U** , the "universe of discourse".

Difference ($A - B$)

Elements in A that are NOT in B .

Hidden Aspect $\rightarrow A - B = A \cap B^c$.

Symmetric Difference ($A \Delta B$)

Elements in $A \cup B$ but NOT in $A \cap B$.

Hidden Aspect \rightarrow XOR of sets.

2 LOGICAL STATEMENTS & TRUTH TABLES

Q		And		$\neg P$		P		$P \rightarrow Q$		$\neg P \wedge (P \rightarrow Q)$	
		p	q	p	q	p	q	p	q	$p \vee q$	
T	T	T	T	F	T	T	F	T	T	T	F
	T	F	T	F	F	T	F	T	F	T	
	F	T	F	F	F	F	T	F	F	F	
F	F	F	F	F	F	F	F	F	F	F	F
		If ... then		Not							
T	T	T	T	T	T	T	T	T	T	T	T
	F	T	F	F	T	F	F	F	F	F	
F	T	F	F	F	F	F	F	F	F	F	T
	F	T	T	F	F	T	F	T	F	T	
	F	F	F	F	F	F	F	F	F	F	

□

Proposition

A statement that is either TRUE or FALSE.

Hidden Aspect → propositions must be *unambiguous* and *cannot be both simultaneously*.

Examples:

- "2 < 5" ✓
- "x + 2 = 5" ✗ (not a proposition — depends on x)

Logical Connectives

Symbol	Meaning	Hidden Aspect
$\neg P$	NOT	flips truth
$P \wedge Q$	AND	true only if both true
$P \vee Q$	OR	inclusive OR (true if any true)
$P \rightarrow Q$	implication	only false when P true and Q false
$P \leftrightarrow Q$	equivalence	both true or both false

The most misunderstood: Implication ($P \rightarrow Q$)

Truth table:

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Hidden Aspect →

If P is false, $P \rightarrow Q$ is automatically **true**.

This is called **vacuous truth**.

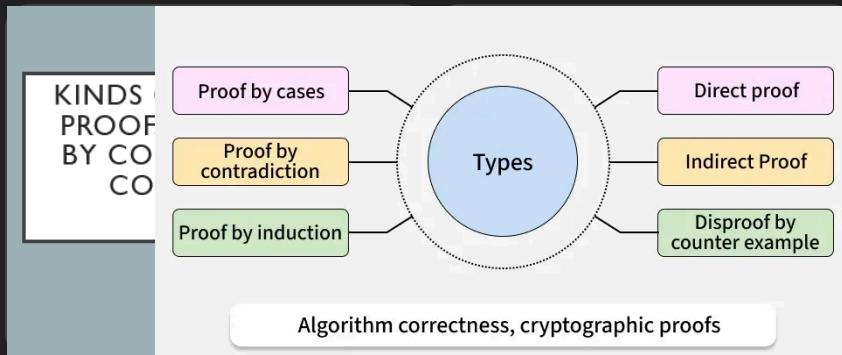
Example:

"If pigs fly, then $2+2=5$."

Since pigs don't fly, the whole implication is TRUE.

This confuses beginners — but it's essential in proofs!

3 PROOF METHODS (THE REAL ENGINE OF MATH)



We cover **three proof methods**:

3.1 Direct Proof

Structure:

1. Assume hypotheses are true
2. Use definitions & algebra
3. Conclude the result

Example skeleton:

To prove: *If x is even, then x^2 is even.*

- Assume $x = 2k$
- Then $x^2 = 4k^2 = 2(2k^2) \rightarrow$ even
- Done

Hidden Aspect →

Direct proofs rely heavily on **definitions** (even, odd, prime, subset, etc.).

3.2 Proof by Contradiction

Prove P by assuming $\neg P$ and showing a contradiction occurs.

Structure:

1. Assume P is false
2. Deduce an impossible situation
3. Conclude P must be true

Example skeleton:

To prove: $\sqrt{2}$ is irrational

- Assume $\sqrt{2} = p/q$ in lowest terms
- Show p and q must both be even → contradiction
- Therefore $\sqrt{2}$ is irrational

Hidden Aspects →

- Contradiction proofs **use the logic truth table of implication**
 - If assuming $\neg P$ leads to contradiction, then $P \rightarrow$ contradiction is false → so P is true.
-

3.3 Mathematical Induction

Used for any statement involving **natural numbers**.

