Discussion with Prof. Chi-Jen Lu (Election Game)

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Outline

The Setting (1/4)

- The society (voters): $V = V_A \dot\cup V_B$.
- Strategy profile for the two parties: $z = (z_A, z_B)$.
  - Party A's strategy z_A : $z_A \in [-1, 1]^k$.
  - Party B's strategy z_B : $z_B \in [-1, 1]^k$.
  - ⋆ E.g., $k = 10$.
- Utilities:
  - z_A for for A's supporters: $u_A(z_A) = \sum_{v \in V_A} \langle z_A, v \rangle$.
    z_A for B's supporters: $u_B(z_A) = \sum_{v \in V_B} \langle z_A, v \rangle$.
  - z_B for B's supporters: $u_B(z_B) = \sum_{v \in V_B} \langle z_B, v \rangle$.
    z_B for for A's supporters: $u_A(z_B) = \sum_{v \in V_A} \langle z_B, v \rangle$.
- Let $u(z_A) := u_A(z_A) + u_B(z_A)$ and $u(z_B) := u_A(z_B) + u_B(z_B)$.

The Setting (2/4)

- Let p_{AwB} denote the probability that A wins against B.
  We set $p_{AwB} := \frac{1}{2} + \frac{u(z_A) - u(z_B)}{2|V|}$.
  - $|V|$: the number of total voters.
- The reward (or payoff) of A:

$$r_A(z) = p_{AwB} \cdot u_A(z_A) + (1 - p_{AwB}) \cdot u_A(z_B).$$

- The reward (or payoff) of B:

$$r_B(z) = (1 - p_{AwB}) \cdot u_B(z_B) + p_{AwB} \cdot u_B(z_A).$$

The Setting (3/4)

- The gradient of z_A:

$$
\begin{aligned}
\frac{\partial r_A(z)}{\partial z_A} &= p_{AwB}\frac{\partial u_A(z_A)}{\partial z_A} + u_A(z_A)\frac{\partial p_{AwB}}{\partial z_A} \\
&+ \quad (1-p_{AwB})\frac{\partial u_A(z_B)}{\partial z_A} - u_A(z_B)\frac{\partial p_{AwB}}{\partial z_A} \\
&= \quad p_{AwB}\sum_{v\in V_A} v + u_A(z_A)\frac{1}{2|V|}\sum_{v\in V} v - u_A(z_B)\frac{1}{2|V|}\sum_{v\in V} v \\
&= \quad \frac{1}{2}\sum_{v\in V_A} v + \frac{\sum_{v\in V_A} v}{2|V|}(u(z_A)-u(z_B)) + \frac{\sum_{v\in V} v}{2|V|}(u_A(z_A)-u_A(z_B)).
\end{aligned}
$$

- Policy updating by gradient ascent (GA): $z_A = z_A + \eta\frac{\partial r_A(z)}{\partial z_A}$
  - Maybe take $\eta = 0.0001$ at first.

The Setting (4/4)

Remark & Issues:

- The two party update their policy simultaneously (based on the state in the previous round).
- **The issue:** The updated policy needs to be projected into $[-1, 1]^k$ and satisfy $||z_A||, ||z_B|| \leq 1$.

To Do List

- Try to derive the gradient of z_B. Make clear the deductions and also verify the given equations.
- Investigate how to do the optimization task: project the updated policy onto the feasible solution space.
  - $[-1, 1]^k$.
  - $||z_A||, ||z_B|| \leq 1$.