Classical Mechanics(H1) (SC1.102)

IIIT-H, Semester Winter 24, Assignment 1

Submission deadline: 2nd February 2024

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1. Masses are fixed at each end of a massless rod of length ℓ. The mid point of the rod moves without friction on a surface of sphere. Set up the generalized coordinates. Assuming no gravity, write the kinetic term of the Lagrangian of the system

2. Consider a pendulum of mass m_1 and suspended from a rigid support from a massless wire of length ℓ_1. Another pendulum of mass m_1 is suspended by another massless wire of length ℓ_2. The system oscillates in the vertical plane. Calculate the number of degrees of freedom of the system. Set up the Lagrangian and obtain the equations of motion for the generalized coordinates (you are not required to solve them).

3. A pendulum of mass m is suspended from a rigid support with a massless wire of length ℓ. Unlike ordinary pendulum this pendulum is not constrained to oscillate in a vertical plane. Set up the generalized coordinates and the Lagrangian for the system. Write down the equations of motions for the generalized coordinate.

4. A massless tube can rotate in a vertical plane about one of its ends. A mass m slides inside the tube without any friction. Set up the equation of motion of the system considering the gravitational acceleration as g.

5. Suppose there is a Lagrangian of a system denoted by L. To this Lagrangian we add a total time derivative dF/dt of an arbitrary function F and form a new Lagrangian $L' = L + dF/dt$. Prove that L and L' have the same equations of motion.

6. A particle moves under a central force about the point $r = 0$. The equation of the orbit is given by $r = e^{-\theta}$. Determine the force. Also calculate the total energy of the system.

7. Determine the equation of orbit for a particle moving under the force

$$F(r) = -\frac{\ell^2}{\mu}\left(\frac{1}{r^3} + \frac{2K^2}{r^5}\right),$$

where K is a constant and the rest of symbols have their usual meanings.