CS 302.1 - Automata Theory

Lecture 09

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Quick Recap

Pumping Lemma for CFL: If L is Context Free, then there exists $p > 0$ (pumping length), such that, for any $w \in L$ of length $|w| \geq p$, w can be split into five parts, i.e. $w = uvxyz$
satisfying the following conditions:

- $|vy| \geq 1$
- $|vxy| \leq p$
- $uv^ixy^iz \in L, \ \forall i \geq 0$

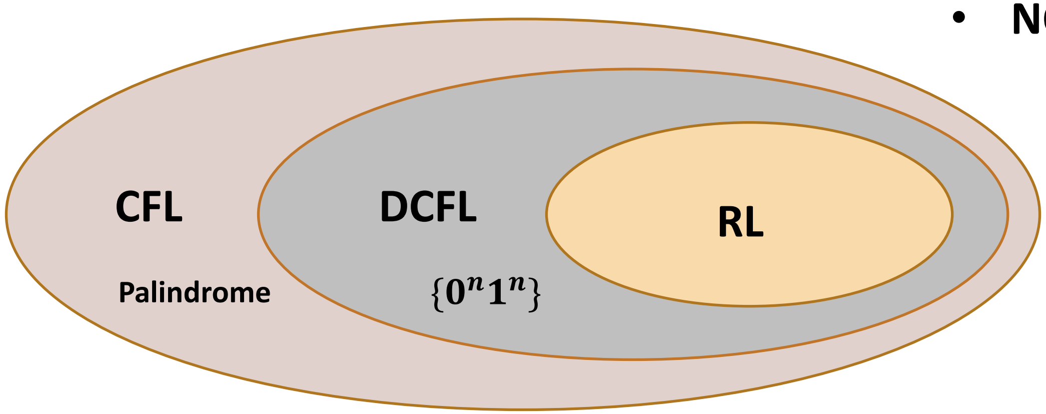
Closure properties of CFLs

CFLs are closed under

- **Union**
- **Star**
- **Concatenation**

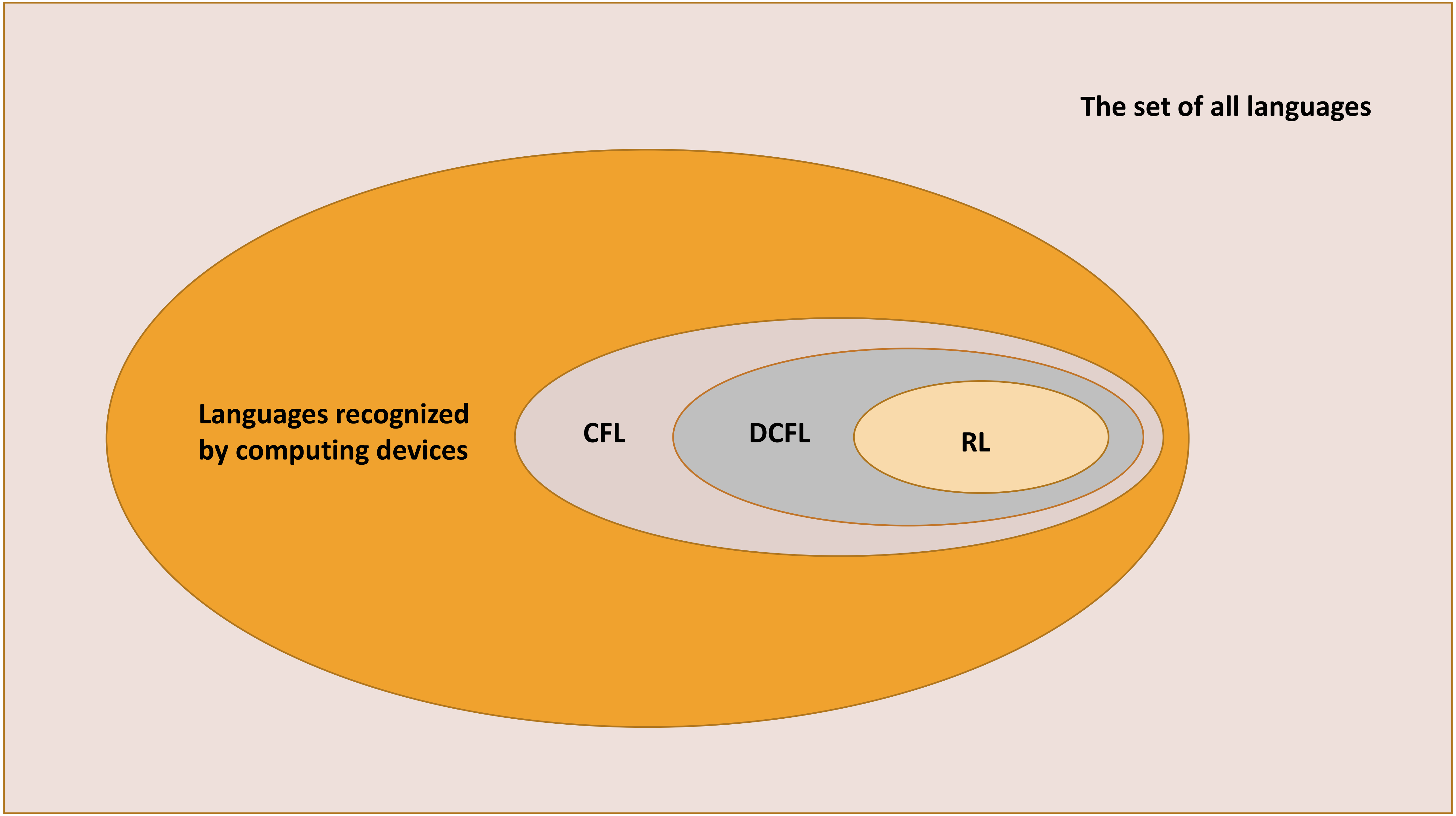
CFLs are NOT closed under

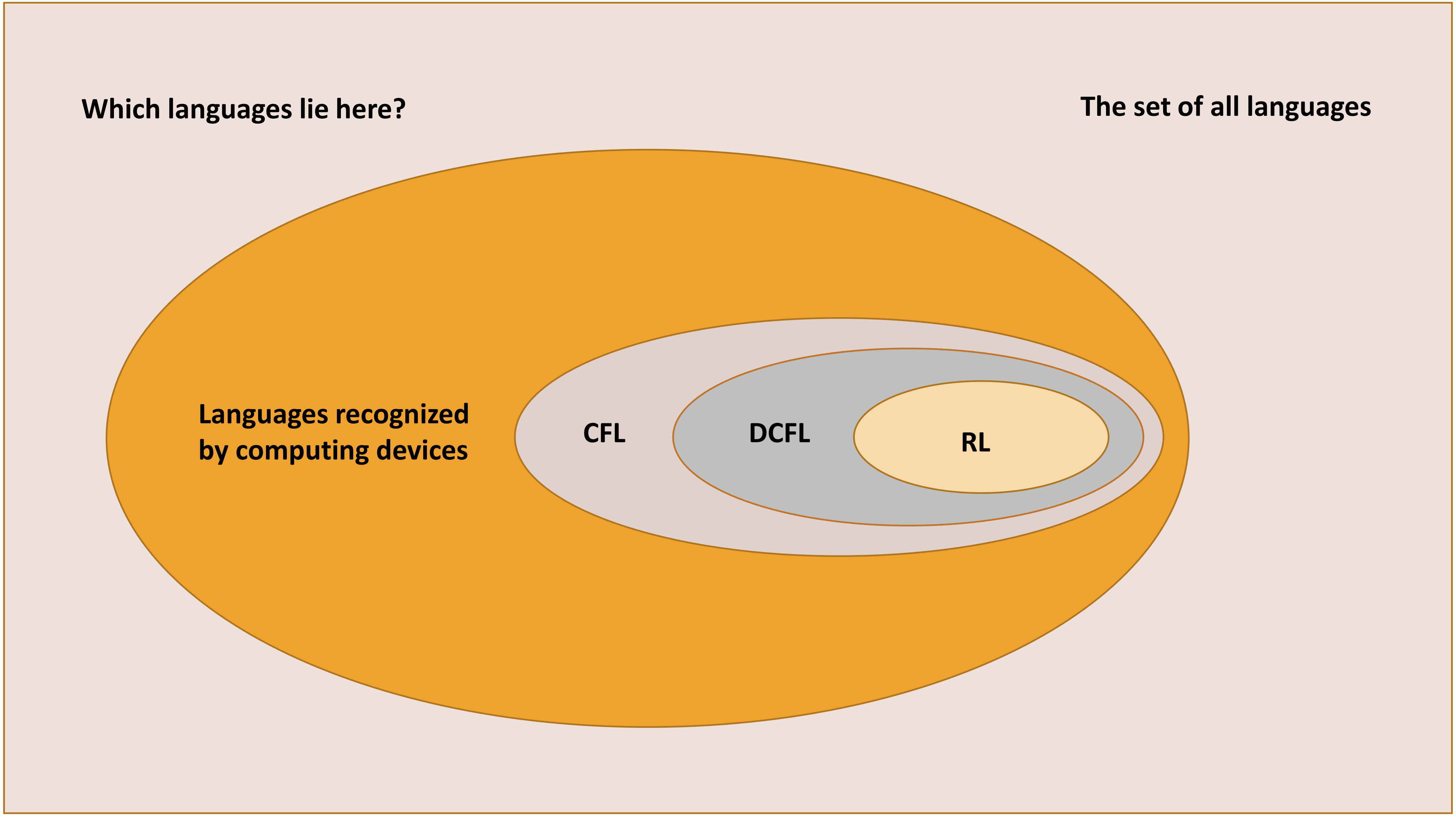
- **Complementation**
- **Intersection**



For DCFLs

- **NOT closed Union**
- **Closed under complementation**
- **NOT closed under intersection**


The set of all languages
Languages recognized
by computing devices
CFL
DCFL
RL


Which languages lie here?
The set of all languages
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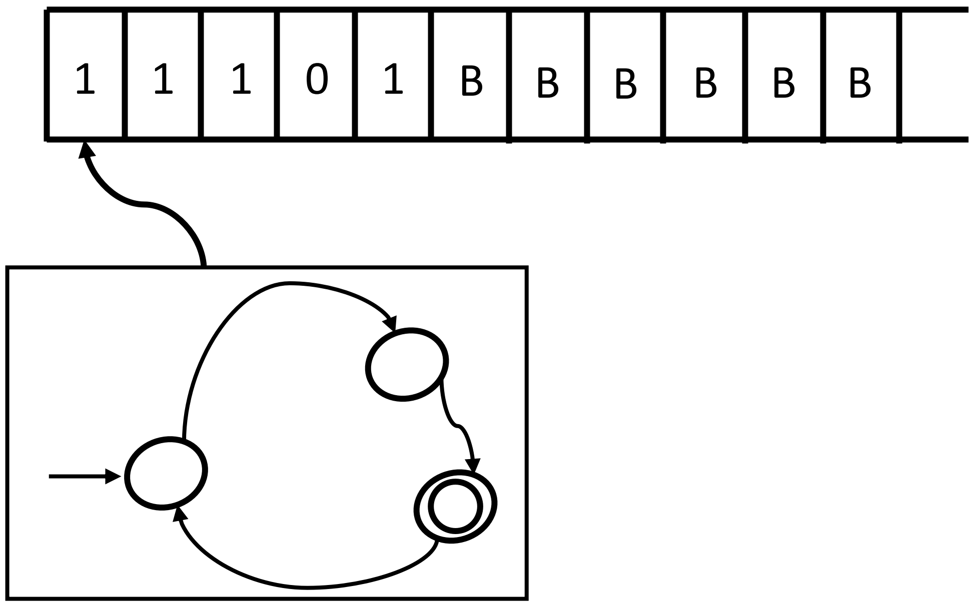
Turing Machines

- A Turing machine is a FSM that has access to a infinite tape as its memory.
- The infinite tape contains in it, the input string followed by Blanks (indicated by B)
- The Turing machine can both read from the tape and write in it – one cell at a time, using a Read/Write head.
- The Read/Write head can move to the Left or to the Right – again one cell at a time.



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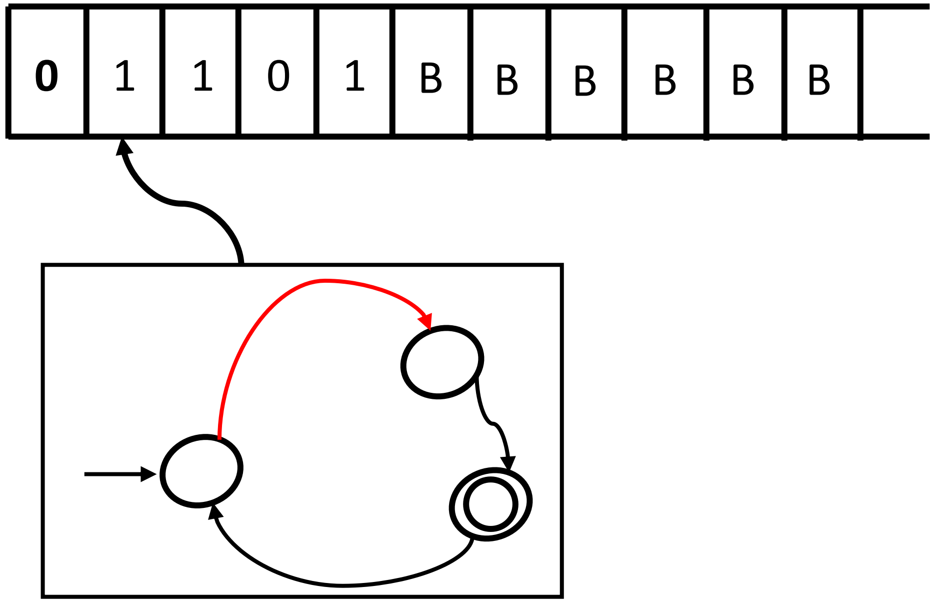

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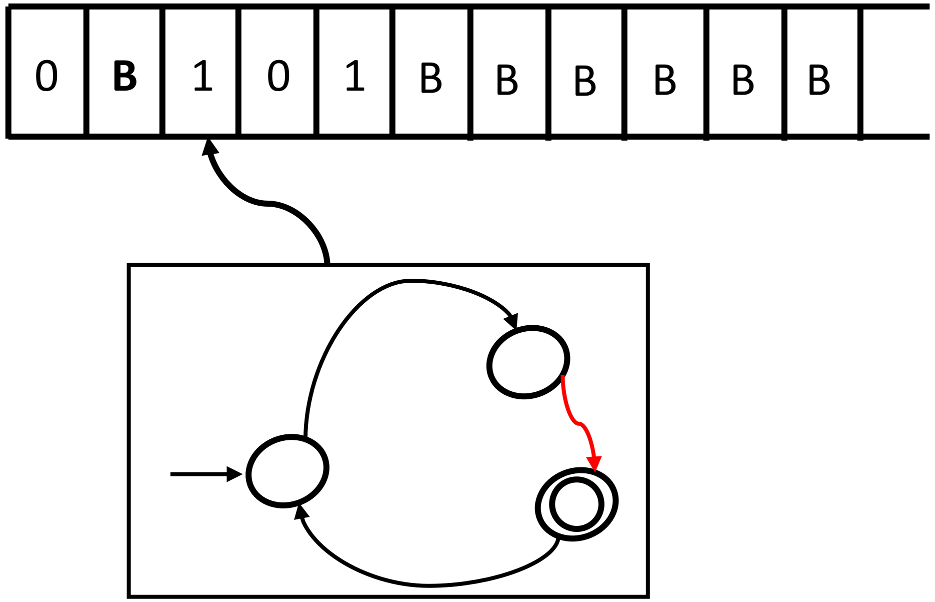

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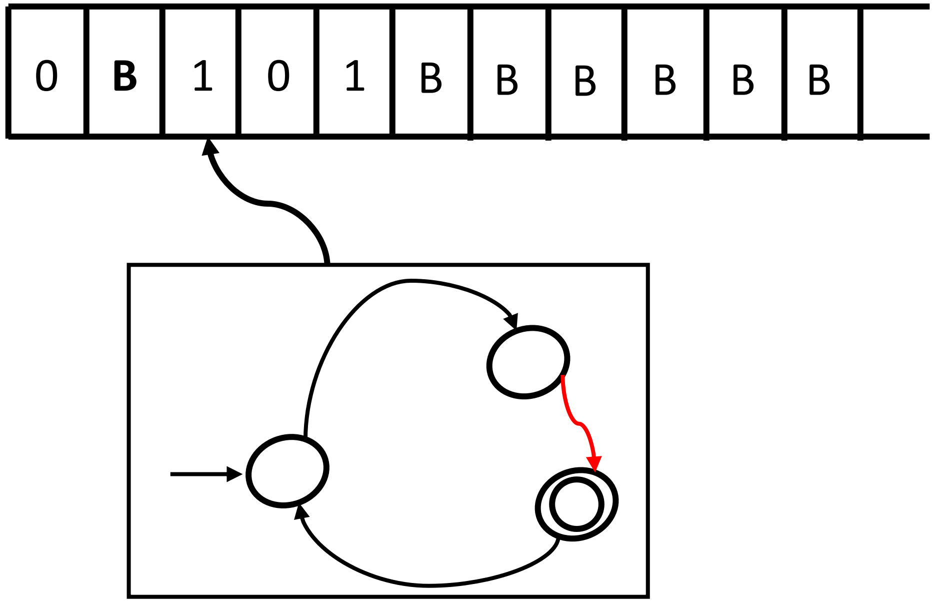

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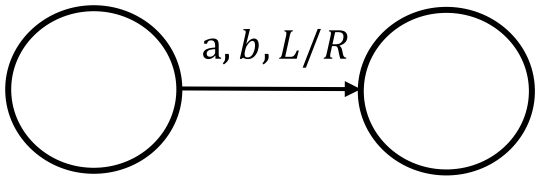
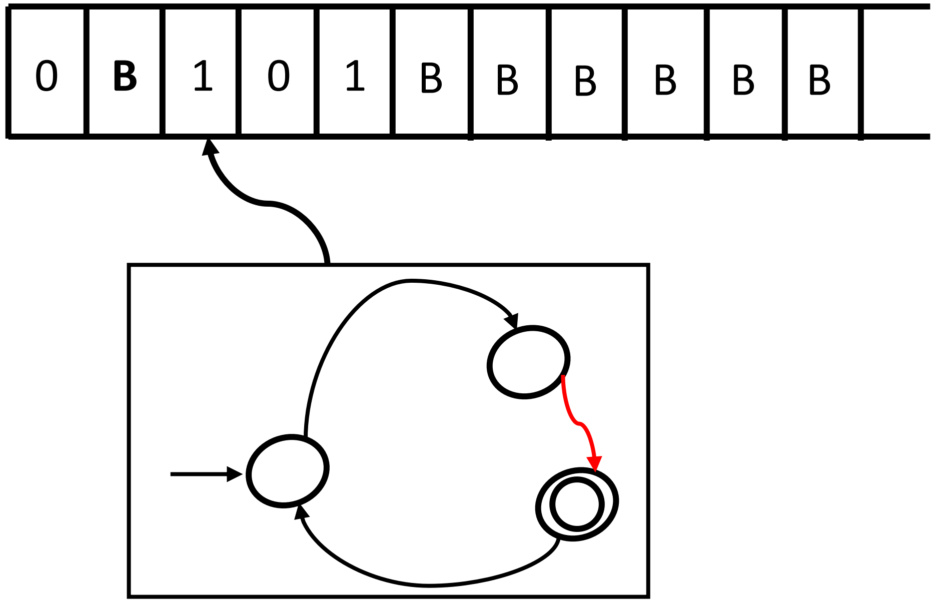
- In a way these "added features" give TMs their power. (eg: ability to write on the tape)
- Notice: acceptance/rejection of a run is not tied to the input.
- Auxiliary computation can be performed – as much as needed, even when the input string has been scanned

Turing Machines



Transition $a, b, L/R$: Read a from the tape, replace with b and move L/R



TM may never halt – it may loop forever

Accept state

Reject state

TM halts and **accepts/rejects** on reaching these states

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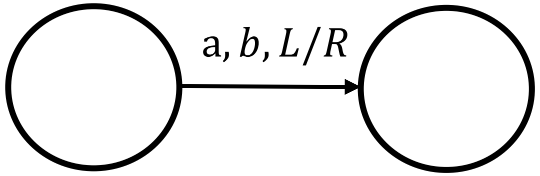

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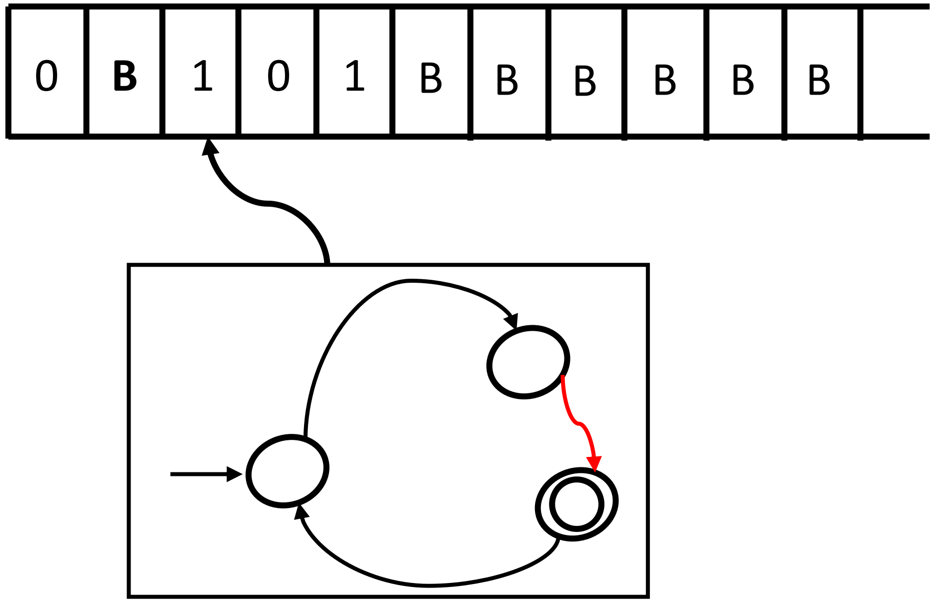

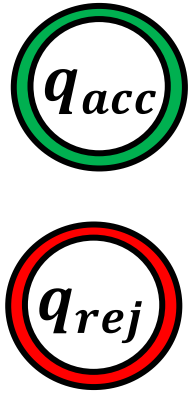

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So, given a TM M and an input ω,

$M(\omega)$ **accepts** if $\omega \in L(M)$
$M(\omega)$ **rejects** if $\omega \notin L(M)$
$M(\omega)$ **runs infinitely** if $\omega \notin L(M)$

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Turing machines are named after **Alan Turing**. In 1936, he gave a negative answer to Hilbert's *Entscheidungsproblem* (Decision problem) – *Are all decision problems decidable?*

Turing claimed that anything "humanly computable" can be simulated by his machines

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By A. M. TURING.

[Received 28 May, 1936.—Read 12 November, 1936.]

