Dynamic Programming (Contd.)

Interval Scheduling:

Requests $R_1, \dots, R_n$

Priority $w_1, \dots, w_n$

Objective:

$$\max_{S \subseteq \{R_1, \dots, R_n\}} \left(\sum_{i \in S} w_i \right)$$

$R_i \to start_i$

$R_i \to finish_i$

Optimal $(R_1, \dots, R_n)$

Case-1: R_n may belong to the optimal seq.

$R_1 \quad R_2 \quad \dots \quad R_n$ (Sorted in the order of finish times)

* Remove all incompatible w/ R_n requests.

For a compatible job R_i, we must have $finish_i < start_n$.

$$p(j) = \max \{ i \mid R_i \text{ is compatible with } R_j \}, \quad i < j$$

max Request whose finish time is less than start time of j

$R_1, \dots, R_{p(n)}$ are compatible with R_n but not $R_{p(n)+1}, \dots, R_{n-1}$.

$p(n)$ may not be $\in [1, n-1]$.

Compute Optimal $(R_1, \dots, R_{p(n)})$ $\Big\}$ $w_n +$ Optimal $(R_1, \dots, R_{p(n)})$

or $w_n + \{\}$.

Case-2: R_n may not belong to optimal seq.

Compute Optimal $(R_1, \dots, R_{n-1})$.

$$\text{Optimal}(R_1, \dots, R_n) = \max \begin{cases} \text{Optimal}(R_1, \dots, R_{n-1}) \\ w_n \\ w_n + \text{Optimal}(R_1, \dots, R_{p(n)}) \end{cases}$$

$$\text{Optimal}(R_1) = \max \begin{cases} 0 \\ w_1 \end{cases}$$

W.L.O.G Assume that weights are positive.

0-1 knapsack.

Items	$w_1, w_2, \dots, w_n$ ($\mathbb{Z}$)	
Values	$v_1, v_2, \dots, v_n$	

$$\max_{S \subseteq [n]} \sum_{i \in S} v_i$$

subject to

$$\sum w_i \leq \underline{W}$$

Either $\text{Item}_n \in \text{Opt}$ or not:

$\hookrightarrow$ $\text{Item}_n \in \text{Opt}$: $w_n \overset{?}{\leq} W$

$\hookrightarrow$ Yes

$\text{Opt}([1, n-1], W - w_n)$

$\downarrow$

$\text{Opt}([1, n-1], W)$

$$\text{Opt}([1, n], W) = \max \begin{cases} \text{Opt}([1, n-1], W - w_n) + v_n \\ \text{Opt}([1, n-1], W) \end{cases}$$

w' can take 2^n many values.

$\underline{\underline{n \times W}}$

$\text{Opt}([1, i], w')$

i: $1, n$

Say W is very very large

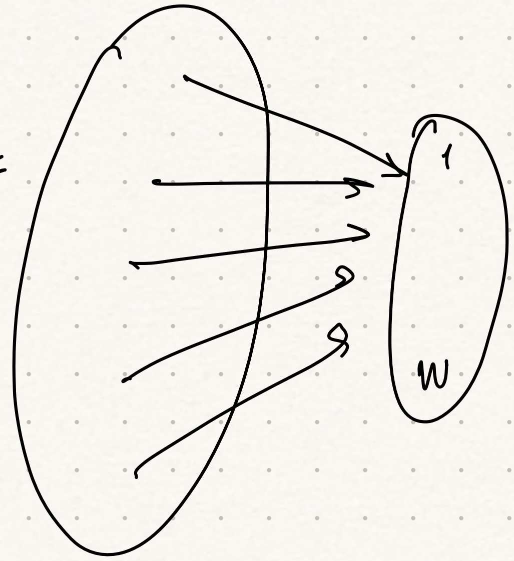
$a_i \in \{0,1\}^n$

$$W' = \sum_{i=1}^{n} a_i w_i$$

If W is small,

$\bar{a}$ s.t.

$$\sum_{i: a_i = 1} w_i \le \boxed{W}$$



$$Opt\Big(\underline{[1, i]},\ \underbrace{W - \sum_{j=i+1}^{n} \alpha_j \cdot w_j}_{2^{n-i}}\Big)$$

