Theory Assignment II

Automata Theory Monsoon 2024, IIIT Hyderabad

August 29, 2024

Total Marks: 40 points

Due date: **16/09/24 11:59 pm**

General Instructions: All symbols have the usual meanings. (example: $\mathbb{R}$ is the set of reals etc.). FSM stands for finite state machine. DFA stands for deterministic finite automaton. TM stands for Turing machine. PDA stands for push down automata. RE stands for Recursively Enumerable.

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1. [3 points] Show that the language $L = \{xy \mid x, y \in \{0,1\}^*, |x| = |y|, x \neq y\}$ is a context-free language. **[CO 2, CO 3]**

2. [6 points] State and Prove whether the following languages are context-free or not using Pumping Lemma.

    1. $L = \{a^n b^m \mid m = n^2\}$
    2. $L = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$

    [CO 2, CO 3]

3. [2 points] Show that the following grammar is ambiguous.

    1. $E \to I$
    2. $E \to E + E$
    3. $E \to E * E$
    4. $E \to (E)$
    5. $I \to \varepsilon \mid 0 \mid 1 \mid 2 \mid \ldots \mid 9$

    [CO 1, CO 2]

4. [4 points] Convert the following CFG into Chomsky normal form:

    $$A \to BAB \mid B \mid \epsilon$$
    $$B \to 00 \mid \epsilon$$

    [CO 1, CO 2]

5. [5 points] Is a PDA equipped with two stacks computationally more powerful than a PDA with a single stack? Provide a formal proof for your argument. **[CO 1, CO 2, CO 3, CO 4]**

6. [6 points] Given the alphabet $\Sigma = \{\#, 0, 1\}$, provide a description of a TM that computes the function $f(\#\langle x\rangle) = \begin{cases} \#\langle \frac{x}{2}\rangle & \text{if } x \text{ is even} \\ \#\langle 3x+1\rangle & \text{otherwise} \end{cases}$ where $\langle x\rangle$ stands for the binary representation of the number x. (For example, if the TM starts with #100 on the tape it should halt with #10 on the tape; if it starts with #11, it should halt with #1010.) You may use variants of TM which are equivalent in power. If the variant has more than one tape then the output should be written on the first tape. **[CO 1]**

7. [4 points] Let R_1, R_2 be two recursive languages, and RE_1, RE_2 be two recursively enumerable languages. State the class the following languages belong to with reasons.

   1. $R_1 \cup R_2$
   2. $RE_1 \cup RE_2$
   3. $R_1 \cup RE_2$
   4. $R_1 \cap RE_2$

   [CO 3, CO 4]

8. [3 points] Let L_1 and L_2 be languages recognised by TMs M_1 and M_2 respectively. Let s_1 and s_2 be strings which belong to L_1 and L_2.

$$s_3 = s_1 s_2 s_1 s_2 ...$$

   Generate a TM M_3 which can, given s_3, separate s_1 and s_2
   For example:

$$s_1 = chipi$$

$$s_2 = chapa$$

   Then:

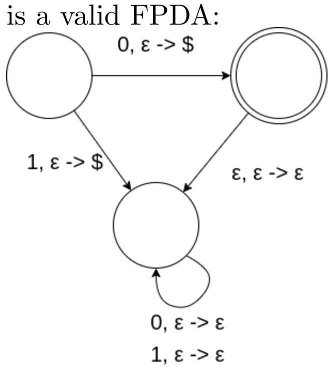
$$s_3 = chipichapachipichapa...$$

   Your task is to extract *chipi* and *chapa*
   Assume the following to be true:

   - If s belongs to either language, no prefix of s belongs to either language
   - You are allowed to call M_1 and M_2 with its input on a separate tape.

   [CO 1, CO 3]

9. [3 points] Consider a 'Fully Deterministic' PDA (FPDA), where no crash is allowed and every pair of input, stack variable has exactly one transition possible per state.
   Example: Consider the FPDA that accepts 0,1 as input letters and the stack can have 0,1,\$. Following is a valid FPDA:



Construct an FPDA that accepts the Language, $L = \{0^n 1^n \mid n >= 0\}$ **[CO 1, CO 2]**

10. [4 points] Let $\Sigma = \{0, 1\}$ and let B be the collection of strings that contain at least one 1 in their second half. In other words, B is defined as:

$$B = \{uv \mid u \in \Sigma^*, v \in \Sigma^* 1 \Sigma^*, \text{ and } |u| \geq |v|\}.$$

Construct a CFG that generates B. **[CO 1, CO 2]**