(CS1.301) Algorithm Analysis & Design (Monsoon 2024)

Practice Problem Set 5

Instructions:

- The following problem set is not graded and is for practice.
- Some of the problems will be covered in the tutorial.

Question 1

Let $G = (V, E)$ be a directed graph, with source $s \in V$, sink $t \in V$, and non-negative edge capacities c_e. Give a polynomial-time algorithm to decide whether G has a unique minimum s-t cut (i.e., an s-t cut of capacity strictly less than that of all other s-t cuts).

Question 2

Consider a round-robin scenario in league matches. The league comprises N teams. At a certain stage during the season, team i has accumulated $w[i]$ wins and has $g[i][j]$ games left to play against team j. A team faces elimination if the team cannot secure first place, except when two teams may be tied for first place, in which case neither of the two teams is eliminated.

Design an algorithm that, given team i at a specific point (described by the arrays w and g) in the season, can determine whether the team is subject to elimination or not. It is assumed that no matches result in ties and that the final rankings are solely based on the number of matches won.

Question 3

Give an algorithm for computing the max-flow with the following additional constraints. (Provide the solutions to both the parts separately)

1. Each edge *e* has a lower bound *l(e)* on the flow through it.
2. There are multiple sources and sinks, and the flow value is computed as the total flow out of all the sources (equivalent to the total flow into all of the sinks).

Question 4

For each pixel i in an image, we have a likelihood a_i that it belongs to the foreground and a likelihood b_i that it belongs to the background. We aim to label each pixel i as belonging to the foreground if $a_i > b_i$ and to the background otherwise. However, if many of a pixel's neighbors are labeled as background, we would also be inclined to label that pixel as background. Therefore, for each pair of pixels (i, j) that are neighbors, there's a separation penalty $p_{ij} \geqslant 0$ for placing one in the background and the other in the foreground.

Design an algorithm to determine if each pixel belongs to the foreground or background.