

START WRITING ANSWERS FROM HERE:

$$A) \begin{pmatrix} 1 & 1 & -4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}$$

$\uparrow \quad \uparrow \quad \uparrow$   
 $x_1 \quad x_2 \quad x_3$

$$u_1 = x_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad u_2 = x_2 - \left( \frac{u_1 \cdot x_2}{u_1 \cdot u_1} \right) u_1$$

$$\Rightarrow u_2 = \begin{pmatrix} 1 \\ 4 \\ 4 \\ -1 \end{pmatrix} - \frac{8}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 2 \\ -3 \end{pmatrix}$$

$$\Rightarrow u_3 = x_3 - \left( \frac{u_1 \cdot x_3}{u_1 \cdot u_1} \right) u_1 - \left( \frac{u_2 \cdot x_3}{u_2 \cdot u_2} \right) u_2$$

$$u_3 = \begin{pmatrix} -4 \\ -2 \\ 2 \\ 0 \end{pmatrix} - \frac{24}{18} \begin{pmatrix} -1 \\ 2 \\ 2 \\ -3 \end{pmatrix} - \left( \frac{-4}{4} \right) \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$u_3 = \begin{pmatrix} -4 \\ -2 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{2}{9} \\ \frac{-4}{9} \\ \frac{-4}{9} \\ \frac{6}{9} \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{-25}{9} \\ \frac{-10}{9} \\ \frac{23}{9} \\ \frac{15}{9} \end{pmatrix}$$



$$e_1 = \frac{\mu_1}{|\mu_1|} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \quad e_2 = \frac{\mu_2}{|\mu_2|} = \begin{pmatrix} \frac{-1}{3\sqrt{2}} \\ \frac{2}{3\sqrt{2}} \\ \frac{2}{3\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$$

$$e_3 = \frac{\mu_3}{|\mu_3|} = \frac{2\sqrt{43}}{3}$$

$$e_3 = \begin{pmatrix} \frac{-25 \times 3}{9 \times 2\sqrt{43}} \\ \frac{-13 \times 3}{9 \times 2\sqrt{43}} \\ \frac{23 \times 3}{9 \times 2\sqrt{43}} \\ \frac{15 \times 3}{9 \times 2\sqrt{43}} \end{pmatrix} = \begin{pmatrix} \frac{-25}{6\sqrt{43}} \\ \frac{-13}{6\sqrt{43}} \\ \frac{23}{6\sqrt{43}} \\ \frac{15}{\sqrt{43}} \end{pmatrix}$$

$$Q = \begin{bmatrix} \frac{1}{2} & \frac{-1}{3\sqrt{2}} & \frac{-25}{6\sqrt{43}} \\ \frac{1}{2} & \frac{2}{3\sqrt{2}} & \frac{-13}{6\sqrt{43}} \\ \frac{1}{2} & \frac{2}{3\sqrt{2}} & \frac{23}{6\sqrt{43}} \\ \frac{1}{2} & \frac{-1}{\sqrt{2}} & \frac{15}{\sqrt{43}} \end{bmatrix}$$



$$Q^T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{3\sqrt{2}} & \frac{2}{3\sqrt{2}} & \frac{2}{3\sqrt{2}} & \frac{-3}{3\sqrt{2}} \\ -\frac{25}{6\sqrt{43}} & -\frac{13}{6\sqrt{43}} & \frac{23}{6\sqrt{43}} & \frac{15}{6\sqrt{43}} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & -4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

3x4

$$R = \cancel{Q^T}$$

$$A = QR$$

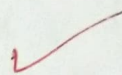
$$QQ^T = I, Q^T = Q^{-1}$$

$$\cancel{Q^T} Q^{-1} A = R$$

$$Q^T A = R$$

$$R =$$

$$\begin{bmatrix} 2 & 4 & -2 \\ 0 & \frac{6}{\sqrt{2}} & \frac{4}{3\sqrt{2}} \\ 0 & 0 & \frac{172}{6\sqrt{43}} \end{bmatrix}$$



5