PSA-MOCK

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Tulsidas Mukherjee

Each question carries 4 marks

1. The digit in the unit place of the integer $1! + 2! + ... + 99!$ is

(A) 3
(B) 0
(C) 1
(D) 7

2. The number of ways in which 8 persons numbered 1,2,...,8 can be seated in a ring so that 1 always sits between 2 and 3 is

(A) 240
(B) 360
(C) 72
(D) 120

3.The value of $\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{y} e^{-\frac{1}{2}(x^2+2y^2)}dxdy$ is

(A)0
(B)0.687
(C)0.5
(D) 0.25

4. Consider the statement : $x(a-x) < y(a-y)$ for all x,y with 0<x<y<1. The statement is true

(A) If and only if $a \geq 2$
(B) If and only if $a > 2$
(C) If and only if $a < -1$
(D) For no values of a

5. The infinite series $\sum\limits_{n=1}^{\infty} \frac{a^n log_e n}{n^2}$ converges iff

(A) $a \in [-1, 1]$
(B) $a \in (-1, 1]$
(C) $a \in (-1, 1)$
(D) $a \in (-\infty, \infty)$

6. X_1, X_2, X_3, X_4, X_5 are Bernoulli($\frac{1}{i+1}$),i=1(1)5 distributions and they are all independent. Then, $P(X_i + X_{6-i} > 0 \ \ \forall i)$ is equal to

(A) $\frac{1}{12}$
(B) $\frac{1}{15}$
(C) $\frac{1}{48}$
(D) $\frac{1}{720}$

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7. The coefficients of three consecutive terms in the expansion of $(1+x)^n$ are 165 , 330 , 462. Then the value of n is

(A) 10
(B) 12
(C) 13
(D) 11

8. Let A be a finite set of real numbers having m ($\geq$ 2) elements. Define a function $f : \mathbb{R} \to \mathbb{R}$, given by

$$f(x) = min\{|a - x| : a \in A\}$$

Then,

(A) f is continuous only at finitely many points
(B) f has m discontinuities
(C) f is discontinuous everywhere
(D) f is continuous everywhere

9. Three ladies have each brought a child for admission to a school. The head of the school wishes to interview the six people one by one, taking care that no child is interviewed before its mother. In how many different ways the interview can be arranged?

(A) 6
(B) 36
(C) 72
(D) 90

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10. What is the limit of the sequence $\{x_n\}$ as $n \to \infty$, where $x_n = \sum_{k=1}^{n} \frac{1}{\sqrt{n^2+k}}$?

(A) $tan^{-1}(1)$
(B) 1
(C) ∞
(D) $\frac{1}{e}$

11. If α and β are roots of the equation $x^2 + ax + b = 0$,where $b \neq 0$, then the roots of the equation $bx^2 + ax + 1 = 0$ are

(A) $1/\alpha$,$1/\beta$
(B) α^2, β^2
(C) $1/\alpha^2$,$1/\beta^2$
(D) $\beta/\alpha, \alpha/\beta$

12. The equation $r = 2acos\theta + 2bsin\theta$, in polar coordinates, represents

(A) A circle passing through the origin
(B) A circle with the origin lying outside it
(C) A circle with radius $2/\sqrt{a^2+b^2}$
(D) A circle with centre at the origin

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13. A shopkeeper has 12 bulbs of which 3 are defective. She sells the bulbs by selecting them at random one at a time. What is the probability that the seventh bulb sold is the last defective one?

(A) 3/44
(B) 9/44
(C) 13/44
(D) 7/44

14. The system of linear equations

$$\begin{array}{ccccccccc} kx_1 & + & \lambda x_2 & + & \lambda x_3 & + & \lambda x_4 & = & 0 \\ \lambda x_1 & + & kx_2 & + & \lambda x_3 & + & \lambda x_4 & = & 0 \\ \lambda x_1 & + & \lambda x_2 & + & kx_3 & + & \lambda x_4 & = & 0 \\ \lambda x_1 & + & \lambda x_2 & + & \lambda x_3 & + & kx_4 & = & 0 \end{array}$$

has unique solution if and only if

(A) $k - \lambda \neq 0$
(B) $k + 3\lambda \neq 0$
(C) $(k - \lambda)(k + 3\lambda) \neq 0$
(D) None of the above .

