

# Longest Increasing Subsequence.

$a_1 \quad a_2 \quad \dots \quad a_n$  (distinct)

$i_1 < i_2 < \dots < i_k$  s.t.  $a_{i_1} < a_{i_2} < a_{i_3} < \dots < a_{i_k}$

1 3 2 4 0 5 -2

1, 2, 4, 5

1, 3, 4, 5

LIS  $([1, n])$

$2^n$

$\swarrow$   $\searrow$  max  
 $\text{LIS}([1, n-1])$   $\hat{\text{LIS}}([1, n-1], a_n) + 1$

} LIS s.t.  
all elements are  
smaller than  $a_n$

$\hat{\text{LIS}}([1, i], x)$ :

if  $i = 0$   
return 0

$m = \hat{\text{LIS}}([1, i-1], x)$

if  $a_i < x$ :

$m \leftarrow \max \{m, 1 + \hat{\text{LIS}}([1, i-1], a_i)\}$

return  $m$

$\text{LIS}([1, n])$ :  
return  $\hat{\text{LIS}}([1, n], \infty)$

Obs:  $x$  takes only the values of the seq + 1 more.

Space:  $O(n^2)$  entries and  $\leq 2$  lookups per entry

$\hookrightarrow O(n^2)$

