Important Numerical from Waves and Oscillation (Chapter – 1)

1) A 40 kg boy jumps from a height 4m on to a platform mounted on a spring. As the spring compress the platform is pushed down a maximum distance of 0.2m below its initial position and it then rebound. What is the boy's speed at the instant when platform is dipressed 0.1m.
[Ans:7.93 m/s]

2) A solid cylinder of radius 2 cm and mass 200 g is rigidly connected with its axis vertical to the lower end of the fine wire. The period of oscillation of the cylinder under the influence of torsion wire is 2 sec. Calculate the torsional couple necessary to twist it through one complete turn. **[Ans:2.48×10^{-3} Nm]**

3) A particle executes SHM given by the equation, $y = 12\, sin\left[\frac{2\pi t}{10} + \frac{\pi}{4}\right]$. Calculate

(a) amplitude (b) frequency (c) epoch (d) displacement at t=1.25 sec (e) velocity at 2.5 sec and (f) acceleration at t=5 sec.

[Ans: (a) 12 units (b) 0.1 Hz (c) $\frac{\pi}{4}$ (d) 12 units (e) 5.552 units (f) 3.35 units]

4) A spring is hung vertically and loaded with a mass of 100g and allowed to oscillate. Calculate (i) the time period (ii) frequency of oscillation. It is noted that the spring extends by 10 cm when it is loaded with 200 g. **[Ans: (i) 0.449 sec (ii) 2.22 Hz]**

5) For a particle vibrating simple harmonically, the displacement is 8 cm at the instant when the velocity is 6 cm/s and, the displacement is 6 cm at the instant when the velocity is 8 cm/s. Calculate (i) amplitude (ii) frequency and (iii) time period.

[Ans: (i) 10 cm (ii) $\frac{1}{2\pi}$ sec^{-1} (iii) 2π sec]

6) A block of mass 1 kg is suspended from a spring of spring constant 25 N/m. If the undamped frequency is $\frac{2}{\sqrt{3}}$ times the damped frequency, what will be the damping factor?

[Ans: 5 kg s^{-1}]

7) A simple pendulum of length 40 cm and mass 5o g is suspended in a car that is travelling with a constant speed 40 m/s around a circle of radius 100 m. if the pendulum undergoes small oscillations in a radial direction about its equilibrium position, what will be its frequency of oscillation? **[Ans: 1.09 Hz]**

8) If the relaxation time of a damped harmonic oscillator is 50 s, find the time in which the amplitude falls to $\frac{1}{e^3}$ times the initial value and the energy of the system falls to $\frac{1}{e^4}$ of its initial value. **[Ans: 150 s and 100 s]**

9) In SHM, when the displacement is one-half the amplitude what fraction of total energy is KE and what fraction is PE? At what displacement is the energy half KE and half PE?

[Ans: 3E/4; E/4; $\frac{a}{\sqrt{2}}$]

10) Find whether the discharge of a capacitor through the following parameter L = 1 mH, C = 0.1 μF and R = 200 Ω is oscillatory? If the circuit is oscillatory, calculate its frequency.

[Ans: Oscillatory; 0 Hz]

11) A circuit has L = 10 mH and C = 10 μF. How much resistance should be added to circuit so that the frequency of oscillation will be 1 % less than that of free oscillation?

[Ans: 8.69 Ω]

12) A 1.5 μF capacitor is charged to 57 V. The charging battery is then disconnected and a 12 mH coil is connected across the capacitor so that LC oscillation occurs. Calculate maximum current in the coil and the energy stored in the system.

[Ans: 0.637 A; 2.43675×10^{-3} J]

13) A metal disc of radius 0.5 m oscillates in its own plane about an axis passing through a point on its edge. Find the length of equivalent simple pendulum.

14) At t = 0, the displacement x(0)of the block in linear oscillator is – 8.50 cm. The block's velocity v(0) then is – 920 m/s and its acceleration a(0) is 47 m/s^2. Find (a) angular frequency (b) phase constant. [TU 2075]

15) Prove that the time period for a physical pendulum will be minimum when the point of suspension and the point of oscillation are equidistance from the centre of gravity.

Hint : The time period of a physical pendulum is given by $T = 2\pi\sqrt{\frac{l_1+K^2/l_1}{g}}$ where l_1 is the distance of point of suspension from C.G.. Here, the time period will be minimum if the numerator, i.e. $l_1 + K^2/l_1$ is minimum. On differentiating $l_1 + \frac{K^2}{l_1}$ with respect to l_1 , we have $\frac{d}{dl_1}\left[l_1 + \frac{K^2}{l_1}\right] = 1 - \frac{K^2}{l_1^2}$

For the time period to be minimum, the differentiated quantity should be equal to zero,

i.e. $1 - \frac{K^2}{l_1^2} = 0$

i.e. $\frac{K^2}{l_1^2} = 1$

But, $K^2 = l_1 l_2$, where l_2 is the distance of the point of oscillation from C.G.

So, $\frac{l_1 l_2}{l_1^2} = 1$

i.e. $l_1 = l_2$