Classical Mechanics(H1) (SC1.102)
IIIT-H, Semester Winter 24, Assignment 1

*Submission deadline: 14*th *January 2024*

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1. The infinitesimal volume element dV, in spherical polar coordinate is

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi \,.$$

   Calculate the volume of a hemisphere of radius is R lying on xy plane and centered at the origin.

2. Calculate the divergence of the position vector $\vec{r}$ of a particle in spherical polar coordinate. Also calculate the gradiant of the magnitude of the position vector.

3. Find the work done by a force field $\vec{F} = \hat{i}x\sin y + \hat{j}y$ for moving an object along the curve $\vec{r} = (1+t)\hat{i} + t^3\hat{j}$ for $t = 1$ to $t = 2$.

4. *The brachistochrone problem:* Using calculus of variation, find the equation of the path between two points along which a particle falling from the rest under gravity travels from the higher point to the lowest point in the least amount of time.

5. A particle moving along a curve $y = y(x)$ with velocity $ds/dt = x$ travels from a point $(0,0)$ to a point $(1,1)$. If the time for this travel is minimum, then what kind of path the particle has followed.

6. Consider that the cost of flying an aircraft at height (vertically up) z is $e^{-\kappa z}$ per unit distance of flight path, where κ is a positive constant. Assume that the aircraft flies in the $x - z$ plane. Find the differential equation for z so that the flying cost will be minimum. You don't have to solve the equation.