MA2.101 Linear Algebra
Spring'24

Assignment 3

Deadline: 25th January, 11:59pm

Instructions:

1) This assignment consists of 4 problems. All problems are compulsory.
2) Mention all assumptions while answering the questions.
3) Be clear in your arguments. Vague arguments shall not be given full credit.
4) Only Handwritten Submissions are allowed. Scan and upload it on moodle.

Problems:

1. If A, B, C are matrices over a field F such that the products BC and $A(BC)$ are well defined, and so are the products AB and $(AB)C$, then prove $A(BC) = (AB)C$.

2. Let e be an elementary row operation and let E be an elementary matrix of size $m \times m$ such that $E = e(I_{m\times m})$ then prove that $e(A) = EA$ holds $\forall$ matrices A of size $m \times n$.

3. If A is an $n \times n$ matrix, prove that the following statements are equivalent.

   (a) A is invertible.

   (b) A is row-equivalent to the $n \times n$ identity matrix.

   (c) A is a product of elementary matrices.

4. Let A be a $n \times m$ matrix and B be a $n \times 1$ vector with real entries, suppose the equation $AX = B$ (here, X belonds to R^m) admits a unique solution, then we can conclude that:

   (a) m >= n

   (b) n >= m

   (c) m = n

   (d) n > m

   Support your answer with appropriate reasoning.