Reduction Formula

$$I_n = \int \cos^n x \; dx$$

$$I_n = \int \cos x . \cos^{n-1} x \; dx$$

$$I_n = \cos^{n-1}x.(\sin x) - (n-1)\int \cos^{n-2}x.(-\sin x).\sin x \; dx$$

$$I_n = \cos^{n-1}x.(\sin x) + (n-1)\int \cos^{n-2}x.\sin^2 x \; dx$$

$$I_n = \cos^{n-1}x.(\sin x) + (n-1)\int \cos^{n-2}x.(1-\cos^2 x) \; dx$$

$$I_n = \cos^{n-1}x.(\sin x) + (n-1)(I_{n-2} - I_n)$$

reduction formula

$$\boxed{I_n = \frac{\cos^{n-1}x.(\sin x)}{n} + \frac{(n-1)}{n} I_{n-2}}$$

$$J_n = \int_0^{\pi/2} \cos^n x \; dx$$

reduction formula

$$\boxed{J_n = \frac{(n-1)}{n} J_{n-2}}$$

Case I: When n is even.

$$\boxed{J_n = \frac{(n-1)(n-3)(n-5)\ldots\ldots\ldots 1}{n(n-2)(n-4)\ldots\ldots\ldots\ldots 2} J_0 = \frac{(n-1)(n-3)(n-5)\ldots\ldots\ldots 1}{n(n-2)(n-4)\ldots\ldots\ldots\ldots 2}\pi/2}$$

Case II: When n is odd.

$$\boxed{J_n = \frac{(n-1)(n-3)(n-5)\ldots\ldots\ldots 2}{n(n-2)(n-4)\ldots\ldots\ldots\ldots 3} \overset{1}{J_1} = \frac{(n-1)(n-3)(n-5)\ldots\ldots\ldots 2}{n(n-2)(n-4)\ldots\ldots\ldots\ldots 3}}$$

$$I_n = \int_0^{\pi/4} \tan^n x \; dx$$

$$I_n = \int_0^{\pi/4} \tan^2 x . \tan^{n-2} x \; dx$$

$(\sec^2 x - 1)$

$$I_n = \int_0^{\pi/4} \sec^2 x . \tan^{n-2} x \; dx - \int_0^{\pi/4} \tan^{n-2} x \; dx$$

Now, $d(\tan^{n-1} x) = (n-1) \tan^{n-2} x \sec^2 x \, dx$

$$I_n = \frac{1}{n-1} \int_0^{\pi/4} d(\tan^{n-1} x) - I_{n-2}$$

$$\boxed{I_n = \frac{1}{n-1} - I_{n-2}}$$

Reduction Formula