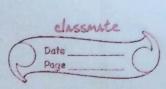
Raunah Repsocia 188 C 2023113019 LA - Assignment - 1 1. RTP (5,+, .) to must contain all rational numbers. We start off by considering a field F over C. We want to reach a field from the set F, while starting minimizing no. of elements in set F. Since Du the identity element of + , and 1 is the identity element of . , F= (0, 1 EF +, . , by definition, are always associative & commutative. let us assume F = do, 19 checking for if Fin a (f,+,.) is a field. Assuming 2 in also EF , : @ Fi not doced where Aleced.

Also, similarly, 1+2 = 3 & F. F= (0,1,2,3,.... 3 under + 2denti-For additive inverse of 1 to exist, -1 EF [1+1-1)=0] linday € -2EF, -3EF6, ... :. F= \(\dots, -3, -2, -1, 0, 1, 2, 3\),} . O F - new = It was projected of additive & derive, identity inverse present here. 42 € F, 1/2 € F for multiplicative where to exist [2(1/2)=1] similary, V N; € F {N; € Z 3, 2/N; € F for multiplicative inverse to exits : F= ((i, i & #) 41/18, i = # } kut poe clarine on , 200, 1/16, i, je7

i, 1/jeff, i, je7 exist. :. if EF (for downs on.)



and the Personal Property and Personal Property and Personal Property	Page
	Taking this as a general form that includes i, i \ \max\rights when i = 1,
- Contractor	when i=1 bdy, i e # 4 when i=1,
-	the set t can be represented as
	F= { plq , p, q \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
	·it is
The state of the law of	but for F to nave no duplicated Eset?, p, a smust be co-prime to each.
The Party of the P	ke co-prime to each.
-	
-	: F = f p/q; p, q & #; 1, q coprine}
	Large Language Language
	= (S) 11 + 11 bran (12) o 100
	p now
	The cet & thus measuril entisfies multiplicative inverse
	thereing substite,
	Checking for all properties for a field, addition has formed
	Checking for all properties for a field, addition has formed
	an abelian group, with the additional property of
	, being distributive oner f.
	of C
	We have ground that the imilest subfield possible is
	the right of all butional now!
	We have also proved that they if any field contains YO, 13 it must contain Os. (Fit,) is a field O, 18 F V F C C, every subfull of (C,+;)
	90,19 it must contain Os. (Fit,) is full
	0,1 EF V FC C, every subfull & of (1,+;)
	must certain an settlemat mis
	[Hence proved]
	Carrier and Carrie
	@ RTP in Freque y 12: n, y & Os is a subfield of
	@ RTP: y F={n, y \(\frac{1}{2}\): n, y \(\infty\) is a subfield of \((\infty\).



sol" w F = fn+ y /2: a, y & B}

Firstly, we note that F & BREC because Fet JEF, JER, but BER, 13 EF (F, +, ')

i He have to Provey & is a field will be sufficient to prove f is a subfield of (4, +,)

For F to be a field :

Amplied Properties:

ODF is closed under +

For 11, 62 E F 1 = N1 + y1 12 12 = N2 + y2 12 (1+6) = (n, + n2) + (y1 + y2) & V2

i. (n, 1 Me), (y, + y,) ∈ S if n, n2, y, y, eQ ut us call them M3, y,

- fit fre = ng + yg 12 ; ng jyz E &

1 closure under ?

For 61, 62 E F

61 = M1+ V24, M1, 4, E B

62 = M2+ V2 42, M2, 42 E B

616 = 1 M1+ V24, (M2+ V242)

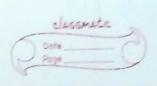
= C6100 + 211 + 12 42

= & 61, 12 + 24, 42) + 52 (11, 4, + 1124)

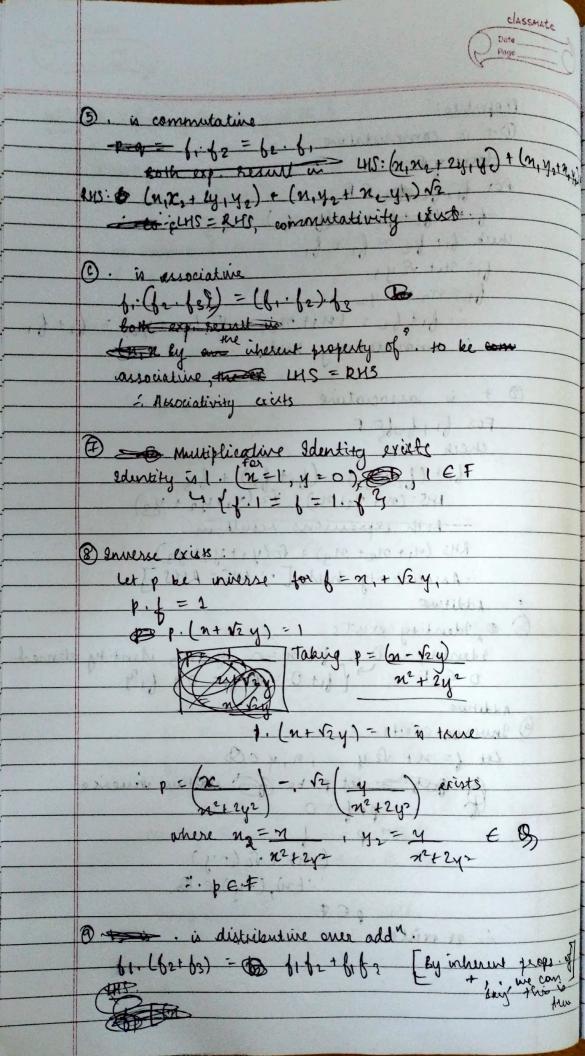
Let M3 = 11, 11 + 124, 4 = EBB

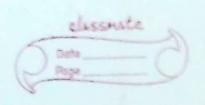
43 = 11, 4, + 124 & EBB

· fifz EF, closure holds.



Properties:
D+ is commutative
- CMITY VE 20
Por firbridg EP
-61+ (62+63) = (61+62) +-63
Check 6, + 62 = 62 + 6,
12=N2+V242
bit be = (nit Me) + V2 (y 1+ y2) = 62+ 61
: commutativity holds-
O + is associative
For fifting F
bi+ (b2+63) = (b1+62) + 63
HIS=(MI+ M2+ M2) + V2 (41+42+43)
- both expressions result in
RHS (M1+ M2+ M3) + (2 (y1+ y2+ y3)
- Associativity holds [: LHS = RHS]
Additive
5 and Adentity exists
edentity is 0, for n =0, y=0; identity element
0 E F 9 / 1+0 = 0 + 1,3
Additivé
1 Inverce exists
Let (= n+ v2y, n, y eD)
A bos sut p be 6 additure unuerice
B and the comments of the comm
P+ P+
56.1.56
= (-y) + 1/2 (-y)
::(n),(-y) e @
- graits





[M1+ (241) ((M2+ 23) + 12 (42+ 43))

(n, (n2+n2) + 2(y,)(y2+43))

+(x)(n,14,143)+ 2000 4,(ne+n3))

= (M1 M2 + M1 X3 + 24143) + 22 (M1 42 + M1 43 + 2418 + 241)

Similarly,

RHS=(n,n2+ n,n2+24,142)

TV2 (n,y2+n,43+n24+1 n341)

-: LKS = RHS -- Property holls.

Thing properties D-0, we can say that (E, +, ') is a field mining our initial argument = (F, +, ·) is a subfield of (f, +, ·)