Métodos Numéricos Computacionais - P2

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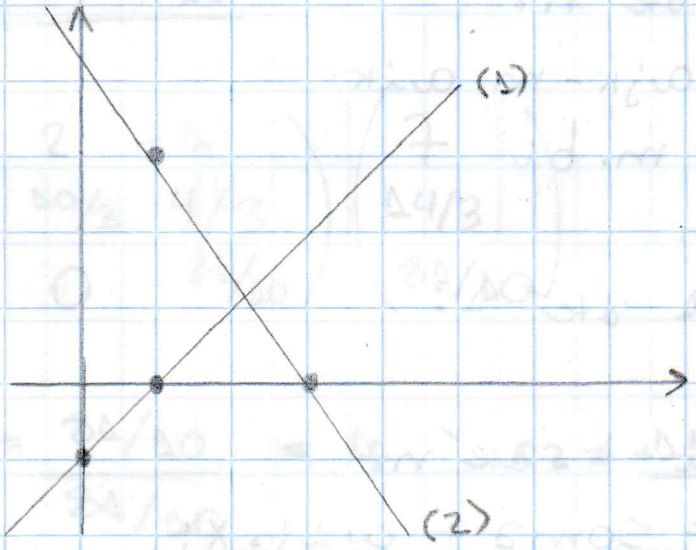
① $\begin{cases} 1x_1 - 1x_2 = 1 & (1) \\ 1x_1 + 1x_2 = 3 & (2) \end{cases}$

1) Desenho:

(1) $x_1 = 0$ e $x_2 = -1$ (2) $x_1 = 0$ e $x_2 = 3$

$x_1 = 1$ e $x_2 = 0$ $x_1 = 3$ e $x_2 = 0$



2) Resolver:

$\begin{cases} x_1 - x_2 = 1 \\ x_1 + x_2 = 3 \end{cases} + \Rightarrow \begin{cases} x_1 - x_2 = 1 \\ 2x_1 = 4 \end{cases} \begin{matrix} \Rightarrow x_2 = 1 \\ \Rightarrow x_1 = 2 \end{matrix}$

Logo: $x_1 = 2$ e $x_2 = 1$

3) Interpretação:

Logo, o vetor $\bar{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ satisfaz as duas equações, e é o ponto em que as retas se interceptam.

② Dado A, n, x, b:

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para i = 1 até n-1:
  para j = i+1 até n:
    m = aji/aii
    para k = j até n:
      ajk = ajk - m.aik
    bj = bj - m.bi
xn = bn/ann
para i = n-1 até 1:
  soma = 0
  para j = i+1 até n:
    soma = soma + aij.xj
  xi = (bi - soma)/aii
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③ $\begin{cases} 6x_1 + 2x_2 - 1x_3 = 7 \\ 2x_1 + 4x_2 + 1x_3 = 7 \\ 3x_1 + 2x_2 + 8x_3 = 13 \end{cases}$

$$\begin{pmatrix} 6 & 2 & -1 \\ 2 & 4 & 1 \\ 3 & 2 & 8 \end{pmatrix} \begin{pmatrix} 7 \\ 7 \\ 13 \end{pmatrix}$$ ③

$m_{21} = 2/6 = 1/3$ e $m_{31} = 3/6 = 1/2$

$$\begin{pmatrix} 6 & 2 & -1 \\ 0 & 10/3 & 4/3 \\ 0 & 1 & 17/2 \end{pmatrix} \begin{pmatrix} 7 \\ 14/3 \\ 19/2 \end{pmatrix}$$

$m_{32} = 3/10$

$$\begin{pmatrix} 6 & 2 & -1 \\ 0 & 10/3 & 4/3 \\ 0 & 0 & 81/10 \end{pmatrix} \begin{pmatrix} 7 \\ 14/3 \\ 81/10 \end{pmatrix}$$

$x_3 = \dfrac{81/10}{81/10} = 1$ $\quad x_2 = \dfrac{14/3 - 4/3 \cdot 1}{10/3} = 1$

$x_1 = \dfrac{7 - (-1).1 - 2.1}{6} = 1$

Logo: $x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

4)
$$\begin{cases} 2x_1 + 2x_2 - 1x_3 = 3 \\ 3x_1 + 3x_2 + 1x_3 = 7 \\ 1x_1 - 1x_2 + 5x_3 = 5 \end{cases}$$

$$\begin{pmatrix} 2 & 2 & -1 \\ 3 & 3 & 1 \\ 1 & -1 & 5 \end{pmatrix} \begin{pmatrix} 3 \\ 7 \\ 5 \end{pmatrix}$$

$m_{21} = 3/2$ e $m_{31} = 1/2$:

$$\begin{pmatrix} 2 & 2 & -1 \\ 0 & 0 & 5/2 \\ 0 & -2 & 11/2 \end{pmatrix} \begin{pmatrix} 3 \\ 5/2 \\ 7/2 \end{pmatrix}$$

$m_{32} = -2/0$ como o pivô é 0, não é possível achar a solução por esse método.

