CS 302.1 - Automata Theory

Lecture 10

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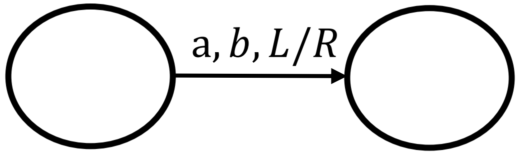
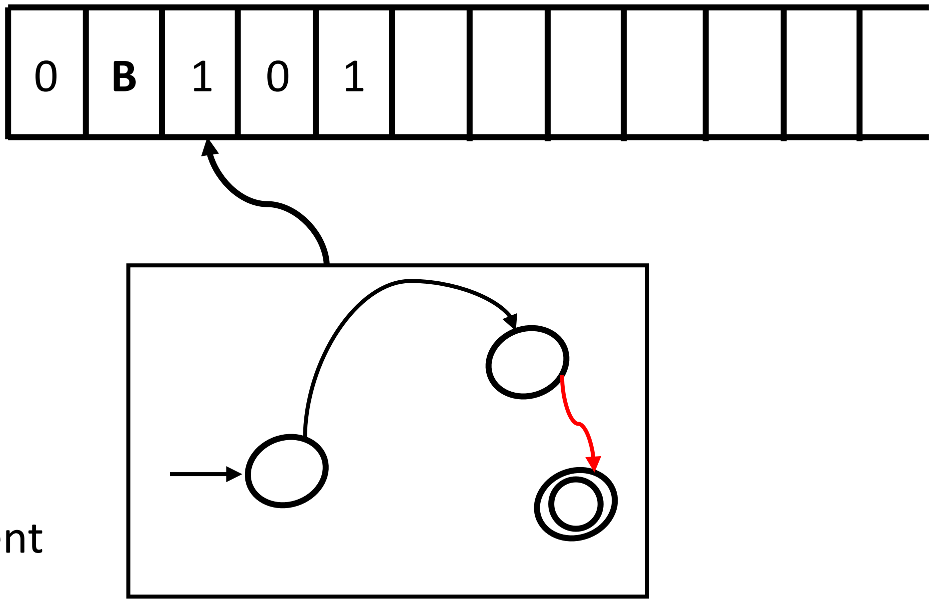
Quick Recap





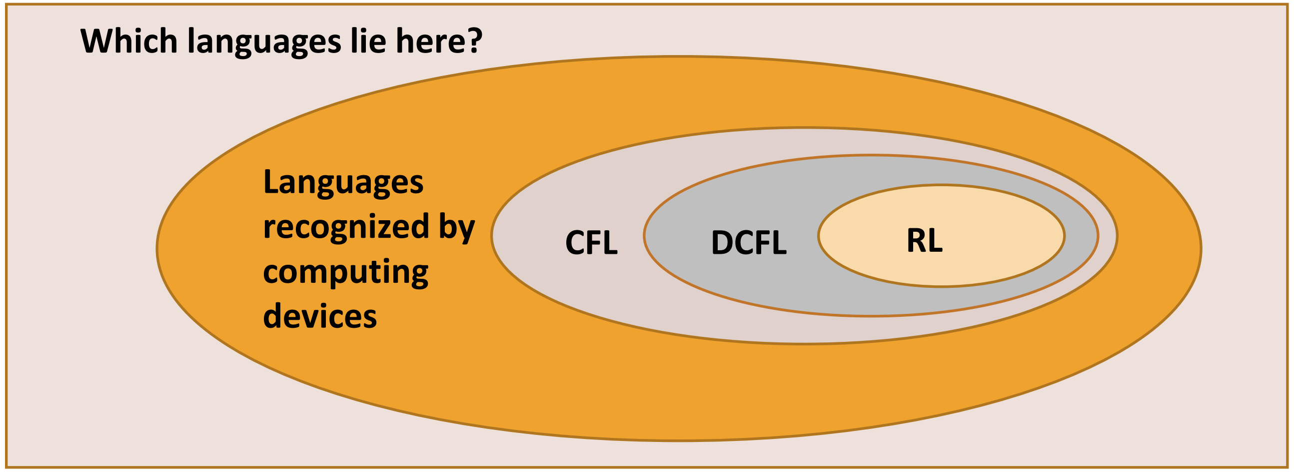
Turing Machines:

- TM **halts** and **accepts/rejects** on reaching the **accept** or **reject** states
- TM may never halt – it may loop forever.

Configuration of a TM: Combination of the current tape contents, the current state and the current head location. $X\ 0\ 0\ \underset{\uparrow q_1}{1}\ 1\ B\ B\ B\ B\ \ldots$

A TM $\boldsymbol{M}$ **accepts** $\boldsymbol{w}$ if there exists a sequence of configurations C_1 to C_k, where

- C_1 is the start configuration M on w.
- Each C_i yields C_{i+1}.
- C_k is an accepting configuration



Variants of Turing Machine models

A TM model $\mathcal{M}_1$ **is equivalent to another model** $\mathcal{M}_2$:

- $\mathcal{M}_2$ **can be simulated by** $\mathcal{M}_1$ **and vice versa**.

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Is the standard TM M more powerful/equivalent to the following TM models where

- The head can move left, right or stay put?
- We have k read/write tapes instead of one?
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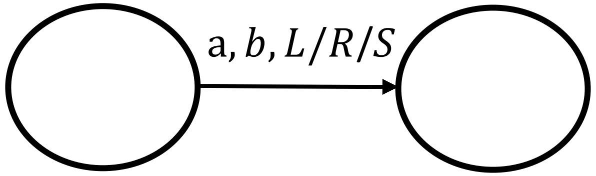
Lazy Turing Machine: The head can either move left, move right or stay put (L, R, S)



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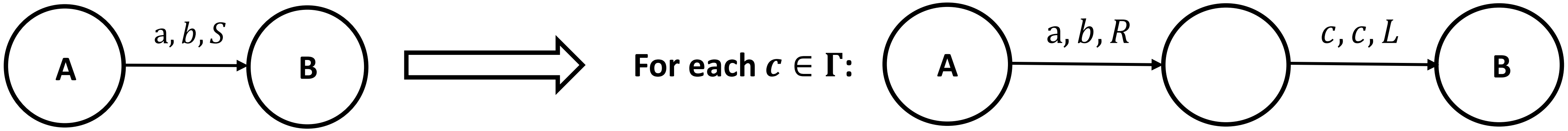

Hence a *lazy Turing machine* model **is equivalent** to a standard Turing Machine model.

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$\boldsymbol{k}$-read/write tapes instead of one: What does a k-tape TM, S look like? k-tape TM also has k heads, each associated with a tape.

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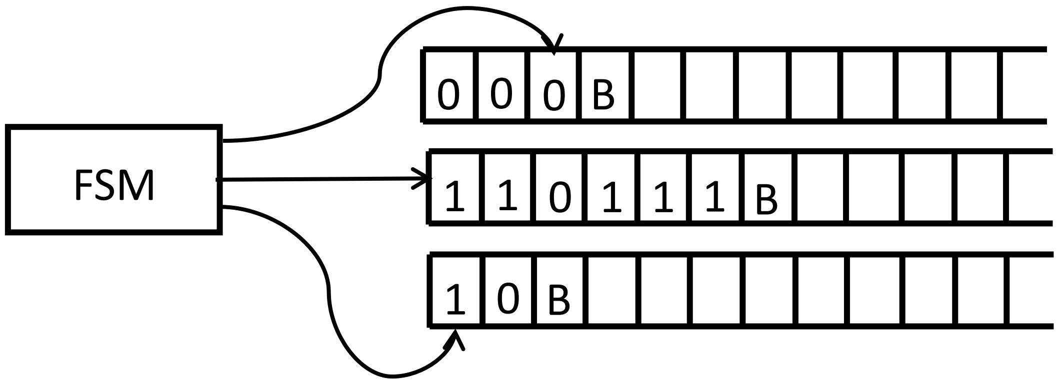
k-read/write tapes instead of one: What does a k-tape TM look like? k-tape TM also has k heads, each associated with a tape. New transition function

$$\delta_S: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R\}^k, \text{ i.e. } \delta_S(q_i, a_1, \cdots, a_k) = (q_j, b_1, \cdots, b_k, L, R, \cdots, L)$$