Template

1. Base Case (check $n=1$ or $n=0$)
2. Inductive Assumption (assume true for $n=k$)
3. Inductive Step (prove for $n=k+1$)

Hidden Aspect →

Induction is just proof of implication chains:

$P(1) \rightarrow P(2) \rightarrow P(3) \rightarrow \dots$

If each implication holds, the entire infinite chain holds.

Another Hidden Aspect →

Induction is equivalent to the well-ordering principle of natural numbers.

🔗 How All These Connect (Hidden big picture)

- **Sets** give you *objects*
- **Logic** gives you *rules*
- **Proofs** give you *trust*

And in advanced math + ML:

- Set theory forms the base of **functions, vector spaces, sigma-algebras, probability spaces**
- Logic underlies **Boolean algebra, algorithms, circuits, SAT solvers**

- Proof techniques are used everywhere in **optimization proofs, convergence proofs, ML guarantees**

You cannot learn real mathematics or ML theory without mastering these.



What's next?

I can continue with:

👉 Practice problems

👉 Visual animations of logic and sets

👉 Step-by-step proofs for each method

👉 Connections to probability, ML, and linear algebra

Just tell me "continue" or choose the next topic.

◻ ⌂ ❓ ⏪ ...

Go deeper in above topics

Perfect. Now we go **much deeper** — including hidden identities, subtle definitions, advanced aspects, and the “mathematical engine” behind sets, logic, and proofs.

This is the level required for **real analysis, linear algebra, probability theory, and ML theory**.

● PART 1 — DEEP SET THEORY (much deeper than school level)

Venn Diagrams

Union $A \cup B$	Intersection $A \cap B$	Absolute Complement A'
Set Minus $A \setminus B$	Complement Intersection $A' \cap B'$	Absolute Union $A \cup B'$
Powerset $\mathcal{P}(A)$	Universal Complement $(A \cap B)' = (A' \cup B')$	Universal Union $(A \cup B)' = (A' \cap B')$

1.
2.
3.
4.
5.
6.

1	2	3
x	y	z
(x,1)	(x,2)	(x,3)
(y,1)	(y,2)	(y,3)
(z,1)	(z,2)	(z,3)

1.1 What exactly is a Set? (Deep definition)

A set is defined in ZF(Zermelo–Fraenkel) axioms:

Axiom of Extensionality

If two sets have exactly the same elements, they are the same set.

Hidden Aspect →

$$A = B \iff (\forall x)(x \in A \leftrightarrow x \in B)$$

This is why order and duplicates don't matter.

Axiom of Separation

You **cannot** take any arbitrary rule to create a set inside the universe — some restrictions apply (to avoid Russell's paradox).

This is why "set of all sets not containing themselves" is illegal.

1.2 Operations on Sets — Full Algebra

Union

$$A \cup B = \{x : x \in A \vee x \in B\}$$

Intersection

$$A \cap B = \{x : x \in A \wedge x \in B\}$$

Difference

$$A - B = \{x : x \in A \wedge x \notin B\}$$

Complement

$$A^c = U - A$$

1.3 Hidden Identities (Rarely Taught but Necessary)

1. Distributive Laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Identity used often in probability proofs.

2. De Morgan's Laws

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

This connects NOT with AND/OR.

3. Idempotent Laws

$$A \cup A = A, \quad A \cap A = A$$

4. Absorption Laws

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

Used in simplifying logic circuits and Boolean algebra.

5. Symmetric Difference Algebra

$$A \Delta B = (A - B) \cup (B - A)$$

$$A \Delta B = (A \cup B) - (A \cap B)$$

$$A \Delta A = \emptyset$$

This is the "XOR" of sets.

1.4 Power Set (Very Important)

$$\mathcal{P}(A) = \{B : B \subseteq A\}$$

If A has n elements,

$$|\mathcal{P}(A)| = 2^n$$

Hidden Aspect →

Power sets are the foundation of:

- Sigma-algebras (Probability theory)
- Function spaces
- Topologies
- Logic semantics

1.5 Cartesian Product

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Hidden Aspect →

Cartesian products generate:

- Functions → subsets of $A \times B$
- Relations → directed edges
- Matrices → finite Cartesian products
- Neural networks → functions between Cartesian product spaces