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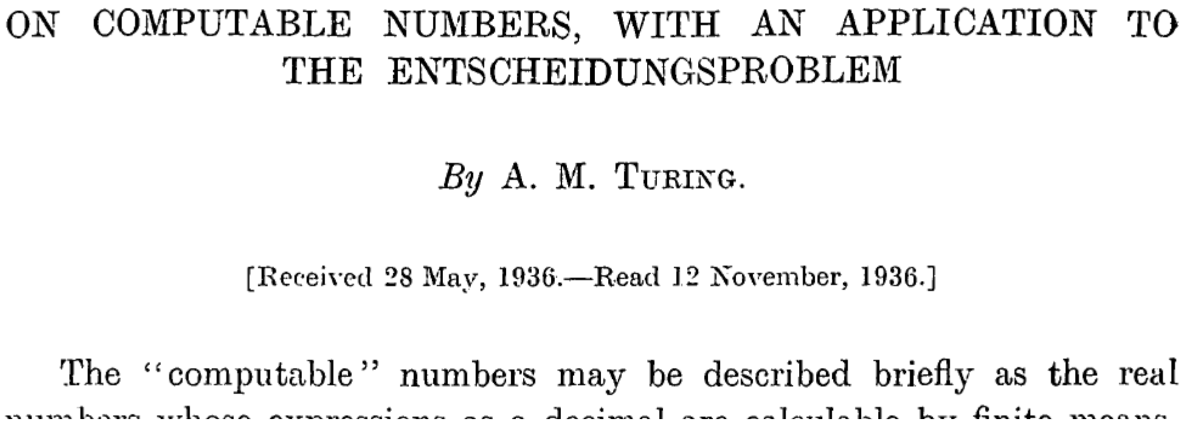
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- Turing assumed that the human brain to be a finite state machine with a finite number of states
- Consider such a human being working on a problem with a notebook, pencil and an eraser.
- The pages of the notebook are laid out on the tape – each cell consists of one page, with a finite amount of information.
- Whatever the human being does with the notebook, can be simulated on the TM: reading, writing, erasing (writing a blank), moving left or right to a new page etc.

Turing Machines

Example: Let $\boldsymbol{L = \{0^n 1^n | n \geq 1\}}$

We will try to develop the basic idea in designing the Turing Machine for this language. Note that $L = CFL$.

Idea: An accepting run of a TM for L could look something like this:

- Mark the first 0 (by replacing it with some special symbol say X)
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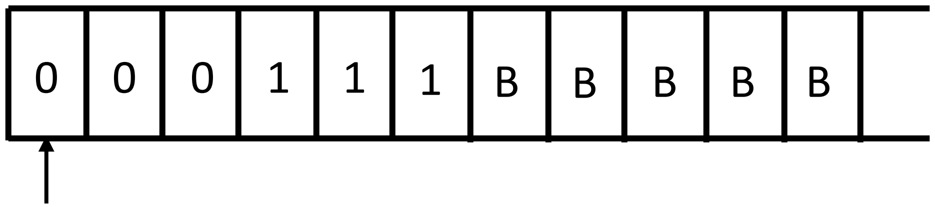

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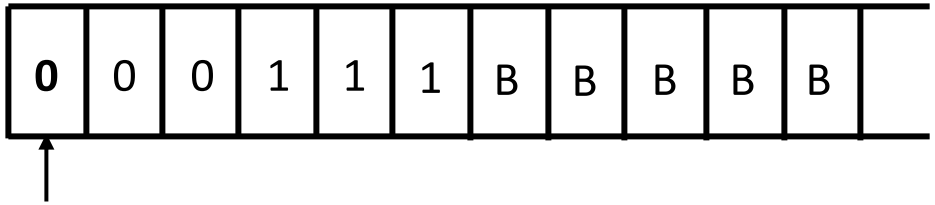

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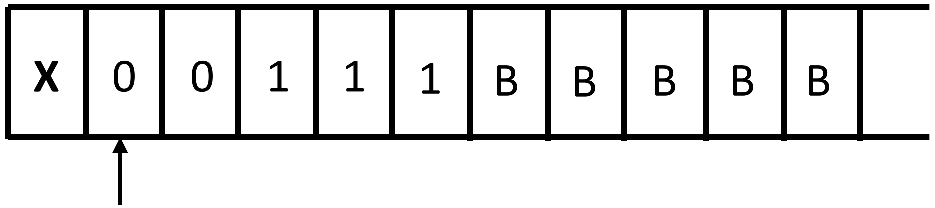

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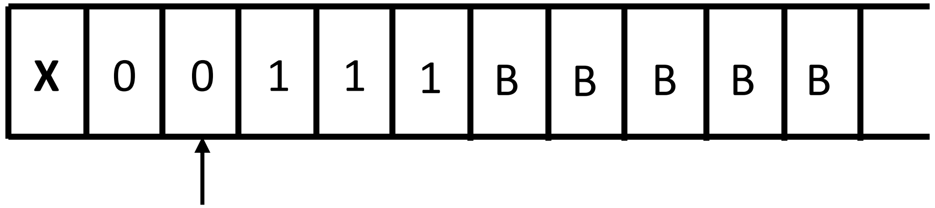

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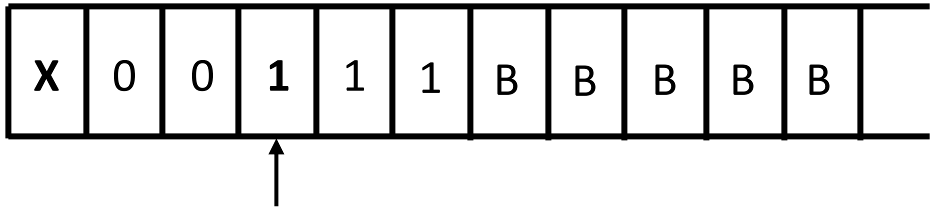

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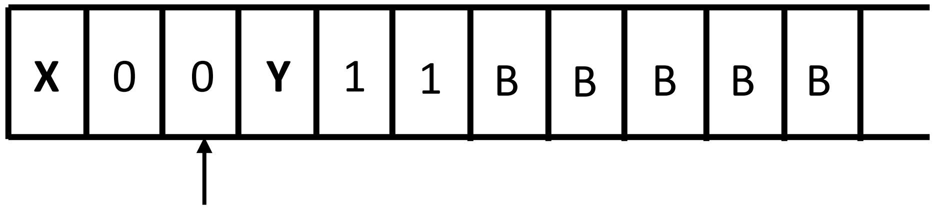

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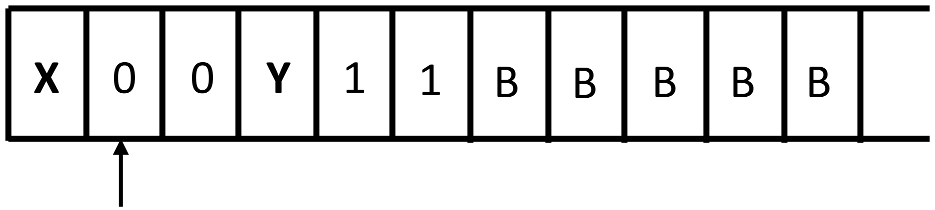

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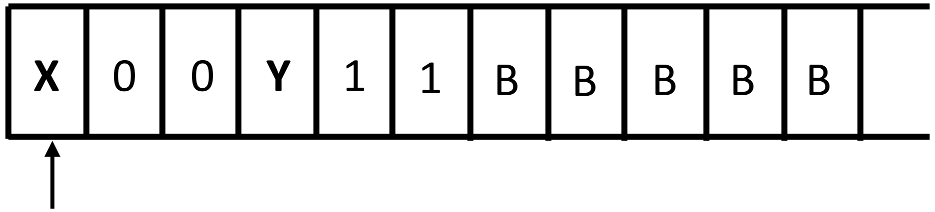

While moving left, when an X is encountered, the head should move right until the next 0 to be marked is encountered $\Rightarrow$ **We need rules like (X, X, R)**

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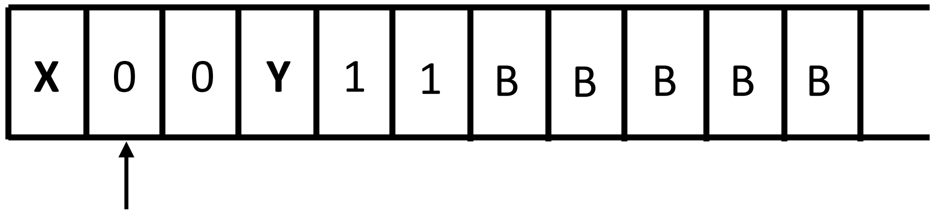

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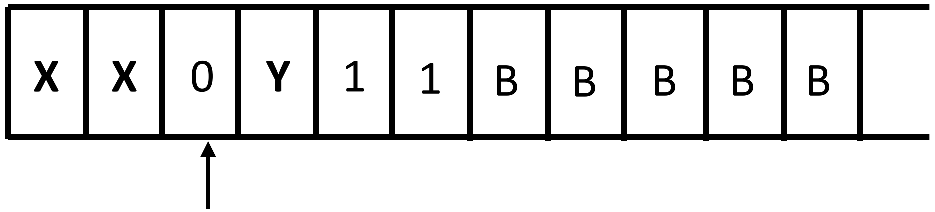

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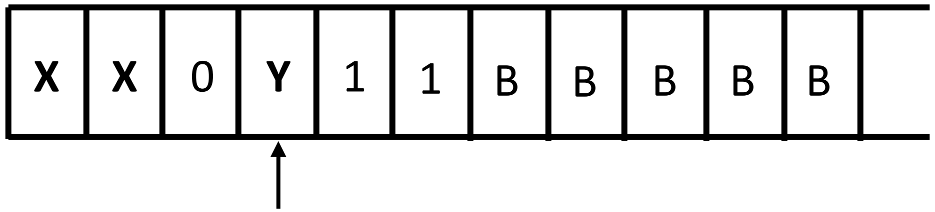

While moving right, when a Y is encountered, the head should move right as that’s where the next 1 to be marked is $\Rightarrow$ **We need rules like $(\boldsymbol{Y}, \boldsymbol{Y}, \boldsymbol{R})$**

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X	**X**	0	**Y**	**Y**	1	B	B	B	B	B	

While moving left, when a Y is encountered, the head should keep moving left as those $1'$s have been marked already $\Rightarrow$ **We need rules like** $(\boldsymbol{Y}, \boldsymbol{Y}, \boldsymbol{L})$

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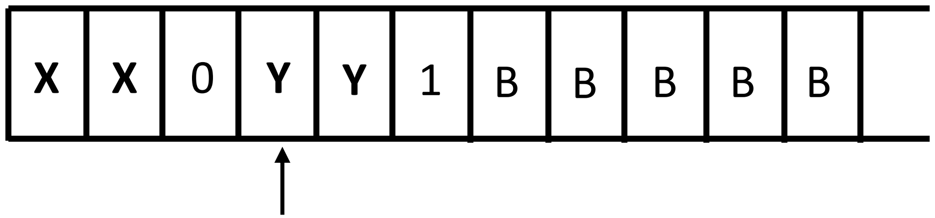

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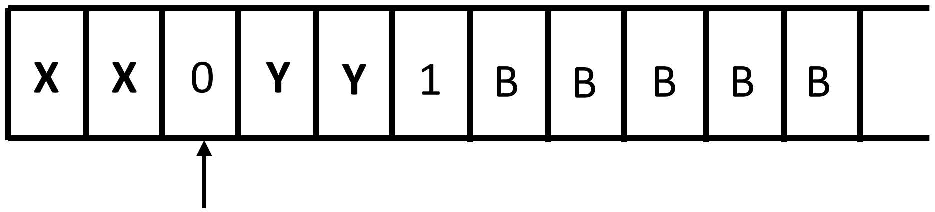

Turing Machines

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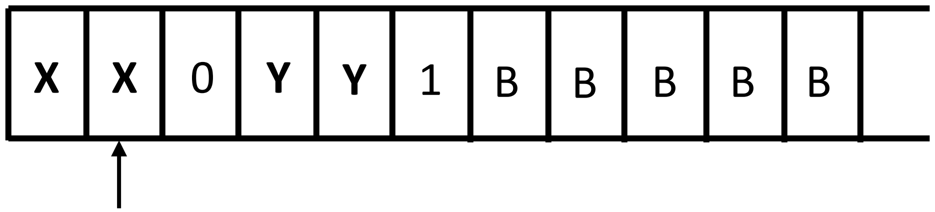

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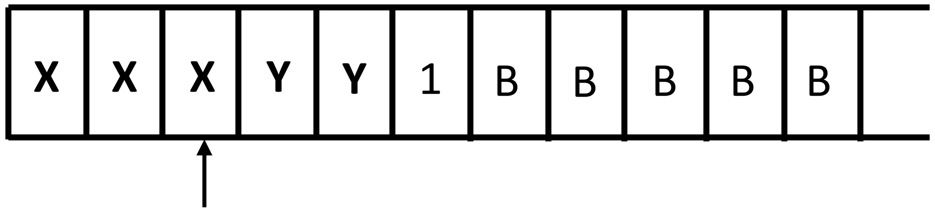

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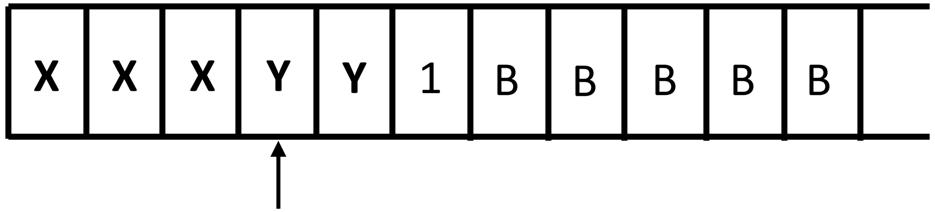

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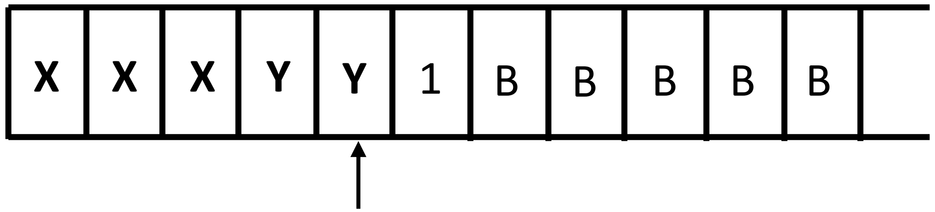

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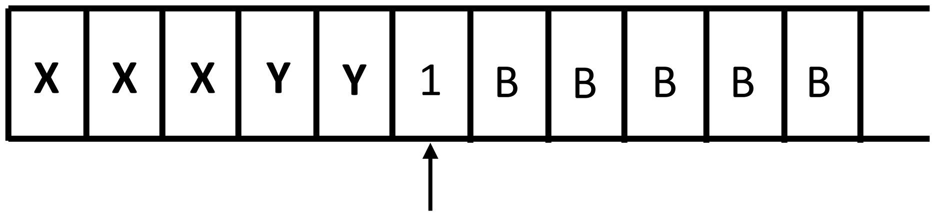

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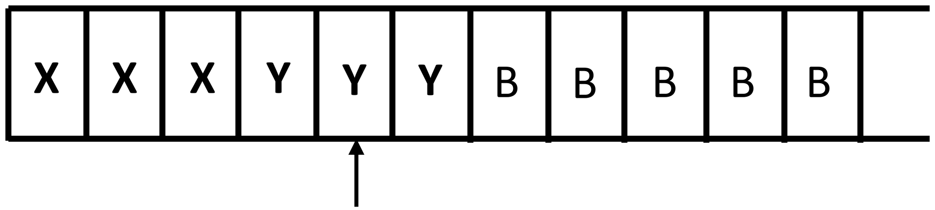

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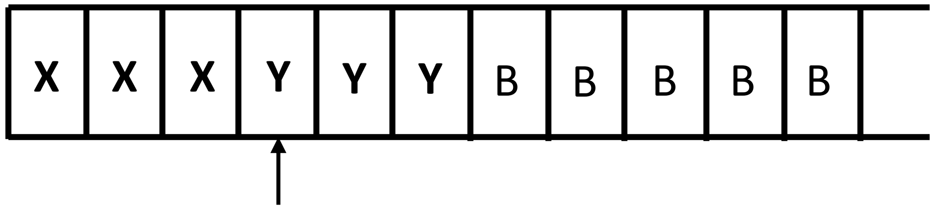

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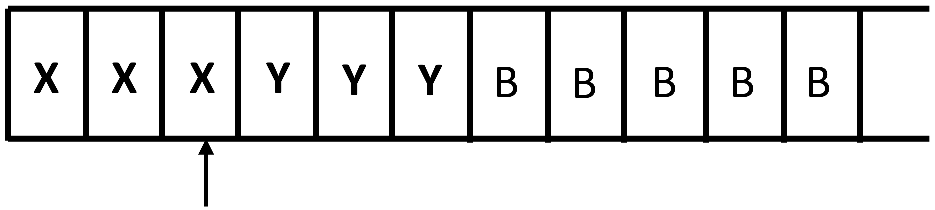

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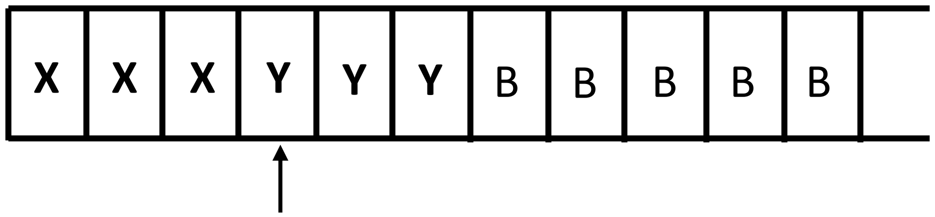

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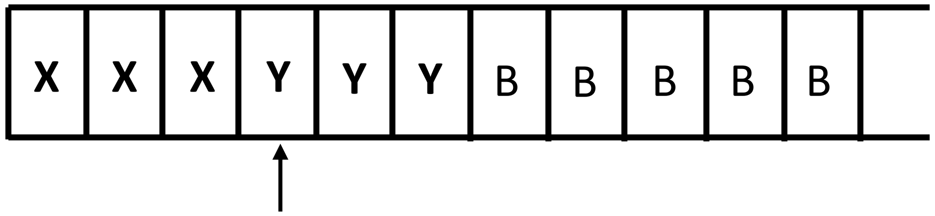

The head keeps moving right until it finds a B

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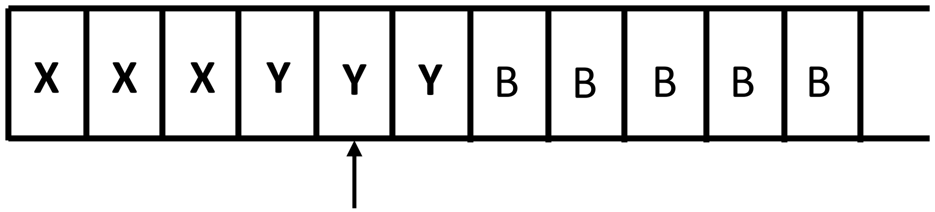

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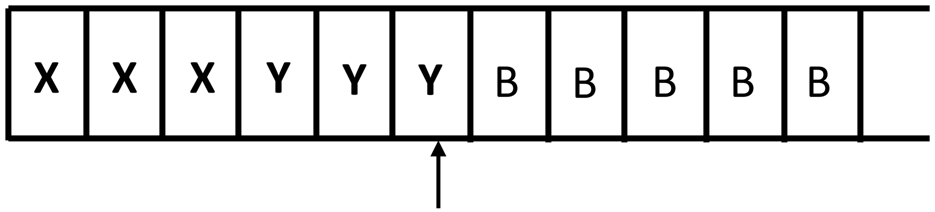

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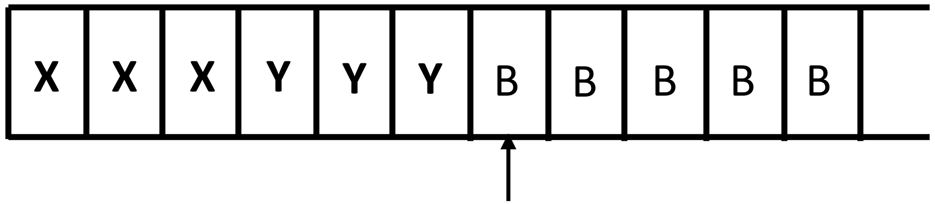

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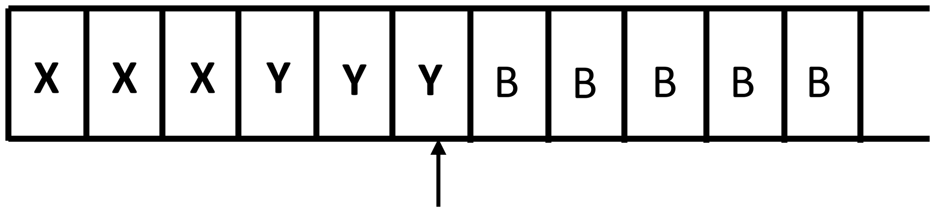

This is when the TM decides to accept the input string.

Turing Machines

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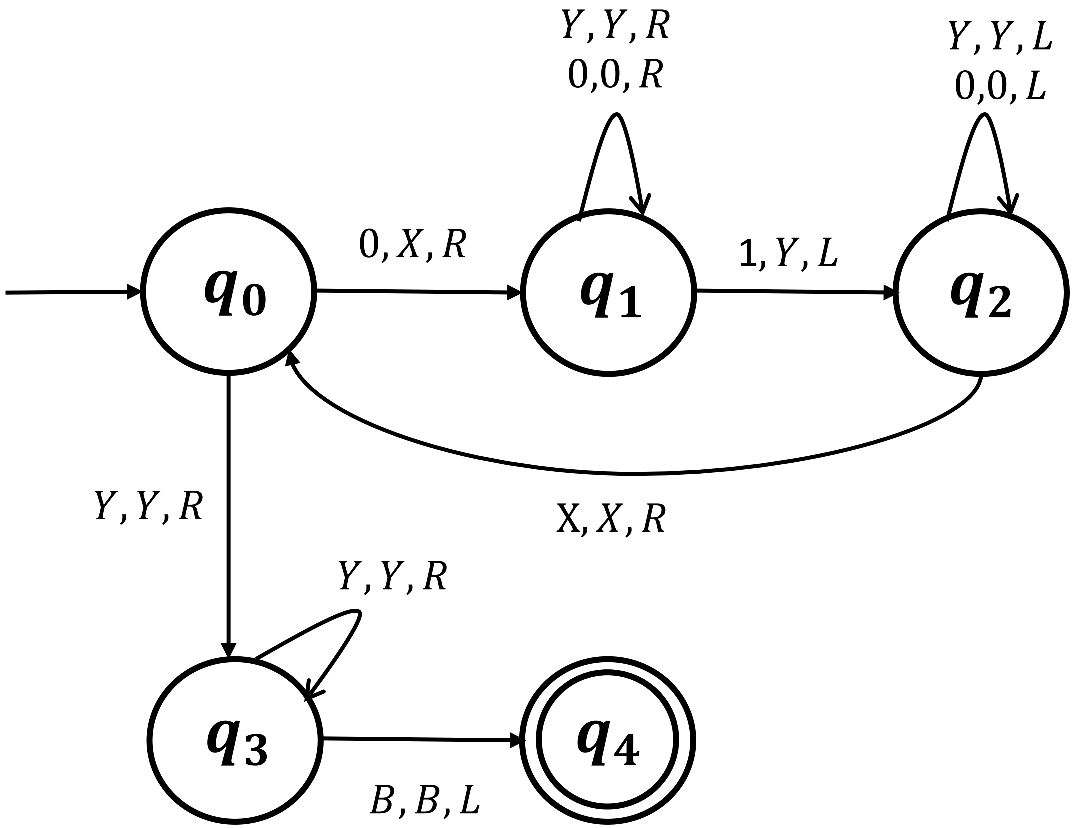

All missing transitions lead to the reject state and the input is rejected when this state is reached.