$\boldsymbol{k = 3}$**:**



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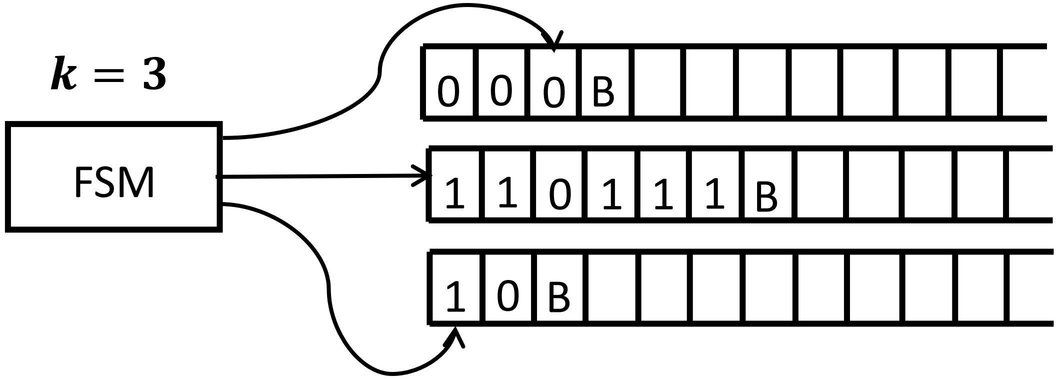

- To simulate S with M, we store the entire information of the k tapes in one single tape.
- M uses $ to separate the contents of the k tapes.
- To keep track of the locations of the k heads, M marks the symbols where the heads would be, with a '_'.

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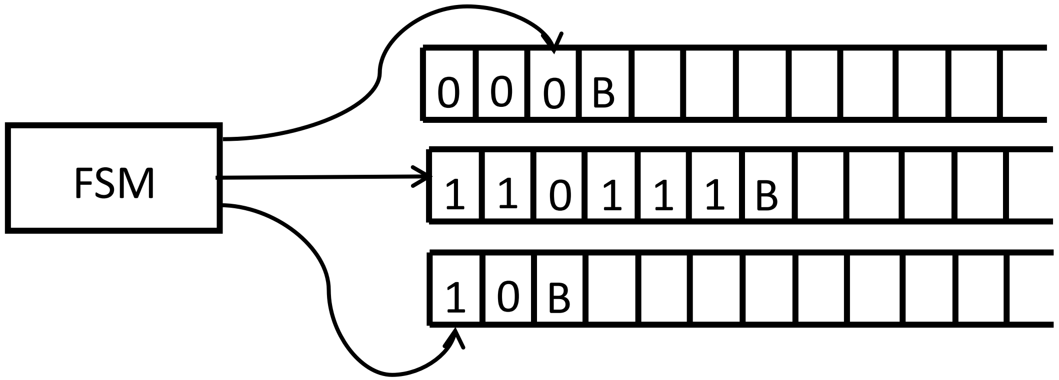
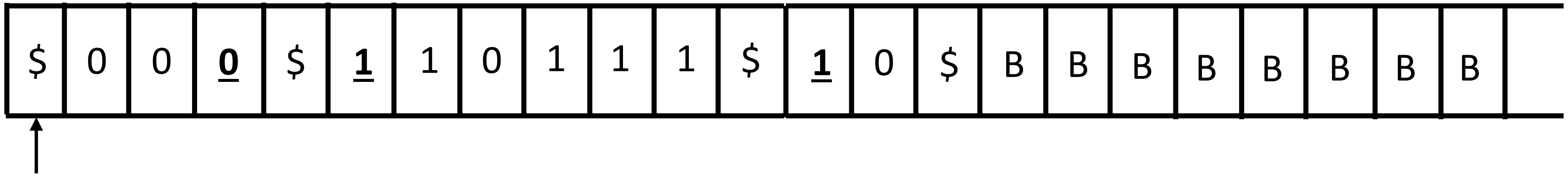


***k*-read/write tapes instead of one:** What does a k-tape TM S look like? k-tape TM also has k heads, each associated with a tape. New transition function $\delta_S: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R\}^k$, i.e. $\delta_S(q_i, a_1, \cdots, a_k) = (q_j, b_1, \cdots, b_k, L, R, \cdots, L)$



- Single tape TM ***M*** **first scans the entire tape from leftmost** $ **to rightmost** $ ($k + 1$ in all) to find the locations of the virtual heads. Then it makes a second pass to update the tape according to δ_S.

- If it so happens that ***M*'s head needs to go to the right of any of the intermediate $** $\Rightarrow S$ has moved the head on the corresponding tape to the unread blank symbols. Starting from this cell to the rightmost $, **shift one cell to the right to make space to write a blank on the empty tape cell** and simulate as before.

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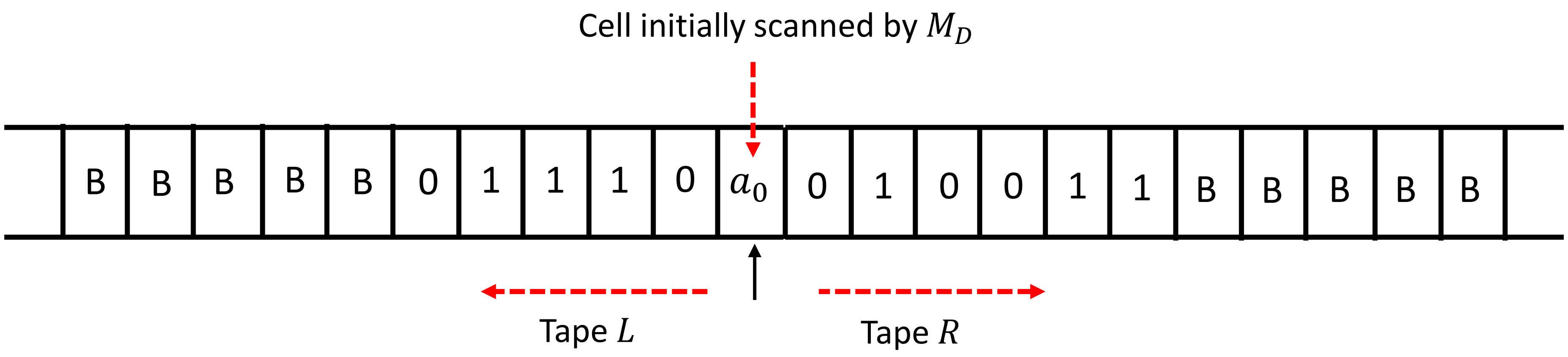
A TM model $\mathcal{M}_1$ is equivalent (as powerful as) to another model $\mathcal{M}_2$ if $\mathcal{M}_2$ can be simulated by $\mathcal{M}_1$ and vice versa.

Two-way infinite Tape: Let M_D be the TM equipped with this power.



