

For Q1.

1. Just stating that, when the probabilities are equal then variance is maximised, will not give any marks.
2. If you are applying AM-GM, then you need to apply it on p_i^2 and not on p_i .
3. If you are applying Cauchy-Schwarz inequality, then you need to explicitly mention at which condition the equality will hold and when the lower bound will be achieved. Directly stating is not sufficient.

COMMON MISTAKES

Q2: You're supposed to prove that $P(X > m+n | X > m) = P(X > n)$ implies X is a geometric RV, not the converse. Some of you've assumed X is geometric RV and proved above equation for X which was not the required solution.

From the equation $P(X > m+n) = P(X > m)P(X > n)$, to prove that X can only be a geometric random variable, it is not sufficient to verify this equation for geometric random variable. You need to deduce that this equation implies that $P(X=x)$ is the equation for pmf of geometric RV.

Common Mistakes

Q3. Example Uncorrelated but Not Independent Random Variables

Example

Let X be a discrete random variable that takes values -1 and 1 with equal probabilities:

$$P(X = -1) = P(X = 1) = \frac{1}{2}.$$

Now, define Y as:

$$Y = X^2.$$

Since X^2 equals 1 for both values of X , we have $Y = 1$ with probability 1.

To show: Uncorrelated but Not Independent

We now calculate the covariance between X and Y .

$$\begin{aligned} E[X] &= (-1) \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = 0, \\ E[Y] &= 1, \\ E[XY] &= (-1) \cdot 1 \cdot \frac{1}{2} + 1 \cdot 1 \cdot \frac{1}{2} = 0. \end{aligned}$$

The covariance is given by:

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y] = 0 - (0 \cdot 1) = 0.$$

Thus, X and Y are **uncorrelated**.

However, X and Y are **independent** as Y is always 1. Hence, the above example is incorrect.

Question 4 , Common Mistakes :

1. Equality condition not proven in both directions [iff]
2. Many of you took Discriminant ≥ 0 rather than ≤ 0 for Quadratic $f(\alpha) \geq 0$

Common Mistakes:

Question 5:

Most people did this question correctly, however the few errors i saw were:

1. Incorrect/invalid summation switchings.
2. People directly using law of iterated expectation, while it being a special case of the proof to be proved in the question. (Note: evaluating $\phi(Y)$ directly is fine).

Question 6:

Most people didnt make an error in this question as well, however the only error that was spotted somewhat often was :

1. Incorrect evaluation of the expression $E[1_A X]$.

Question 7

Common mistakes:

mostly every did it the question correctly, But some mistakes made by few students

- Using too few simulations may lead to inaccurate PMF estimates or wrong expected values
- Errors in visualizing the PMF (wrong axis labels, unclear scales, or not interpreting the graph well).