START WRITING ANSWERS FROM HERE:

9)
$$\begin{pmatrix} 1 & 1 & -4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}$$
$\uparrow x_1$ $\uparrow x_2$ $\uparrow x_3$

$u_1 = x_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $u_2 = x_2 - \left(\frac{u_1 \cdot x_2}{u_1 \cdot u_1}\right) u_1$

$\Rightarrow u_2 = \begin{pmatrix} 1 \\ 4 \\ 4 \\ -1 \end{pmatrix} - \frac{8}{4}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 2 \\ -3 \end{pmatrix}$

$\Rightarrow u_3 = x_3 - \left(\frac{u_1 \cdot x_3}{u_1 \cdot u_1}\right) u_1 - \left(\frac{u_2 \cdot x_3}{u_2 \cdot u_2}\right) u_2$

$u_3 = \begin{pmatrix} -4 \\ -2 \\ 2 \\ 0 \end{pmatrix} - \frac{24}{18}\begin{pmatrix} -1 \\ 2 \\ 2 \\ -3 \end{pmatrix} - \left(\frac{-4}{4}\right)\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

$u_3 = \begin{pmatrix} -4 \\ -2 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{2}{9} \\ -\frac{4}{9} \\ -\frac{4}{9} \\ \frac{6}{9} \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{25}{9} \\ -\frac{10}{9} \\ \frac{23}{9} \\ \frac{15}{9} \end{pmatrix}$

$$e_1 = \frac{u_1}{|u_1|} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \quad e_2 = \frac{u_2}{|u_2|} = \begin{pmatrix} \frac{-1}{3\sqrt{2}} \\ \frac{2}{3\sqrt{2}} \\ \frac{2}{3\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$$

$$e_3 = \frac{u_3}{|u_3|} \quad \& \quad |u_3| = \frac{2\sqrt{43}}{3}$$

$$e_3 = \begin{pmatrix} \frac{-25 \times 3}{9 \times 2\sqrt{43}} \\ \frac{-13 \times 3}{9 \times 2\sqrt{43}} \\ \frac{23 \times 3}{9 \times 2\sqrt{43}} \\ \frac{15 \times 3}{9 \times 2\sqrt{43}} \end{pmatrix} = \begin{pmatrix} \frac{-25}{6\sqrt{43}} \\ \frac{-13}{6\sqrt{43}} \\ \frac{23}{6\sqrt{43}} \\ \frac{15}{6\sqrt{43}} \end{pmatrix}$$

$$Q = \begin{bmatrix} \frac{\pm 1}{2} & \frac{-1}{3\sqrt{2}} & \frac{-25}{6\sqrt{43}} \\ \frac{1}{2} & \frac{2}{3\sqrt{2}} & \frac{-13}{6\sqrt{43}} \\ \frac{1}{2} & \frac{2}{3\sqrt{2}} & \frac{23}{6\sqrt{43}} \\ \frac{1}{2} & \frac{-1}{\sqrt{2}} & \frac{15}{6\sqrt{43}} \end{bmatrix}$$

$$Q^T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{-1}{3\sqrt{2}} & \frac{2}{3\sqrt{2}} & \frac{2}{3\sqrt{2}} & \frac{-3}{3\sqrt{2}} \\ \frac{-25}{6\sqrt{43}} & \frac{-13}{6\sqrt{43}} & \frac{23}{6\sqrt{43}} & \frac{15}{6\sqrt{43}} \end{bmatrix}_{3\times 4}, \quad A = \begin{pmatrix} 1 & 1 & -4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}$$

~~$R = Q^T$~~ $\quad A = QR$

$QQ^T = I$, $Q^T = Q^{-1}$

$Q^T \quad Q^{-1}A = R$

$Q^TA = R$

$$R = \begin{bmatrix} 2 & 4 & -2 \\ 0 & \frac{6}{\sqrt{2}} & \frac{4}{3\sqrt{2}} \\ 0 & 0 & \frac{172}{6\sqrt{43}} \end{bmatrix}$$

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