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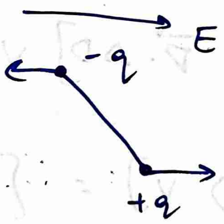
→ $\vec{p} = \alpha \vec{E}$

↳ Tensor.

$p_x = \alpha_{xx} E_x + \ldots$

The dipole tries to aline itself along the electric field.

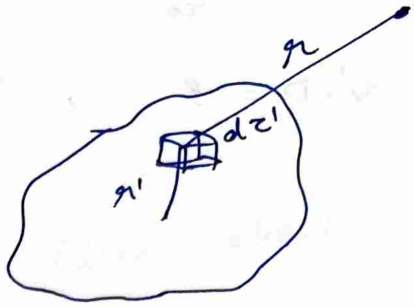


$\vec{\tau} = \vec{p} \times \vec{E}$

Polarization $\vec{P}$ = $\dfrac{\text{Electric dipoles}}{\text{Volume}}$.

$$V_{dip} = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p}\cdot\hat{r}}{r^2}$$

↳ For single dipole.



$$V(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\vec{P}(r')\,\hat{r}}{r^2}\, d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0} \left[\underbrace{\oint_S \frac{\vec{P}\cdot\hat{n}}{r}\, da}_{\text{Eletric pot. given by surface}} - \underbrace{\int_V \frac{\vec{\nabla}\cdot\vec{P}\, d\tau'}{r}}_{\text{Pot. of vol.}} \right]$$

charges

$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0} \oint_S \frac{\sigma_b}{r}\, da + \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho_b}{r}\, d\tau}$$

where $\rho_b = -\vec{\nabla}\cdot\vec{P}$, $\sigma_b = \vec{P}\cdot\hat{n}$ ↳ $\hat{n}$: Unit vector ⊥ler to surface.

→ Gauss's law

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\Rightarrow \varepsilon_0 \vec{\nabla}\cdot\vec{E} = \rho = \rho_f + \rho_b$$

$$= \rho_f - \vec{\nabla}\cdot\vec{P}$$

$$\Rightarrow \vec{\nabla}(\varepsilon_0\vec{E} + \vec{P}) = \rho_f$$

$$\Rightarrow \vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

↳ Displacement vector.

But can't replace $\vec{E}$ with $\vec{D}$.

Because $\vec{\nabla}\times\vec{E} = 0$ (always)

But $\vec{\nabla}\times\vec{D}$ need not be 0.

Because $\vec{\nabla}\times\vec{D} = \vec{\nabla}\times\vec{P}$

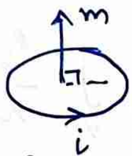
$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$, $\vec{\nabla}\times\vec{E} = 0$. → Here E" was ~~same~~ equal

$\vec{\nabla}\cdot D = \rho$, $\underbrace{\vec{\nabla}\times\vec{D} = \vec{\nabla}\times\vec{P}}$ } → Boundary conditions of $\vec{D}$.

↳ But here the D" will be discontinuous by curl of $\vec{P}$.

• <u>Magnetic field inside metals</u>

→ The cause of magnetic field is electric current.



→ Acts as magnetic dipole.

$\boxed{\vec{m} = IA\hat{z}}$

Tiny dipole with N pole & S pole.

→ Since orientation of these dipoles is completely random (in absence of field), they cancel each other.

Here, unlike electric field & electric dipole → They tend to orient along $\vec{E}$,

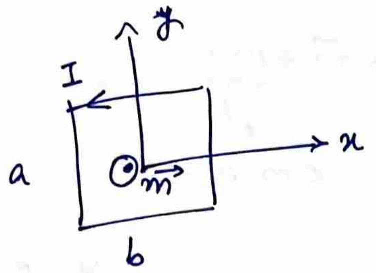
there are ~~two~~ types of materials:

1) Paramagnetic : $\vec{m} \propto \vec{B}$ (Aligns along $\vec{B}$)

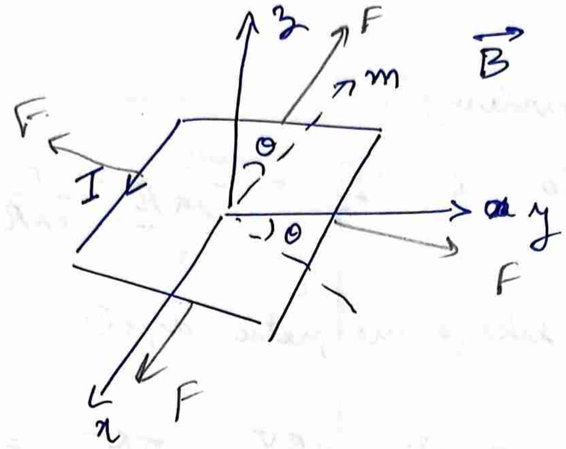
2) Diamagnetic . $\vec{m} \propto -\vec{B}$ (Aligns opp. to $\vec{B}$)