PART 2 — DEEP LOGIC (beyond truth tables)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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TRUTH TABLE						
p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$\neg p$	
T	T	T	T	T	F	
T	F	F	T	F	F	
F	T	F	T	T	T	
F	F	F	F	T	T	

Quantifiers

Count/uncount	Count, nouns	Un
Large quant.	<ul style="list-style-type: none"> • A lot of + noun • Lots of + noun • Verb + a lot. • Plenty of + noun 	many
Small quant.	<ul style="list-style-type: none"> • Some • Any 	(a/very) few
More/less than you need!	<ul style="list-style-type: none"> • (not) enough + noun 	too many
Zero quantity	<ul style="list-style-type: none"> • not...any + noun • no + noun • none 	Un

I (not) adj/adv + enough, too + adj/adv

2.1 Propositions – Deep view

A proposition is a statement with a definite truth value.

Hidden Aspect →

Mathematics requires **bivalence**:

Every statement is either true or false (classical logic)

This is not true in other systems (intuitionistic logic, fuzzy logic), but for mathematics and ML, we use classical.

2.2 Logical Connectives (deep semantics)

Negation

$$\neg P = \text{"P is false"}$$

Conjunction

$$P \wedge Q = \text{true iff both true}$$

Disjunction (Inclusive OR)

$$P \vee Q = \text{true if at least one true}$$

2.3 The Most Misunderstood: Implication

$$P \rightarrow Q \equiv \neg P \vee Q$$

This identity is crucial.

Hidden Aspect →

Implication is NOT causation.

It's "if P is true, Q must be true."

Truth table comes from the identity above.

2.4 Biconditional

$$P \leftrightarrow Q = (P \rightarrow Q) \wedge (Q \rightarrow P)$$

Equivalent means **same truth value**.

2.5 Quantifiers (Real Mathematics Starts Here)

Universal Quantifier

$$\forall x \in A, P(x)$$

Means: P holds for *every* x in A.

Existential Quantifier

$$\exists x \in A, P(x)$$

Means: at least one x satisfies P.

Hidden Transformation Rules

De Morgan for quantifiers:

$$\neg(\forall x P(x)) = \exists x \neg P(x)$$

$$\neg(\exists x P(x)) = \forall x \neg P(x)$$

These rules are used to negate statements in proofs.

Example:

"Not every number is positive"

= "There exists a number that is not positive."

2.6 Predicate Logic (Very Important for ML)

A predicate is a function returning TRUE/FALSE.

Examples:

- $P(x) : x > 0$
- $Even(n) : n \bmod 2 = 0$

Hidden Aspect →

All ML hypothesis classes are collections of predicates.



PART 3 — PROOF METHODS (DEEP VERSION)

Composing a Proof Based on PSMI (2)

Proof (cont'd): $P(n)$: an n -cent postage can be made up using 3-cent and 7-cent stamps

Basic step: We need to verify that $P(12) \wedge P(13) \wedge P(14)$ is true.

Indeed, $12 = 4 \cdot (3)$
 $13 = 2 \cdot (3) + 1 \cdot (7)$
 $14 = 2 \cdot (7)$
which completes the basic step.

Inductive step: We need to prove that
 $\forall n \geq 14 \quad [(P(12) \wedge \dots \wedge P(n)) \rightarrow P(n+1)]$

12

Which Form of Induction Should Be Used?

- We can always use strong induction instead of mathematical induction. But there is no reason to use it if it is simpler to use mathematical induction.
- In fact, the principles of mathematical induction, strong induction, and the well-ordering property are all equivalent.
- Sometimes it is clear how to proceed using one of the three methods, but not the other two.

3.1 Direct Proof (Deep Version)

To prove:

$$P \rightarrow Q$$

Assume P , use logic/definitions to show Q must hold.

Hidden Aspect →

Direct proof always uses the implication identity:

$$P \rightarrow Q \equiv \neg P \vee Q$$

So proving Q under P satisfies $Q \vee \neg P$.

3.2 Proof by Contrapositive (hidden method)

Instead of proving:

$$P \rightarrow Q$$

Prove:

$$\neg Q \rightarrow \neg P$$

These are logically equivalent.

Example:

"If x^2 is odd, then x is odd."

Contrapositive:

"If x is even, x^2 cannot be odd."

This is simpler.