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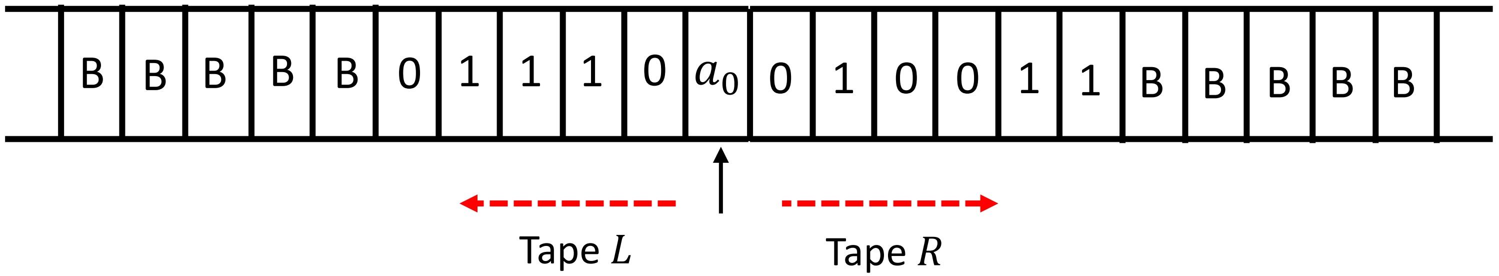

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- Cut the two tapes of M_D into Tape R and (Tape L)^R. We get a two-tape TM.
- Whenever M_D uses the tape to the right of the a_0, Tape R is used.
- When M_D uses the tape to the left of a_0, (Tape L)^R is used.

M_D isn't any more powerful than a one way infinite tape TM.

So a TM with a two-way infinite tape is equivalent to a TM with a one-way infinite tape.

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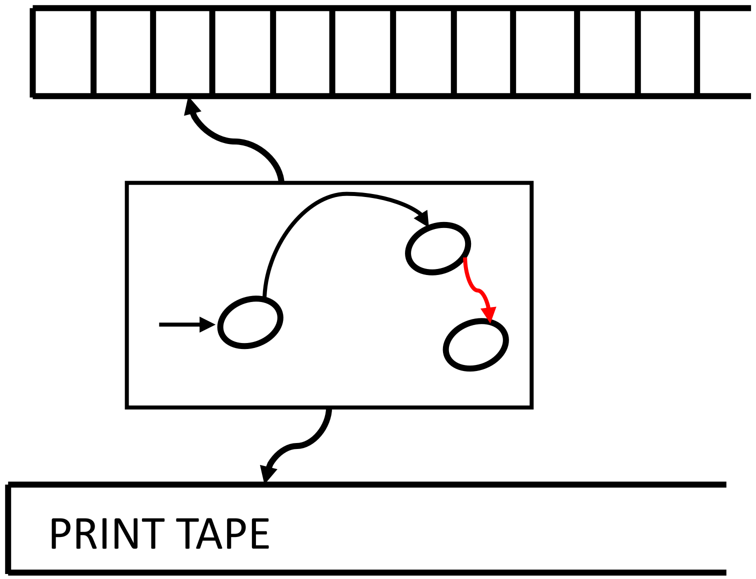
- We introduce non-determinism?

Enumerators: TM attached with a printer



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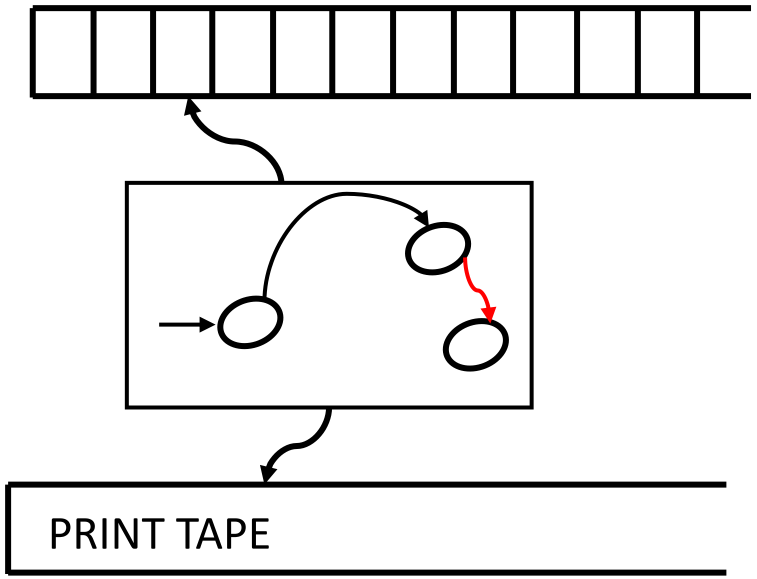
Enumerators: TM attached with a printer

- The Enumerator E uses the print tape to output strings
- The input tape is initially blank
- The language of E is the set of strings that it prints out
- If E does not halt, it may print infinitely many strings in some order



$$\mathcal{L}(E) = \{\mathrm{w} \in \Sigma^* | \mathrm{w \text{ is printed by } E}\}$$

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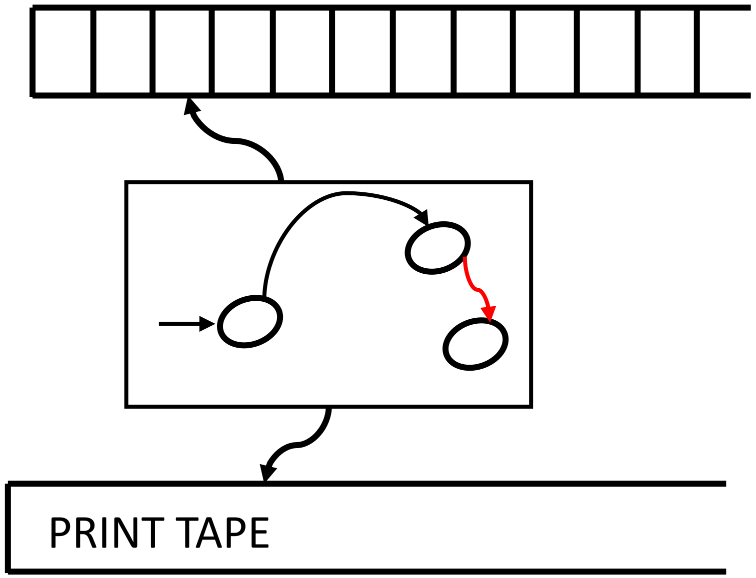

Enumerators: TM attached with a printer

If $L(M)$ is the language recognized by a Turing Machine M then there exists an enumerator E that enumerates it.

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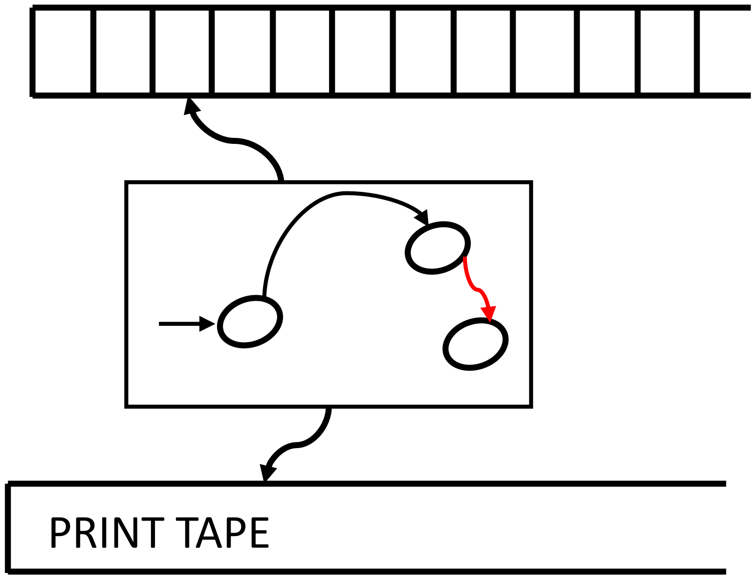

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If $L(M)$ is the language recognized by a Turing Machine M then there exists an enumerator E that enumerates it.

The set of all finite length (binary) strings is **countably infinite**.

