• Shortest path.

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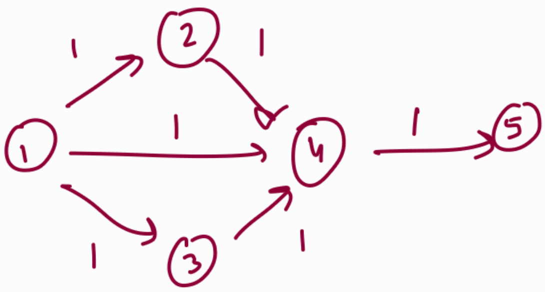
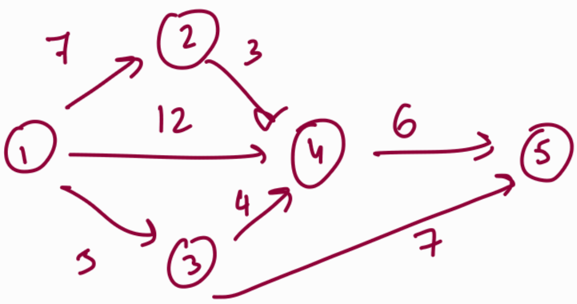


(1, 4).

If all wts. are equal, then BFS gives the shortest path.

Now, edges have wts. BFS no longer gives the shortest path

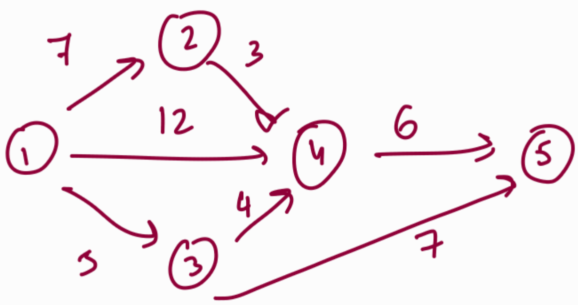


• Shortest paths in DAGs

$G = (V, E)$, $wt : E \to \mathbb{R}$

If P is a path with edges $e_1, \dots, e_k$ in it, then

$wt(P) = wt(e_1) + wt(e_2) + \dots + wt(e_k)$.



Q^n: What is the shortest path from 1 to 5?

• Topological sort.

1) Place the nodes in topological order.

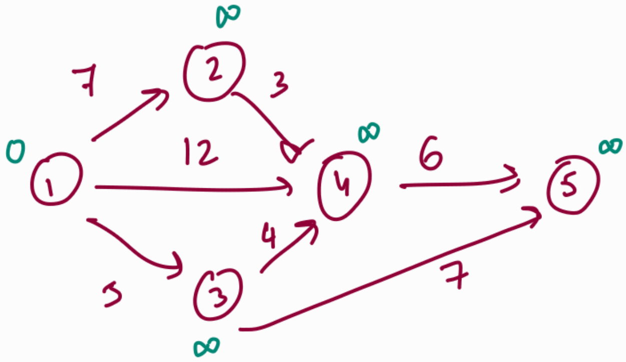
2) $\forall v \in V \setminus \{s\}$, $dist(s, t) \leftarrow \infty$

3) Visited $\leftarrow \{s\}$

4) $\boxed{\text{while visited} \neq V:}$ → Not the exact condition. Have to modify.

for all $v \in N(\text{visited})$:

$$\text{dist}(s,v) = \min\left\{ \{ \text{dist}(s,u) + \text{wt}(u,v) \mid (u,v) \in E,\ u \in \text{visited} \} \cup \{ \text{dist}(s,t) \} \right\}$$



