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- Ohm's law :

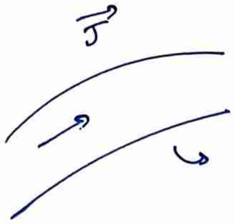
$$\boxed{\vec{J} = \sigma \vec{E}}$$, σ : conductance.

→ Insulators have very low σ , and conductors very high σ.

Can approximate σ to ∞.

$\Rightarrow$ For conductors , $\vec{E} = \dfrac{\vec{J}}{\sigma} = 0$

$\Rightarrow$ Current flow is there but without electric field



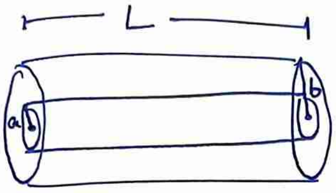
↳ Conductor : $\vec{E} = 0$ inside conductor.

& But there is current flow. (Contradiction!)

How?

Because $\vec{E} = 0$ is valid only for electro STATICS.

Eg:



Both cylinders are charged.

V → pot. diff. b/w a & b.

There'll be current flow radially.

$$E(2\pi r L) = \frac{Q}{\varepsilon_0}$$

$E(2\pi r L)$ → At some dist. r

Q → on cyl. on rad. a

$$\vec{E} = \frac{Q/L}{2\pi \varepsilon_0 r}\,\hat{r}$$

$$V = -\int_b^a \vec{E} \cdot d\vec{l}$$

$$= -\int_b^a \frac{Q}{2\pi \varepsilon_0 L r}\, dr$$

$$= \frac{+Q}{2\pi \varepsilon_0 L} \ln\left(\frac{b}{a}\right)$$

$$= \frac{\lambda}{2\pi \varepsilon_0} \ln\left(\frac{b}{a}\right)$$

$$J = \sigma E = \frac{\sigma \lambda}{2\pi \varepsilon_0 r}.$$

$$\cancel{I = \int J \cdot da = \oint \frac{\sigma\lambda}{2\pi\varepsilon_0 r}\, 2\pi r l = \frac{\sigma\lambda L}{\varepsilon_0}}$$

$$I = \iiint \frac{\sigma\lambda}{2\pi\varepsilon_0 \cancel{r}}\, \cancel{r}\, dr\, d\phi\, dz$$

$$= \frac{\sigma\lambda (b-a) L\, \cancel{2\pi}}{\cancel{2\pi}\varepsilon_0}$$

$$= \frac{\sigma\lambda(b-a)L}{\varepsilon_0}$$

$$\Rightarrow \lambda = \frac{I\varepsilon_0}{\sigma(b-a)L}.$$

$$V = \frac{I\varepsilon_0}{\sigma(b-a)L}\left(\frac{1}{2\pi\varepsilon_0}\right)\ln\left(\frac{b}{a}\right).$$

$$\cancel{\Rightarrow I = \frac{2\pi\sigma L V}{\ln\left(\frac{b}{a}\right)}} \quad \Rightarrow V \propto I$$

→ So if there is electric field, then $F = qE$.

⇒ Charge is accelerated.

So then $i = \frac{dq}{dt}$ should continuously increase.

But this doesn't happen because the charges are continuously colliding with each other.

So they can't accelerate.

~~To~~
~~The~~

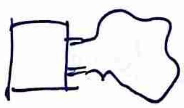
→ Joule's law.

$$\text{Power} = VI \qquad \left(V = \frac{W}{q},\ I = \frac{q}{t} \Rightarrow VI = \underbrace{\frac{W}{t}}_{\text{Energy per unit time}} \right).$$
$$= I^2 R$$

So energy is lost continuously.

How is the energy loss getting compensated?

What is the work done by the $\vec{E}$.



$$\underbrace{\oint \vec{E} \cdot d\vec{l}}_{\int (\vec{\nabla} \times \vec{E}) \cdot d\vec{a} = 0} = 0$$

So the electric field that is driving the charges is not doing any work.

So the energy that is actually driving the current comes from the battery and its called ~~elect~~

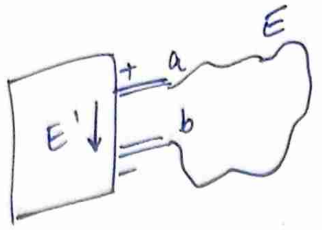
ELECTROMOTIVE FORCE → (Not force, it's energy).

