(CS1.301) Algorithm Analysis & Design (Monsoon 2024)

Practice Problem Set 2

Instructions:

- The following problem set is not graded and is for practice.
- Some of the problems will be covered in the tutorial.

Practice Problems

Question 1

In a country named Valoria, there are n cities numbered from 1 to n. City 1 is the capital of Valoria. There are m roads connecting these cities. A citizen can go from city a_i to city b_i and vise versa using the i^{th} road, the length of this road is x_i. Additionally there are k train routes in Valoria. One can use the i^{th} train route to go from capital of the country to city s_i and vise versa, the length of this route is y_i. Now, assume you are the president of Valoria. You don't want to waste the money of the country. So, you need to close some of the train routes. By doing so, you need to find the maximum number of the train routes which can be closed under the following condition: the length of the shortest path from every city to the capital mustn't change.

Note: It is guaranteed that there is at least one way from every city to the capital. Note, that there can be multiple roads between two cities. Also, there can be multiple routes going to the same city from the capital.

Question 2

Given a list of n natural numbers d_1,d_2...,d_n, show how to decide in polynomial time whether there exists an undirected graph G = (V, E) whose node degrees are precisely the numbers d_1,d_2...,d_n. (That is, if V = v_1,v_2 .....,v_n, then the degree of v_i should be exactly d_i). G should not contain multiple edges between the same pair of nodes, or "loop" edges with both endpoints equal to the same node.

Question 3

Specifically, let G = (V, E) be a connected graph with n vertices, m edges, and positive edge costs that you may assume are all distinct. Let T = (V, E') be a spanning tree of G; we define the bottleneck edge of T to be the edge of T with the greatest cost.

A spanning tree T of G is a minimum-bottleneck spanning tree if there is no spanning tree T' of G with a cheaper bottleneck edge.

(a) Is every minimum-bottleneck tree of G a minimum spanning tree of G? Prove or give a counterexample.

(b) Is every minimum spanning tree of G a minimum-bottleneck tree of G? Prove or give a counterexample

Question 4

Let G = (V, E) be an undirected graph with edge costs $c_e > 0$ on the edges e E. Let T be a Minimal Spanning Tree in G. Let a new edge (v, w) with edge weight c, be added to G. Give an efficient algorithm to test if T remains a Minimal Spanning Tree after addition of (v, w) to G (and not to T). If T is no longer the minimal spanning tree, then give an efficient algorithm to update the tree T to the new Minimum Spanning Tree.

Question 5

Given a list of n jobs $J_1, J_2, \ldots, J_n$ with processing times $p_1, p_2, \ldots, p_n$ and weights $w_1, w_2, \ldots, w_n$. Starting from t = 0, the cost for completing a job J_i is given by $w_i *$ (total time from t = 0 to the time at which job J_i finishes). Give an algorithm to find the order to perform the jobs with minimum total cost.