15. Let $\{a_n\}$ be a sequence of non-negative real numbers such that the series $\sum\limits_n a_n$ converges. If p is real number such that $\sum\limits_n \frac{\sqrt{a_n}}{n^p}$ diverges, then

(A) p must be strictly less than 1/2
(B) p must be strictly less than or equal to 1/2
(C) p must be strictly less that or equal to 1 but can be greater than 1/2
(D) p must be strictly less that 1 but can be greater than 1/2

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16. In the Taylor expansion of the function $f(x) = e^{x/2}$ about x=3, the coefficient of $(x-3)^5$ is

(A) $e^{3/2}\frac{1}{5!}$
(B) $e^{3/2}\frac{1}{2^5 5!}$
(C) $e^{-3/2}\frac{1}{2^5 5!}$
(D) none of the above

17. If $T_1$and T_2be unbiased estimators of $g(\theta)$ with variances $\sigma_1^2 = 1$ and $\sigma_2^2 = 4$ and $Corr(T_1, T_2) = \frac{1}{4}$, then the values of W_1 and $W_2$for which $T = W_1T_1 + W_2T_2$ is BLUE for $g(\theta)$ are respectively

(A) $\frac{7}{8}$ and $\frac{1}{8}$
(B) $\frac{5}{8} and \frac{3}{8}$
(C) $\frac{1}{2} and \frac{1}{2}$
(D) $\frac{1}{8}$ and $\frac{7}{8}$

18. If M is a 3×3 matrix such that
$[0\ 1\ 2]M = [1\ 0\ 0]$ and $[3\ 4\ 5]M = [0\ 1\ 0]$.
Then $[6\ 7\ 8]M$ is equal to

(A) $[2\ 1\ \ -2]$
(B) $[0\ 0\ 1]$
(C) $[-1\ 2\ 0]$
(D) $[9\ 10\ 8]$

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19. Let

$$f(x,y) = \begin{cases} \frac{|x|}{|x|+|y|}\sqrt{x^4+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Then at (0,0),

(A) f is continuous
(B) $\frac{\delta f}{\delta x} = 0$ and $\frac{\delta f}{\delta y}$ does not exist
(C)$\frac{\delta f}{\delta y} = 0$ and $\frac{\delta f}{\delta x}$ does not exist
(D) $\frac{\delta f}{\delta y} = 0 \ and \frac{\delta f}{\delta x} = 0$

20. Let X and Y be independent Bernoulli random variables with
P(X=1)=p and P(Y=1)=1-p
Then the distribution of X+Y-XY is

(A) Binomial$(2, \frac{1}{2}(1-p+p^2))$
(B) Binomial$(2, p^2+(1-p)^2)$
(C) Bernoulli$(2p-2p^2)$
(D) Bernoulli$(1-p+p^2)$

21. Let X be a random variable with the probability density function

$$f(x;\theta) = \begin{cases} \theta e^{\theta(x-\theta)} & if\ x > \theta \\ 0 & otherwise \end{cases}$$

where $\theta > 0$. If a test of size $\alpha = 0.1$ for testing $H_0 : \theta = 1$ vs $H_1 : \theta = 2$ rejects H_0 when X<m , then the value of m is

(A) -ln(0.1)
(B) ln(1.1)
(C) 1-ln(0.1)
(D) 1+ln(1.1)

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22. X is a Bernoulli(1/4) and Y is a Bernoulli(1/2) variable, independent of each other

$$Z = \begin{cases} \frac{X^2+X}{X^3+X} & if \;\; Y = 1 \\ \frac{X^2-X}{X^2+1} & if \;\; Y = 0 \end{cases}$$

Then, E(Z) equals

(A) 3/4
(B) 1/4
(C) 1/2
(D) 0

23. Suppose $\begin{pmatrix} X \\ Y \end{pmatrix}$ is a random vector such that the marginal distribution of X and the marginal distribution of Y are the same and each is normally distributed with mean 0 and variance 1 . Then, which of the following conditions imply independence of X and Y?