3.3 Proof by Contradiction (deep logic)

To prove P:

1. Assume $\neg P$
2. Derive contradiction C
3. Therefore P is true

Hidden Aspect →

Contradiction works because:

$$(\neg P \rightarrow \text{False}) \equiv P$$

Meaning:

If assuming $\neg P$ leads to falsehood, then $\neg P$ is impossible $\rightarrow P$ must be true.

This method is essential in:

- irrationality proofs
- existence proofs
- optimization proofs
- linear algebra (rank-nullity, eigenvalues existence)

3.4 Mathematical Induction (deep)

To prove $P(n)$ for all natural numbers:

Step 1: Base Case

Prove $P(1)$

Step 2: Inductive Hypothesis

Assume $P(k)$ is true

Step 3: Inductive Step

Use $P(k)$ to prove $P(k+1)$

Hidden Aspect →

Induction is equivalent to the **Well-Ordering Principle**:

Every non-empty subset of \mathbb{N} has a smallest element.

Induction \equiv Recursion \equiv Well-ordering
 (These three are mathematically equivalent.)

3.5 Strong Induction

Assume all cases up to k are true:

$$P(1), P(2), \dots, P(k)$$

Then prove $P(k+1)$.

Hidden Aspect \rightarrow

Strong induction = ordinary induction in power, but easier for:

- prime factorizations
 - recurrence relations
 - DP algorithms
-

3.6 Existence Proofs

To prove something *exists*, you can:

- give explicit example (**constructive proof**)
- argue contradiction if it doesn't exist (**non-constructive proof**)

Example:

"There exist irrational a, b such that a^b is rational."

Classic non-constructive proof uses $\sqrt{2}$.

3.7 Uniqueness Proofs

To prove a solution is unique:

1. Assume there are two solutions
 2. Show they must be equal
-

3.8 Common Proof Mistakes (Deep Insight)

- Treating implication as causation

- Assuming what you need to prove
 - Invalid generalization
 - Assuming converse is true (wrong!)
-

✓ NEXT LEVEL AVAILABLE

I can now go even deeper by giving you:

- A) Fully rigorous proofs of all set identities
- B) Deep logical equivalences and quantifier manipulation
- C) Real analysis-level problems using sets & logic
- D) Complete proof examples using each method
- E) Problems + solutions to train your proof skills

Just tell me:

- 👉 "Go deeper into sets"
- 👉 "Go deeper into logic"
- 👉 "Go deeper into proof methods"

◻️ ⌂ ⓘ ⓘ ...

I don't know what symbol stand for what

Got it — before going deep, you must understand **every symbol** used in sets, logic, and proofs.

Here is a **complete symbol dictionary**, explained in simple and clear meaning, with examples.

★ SET THEORY SYMBOLS (with meaning + examples)

Table of set theory symbols

Symbol	Symbol Name	Meaning / definition
{ }	set	a collection of elements
$A \cap B$	intersection	objects that belong to set A and set B
$A \cup B$	union	objects that belong to set A or set B
$A \subseteq B$	subset	subset has less elements or equal to the set
$A \subset B$	proper subset / strict subset	subset has less elements than the set
$A \not\subseteq B$	not subset	left set not a subset of right set
$A \supseteq B$	superset	set A has more elements or equal to the set B
$A \supset B$	proper superset / strict superset	set A has more elements than set B
$A \not\supset B$	not superset	set A is not a superset of set B
2^A	power set	all subsets of A
$A \supset B$	proper superset / strict superset	A is a superset of B, but B is not equal to A
$A \not\supset B$	not superset	set A is not a superset of set B
2^A	power set	all subsets of A
$P(A)$	power set	all subsets of A
$A=B$	equality	both sets have the same members

Symbol	Symbol Name	Meaning / definition	Example
{ }	set	a collection of elements	$A=\{3,7,9,14\}$, $B=\{9,14,28\}$
$A \cap B$	intersection	objects that belong to set A and set B	$A \cap B = \{9,14\}$
$A \cup B$	union	objects that belong to set A or set B	$A \cup B = \{3,7,9,14,28\}$
$A \subseteq B$	subset	subset has less elements or equal to the set	$\{9,14,28\} \subseteq \{9,14,28\}$
$A \subset B$	proper subset / strict subset	subset has less elements than the set	$\{9,14\} \subset \{9,14,28\}$
$A \not\subseteq B$	not subset	left set not a subset of right set	$\{9,66\} \not\subseteq \{9,14,28\}$
$A \supseteq B$	superset	set A has more elements or equal to the set B	$\{9,14,28\} \supseteq \{9,14\}$
$A \supset B$	proper superset / strict superset	set A has more elements than set B	$\{9,14,28\} \supset \{9,14\}$
$A \not\supset B$	not superset	set A is not a superset of set B	$\{9,14,28\} \not\supset \{9,66\}$
2^A	power set	all subsets of A	