$w_{i+1}, \dots, w_n$

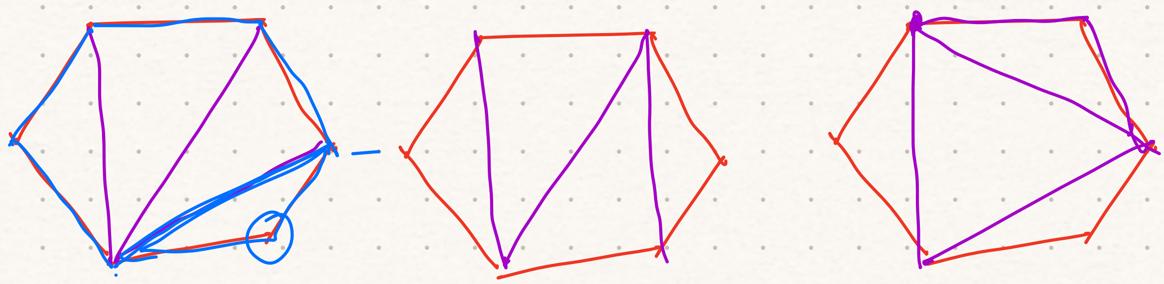
$\underbrace{\bar{\alpha}_{i+1} \quad \bar{\alpha}_n}_{(n-i)}$

$\in \{0,1\}$

$\forall a_1, a_2 \in$ ~~Poly~~ polygon

$\forall \lambda \in [0,1],\ \lambda a_1 + (1-\lambda) a_2 \in$ polygon.

Triangulation of Polygons:



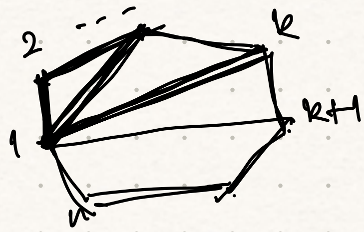
Cost: Sum of the lengths of diagonals introduced. } S

want to minimize this cost.

" Minimize the sum of perimeters of the triangles.

$2S + \underline{C}$ ← polygon

Cost of triangulation

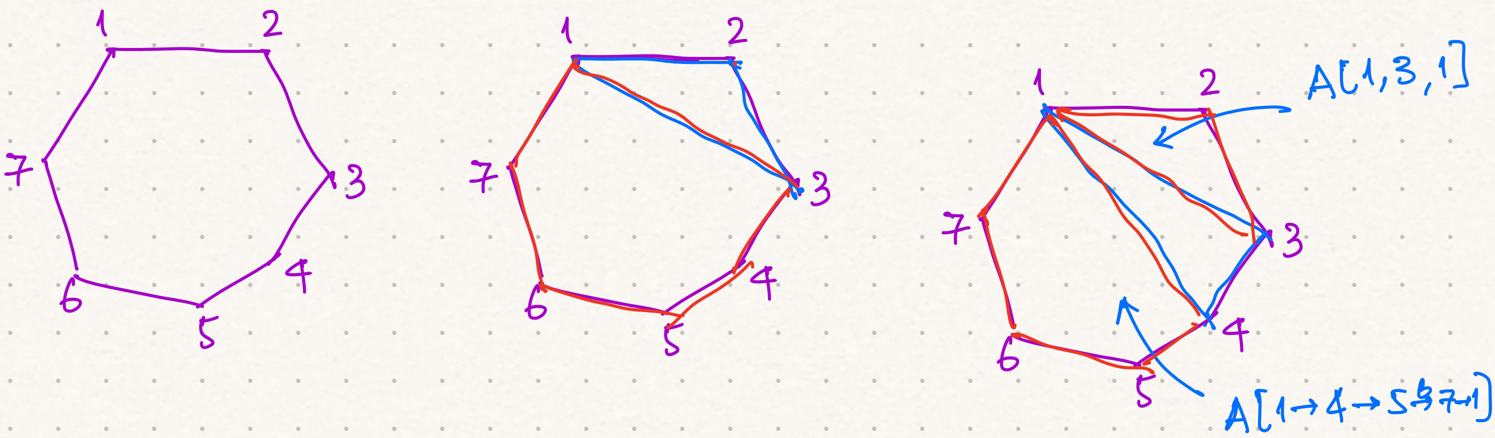


i, j

$A[i,j]$ denote the min cost of triangulation of polygon with vertices $i, i+1, \cdots, j$.

dist(1,k) + dist(1,k+1)

$$A[1, \underset{+1}{n}] = \min_{k} \left\{ A[1,k] + A[k+1, \underbrace{n}_{+1}] + \text{Perimeter of } (1, k, k+1) \right\}$$



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