Recap.

Continuous RV

→ A RV X is said to be continuous if $\underbrace{F_X(x)}_{\text{Prob. of an event } P(X \le x)} = \int_{-\infty}^{x} \underbrace{f_X(u)}_{\text{PDF}}\, du$

≥ 0

So there is no interval with -ve mass.

Properties of PDF

(i) $f_X(x) \geq 0 \quad \forall x.$ ← So

(ii) $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$

↳ Every ω takes some place in $(-\infty, \infty)$,

So $\int_{-\infty}^{\infty} (\ \ ) = P(\Omega) = 1$

$$P(X \in B) = \int_B f_X(u)\, du$$

→ $F_X(x) = \int_{-\infty}^{x} f_X(u)\, du \Rightarrow f_X(x) = F_X'(x)$

Def^n of derivative $\quad F_X'(x) = \lim_{\Delta x \to 0} \dfrac{F_X(x+\Delta x) - F_X(x)}{\Delta x}$

→ In a discrete setting, $E[X]$ → single no.

|||ly in continuous setting,

→ $E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$

→ If X is non-negative RV,

$$E[X] = \int_{0}^{\infty} P(X > x)\, dx \longrightarrow \text{Start with } P(X > x)$$

↓ Expand in terms of f_X

→ $E[g(X)] = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx$

$$P(X > x) = P(X \in (x, \infty))$$

$$= \int_x^\infty f_X(u)\,du \qquad \left(\because P(X \in B) = \int_B f_X(u)\,du \right)$$

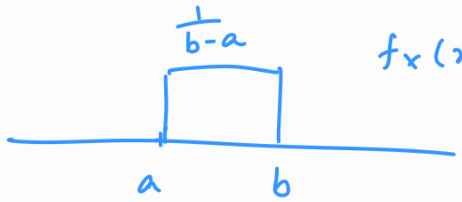
$$E[X] = \underbrace{\int_0^\infty \left(\int_x^\infty f_X(u)\,du \right) dx}$$

$\downarrow$

Change of variables.

(integrals)

Eg: Uniform RV.



$f_X(x)$

$$E[X] = \int_a^b x f_X(x)\,dx$$

$$= \int_a^b \frac{x}{b-a}\,dx$$

$$= \left. \frac{x^2}{2(b-a)} \right]_a^b$$

$$= \frac{b^2 - a^2}{2(b-a)} = \frac{a+b}{2}.$$

$$E[X^2] = \int_a^b x^2 f_X(x)\,dx$$

$$= \int_a^b \frac{x^2}{b-a}\,dx$$

$$= \left. \frac{x^3}{3(b-a)} \right]_a^b = \frac{b^3 - a^3}{3(b-a)}$$

$$= \frac{a^2 + b^2 + ab}{3}.$$

$$\mathrm{var}(X) = \frac{a^2 + b^2 + ab}{3} - \left(\frac{a+b}{2} \right)^2 = \frac{a^2 + b^2 + ab}{3} - \frac{a^2 + b^2 + 2ab}{2}$$

$$= \frac{2a^2 - 3a^2 + 2b^2 - 3b^2 + 2ab - 6ab}{6}$$

$$= \frac{-a^2 - b^2 - 4ab}{6}$$

$$\mathrm{var}(X) = \frac{(b-a)^2}{12}$$

$f_X(x) = \lambda e^{-\lambda x}, \quad x \geq 0.$

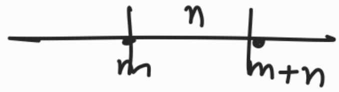
$P(X > x)$

Memoryless property of Geometric RV

$P(X > m+n \mid X > m)$

$= P(X > n)$

Success hasn't occurred in m tosses, the prob. that it occurs in m+n tosses is prob. that it occurs in n tosses.



So it forgets memory of prev. m tosses.

(Indep. of Bernoulli RV involved in Geom RV. across time)

Any i^{th} toss is indep. of prev. $(i-1)$ tosses.