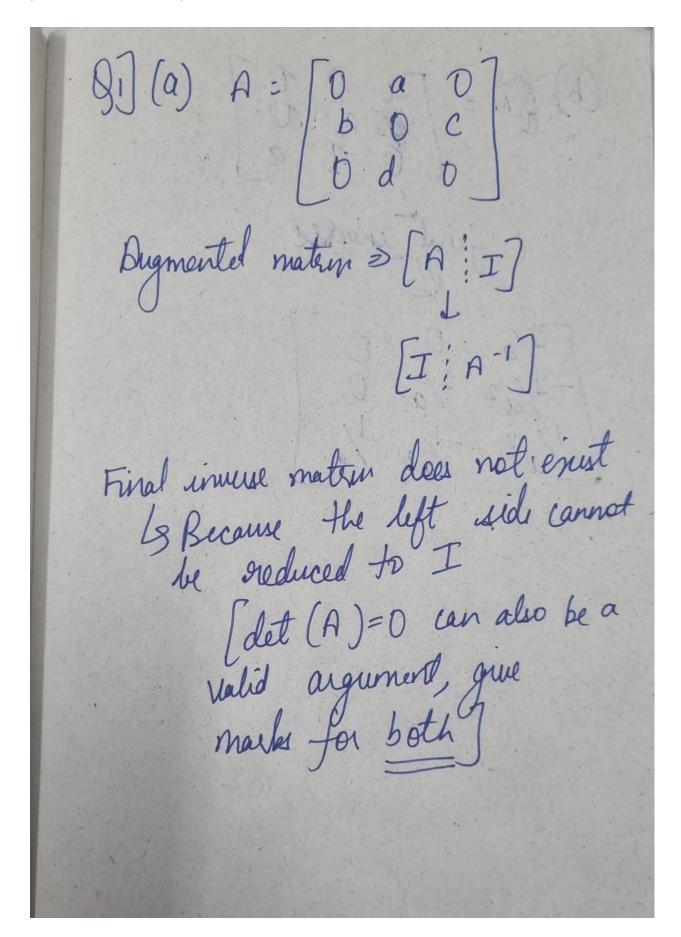
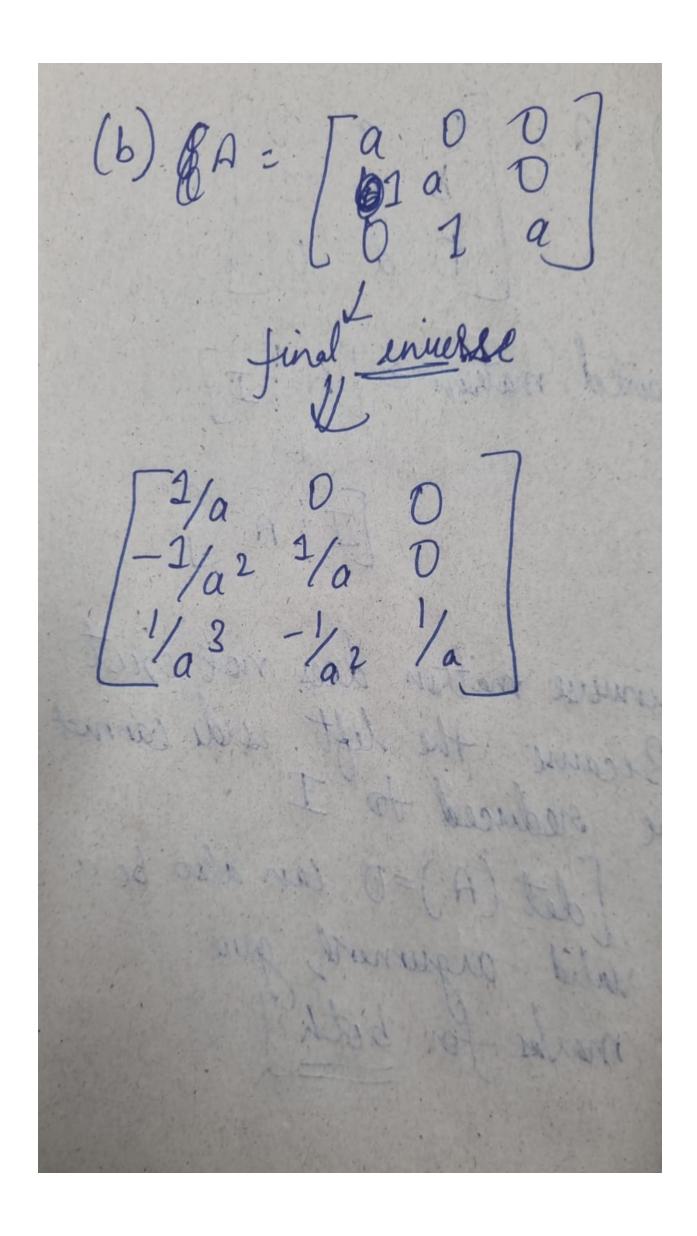
A-1 Answer Key

