<u>Electrodynamics : Assignment 2</u>

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Date
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1. ρ (charge density) $= \dfrac{q}{\pi a^3} e^{-2r/a}$

Q_{enc} in sphere of radius 'r' centered around the nuclei $\Rightarrow q(r) = \int_0^r \rho(r)\,dV$

$dV = 4\pi r^2\,dr$ [for sphere]

$$\therefore q = \int_0^r \frac{q}{\pi a^3} e^{-2r/a} (4\pi r^2\,dr)$$

$$= \frac{4q}{a^3} \int e^{-2r/a} r^2\,dr$$

$$\cancel{= \frac{4q}{a^3} \int e^{-2r/a} \left(\frac{r^2}{a}\right) dr \quad u = (r^2/a) \quad du = 2r/a}$$

$r = -au/2$

let $u = -2r/a$

$du = -\dfrac{2}{a}\,dr$

$$\therefore q = \frac{-4q}{a^3} \int_0^{-ar/2} e^{u} \left(\frac{a^2 u^2}{4}\right)\left(-\frac{a}{2}\,du\right)$$

$$= -\frac{q}{2} \int_0^{-ar/2} e^{u} u^2\,du$$

$$= -\frac{q}{2} \left[e^{u}(u^2 - 2u + 2)\right]_0^{-au/2}$$

$$= -\frac{q}{2} \left[e^{-2r/a}\left(\frac{4r^2}{a^2} + \frac{4r}{a} + 2\right)\right]_0^r$$

$$= -\frac{q}{\cancel{2}} \left[\cancel{2} e^{-2r/a}\left(\frac{2r^2}{a^2} + \frac{2r}{a} + 1\right)\right]_0^r$$

$$= q e^{-2r/a}\left(1 - e^{-2r/a}\left(\frac{2r^2}{a^2} + \frac{2r}{a} + 1\right)\right)$$

Using Gauss's theorem,

$$\oint \vec{E}\cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \vec{E}\oint d\vec{s} = \frac{q}{\varepsilon_0}\left[1 - e^{-2r/a}\left(\frac{2r^2}{a^2} + \frac{2r}{a} + 1\right)\right]$$

[By symmetry, we can take E outside the integral]

$$\Rightarrow \vec{E}_e = \frac{q}{4\pi\varepsilon_0 r^2}\left[1 - e^{-2r/a}\left(\frac{2r^2}{a^2} + \frac{2r}{a} + 1\right)\right]$$

Nucleus is shifted to $r = d$ where $\vec{E}_{ext} = \vec{E}_e$ (at $r = d$)

$$\therefore E_{ext} = \frac{q}{4\pi\varepsilon_0 d^2}\left[1 - e^{-2d/a}\left(\frac{2d^2}{a^2} + \frac{2d}{a} + 1\right)\right]$$

By Taylor's expansion

$$e^{-2d/a} = 1 - 2\cdot\frac{d}{a} + 2\left(\frac{d}{a}\right)^2 - \frac{4}{3}\left(\frac{d^3}{a}\right) \ldots$$

$$e^{-2d/a}\left(\frac{2d^2}{a^2} + \frac{2d}{a} + 1\right) \approx 1 - \frac{4}{3}\left(\frac{d}{a}\right)^3$$ [neglecting powers > 4]

$$\therefore E_{ext} \approx \frac{q}{4\pi\varepsilon_0 d^2}\left[1 - \left(1 - \frac{4}{3}\left(\frac{d}{a}\right)^3\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0 d^2}\left(\frac{4}{3}\frac{d^3}{a^3}\right)$$

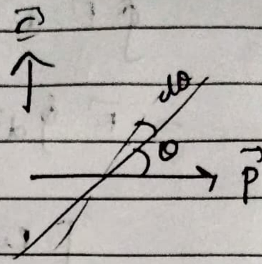
$$= \boxed{\frac{qd}{3\pi\varepsilon_0 a^3}}$$

Atomic polarisability $\alpha = \frac{p}{E}$ (dipole moment)

$$p = qd \Rightarrow \alpha = qd / (qd/3\pi\varepsilon_0 a^3) = \boxed{3\pi\varepsilon_0 a^3}$$

2. Torque on dipole is $\vec{N} = \vec{p} \times \vec{E}$ [We know]

work done by $\vec{N} = \int_{\theta_i}^{\theta_f} \vec{N} \cdot d\vec{\theta}$



If $\theta_i = \pi/2$,

Work done in moving $\vec{p} = \pi/2$

$E_i = E_f$ (By conservation of energy)

