MA2.101 Linear Algebra
Spring'24

Assignment 6

Deadline: 18^{th} February, 11:59pm

Instructions:

1) This assignment consists of 4 problems. All problems are compulsory.
2) Mention all assumptions while answering the questions.
3) Be clear in your arguments. Vague arguments shall not be given full credit.
4) Only Handwritten Submissions are allowed. Scan and upload it on moodle.

Problems:

1. Let m and n be positive integers and let F be a field. Suppose W is a subspace of F^n and $\dim W \leq m$. Then there is precisely one $m \times n$ row-reduced echelon matrix over F which has W as its row space. Prove it.

2. Consider the vector space P_4, which consists of all polynomials of degree 4 or less with real number coefficients. W be the subspace of P_4 given by

$$W = \{p(x) \in P_4 \quad | \quad p(1) + p(-1) = 0 \quad \text{and} \quad p(2) + p(-2) = 0\}.$$

   Our task is twofold:

   (a) Find a basis for the subspace W.

   (b) Determine the dimension of W.

3. Suppose that a set of vectors $S_1 = \{v_1, v_2, v_3\}$ is a spanning set of a subspace V in R^5. If v_4 is another vector in V, then is the set

$$S_2 = \{v_1, v_2, v_3, v_4\}$$

   still a spanning set for V? If so, prove it. Otherwise, give a counterexample.

4. Let V and W be subspaces of R^n such that $V \cap W = \{0\}$ and $dim(V) + dim(W) = n$.

   (a) If $v + w = 0$, where $v \in V$ and $w \in W$, then show that $v = 0$ and $w = 0$.

   (b) If B_1 is a basis for the subspace V and B_2 is a basis for the subspace W, then show that the union $B_1 \cup B_2$ is a basis for R^n.

   (c) If x is in R^n, then show that x can be written in the form $x = v + w$, where $v \in V$ and $w \in W$.

   (d) Show that the representation obtained in part (c) is unique.