ISI Mock 1 PSA

April 13, 2024

1) If the sum, $\sum_{x=1}^{100} x!$, is divided by 36, the remainder is
$a)$ 0
$b)$ 1
$c)$ 9
$d)$ NONE

2) If a, b, c, d are in $G.P.$, then, $(a^3 + b^3)^{-1}, (b^3 + c^3)^{-1}, (c^3 + d^3)^{-1}$ are in
$a)$ Arithmetic Progression
$b)$ Geometric Progression
$c)$ Harmonic Progression
$d)$ NONE

3) The solution set of the inequality $||x| - 1| < 1 - x$ is
$a)$ $(-\infty, 0)$
$b)$ $(-\infty, \infty)$
$c)$ $(0, \infty)$
$d)$ $(-1, 1)$

4) The distance of the curve, $y = x^2$,from the straight line $2x - y = 4$ is maximum at the point
$a)$ $(-1, 1)$
$b)$ $(1, 1)$
$c)$ $(2, 4)$
$d)$ $(\frac{1}{2}, \frac{1}{4})$

5) Let $f(x) = |x-2|^3 + sin|x-2| + |x-2| + cos|x-3|$. $f(x)$ is not differentiable at
$a)$ exactly 1 point
$b)$ exactly 2 points
$c)$ atmost 2 points
$d)$ atleast 1 point

6) Let p and q be two non-zero real numbers and $pq \neq -1$. Suppose the roots of the equation

$$\left(p + \frac{1}{p}\right)x^2 - \left(p + \frac{1}{p} + q + \frac{1}{q}\right)x + \left(q + \frac{1}{q}\right) = 0$$

are distinct and rational. Then,
$a)$ $p = q$
$b)$ p and q must be rational numbers
$c)$ $p + q$ must be a rational number
$d)$ $\frac{p}{q}$ must be a rational number

7) Let A be the set of functions $f : R \to R$ for which $|f(x) - f(y)| \leq |x-y|^2$, for all $x, y \in R$ and $f(0) = 0$. Then, for any $f \in A$,, which one is TRUE?

$a)$ the function $g(x) = e^{f^2(x)} \in A$
$b)$ the functions $g(x) = P(f(x)) \in A$ for every polynomial P
$c)$ the function $g(x) = x + f^2(x) \in A$
$d)$ $NONE$

8) The lengths of the sides of a right-angled triangle are numerically equal to the arithmetic mean, the geometric mean and the harmonic mean of two distinct positive real numbers $a, b, a > b$. Then, $\frac{a}{b}$ is

$a)$ $5 + \sqrt{2}$
$b)$ $2 + \sqrt{3}$
$c)$ $2 + \sqrt{5}$
$d)$ $3 + \sqrt{2}$

9) If $\left(\binom{n}{0} + \binom{n}{1}\right)\left(\binom{n}{1} + \binom{n}{2}\right) .... \left(\binom{n}{n-1} + \binom{n}{n}\right) = k\binom{n}{0}\binom{n}{1}....\binom{n}{n-1}$

$a)$ $\frac{n^n}{n!}$
$b)$ $\frac{(n+1)^n}{n!}$
$c)$ $\frac{(n+1)^n}{n*n!}$
$d)$ $\frac{(n+1)^{n+1}}{n!}$

10) The number of points at which the function

$$f(x) = \begin{cases} max(1+x, 1-x) & x < 0 \\ min(1+x, 1+x^2) & x \geq 0 \end{cases}$$

is not differentiable is

$a)$ 1
$b)$ 0
$c)$ 2
$d)$ $NONE$

11) The set of values of a for which the integral $\int\limits_0^2 (|x-a| - |x-1|)\, dx$ is non-negative is

$a)$ all numbers $a \geq 1$
$b)$ all real numbers
$c)$ all numbers a with $0 \leq a \leq 2$
$d)$ all numbers $a \leq 1$

12) For real numbers a, b, c; let

$$\begin{bmatrix} a & ac & 0 \\ 1 & c & 0 \\ b & bc & 1 \end{bmatrix}.$$

Then, which statement is TRUE?

$a)$ Rank(M) $= 3$ for every $a, b, c \in R$
$b)$ If $a + c = 0$, then, M is diagonalizable for every $b \in R$
$c)$ M has a pair of orthogonal eigenvectors for every $a, b, c \in R$
$d)$ If $b = 0$ and $a + c = 1$, then, M is not idempotent

13) Consider the matrix $A = I_9 - 2u^T u$ with $u = \frac{1}{3}[1,1,1,1,1,1,1,1,1]$, where I_9 is the 9×9 identity matrix and u^T is the transpose of u. If λ and μ are two distinct eigenvalues of A,then, $|\lambda - \mu|$ is

$a)$ 1
$b)$ 2
$c)$ 3
$d)$ $NONE$

14) Suppose M is a 5×5 matrix with real entries and $p(x) = det(xI - M)$. Then, which statement is TRUE?

