Classical Mechanics(H1) (SC1.102)

IIIT-H, Semester Winter 24, Assignment 3

Submission deadline: 3rd March 2024

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1. Three identical pendulum of mass m and length ℓ are hanged side by side on a horizontal straight line. The masses are connected by two massless spring of spring constant k (1st one connected to second, second one with the third). Calculate the eigen frequencies. Determine the normal modes and the motions they correspond to.

2. A tennis ball is dropped from height h on a surface. It bounces a couple of times and then comes to a halt. Draw the phase space portrait/diagram of the ball.

3. You are given a Lagrangian
$$L = \frac{1}{2}ma^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) + mga\cos\theta\,,$$
where θ, ϕ are generalized coordinates and rest of the terms are constant. Which of these two coordinates is cyclic? Calculate the momentum corresponding to it show that it is constant in time. Obtain the Hamiltonian for the system. Write down the Hamilton's EoMs (no need to solve).

4. Using fundamental Poisson brackets find the values of α and β for which the equations
$$Q = q^\alpha \cos\beta p, P = q^\alpha \sin\beta p\,,$$
represents a canonical transformation.

5. (a) The Hamiltonian for a system has the form
$$H = \frac{1}{2}\left(\frac{1}{q^2} + p^2q^4\right),$$
find the equation of motion for q and
(b) find a canonical transformation that reduces H to the form of a harmonic oscillator. Show that the solution for transformed variables is such that the equation of motion in part (a) is satisfied.