

Divide and Conquer (contd.)

Counting Inversions:

$$\begin{matrix} a_1 & a_2 & \dots & a_n \\ b_1 & b_2 & \dots & b_n \end{matrix} \left. \vphantom{\begin{matrix} a_1 \\ b_1 \end{matrix}} \right\} \begin{matrix} \cup a_i \\ = \cup b_i \end{matrix}$$

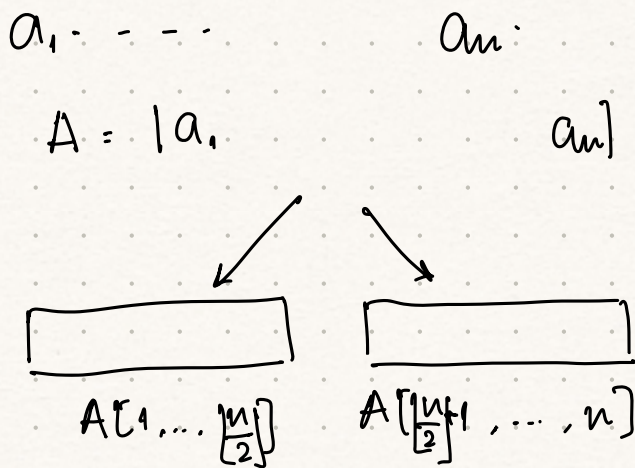
Recap: (in divide & conquer)

1. Integer mult.
2. Matrix mult.
3. FFT
4. Closest pair of points

$a_1 \dots a_n$ } not necessary that they are in sorted order.

(i, j) s.t. $i < j$ and $a_i > a_j$ } inversion pair.

of inversions $\hat{=}$ measure of dist from sortedness.



Det(M)

$$= \sum_{\sigma \in S_n} (-1)^{\text{Sign}(\sigma)} \prod_{i=1}^n M_{i, \sigma(i)}$$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2 \times 2}$

$1 \rightarrow 1$
 $2 \rightarrow 2$

$1 \rightarrow 2$
 $2 \rightarrow 1$

$ad - bc$

[Mahajan-Vinay, SODA 97].

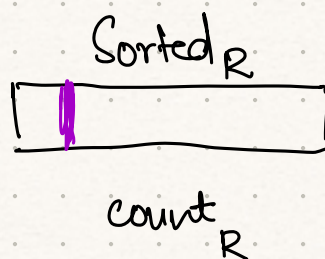
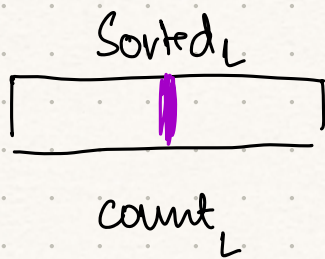
Qn: Can we also count the no. of inv while doing this?

Count Inversions($A[1, \dots, n]$)

Brute force:
 $\binom{n}{2}$ pairs $i < j$
 $a_i > a_j$

P = Count Inversions $A[1, \dots, \lfloor \frac{n}{2} \rfloor]$ } L

Q = Count Inversions $A[\lfloor \frac{n}{2} \rfloor + 1, \dots, n]$ } R



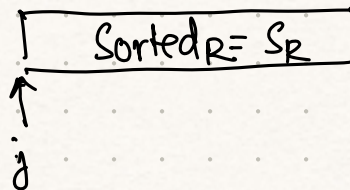
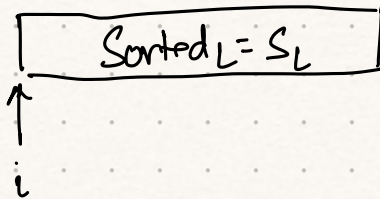
$U = \# \text{ of inversions across } L \text{ and } R$

Total # of inv =

$$P + Q + U.$$

Brute force merge
 $O(n^2)$.

If $a_i > a_j$ then so are the elements to the right of a_i in Sorted_L .



1-indexing

$$i = j = 1 \text{ (init)}$$

$$T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + T\left(\left\lceil \frac{n}{2} \right\rceil\right)$$

$$T(n) = T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + T\left(\left\lceil \frac{n}{2} \right\rceil\right) + \underline{\underline{O(n)}}$$

Getting a count of inv involving a_j .

$$S_L[i] > S_R[j]$$

If yes then $\text{count} += |S_L| - i + 1$
 $j \leftarrow j + 1$

Else, $i \leftarrow i + 1$.

→ Median of Medians } ← Approximate median finding.

Quicksort(A)

$$T(n) = T(|L|) + T(|R|) + O(n).$$

↳ pivot p.

$L = \text{Quicksort}(A_{<p})$

$R = \text{Quicksort}(A_{>p})$

Return L, {p}, R.