Question-1 (6 marks)

For each of the part, in case of correct conversion using the Augmented matrix method, award full 3 marks. There are roughly 6 steps for conversion in either part. Cut 0.5 marks per calculation mistake.



A-1 Answer Key



A-1 Answer Key

Question-2 (2 marks)

Given A is nonsingular and symmetric, show that $A^{-1} = (A^{-1})^T$.

Since A is nonsingular, A^{-1} exists. Since $I = I^T$ and $AA^{-1} = I$,

$$AA^{-1} = (AA^{-1})^T$$
.

Since $(AB)^T = B^T A^T$,

$$AA^{-1} = (A^{-1})^T A^T$$
.

Since $AA^{-1} = A^{-1}A = I$, we rearrange the left side to obtain

$$A^{-1}A = (A^{-1})^T A^T$$
.

Since A is symmetric, $A = A^T$, and we can substitute this into the right side to obtain

$$A^{-1}A = (A^{-1})^T A.$$

From here, we see that

$$A^{-1}A(A^{-1}) = (A^{-1})^T A(A^{-1})$$
$$A^{-1}I = (A^{-1})^T I$$
$$A^{-1} = (A^{-1})^T,$$

Award full 2 marks if all steps are followed along with correct properties being stated. In case that properties are not stated everywhere, cut 0.5-1 marks depending on the number of such occurences.

Question-3 (5 marks)

0.5*6 marks for finding each of the matrices correctly.

0.5 mark for stating A^7=A

1.5 mark for correct manipulations to calculate A^2015

Question-4 (3 marks)

Proof by contradiction, take trace on both sides and disprove the claim.

Cut 0.5-1 marks if vague/missing arguments.

In case of any other solutions provided, give marks based on corectness.

Reference Link: https://yutsumura.com/matrix-xy-yx-never-be-the-identity-matrix/

Question-5 (5 marks)

- 2 things are required to be proved for the 1st part:
- 1) The vectors are orthogonal → 0.5*3 for all 3 checking <ui,uj≥0 for all i,j.
- 2) They form basis of $R^3 \rightarrow 1$ mark for showing that the vectors are spanning

For the 2nd part, if all calculations are correct, award 2.5 marks. Cut 0.5 mark per calculation mistake.

$$x_{1}y_{1} + x_{2}y_{2} + x_{3}y_{3} = \omega$$
 $x_{1} + x_{2} + x_{3} = y$
 $x_{1} - x_{2} + x_{3} = 6$
 $x_{1} - 2x_{3} = 6$
 $x_{1} = 5, x_{2} = 3 = 1$

Question-6 (4 marks)

For upper-triangular matrix, we only need to concern with proving that the resulting multiplication would lead to matrix C having cij=0 for all i>j:

Definition. A $n \times n$ matrix A is upper triangular if $a_{ij} = 0$ for i > j.

Now consider A, B upper triangular and C = AB. Then consider C_{ij} entry from C with i > j, given by

$$C_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

$$= \sum_{k=1}^{i-1} a_{ik} b_{kj} + \sum_{k=i}^{n} a_{ik} b_{kj}$$

$$= 0 + 0 = 0$$

since

$$k \le i - 1 \implies a_{ik} = 0$$

 $k \ge i \implies b_{kj} = 0$

for this particular combination of (i, j). We conclude that $C_{ij} = 0$ if i > j hence C is upper triangular.

The above proof should be used as reference, more elaborate reasoning is expected from the students in their answers.

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