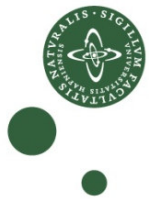
Code-Optimization Motivation and Requirements

Compiler's target code is still not as good as the hand-tuned code of the assembly-language expert, but is better than what most programmers would generate, even if they wanted to.

Achieved via code transformations, albeit the result is hardly optimal.



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Achieved via code transformations, albeit the result is hardly optimal.

An optimization, a.k.a., code transformation, needs to:

- preserve the observable behavior of the original prg (semantics),
- speed-up the program by a measurable amount (on average),
- prg's size not an issue but may affect instr-cache performance,
- be worth the effort, e.g., compile time, maintainability, etc.

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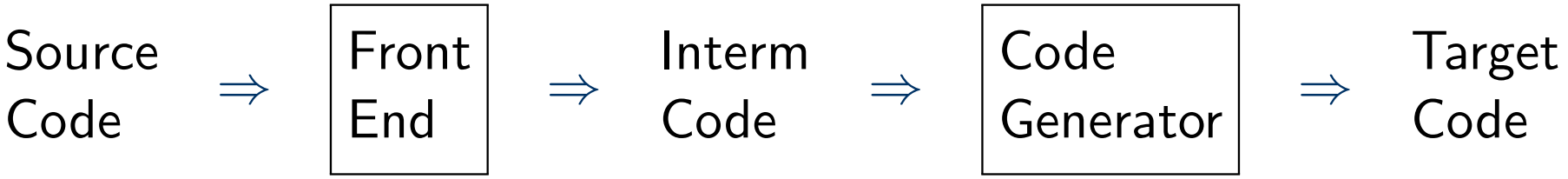
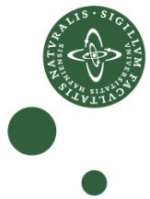
Strategy: spend the compiler effort in hot areas, e.g., loop nests:

- programers can write clear code in high-level languages
- while still getting efficient execution.

Optimization Break Down



User can:	Compiler Can:	Compiler Can:
profile program	improve loops	use register
change algorithm	re-order/pipeline loops	select/reorder instrs
transform loops	optimize procedure calls	peephole transfs
	eliminate procedure calls	
	straighten code	
	recognize common subexps	
	compute constant subexps	
	propagate constants	
	... and lots more	



Optimizations at Three-Address-Code (TAC) Level

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Three Address Code (TAC) is a low-level IL:

- instruction has at most 2 operands & one result, e.g., `s := s+i`
- Jump labels, `goto`, conditional jump (`if`), e.g., `if i=0 goto L2`
- Memory load/store, function call/return (not used here).

Three Address Code Example

```
    i := 20
    s := 0
L1: if i=0 goto L2
    s := s + i
    i := i - 1
    goto L1
L2: ...
```

Interm-lang optimizations, e.g., TAC, portable to various backends, ... but Can we rebuild the control flow structure from TAC?

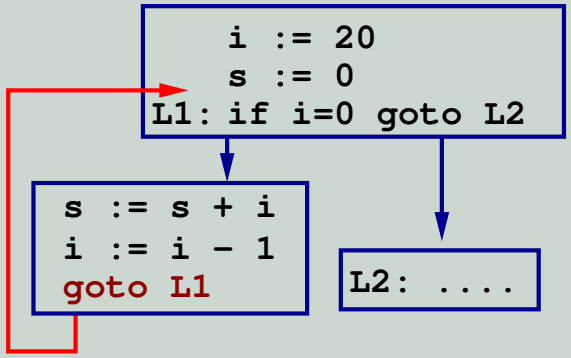
Basic Blocks (BB)

Basic Block: intuitively the maximal sequence of (consecutive) TAC instructions in which flow can enter/exit only via the first/last instr.

Code Example

```
    i := 20
    s := 0
L1: if i=0 goto L2
    s := s + i
    i := i - 1
    goto L1
L2: ...
```

Incorrect Basic Block



Identifying Basic Blocks (BB)

- Statements that start a basic block (BB):
  - first statement of any function
  - any labeled statement that is the target of a branch
  - any statement following a branch (conditional or unconditional)
- for each statement starting a BB, the BB consists of all stmts up to, but excluding, the start of a BB or the end of the program!

