PSA-MOCK

April 13, 2024

Each question carries 4 marks

1. For a pair (A,B) of subsets of the set X={1,2,...,100}, let $A\Delta B$ denote the set of all elements of X which belong to exactly one of A or B. The number of pairs (A,B) of subsets of X such that $A\Delta B = \{2, 4, 6, ..., 100\}$ is

(A) 2^{151}
(B) 2^{52}
(C) 2^{100}
(D) 2^{101}

2. If $\alpha_1, \alpha_2, ..., \alpha_n$ be the roots of the equation $x^n + 1 = 0$, then $(1 - \alpha_1)(1 - \alpha_2)...(1 - \alpha_n)$ is equal to

(A) 1
(B) 0
(C) n
(D) 2

3.The number of real solutions of the equation

$$x^7 + 5x^5 + x^3 - 3x^2 + 3x - 7 = 0$$

is –

(A) 5
(B) 7
(C) 3
(D) 1

4. Let $X_1, X_2, ..., X_n$ be a random sample from a population with probability density function

$$f_\theta(x) = \begin{cases} \theta e^{-\theta x}, & x > 0 \\ 0, & otherwise \end{cases}$$

where $\theta > 0$ is an unknown parameter.
Then the uniformly minimum variance unbiased estimator for θ is

(A) $\frac{1}{\bar{X}_n}$
(B) $\sum\limits_{i=1}^{n} X_i$
(C) $\bar{X}_n$
(D) $\frac{n-1}{\sum\limits_{i=1}^{n} X_i}$

5. Let X be a random variable with the probability density function

$$f(x|\theta) = \begin{cases} 2\theta x + 1 - \theta, & 0 < x < 1,\ -1 \leq \theta \leq 1, \\ 0, & otherwise, \end{cases}$$

Based on a sample of size 1 , the most powerful critical region (rejection region) for testing $H_0 : \theta = 0$ against $H_1 : \theta = 1$ at level $\alpha = 0.2$ is given by

(A) $X > \frac{4}{5}$
(B) $X \leq \frac{2}{5}$
(C) $X > \frac{8}{5}$
(D) $X < \frac{4}{5}$

6. A necessary and sufficient condition for the quadratic function $ax^2 + bx + c$ to take both positive and negative values is

(A) $ab \neq 0$
(B) $b^2 - 4ac > 0$
(C) $b^2 - 4ac \geq 0$
(D) None of the above

7. The coefficient of x^4 in the expansion of $(1+x-2x^2)^7$ is

(A) -81
(B) -91
(C) 81
(D) 91

8. The rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{bmatrix}$ is less than 4 if and only if

(A) a=b=c=d
(B) atleast 2 of a,b,c,d are equal
(C) atleast 3 of a,b,c,d are equal
(D) a,b,c,d are distinct real numbers

9. Let w denote a complex fifth root of unity. Define

$$b_k = \sum_{j=0}^{4} j w^{-kj},$$

for $0 \le k \le 4$.Then $\sum\limits_{k=0}^{4} b_k w^{-kj}$ is equal to

(A) 5
(B) $5w$
(C) $5(1+w)$
(D) 0

10. If $f(x) = x^2$ and $g(x) = xsinx + cosx$ then

(A) f and g agree at no points
(B) f and g agree at exactly 1 point
(C) f and g agree at exactly 2 points
(D) f and g agree at more than 2 points

11. Let $\lambda_1, \lambda_2, \lambda_3$ denote the eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cost & sint \\ 0 & -sint & cost \end{pmatrix}.$$

If $\lambda_1 + \lambda_2 + \lambda_3 = \sqrt{2} + 1$, then the set of possible values of t , $-\pi \leq t < \pi$, is

(A) Empty set
(B) $\{\frac{\pi}{4}\}$
(C) $\{-\frac{\pi}{4}, \frac{\pi}{4}\}$
(D) $\{-\frac{\pi}{3}, \frac{\pi}{3}\}$

12. The values of η for which the following system of equations

$$\begin{array}{ccccccc} x & + & y & + & z & = & 1 \\ x & + & 2y & + & 4z & = & \eta \\ x & + & 4y & + & 10z & = & \eta^2 \end{array}$$

has a solution, are

(A) $\eta = 1, -2$
(B) $\eta = -1, -2$
(C) $\eta = 3, -3$
(D) $\eta = 1, 2$

13. For $x > 0$, let $f(x) = \lim_{n\to\infty} n(x^{\frac{1}{n}} - 1)$. Then

(A) $f(x) + f(1/x) = 1$
(B) $f(xy) = f(x) + f(y)$
(C) $f(xy) = xf(x) + f(x)$
(D) None of the above

14. If $a = \lim_{n\to\infty}(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n})$ and $b = \lim_{n\to\infty}\frac{1}{n}(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n})$, then

(A) both $a = \infty$ and $b = \infty$
(B) $a = \infty$ and $b = 0$
(C) $a = \infty$ and $b = 1$
(D) None of the above .