→ Inside the battery, charge gets acculumated on both the terminal. So $\vec{E}$ exists inside battery.

$$\Rightarrow \vec{J} = \sigma (E + \underbrace{E'}_{\text{(Inside battery)}})$$

If ideal conductor,

$$\frac{J}{\sigma} = 0 \Rightarrow \vec{E} + \vec{E'} = 0 \Rightarrow \boxed{\vec{E} = -\vec{E'}}$$



Pot. diff. in battery / Voltage of battery =

Voltage of battery = The energy lost by the battery to drive a charge from a to b

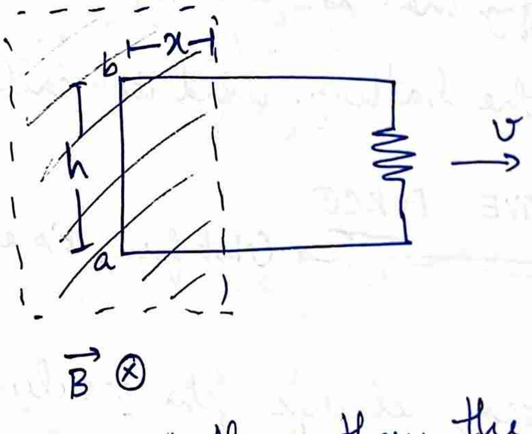
$$V = -\int_a^b \vec{E}\cdot d\vec{l} = \oint \vec{E'}\cdot d\vec{l} = \mathcal{E}$$

E' is there only inside the battery. So can be written as ~~full path~~ $\oint$ ($\because E' = 0$ outside)

→ Other examples of electromotive force:

Generator.

• Principle of generators



b − a = h

Current flows thru the loop.

Because of Lorentz force, ~~the curren~~ the charge experience force, and current flows thru the wire.

$$\mathcal{E} = \int \vec{E}_{mg} \cdot d\vec{l}$$

$$= \int \frac{\text{Force}}{q} \cdot d\vec{l} = \int \frac{\cancel{q}(\vec{v}\times\vec{B})\cdot d\vec{l}}{\cancel{q}}$$

$$= \int (\vec{v}\times\vec{B})\cdot d\vec{l}$$

Here $\vec{v} \perp \vec{B}$. $\Rightarrow$ $\vec{v}\times\vec{B} = vB$.

$$\mathcal{E} = vB(b-a)$$

$$\boxed{\mathcal{E} = vBh}$$

→ The magnetic field doesn't do any work.

→ The work is actually done by the mechanical force which is pulling the loop to give it a velocity v.

(See proof in Griffiths)

Φ → Total flux of magnetic field.

$$\Phi = \int \vec{B}\cdot d\vec{a}$$

$$= Bhx$$

$$\Rightarrow \frac{d\Phi}{dt} = Bh\frac{dx}{dt} = -vBh = -\mathcal{E}$$

$$\Rightarrow \boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$ → Faraday's law

→ The ~~rate of~~ change of magnetic field produces electric field
↳ Faraday's.

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$$

E.dl thru the entire loop

$$\int(\vec{\nabla}\times\vec{E})\cdot d\vec{a} = \cancel{-\frac{d}{dt}\int} -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a}$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ → This is for moving charges.
In electrostatics, $\vec{\nabla}\times\vec{E} = 0$

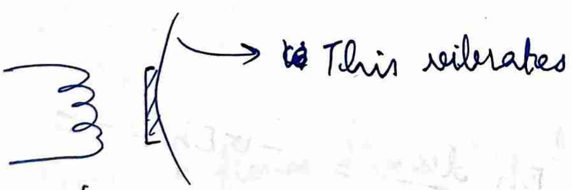
↳ FARADAY'S LAW.

→ Two methods to generate $\vec{E}$:

1) → Put a set of charges together
2) → Take a closed loop, and change the magnetic field through the loop. ~~can get~~

→ $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$, Here $\vec{E}$ is the electric field inside the conductor.

→ Microphone works on this principle.



Video

So then the flux thru the closed loop change. So $\vec{E}$ is generated & hence mechanical energy is converted to electrical energy.