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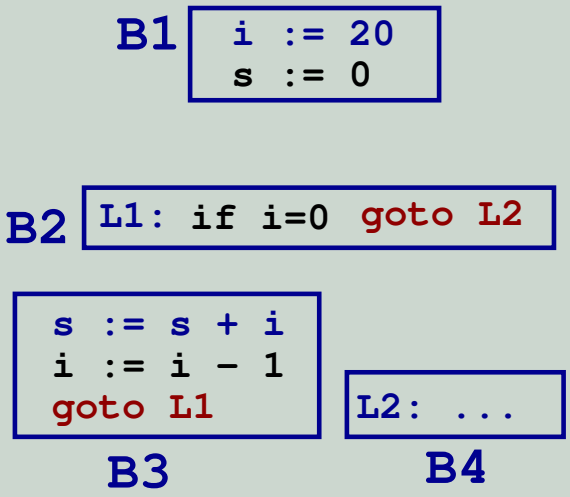
Basic Blocks

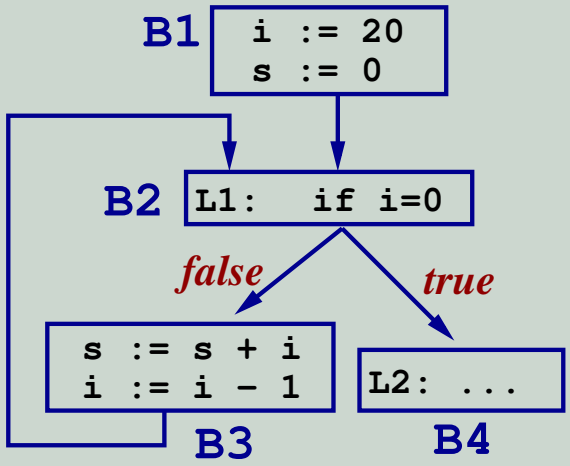


Building The Control-Flow Graph

Place an arrow from node A to node B if it is possible for control to "flow" from A to B (and remove gotos).

Code Example and its Control-Flow Graph (CFG)

```
    i := 20
    s := 0
L1: if i=0 goto L2
    s := s + i
    i := i - 1
    goto L1
L2: ...
```



Identifying Loops, Preliminaries

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Defition: A loop, L, is a subgraph of the CFG such that:

- all nodes of the loop are strongly connected, i.e., the loop contains a path between any two loop nodes.
- the loop has an unique entry point, named header, such that
- the only way to reach a loop node is through the entry point.

A loop that contains no other loop is called an *innermost loop*.

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Dominator Definition: a node p dominates node q if all paths from the start of the program to q go through p.

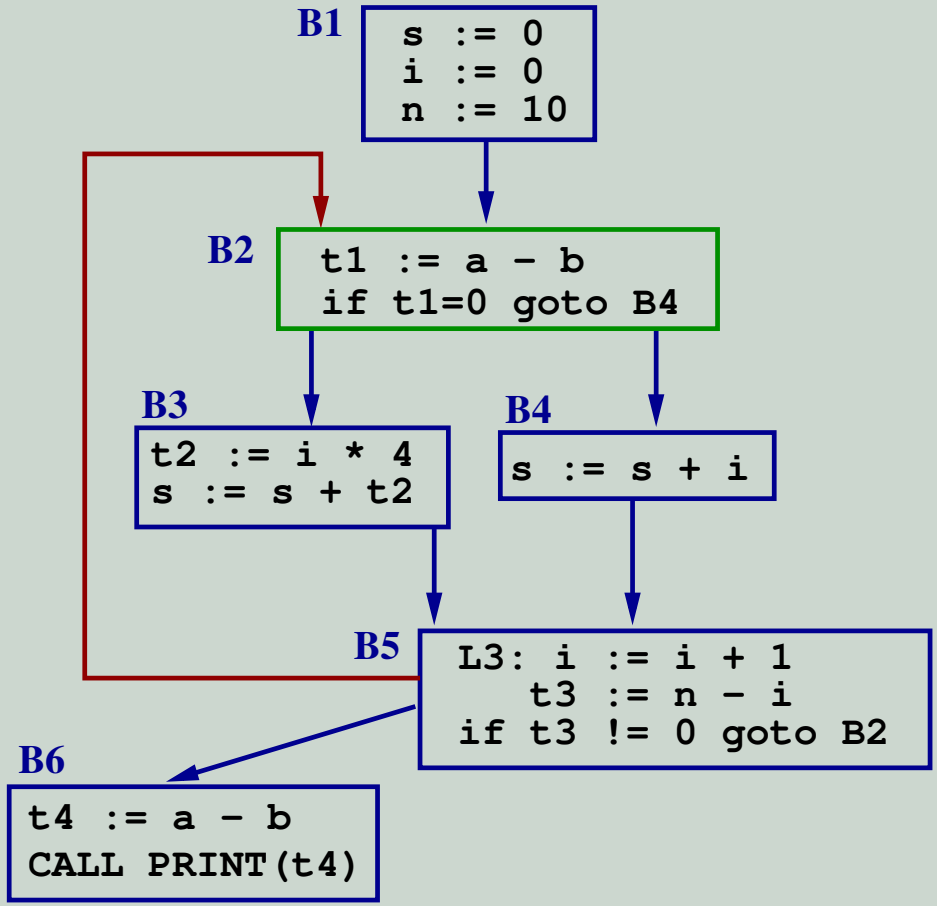
Identifying loops requires finding their "back edges":

- edges in the program in which the destination node dominates the source node.
- a loop must have an unique header, and one or more backedges.
- header dominates all blocks in the loop, otherwise not unique.

