# Breadth First Search.

Connected graph (undirected): For any pair of vertices $u, v \in V$, there is a path from $u$ to $v$

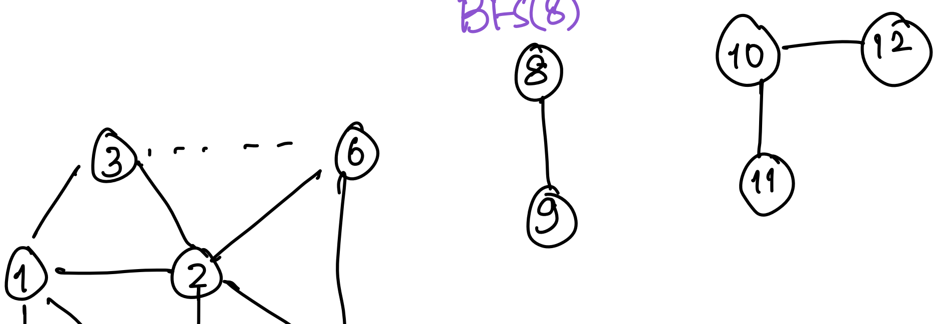
(Can be checked using BFS)

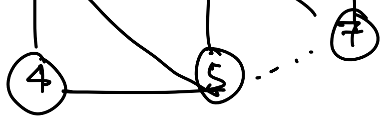
```
BFS(s):

Discovered [s] = True                    } Init
For all v ∈ V \ {s}
      Discovered[v] = False
L[0] ← {s}
 i ← 0
T ← ϕ
While L[i] is not empty:
      L[i+1] ← [ ]                       } init
      for each u ∈ L[i] :
          For each edge (u,v) ∈ E (incident on u):
              if Discovered [v] == false:
                  Discovered [v] ← True
                  T ← T ∪ {(u,v)}
                  L[i+1]. append (v) .
```

Qu: Compute Connected components

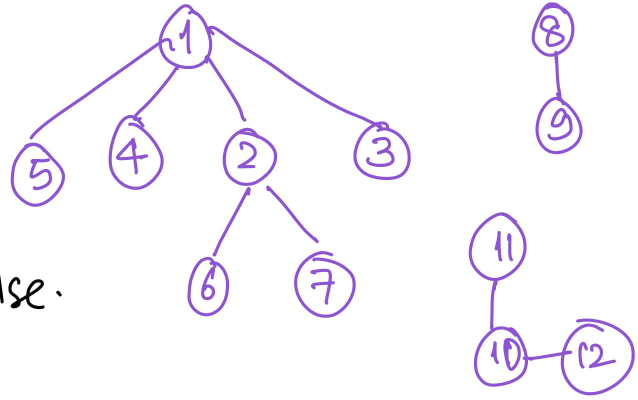
BFS(8) BFS(11)

BFS(1)





Init: For all $v \in V$:
Discovered $[v] \leftarrow$ false.



Pick a vertex: (say $u$)
Run BFS $[u]$ ← Connected component containing $u$.

## Testing bipartiteness

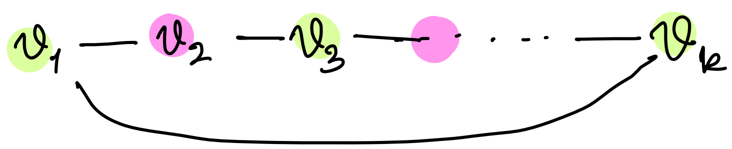
Bipartite graphs
$\cong$
2-colourable graphs.

(Undirected)
Lemma: A graph is bipartite if and only if it has no odd cycles.

Cycles with odd no. of edges.

→ Bipartite ⇒ no odd cycles.