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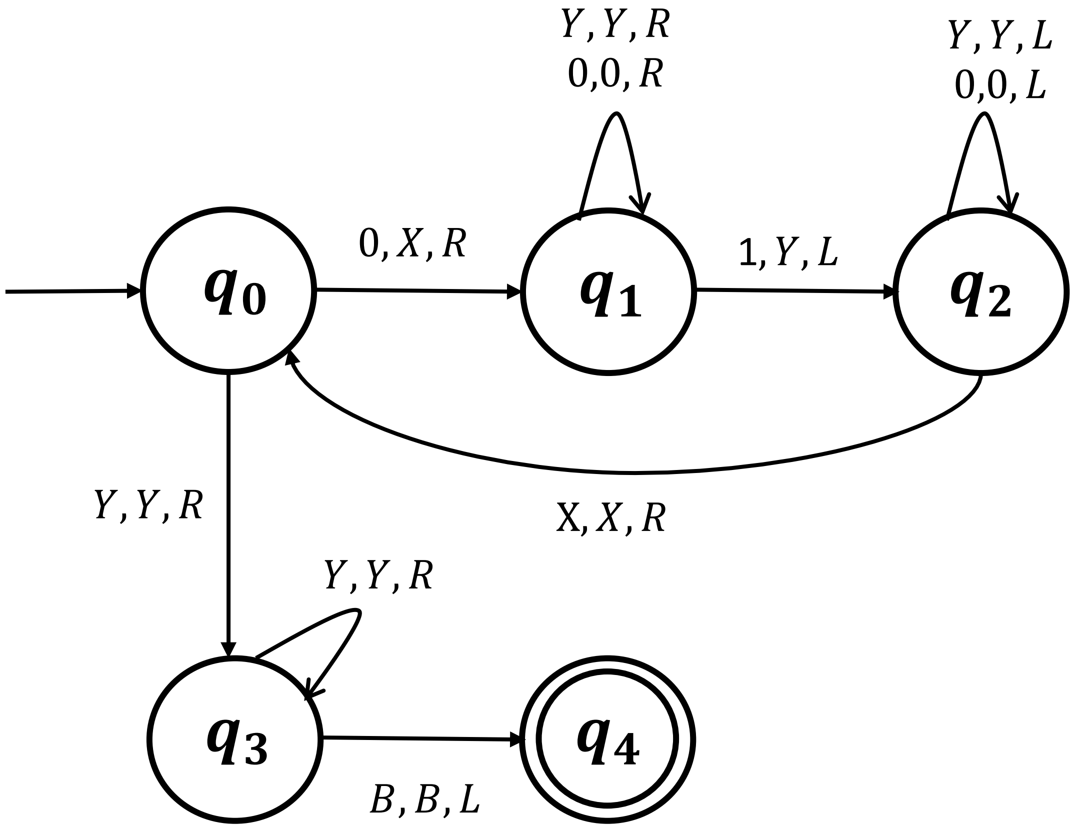

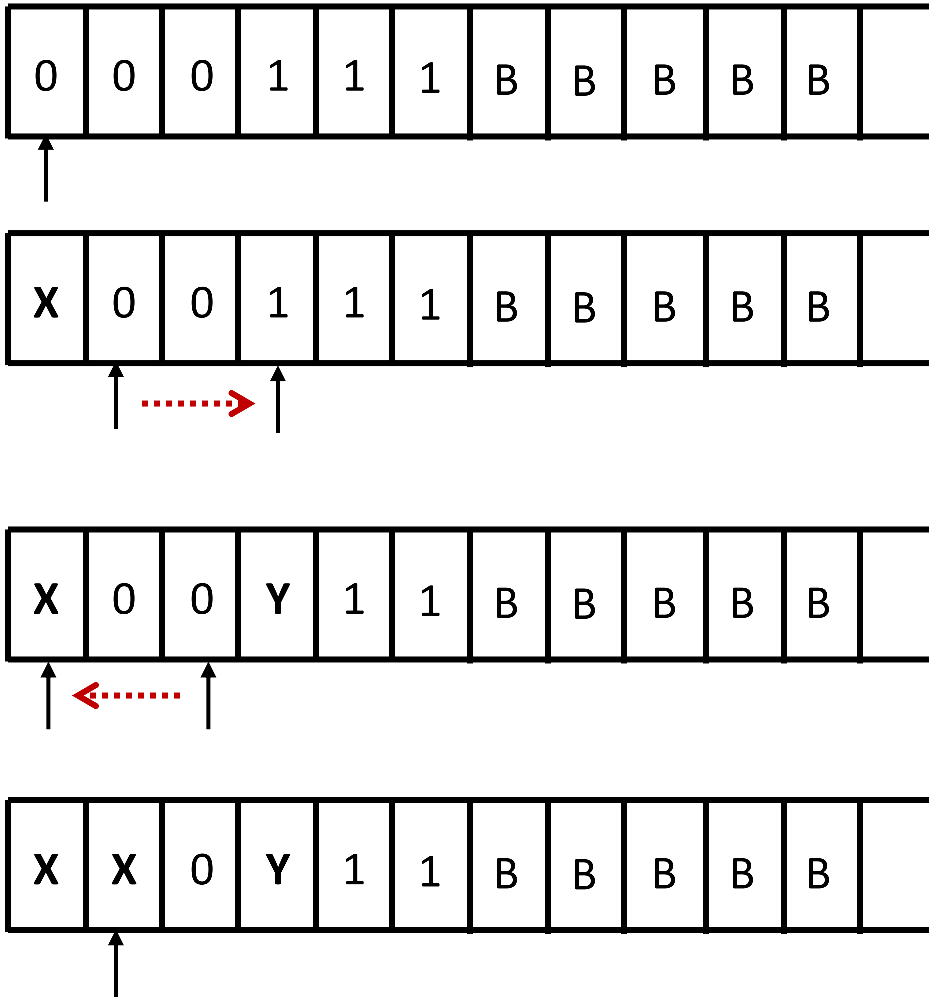

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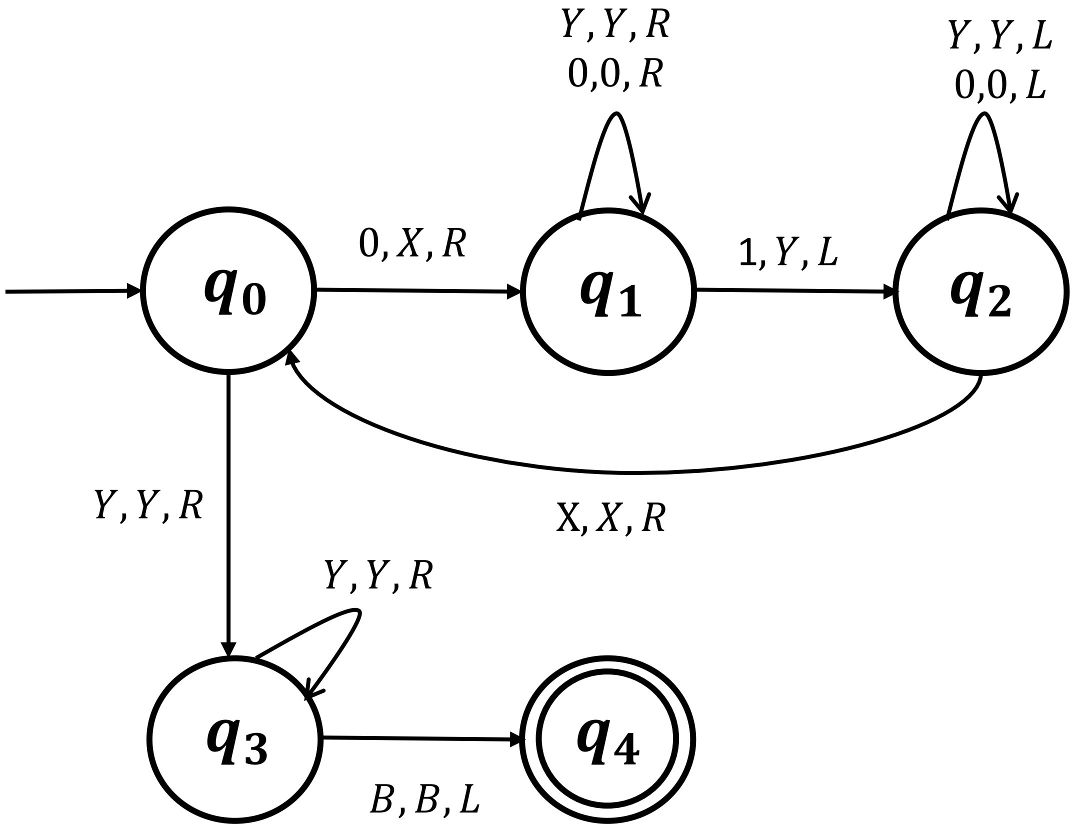

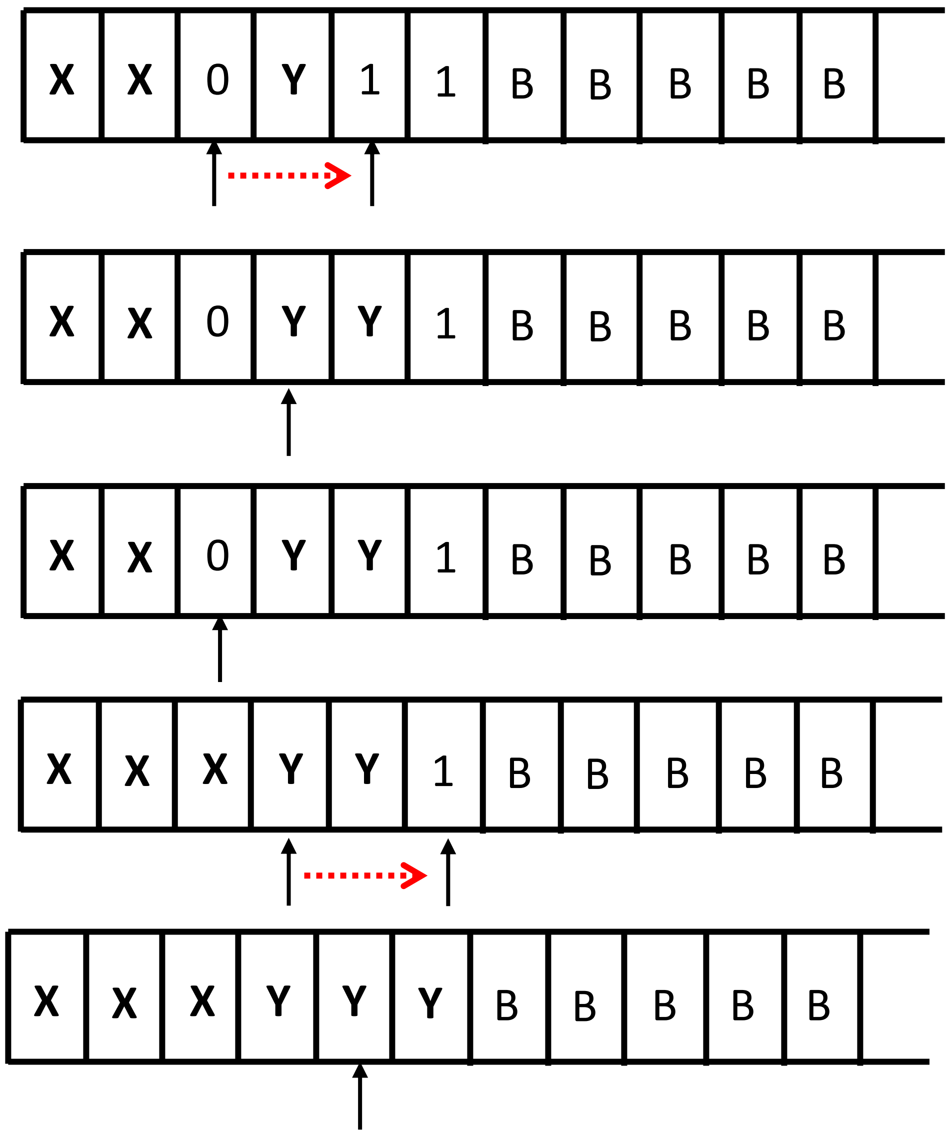

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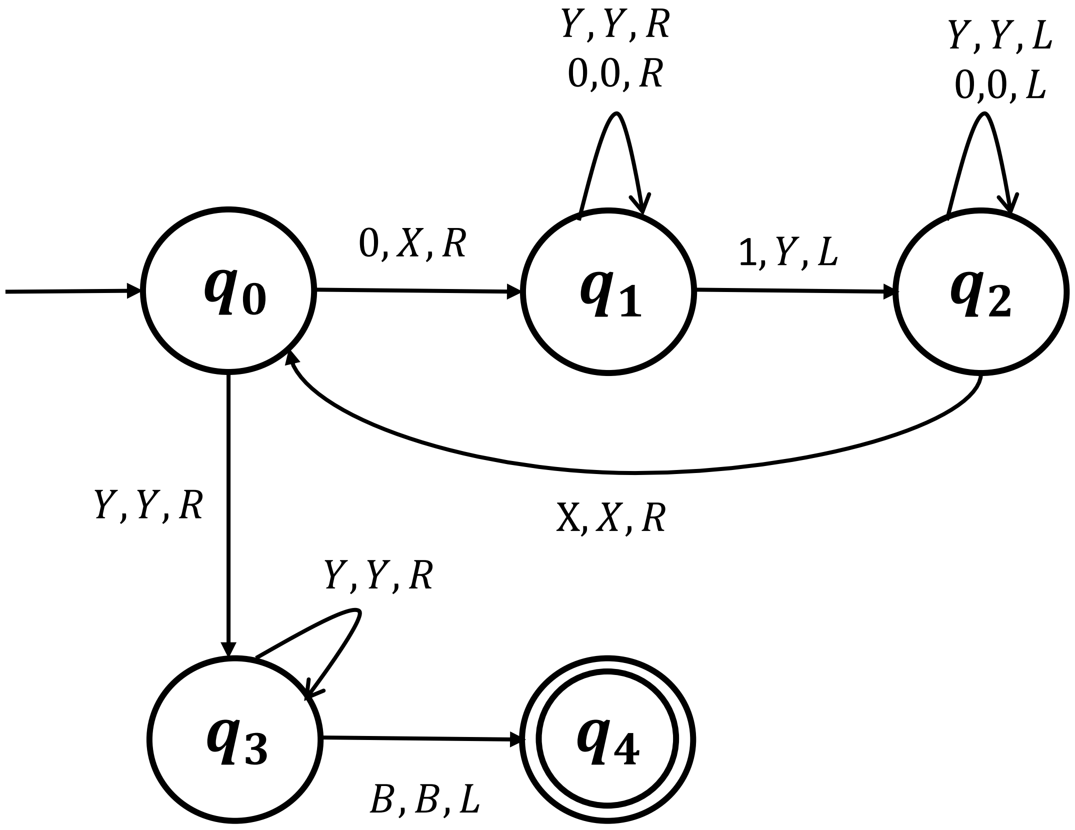

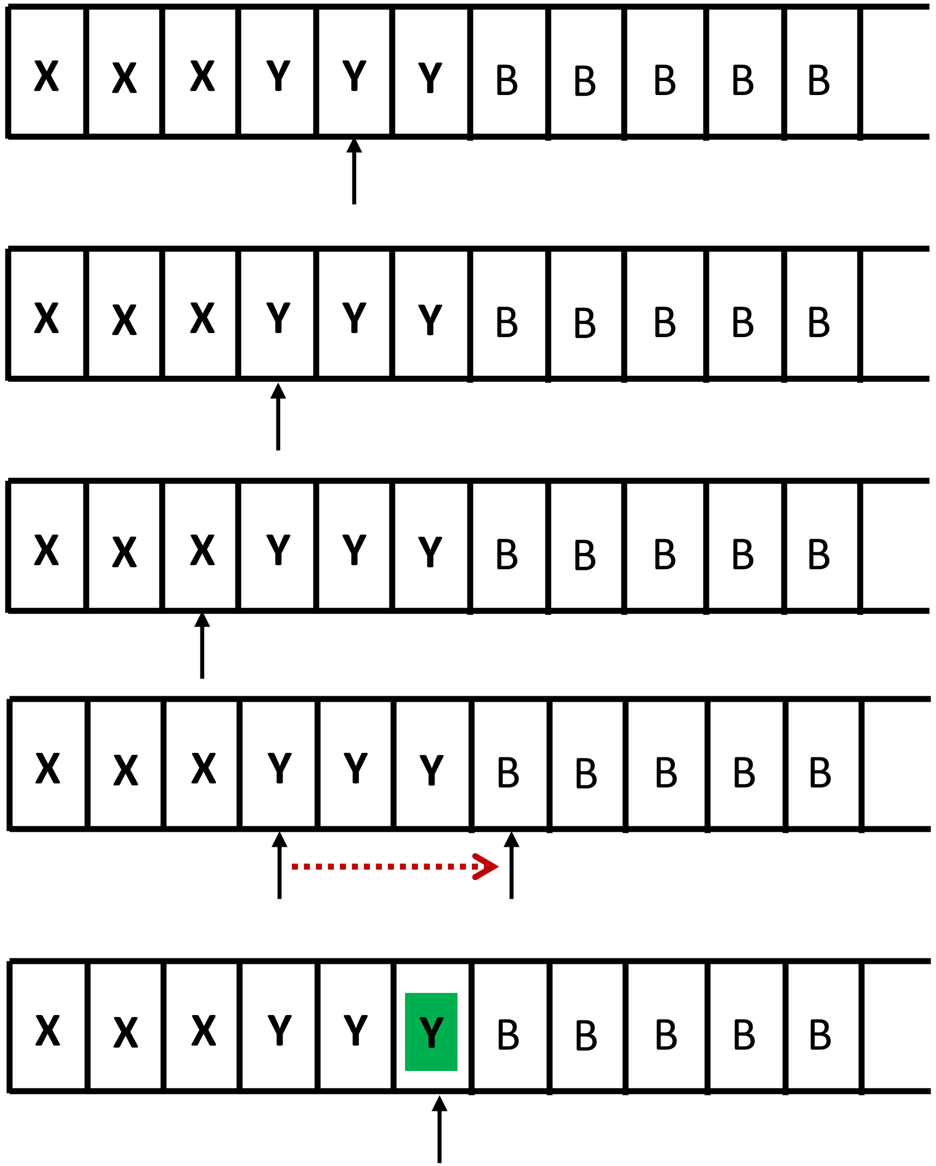

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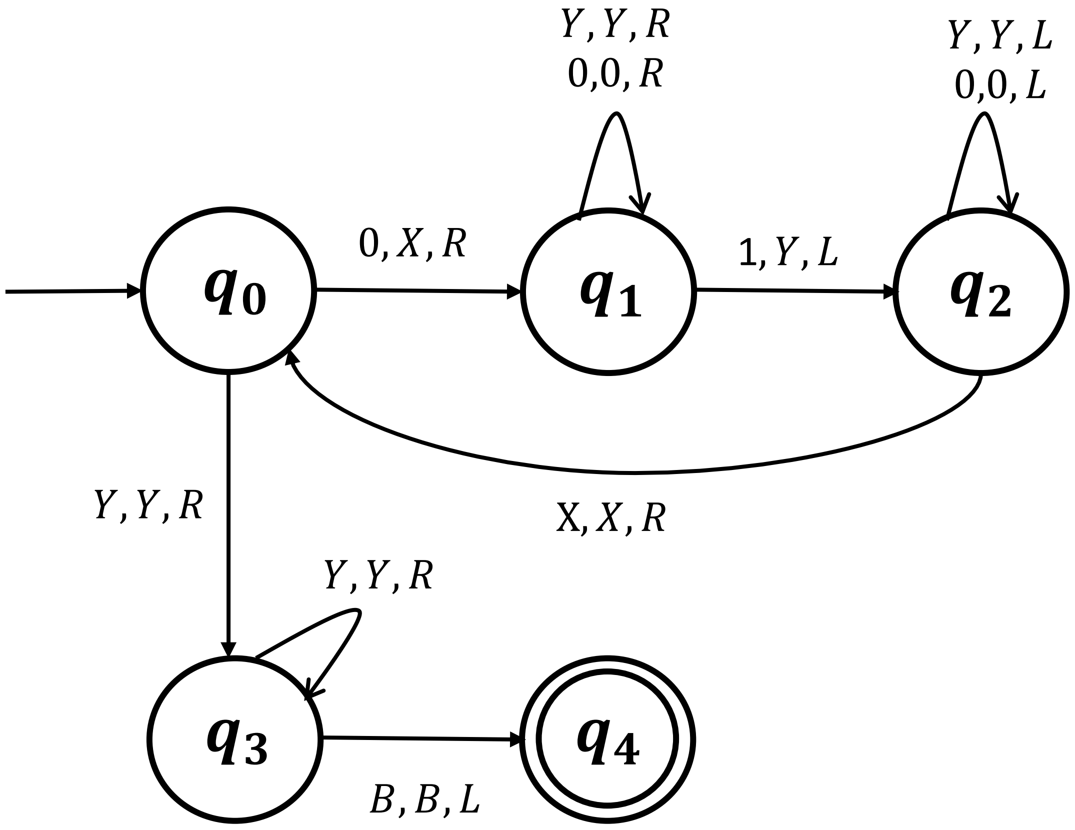
Example: Let $L = \{0^n1^n2^n | n \geq 1\}$



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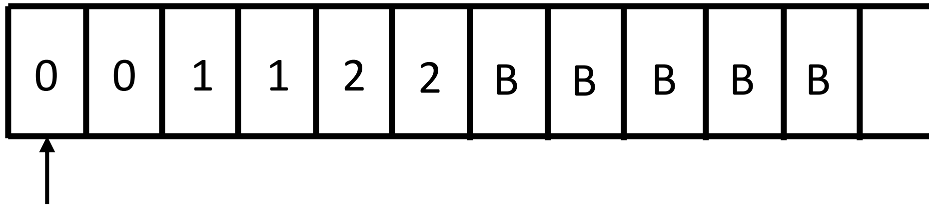

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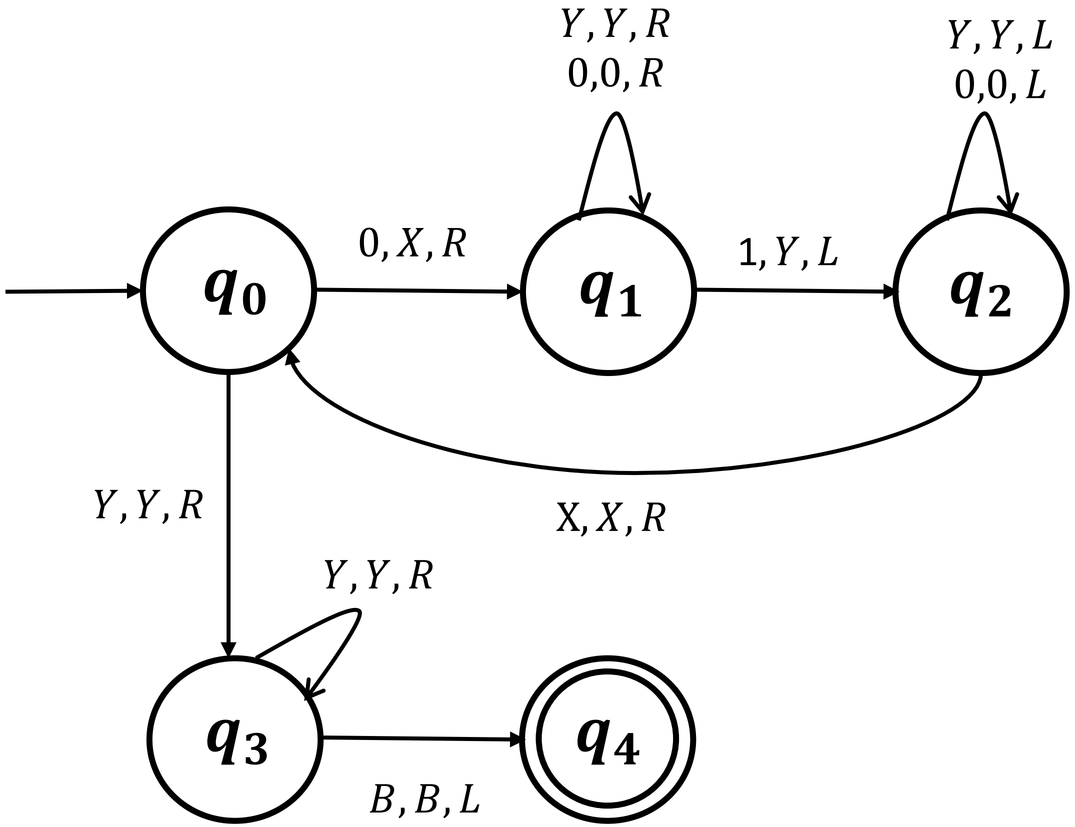