Example of a Loop CFG

Example: Finding Basic Blocks and The Control-Flow Graph (CFG)

```
    s := 0
    i := 0
    n := 10
L1:
    t1 := a - b
    if t1 = 0 goto L2
    t2 := i * 4
    s := s + t2
    goto L3
L2: s := s + i
L3: i := i + 1
    t3 := n - i
    if t3 != 0 goto L1
    t4 := a - b
    CALL PRINT(t4)
```



Identifying Loops

Algorithm for Dominators. $D(n)$ is the set of dominators of block n.

Input: CFG with node set N, initial node n_0. **Output**: $D(n), \forall n \in N$

$D(n_0) := \{n_0\}$
for $n \in N - \{n_0\}$ do $D(n) := N$

while changes to any $D(n)$ occur do
    for $n \in N - \{n_0\}$ do
        $D(n) := \{n\} \cup (\cap_{p \in pred(n)} D(p))$

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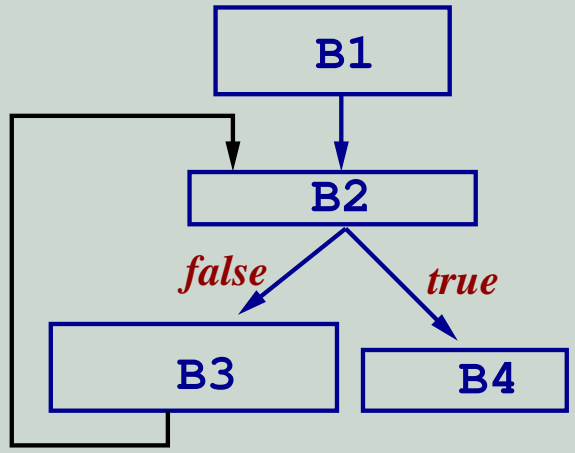
High-Level Algorithm: With each backedge $n \rightarrow d$ (d is the loop header), we associate a natural loop (of $n \rightarrow d$) consisting of node d and all nodes that can reach n without going through d.

Intuition: since d is the only entry to the loop, a path from any block outside the loop must pass through d.

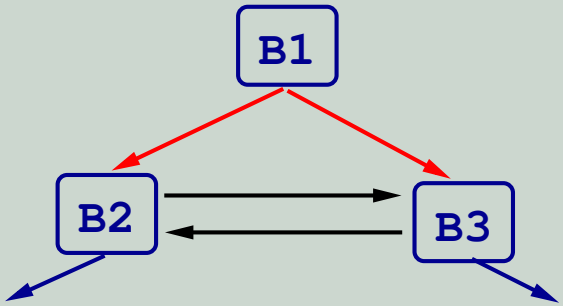
Reducible Control-Flow Graphs (CFG)

A CFG is reducible if it can be partitioned in forward and backward edges, and the forward edges form a directed-acyclic graph.

Reducible CFG



Irreducible CFG

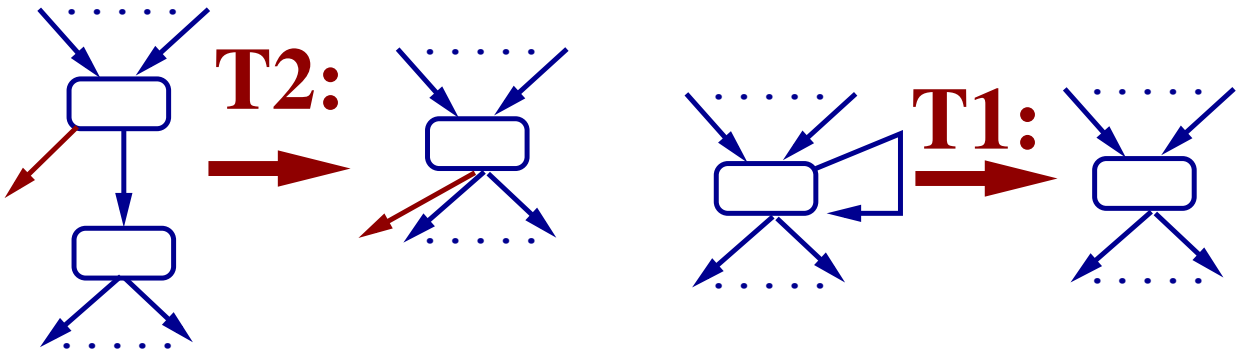


Irreducible CFG: due to unstructured GOTOs, e.g., jumps in the middle of a loop.

