(CS1.301) Algorithm Analysis & Design (Monsoon 2024) Due: 10pm, 16.09.2024

Problem Set 3

Instructions:

- Discussions amongst the students are not discouraged, but all writeups must be done individually and must include names of all collaborators.
- Referring sources other than the lecture notes is discouraged as solutions to some of the problems can be found easily via a web search. But if you do use an outside source (eg., text books, other lecture notes, any material available online), do mention the same in your writeup. This will not affect your grades. However dishonesty of any sort when caught shall be heavily penalized.
- Be clear in your arguments. Vague arguments shall not be given full credit.
- Total marks for this problem set are 20.

Question 1

You are given an array of length n, initially sorted in ascending order. This array is rotated an unknown number of times. Here, if an array $[a_0, a_1, a_2, \cdots, a_{n-1}]$ is rotated 1 time, it becomes $[a_{n-1}, a_0, a_1, a_2, \cdots, a_{n-2}]$. Find the index of the smallest element. [2 marks]

Question 2

Describe and analyze a variant of Karatsuba's algorithm that multiplies any m-digit number and any n-digit number, for any $n > m$, in $O(m^{\log_2 3 - 1} n)$. [2 marks]

Question 3

This question consists of two parts:

- What is the FFT of $[1, 0, -1, 0]$? [1.5 marks]
- Of which sequence is $[1, 0, -1, 0]$ the FFT? [1.5 marks]

Question 4

We draw N line segments between two parallel lines at $Y = 0$ and $Y = 5$. Each line segment starts from one of the parallel lines and ends at the other. Find the number of unordered pairs of line segments that intersect with each other.

Note: Even if three lines, say $line1$, 2 and 3 intersect at the same point, this is to be considered as 3 pairs, namely $(line1, line2)$, $(line2, line3)$ and $(line1, line3)$.

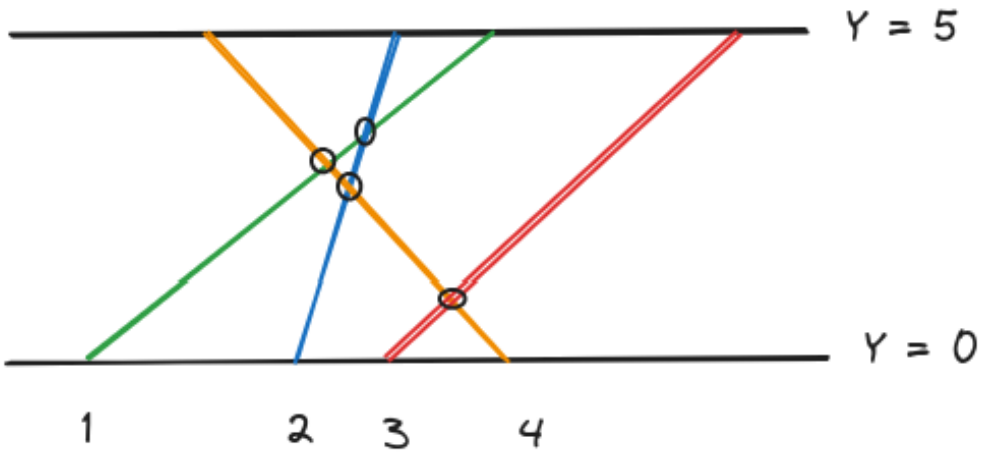

Figure 1: In the above image, there are 4 points of intersection, denoting 4 line segments that intersect with each other, namely (1, 4), (2, 4), (1, 2) and (3, 4).

Propose a divide-and-conquer solution. [4 marks]

Question 5

Imagine you're managing a collection of n encrypted digital files stored on various cloud servers. Some of these files might be of the same type, but you can't determine this by looking at the files directly. However, you have access to a specialized tool, called a "data comparator," which can compare any two files and tell you if they are of the same type.

Your task is to find out if there is a group of more than $\frac{n}{2}$ files that are of the same type. The only operation you can perform is using the data comparator to compare two files at a time.

You need to determine the answer using no more than $O(n \log n)$ comparisons with the data comparator. [4 marks]

Question 6

Imagine you're working on a mission to uncover a hidden treasure buried somewhere within two vast archives. Each archive holds n unique artifacts, for a total of $2n$ artifacts between the two archives. Your goal is to find the treasure, which is hidden with the artifact that is exactly in the middle when all $2n$ artifacts are sorted by their value—this is the median artifact, the n^{th} smallest in the combined collection.

However, there's a catch: you can't examine all the artifacts directly. Instead, you can only query each archive individually. When you make a query, you can ask an archive for the artifact that ranks k^{th} in value within that archive, and it will reveal just that one artifact to you.

Since each query is costly and time-consuming, you want to figure out the median artifact with the least number of queries possible. Your challenge is to design an efficient strategy to find the median using at most $O(\log(n))$ queries. [5 marks]