Métodos Numéricos Computacionais - P3

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1)

	k	0	1	2	3
i	x_i	$\Delta^0 y_i$	$\Delta^1 y_i$	$\Delta^2 y_i$	$\Delta^3 y_i$
0	1,2	0,932			
			1		
1	1,3	0,964		4	
			2		6
2	1,4	0,985		5	
			3		
3	1,5	0,997			

1 $\dfrac{0,964 - 0,932}{1,3 - 1,2} = 0,32$

2 $\dfrac{0,985 - 0,964}{1,4 - 1,3} = 0,21$

3 $\dfrac{0,997 - 0,985}{1,5 - 1,4} = 0,12$

4 $\dfrac{0,21 - 0,32}{1,4 - 1,2} = -0,55$

5 $\dfrac{0,12 - 0,21}{1,5 - 1,3} = -0,45$

6 $\dfrac{-0,45 + 0,55}{1,5 - 1,2} \approx 0,333$

$\therefore\ p_3(x) = y_0 + (x - x_0)\left(\Delta^1 y_0 + (x - x_1).\left(\Delta^2 y_0 + (x - x_2).\right.\right.$
$\left.\left.(\Delta^3 y_0)\right)\right)$

$\Rightarrow p_3(x) = 0{,}932 + (x - 1{,}2).(0{,}32 + (x - 1{,}3).(-0{,}55 + (x - 1{,}4).(0{,}333)))$ ②

$p_3(1{,}38) = 0{,}932 + (1{,}38 - 1{,}2).(0{,}32 + (1{,}38 - 1{,}3).(-0{,}55 + (1{,}38 - 1{,}4).(0{,}333)))$

$= 0{,}932 + 0{,}18(0{,}32 + (0{,}08).(-0{,}55 + (-0{,}02).(0{,}333)))$

$\approx 0{,}98158$

②

i	K: x_i	0: $\Delta^0 y_i$	1: $\Delta^1 y_i$	2: $\Delta^2 y_i$	3: $\Delta^3 y_i$
0	1,2	0,932	①		
				④	
1	1,3	0,964	②		⑥
				⑤	
2	1,4	0,985	③		
3	1,5	0,997			

① $0{,}964 - 0{,}932 = 0{,}032$

② $0{,}985 - 0{,}964 = 0{,}021$

③ $0{,}997 - 0{,}985 = 0{,}012$ $\qquad h = (1{,}3 - 1{,}2)$

④ $0{,}021 - 0{,}032 = -0{,}011$ $\qquad = 0{,}1$

⑤ $0{,}012 - 0{,}021 = -0{,}009$

⑥ $-0{,}009 + 0{,}011 = 0{,}002$

$\therefore p_3(x) = y_0 + (x - x_0).\left(\frac{\Delta^1 y_0}{1!\,h} + (x - x_1).\left(\frac{\Delta^2 y_0}{2!\,h^2} + (x - x_2).\left(\frac{\Delta^3 y_0}{3!\,h^3}\right)\right)\right)$

$$\Rightarrow p_3(x) = 0{,}932 + (x-1{,}2).\left(\frac{0{,}032}{0{,}1} + (x-1{,}3).\left(\frac{-0{,}011}{2.0{,}01} + (x-1{,}4).\left(\frac{0{,}002}{6.0{,}001}\right)\right)\right)$$ ③

$$= 0{,}932 + (x-1{,}2).(0{,}32 + (x-1{,}3).(-0{,}55 + (x-1{,}4).(0{,}333)))$$

Logo: $p_3(x) = 0{,}932 + (x-1{,}2).(0{,}32 + (x-1{,}3).(-0{,}55 + (x-1{,}4).(0{,}333)))$ //

$$p_3(1{,}38) = 0{,}932 + (1{,}38-1{,}2).(0{,}32 + (1{,}38-1{,}3).(-0{,}55 + (1{,}38-1{,}4).(0{,}333)))$$

$$\approx 0{,}98158$$ //

③ Os resultados obtidos são idênticos, pois Newton-Gregory gera o mesmo polinômio que Newton, com a necessidade que os pontos sejam igualmente espaçados.