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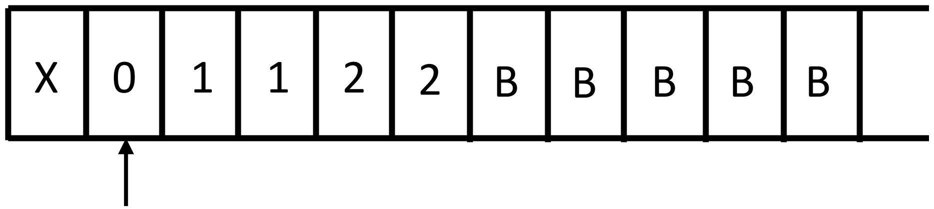

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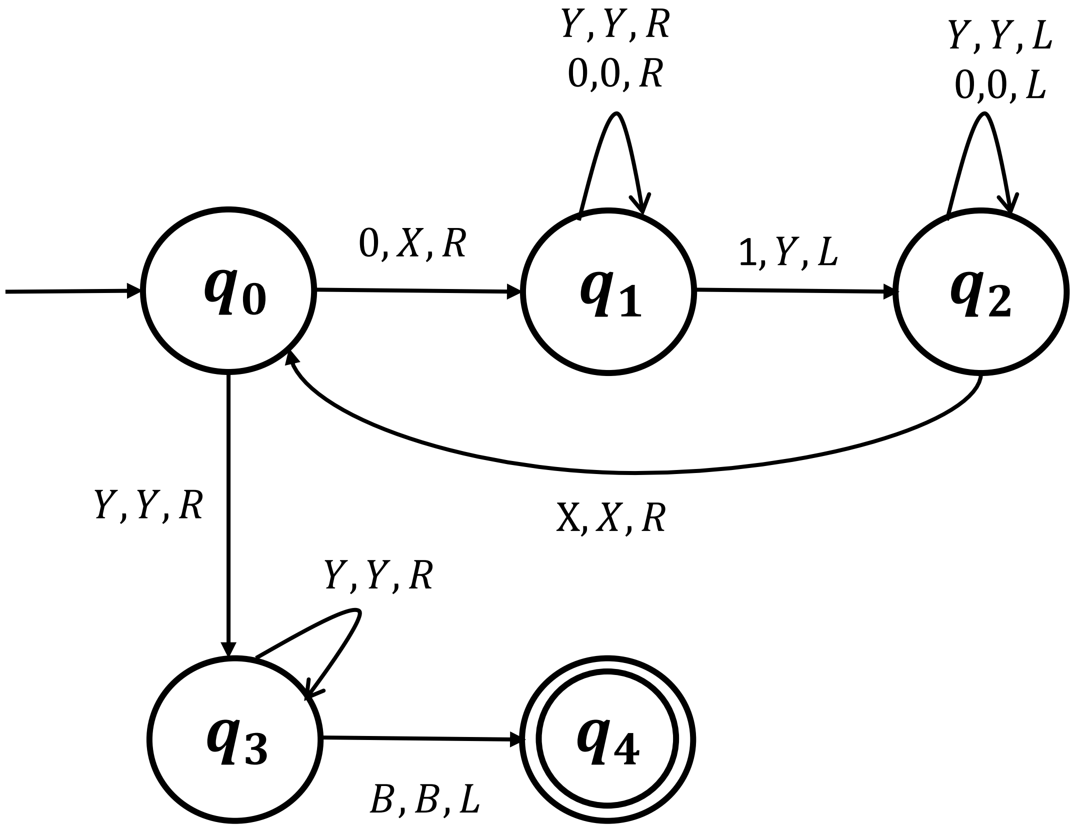

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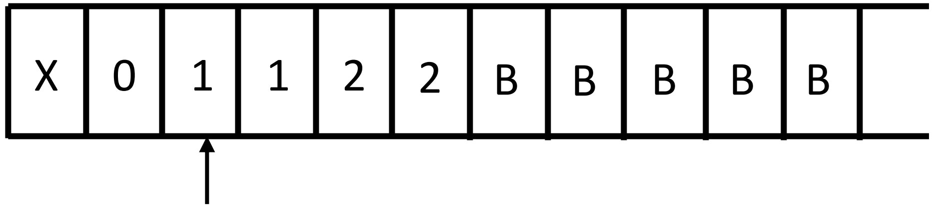

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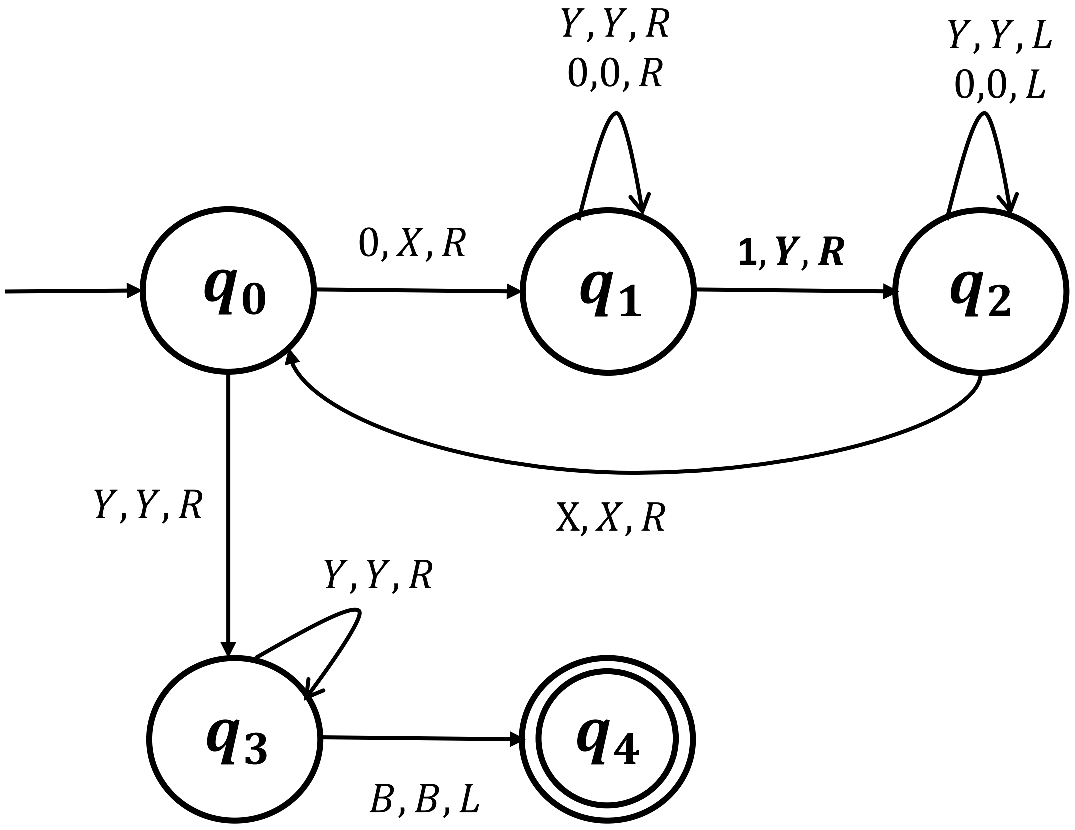

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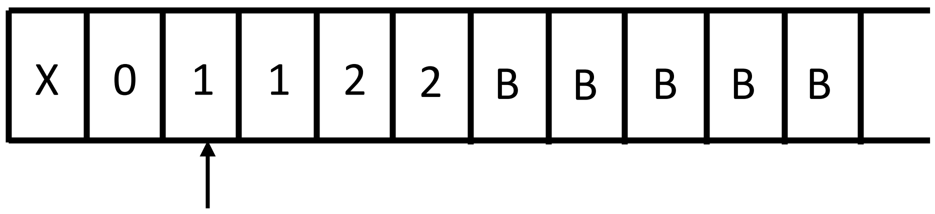

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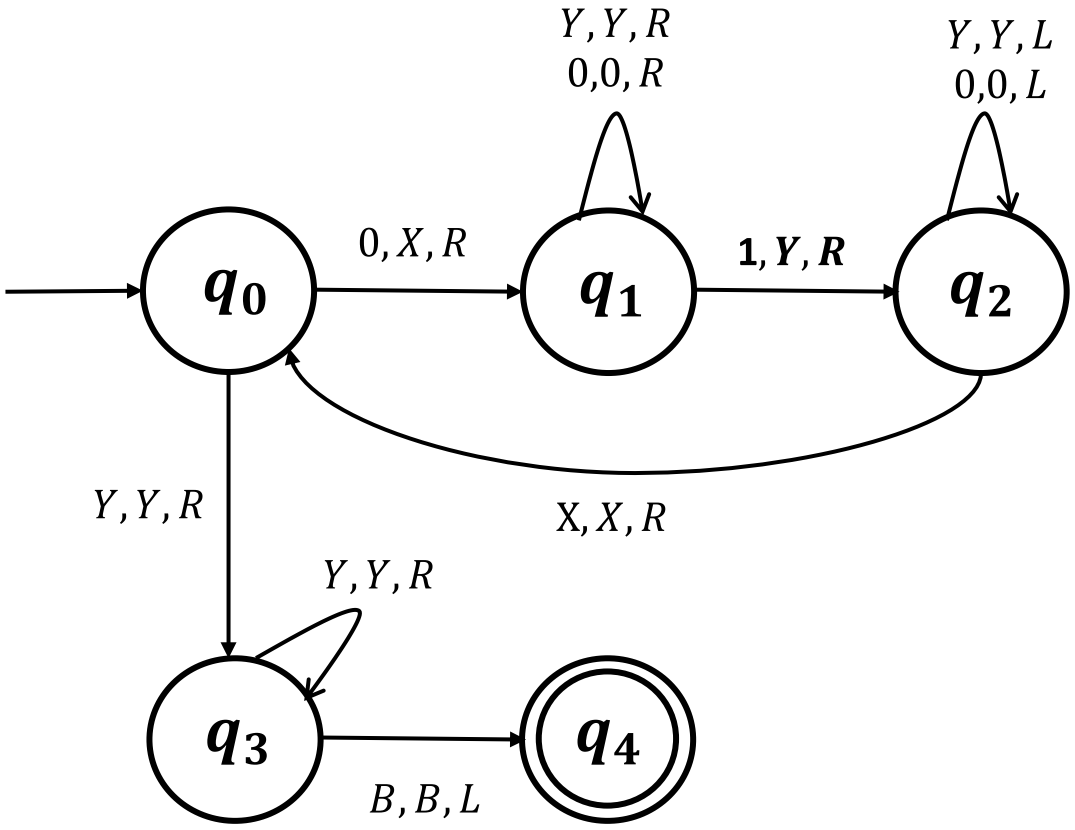

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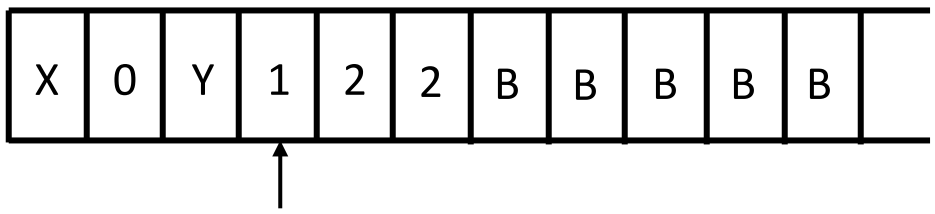

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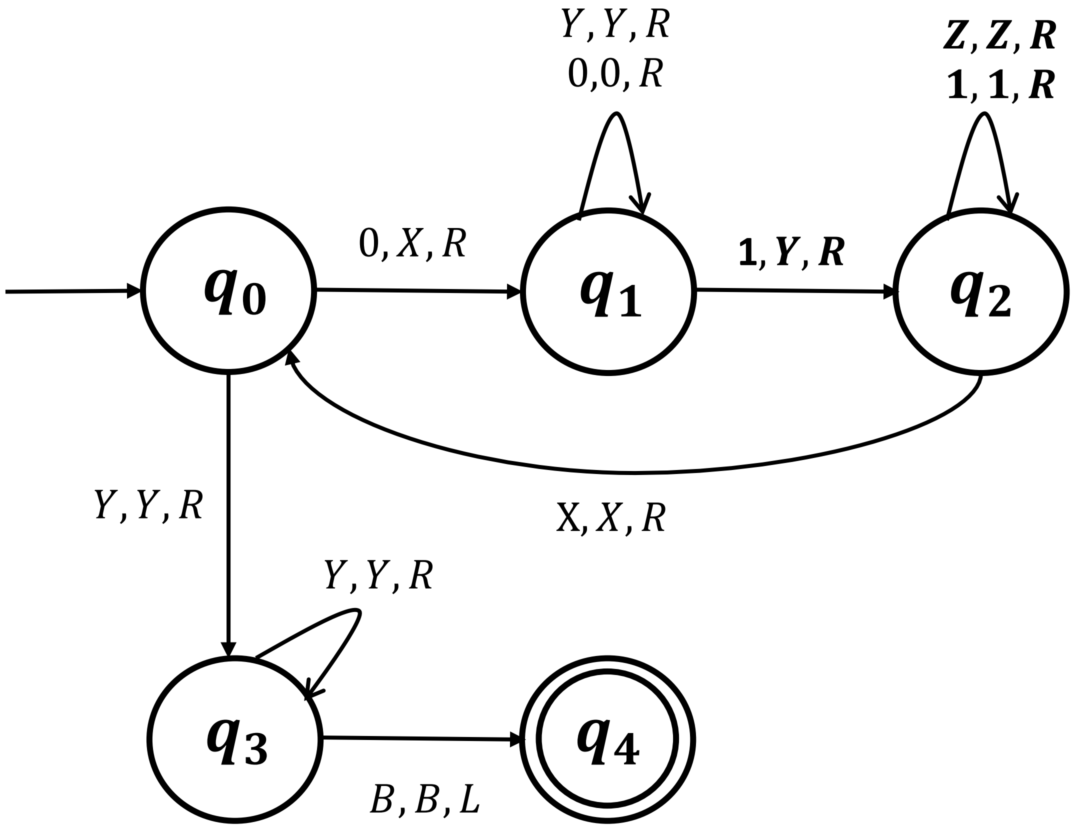

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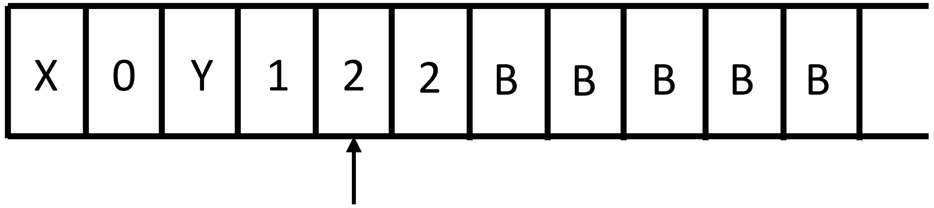

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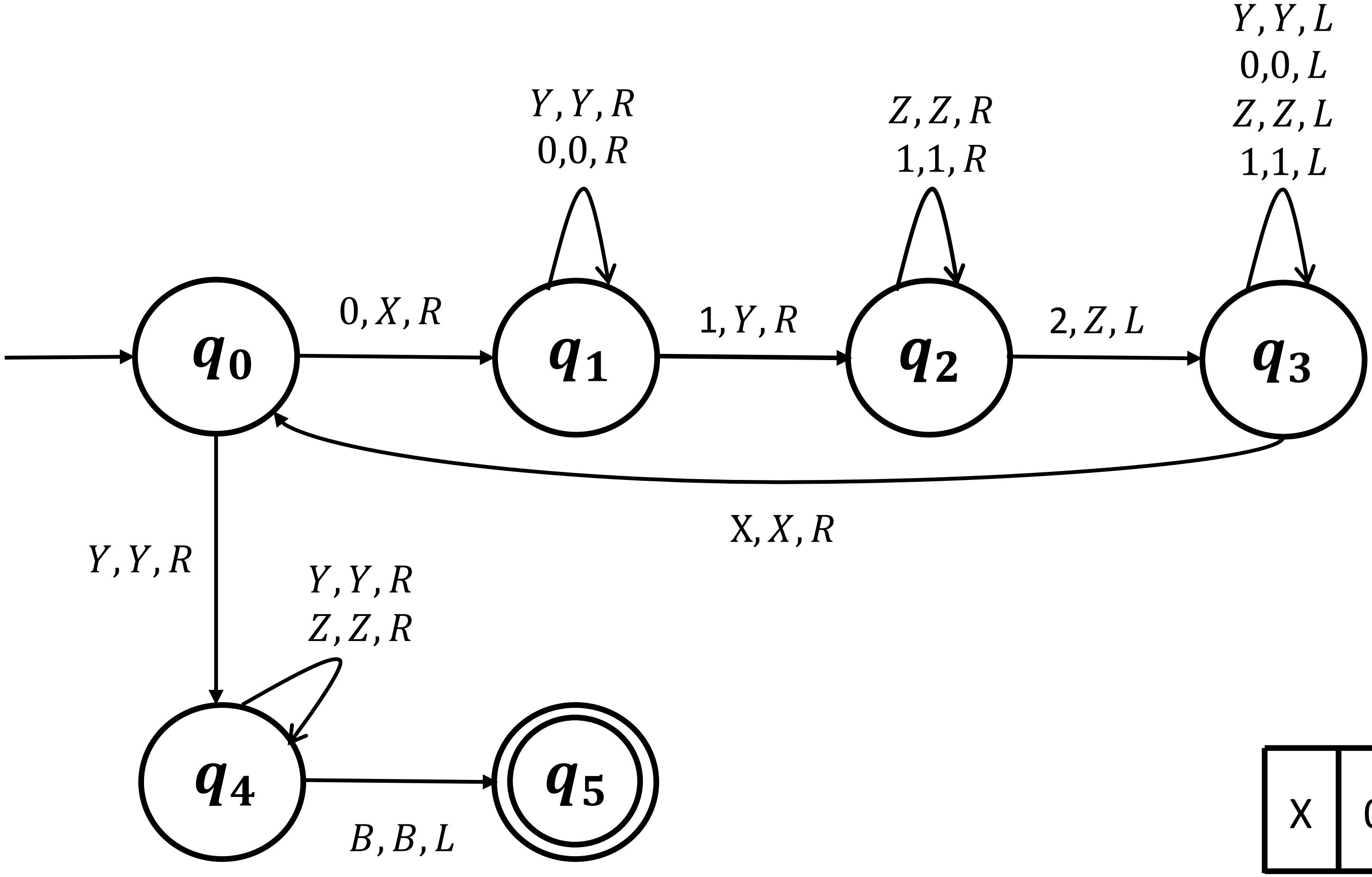
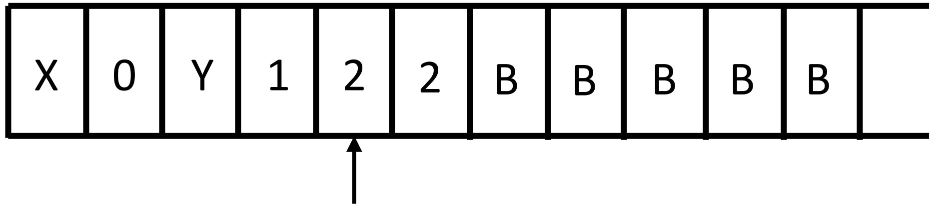


- Continue to go right to mark the next 2 with a Z.
- Skip all the $1'$s and move right/ Also, move across (to the right) the Z's already marked.
- Mark a new 2 with a Z and start moving left.
- Keep moving left until an X is encountered.



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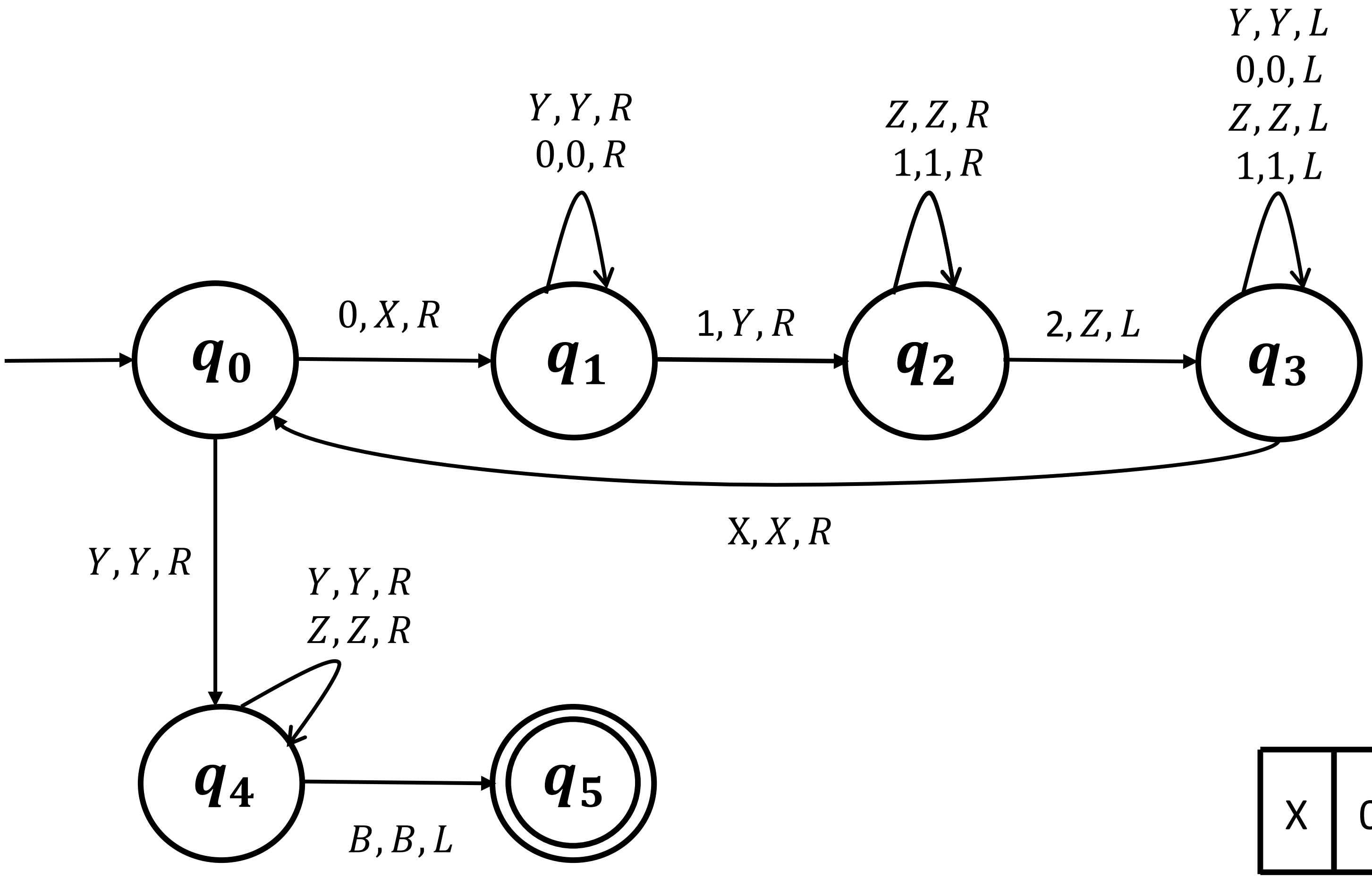

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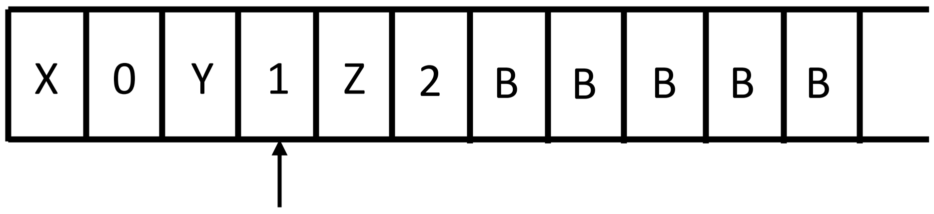

