PRP Quiz 1 Solution

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1 Question 1

We can count the number of disjoint sets that can be formed according to the question. The power set of that set will be my maximum possible number of elements in the smallest σ field.

Part 1: $\{A\backslash(A\cap C), B\backslash(B\cap C), C\backslash A\backslash B, A\cap C, B\cap C, (A\cup B\cup C)^C\}$
So, the answer is 2^6.

Part 2: $\{A, B\backslash C, C, (A\cup B)^C\}$
So, the answer is 2^4.

2 Question 2

The condition provided is:

$$P(A \mid B) = P(A \mid B^c).$$

Using the definition of conditional probability, this can be written as:

$$\frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B^c)}{P(B^c)},$$

where $P(B^c) = 1 - P(B)$.

Rewriting this, we get:

$$P(A\cap B)\cdot(1-P(B)) = P(A\cap B^c)\cdot P(B).$$

Expressing $P(A)$

We can express $P(A)$ in terms of $A\cap B$ and $A\cap B^c$:

$$P(A) = P(A\cap B) + P(A\cap B^c).$$

Substituting $P(A\cap B^c) = P(A) - P(A\cap B)$ into the earlier equation gives:

$$P(A\cap B)\cdot(1-P(B)) = (P(A) - P(A\cap B))\cdot P(B).$$

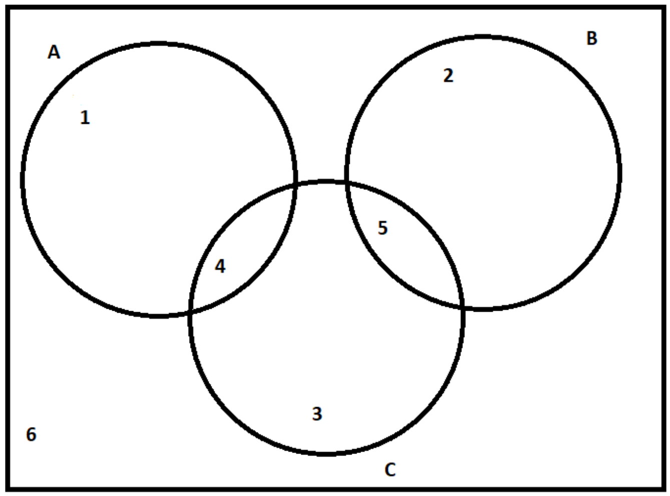


Figure 1: Part 1

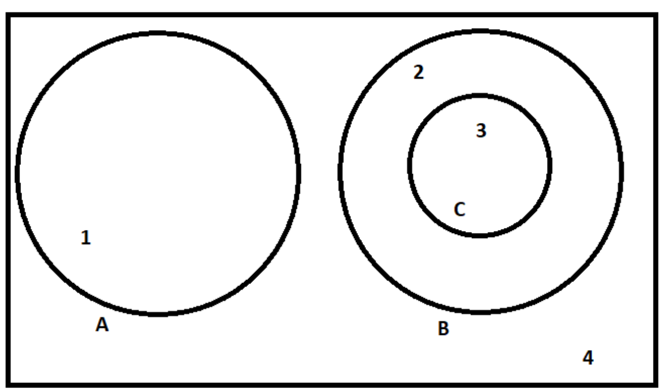


Figure 2: Part 2

Simplifying the Condition

Expanding and simplifying:

$$P(A \cap B) - P(A \cap B) \cdot P(B) = P(A) \cdot P(B) - P(A \cap B) \cdot P(B).$$

Combine like terms:

$$P(A \cap B) = P(A) \cdot P(B).$$

Hence, they are independent.

3 sol: Let Y be a random variable with CDF

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ y & 0 \le y < 1 \\ 1 & y \geq 1 \end{cases}$$

Define RVS

$$X_1 = F_1^{-1}(Y)$$
$$X_2 = F_2^{-1}(Y)$$

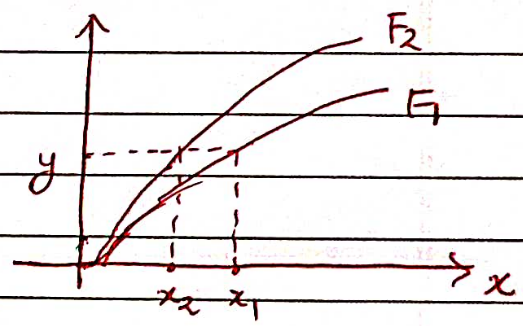
$$P(F_i^{-1}(Y) \le x_i) = P(Y \le F_i(x_i))$$
$$= F_i(x_i)$$

So the CDF of RVS x_1 & x_2 are F_1 & F_2, respectively.

For $y \in [0,1]$,

let $F_1^{-1}(y) = x_1$

$F_2^{-1}(y) = x_2$



Suppose $x_1 \le x_2$

$\Rightarrow F_1(x_1) \le F_1(x_2) < F_2(x_2)$

$\Rightarrow y = F_1(x_1) \le F_1(x_2) < F_2(x_2) = y$

This is a contradiction

So, $x_1 > x_2$, i.e. $F_1^{-1}(y) > F_2^{-1}(y)$,

$\forall y \in [0,1]$

$$X_1 = F_1^{-1}(Y) > F_2^{-1}(Y) = X_2$$