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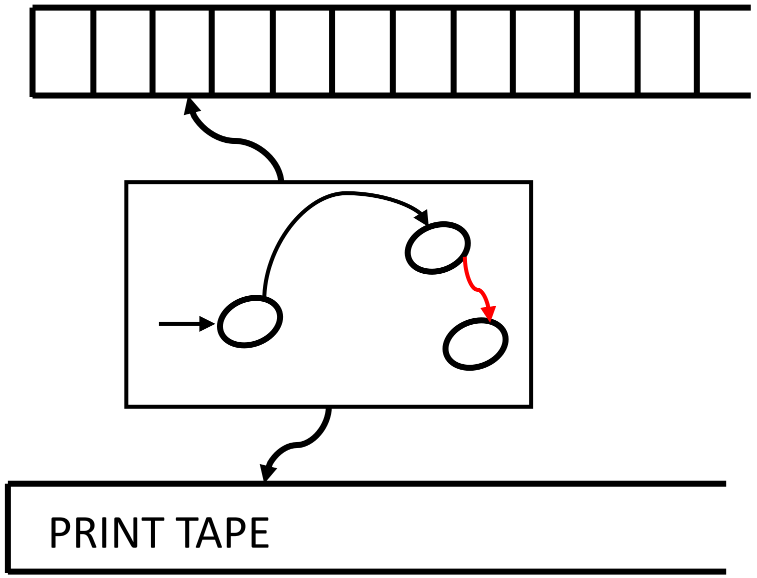

Enumerators: TM attached with a printer

If $L(M)$ is the language recognized by a Turing Machine M then there exists an enumerator E that prints it.

The set of all finite length (binary) strings is **countably infinite**.

- Lexicographically generate all binary strings one after the other. There exists a one-one correspondence with $\mathbb{N}$.
- We can lexicographically generate all (binary) strings and number them:

$$s_1 = 0, s_2 = 1, s_3 = 00, \cdots$$

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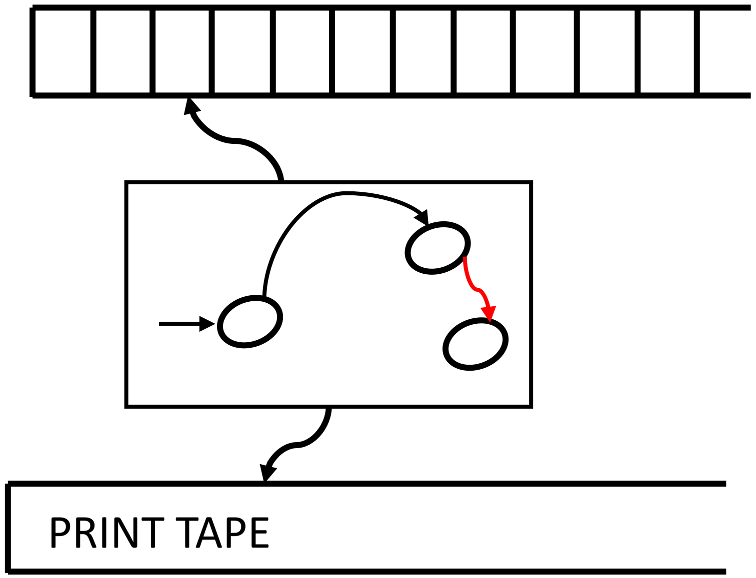

Enumerators: TM attached with a printer

If $L(M)$ is the language recognized by a Turing Machine M then there exists an enumerator E that prints it.

Proof:

For $i = 1, 2, \cdots$

  For $j = 1, 2, \cdots, i$

    Run M with string s_j for i steps.

    If any string is accepted, then PRINT it.

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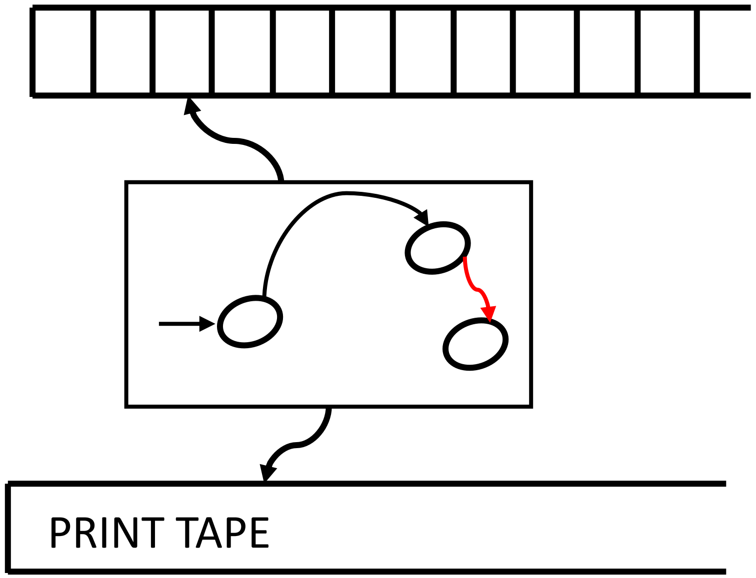

Enumerators: TM attached with a printer

If there exists an Enumerator $\boldsymbol{E}$, then there exists a Turing Machine $\boldsymbol{M}$ such that $\boldsymbol{L(M) = L(E)}$.

Proof: ???

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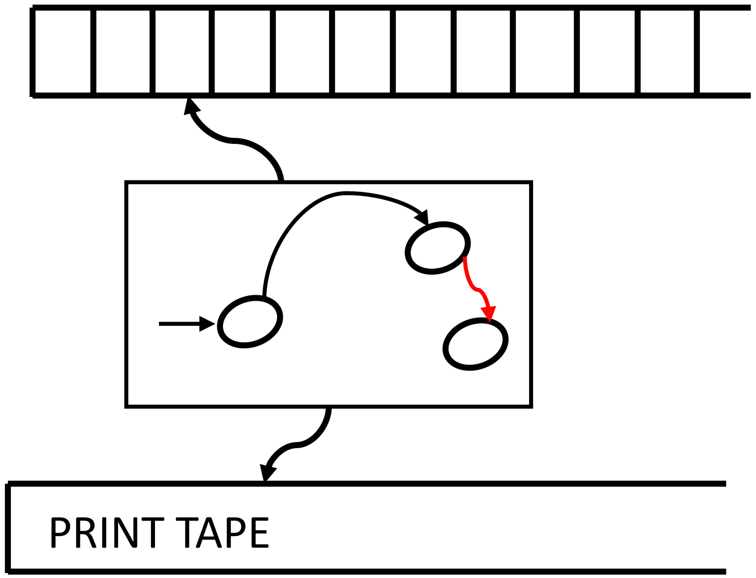

Enumerators: TM attached with a printer

If there exists an Enumerator E, then there exists a Turing Machine M such that $L(M) = L(E)$.

Proof:

$M =$ On input w:

1. Run E. Every time E prints some string, compare it with w.
2. If they match, ACCEPT.

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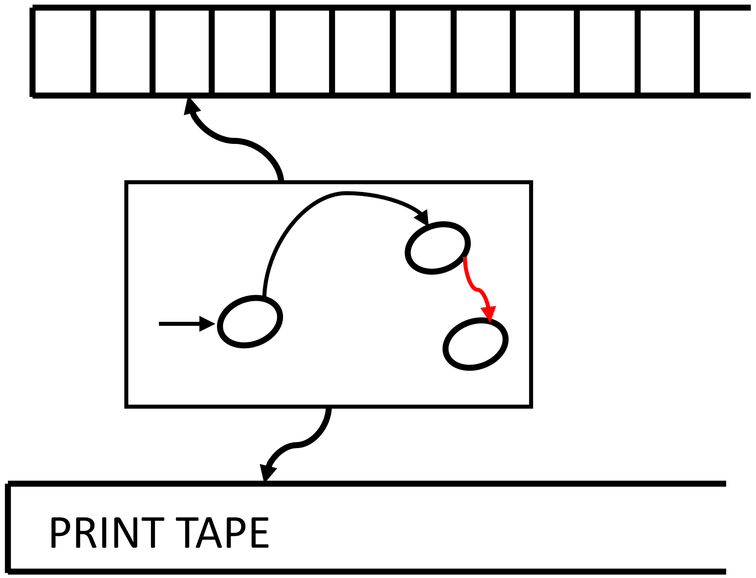

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$\boldsymbol{E}$ and $\boldsymbol{M}$ are equivalent

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Non-deterministic Turing Machines (NTM): In a deterministic Turing machine, from a given configuration, exactly one configuration available to it at any stage. For an NTM however,

- At any point in the computation, several possible configurations are available.