$\Rightarrow U_i + K_i = U_f + K_f + W$

$\because$ final & initial $\omega = 0$

$K_i = K_f = 0$

$\Rightarrow U_f - U_i = W$

If our initial pos^n was considered as 0 pot. (arbitrary)

$$\Rightarrow U_f = W_\theta = \int_{\pi/2}^{\theta} \vec{N} \cdot d\vec{\theta} = \int_{\pi/2}^{\theta} |N| \, |d\theta|$$

$$= \int_{\pi/2}^{\theta} |p||E| \sin\theta \, d\theta$$

$$= pE(-\cos\theta)$$

$$= -\vec{p} \cdot \vec{E}$$

$$\therefore \boxed{U = -\vec{p} \cdot \vec{E}}$$

(Radius R)

3. For uniformly charged sphere with charge density $= \rho$

Case ① $r \le R$

Assuming Gaussian surf. of radius 'r' centered at origin (centre of our charged sphere)

$$\oint \vec{E}_{inside} \cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0} \quad \text{[Gauss's Theorem]}$$

By spherical symmetry, $|\vec{E}_{inside}|$ is a constant on our Gaussian surf. & points radially outwards/inwards.

$$\therefore \oint \vec{E} \cdot d\vec{s} = |E| \oint |d\vec{s}| = |E| (4\pi r^2)$$

$$\therefore |E| = \frac{Q_{enc}}{4\pi \varepsilon_0 r^2}$$

$$Q_{enc} = \int \rho \, dV$$

$$= \int_0^r \rho \, 4\pi r^2 \, dr$$

But ρ is constant

$$\therefore Q_{enc} = \rho \left(\frac{4\pi r^3}{3} \right)$$

$$\therefore |\vec{E}_{inside}| = \frac{\rho (\cancel{4\pi} r^{\cancel{3}})}{3 (\cancel{4\pi} \varepsilon_0 \cancel{r^2})}$$

$$= \frac{\rho r}{3 \varepsilon_0}$$

$$\therefore \vec{E}_{inside} = \boxed{\frac{\rho |\vec{r}|}{3 \varepsilon_0} \hat{r}}$$

For $r > R$

$$|\vec{E}_{outside}| \oint d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$$

vol. of sphere

$$Q_{enc} = \text{total charge} = \rho\left(\frac{4}{3}\pi R^3\right)$$

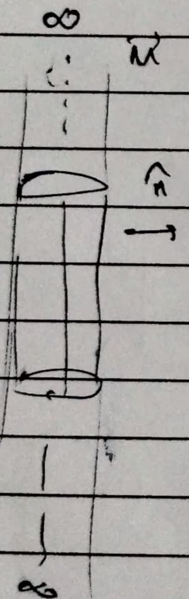
$$\therefore |\vec{E}| = \rho\left(\frac{4}{3}\pi R^3\right)\left(\frac{1}{\varepsilon_0}\right)\left(\frac{1}{4\pi r^2}\right)$$

$$= \frac{\rho R^3}{3\varepsilon_0 r^2}$$

$$\therefore \vec{E} = \boxed{\frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r}}$$

[using Radial symmetry similarly]

4.



$\because \vec{M}$ is magnetisation,

$J_b = \nabla \times M = 0$

pot. of vol. current

$K_b = M \times \hat{n} = M\hat{\phi}$

is pot of surf. current

$\therefore K_b$ = current in solenoid

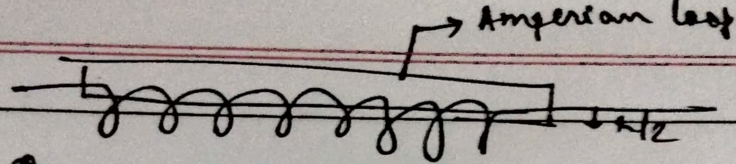
$\therefore$ outside solenoid, $\vec{B} = 0$

inside, $\boxed{B = \mu_0 M}$ $(\because B = \mu_0 K_b)$

<u>Magnetic field of solenoid.</u>

For N no. of turns/length, I current,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \quad \text{[By ampere's law]}$$



$\oint \vec{B}\cdot d\vec{l} = BL$ $\left[\vec{B}_{outside} \approx 0\right.$
& const. by sym
& length of solenoid$\left.\right]$

$= \mu_0 N I L$

$\therefore B = \mu_0 N I$

$\therefore NI = M$ [surf. charge density]

Hence $\boxed{B = \mu_0 M}$