$a)$ $p(0) = det(M)$
$b)$ every eigenvalue of M is real if $p(1) + p(2) = 0 = p(2) + p(3)$
$c)$ $M^{-1}$is necessarily a polynomial in M of degree 4 if M is invertible
$d)$ M is not invertible if $M^2 - 2M = 0$

15)$\lim\limits_{x \to 0} \left(\frac{sin(x)}{x}\right)^{\frac{1}{x^2}}$ is

$a)$ 0
$b)$ 1
$c)$ $e^{-\frac{1}{2}}$
$d)$ $e^{-\frac{1}{6}}$

16) The number of ways of distributing 12 identical oranges among 4 children so that every child gets atleast one and no child more than 4 is

a) 31
b) 52
c) 35
d) 42

17) Let $X_1, X_2, .., X_n$ be a random sample of size $n(\geq 2)$ from an exponential distribution with the probability density function

$$f\left(x;\theta\right)=\begin{cases}e^{-(x-2\theta)} & x>2\theta\\ 0 & otherwise\end{cases}$$

where $\theta\in(0,\infty)$. If $X_{(1)}=min\left\{X_1,..,X_n\right\}$,then, the conditional expectation

$$E\left[\frac{1}{\theta}\left(X_{(1)}-\frac{1}{n}\right)|X_1-X_2=2\right]\ is$$

a) 0
b) 1
c) 2
d) *NONE*

18) Let (X, Y) be a random vector such that, for any $y>0$, the conditional probability density function of X given $Y=y$ is

$$f_{X|Y=y}\left(x\right)=\begin{cases}ye^{-yx} & x>0\\ 0 & otherwise\end{cases}.$$

If the marginal probability density function of Y is

$$g(y)=\begin{cases}ye^{-y} & y>0\\ 0 & otherwise\end{cases};$$

then, $E(Y|X=1)$ is

a) 1
b) 1.5
c) 2
d) NONE

19) Let $X_1, X_2, ..., X_n$ be a random sample from an exponential distribution with probability density function

$$f(x;\theta) = \begin{cases} \theta e^{-\theta x} & x > 0 \\ 0 & otherwise \end{cases}.$$

where $\theta \in (0, \infty)$ is unknown. Let $\alpha \in (0,1)$ be fixed and let β be the power of the most powerful test of size α for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$. Consider the critical region

$$R = \left\{ (x_1, x_2, .., x_n) \in R^n : \sum_{I=1}^{n} x_i > \frac{1}{2}\chi^2_{2n}(1-\alpha) \right\},$$

where for any $\gamma \in (0,1)$, $\chi^2_{2n}(\gamma)$ is a fixed point such that $P(\chi^2_{2n} > \chi^2_{2n}(\gamma)) = \gamma$. Then, the critical region R corresponds to the

$a)$ most powerful test of size α for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$
$b)$ most powerful test of size $1-\alpha$ for testing $H_0^* : \theta = 2$ against $H_1^* : \theta = 1$
$c)$ most powerful test of size β for testing $H_0^* : \theta = 2$ against $H_1^* : \theta = 1$
$d)$ most powerful test of size $1-\beta$ for testing $H_0^* : \theta = 2$ against $H_1^* : \theta = 1$

20) Let (X, Y) be a random vector with joint moment generating function

$$M(t_1, t_2) = \frac{1}{(1-(t_1+t_2))(1-t_2)}, -\infty < t_1 < \infty, -\infty < t_2 < min\{1, 1-t_1\}.$$

Let $Z = X + Y$. Then, $Var(Z)$ is equal to

$a)$ 3
$b)$ 4
$c)$ 5
$d)$ 6

21) A fair die is rolled twice independently. Let X and Y denote the outcomes of the first and second roll respectively. Then, $E(X+Y|(X-Y)^2 = 1)$ is

$a)$ 0
$b)$ 1
$c)$ 2
$d)$ $NONE$

22) If the marginal pdf of the k^{th}order statistic of a random sample of size 8 from a uniform distribution on $[0, 2]$ is

$$f(x) = \begin{cases} \frac{7}{32}x^6(2-x) & 0 < x < 2 \\ 0 & otherwise \end{cases}$$

