MA2.101 Linear Algebra
Spring'24

Tutorial 2

1. Let A be an $n \times n$ matrix over a field $\mathcal{F}$. Prove the following two statements:
   - If A is invertible and $AB = 0$ for some $n \times n$ matrix B, then $B = 0$.
   - If A is not invertible, then there exists an $n \times n$ matrix B such that $AB = 0$ but $B \neq 0$.

2. Find all solutions to the following system of equations:

$$x_1 + 2x_2 + x_3 + x_4 = 7 \tag{1}$$

$$2x_1 + 4x_2 + 4x_3 - 2x_4 = 24 \tag{2}$$

$$3x_1 + 6x_2 + 9x_4 = 6 \tag{3}$$

   Notice that the system of linear equations is of the form $AX = Y$, where A and Y are known and one needs to solve for X. Use elementary row operations to derive row-reduced echelon form for A in order to solve for X.

3. a) Mention the conditions for matrix to be in row echelon form and in reduced row echelon form respectively.
   b) Augment the following set of equations in matrix form and find its reduced row echelon form. What can you infer from the reduced row echelon matrix?

$$x_2 + 5x_3 = -4 \tag{4}$$

$$x_1 + 4x_2 + 3x_3 = -2 \tag{5}$$

$$2x_1 + 7x_2 + x_3 = -2 \tag{6}$$

4. If $A_1, A_2, \ldots, A_r$ are invertible matrices then $B = A_1 A_2 \ldots A_r$ is also invertible.

5. Let $\mathbf{b}^T = [1, 2, -1, -2]$. Suppose A is a 4×4 matrix such that the linear system $Ax = b$ has no solution. Mark each of the statements given below as TRUE or FALSE?

   (a) The homogeneous system $Ax = 0$ has only the trivial solution.

   (b) The matrix A is invertible.

   (c) Let $c^T = [-1, -2, 1, 2]$. Then, the system $Ax = c$ has no solution.

   (d) Let $B = RREF(A)$. Then,
   i) B[4,:] = [0,0,0,0].
   ii) B[4,:] = [0,0,0,1].