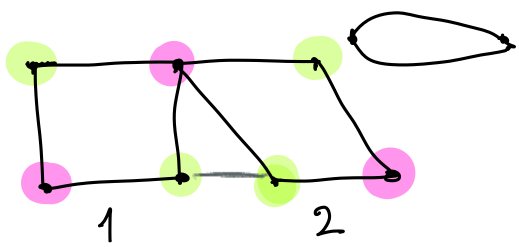
For the sake of contradiction: Say there is an odd cycle



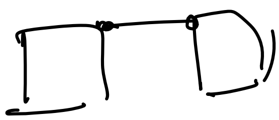

$k \leftarrow$ odd.

But $(v_1, v_k)$ edge is monochromatic contradicting the 2-colourability of the entire graph.

no Odd cycles ⇒ Bipartite




Start w/ a vertex.

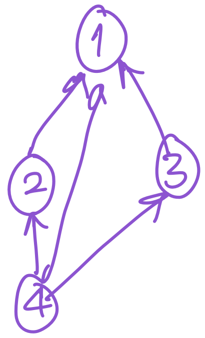


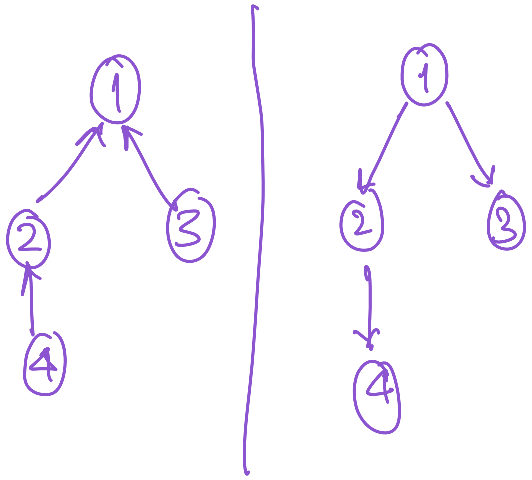
→ Even cycles

→ Acyclic.

undirected acyclic graph is a tree.

(directed acyclic graph need not be a directed tree).

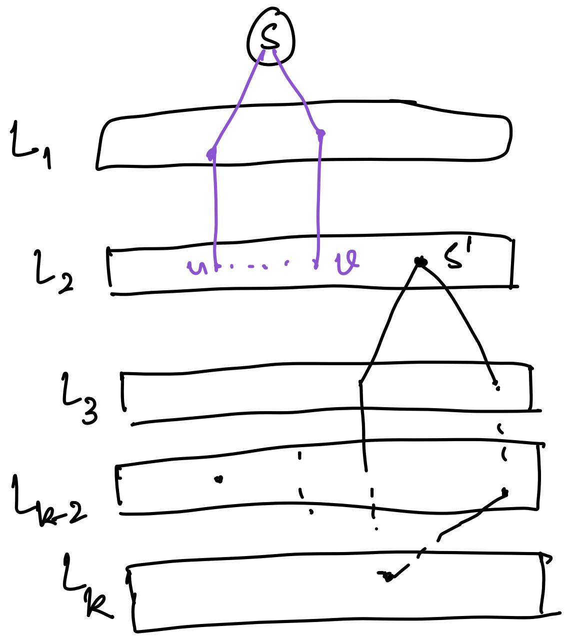




Qu: Suppose we have BFS tree and Layer computed for a given connected graph. Using this, can we infer bipartiteness?

Cross edges in the same layer

↳ s-



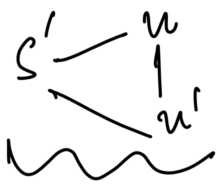

BFS(s)