How to test CFG reducibility?

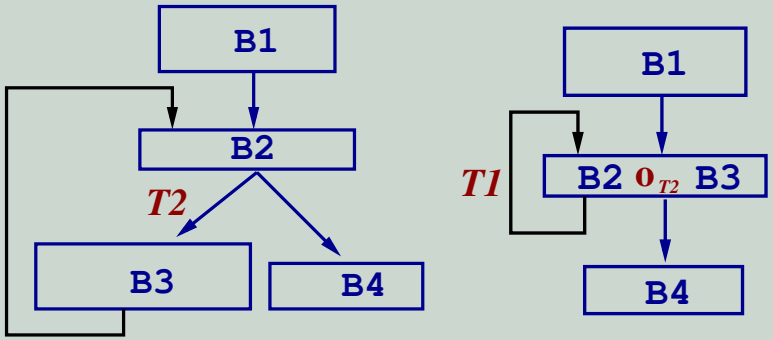
Testing CFG Reducibility via T1-T2 Transformation

Why Is Reducibility Important? 1. A reducible CFG can be written only in terms of (while/do) loops, if statements and function calls.



Reducible CFG *iff* can be reduced to 1 node via T1/T2 applications.

Reducible CFG via T2



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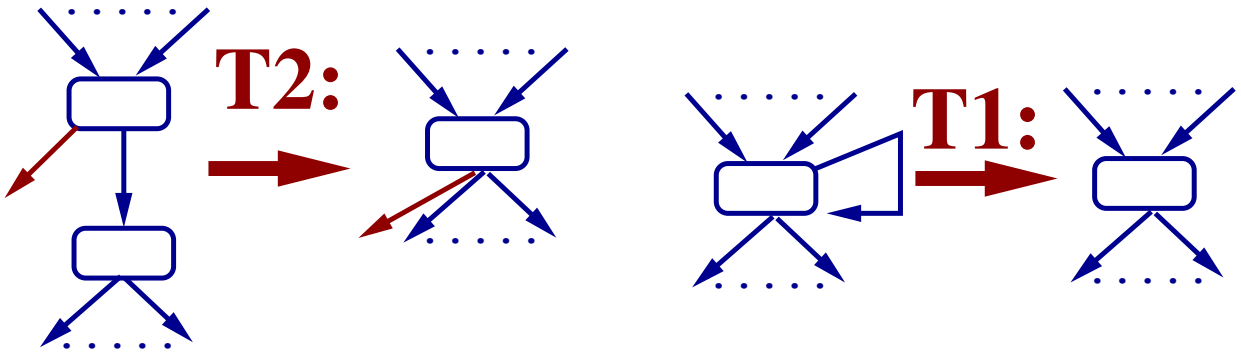

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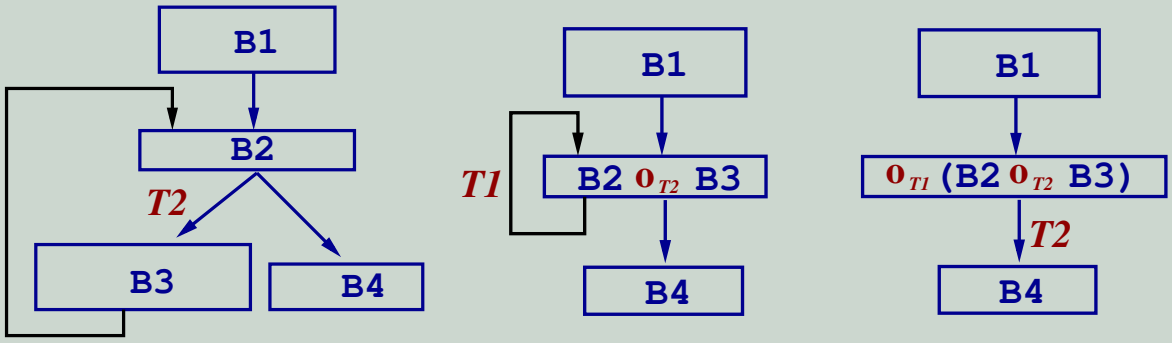
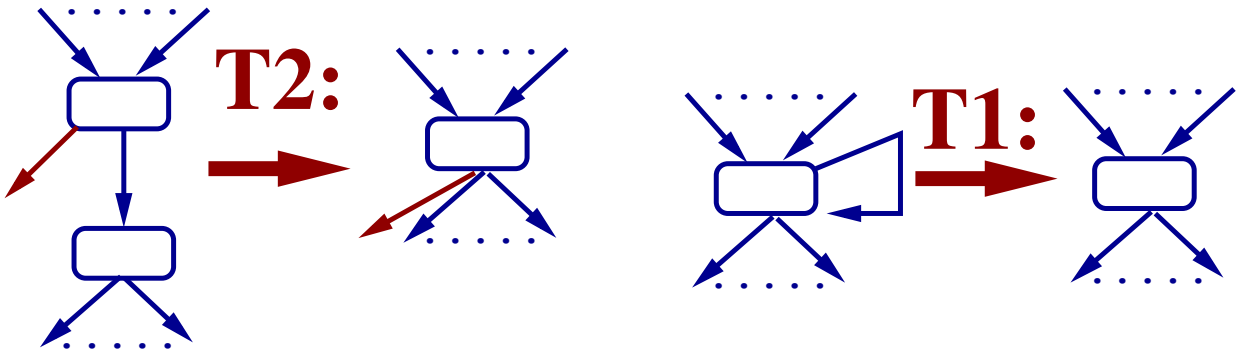
Reducing CFG via T1