(A) $Cov(X, Y) = 0$
(B) aX+bY is normally distributed with mean 0 and variance $a^2 + b^2$ for all real a and b.
(C) $P(X \leq 0, Y \leq 0) = \frac{1}{4}$
(D) $E[e^{itX+isY}] = E[e^{itX}]E[e^{isY}]$ for all real s and t.

24. Consider a random sample $X_1, X_2, ..., X_n \sim U(-\theta, \theta)$, then T=$\underset{i}{max}|X_i|$.

$$\phi(T) = \begin{cases} 1, & T < c \\ 0, & T > c \end{cases}$$

is a function with expectation α under H_0 and , $H_0 : \theta = 2$ vs $H_1 : \theta < 2$
Then

(A) ϕ is a Most Powerful test of size α
(B) $1 - \phi$ is a Most Powerful test of size α
(C) $1 - \phi$ is a Most Powerful test of size $1 - \alpha$
(D) None of the above

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25. In order to compare between $t(\geq 2)$ treatments, one carries out t replications of an experiment using a randomized block design with t blocks. The error degrees of freedom for testing the equality of treatment effects in such a design is

(A) $t+1$
(B) $3(t-1)$
(C) $(t-1)(t^2+t-1)$
(D) $t(t-1)(t+1)$

26. If a fair coin is tossed 5 times , what is the probability of obtaining at least 3 consecutive heads?

(A) 1/8
(B) 5/16
(C) 1/4
(D) 3/16

27. Suppose an SRSWOR of size n has been drawn from a population labelled 1,2,...,N, where the population size N is unknown . Then

(A) MLE of N is also an unbiased estimator for N.
(B) MLE of N is not unbiased but a linear function of an unbiased estimator of N
(C) MLE of N is biased and inconsistlent for N
(D) MLE of N is non-unique and consistent.

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28.Consider data on body mass index (BMI) and fasting blood sugar levels (fgl) for two groups of individuals. Suppose that the correlation coefficient between BMI and fgl is equal to 0.5 in each of the two groups. If the two groups are combined together, the correlation coefficient between BMI and fgl :

(A) can be negative
(B) will be equal to 0.5
(C) will be positive but not necessarily equal to 0.5
(D) can be 0 but not negative

29. Let $X_1, X_2, ...$ be a sequence of independent random variables . Suppose for k=1,2,...

$$P(X_{2k-1} = 1) = P(X_{2k-1} = -1) = \frac{1}{2}$$

and the probability density function of X_{2k} is

$$f(x) = \frac{1}{\sqrt{2}} e^{-\sqrt{2}|x|}, |x| < \infty$$

Then $\underset{n\to\infty}{lim} P(\frac{X_1+X_2+...+X_{2n}}{\sqrt{2n}} \geq 1)$ is

(A) $\Phi(1)$
(B) $\frac{1}{2}$
(C) $\Phi(-1)$
(D) 1

30. Let $\frac{X}{\sigma_1^2}$ and $\frac{Y}{\sigma_2^2}$ be two independent chi-square variates with 2 and 6 degrees of freedom respectively. An unbiased estimator of $\frac{\sigma_1^2}{\sigma_2^2}$ is given by

(A) $\frac{X}{Y}$
(B) $\frac{2X}{Y}$
(C) $\frac{3X}{Y}$
(D) $\frac{4X}{Y}$

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