# Algorithm Analysis and Design.

Bubble sort $O(n^2)$

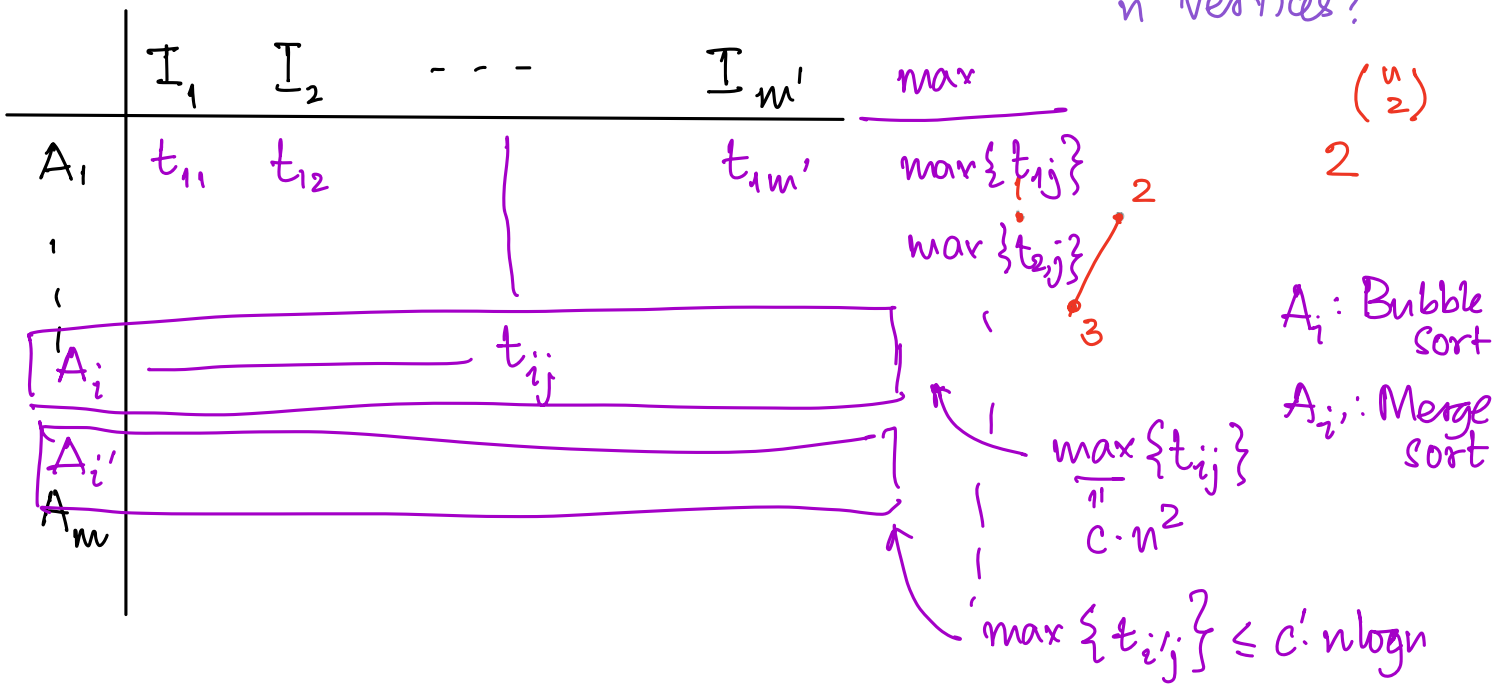
"Worst case"

Worst case analysis of algorithms.

How many graphs are possible on n vertices?

$\binom{n}{2}$

2

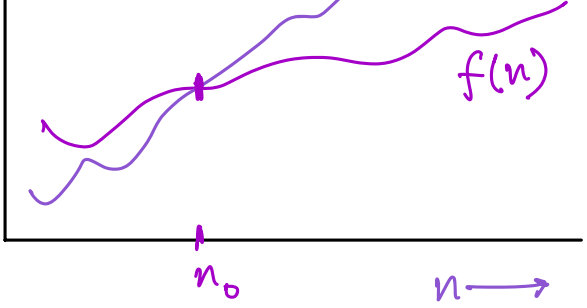



## Asymptotics

### Big Oh notation.

Let $f$ and $g$ be functions from $\mathbb{R}_{>0} \to \mathbb{R}_{>0}$. We say that $f(n) = O(g(n))$ if $\exists$ a constant $c$ and $n_0$ large enough s.t $\forall n \geq n_0$

$f(n) \leq c \cdot g(n)$




# Basic Graph Algorithms.

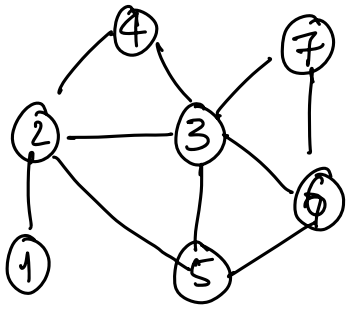
Klienberg
Tardos

Erikson (UIUC)