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- At any point in the computation, several possible configurations are available.
- Its transition function is $\delta_N\colon \mathrm{Q} \times \Gamma \to \mathcal{P}(Q \times \Gamma \times \{L, R\})$, i.e. $\delta(q_i, a) \to \{(q_j, b, R), (q_k, c, L) \cdots\}$

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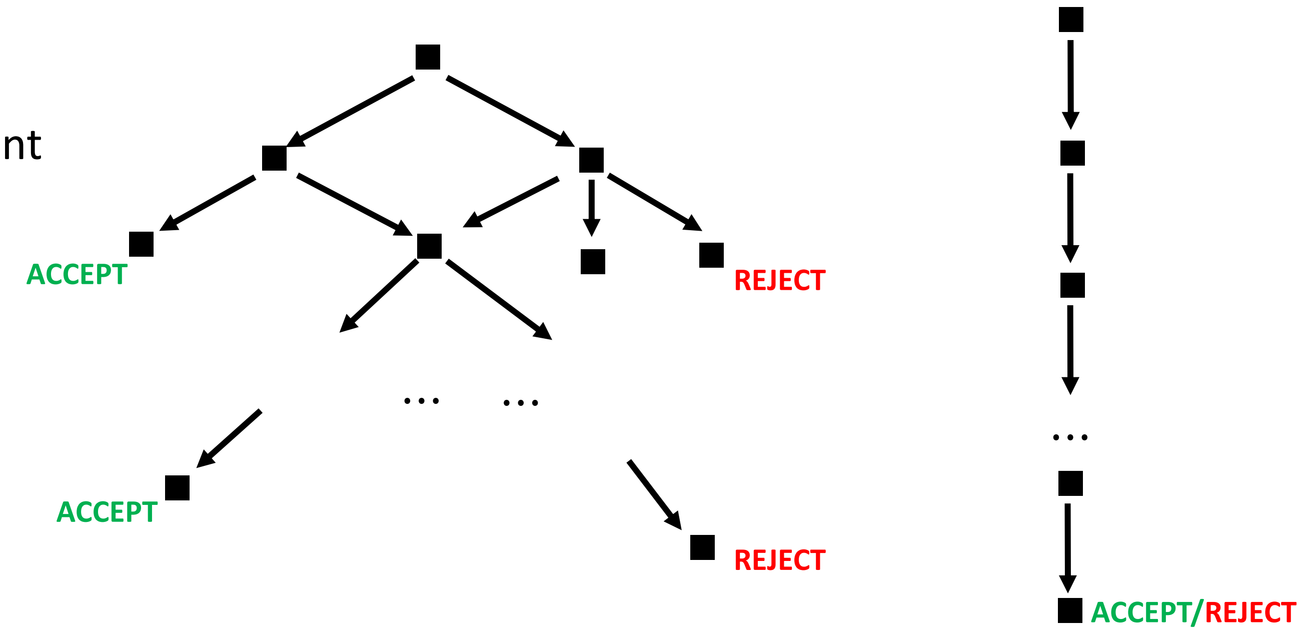

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- The computation corresponds to a configuration tree: From the starting configuration, the computation has several branches, each of which leads to a different configuration.
- If any branch of the computation, leads to an accepting configuration, the NTM accepts. Immediately, DTM is a special case of NTM

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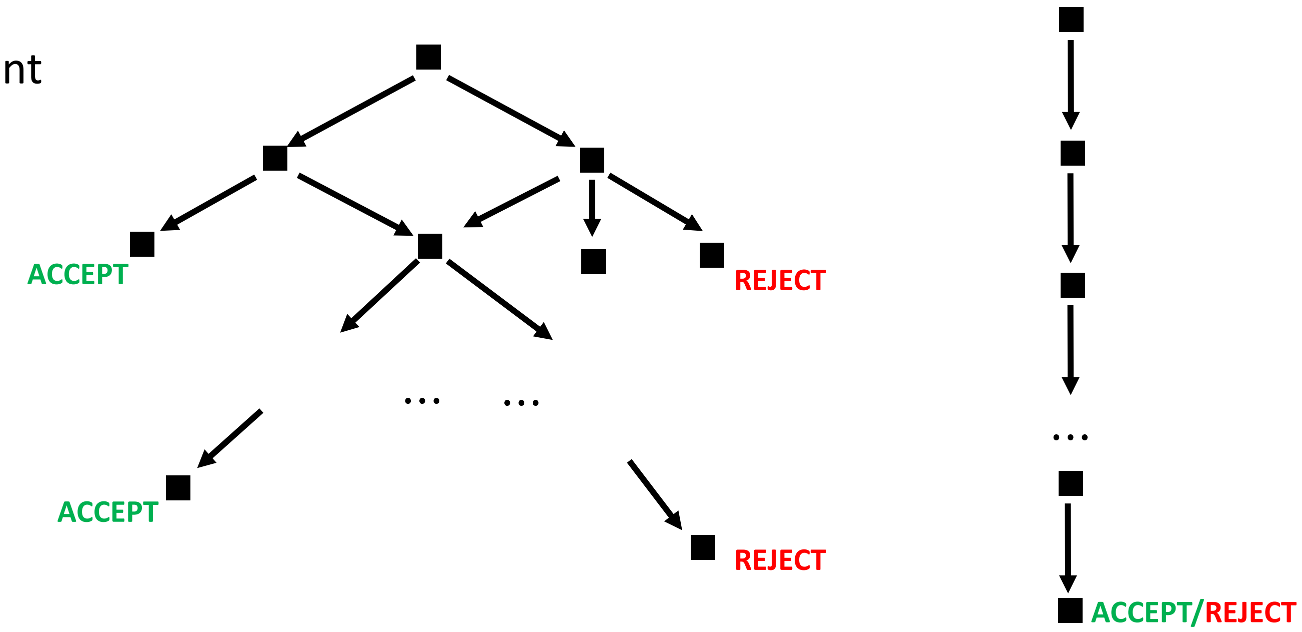

Non-deterministic Turing Machines (NTM):

- For a DTM, from a given configuration, exactly one configuration available to it at any stage.
- For an NTM, any point in the computation, several possible configurations are available.
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Are NTMs more powerful than DTMs? No. Any NTM can be simulated by a DTM. How?

