MA2.101 Linear Algebra
Spring'24

Assignment 4

Deadline: 1st February, 11:59pm

Instructions:

1) This assignment consists of 3 problems. All problems are compulsory.
2) Mention all assumptions while answering the questions.
3) Be clear in your arguments. Vague arguments shall not be given full credit.
4) Only Handwritten Submissions are allowed. Scan and upload it on moodle.

Problems:

1. On R^n, define two properties: $\overline{\alpha} \oplus \overline{\beta} = \overline{\alpha} - \overline{\beta}$ and $c\overline{\alpha} = -c\overline{\alpha}$.
   Which of the axioms for the vector space are satisfied by $(R^n, \oplus, \cdot)$?

2. Let V be the set of all complex-valued functions f on the real line such that $\forall t \in R$, $f(-t) = f((t))^* = f^*(t)$, where $f^*(t)$ denotes the complex conjugation of $f(t)$.

   (a) Show that V with operations $(f+g)(t) = f(t) + g(t)$ and $(cf)(t) = cf(t)$ is a vector space over the field R.

   (b) Give an example of a function f_n in V which is not real-valued.

3. Prove the following theorem:

   A non-empty subset W of vector space V is a subspace of V if and only if, for each pair of vectors $\overline{\alpha}, \beta \in W$ and each scalar $c \in F$, the vector $c\overline{\alpha} + \beta \in W$.