Divide and Conquer (contd.)

Recap: (in divide & conquer)

1. Integer mult.
2. Matrix mult.
3. FFT
4. Closest pair of points

Counting Inversions:

$$\left.\begin{matrix} a_1 & a_2 & \cdots & a_n \\ b_1 & b_2 & \cdots & b_n \end{matrix}\right\} \quad \begin{matrix} \cup a_i \\ = \cup b_i \end{matrix}$$

$a_1 \cdots a_n$ } not neccessary that they are in sorted order.

(i,j) s.t $i<j$ and $a_i > a_j$ } inversion pair.

\# of inversions $\approx$ measure of dist from sortedness.

$a_1 \cdots a_n$.

$A = [a_1 \quad a_n]$

$A[1, \ldots, \lfloor \frac{n}{2} \rfloor]$ $\quad$ $A[\lfloor \frac{n}{2} \rfloor + 1, \ldots, n]$

$\mathrm{Det}(M)$ $\quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$ $\quad$ $1 \to 1$, $2 \to 2$ $\quad$ $1 \nearrow 1$, $2 \searrow 2$

$$= \sum_{\sigma \in S_n} (-1)^{\mathrm{Sign}(\sigma)} \prod_{i=1}^{n} M_{i\,\sigma(i)}$$

Sign(σ)

$ad - bc$

[Mahajan-Vinay, SODA 97].

Qu: Can we also count the no. of inv while doing this?

Brute force: $\binom{n}{2}$ pairs $i<j$, $a_i \overset{?}{>} a_j$

CountInversions$(A[1, \cdots, n])$

P = CountInversions $A[1, \ldots, \lfloor \frac{n}{2} \rfloor]$ } L

Q = Count Inversions $A[\lfloor \frac{n}{2} \rfloor + 1, \ldots, n]$ } R

Sorted_L Sorted_R

count_L count_R

$a_i \in L$ $a_j \in R$.

U: # of inversions across L and R

Total # of inv: $\underline{P+Q+U}$.

Brute force merge $O(n^2)$.

If $\underset{\sim\sim}{a_i > a_j}$ then so are the elements to the right of a_i in Sorted_L.

$\text{Sorted}_L = S_L$ $\text{Sorted}_R = S_R$

$\uparrow$ i $\uparrow$ j

1-indexing

$\underline{i=j=1}$ (init)

$S_L[i] \overset{?}{>} S_R[j]$

If yes then count += $|S_L| - i + 1$; $j \leftarrow j+1$. } Getting a count of inv involving $\underline{a_j}$.

Else, $i \leftarrow i+1$.

$$T(n) = T\left(\left\lfloor \tfrac{n}{2} \right\rfloor\right) + T\left(\left\lceil \tfrac{n}{2} \right\rceil\right) = T\left(\left\lfloor \tfrac{n}{2} \right\rfloor\right) + T\left(\left\lfloor \tfrac{n}{2} \right\rfloor + 1\right) + \underline{O(n)}$$

→ Median of Medians $\leftarrow$ Approximate median finding.

Quicksort(A)

↳ pivot p.

$L = \text{Quicksort}(A_{<p})$

$R = \text{Quicksort}(A_{>p})$

Return $L, \{p\}, R$.

$$T(n) = T(|L|) + T(|R|) + O(n).$$