Q1. $X \sim Bin(n = 100, \; p = 0.01)$

$P_X(5) = ?$ (n large).

Sol: $\because$ n is large, we can use Poisson approximation.

We use to avoid computing factorials of large nos. which show up in bin(n,p).

$$\lambda = np = 1$$

$$P_X(5) = \frac{e^{-\lambda}\lambda^5}{5!} = \underline{\underline{\frac{e^{-1}}{5!}}} = \frac{1}{(120)e} \approx \frac{0.3}{100} \approx 0.003$$

Q2. $X \sim Geometric(p)$

$P(X > n) = ?$ $n = 1, 2, \ldots$

Sol: $P_X(x) = (1-p)^{x-1}p$

$$P(X > n) = \sum_{i=n+1}^{\infty} P_X(i) = \sum_{i=n+1}^{\infty}(1-p)^{i-1}p$$

$$= (1-p)^n \quad ?$$

$P(X > m+n \mid X > m) = ?$, $m, n \in \mathbb{N}$

Sol:
$$= \frac{P((X > m+n) \cap (X > m))}{P(X > m)}$$

$$= \frac{P(X > m+n)}{P(X > m)} = \frac{(1-p)^{m+n}}{(1-p)^m} = \underline{\underline{(1-p)^n}} = P(X > n)$$

*Here this is true only for infinite geometric (so only geometric RV).

$\Rightarrow^{*}$ $\underbrace{P(X > m+n \mid X > m) = P(X > n)}$. $\rightsquigarrow$ This can be interpreted as until m trials there is no success, so then it's another geometric RV starting at n.

MEMORY LESS RV.

There is no memory of m $\therefore$ it's called memoryless RV!

$P(X > 2 \mid X > 1) = P(X > 2)$

1 2

That means there was no heads in

the first toss, and now we are seeing success from $\underline{x=2}$.

Basically we proved if X is Geometric RV,

then $P(X > m+n \mid X > m) = P(X > n)$.

But it is also true that if $P(X > m+n \mid X > m) = P(X > n)$,

then X is a geometric RV. (Given the RV satisfying the above condition is discrete).

Q3. X_1, X_2, X_3 are independent. Take a test thrice and awarded best.

$$X = \max\{X_1, X_2, X_3\} \text{ and } P_{X_i}(k) = \frac{1}{10}, \; k \in [1:10], \; i \in [1:3]$$

Find PMF of X.

Sol:

Suppose we have 2 attempts, X_1, X_2

$$X' = \max\{X_1, X_2\} = k.$$

If we fix $X_1 = k$, then X_2 can take $k-1$ values.

" " " $X_2 = k$, then X_1 can take k values (X_1 can also be k, max would still be k).

$$\Rightarrow P_{X'}(k) = \frac{2k-1}{10^2}$$

Can similarly do it for three.. (counting).

* Counting works only if X is uniform.

A more general method is : (Applicable also for non-uniform RV).

$$P(X \le k) = P(\max\{X_1, X_2, X_3\} \le k)$$

$$= P(X_1 \le k, X_2 \le k, X_3 \le k)$$

$$= P(X_1 \le k)\, P(X_2 \le k)\, P(X_3 \le k) \quad (\because X_1, X_2, X_3 \text{ indep.})$$

$$= (F(k))^3 \quad \text{where } F(x) = P(X_i \le x)$$

(Here $\forall i$ it's the same $\because$ PMF is same)

$$F_X(k) = (F(k))^3$$

$\Rightarrow$ max of RV is also a RV.

$$P_X(k) = P(X = k) = P(X \le k) - P(X \le k-1)$$

$$= F_X(k) - F_X(k-1)$$