"is an element of"

Example:

$$3 \in \{1, 2, 3\}$$

∉

"is NOT an element of"

$$5 \notin \{1, 2, 3\}$$

2. Subsets

⊆

"is a subset of (can be equal)"

$$\{1, 2\} \subseteq \{1, 2, 3\}$$

⊂

"proper subset" (strictly smaller)

$$\{1, 2\} \subset \{1, 2, 3\}$$

⊄

"is NOT a subset of"

3. Set Operations

∪

Union = elements in A OR B

$$A \cup B$$

\cap

Intersection = elements in BOTH A AND B

$$A \cap B$$

*– *or* *

Set difference = elements in A but NOT in B

$$A - B$$

Δ

Symmetric Difference (elements in either A or B, but not both)

$$A \Delta B$$

c or $-$

Complement (everything not in the set)

$$A^c$$

4. Important Sets

\mathbb{N}

Natural numbers = {1, 2, 3, ...}

\mathbb{Z}

Integers = {..., -2, -1, 0, 1, 2, ...}

\mathbb{Q}

Rational numbers = “fractions”

\mathbb{R}

Real numbers

\mathbb{C}

Complex numbers

5. Power Set

$\mathcal{P}(A)$

Set of all subsets of A.

Example:

$$\mathcal{P}(\{1, 2\}) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$$

6. Empty Set

\emptyset

Set with no elements.

$$\emptyset \subseteq A$$

7. Cartesian Product

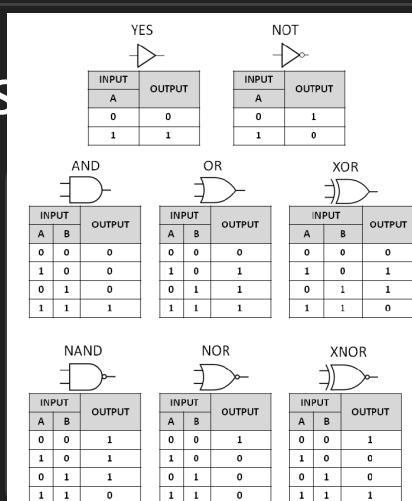
$A \times B$

All ordered pairs (a, b) where $a \in A$ and $b \in B$.

★ LOGIC SYMBOLS

And		Or	
p	q	$p \cdot q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

If... then		Not	
p	q	$p \supset q$	$\sim p$
T	T	T	F
T	F	F	T
F	T	T	
F	F	T	



1. Logical Connectives

$\neg P$

NOT P

(negation)

P \wedge Q

P AND Q

(both must be true)

P \vee Q

P OR Q

(at least one true)

P \rightarrow Q**Implication**

"If P then Q"

Only false when: P = true, Q = false.

P \leftrightarrow Q**Biconditional**

"P if and only if Q"

Both have same truth value.

2. Quantifiers (VERY IMPORTANT)

 \forall **For all**

Example:

$$\forall x \in \mathbb{R}, x^2 \geq 0$$

"For all real numbers x, $x^2 \geq 0$." **\exists** **There exists**

Example:

$$\exists x \in \mathbb{Z}, x^2 = 4$$

"There exists an integer whose square is 4."

\nexists

There does NOT exist

★ PROOF SYMBOLS

1. Therefore

 \therefore

Conclusion

$$x = 2 \Rightarrow \therefore x^2 = 4$$

2. Because / Since

 \because

3. Equivalent

 \equiv

Means "identically equal" or "logically equivalent".

Example:

$$P \rightarrow Q \equiv \neg P \vee Q$$

4. Equality sets

 $=$

Both sides exactly equal as sets or numbers.

 \neq

Not equal.

★ FUNCTION & RELATION SYMBOLS

$f : A \rightarrow B$

Function from A to B.

$f(a)$

Value of function at a.

$a \sim b$

a is related to b under relation ~

★ ORDER SYMBOLS

$<$, $>$, \leq , \geq

Less than, greater than, less or equal, greater or equal.