$a)$ 1
$b)$ 2
$c)$ 7
$d)$ $NONE$

23) Let X be a random variable having moment generating function

$$M(t) = \frac{e^t - 1}{t(1-t)}, t < 1.$$

Then, $P(X < 1)$ is
$a)$ e^{-1}
$b)$ $1 - e^{-1}$
$c)$ e^{-2}
$d)$ $NONE$

24) Let X and Y be two independent exponential random variables with $E(X^2) = \frac{1}{2}$ and $E(Y^2) = \frac{2}{9}$. Then $P(X < 2Y)$ (rounded off to two decimal places) is equal to
$a)$ $\frac{1}{3}$
$b)$ $\frac{4}{7}$
$c)$ $\frac{2}{9}$
$d)$ NONE

25) Let $X_1, X_2, .., X_n$ be a random sample from a population $f(x;\theta)$, where θ is a parameter. Then which one of the following statements is NOT true?

a) $\sum_{i=1}^{n} X_i$ is a complete and sufficient statistic for θ, if

$$f(x;\theta) = \frac{e^{-\theta}\theta^x}{x!}, x = 0, 1, 2, .. \ and\ \theta > 0$$

b) $\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2\right)$ is a complete and sufficient statistic for θ, if

$$f(x;\theta) = \frac{1}{\sqrt{2\pi}\theta} e^{-\frac{1}{2\theta^2}(x-\theta)^2}, -\infty < x < \infty \ and\ \theta > 0$$

c) $f(x;\theta) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0$ has monotone likelihood ratio property in $\prod_{i=1}^{n} X_i$

d) $X_{(n)} - X_{(1)}$ is ancillary statistic for θ if $f(x;\theta) = 1, 0 < \theta < x < \theta + 1$, where $X_{(1)} = min\{X_1, X_2, .., X_n\}$ and $X_{(n)} = max\{X_1, X_2, .., X_n\}$

26) A company sometimes stops payments of quarterly dividends. If the company pays the quarterly dividend, the probability that the next one will be paid is 0.7. If the company stops the quarterly dividend, the probability that the next quarterly dividend will not be paid is 0.5. Then the probability (rounded off to three decimal places) that the company will not pay quarterly dividend in the long run is

a) 0.1
b) 0.3
c) 0.5
d) *NONE*

27) Given observed $(y_1, x_1), (y_2, x_2), .., (y_n, x_n)$,the sum of squares

$$\sum_{i=1}^{n} (y_i - \alpha - \beta x_i)^2$$

is minimised with respect to α and β to obtain estimators $\hat{\alpha}$ and $\hat{\beta}$. Suppose that the actual model is

$$E(Y_i|X_i = x_i) = \alpha_0 + \frac{\beta_0}{c} x_i$$

for all i, where $c \in (0, 1)$ is a fixed number and $\alpha_0 > 0$ and $\beta_0 > 0$ are unknown parameters. Then,

a) $E(\hat{\alpha}) = \alpha_0$ and $E(\hat{\beta}) = \beta_0$
b) $E(\hat{\alpha}) \neq \alpha_0$ and $E(\hat{\beta}) > \beta_0$
c) $E(\hat{\alpha}) = \alpha_0$ and $E(\hat{\beta}) > \beta_0$
d) $E(\hat{\alpha}) \neq \alpha_0$ and $E(\hat{\beta}) = \beta_0$

28) Consider a finite population of size $N > 1$ with units $U_1, U_2, .., U_N$. The following sampling method is used to select a sample: either the sample consists of only one unit U_j with probability $\frac{1}{N+1}$ for any $j = 1, 2, .., N$,or it consists of the whole population with probability $\frac{1}{N+1}$. Then, the expected sample size is

$a)$ $\frac{N+2}{N+1}$
$b)$ $\frac{2N+1}{N+1}$
$c)$ $\frac{2N}{N+1}$
$d)$NONE

29) Let X and Y be RVs with joint cumulative distribution function $F(x, y)$. Then, which are sufficient for $(x, y) \in R^2$ to be a point of continuity of F?

$a)$ $P(X = x, Y = y) = 0$
$b)$ Either $P(X = x) = 0$ or $P(Y = y) = 0$
$c)$ $P(X = x) = 0$ and $P(Y = y) = 0$
$d)$ $P(X = x, Y \leq y) = 0$ and $P(X \leq x, Y = y) = 0$

30) Suppose X and Y are independent RVs where Y is symmetric about 0. Let $U = X + Y$ and $V = X - Y$. Then, which is TRUE?

$a)$ U and V are always independent.
$b)$ U and V have same distribution
$c)$ U is always symmetric about 0
$d)$ V is always symmetric about 0