• Inductance :

$$I_2 = M_{21} I_1$$

$$I_1 = M_{12} I_2 .$$

$$\underbrace{M_{21} = M_{12}}_{\text{Mutual inductance.}}$$

$$\Phi = \int \vec{B} \cdot d\vec{a}$$

$$B = \frac{\mu_0}{4\pi} I \int \underbrace{(\quad)}_{\text{dependent on geometry of system.}}$$

So then if geometry of the system remains same.

So $M_{12} = M_{21}$.

Self inductance : $\Phi = \underbrace{L}_{\text{Self inductance}} I$

Applications: Transformers

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$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Divergence of curl of a vector is 0.

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0 .$$

~~IIIly curl of divergence of vector~~

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = -\vec{\nabla}\cdot\left(\frac{\partial \vec{B}}{\partial t}\right)$$

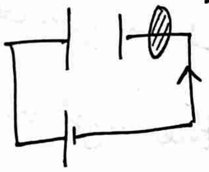
$$0 = 0 \quad (\checkmark)$$

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0$ But $\mu_0\vec{\nabla}\cdot\vec{J}$ need not be 0.

$\mu_0\vec{\nabla}\cdot\vec{J} \rightarrow 0$ only when $\vec{J} = 0$ $\left(\mu_0\vec{\nabla}\cdot\vec{J} = \mu_0\frac{\partial \rho}{\partial t}\right)$

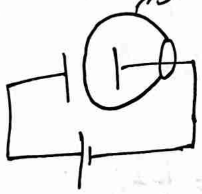
→ Fundamental problems in Ampere's law. :(

Experiment.



I going thru surface

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}.$$
$$I \neq 0$$



Surface

But now thru this surface, there is no current.

So $I_{enc} = 0$.

Contradiction

So correction term req.

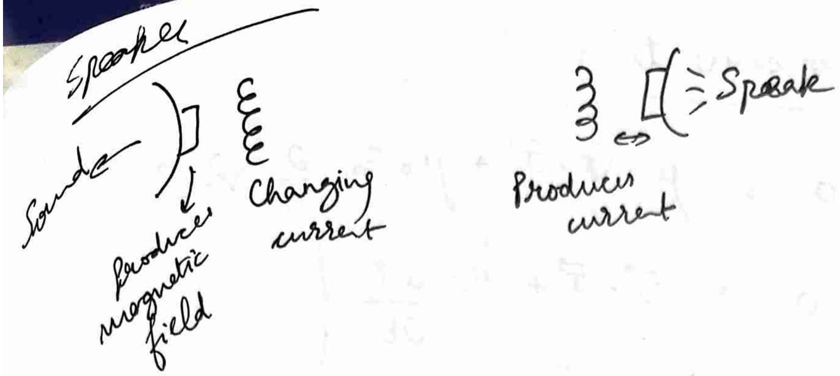
$$\mu_0(\vec{\nabla}\cdot\vec{J}) = -\mu_0\frac{\partial \rho}{\partial t} = -\mu_0\frac{\partial}{\partial t}(\varepsilon_0\vec{\nabla}\cdot\vec{E})$$
$$= -\vec{\nabla}\cdot\left(\mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}\right)$$

⇒ Corrected Ampere's law:

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}} \rightarrow \text{CORRECTED AMPERE'S LAW}$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\cdot(\mu_0\vec{J}) + \vec{\nabla}\left(\mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}\right) = 0$$

Physical significance of the corrected term is changing electric field produces magnetic field.



$\rightarrow J_d = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$.

- **MAXWELL'S EQ^{NS}:**

$$\boxed{\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\varepsilon_0} \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \times \vec{B} &= \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \end{aligned}}$$

$\rightarrow$ All these eq^{ns}: medium: vacuum

In other media,

$$\vec{E} \rightarrow \vec{D} = \varepsilon_0 \vec{E}$$
$$\vec{B} \rightarrow \vec{H} = \frac{1}{\mu_0} \vec{B}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

- Charge and energy conservation

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0 = \mu_0 \vec{\nabla}\cdot\vec{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\vec{\nabla}\cdot\vec{E}$$

$$\boxed{0 = \vec{\nabla}\cdot\vec{J} + \cancel{\varepsilon_0}\frac{\partial \rho}{\partial t}}$$