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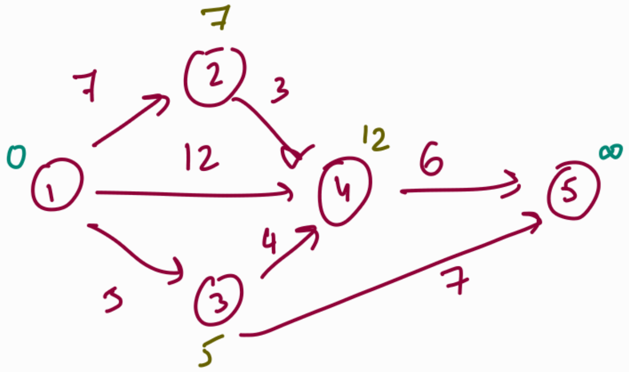
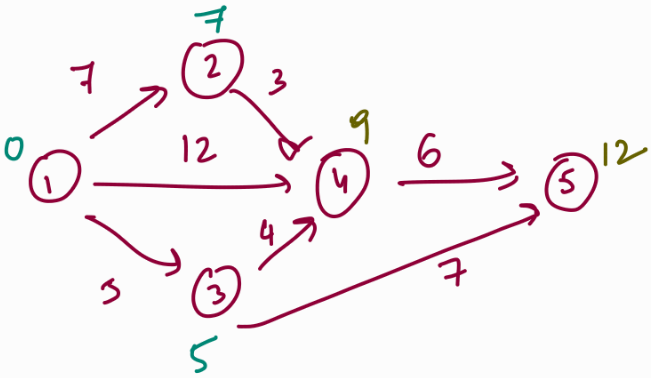


$N_{out}(1,2,3,4) = \{2,3,4,5\}$.

$$\text{dist}(s,4) = \min\{ \overset{12}{\text{dist}(1,4)},\ \underset{10}{\text{dist}(1,2)+\text{wt}(2,4)},\ \underset{9}{\text{dist}(1,3)+\text{wt}(3,4)} \}$$

$$\text{dist}(s,5) = \min\{ \text{dist}(1,4)+6,\ \text{dist}(1,3)+7,\ \infty \}$$



Direct acyclicness guarantees that the algo ends at some point of time.

→ Even if there are -ve wts., it'll still work because of the acyclicness.

→ Assumed DAG because if it is cyclic, then a cyclic dependency forms in recursion.

Eg: If dist(1,4) req. calc. of dist(1,5) and dist(1,5) req. dist(1,4). Basically a cycle. Unresolvable recursion.

See notes for precise code. Above code not precise. (Recursion better instead of while).

- Single source graphs with no negative edges (Can have cycles).
(Works with indirected graphs as well)

- Djikstra's algo :

Try it out using eg.

Visited $\leftarrow \{s\}$, $d[s] = 0$

For every $v \in V \setminus \{s\}$

$d(v) \leftarrow \infty$

while visited $\neq V$:

For all $v \in (V \setminus \text{visited}) \cap N(\text{visited})$:

$$d'[v] = \min_{(u,v) \in E,\ u \in \text{visited}} \{ d(u) + wt.(u,v) \}$$

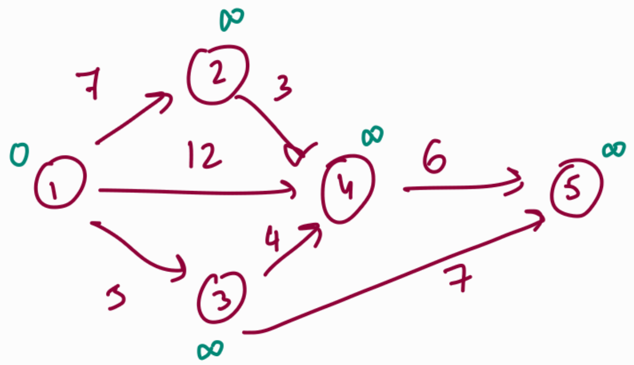
select $v \in (V \setminus \text{visited}) \cap (N(\text{visited}))$ s.t

$d'(v) = \min \{ d'[v] \mid v \in A \}$

set $d[v] \leftarrow d'[v]$

add v to visited.

The while loop runs $(n-1)$ times



→ Does this always give the correct output of shortest path?

(Req. proof of correctedness).

Observations:

1. Visited set grows by 1 vertex in each iteration, but elements once added are not disturbed in the later stage of the algorithm.

2. Shortest dist. once computed are not updated ever again. ($d[u]$ is never updated once it attains a value)

Correctness lemma :

Consider the set visited at an arbitrary point of time in the algo's execution. For all $u \in$ visited , $d[u]$ is the shortest dist. from $s \rightsquigarrow u$.

(Proof in the next doc.)

Dry run with examples.