(CS1.301) Algorithm Analysis & Design (Monsoon 2024) Due: 10pm, 28.08.2024

Problem Set 2

Instructions:

- Discussions amongst the students are not discouraged, but all writeups must be done individually and must include names of all collaborators.
- Referring sources other than the lecture notes is discouraged as solutions to some of the problems can be found easily via a web search. But if you do use an outside source (eg., text books, other lecture notes, any material available online), do mention the same in your writeup. This will not affect your grades. However dishonesty of any sort when caught shall be heavily penalized.
- Be clear in your arguments. Vague arguments shall not be given full credit.
- Total marks for this problem set are 15.

Question 1

Consider an undirected (positively) weighted graph $G = (V, E)$ with a MST T and a shortest path $\pi(s, t)$ between two vertices $s, t \in V$. Find whether T will still be an MST and $\pi(s, t)$ be a shortest path if

(a) Weight of each edge is multiplied by a fixed constant $c > 0$.

(b) Weight of each edge is incremented by a fixed constant $c > 0$.
Explain your answers. [4 marks]

Question 2

Answer the following:

(a) Using Huffman Coding, how many bits are required for encoding the message 'huffmanencoding'?

(b) For the same message as in part a, what is the Average Bit Length? [4 marks]

Question 3

In a city there are N houses, each of which is in need of a water supply. It costs w_i dollars to build a well at house i, and it costs $c_{i,j}$ to build a pipe in between houses i and j. A house can receive water if either there is a well built there or there is some path of pipes to a house with a well. Design an algorithm to find the minimum amount of money needed to supply every house with water. [4 marks]

Question 4

You are given a directed graph G where every edge has a negative weight, and a source vertex s. Your task is to find the shortest-distances from s to every other vertex in G. Your algorithm should handle the following cases:

- If vertex t cannot be reached from s, report the distance dist(t) as ∞.
- If there is a cycle that is reachable from s, and vertex t can be reached from this cycle, then the shortest-path distance from s to t is not well-defined because there are paths with arbitrarily large negative lengths. In this case, report the distance dist(t) as -∞.
- If neither of the previous conditions applies, compute and report the correct shortest-path distance from s to t.

[3 marks]