3) Ferromagnetic : The material gets magnetised (remains magnetic even after magnetic field is removed)

$$\boxed{\vec{M} = \frac{m}{\text{Volume}}}$$

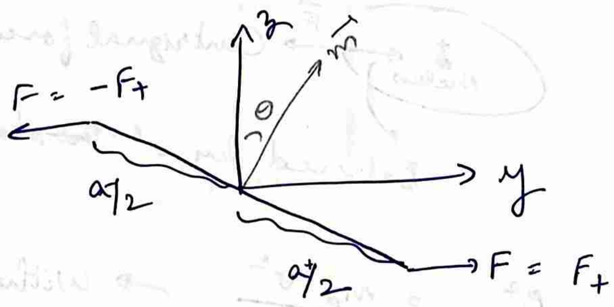


$\vec{m}$ along z.



Tilted it about x axis.

$$\vec{F} = q\,(\vec{v} \times \vec{B})$$



$$\vec{\tau} = \frac{\vec{a}}{2} \times \vec{F} + \left(-\frac{\vec{a}}{2}\right) \times (-\vec{F})$$

$$= \vec{a} \times \vec{F}$$

$$= aF\sin\theta\ \hat{x}$$

$$\vec{F} = Q(\vec{v} \times \vec{B})$$

$$= i(\vec{l} \times \vec{B})$$

$$= ibB$$

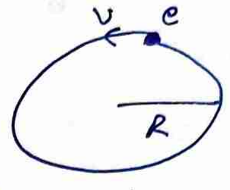
$$\Rightarrow \vec{\tau} = iab\sin\theta\,\hat{x}\ B$$

$$= \vec{m} \times \vec{B}$$

$$\Rightarrow \boxed{\vec{\tau} = \vec{m} \times \vec{B}}$$ → (small) Contributes to paramagnetism

When magnetic field is applied , $\vec{m} \longrightarrow \vec{m} + \underbrace{\Delta m}$

$\Delta m \propto -B$.



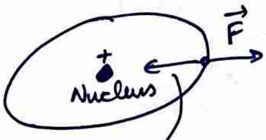
Atoms, electron revolving.

So $i = \frac{-e}{T} = \frac{-e}{\frac{2\pi R}{v}} = \frac{-ev}{2\pi R}$

Atom Acts like a magnetic dipole.

$$\vec{m} = I\pi R^2 = \frac{-ev}{2\pi R} \cdot \pi R^2 = -\frac{evR}{2} \hat{z}$$

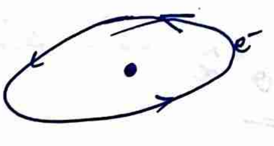
So



$\vec{F}$ Centrifugal force

Balanced by ~~electrostatic~~ attraction

$$\frac{1}{4\pi\varepsilon_0} \frac{e^2}{R} = m_e \frac{v^2}{R} \rightarrow \text{Without magnetic field.}$$



$\uparrow z \uparrow \vec{B}$

Lorentz force.

$\underbrace{-e(\vec{v} \times \vec{B})} \rightarrow$ Extra force

So velocity no longer remains same

So, $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R} + ev'B = m_e\frac{v'^2}{R}$

$v' > v$ (has to be).

So m will also ~~incre~~ increase.

$\Delta m \uparrow$ ses.

$ev'B = \frac{m_e(v'^2 - v^2)}{R} = \frac{m_e}{R}(v'+v)\Delta v$

v' & v are almost same.

So Δv is proportional to v'

$\Rightarrow \Delta v \approx \frac{eBR}{m_e}$

$\Rightarrow \Delta m \propto -\vec{B}$ } $\rightarrow$ Contribution to diamagnetism.

- Magnetic vector potential, $A(r) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$

$$A(r) = \frac{\mu_0}{4\pi}\int\frac{\vec{M}(r')\times\hat{r}}{r^2}d\tau$$

$\frac{1}{r^2} = \nabla\left(\frac{-1}{r}\right)$

Then integration by parts.

$$A(r) = \frac{\mu_0}{4\pi}\left[\int\frac{1}{r}\underbrace{\nabla'\times M(r')}_{\substack{\text{Volume current}\\(J_b)}}d\tau' + \oint\frac{1}{r}\underbrace{\vec{M}(r')\times d\vec{a}'}_{\substack{\text{Surface current}\\(K_b)}}\right]$$

$J_b = \nabla'\times M(r')$ & $K_b = \vec{M}(r')\times\hat{n}$

- Magnetic field inside diamagnetic material

$$\vec{\nabla} \times \vec{B} = \mu_0 J$$

$$\frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) = J_{free} + J_{bound}$$

$$= J_{free} + \vec{\nabla} \times \vec{M}$$

$$\Rightarrow \vec{\nabla} \times \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = J_f$$

Ampere's law inside magnetised obj.

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

External magnetic field

When material is magnetised.