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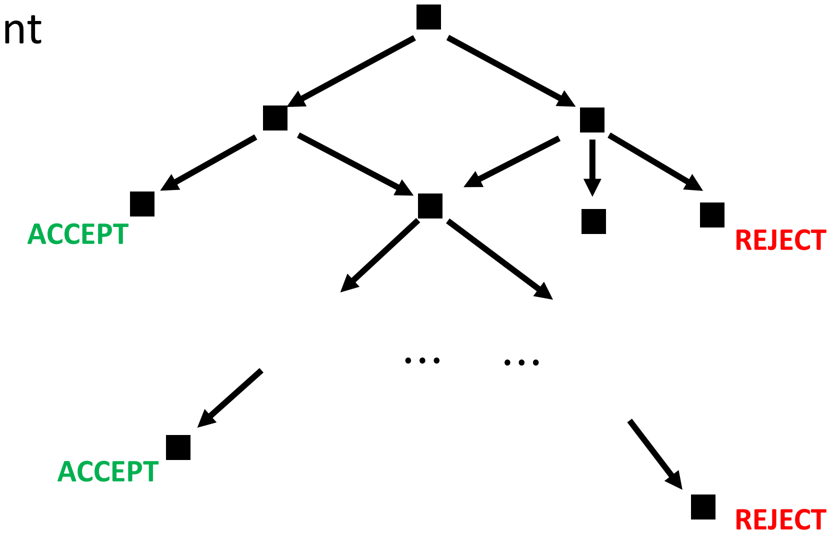
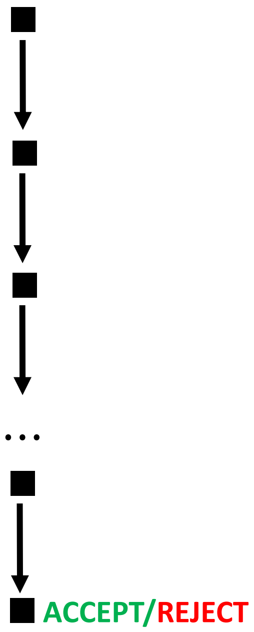
Any NTM can be simulated by a DTM





- The DTM searches from among the configurations of the NTM for an accepting configuration.

- Clearly an ordinary Depth First Search shouldn't work

- A branch of the configuration tree can be of infinite depth (when the TM loops forever for that sequence of configuration) and hence the DTM can miss a much shorter accepting configuration.

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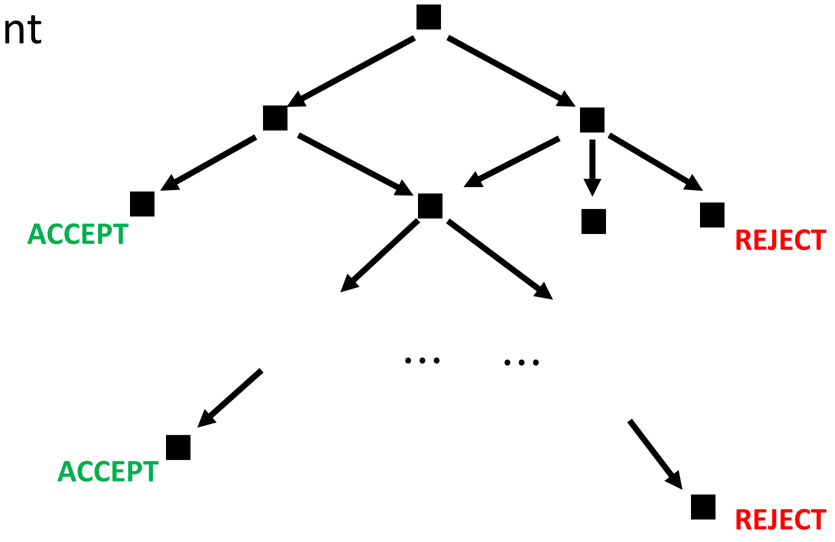
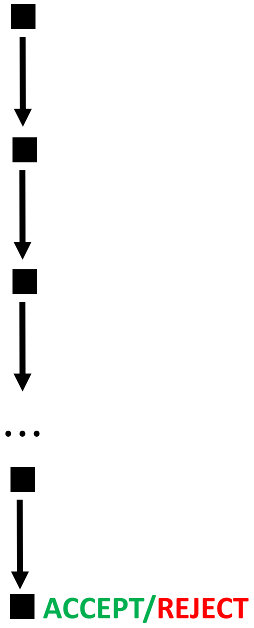
Any NTM can be simulated by a 3-tape DTM





Input string w

Generate runs lexicographically

Simulate the run for i/p w

- Tape 1 holds the input string w.
- As for the content of Tape 2, note that we can always obtain a bound for the maximum number of nodes at any level of the configuration tree (say b).

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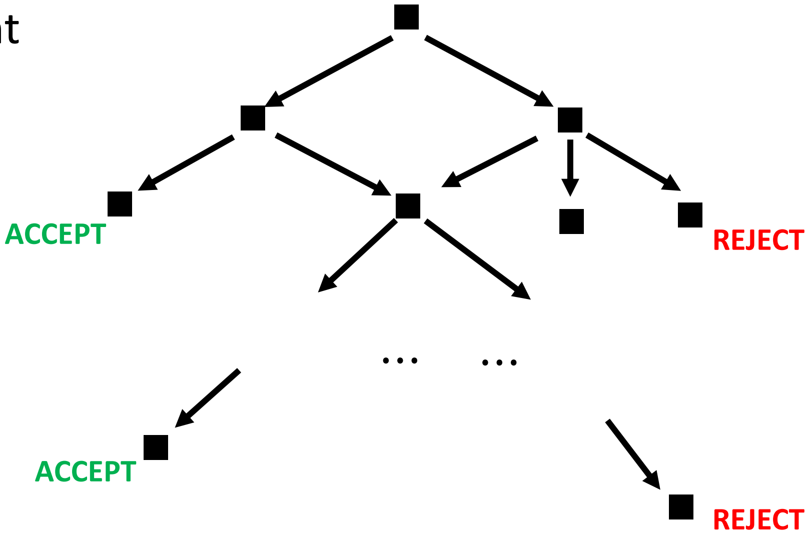
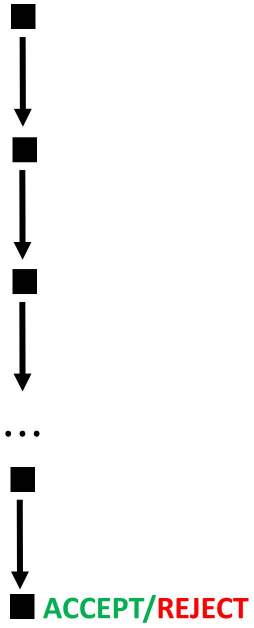
Any NTM can be simulated by a 3-tape DTM





Input string w

Generate runs lexicographically

Simulate the run for i/p w

- Tape 1 holds the input string w.
- As for the content of Tape 2, note that we can always obtain a bound for the maximum number of nodes at any level of the configuration tree (say b).
- Let $C = \{1,2,\cdots,b\}$, then we can define a run by a string $s \in C$. E.g: 122: choose the first node from level 1, second node from level 2, third node from level 3.

Variants of Turing Machine Models

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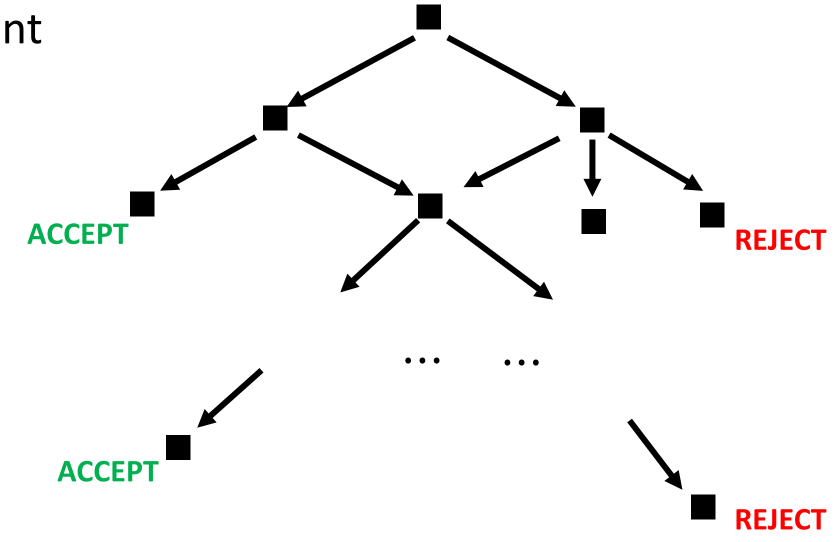

