Assignment 5

(MA6.102) Probability and Random Processes, Monsoon 2024

Release date: 10 October 2024, Due date: 18 October 2024

INSTRUCTIONS

- Discussions with other students are not discouraged. However, all write-ups must be done individually with your own solutions.
- Any plagiarism when caught will be heavily penalised.
- Be clear and precise in your writing.

Problem 1. Let X be an exponentially distributed random variable with parameter λ. Let $Y = \lfloor X \rfloor$, which is the integer part of X, and let $R = X - \lfloor X \rfloor$. Find the PMF of Y and the PDF of R.

Problem 2. Let X and Y be two discrete random variables taking values in integers. Prove that X and Y are independent if and only if $F_{X,Y}(x,y) = F_X(x)F_Y(y)$, for all x, y.

Problem 3. Let X_1, X_2, and X_3 be independent and identically distributed continuous random variables. A family has three children, A, B, and C of heights X_1, X_2, and X_3 respectively. Compute the probabilities: $P(A \text{ is the tallest child})$ and $P(A \text{ is taller than } B \mid A \text{ is taller than } C)$.

Problem 4. Let X be a continuous random variable with PDF f_X, and Y be a function of X defined as

$$Y \triangleq \begin{cases} X, & \text{if } X \geq 0, \\ X^2, & \text{if } X \leq 0. \end{cases}$$

Compute the PDF of Y in terms of f_X.

Problem 5. Let X_1, X_2, and X_3 be independent and uniformly distributed random variables on $[0,1]$. Find the joint density function of $X = X_1X_2$ and $Y = X_3^2$, and show that $P(X \geq Y) = \frac{4}{9}$.

Problem 6. Let X and Y be independent exponential random variables with parameter λ. Find the joint PDF of

$$Z = X + Y \text{ and } W = \frac{X}{Y}$$

and show that they are independent.