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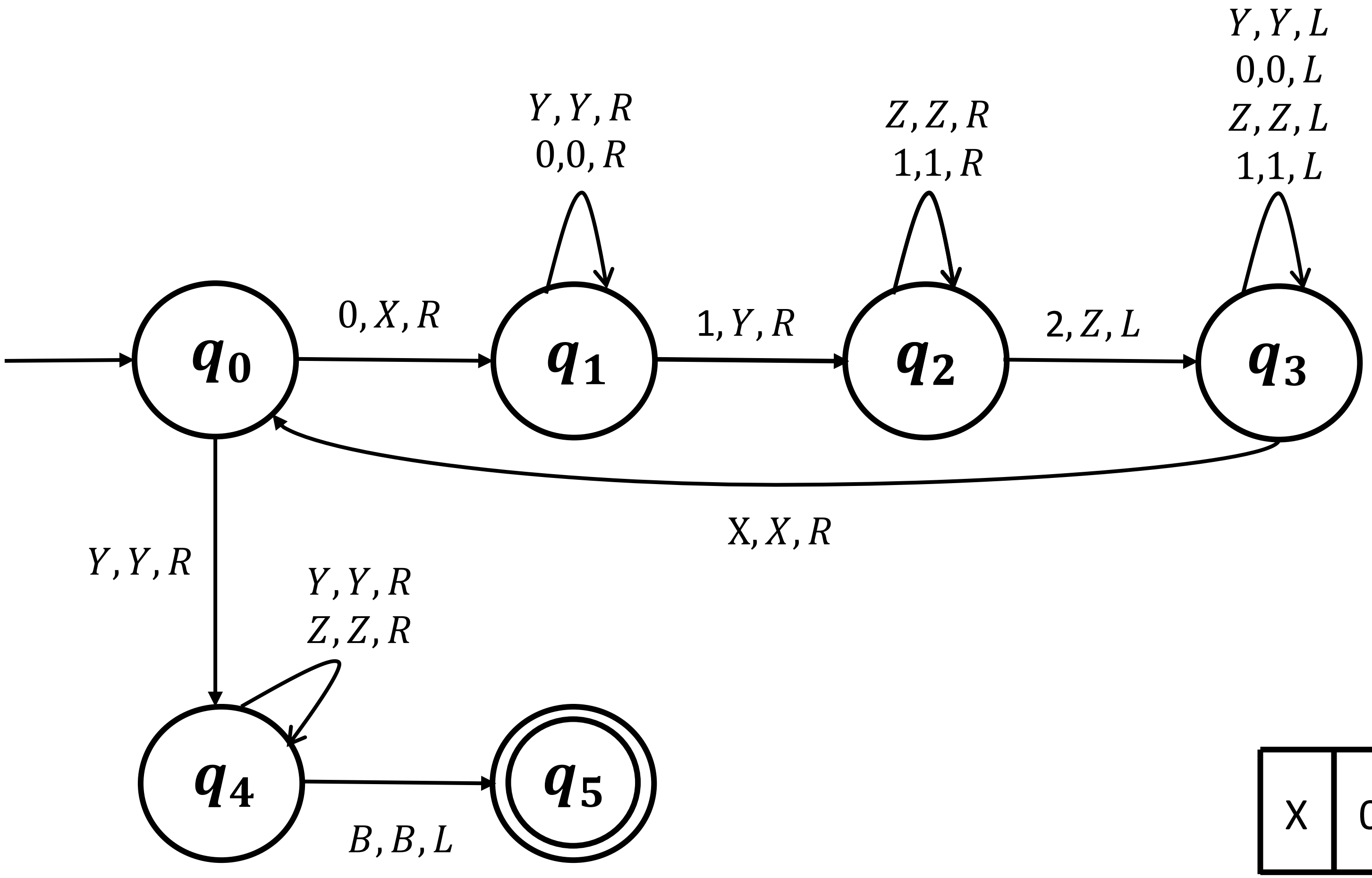

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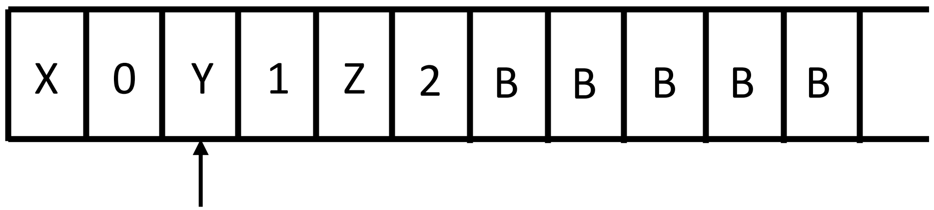

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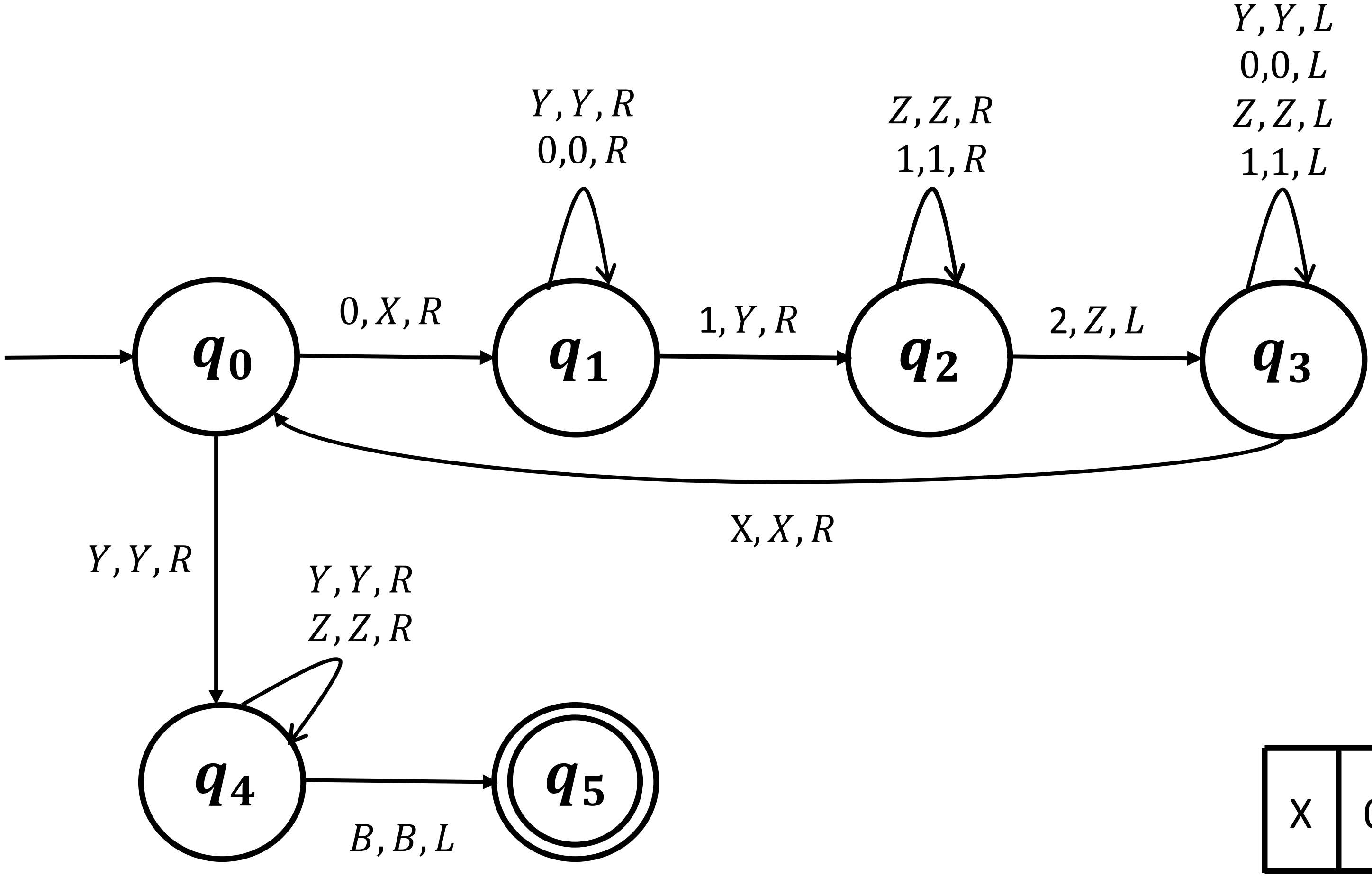

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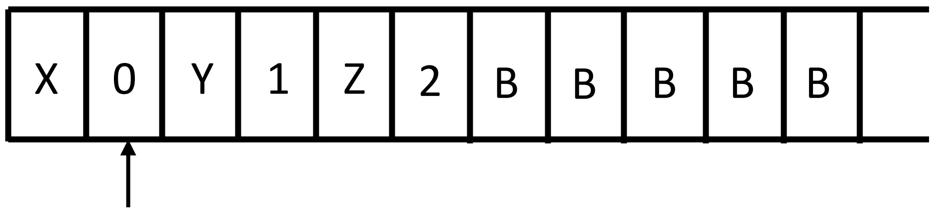

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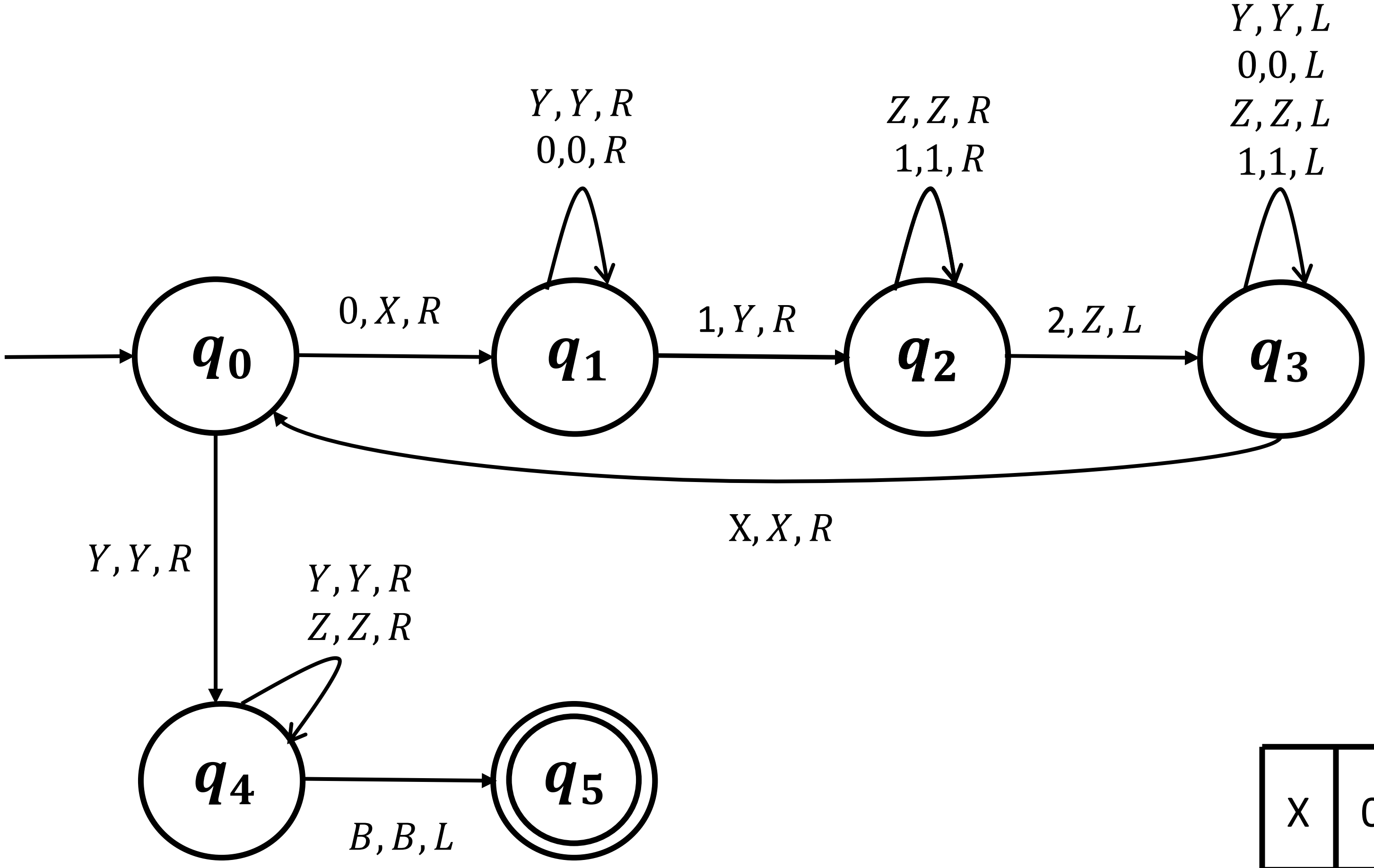

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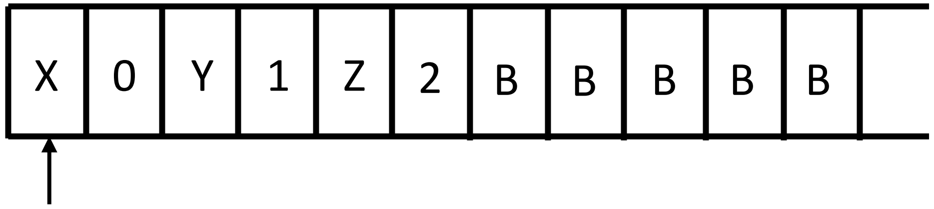

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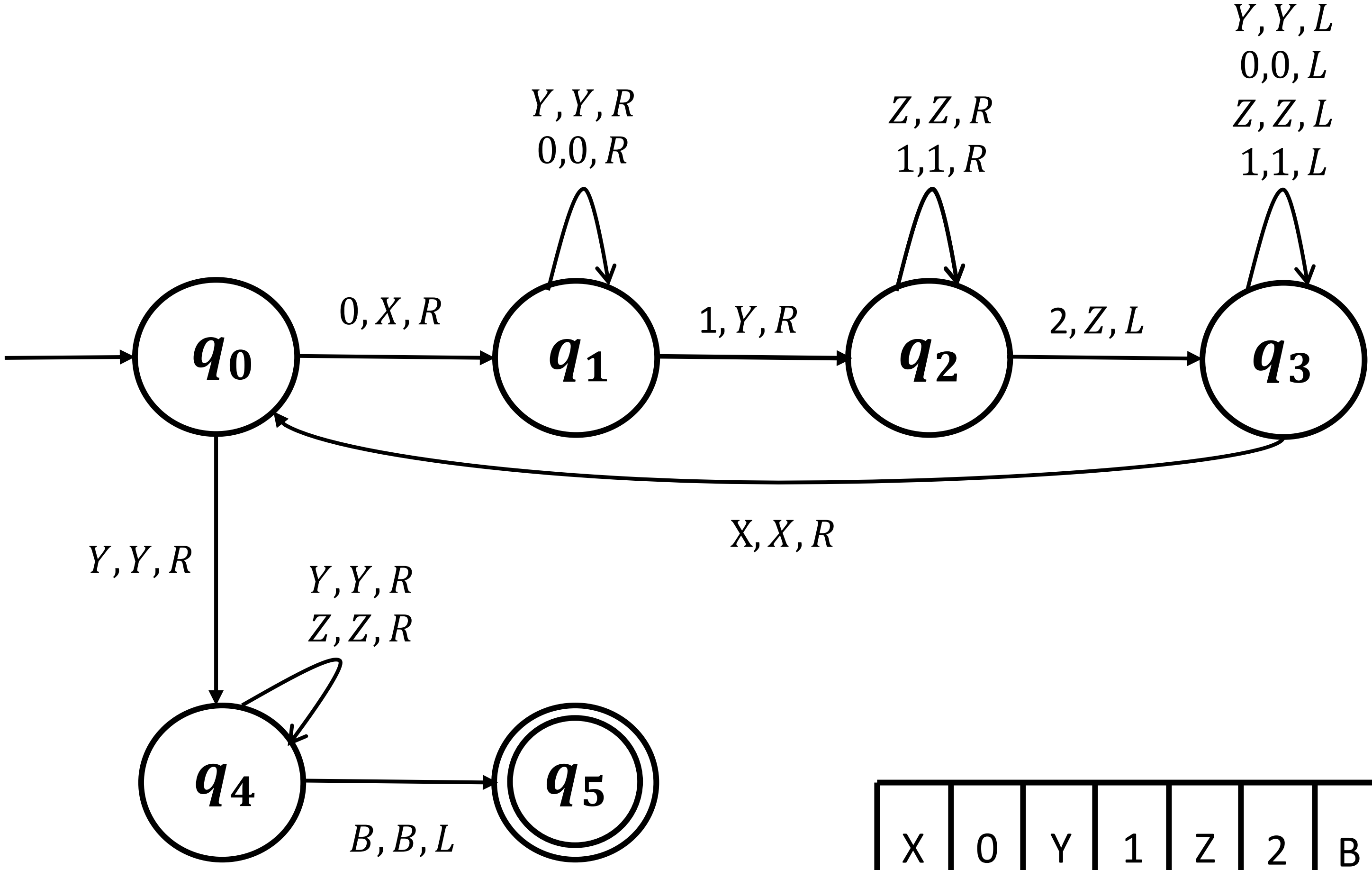

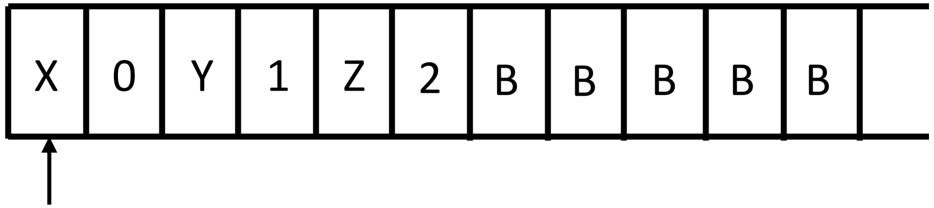

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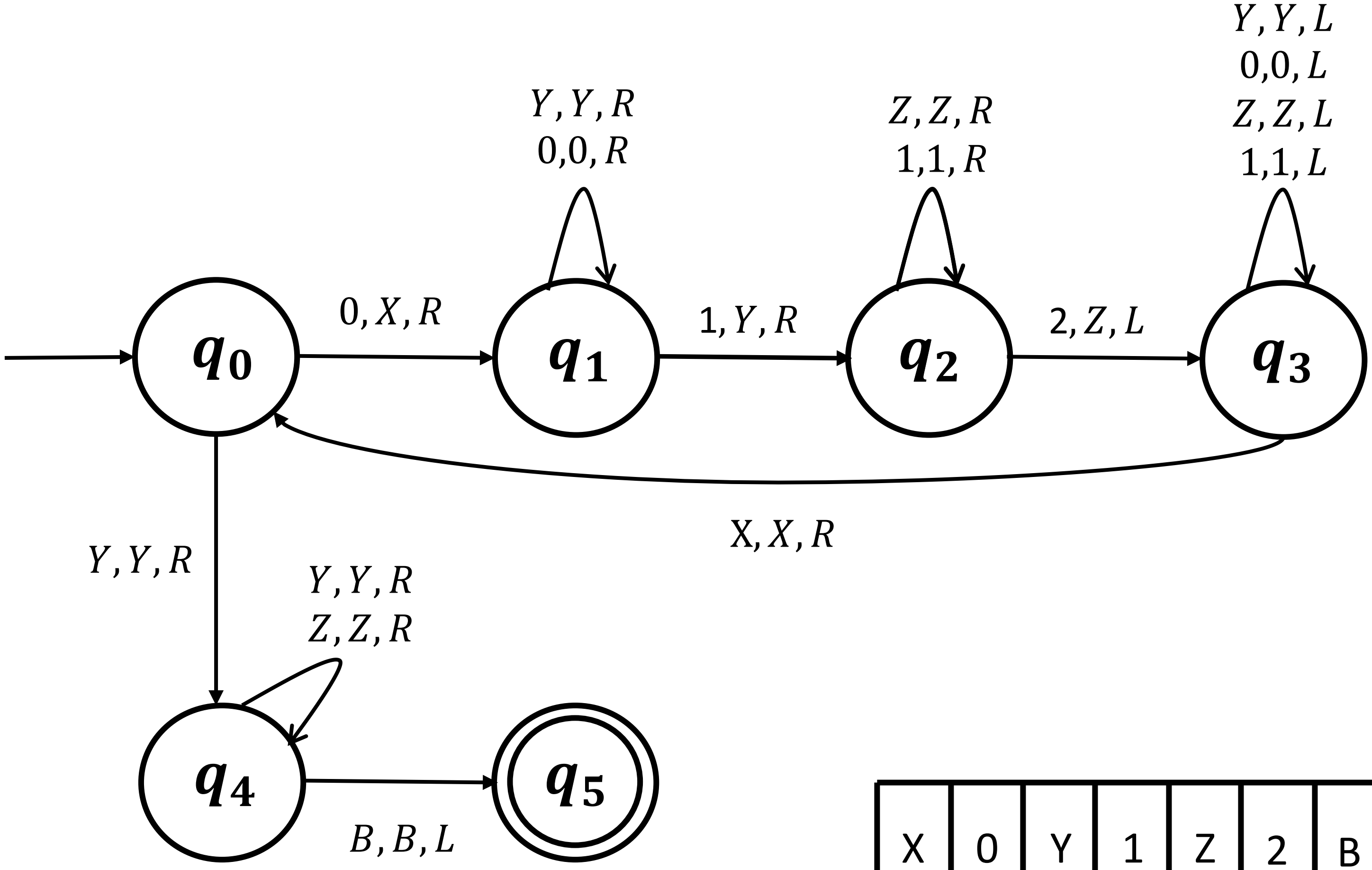

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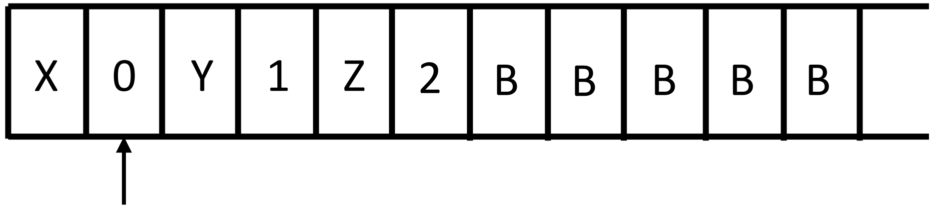

