

Derivatives Securities Assignment 4: Exotic options

Problem 1: Barrier options

On November 17, 2016 at 12:00 noon, we observed that the SPDR S&P500 ETF is trading at $SPY=218.78/218.79$, and that the term structure of ATM options is

Expiration Date	ATM Volatility (%)
Dec 30, 2016	11.5
Jan 20, 2017	11.9
Feb 17, 2017	12.9
Mar 31, 2017	13.6
Jun 30, 2017	14.9
Sep 29, 2017	15.7
Dec 15, 2017	16.0
Jan 19, 2018	16.0

Assume a continuous dividend yield of 2.08% and term-structure of interest rates

Tenor	Rate (%)
1 month	0.30
3 months	0.50
6 months	0.60
1 year	0.70

1. Calculate a *daily forward implied volatility curve* from the data going out 1 year from Nov 17, 2016. For this purpose, interpolate linearly the squares of the volatilities at different tenors and then derive forward 1-day variances and volatilities. The result should be a *piece-wise constant* forward volatility, which jumps at the dates in the table.
2. Same as question 1, for interest rates.
3. Build a 1-year trinomial tree using the forward rates, volatilities and the dividend yield. (Use daily time-steps if possible).
4. Price a European-style call on SPY with strike 225 maturing on November 17, 2017.
5. Price an American-style call on SPY with strike 225 maturing on November 17, 2017
6. Price an up-and-out call on SPY with strike 225 and knock-out level 235 maturing on

November 17, 2017

7. Price a down-and-out call on SPY with strike 225 and knockout at 210.

For problem 4, compare your answer with Black Scholes with the appropriate constant volatility and rate. For problem 5, 6, 7 compare the result of using a term structure of volatility with using the constant, 1-year term volatility in the pricer. Also, for 5 and 6, compare with the explicit pricing formulas in Hull (for constant parameters).

Problem 2: Range accrual option

Using the same data as above, price a derivative security which pays the holder \$1,000 dollars for each day that SPY is in the range $\$210 < \text{SPY} < \230 , with the following conditions:

- Payments stop if the option crosses any of the barriers and the option expires.
- The option expires on May 17, 2017.

Notes: The pricing date for the options is November 17, 2016. You must use the exact number of days between the pricing date and the maturity of the option or rate. Holidays will be neglected. (For example **networkdays()** in Excel would give the number of days.)