

**COURSE: LINEAR ALGEBRA**  
**Course Code: MA2.101**

**Spring-2024**

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**Assignment 2: [Released date: 9.04.2024] [Submission Date: 19.04.2024]**

**Full Marks- 25**

**1.** Find all the real values of  $k$  for which the following matrices are diagonalizable:

**a)**  $\begin{pmatrix} 1 & k & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  **b)**  $\begin{pmatrix} 1 & 1 & k \\ 1 & 1 & k \\ 1 & 1 & k \end{pmatrix}$  **[CO-2][4+4=8]**

**2. (a)** Let  $\mathbf{A}$  be an invertible matrix. Prove that if  $\mathbf{A}$  is diagonalizable so is  $\mathbf{A}^{-1}$ .

**(b)** Compute the indicated power of the following matrix:

$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}^{2015}$  **[CO-2][2+5=7]**

**3. (a)** Consider the vectors  $\mathbf{a} = (1-i, 1+2i)$ ,  $\mathbf{b} = (2+i, z)$  in  $\mathbb{C}^2$ . Determine the complex number  $z$  such that  $\{\mathbf{a}, \mathbf{b}\}$  is an orthogonal set of vectors, and hence obtain an orthonormal set of vectors in  $\mathbb{C}^2$ .

**(b)** The Gram-Schmidt process applied to the vectors  $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$  yields the same basis as the Gram-Schmidt process applied to the vectors  $\{\mathbf{a}_3, \mathbf{a}_2, \mathbf{a}_1\}$ . **[CO-2][3+2=5]**

**4.** Let  $\mathbf{M}$  be a square matrix. Either proof or give counterexample.

a) If  $\mathbf{M}$  is diagonalizable, then so is  $\mathbf{M}^2$ .

b) If  $\mathbf{M}^2$  is diagonalizable, then so is  $\mathbf{M}$ .

**[CO-2][2.5+2.5=5]**

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