(CS1.301) Algorithm Analysis & Design (Monsoon 2024) Due: 10pm, 12.10.2024

Problem Set 4

Instructions:

- Discussions amongst the students are not discouraged, but all writeups must be done individually and must include names of all collaborators.
- Referring sources other than the lecture notes is discouraged as solutions to some of the problems can be found easily via a web search. But if you do use an outside source (eg., text books, other lecture notes, any material available online), do mention the same in your writeup. This will not affect your grades. However dishonesty of any sort when caught shall be heavily penalized.
- Be clear in your arguments. Vague arguments shall not be given full credit.
- Total marks for this problem set are 30.
- **Note: For each problem you must do the following:**
  (a) Provide and analyze the run time complexity.
  (b) Neatly dry run on a non-trivial example. This should clearly show your solution's working. The example should not be the given sample.
  (c) Any observations in the problem that led you to this solution.

Problem 1

You are playing a game, in which to win you need to score **exactly** T points. In this game, there are N tasks, the i^{th} of which gives you a_i points. Doing any task once takes you 1 second to complete. You can do any task **any number** of times. You want to find the **minimum** time required to win the game.

For example, if there are 3 tasks with points as {5,8,10} and T is 21, the minimum time you can win the game in is 3 seconds as $8 + 8 + 5 = 21$

Provide an algorithm which finds the minimum time required to win the game. If it is not possible to win the game return -1. [5 marks]

Problem 2

Let G = (V, E) be an undirected path graph with n nodes. A graph is a path if the graph is connected, and the nodes can be put in a row so that all edges are between adjacent nodes. For example, here's a path graph on 4 nodes:

A subset of nodes is called an independent set if no two nodes in the subset are connected by an edge. For example, in the path graph above the first and third nodes form an independent set,but the first and second nodes do not.

We will also associate with each node a positive integer weight (note that these weights are attached to nodes, not to edges). For example, here's a path graph with node weights:

In the **path weighted independent set problem**, we have:

Input: An undirected, unweighted path graph G = (V, E) with $|V| = n$ nodes, and integer weight $w_i > 0$ on each node i

Output: A subset of nodes S such that no two nodes in S have an edge between them, and the total weight of the nodes in S is maximized

Give an algorithm that takes an n-node path G with weights and returns an independent set of maximum total weight. The running time should be polynomial in n, independent of the values of the weights. [5 marks]

Problem 3

In a distant land, a traveler finds themselves lost in a dangerous landscape in the form of a 2D grid consisting of M × N zones. Each zone has different effects on the traveler's strength — some areas drain their strength, while others offer moments of relief, restoring their strength. The traveler begins their journey at the north-west (top left) point of the land and must reach the south-east (bottom right) point to find their way home.

The traveler can only move southward (down) or eastward (right) at each step of the journey. As they move through the zones, they lose or gain strength based on the value of the zone in the grid. (Negative values will decrease the strength, and Positive values will increase the strength, by the given value). The challenge is that the traveler's strength can never drop to zero or below, or they will not survive the journey.

The task is to find the minimum amount of initial energy needed to complete the journey.

Consider the following map of the zones, where each number represents the effect on the traveler's energy:

-2	-4	15
-1	-5	-4
-4	20	-1

If the traveler chooses the path EAST -> EAST -> SOUTH -> SOUTH, the initial energy needed to complete the journey and reach home is 7. This is the minimum possible initial energy required.

<u>Note</u>:

1. The traveler's energy has no maximum limit.
2. All zones in the landscape either drain energy or restore it, including the starting point (the top left zone) and the final destination (the bottom right zone).

Describe an algorithm to determine the **minimum** amount of energy the traveler must start with to ensure they can complete their journey safely. [10 marks]

Problem 4

In a faraway kingdom, the royal chef is tasked with preparing a series of dishes for a grand feast. Each dish can either be savory (type 1) or sweet (type 2), and there are K serving platters available to present the dishes. The chef must carefully arrange the dishes onto the platters, while adhering to the following rules:

1. The order in which the dishes are served must be maintained. A dish that appears earlier in the sequence cannot be placed on a platter after a dish that appears later.
2. Every platter must have at least one dish, and all dishes must be placed on the platters.
3. For each platter, the chef calculates the **interaction score** by multiplying the number of savory dishes with the number of sweet dishes on that platter and summing this across all platters. The goal is to **minimize** the interaction score.

Consider the example below for better clarity:
Dishes: $\{12112\}$, $\mathsf{K} = 2$
<u>Explanation</u>: The chef has 4 different ways to arrange the dishes onto the platters:

(a) $\{1\},\{2112\}$: The total score = $1 \times 0 + 2 \times 2 = 4$

(b) $\{12\},\{112\}$: The total score = $1 \times 1 + 2 \times 1 = 3$

(c) $\{121\},\{12\}$: The total score = $2 \times 1 + 1 \times 1 = 3$

(d) $\{1211\},\{2\}$: The total score = $3 \times 1 + 0 \times 1 = 3$

In the above example the minimum possible interaction score is 3.
Develop an algorithm to determine the **minimum possible total interaction score**, while adhering to the specified constraints. If it's not possible to serve the dishes on K platters while following the rules, return -1. [10 marks]