$\hookrightarrow$ Local charge conservation

- Poynting's theorem

$$U = \begin{matrix} U_e \\ + \\ U_m \end{matrix} = \int_V d^3x \left(\frac{1}{2}\varepsilon_0 \vec{E}\cdot\vec{E}\right) + \int_V d^3x \left(\frac{1}{2\mu_0}\vec{B}\cdot\vec{B}\right)$$

$$\frac{dU}{dt} = \int d^3x \left(\frac{1}{2}\varepsilon_0 \frac{\partial}{\partial t}(\vec{E}\cdot\vec{E}) + \frac{1}{2\mu_0}\frac{\partial}{\partial t}(\vec{B}\cdot\vec{B})\right)$$

$$= \int d^3x \left(\varepsilon_0 \vec{E}\cdot\frac{\partial \vec{E}}{\partial t} + \frac{1}{\mu_0}\vec{B}\cdot\frac{\partial \vec{B}}{\partial t}\right)$$

$$= \int d^3x \left[\frac{1}{\mu_0}\vec{E}\cdot(\vec{\nabla}\times\vec{B}) - \vec{E}\cdot\vec{J} - \frac{1}{\mu_0}\vec{B}\cdot(\vec{\nabla}\times\vec{E})\right]$$

$$\left(-\vec{\nabla}\cdot(\vec{E}\times\vec{B}) = \vec{E}\cdot(\vec{\nabla}\times\vec{B}) - \vec{B}\cdot(\vec{\nabla}\times\vec{E})\right)$$

$$= -\int d^3x\,(\vec{E}\cdot\vec{J}) - \frac{1}{\mu_0}\underbrace{\int \vec{\nabla}\cdot(\vec{E}\times\vec{B})\,d^3x}_{\text{Divergence th.}}$$

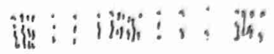
$$= \underbrace{-\int d^3x\,(\vec{E}\cdot\vec{J})}_{\text{Work done on charges}} - \underbrace{\frac{1}{\mu_0}\oint(\vec{E}\times\vec{B})\,d\vec{S}}_{\text{Energy diverging out of surface}}$$

$$\delta W = q\vec{E}\cdot\vec{v}\,\delta t$$
$$= \vec{E}\cdot\vec{I}\,\delta t$$
↳ Current

$$\left(\frac{\delta W}{\delta t} = \vec{E}\cdot\vec{J}\right)$$

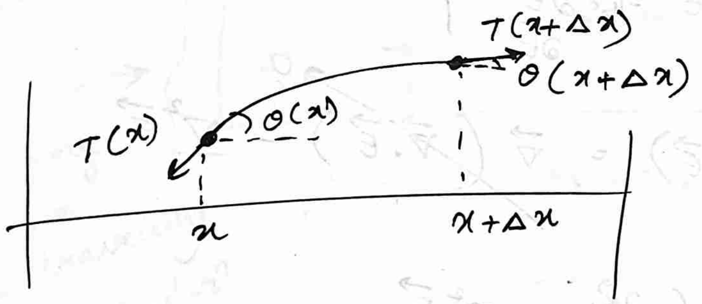
$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$ → Poynting vector.
Physical signif. : Dir^n of the energy

Waves

• Types of waves : Longitudinal 
Transverse. 

$u(x,t)$
↳ Represents displacement.

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u(x,t)}{\partial t^2}$$ → Can get Wave eq^n from this



$$T(x)\cos(\theta(x)) = T(x+\Delta x)\cos(\theta(x+\Delta x))$$
$$\cancel{T(x)\sin(\theta(x)) = T(x+\Delta x)\sin(\theta(x+\Delta x))}$$

$$\cancel{\rho\Delta x\,\frac{\partial^2 u}{\partial t^2}}$$

$$\rho\,\Delta x\,\frac{\partial^2 u}{\partial t^2} = T(x+\Delta x)\sin(\theta(x+\Delta x)) - T(x)\sin(\theta(x)).$$

When θ is very small,

$\sin\theta \approx \tan\theta$

$$\cancel{\rho\Delta x\,\frac{\partial^2 u}{\partial t^2}}$$

Solutions : $u = A\sin(kz - \omega t)$, $k = \frac{2\pi}{\lambda}$, $\omega = \frac{v}{k}$

Amplitude

In general, amplitudes are complex nos.

