Closest Pair of Points

Question: Given a set of points on a real line, find a pair of closest points.

Say input $= \{ P_1, \ldots, P_m \}$ where each $P_i \in \mathbb{R}$.

$\underset{\curvearrowright}{}$ provided in an arbitrary order.

$\rule{10cm}{0.4pt}$

→ Sort them and record distances between consecutive points.

$\hookrightarrow$ pick the pair of consecutive points that attain min

2-dim:

$\mathbb{R}^2$

Given a set of points in a plane, find a pair that attains closest distance.

$\{ P_1, \ldots, P_n \}$ s.t $P_i \in \mathbb{R}^2$

P_i, P_j

$\swarrow \qquad \searrow$

$(x_1, y_1) \qquad (x_2, y_2)$

Trivial: Compute distances for all $\binom{n}{2}$ pairs and pick the min.

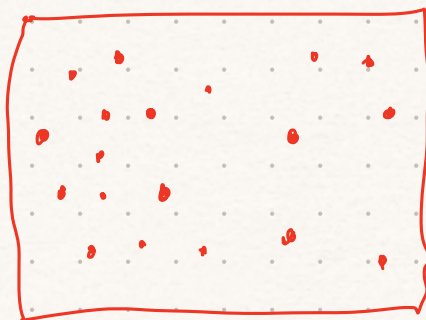$\left| \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \right|$

$O(n^2)$

· Sort by x and get cons. dist.

$P = \{ P_1', \ldots, P_n' \}$ / Sort w/ y-values.

· Obtain the median x value and divide the space into 2 parts. P_L and P_R

· Obtain "recursively" closest pair of points in each of P_L and P_R.

δ_L = dist attained by closest pair of points in P_L.

δ_R:

P_R
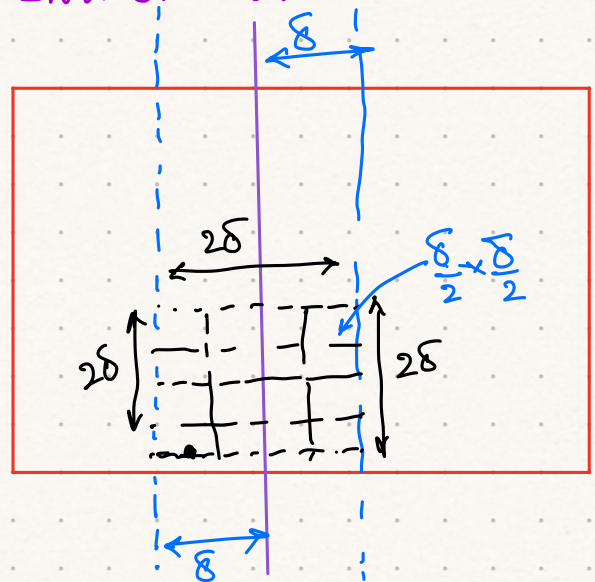
$\delta = \min \{\delta_L, \delta_R\}$.

Want to test if any pair of points with one on each side of the partition, attain a value smaller than δ.

Obs: This band could contain all the points $\{P_1, \ldots, P_n\}$.

Brute force for points in band:

For all pairs of points find their dist and check if their min $\leq \delta$.

$\cdot$ Sort the points in the band wrt their y-coordinates.

Claim: # of points in this grid ≤ 16.

$\hookrightarrow$ suff to prove that each $\frac{\delta}{2} \times \frac{\delta}{2}$ square contains at most one point.

(Both these points are on same side)

Suppose not and there are 2 points in a square of size $\frac{\delta}{2} \times \frac{\delta}{2}$. $\Rightarrow$ dist between them is at most $(\frac{\delta}{2})\sqrt{2}$

This contradicts that δ is min dist (min of) across L and R.

$\curvearrowleft$ length of a diag in $\frac{\delta}{2} \times \frac{\delta}{2}$ square.

From the above arguments, we get that for each point in the band (points considered in the order of incr. y values) we only need to compare with next 15 points.

$\hookrightarrow$ Record the min over all the computations thus made and

Check if it is $< \delta$.

   If yes: report the corr pair as closest set of points.

   Else:

       Report the points that achieve δ in L or R.

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + 15n \Rightarrow O(n \log n) \text{ arithmetic operations.}$$

+ Preprocessing of $O(n \log n)$.

             ⎰ maintain sorted list w/ x-values

                                    y-values.