DasGupta
Papadimitrou
Vazirani

Goodrich
Tamassia

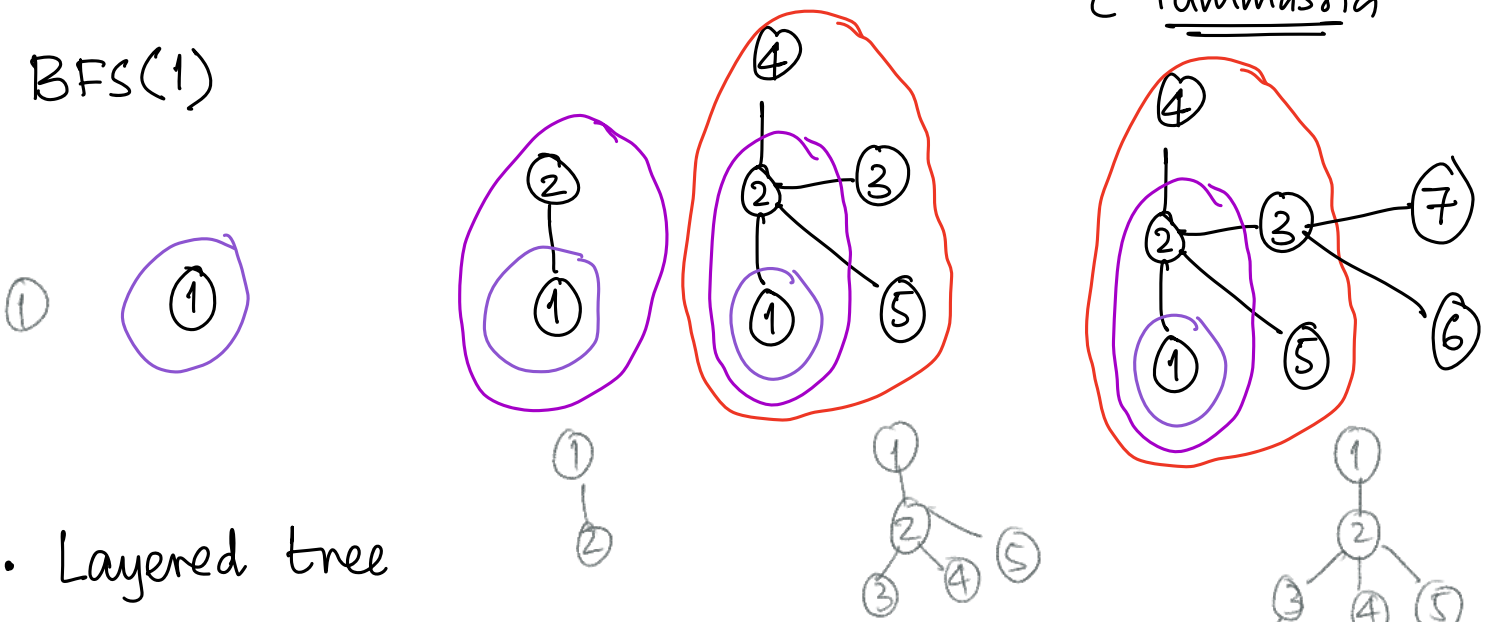
## Breadth First Search.




Visited

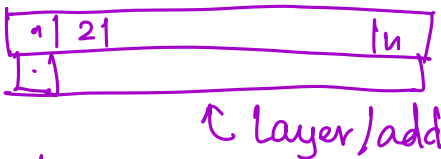
Neighbours

BFS(1)




- Layered tree
  - Layer 0 contains the start node.
  - Layer 1 contains the neighbours of start node.
  - $\forall j \geq 2$, Layer $L_j$ contains all vertices
    - that are not already in $L_1, \ldots, L_{j-1}$
    - and those that have an edge to a vertex in $L_{j-1}$

Bookkeeping : Explored/Visited ?



↑ Layer/address.

for every vertex visited, we examine the neighbours

$v$, $d_v$.

$n \leftarrow$ denotes # of vertices

$m \leftarrow$ denotes # of edges.

$$\sum_v d_v = 2|E| = 2m$$

$O(n+m)$ ← Running time

Obs: Layer no.s implicitly hold the information about "shortest distance of a vertex" from the root.

Say this is false for the sake of contradiction.

Thm: All elements in Layer $L_j$ have a shortest dist of $j$ are at a distance of $j$ from the root.

Base case: Layer $j = 1$. Contents of Layer 1 are neighbours of the root node.

I.H: $L_{j-1}$

Inductive step: $v \in L_j$, $u$ is a predecessor of $v$ s.t $u \in L_{j-1}$ and $(u,v) \in E$

$v \notin L_1 \cup \cdots \cup L_{j-1}$.