ASSIGNMENT 4

D.Ravalika

Latex-tikz codes from

https://github.com/Ravalika1630/Assignment−4/blob/main/Assignment%204.tex

1 Question No 2.41

Find the equation of the plane through the intersection of the planes

$$\begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{x} = 1 \tag{1.0.1}$$

$$\begin{pmatrix}2 & 3 & 4\end{pmatrix}\mathbf{x} = 5 \tag{1.0.2}$$

and which is perpendicular to the plane

$$\begin{pmatrix}1 & -1 & 1\end{pmatrix}\mathbf{x} = 0 \tag{1.0.3}$$

2 SOLUTION

Equation (1.0.1),(1.0.2) and (1.0.3) can be written as,

$$\mathbf{n_1^T x} = c_1 \tag{2.0.1}$$

$$\mathbf{n_2^T x} = c_2 \tag{2.0.2}$$

$$\mathbf{n_3^T x} = c_3 \tag{2.0.3}$$

Where,

$$\mathbf{n_1} = \begin{pmatrix}1\\1\\1\end{pmatrix}, \mathbf{n_2} = \begin{pmatrix}2\\3\\4\end{pmatrix}, \mathbf{n_3} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$$c_1 = 1, c_2 = 5, c_3 = 0 \tag{2.0.4}$$

Required equation of plane containing (2.0.1) and (2.0.2) is,

$$\mathbf{n_1^T x} + \lambda \mathbf{n_2^T x} = c_1 + \lambda c_2 \tag{2.0.5}$$

$$\implies (\mathbf{n_1^T} + \lambda \mathbf{n_2^T})\mathbf{x} = c_1 + \lambda c_2 \tag{2.0.6}$$

But (2.0.6) is perpendicular to (2.0.3).So,

$$(\mathbf{n_3^T})^T(\mathbf{n_1^T} + \mathbf{n_2^T}\lambda) = 0 \tag{2.0.7}$$

$$\implies \lambda = -\frac{\mathbf{n_3 n_1^T}}{\mathbf{n_3 n_2^T}} \tag{2.0.8}$$

$$\implies \lambda = \frac{-1}{3} \tag{2.0.9}$$

$\therefore$ By substituting λ,n_1,n_2,c_1,c_2 values in (2.0.6) we get required plane equation as,

$$\begin{pmatrix}1 & 0 & 1\end{pmatrix}\mathbf{x} = 2 \tag{2.0.10}$$