~~to~~ (Imaginary nos. are very REAL then :))

$$u(z,t) = Ae^{i(kz-\omega t)}$$

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- EM Waves : Free space : $\rho = 0$, $J = 0$

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\left\{ \begin{aligned} \vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \\ \vec{\nabla}\times\vec{B} &= \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \end{aligned} \right\}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\cancelto{0}{(\vec{\nabla}\cdot\vec{E})} - \nabla^2\vec{E}$$

$$\vec{\nabla}\times\left(\frac{-\partial\vec{B}}{\partial t}\right) = -\nabla^2\vec{E}$$

$$-\nabla^2\vec{E} = \vec{\nabla}\times\left(-\frac{\partial B}{\partial t}\right)$$

$$= -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$= -\frac{\partial}{\partial t}\left(\frac{\partial\vec{E}}{\partial t}\right)\mu_0\varepsilon_0$$

$$= -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

Solve it for some medium

$$\boxed{\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}$$ → Very similar to wave eqn.

Wave eqⁿ $\dfrac{\partial^2 v}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 v}{\partial t^2}$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 3\times 10^8 \, m/s .$$

The same thing if we do it for $\vec{B}$, then.

$$\boxed{\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$$

Both electric & magnetic field travel with the speed of light.

We still don't know how ~~&~~ $\vec{E}$ & $\vec{B}$ are oriented.

Solⁿˢ : $$\boxed{\begin{aligned}\vec{E} &= \vec{E_0}\, e^{i(kz-\omega t)} \\ \vec{B} &= \vec{B_0}\, e^{i(kz-\omega t)}\end{aligned}}$$

Assuming the wave is travelling along z-dirⁿ.

$\vec{E_0}$ & $\vec{B_0}$ are amplitudes (Const.)

$\boxed{k = \dfrac{2\pi}{\lambda}}$, $\cancel{\omega = \frac{c}{k}}$ $\boxed{\omega = kc}$

$$\vec{\nabla}\cdot\vec{E} = 0 \Rightarrow \vec{\nabla}\cdot\left(\vec{E_0}\, e^{i(kz-\omega t)}\right) = 0$$

$$\cancel{\frac{\partial}{\partial z}\left(\vec{E_0}\, e^{i(kz-\omega t)}\right) = 0}$$

$$\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\left(\vec{E_0}\, e^{i(kz-\omega t)}\right) = 0$$

$$\hat{z}\cdot\vec{E_0}\,(ik) = 0 \Rightarrow \vec{E} \text{ is } \perp\text{lar to } \hat{z}.$$

$$\hat{z}\cdot\vec{E_0}\, e^{i(kz-\omega t)}\, ik = 0 .$$

|||ly $\vec{B}$ is also ⊥lar to $\hat{z}$.

→ When an EM wave is propogating, then $\vec{E}$ & $\vec{B}$ are always ⊥lar to direction of propogation

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cancel{\vec{E}_x} \to 0 & \cancel{\vec{E}_y} \to 0 & \cancel{\vec{E}_z} \end{vmatrix} \Rightarrow e^{i(kz-\omega t)} = -\frac{\partial}{\partial t} B_z \hat{z}\, e^{i(kz-\omega t)}$$

E_x & E_y are 0.

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x e^{i(kz-\omega t)} & E_y e^{i(kz-\omega t)} & 0 \end{vmatrix} = \hat{x}\left(-\frac{\partial}{\partial z}\left(E_y e^{i(kz-\omega t)}\right)\right) - \hat{y}\left(-\frac{\partial}{\partial z}\left(E_x e^{i(kz-\omega t)}\right)\right)$$

$$= -\hat{x}\left(E_y e^{i(kz-\omega t)} \cdot ik\right) + \hat{y}\left(E_x e^{i(kz-\omega t)} \cdot ik\right)$$

$$-\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}\left(\vec{B}_0 e^{i(kz-\omega t)}\right) = -\vec{B}_0 e^{i(kz-\omega t)} \cdot (-i\omega)$$

$$= i\omega\left(\hat{x} B_{0x} + \hat{y} B_{0y}\right) e^{i(kz-\omega t)}$$

$$\Rightarrow \left[-\hat{x}(E_y ik) + \hat{y}(E_x ik)\right] e^{i(kz-\omega t)}$$
$$= i\omega(\hat{x} B_x + \hat{y} B_y) e^{i(kz-\omega t)}$$