Testing CFG Reducibility via T1-T2 Transformation

Why Is Reducibility Important? 2. If AbSyn guarantees reducible CFG then data-flow rules associated with each AbSyn node; the result is composed in one program traversal, rather than fix-point iter.



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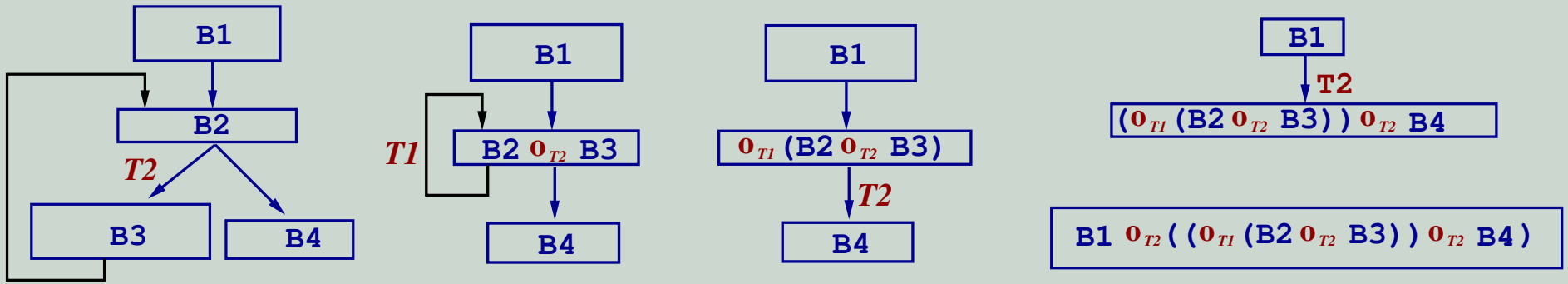
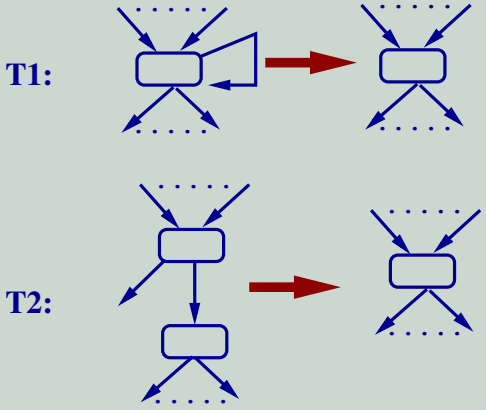
Reducing CFG via T2



Testing and Solving Ireducible CFG

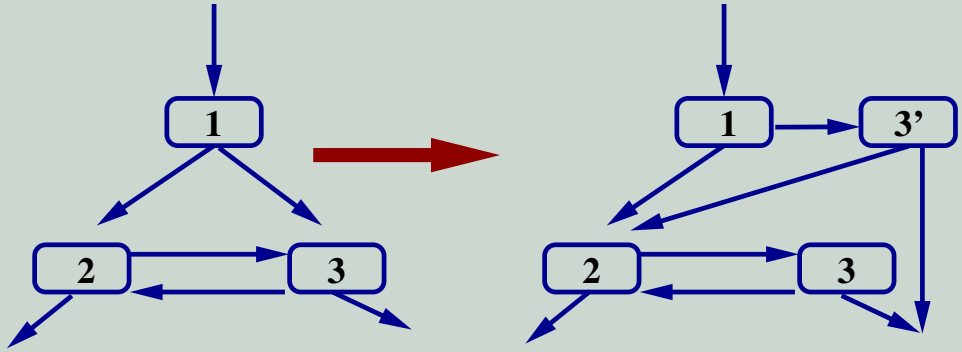
Alg for Testing Reducibility

In a copy of the CFG, apply T1 and T2 to a fixpoint. If the result is a single node than the CFG is reducible.