★ INTERVAL NOTATION

(a, b)

Open interval ($a < b$) excludes endpoints.

$[a, b]$

Closed interval includes endpoints.

★ Would you like a VISUAL explanation of each symbol with diagrams?

Or I can continue with:

- ✓ Deep explanation of union/intersection using number lines
- ✓ Visual truth tables for AND/OR/NOT
- ✓ Real-life examples for each logical operator
- ✓ How all these symbols are used in proofs

Tell me what you want next:

"Explain symbols visually"

or

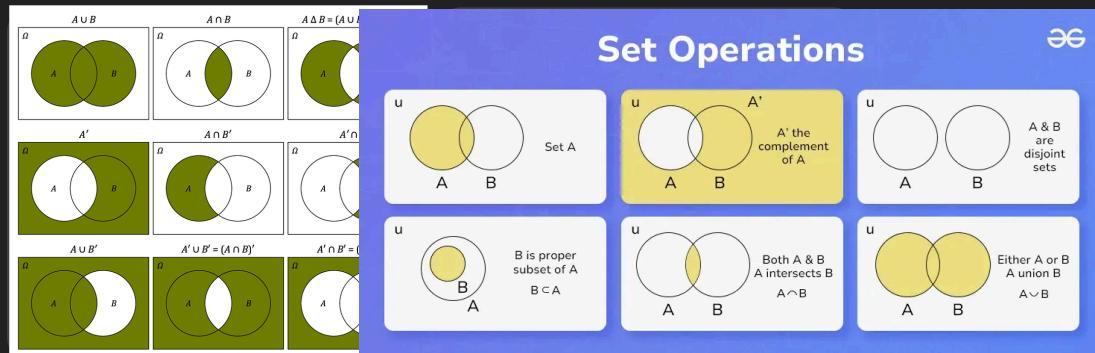
"Start teaching sets deeply now that I know symbols."



Sure

Great — since you said "Sure", I will now give you **VISUAL + INTUITIVE EXPLANATIONS** of every important symbol in **sets, logic, and proofs**, using diagrams-in-words and simple examples so you never get confused again.

★ PART 1 — SET SYMBOLS (VISUALIZED)



1 \in — "is an element of"

Think of a **bag** (set).

If the number is *inside* the bag $\rightarrow \in$

Example:

Set A = {2, 4, 6}

- $4 \in A$ ✓
- $5 \in A$ X (false)

Visual:

vbnet

A: [2 4 6]

↑

4 is inside A $\rightarrow 4 \in A$

 Copy code

2 \notin — “is NOT an element of”

If the element is *outside* the bag $\rightarrow \notin$

css

A: [2 4 6] 5 \rightarrow NOT inside $\rightarrow 5 \notin A$

 Copy code

3 \subseteq — “subset of (can be equal)”

Set B is inside set A.

vbnnet

A: [1 2 3 4]
[2 3] \leftarrow B is fully inside A

B \subseteq A

 Copy code

4 \subset — “proper subset (strictly smaller)”

Same as subset but **cannot be equal**.

css

A: [1 2 3 4]
[2 3] \leftarrow B smaller

B \subset A

 Copy code

5 \cup — “union” = OR

Take EVERYTHING in A or B or both.

Visual Venn Diagram:

css



Union = total shaded area.

 Copy code

Example:

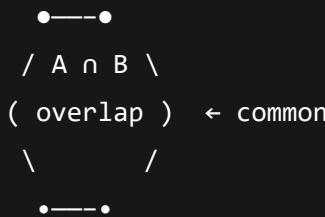
$$A = \{1,2,3\}, B = \{3,4\}$$

$$A \cup B = \{1,2,3,4\}$$

6 \cap — “intersection” = AND

Only the common part.

css



$A \cap B$ = elements in both A and B.

 Copy code

Example:

$$A = \{1,2,3\}, B = \{3,4\}$$

$$A \cap B = \{3\}$$

7 $A - B$ — “difference”

Everything in A **except** what B also has.

makefile

A: [1 2 3 4]

B: [3 4]

A - B = [1 2]

Copy code

You remove elements of B from A.

8 A^c — “complement”

Everything NOT in A (relative to a universal set U).

makefile

U: [1 2 3 4 5 6 7]

A: [3 4]

A^c = [1 2 5 6 7]

Copy code

9 Ø — empty set

A set with NO elements.

