Matrix Chain Multiplication

$M_1 \; M_2 \; \ldots \; M_t \; \} \; n \times n$

M_{12}

$\overline{M_1 M_2}$

$k \times l \quad l \times s$

$O(kls)$

$M_i \leftarrow n_i \times n_{i+1}$

A	B	C
6×2	2×4	4×3

(AB) C

$(6 \times 2 \times 4) + (6 \times 4 \times 3)$

$48 + 72$

A (BC)

$6 \times 2 \times 3 + 2 \times 4 \times 3$

6×10

M_1	M_2	…	M_t
$n_1 \times n_2$	$n_2 \times n_3$		$n_t \times n_{t+1}$.

$M_k \quad M_{k+1}$

MCM: Matrix Chain Mult.

MCP: Matrix Chain Product.

$\mathrm{MCM}(1,n) \xrightarrow{\quad} \mathrm{MCM}(1,k) \cdot \mathrm{merge} \cdot \mathrm{MCM}(k+1,t)$

$k \in [1, t-1]$

$\boxed{\mathrm{MCP}(1,k) \cdot \mathrm{MCP}(k+1,t)}$

$T(1,k) + T(k+1,t)$

$+ \; n_1 \times n_{k+1} \times n_{t+1}$.

$$T(1,t) = \min_{k \in [1,t-1]} \left\{ T(1,k) + T(k+1,t) + n_1 \times n_{k+1} \times n_{t+1} \right\}.$$

A B C

1 2 3

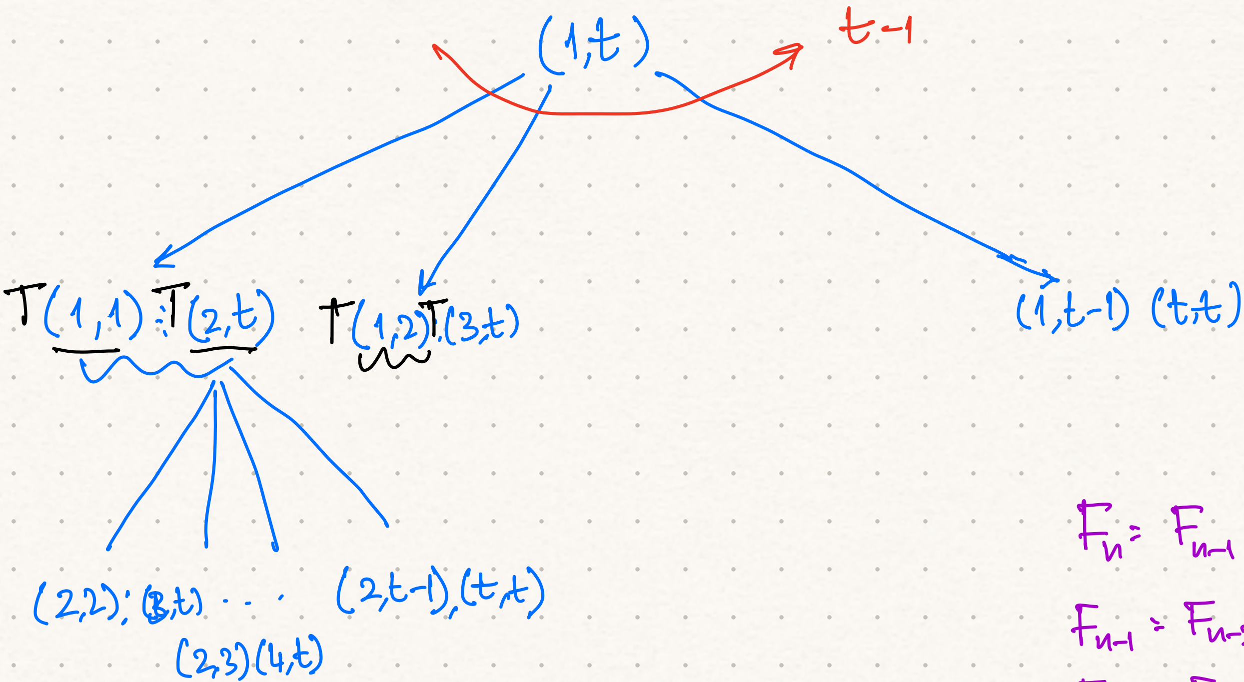
$n_1 \times n_2$

$n_2 \times n_3$

$n_3 \times n_4$

$n_1 = 6,\ n_2 = 2,\ n_3 = 4,\ n_4 = 3.$

$$T(1,3) = \min \{ T(1,1) + T(2,3) + n_1 \times n_2 \times n_4 ;$$
$$T(1,2) + T(3,3) + n_1 \times n_3 \times n_4 \}$$