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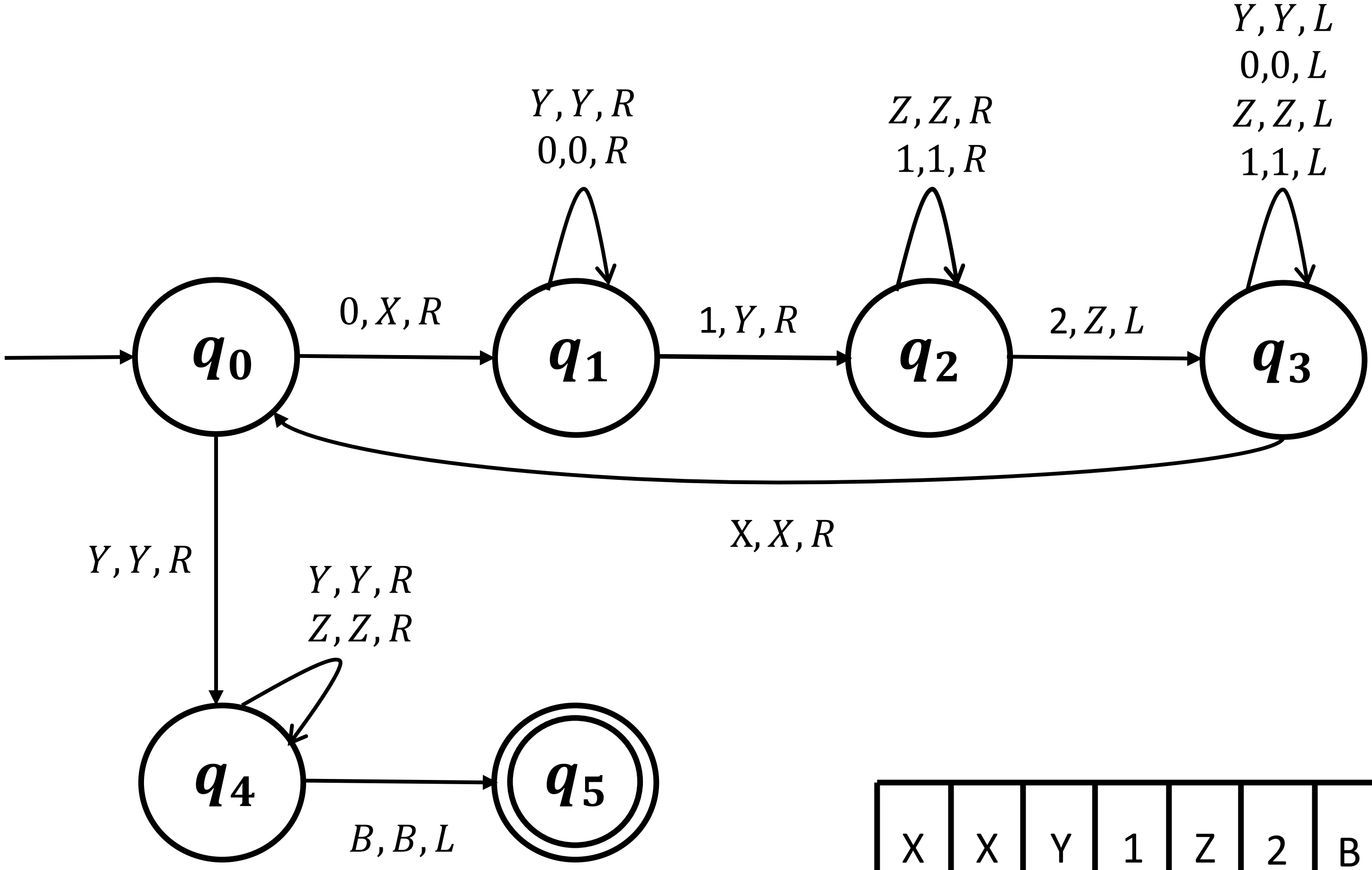

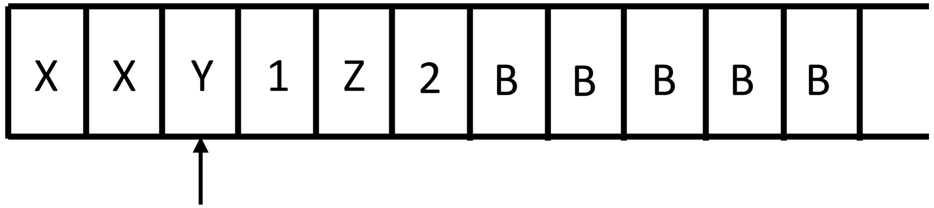

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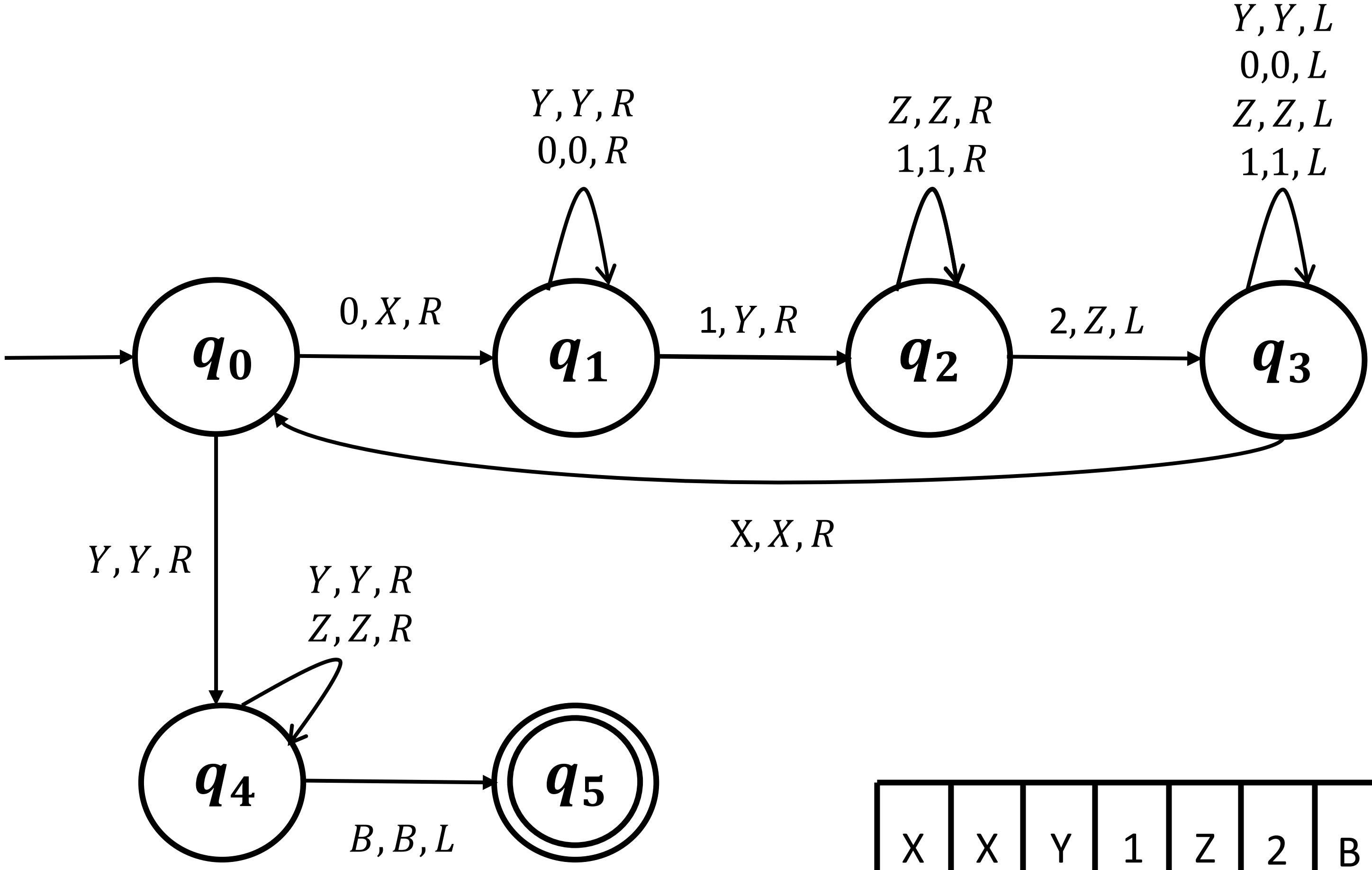

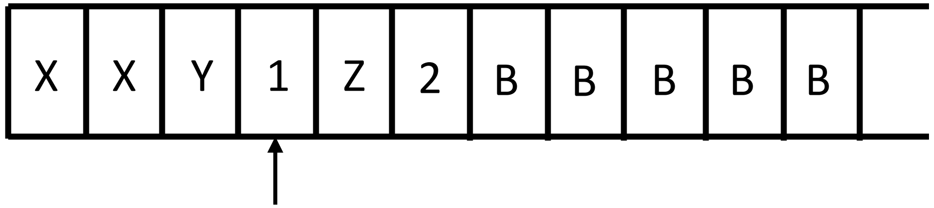

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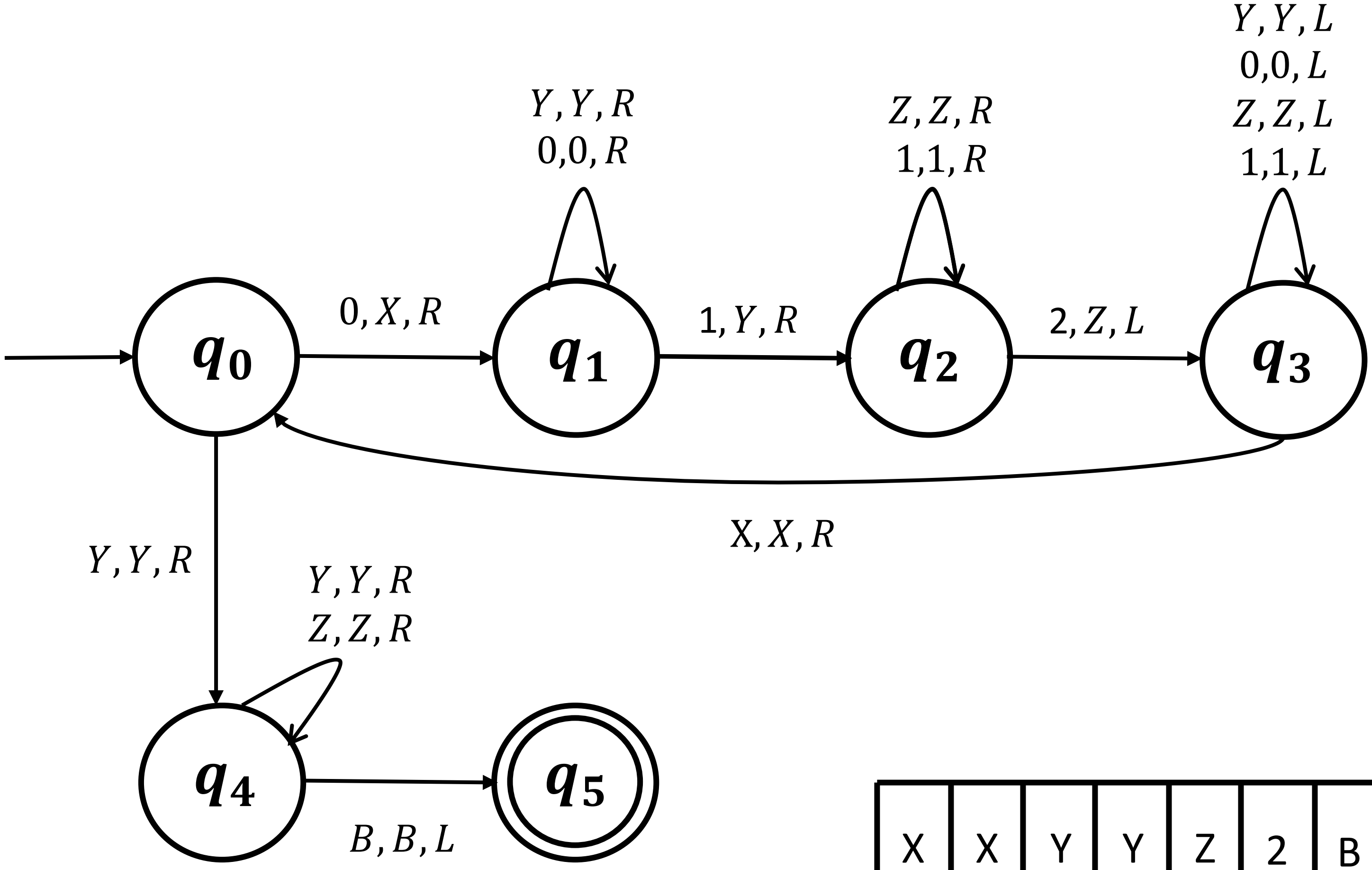

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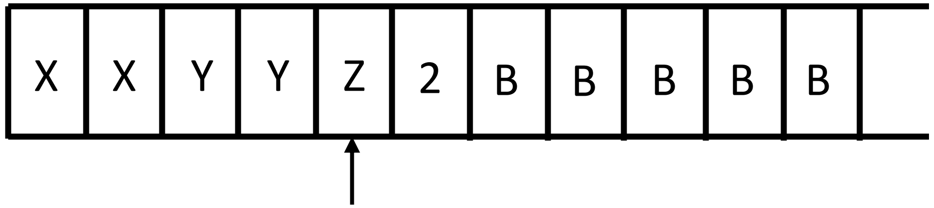

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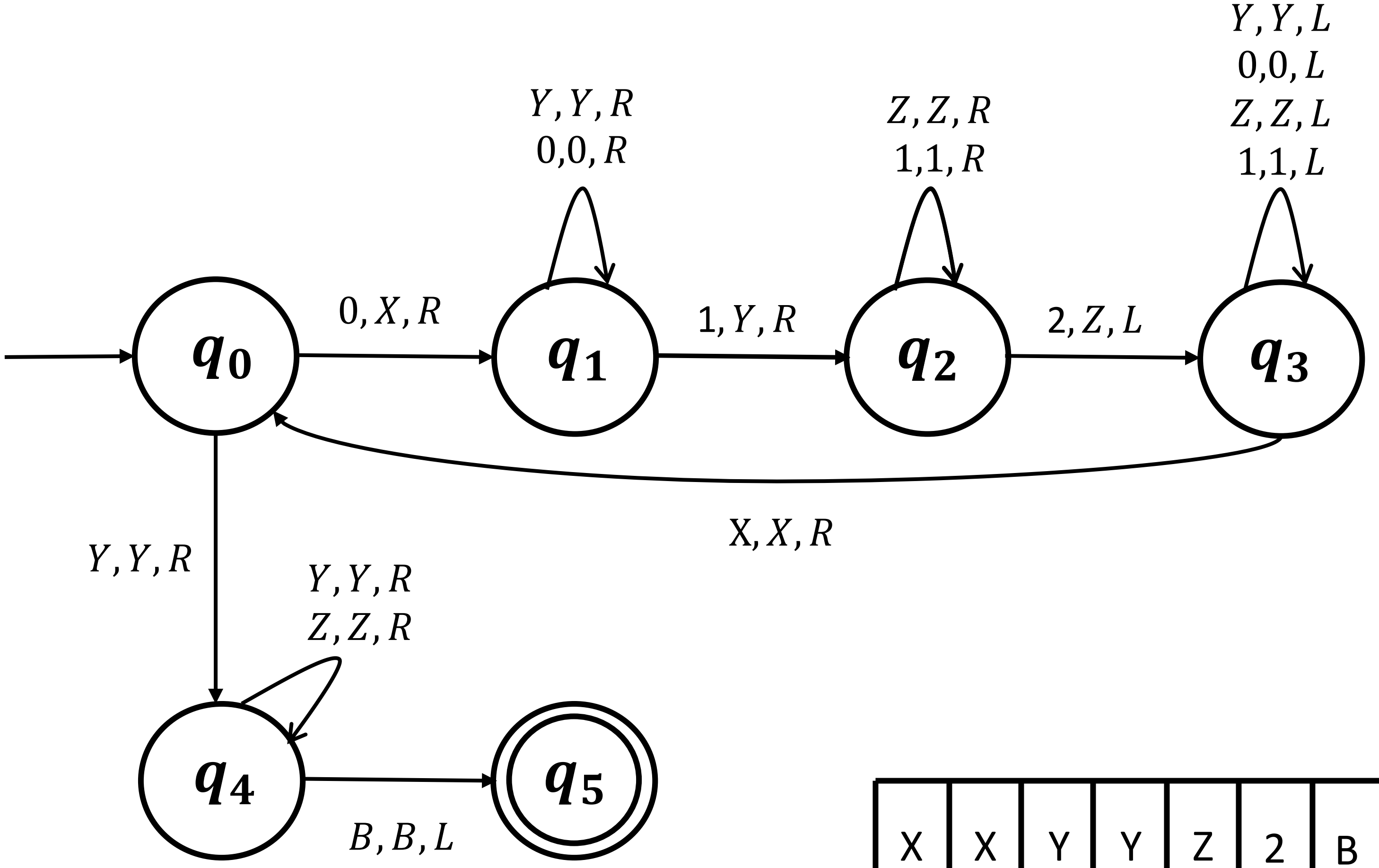

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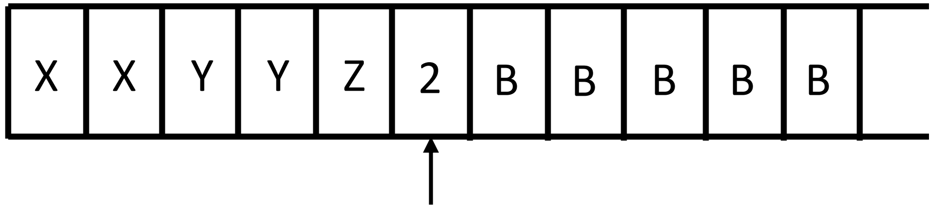

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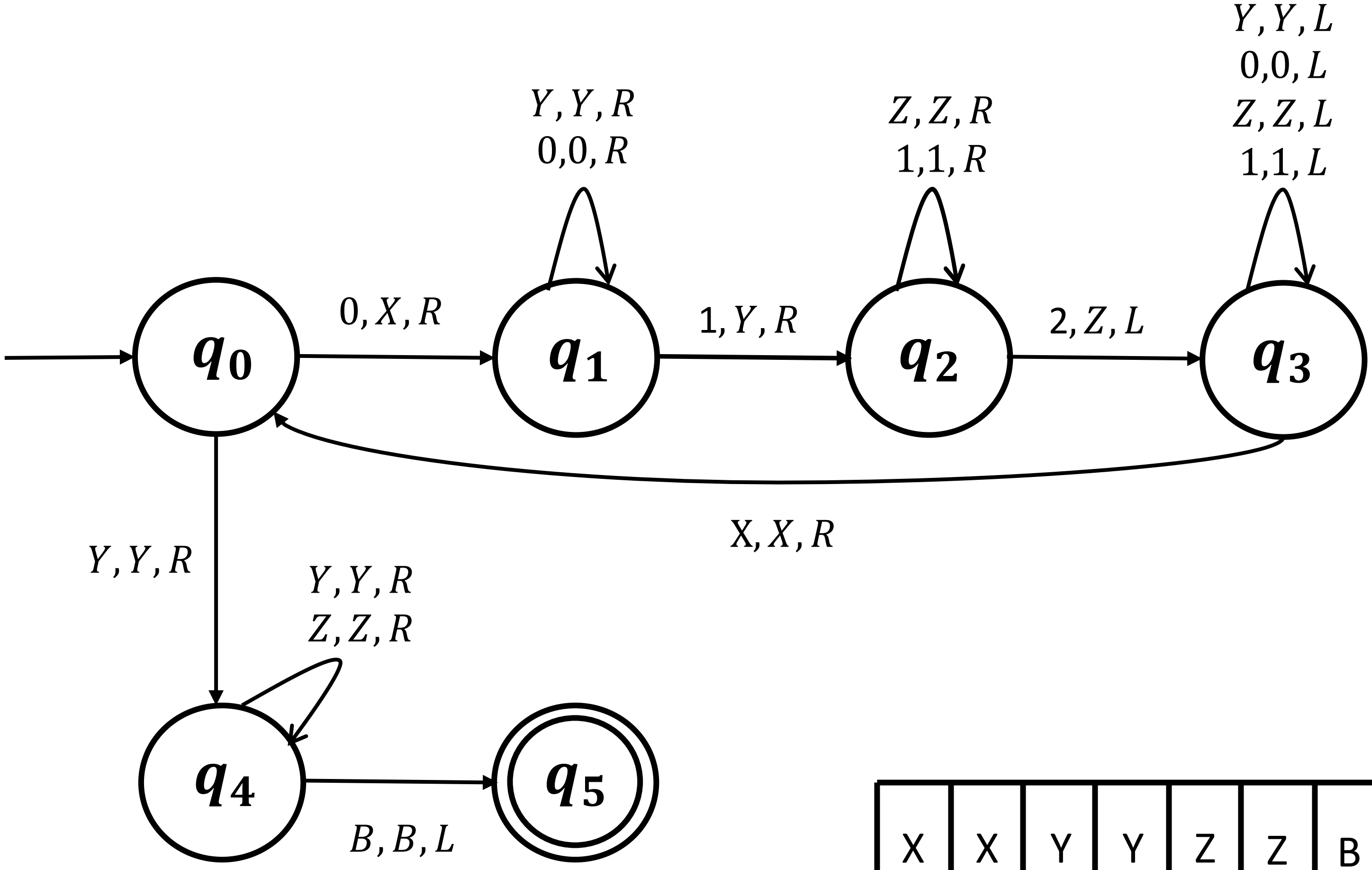

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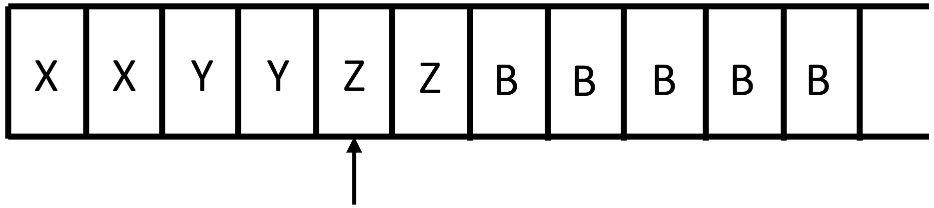

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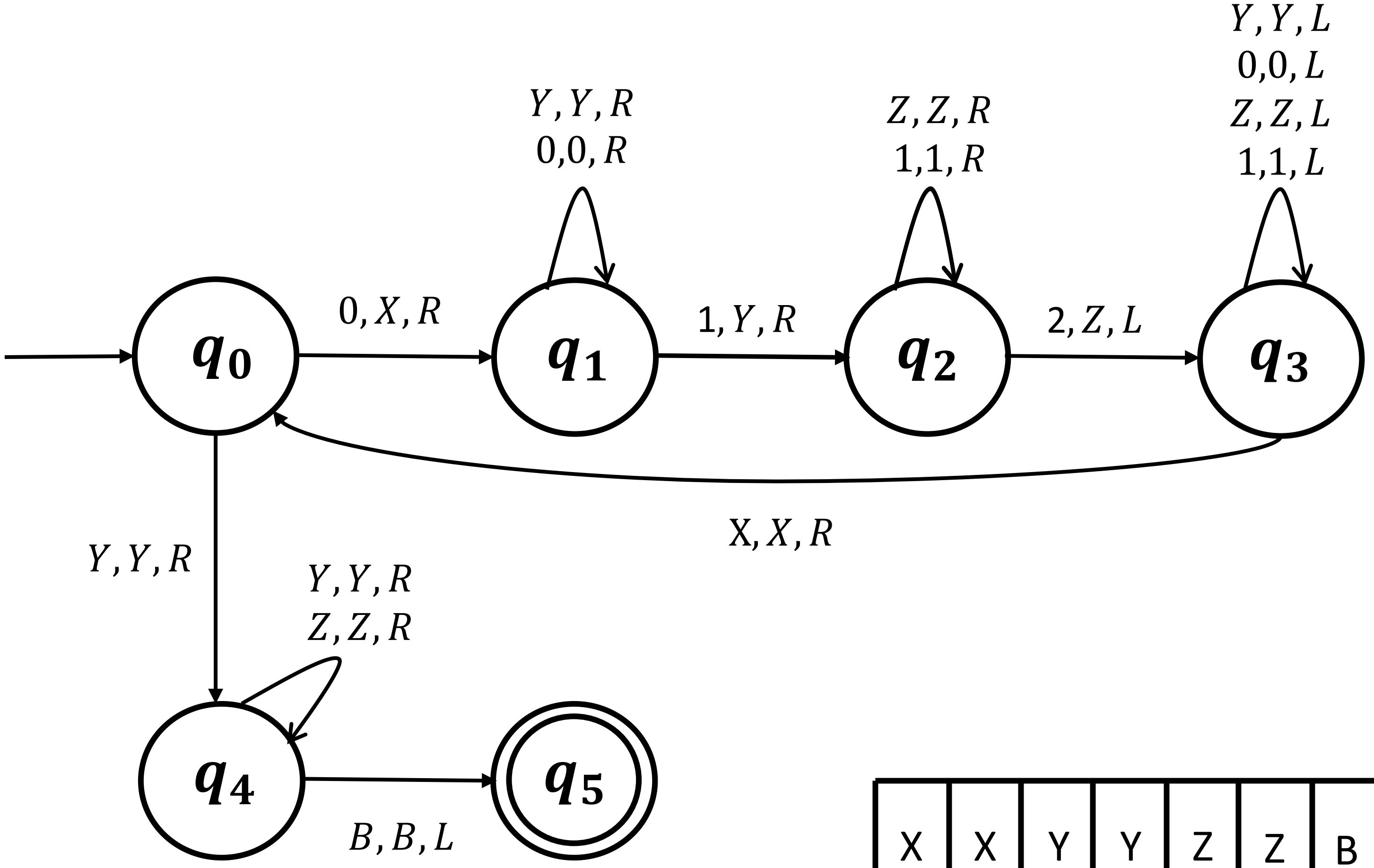