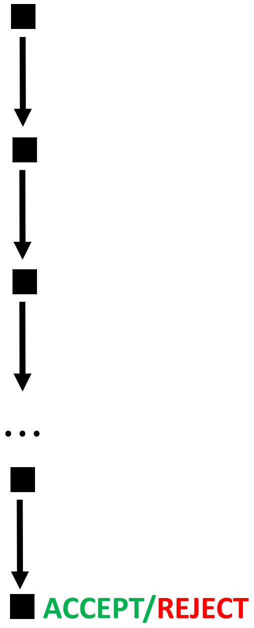

$1100011BB$
121
Simulate the run for i/p w

- Tape 1 holds the input string w.
- Tape 2 generates a string in $C = \{1,2,\cdots,b\}$ lexicographically: Generate all strings of length 1, then strings of length 2 and so on, i.e. $\{1,2,\ldots, b, 11, 12, 21, 22, \ldots\}$.
- Some of these runs may be invalid or too short to lead to any accept/reject state.

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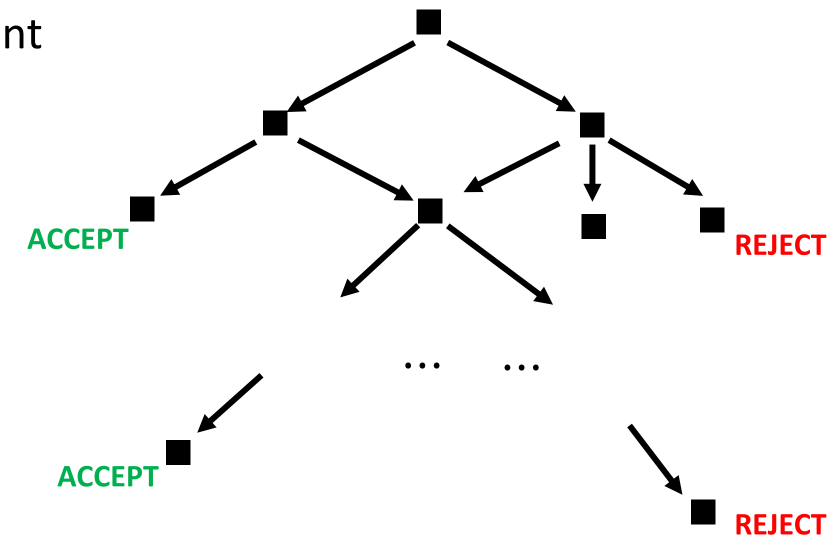

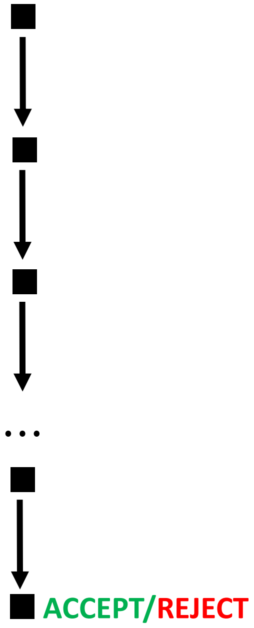

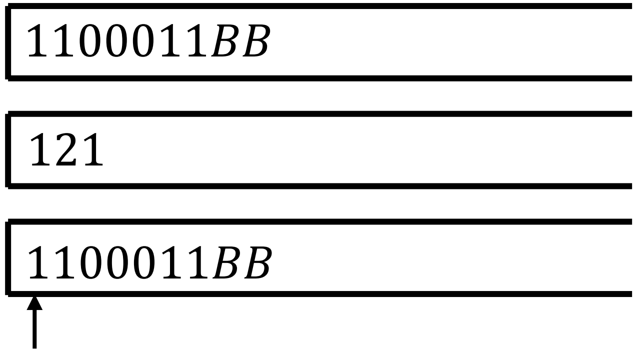

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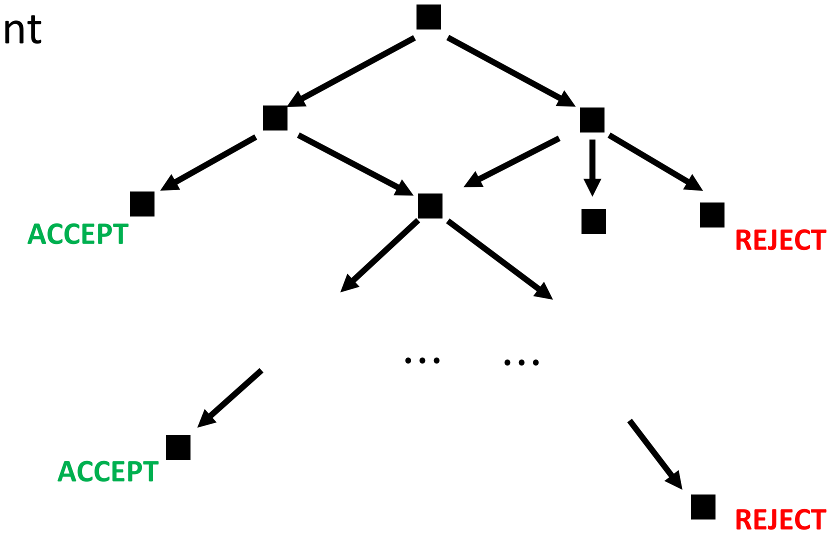

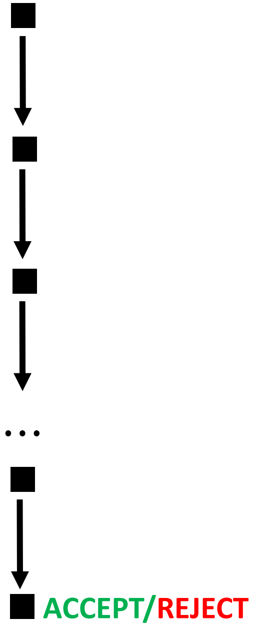

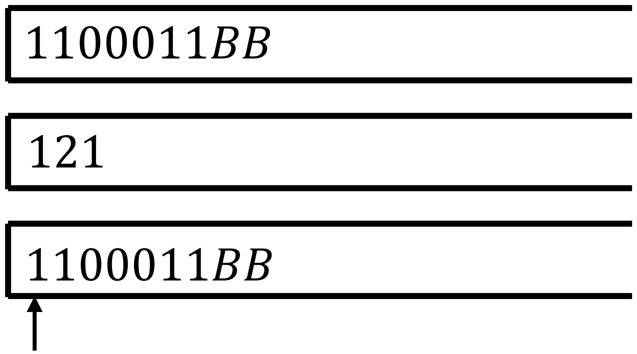

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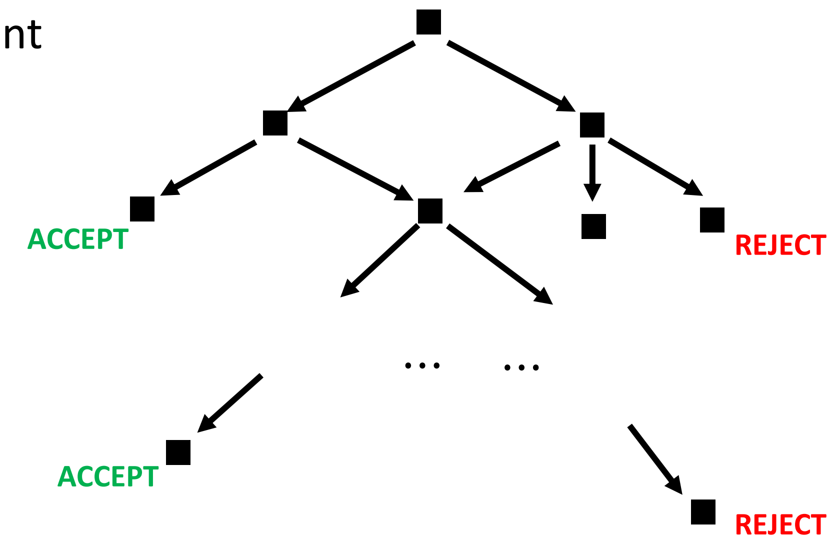