4

							Σ
x	1	2	3	4	5	6	21
x_2	1	4	9	16	25	36	91
y_i	1	4	9	16	25	36	91
$x_i y_i$	1	8	27	64	125	216	441

$$f(x) = \underbrace{a}_{B}x + \underbrace{b}_{A}$$

$$\hat{y} = A + Bx$$

$x \cdot 3{,}5$

$$\begin{pmatrix} 6 & 21 \\ 21 & 91 \end{pmatrix} \cdot \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 91 \\ 441 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 21 \\ 0 & 17{,}5 \end{pmatrix} \begin{pmatrix} 91 \\ 122{,}5 \end{pmatrix} \quad B = 7 \Rightarrow a = 7$$

$$A = -9{,}333 \Rightarrow b = -9{,}333$$

$$\therefore \quad \underline{f(x) = 7x - 9{,}333}$$

5 $f(x) = ax^b \Rightarrow \ln(y) = \ln(ax^b)$

$\Rightarrow \ln(y) = \ln(a) + \ln(x^b)$

$\Rightarrow \underbrace{\ln(y)}_{Y} = \underbrace{\ln(a)}_{A} + b \cdot \underbrace{\ln(x)}_{X}$

$A = \ln(a) \Rightarrow a = e^A$

$Y = \ln(y)$ e $X = \ln(x)$

							Σ
X	0	0,693	1,099	1,386	1,609	1,792	6,579
X^2	0	0,480	1,208	1,921	2,589	3,211	9,409
Y	0	1,386	2,197	2,773	3,219	3,584	13,159
XY	0	0,960	2,415	3,843	5,179	6,422	18,819

$$\overset{1,0965}{\curvearrowleft}\begin{pmatrix} 6 & 6,579 \\ 6,579 & 9,409 \end{pmatrix} \cdot \begin{pmatrix} A \\ b \end{pmatrix} = \begin{pmatrix} 13,159 \\ 18,829 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 6,579 \\ 0 & 2,195 \end{pmatrix} \begin{pmatrix} 13,159 \\ 4,390 \end{pmatrix} \quad \begin{array}{l} b = 2 \\ A = 0,00016 \end{array}$$

$$a = e^{A} \Rightarrow a = e^{0,00016} \Rightarrow \underline{a = 1,00016}$$

$$\therefore \underline{f(x) = 1,00016 \cdot x^2}$$

⑥ $f(x) = a \cdot e^{bx} \Rightarrow \ln(y) = \ln(a) + \ln(e^{bx})$

$$\Rightarrow \ln(y) = \ln(a) + bx \cdot \cancel{\ln(e)}^{1}$$

$$\Rightarrow \underbrace{\ln(y)}_{Y} = \underbrace{\ln(a)}_{A} + bx \qquad \begin{array}{l} A = \ln(a) \\ a = e^{A} \end{array}$$

							Σ
x	1	2	3	4	5	6	21
x^2	1	4	9	16	25	36	91
Y	0	1,386	2,197	2,773	3,219	3,584	13,159
xY	0	2,772	6,591	11,092	16,095	21,504	58,054

$$\curvearrowleft\begin{pmatrix} 6 & 21 \\ 21 & 91 \end{pmatrix} \cdot \begin{pmatrix} A \\ b \end{pmatrix} = \begin{pmatrix} 13,159 \\ 58,054 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 21 \\ 0 & 17,5 \end{pmatrix} \begin{pmatrix} 13,159 \\ 11,9975 \end{pmatrix} \quad \begin{array}{ll} A = -0,208 & a = 0,812 \\ b = 0,686 & \end{array}$$

$\therefore\ f(x) = 0{,}812 \cdot e^{0{,}686x}$

(7)

							Σ
y	1	4	9	16	25	36	91
y^2	1	16	81	256	625	1296	2275

(4)

e	3,333	-0,667	-2,667	-2,667	-0,667	3,333
e^2	11,109	0,445	7,113	7,113	0,445	11,109

$\Sigma = 37{,}334$

$$R^2 = 1 - \left(\frac{6 \cdot 37{,}334}{6 \cdot 2275 - 91^2}\right) = 0{,}958$$

(5)

e -0,00016 -0,00064 -0,00144 -0,00256 -0,004 -0,00576

e^2 0,0000000256 0,0000004096 0,0000020736 0,0000065536 0,000016 0,0000332

$\Sigma e^2 = 0{,}0000573$

$$R^2 = 1 - \left(\frac{6 \cdot 0{,}0000573}{6 \cdot (2275) - 91)^2}\right) = 0{,}9999\ldots$$

(6)

e	$-0,611$	$0,798$	$2,642$	$3,374$	$-0,072$	$-13,787$
e^2	$0,375$	$0,637$	$6,980$	$11,384$	$0,005$	$190,081$

$\sum e^2 = 209,462$

$$R^2 = 1 - \left(\frac{6 \cdot 209,462}{6 \cdot 2275 - 91^2}\right) = \underline{0,766} //$$

(8) O coeficiente de determinação R_2 do ajuste $f(x) = 1,00016 \cdot x^2$ é igual a 0,999... o que indica que o ajuste passa por quase todos os pontos e a função original é muito próxima desse ajuste (função quadrática).