

### Q3

Let  $X_1$  be a random variable with CDF  $F_1(x)$ , and  $X_2$  be a random variable with CDF  $F_2(x)$ . By definition:

$$P(X_1 \leq x) = F_1(x) \text{ and } P(X_2 \leq x) = F_2(x).$$

We need to show that  $P(X_1 > X_2) > 0 \Rightarrow 1 - P(X_1 \leq X_2) > 0$  or  $P(X_1 \leq X_2) < 1$

#### **Proof by Contradiction**

Suppose to the contrary, that  $P(X_1 \leq X_2) = 1$ . This would imply that  $X_1$  is always less than or equal to  $X_2$ , meaning:

$F_1(x) = P(X_1 \leq x) \geq P(X_2 \leq x) = F_2(x)$ . But we know that  $F_1(x) < F_2(x)$  for all  $x$ , which contradicts the assumption.

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**For Q3.**

**The above approach is not correct because:**

**1. Understanding the Problem:**

- **Original Goal:** The aim is to show that a certain probability,  $P(X_1 > X_2)$ , equals 1.
- **Incorrect Approach:** The above approach shows that  $P(X_1 > X_2)$  is greater than 0, which is insufficient

**2. Specific Issues with the Approach:**

- The flawed approach demonstrates that there exists some  $w$  under which  $P(X_1 > X_2) > 0$ .
- However, the problem demands showing  $X_1 > X_2$  for *all* possible  $w$ .

**Other Common Mistakes:**

- The question requires a formal proof, not just an intuitive argument. Examples, such as tossing a coin, can help verify a claim but do not constitute proof.

**For Q2**

Common mistakes;

1. Assuming A and B were independent events, and using that to try to prove their independence via proving some tautology or ground truth. (ie : using it to prove  $P(A) = P(A)$ ).  
This is invalid as a proof technique.  
Let's say you have statement q, you'd like to prove.  
If, under the assumption q, you establish the proof of some ground truth, ie:  
Some tautology, the statement you've proved is:  
Q implies T. (Q implies a Tautology).  
This does not mean Q is true. Q can still be false and :  
Q implies T, would still be true.
2. Assuming A and B were independent events, and using that to prove  $P(A \text{ intersection } B) = P(A)P(B)$ . This is the definition of independent events.