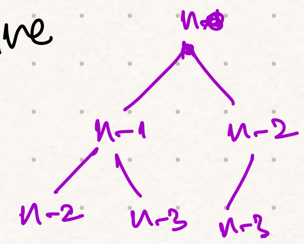
$F_1 = F_0 = 1$

$F_n = F_{n-1} + F_{n-2}$

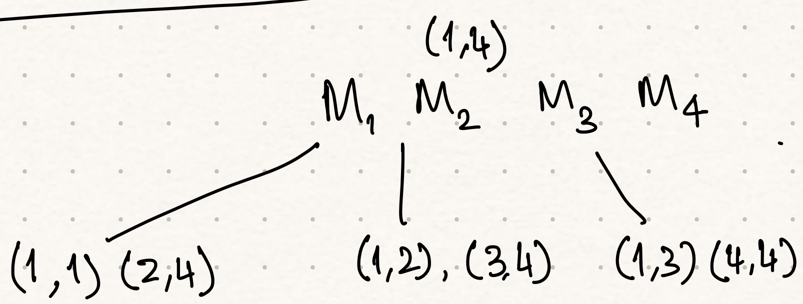
$F_{n-1} = F_{n-2} + F_{n-3}$

$F_{n-2} = F_{n-3}$

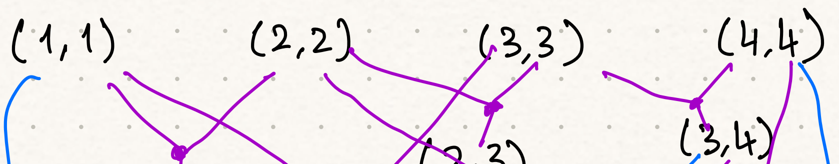
"Make good use of space to save time".

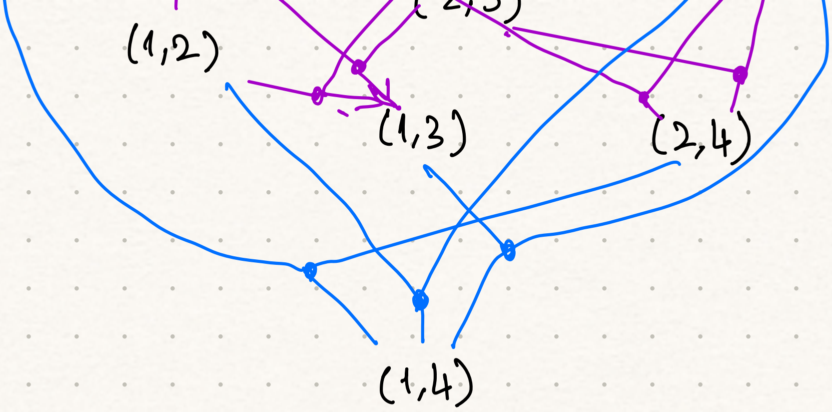


$\boxed{T(i,j) \qquad i \le j}$ # of such cases : $\binom{t}{2} + t$.



(1,1)
(1,2)
(1,3)
(2,2)
(2,3)
(2,4)
(3,3)
(3,4)
(4,4)





"We are building a look up table for computations for efficient reuse"

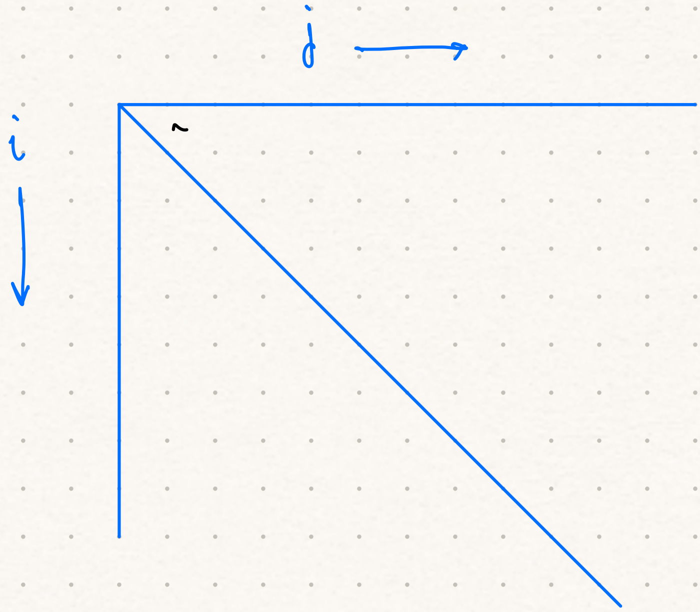
- Memoization.

$\left(\binom{t}{2}+t\right)$

entries

and for each entry we make at most $\underline{\underline{2t}}$ lookups and $\le t$ arithmetic computations

$\le 2(j-i) \quad + \quad (j-i)$



$$T(i,j) = \min_{k\in[i,j-1]} \left\{ T(i,k) + T(k+1,j) + n_i \times n_{k+1} \times n_{j+1} \right\}.$$

Ref: Aho, Hopcraft, Ullman.