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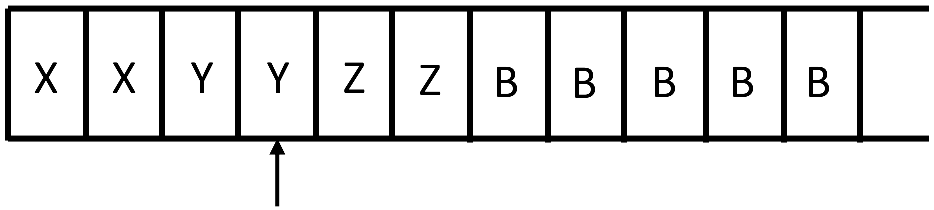

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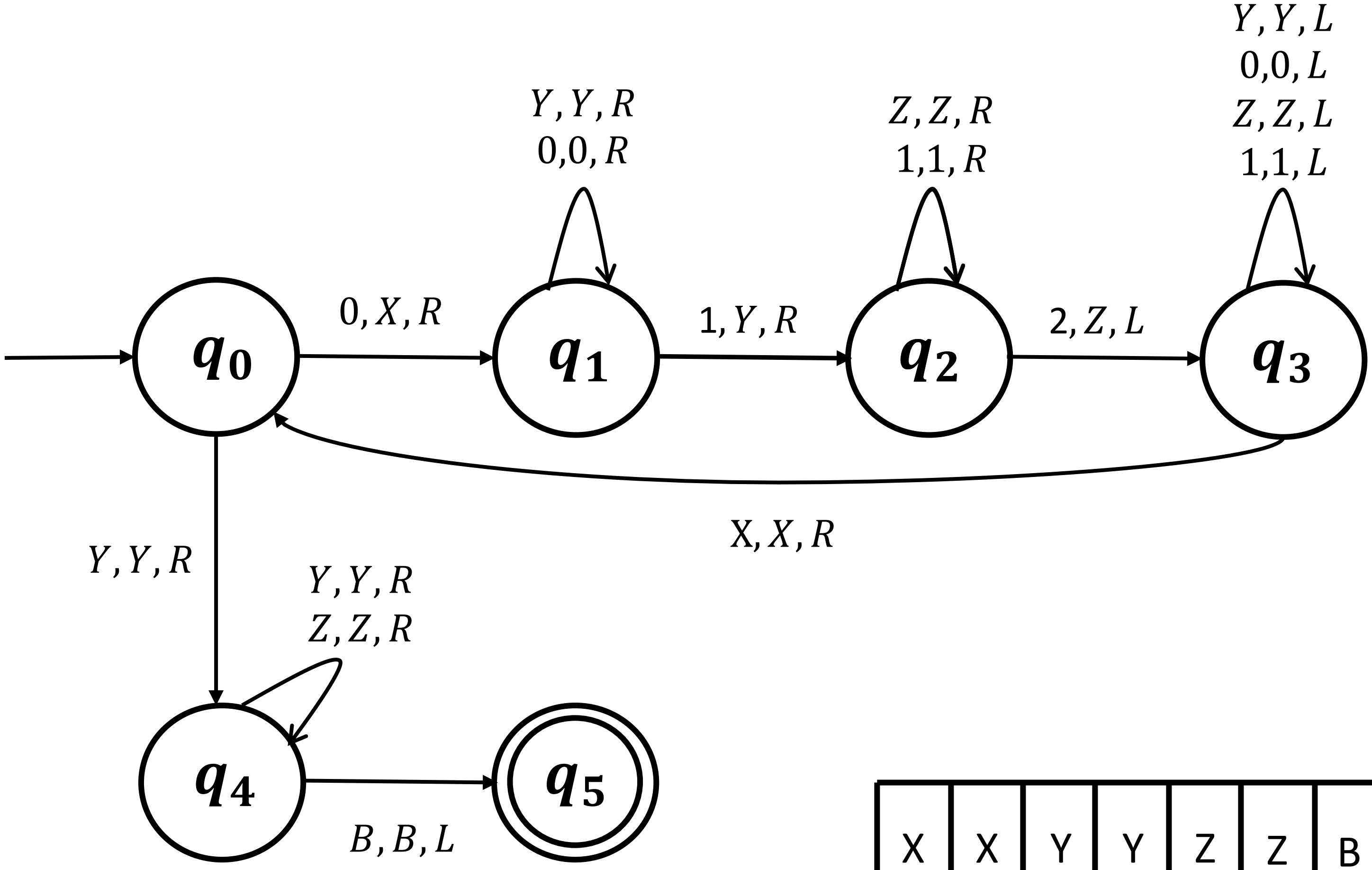

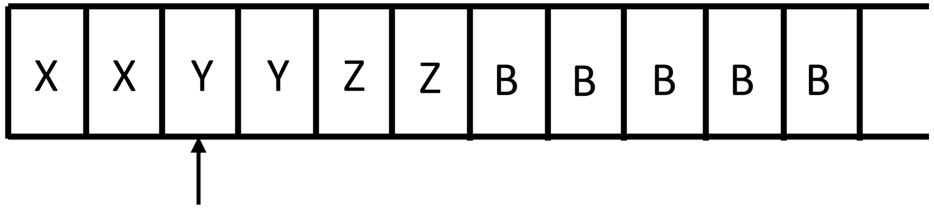

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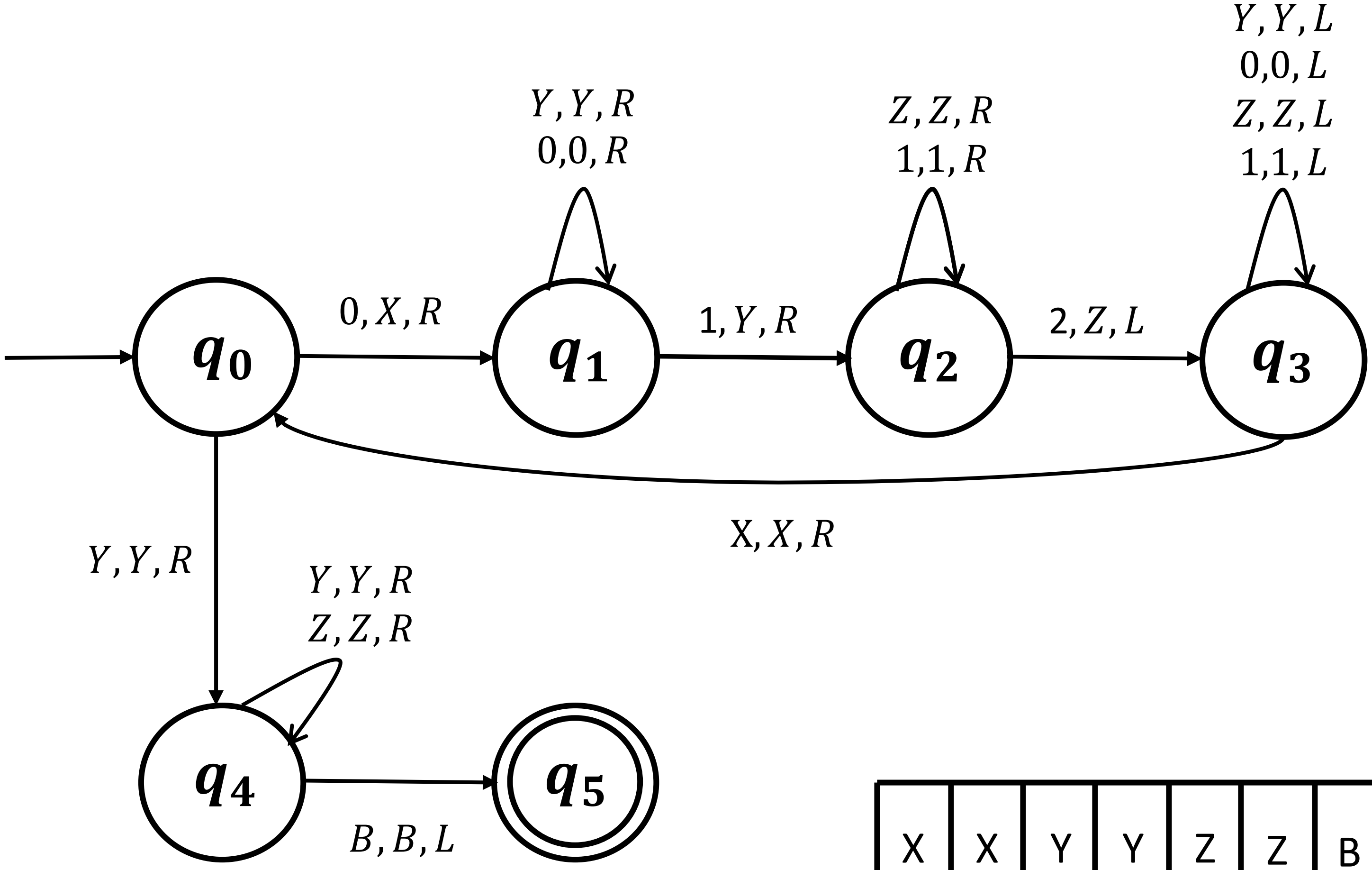

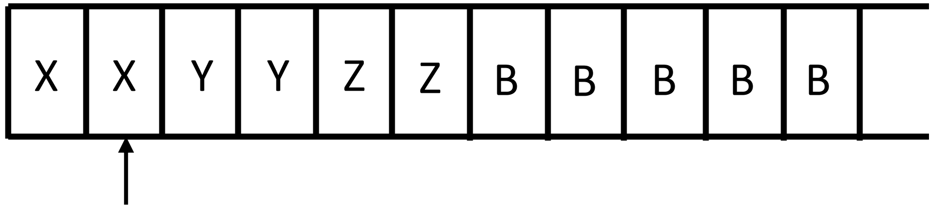

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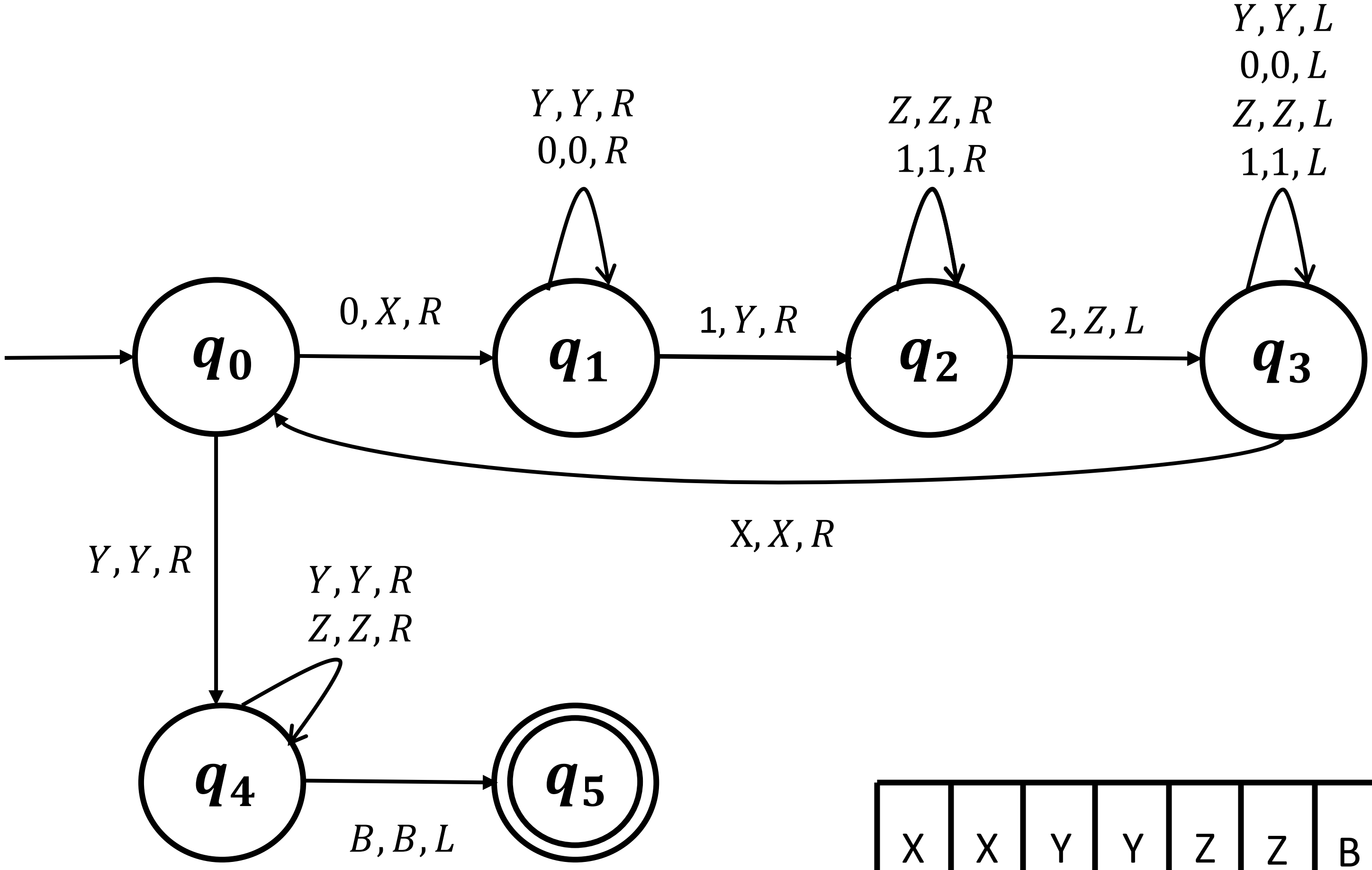

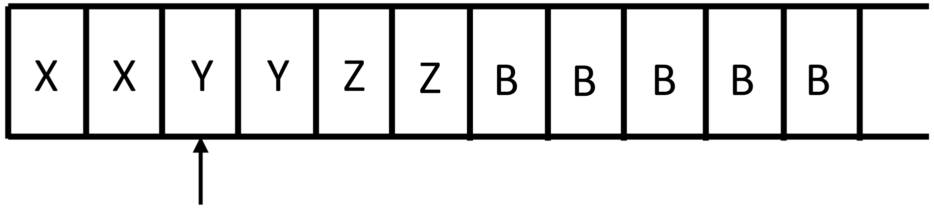

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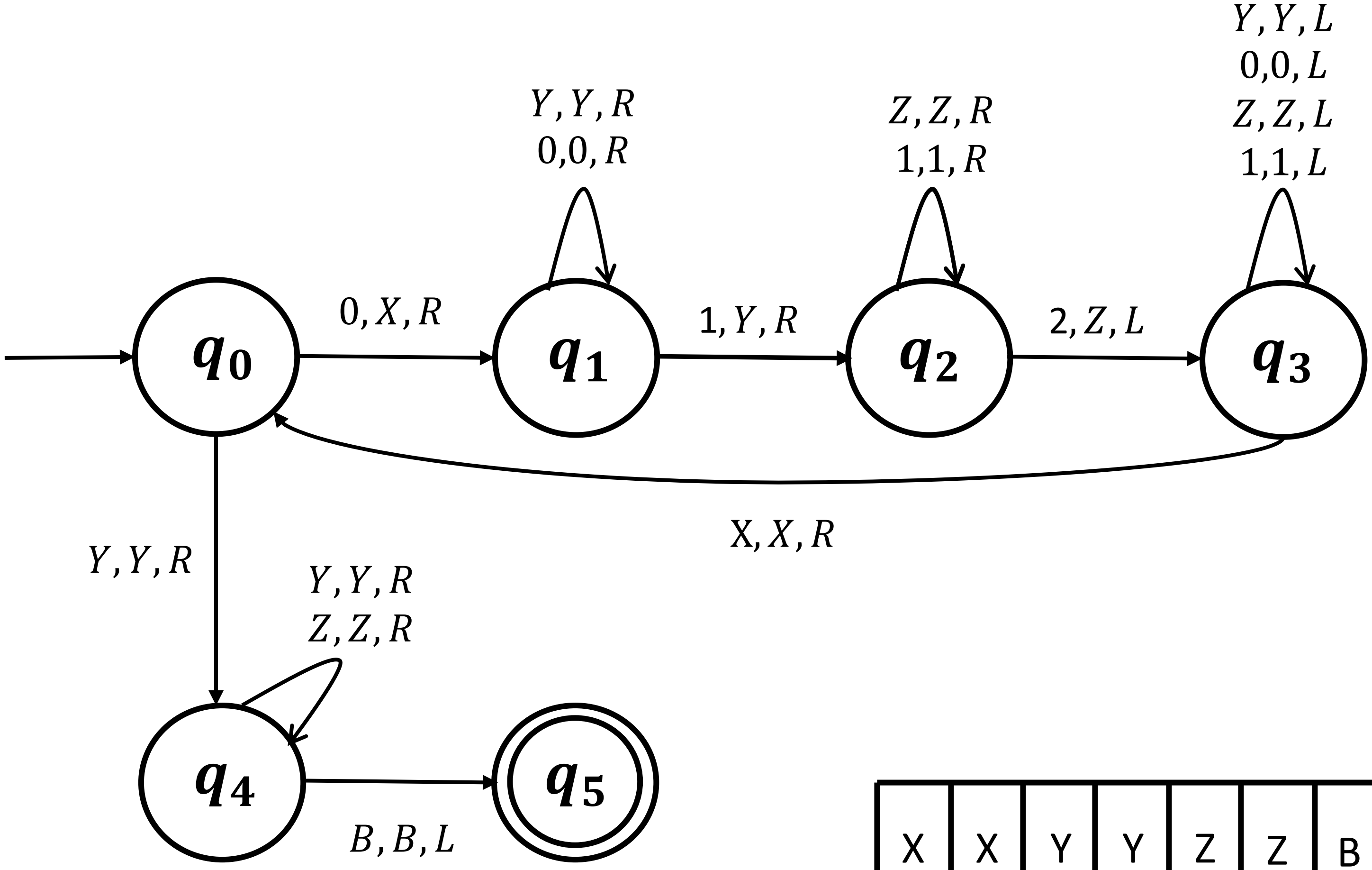

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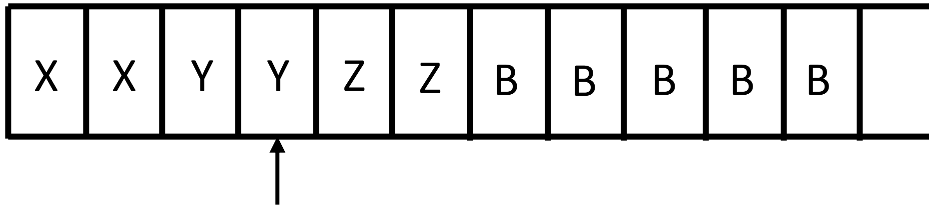

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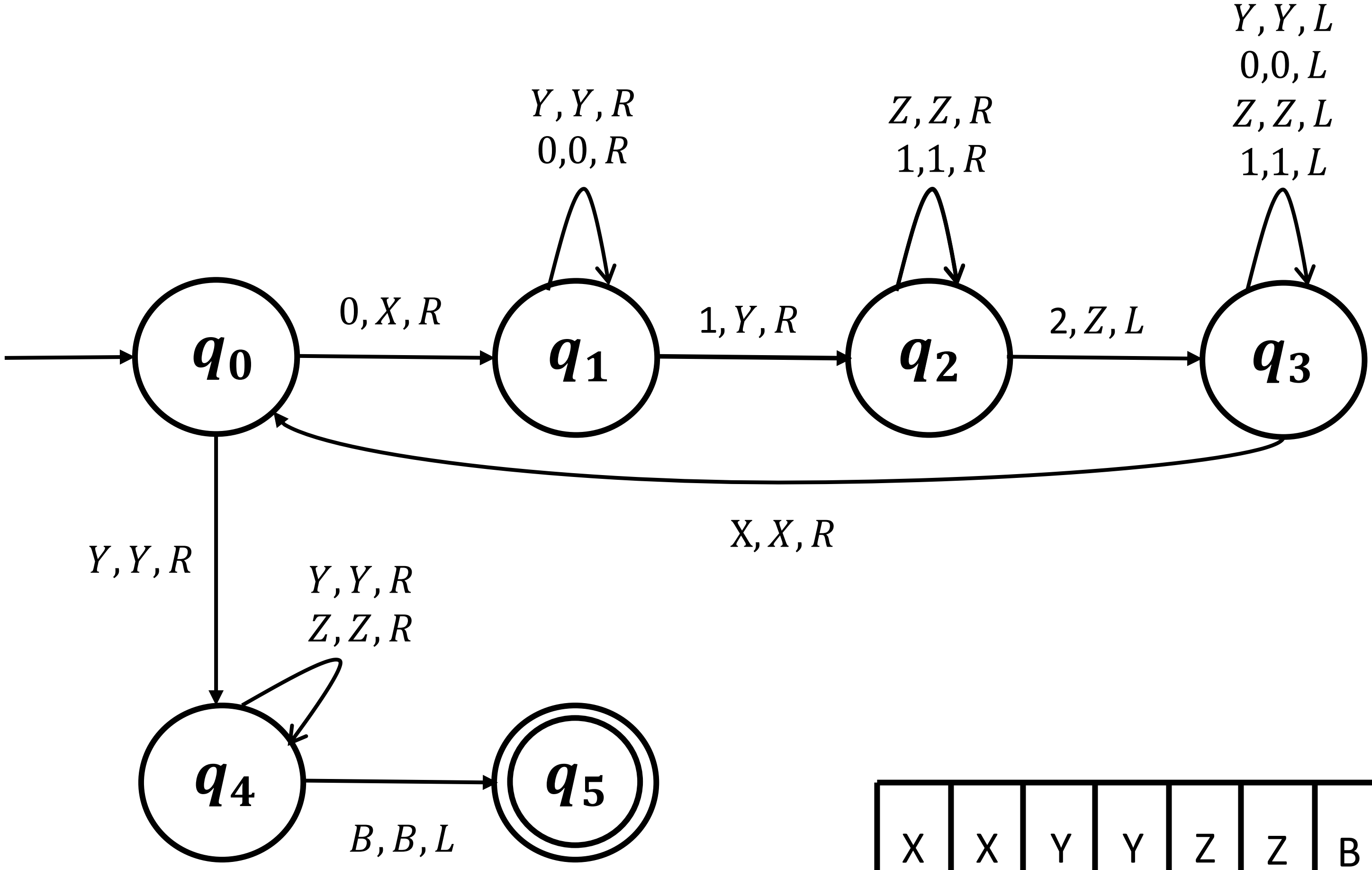

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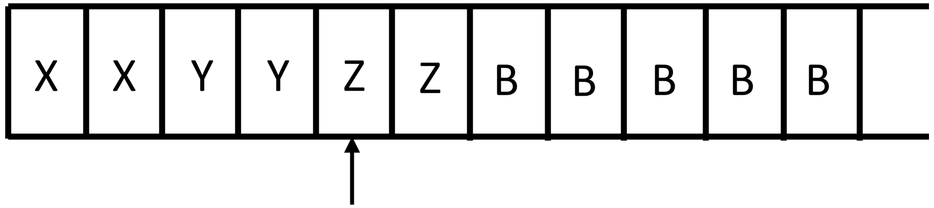