Think: “a bag with nothing inside.”

Ø = { }

Copy code

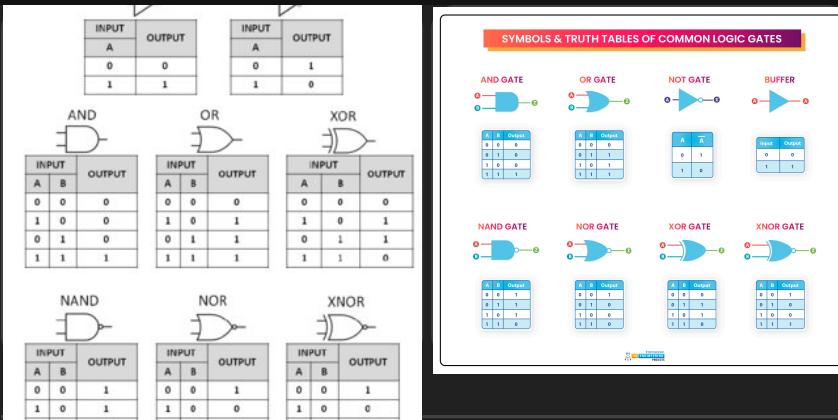
10 P(A) — power set (set of all subsets)

Example: A = {1,2}

P(A) = { Ø, {1}, {2}, {1,2} }

Copy code

★ PART 2 — LOGIC SYMBOLS (VISUALIZED)



1 $\neg P$ — NOT P

Flips the truth.

yaml

```
P : TRUE
¬P: FALSE
```

Copy code

2 $P \wedge Q$ — AND

Both must be true.

vbnnet

```
P: TRUE      Q: TRUE
P ∧ Q = TRUE
```

Copy code

Even one false \rightarrow result is false.

3 $P \vee Q$ — OR (inclusive)

At least one true = true.

graphql

```
TRUE ∨ FALSE = TRUE
FALSE ∨ FALSE = FALSE
```

Copy code

4 $P \rightarrow Q$ — “if P then Q” (implication)

ONLY FALSE when:

ini

$P = \text{TRUE}$

$Q = \text{FALSE}$

Copy code

Visual truth table:

r

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Copy code

Remember:

When P is false \rightarrow implication is automatically true

(called “vacuous truth”).

5 $P \leftrightarrow Q$ — equivalence

True if P and Q have the **same** truth value.

r

$T \leftrightarrow T = \text{TRUE}$

$F \leftrightarrow F = \text{TRUE}$

Copy code

6 \forall — “for all”

Example:

$$\forall x \in \mathbb{R}, \quad x^2 \geq 0$$

Copy code

Meaning: Every real number has non-negative square.

7 \exists — “there exists”

Example:

$\exists x \in \mathbb{Z}, x^2 = 9$

 Copy code

Meaning: There exists an integer whose square is 9.

★ PART 3 — PROOF SYMBOLS (VISUAL)

1 \therefore — therefore

Used for conclusions.

makefile

$x = 2$
 $\therefore x^2 = 4$

 Copy code

2 \because — because

Used for reasons.

$x^2 = 4 \because x = 2$

 Copy code

3 \equiv — logically equivalent

Example:

css

$P \rightarrow Q \equiv \neg P \vee Q$

 Copy code

Meaning: THEY MEAN THE SAME THING logically.

★ PART 4 — QUICK SUMMARY TABLE

Symbol	Meaning
\in	element of
\notin	not element of
\subseteq	subset
\subset	proper subset
\cup	union (OR)
\cap	intersection (AND)
$-$	difference
c	complement
\emptyset	empty set
$\mathcal{P}(A)$	power set
\neg	NOT
\wedge	AND
\vee	OR
\rightarrow	implies
\leftrightarrow	equivalent
\forall	for all
\exists	there exists
\therefore	therefore
\because	because

Symbol**Meaning** \equiv

equivalent

 **Want the next step?**

I can now give you ANY of the following:

- A) Symbol drills (practice problems)
- B) Symbol-to-picture mapping (you tell symbol → I draw visual meaning)
- C) Deep set theory now that symbols are clear
- D) Deep logic + truth tables
- E) Begin proof methods with step-by-step examples

Just tell me:

"Continue with A/B/C/D/E."

∅ ⊆ ⊇ ⊃ ...