Node Splitting

Irreducible CFGs are difficult to optimize. It is always possible to solve irreducibility, but, in the worst case, at the cost of an exponential-code explosion:



Key property: if a CFG is reducible then all cycles are (regular) loops, and identifying the backedges is enough to find all loops.

CFG Reducibility via T1-T2 Transformation

Why Is Reducibility Important? 2. If AbSyn guarantees reducible CFG then equations associated with each AbSyn node; the result is composed in one program traversal, rather than fix-point iteration.

This allows optimizations to be implemented in the simpler, intuitive case-analysis style, e.g., live-function computation:

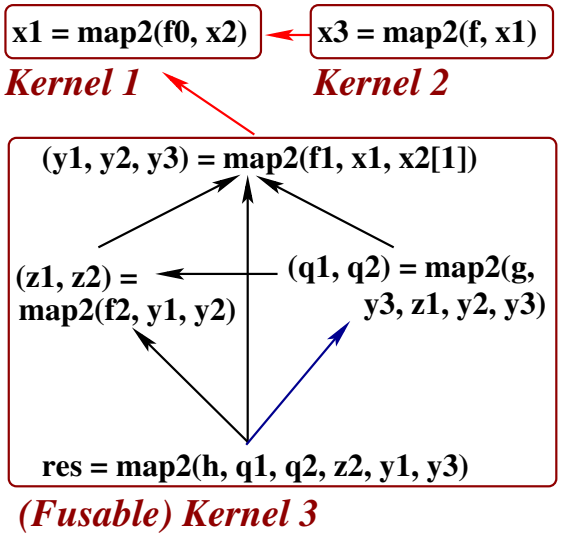
Pseudocode for computing the live functions (names)

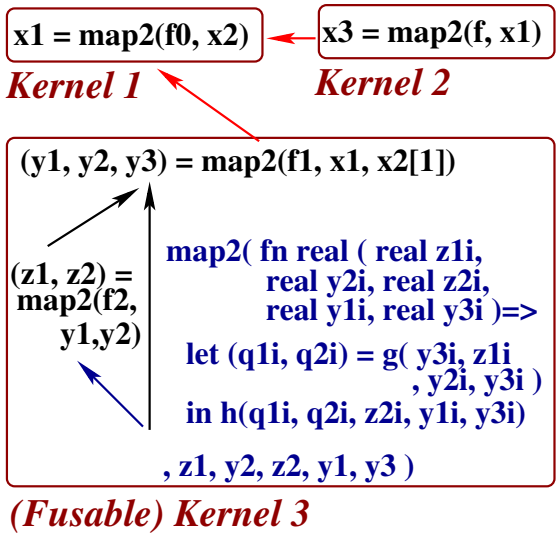
```
fun live_funs ( exp, ftab, livefs : string list ) : string list =
  case exp of
    Plus (e1, e2, pos) =>
        live_funs(e2, ftab, live_funs(e1, livefs, ftab) )
  | ...
  | FunApp(fid, args, pos) =>
        let val elives = foldl ( fn(e,lives)=>live_funs(e, ftab, lives) )
                         livefs args
        in  if( fid ∈ elives ) then elives
            else live_funs( fid's body, ftab, fid::elives )
        end
```

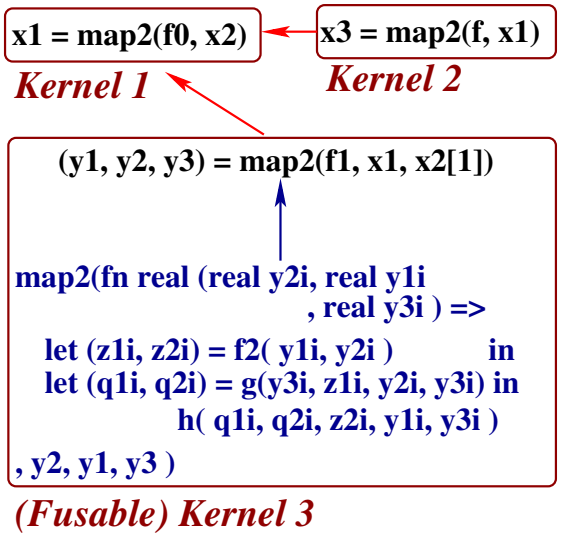
T1-T2 Graph Reduction

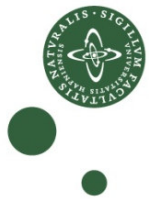
Why Is Reducibility Important? 3. Other less-conventional applications, such as map fusion without duplicating computation.