$L_0 \quad L_1 \quad L_2 \quad - - - \quad L_k$

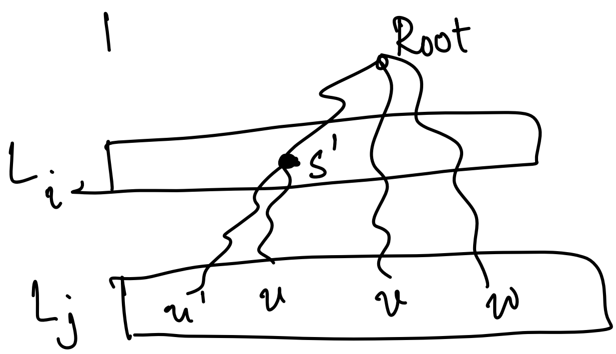
Part 1: $\boxed{L_0 \cup L_2 \cup \cdots \cup L_k}$

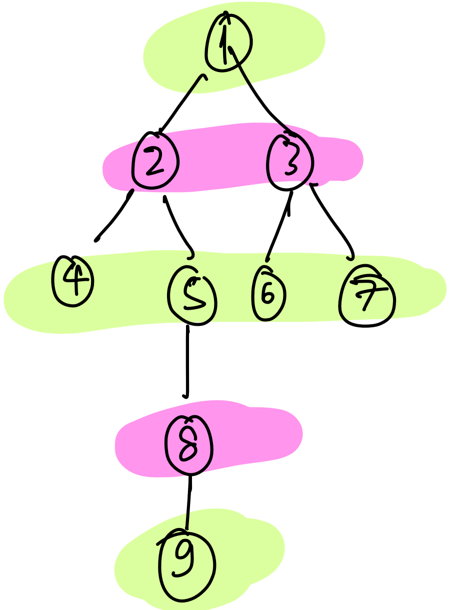
Part 2: $\boxed{L_1 \cup L_3 \cup \cdots \cup L_{k-1}}$

$\text{dist}(s', u)$
$= \text{dist}(s', u')$




Odd cycle.

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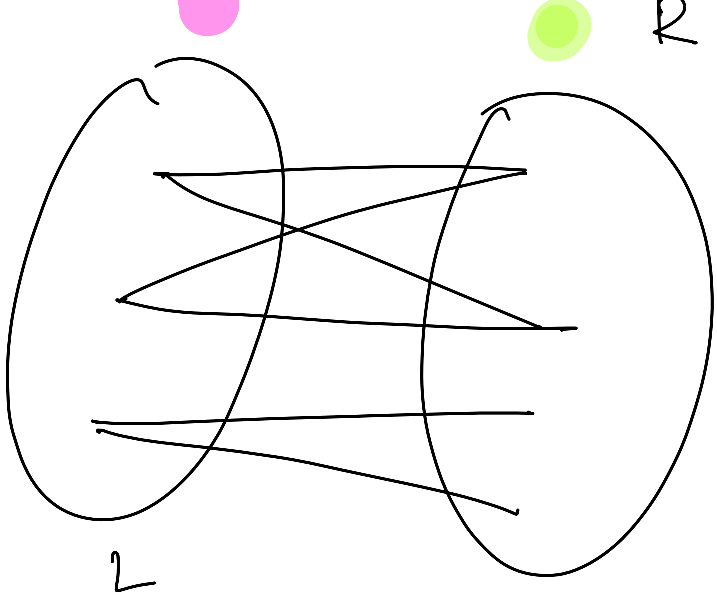
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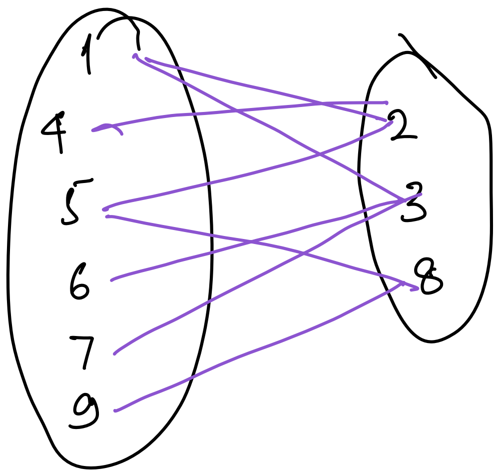
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