15. If $a_1, a_2, ..., a_n$ are positive real numbers , then

$$\frac{a_1}{a_2} + \frac{a_2}{a_3} + ... + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1}$$

is always

(A) $\geq n$
(B) $\leq n$
(C) $\leq n^{\frac{1}{n}}$
(D) None of the above

16. Suppose (X,Y) follows Bivariate Normal Distribution with $E(X) = 6$, $E(Y) = 3$, $Var(X) = 4$, $Var(Y) = 1$. Then $P(X > 2Y)$ is

(A) 1/2
(B) 1/4
(C) 1/3
(D) depends on Cov(X,Y)

17. Suppose a fair coin is tossed 2n times. If p_n denotes the probability of getting an equal number of heads and tails then $\underset{n\to\infty}{lim}\sqrt{n}.p_n$ equals

(A) 0
(B) $\sqrt{\pi}$
(C) $\sqrt{1/\pi}$
(D) 1

18. X is a random variable having the moment generating function $M_X(t) = (\frac{1+e^t}{2})^5$. Then $P(X \leq 2)$ is

(A) 3/16
(B) 4/19
(C) 6/12
(D) 1/3

19. For each n=1,2,...., Y_n follows Poisson(4n). Then $\frac{(Y_n-4n)}{\sqrt{n}}$ converges in probability to

(A) N(0,1)
(B) N(0,2)
(C) N(0,4)
(D) N(0,8)

20. The pages of a book are numbered consecutively starting from 1. Total 2989 digits were used to number the pages, then the total number of pages in the book is divisible by –
(A) 2
(B) 3
(C) 5
(D) 7

21. Three tickets are drawn randomly without replacement from a set of tickets numbered 1 to 100. Then the probability that the selected tickets are in Geometric Progression is

(A) $\frac{111}{300}$
(B) $\frac{19}{\binom{100}{3}}$
(C) $\frac{100}{\binom{100}{2}}$
(D) $\frac{1}{3}$

22. Which of the following is not a property of Moment Generating Function ?

(A) It exists if Expectation of X exists
(B) It uniquely determines the distribution of X.
(C) It is defined as the expected value of e^{tX} .
(D) It is always finite for any random variable X.

23. Consider a finite population of size $N > 1$ with units $U_1, U_2, \ldots.., U_N$. The following sampling method is used to select a sample : either the sample consists of only one unit U_j with probability $\frac{1}{N+1}$ for any j=1,2,..,N, or it consists of the whole population with probability $\frac{1}{N+1}$. Then the expected sample size is

(A) $\frac{N+2}{N+1}$
(B) $\frac{2N+1}{N+1}$
(C) $\frac{2N}{N+1}$
(D) 2

24. A box contains 2016 balls labelled 1,2,3,.....,2016. Two balls are selected at random by sampling without replacement . Let X_1 and X_2 be the labels on first and second balls respectively. Then

(A) $P(X_1 < X_2) > 1/2$
(B) $E(X_2|X_1 = 1008) > 1008$
(C) $E(X_1) \neq E(X_2)$
(D) X_1, X_2 are independent.

25. Consider a confounded 2^5 factorial design with factors A,B,C,D,E arranged in four blocks each of size eight. If the principal block of this design consists of the treatment combinations (1), ab, de, ace, and four others, then the combined factorial effects would be

(A) AB , DE , ABDE
(B) ABC , CDE , ABDE
(C) AB , CDE , ABCDE
(D) ABC , DE , ABCDE

26. Suppose X and Z are two independent random variables such that $E(X) = \mu > 0$, $Var(X) = \sigma^2 > 0$ and $P(Z = 1) = P(Z = -1) = \frac{1}{2}$. Let Y=ZX. Then

(A) $Var(Y) = \mu^2 + \sigma^2$
(B) X and Y have the same distribution
(C) $E(Y) > \mu$
(D) X and Y are always independent .

27. $A_1, A_2, ..., A_n$ is a decreasing sequence with $\bigcap\limits_i A_i = \phi$. Then, $\lim\limits_{n\to\infty} P(A_n)$ is equal to

(A) 0
(B) $P(A_1)$
(C) $P(\bigcup\limits_i A_i)$
(D) can not be said

28. If $X_1, X_2, ..., X_n \sim Exp(1)$, then expectation of $(\frac{X_1^2}{X_1^2+X_2^2+...+X_n^2})$ is

(A) $\frac{1}{n+1}$
(B) $\frac{1}{n}$
(C) $\frac{n-1}{n+1}$
(D) $\frac{1}{n-1}$

29. Let A be a 2×2 matrix with real entries. If $5 + 3\sqrt{-1}$ is an eigenvalue of A, then the determinant of A equals –

(A) 16
(B) 8
(C) 4
(D) 34

30. Suppose X is a $N(\mu, \sigma^2)$ random variable , and $Y = \Phi(X)$, where Φ is the cumulative distribution function of a standard normal random variable. What is E(Y) ?

(A) $\Phi(\mu/\sqrt{2+\sigma^2})$
(B) $\Phi(\mu/\sqrt{1+\sigma^2})$
(C) $\Phi(\mu/\sigma)$
(D) $\Phi(\mu/\sqrt{4+\sigma^2})$