Troels Henriksen and C. Oancea, A T2 Graph-Reduction Approach To Fusion, *Procs. ACM Workshop on Funct. High Perf. Comp.* 2013.









Example of Optimizations

We have already seen **liveness analysis & register allocation**.

Common-Subexpression Elimination (CSE): if the same expression e is computed twice, replace if possible the second occurrence of e with the temporary that holds the value of the first computation of e.

Copy Propagation (CP): after a statement `x := y`, we know that `x` and `y` have the same value, hence we can replace all occurrences of `x` with `y` between this assignment and the next definition of `x` or `y`.

Dead-Code Elimination (DC) (one reason is copy propagation):

- can safely remove any statement that defines a dead variable,
- a branch to dead code moved to whatever follows the dead code,
- if a branch-condition value is statically known $\Leftrightarrow$ merge two BBs.

Constant Folding and Propagation (CtF/P): if possible, expressions should be computed at compile time, and the constant result should be propagated. **And these are just a few!**

Example: Common-Subexpression Elimination (CSE) and Copy Propagation (CP)

Original	After CSE1	After CP1	After CSE2 & CP2
t1 := 4 - 2	t1 := 4 - 2	t1 := 4 - 2	t1 := 4 - 2
t2 := t1 / 2	t2 := t1 / 2	t2 := t1 / 2	t2 := t1 / 2
t3 := a * t2	t3 := a * t2	t3 := a * t2	t3 := a * t2
t4 := t3 * t1	t4 := t3 * t1	t4 := t3 * t1	t4 := t3 * t1
t5 := t4 + b	t5 := t4 + b	t5 := t4 + b	t5 := t4 + b
t6 := t3 * t1	t6 := t4	t6 := t4	t6 := t4
t7 := t6 + b	t7 := t6 + b	t7 := t4 + b	t7 := t5
c := t5 * t7	c := t5 * t7	c := t5 * t7	c := t5 * t5

Copy propagation makes further common-subexpression elimination possible and the reverse.

Example Continuation: Constant Folding (CFP) & Copy Propagation (CP) & Dead Code Elim (DCE)

Original	After CtFP	After CP	After DCE
`t1 := 4 - 2`	`t1 := 2`	`t1 := 2`	
`t2 := t1 / 2`	`t2 := 1`	`t2 := 1`	
`t3 := a * t2`	`t3 := a`	`t3 := a`	
`t4 := t3 * t1`	`t4 := t3 * 2`	`t4 := a * 2`	`t4 := a * 2`
`t5 := t4 + b`	`t5 := t4 + b`	`t5 := t4 + b`	`t5 := t4 + b`
`t6 := t4`	`t6 := t4`	`t6 := t4`	
`t7 := t5`	`t7 := t5`	`t7 := t5`	
`c := t5 * t5`	`c := t5 * t5`	`c := t5 * t5`	`c := t5 * t5`

One of the main difficulties is deciding in which order to apply these optimizations, and how many times to apply them...

Data-Flow Analysis

Global analysis is used to derive information that can drive optimizations. Example: *Liveness analysis.*

Information can flow forwards or backwards through the program. Typical Structure:

- Successor/predecessor on basic blocks ($succ[B_i]$ or $pred[B_i]$).
- Define $gen[i]$ and $kill[i]$ sets.
- Define equations for $in[i]$ and $out[i]$.
- Initialize $in[i]$ and $out[i]$.
- Iterate to a fix point.
- Use $in[i]$ or $out[i]$ for optimizations.

Reaching Definitions at Basic-Block Level

What variable definitions might reach a certain use of the variable?

TAC Statement γ, of shape `a := b op c`, generates a new definition for `a` and kills all previous definitions of `a`:

$gen_r[\gamma] = \{\gamma\}, kill_r[\gamma] = D_a - \{\gamma\}$, where $D_a =$ set of all defs of a.

Intuitively, at basic-block level, we just add (union) together the generated definitions for all statements in the basic lock, except for the case when a following statement kills a previous definition!

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Formally, for basic block B, denoting $\beta > \gamma$ if stmt β follows γ:

$$gen_r[B] = \cup_{\gamma \in B}(gen_r[\gamma] - \cup_{\beta \in B}^{\beta > \gamma} kill_r[\beta])$$

$$kill_r[B] = \cup_{\gamma \in B}(kill_r[\gamma] - \cup_{\beta \in B}^{\beta > \gamma} gen_r[\beta])$$

Reaching Definitions at Program Level (Global)

Global-Reaching-Definitions Example

$pred_{B_i}$: the set of blocks B_j s.th. $\exists$ edge from B_j to B_i.