5)
$$\begin{pmatrix} 2 & 2 & -1 \\ 3 & 3 & 1 \\ 1 & -1 & 5 \end{pmatrix} \begin{pmatrix} 3 \\ 7 \\ 5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3 & 3 & 1 \\ 2 & 2 & -1 \\ 1 & -1 & 5 \end{pmatrix} \begin{pmatrix} 7 \\ 3 \\ 5 \end{pmatrix}$$

$m_{21} = 2/3$ e $m_{31} = 1/3$

$$\begin{pmatrix} 3 & 3 & 1 \\ 0 & 0 & -5/3 \\ 0 & -2 & 14/3 \end{pmatrix} \begin{pmatrix} 7 \\ -5/3 \\ 8/3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3 & 3 & 1 \\ 0 & -2 & 14/3 \\ 0 & 0 & -5/3 \end{pmatrix} \begin{pmatrix} 7 \\ 8/3 \\ -5/3 \end{pmatrix}$$

$$x_3 = \frac{-5/3}{-5/3} = 1 \qquad x_2 = \frac{8/3 - 14/3}{-2} = 1$$

$$x_1 = \frac{7 - 1 - 3}{3} = 1$$

Logo: $x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

⑥ $$\begin{cases} 5x_1 + 2x_2 + 1x_3 = 0 \\ 3x_1 + 1x_2 + 4x_3 = -7 \\ 1x_1 + 1x_2 + 3x_3 = -5 \end{cases}$$

$$\begin{pmatrix} 5 & 1 & 1 \\ 3 & 4 & 1 \\ 1 & 3 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \cdot \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

$$5 = 1 . u_{11} + 0.0 + 0.0 \Rightarrow u_{11} = 5$$

$$1 = 1 . u_{12} + 0 . u_{22} + 0.0 \Rightarrow u_{12} = 1$$

$$1 = 1 . u_{13} + 0 . u_{23} + 0 . 33 \Rightarrow u_{13} = 1$$

$$3 = l_{21} . 5 + 1.0 + 0.0 \Rightarrow l_{21} = 3/5$$

$$1 = l_{31} . 5 + l_{32} . 0 + 1.0 \Rightarrow l_{31} = 1/5$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 3/5 & 1 & 0 \\ 1/5 & l_{31} & 1 \end{pmatrix} \cdot \begin{pmatrix} 5 & 1 & 1 \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

$$4 = 3/5 . 1 + 1 . u_{22} + \cancel{0 . u_{23}} \Rightarrow u_{22} = 17/5$$

$$1 = 3/5 . 1 + 1 . u_{23} + \cancel{0 . u_{33}} \Rightarrow u_{23} = 2/5$$

$$3 = 1/5 . 1 + l_{31} . 17/5 + \cancel{1 . 0} \Rightarrow l_{31} = 14/17$$

$$6 = 1/5 . 1 + 14/17 . 2/5 + u_{33} \Rightarrow u_{33} = 93/17$$

$$\Rightarrow \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 3/5 & 1 & 0 \\ 1/5 & 14/17 & 1 \end{pmatrix}}_{L} \cdot \underbrace{\begin{pmatrix} 5 & 1 & 1 \\ 0 & 17/5 & 2/5 \\ 0 & 0 & 93/17 \end{pmatrix}}_{U}$$

7

$$\begin{pmatrix} 1 & 0 & 0 \\ 3/5 & 1 & 0 \\ 1/5 & 14/17 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -7 \\ -5 \end{pmatrix}$$

$y_1 = 0$
$y_2 = -7$
$y_3 = -5 - 98/17 = -183/17$

$$\begin{pmatrix} 5 & 1 & 1 \\ 0 & 17/5 & 2/5 \\ 0 & 0 & 93/17 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -7 \\ -183/17 \end{pmatrix}$$

$x_3 = -183/97$
$x_2 = -2/5 \cdot (-183/97) - 7 = -3029/1649$
$x_1 = (0 + 183/97 + 3029/1649)/5 =$
$= 1228/1649$

Logo: $x = \begin{pmatrix} 1228/1649 \\ -3029/1649 \\ -183/97 \end{pmatrix}$ //

7 $\begin{cases} 5x_1 + x_2 + 1x_3 = 5 \\ 3x_1 + 4x_2 + x_3 = 6 \\ 3x_1 + 3x_2 + 6x_3 = 0 \end{cases}$

$x_1 = \dfrac{5 - x_2 - x_3}{5}$ $\quad x_2 = \dfrac{6 - 3x_1 - x_3}{4}$

$x_3 = \dfrac{-3x_1 - 3x_2}{6}$

x^0	x^1	x^2	x^3
0	1	1,1125	1,01187
0	0,75	0,99375	1,0043
0	-1,3125	-1,0531	-1,0080

1,8125

3ª iteração

0,40872

0,10124

Logo, $x \approx \begin{pmatrix} 1,01187 \\ 1,0043 \\ -1,008 \end{pmatrix}$

8 $A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \begin{matrix} m = 2 \\ m = 1 \end{matrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1/2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad m = 1/2$$

$$\begin{array}{ccc} C_1 & C_2 & C_3 \\ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} & \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} & \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \end{array}$$

$$\therefore \underline{A^{-1}} = \underline{\begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ 0 & -1 & 2 \end{pmatrix}}$$