LA Assignment 7 paunal Sehearia 2025113019 the Hoffmann & kince] a let N be the north space for T the hosis for N be it at

There are vectors taken, a a bour of for v We know, Tati, Itan span the range of T une range of The State of Total State of Total State of Total State of Total State of St checking if Tarket! Tan are independent a fore have except a (Ci CF) s. that have icalor ci (ci CF) s. that = k+1 ei (Tai) =0 =) T(= c; x;) = 0 by definition of multispace; one vector or crest = = c; di must belong in N. ent y 2 EBN, EPP the of can be represented in finely combination of the basis nectors of m, x2, ... , xk To be di to that Franco (1) & Q.

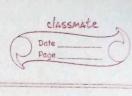
E (i $\alpha_i = 0$)

i. i. Let $\alpha_i = 0$ i. we know $\{d_1, \dots, \alpha_k\}$ and $\{\alpha_{k+1}, \dots, \alpha_n\}$ are in dependent

i. $b_1 = b_2 = \dots = b_k = C_{k+1}$ The property of the property to properly and a prost to

& same dance Sorm a basis of for the nange of T, and a basis of for the nange of T, and a directly of T, a nank (T) + nullity (T) = ain (Y) Etlence proved? 0. (: 1:1:0: B = {\array, \cdots, \array \a be the ordered basis of for VI W nespectively. Her each P, y; 15pcm; 15q, n; P, q & T are define a linear transformation $E^{p,q}$ from V into W by $E^{p,q}(\alpha_i) = 0 \quad \text{i.t.}$ Sin Pr We know that there is a might linear transformation from V to W that latisfy there given conditions whether the mn' thanfanation. checking & EP. 9 forms a back & for L(V, W) Let The a linear transformation from Vinto W. Hoe each y, 15 j & n, let the ordered basis B is Taj is

Taj = \(\frac{\infty}{I} \) App \(\frac{\infty}{I} \) \(\



we wish to show that T = ZZ Apg EP. Q Let V be the linear transformation in right hard member of D. Then, Vj.

Va; = \(\frac{\times}{\times} \) App \(\frac{\times}{\times} \)) = E E App Sig Bp = E Apilp : U = T [& we have aldready noted that union than formations are vivigue] E ?? spano L (V,W) & BUZES A EPA is the Tero transformation Ua: = 0 41, 10 E AND PO TO But : fr & B'Charing W)

Fach of fr is independent. :. Ap; = 0 for 3 to hall 4 p; : £ P.9 is independent & also spans L(V,W)

: £ P.9 is the basis for L(V,W)

but, we know, £ is a my transfer mation

: dum(L(V,W)) = mn & = dum(V) x dim(W)

Heo, since m, n are finite, mn is also finite. : L(

junite dimensional.