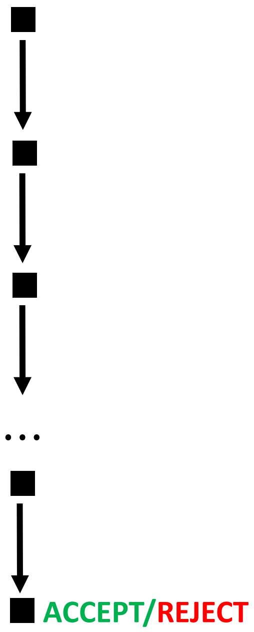

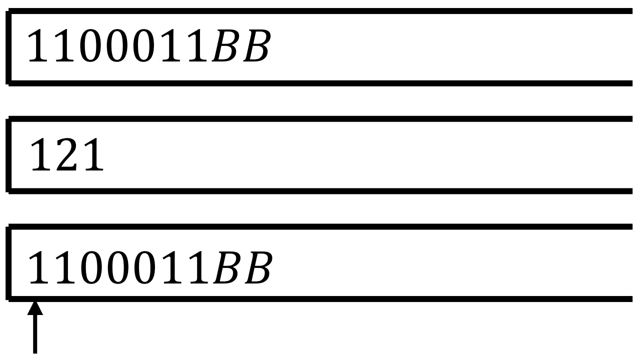

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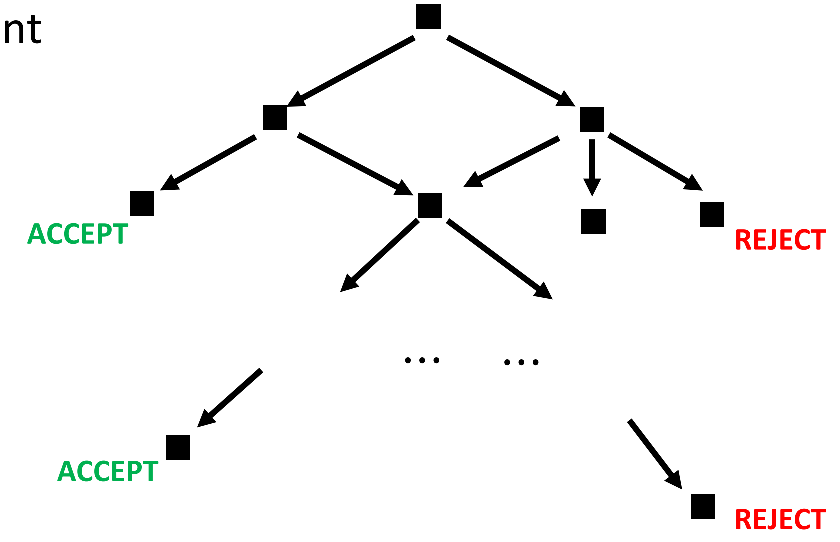

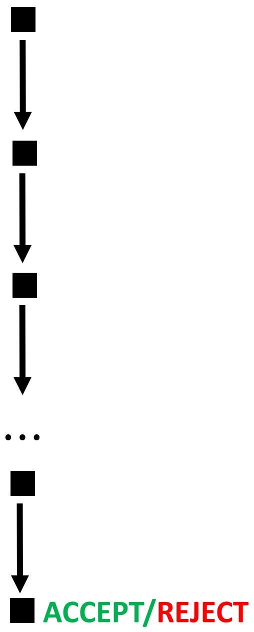

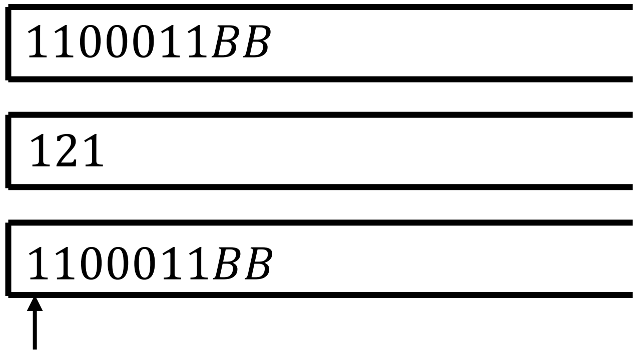

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- If the simulation leads to an accept state – accept the computation
- During the simulation, if the run in Tape 2 is found to be invalid, abort and generate the next lexicographic string

3-tape DTM $\equiv$ 1-tape DTM $\Rightarrow$ NTM $\equiv$ DTM

Variants of Turing Machine Models

Variants of TM:

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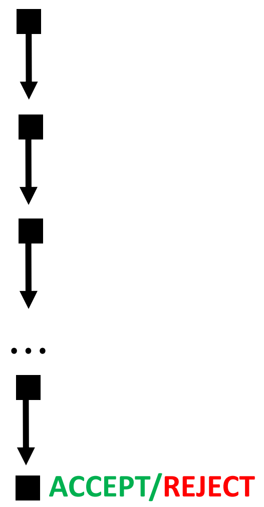
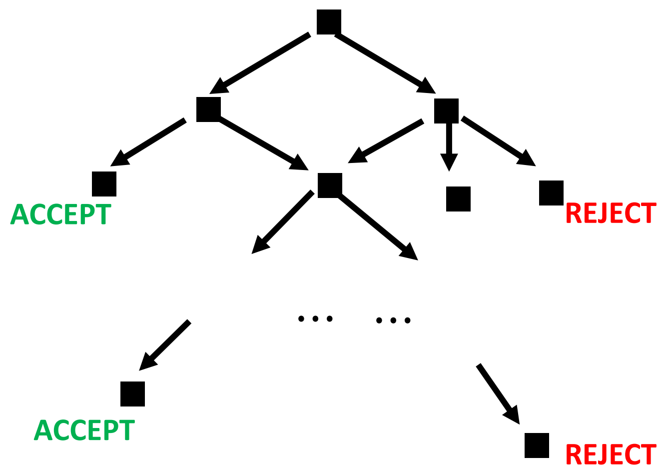
Key takeaway: A standard TM is quite robust. Adding extra power seems to make no difference in computing power

Variants of Turing Machine Models

- As an aside, although $NTM \equiv DTM$, a DTM may require several more steps to perform the same computation.
- For a moment, consider problems that are computable (TM halts on all inputs).
- For a given decision problem L, let for all input strings $|w| = n$, suppose $\exists$
- **DTM that halts within $\boldsymbol{DTIME}$ steps and outputs YES if $\boldsymbol{w \in L}$ and NO if $\boldsymbol{w \notin L}$.**





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$\boldsymbol{P}$ = Set of all problems for which $DTIME$ is a polynomial in n
(e.g.: $n^{1000} + 3n^2$)

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Variants of Turing Machine Models

- For a given decision problem L, let for all input strings w such that $|w| = n$, suppose $\exists$
- **DTM that halts within $DTIME$ steps and outputs YES if $w \in L$ and NO if $w \notin L$.**
- **NTM that halts within $NTIME$ steps and outputs YES if $w \in L$ and NO if $w \notin L$.**

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$\boldsymbol{NP}$ = Set of all problems for which $NTIME$ is a polynomial in n.

Clearly, $P \subseteq NP$. However, is $P = NP$?

A million dollar problem

Thank You!