Comparing comp.

$$\Rightarrow -E_y \cancel{i}k = \cancel{i}\omega B_x \quad , \quad E_x \cancel{i}k = \cancel{i}\omega B_y$$

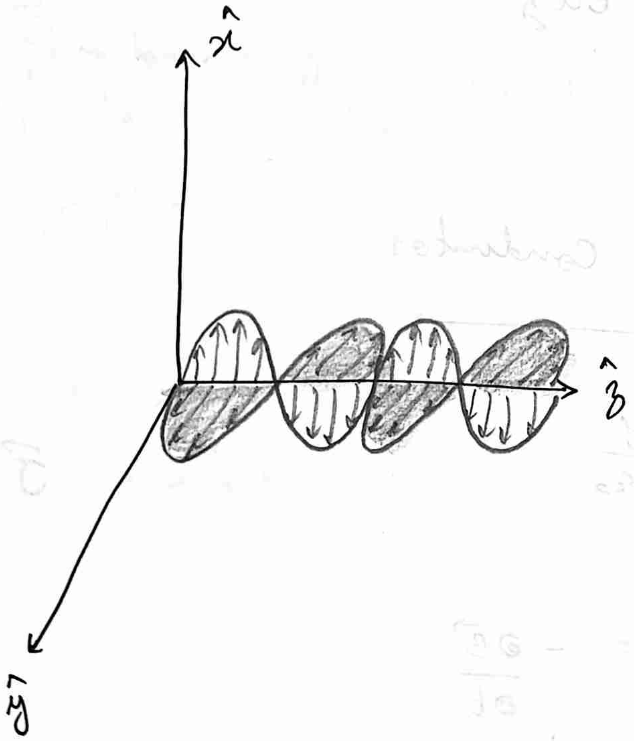
$$\Rightarrow \frac{k}{\omega} = -\frac{B_x}{E_y} \quad , \quad \frac{k}{\omega} = \frac{B_y}{E_x}$$

$$\Rightarrow E_x B_x + E_y B_y = 0.$$

$$\Rightarrow \vec{B_0} = \frac{k}{\omega} \hat{z} \times \vec{E_0}$$

$\hookrightarrow$ Tells us that $\vec{B}$, $\hat{z}$, $\vec{E}$ are mutually perpendicular.

$$\Rightarrow B_0 = \frac{k}{\omega} E_0 = \frac{E_0}{c}$$



$$\vec{E_0} e^{i(kz-\omega t)}$$
$$= \vec{E_0}\left(\cos(kz-\omega t) + i\sin(kz-\omega t)\right)$$

→ If a phase is added $\vec{E}_0 e^{i(kz-\omega t+\delta)}$

then $\vec{E}$ & $\vec{B}$ won't be in phase.

There'll be a phase difference

$$\text{Energy} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

$$= \varepsilon_0 E^2 \cos^2(kz-\omega t)$$

↳ Density

Total energy = avg. over time

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$

After a lot of math !!

$$= \underline{\underline{cu\hat{z}}}$$

↳ Energy is carried with speed of light along $\hat{z}$.

EM wave in Conductor

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}$$

$$\vec{J} = \sigma\vec{E}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}.$$

$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$. $\rightarrow$ local charge cons.

$\vec{J} = \sigma \vec{E} \Rightarrow \sigma \vec{\nabla}\cdot\vec{E} = -\frac{\partial \rho}{\partial t}$

$\Rightarrow \sigma \left(\frac{\rho}{\varepsilon_0}\right) = -\frac{\partial \rho}{\partial t}$

$\Rightarrow -\frac{\sigma}{\varepsilon_0} t = \ln \rho$

$\Rightarrow \rho(t) = \rho_0 e^{-\sigma/\varepsilon_0 t}$

Conductors, $\sigma \rightarrow$ very large.

$\rho(t) \approx 0$

$\Rightarrow$ Even if we start with free charges, & ~~so~~ the free charges spread out on the boundary of the conductor.

So we don't have to worry about the free charges inside the conductor.

Eq^{ns},
$$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu\sigma \frac{\partial E}{\partial t}$$

Similar to damping oscillation.

So the amplitude keeps on decreasing.