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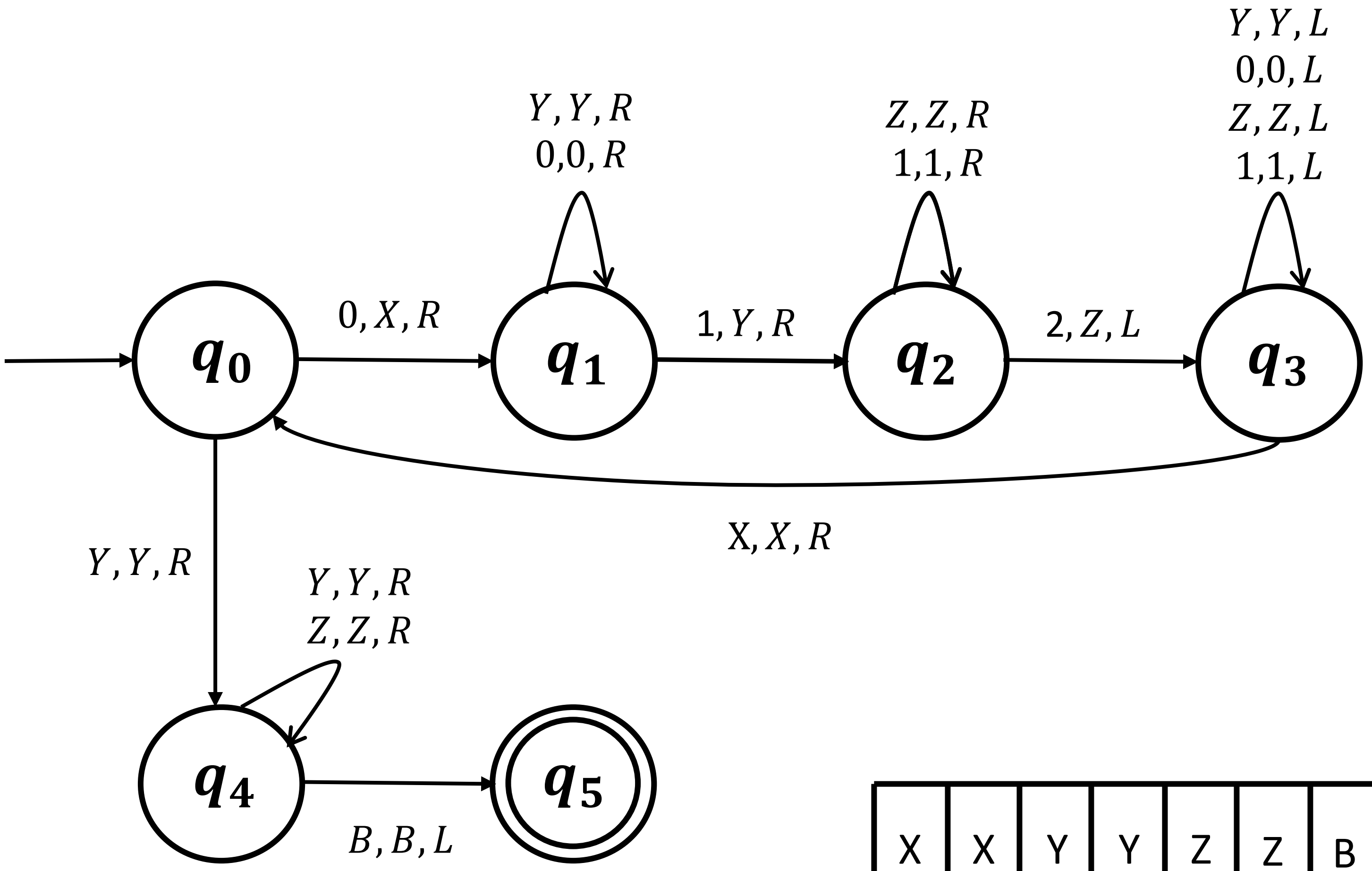

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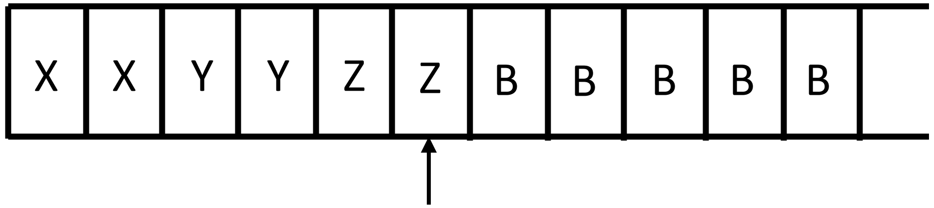

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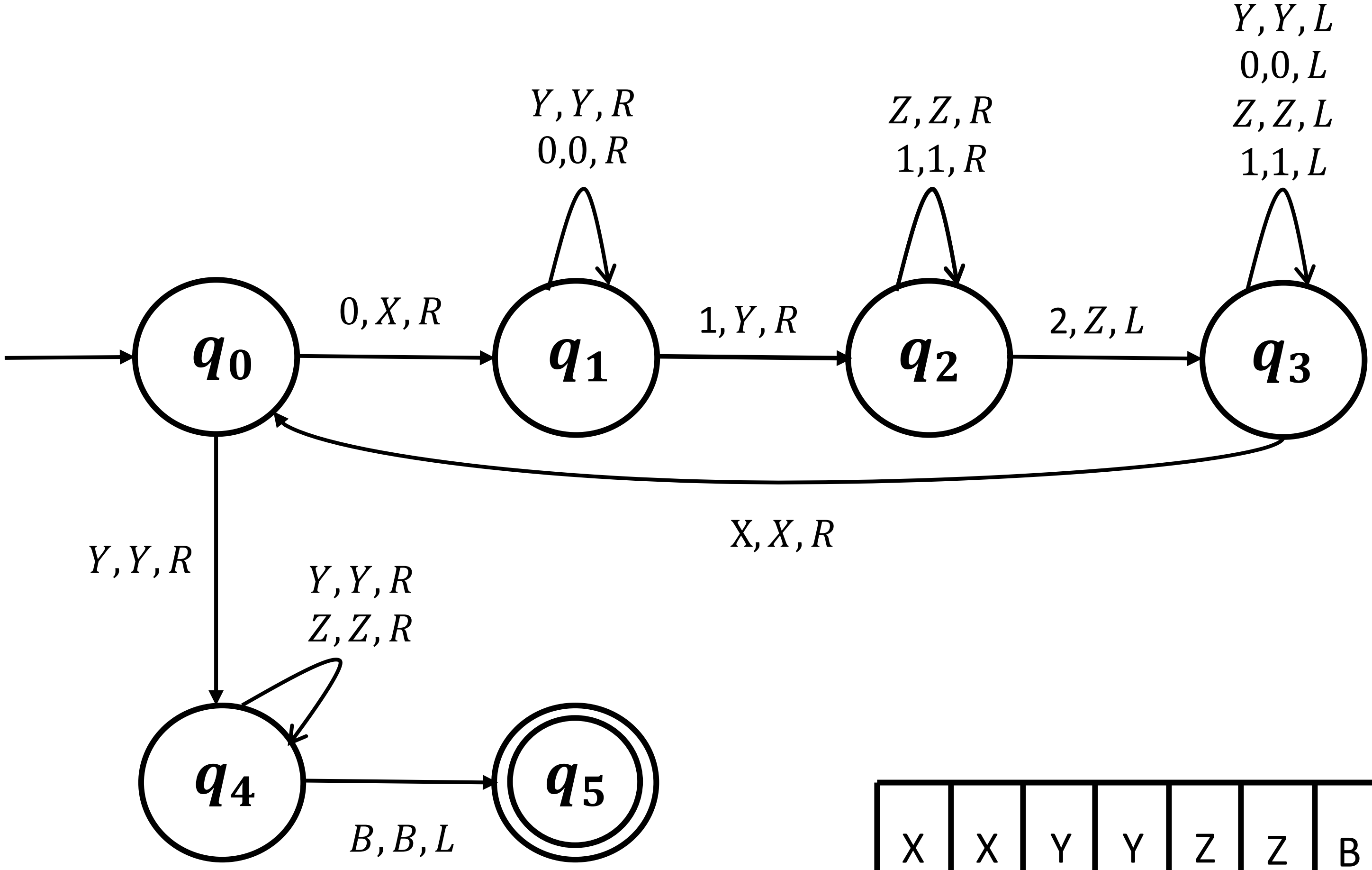

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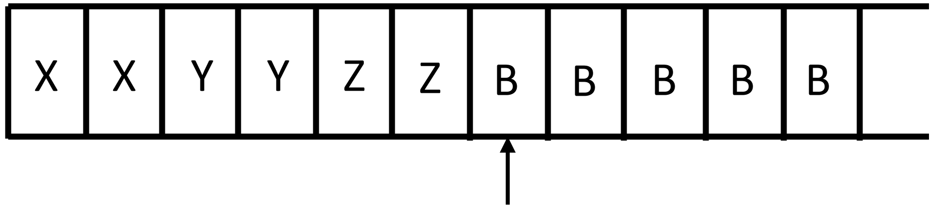

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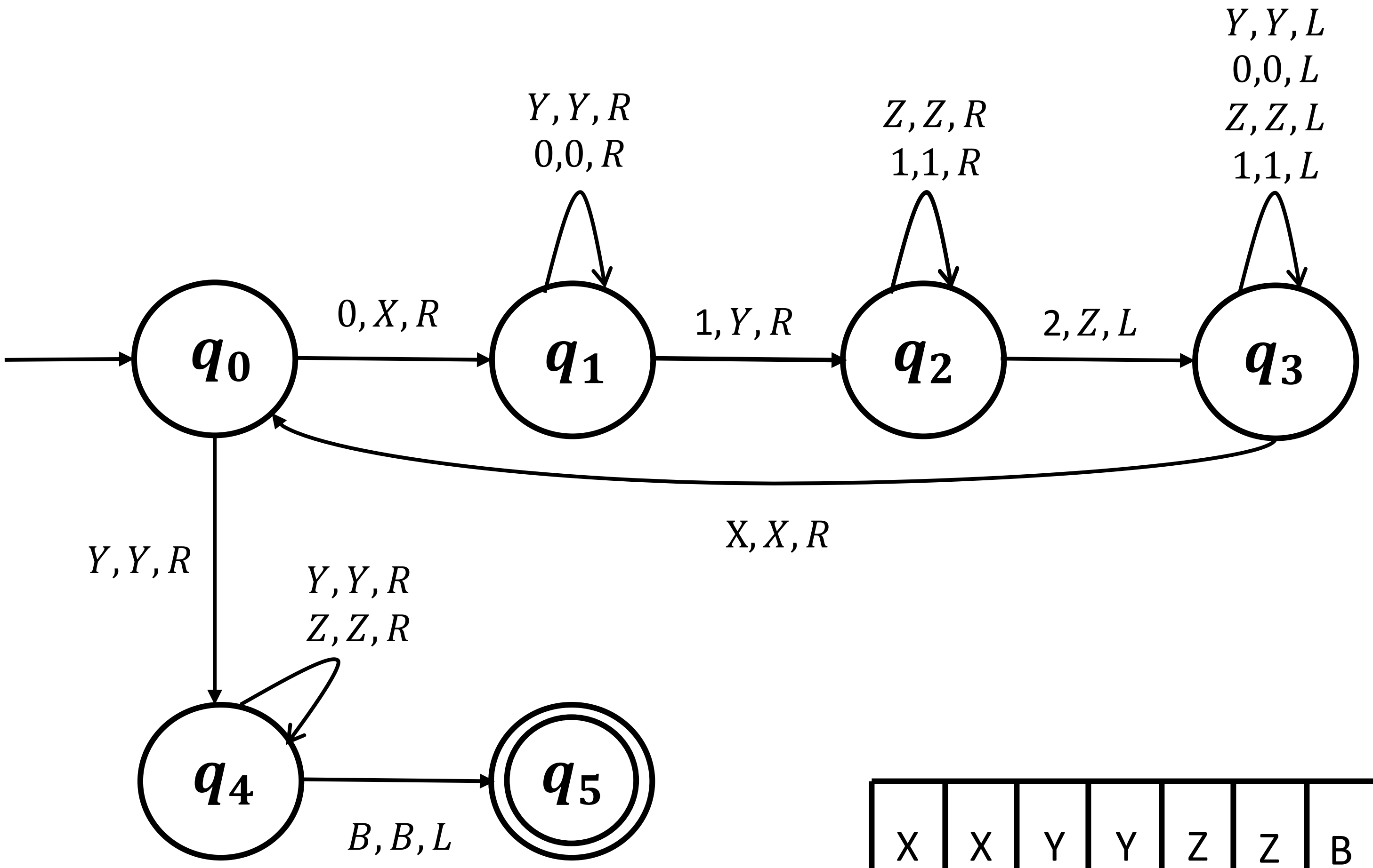

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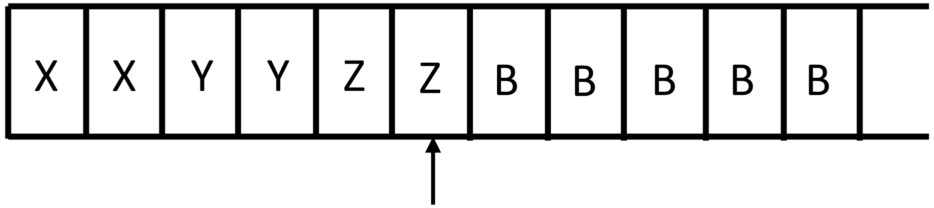

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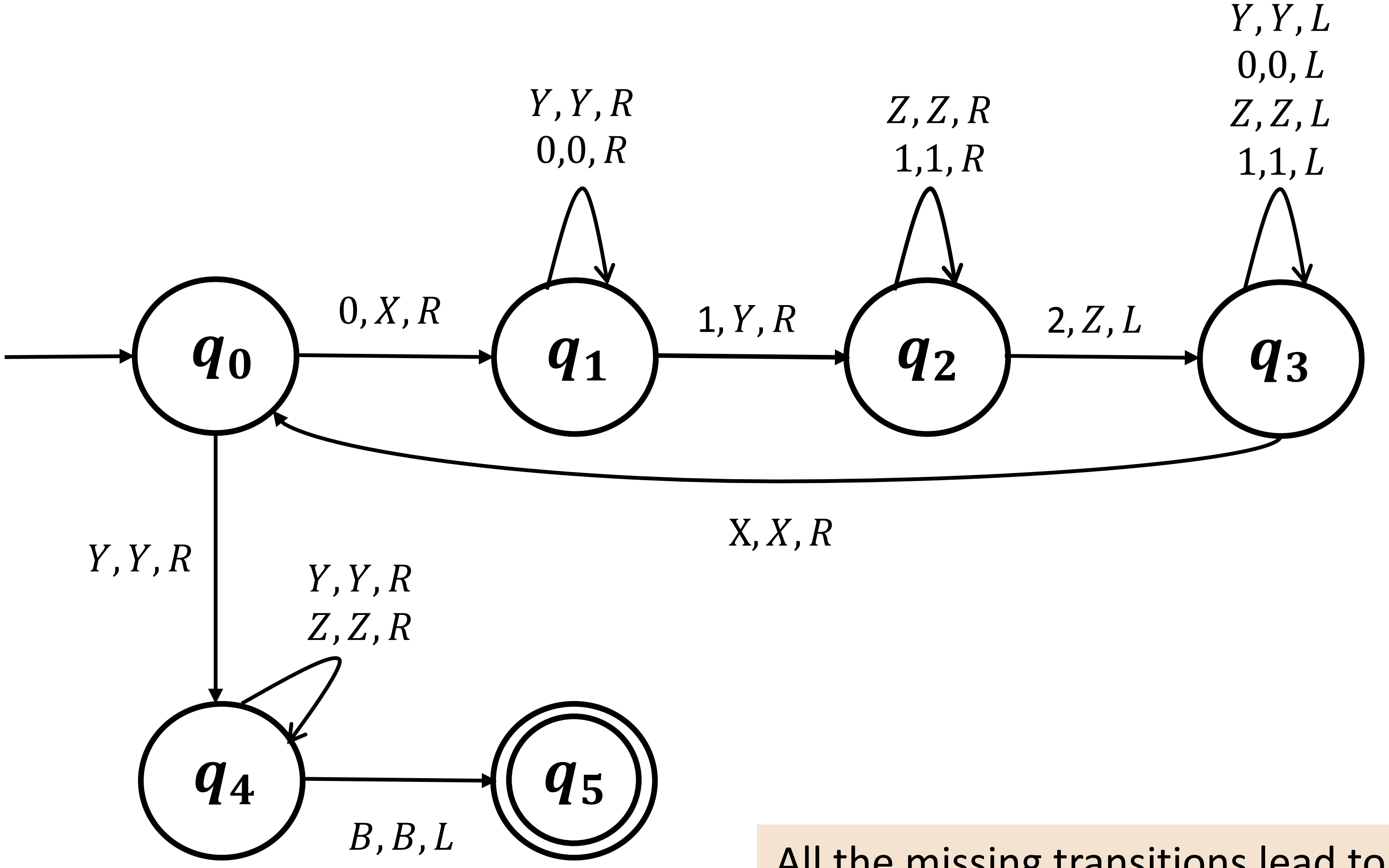

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All the missing transitions lead to a reject state and so any input not of the form $\{0^n 1^n 2^n\}$ is rejected.

Turing Machines

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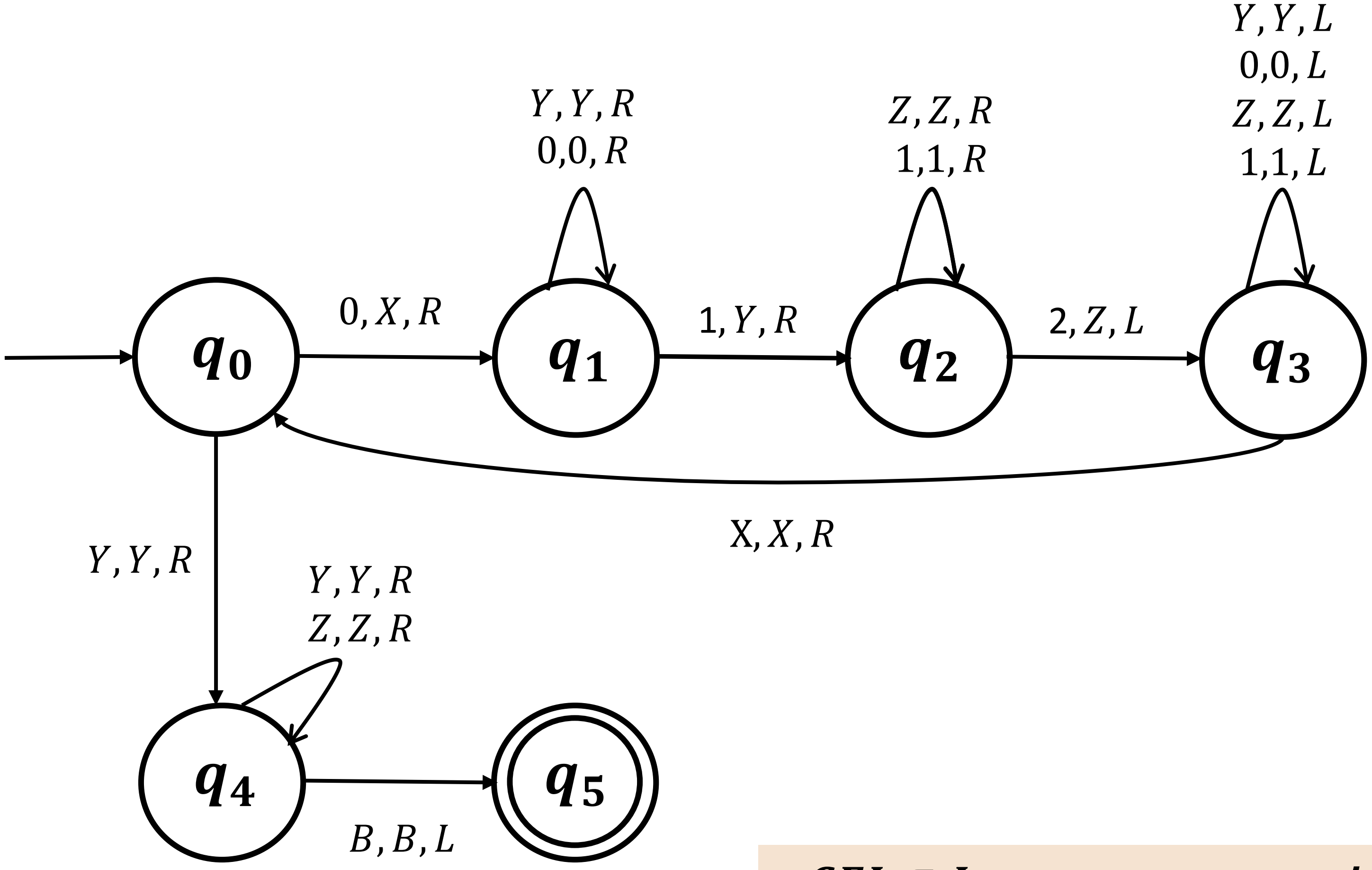

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$CFL \subseteq$ Language recognized by TM

Turing Machines

Formally, a Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ where

- Q is a finite set called the ***states.***
- Σ is the set of input ***alphabets*** not containing the blank symbol B.
- Γ is the ***tape alphabet***, where $B \subseteq \Gamma$ and $\Sigma \subseteq \Gamma$.
- $\delta: Q \times \Gamma \mapsto Q \times \Gamma \times \{L, R\}$ is the ***transition function***
- $q_0 \in Q$ is the ***start state***.
- $q_{accept} \in Q$ is the ***accepting state***.
- $q_{reject} \in Q - \{q_{accept}\}$ is the ***reject state.***

Configuration of a TM: Combination of the current state, the current tape contents and the current head location.

Formally, it is a triple: $(\boldsymbol{q}, \boldsymbol{a}, \boldsymbol{x})$, where $\boldsymbol{q} \in Q$, $\boldsymbol{a} \in \Gamma$, $\boldsymbol{x} \in Z_+$

At each step, the Turing machine configuration changes. We say C_1 **yields** C_2 if the TM changes from C_1 to C_2 in one step.

Turing Machines

Formally, a Turing Machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ where

- Q is a finite set called the ***states.***
- Σ is the set of input ***alphabets*** not containing the blank symbol B**.**
- Γ is the ***tape alphabet***, where $B \subseteq \Gamma$ and $\Sigma \subseteq \Gamma$**.**
- $\delta: Q \times \Gamma \mapsto Q \times \Gamma \times \{L, R\}$ is the ***transition function***
- $q_0 \in Q$ is the ***start state***.
- $q_{accept} \in Q$ is the ***accepting state***.
- $q_{reject} \in Q - \{q_{accept}\}$ is the ***reject state.***

A TM $\boldsymbol{M}$ **accepts** $\boldsymbol{w}$ if there exists a sequence of configurations C_1 to C_k, where

- C_1 is the start configuration M on w.
- Each C_i yields C_{i+1}.
- C_k is an accepting configuration

Language recognized a TM M:

$$\boldsymbol{L}(\boldsymbol{M}) = \{\boldsymbol{w} | \boldsymbol{M} \textbf{ accepts } \boldsymbol{w}\}$$

Configuration of a TM: Combination of the current state, the current tape contents and the current head location.

$X\ 0\ 0\ 1\ 1\ 1\ B\ B\ B\ B\ \ldots$

$\uparrow q_1$

At each step, the Turing machine configuration changes. We say C_1 **yields** C_2 if the TM changes from C_1 to C_2 in one step.

Start configuration:

$0\ 0\ 0\ 1\ 1\ 1\ B\ B\ B\ B\ \ldots$

$\uparrow q_0$

Accept configuration:

$X\ X\ X\ Y\ Y\ Y\ B\ B\ B\ B\ \ldots$

$\uparrow q_{accept}$

Reject configuration:

$X\ X\ X\ Y\ Y\ 0\ B\ B\ B\ B\ \ldots$

$\uparrow q_{reject}$

Turing Machines

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Next Lecture

Various TM model **variants**: Robustness of the standard TM model

Thank You!