Initialization: $\forall B_i$:

$in_r[B_i] = \emptyset$

$out_r[B_i] = gen_r[B_i]$.

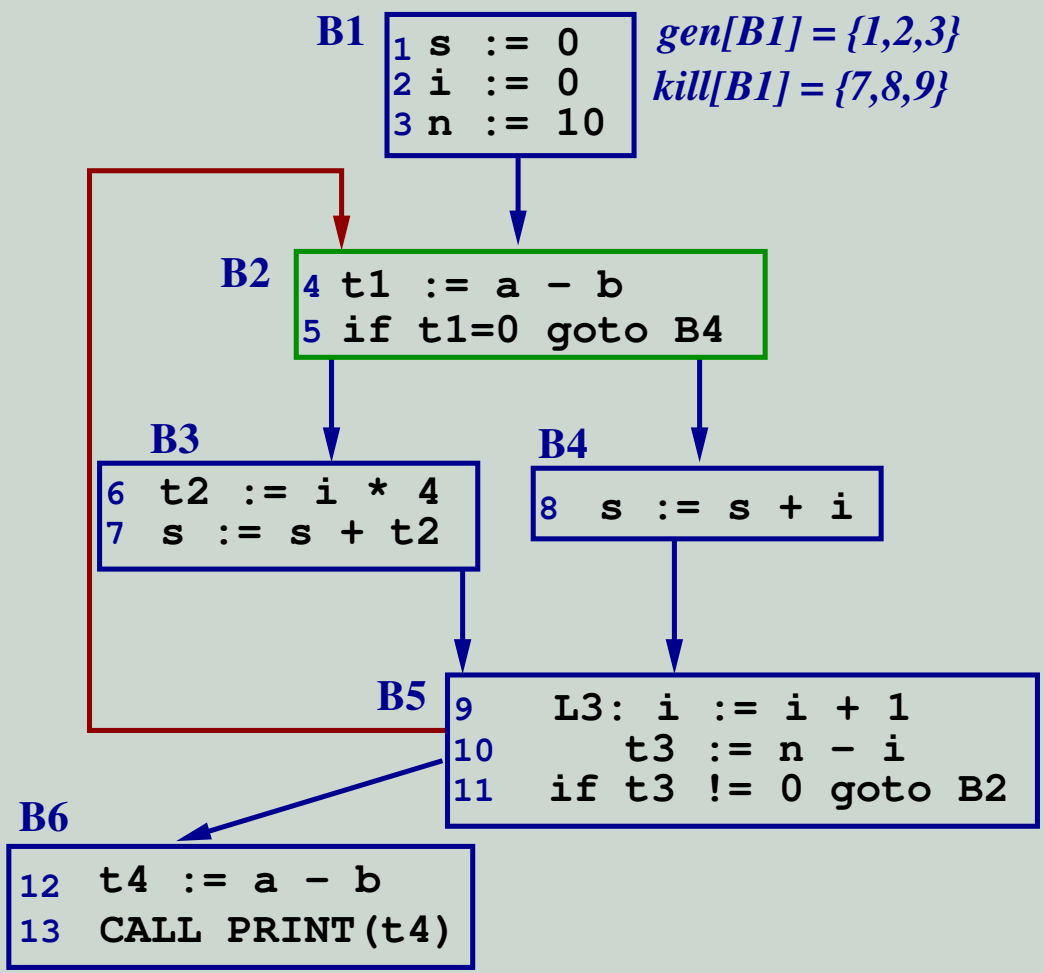
Data-Flow Equations:

$in_r[B_i] = \cup_{B_j \in pred(B_i)}(out_r[B_j])$

$out_r[B_i] =$

$(in_r[B_i] - kill_r[B_i]) \cup gen_r[B_i]$

Solve it $\Rightarrow$



Copy Propagation at Basic-Block Level

Many optimizations and user code introduce copy stmts s: `x := y`

Possible to eliminate such copy stmts s if we determine all stmts u where `x` is used, by substituting `y` for `x`, provided that:

- s is the only definition of `x` reaching u (done that already!)
- any path from s to u exhibits no assignments to `y` (how to do?).

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For each basic-block B:

- $gen_c(B)$ is the set of copy stmts `x := y` for which there is no subsequent assignment to `y` (or `x`) within B.
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Initialization: we start with the *out* set containing all copy statements in the program other than the statements killed in the block itself (except for the entry block, which dominates all others).

Copy Propagation at Program Level (Global)

Global-Copy-Propagation Example

C = set of all copy stmts in the prg. $pred_{B_i}$ = set of blocks B_j s.th. $\exists$ edge from B_j to B_i.

Initialization: $\forall B_i$:

$in_c[B_i] = \emptyset$,

$out_c[B_1] = gen_c[B_1]$,

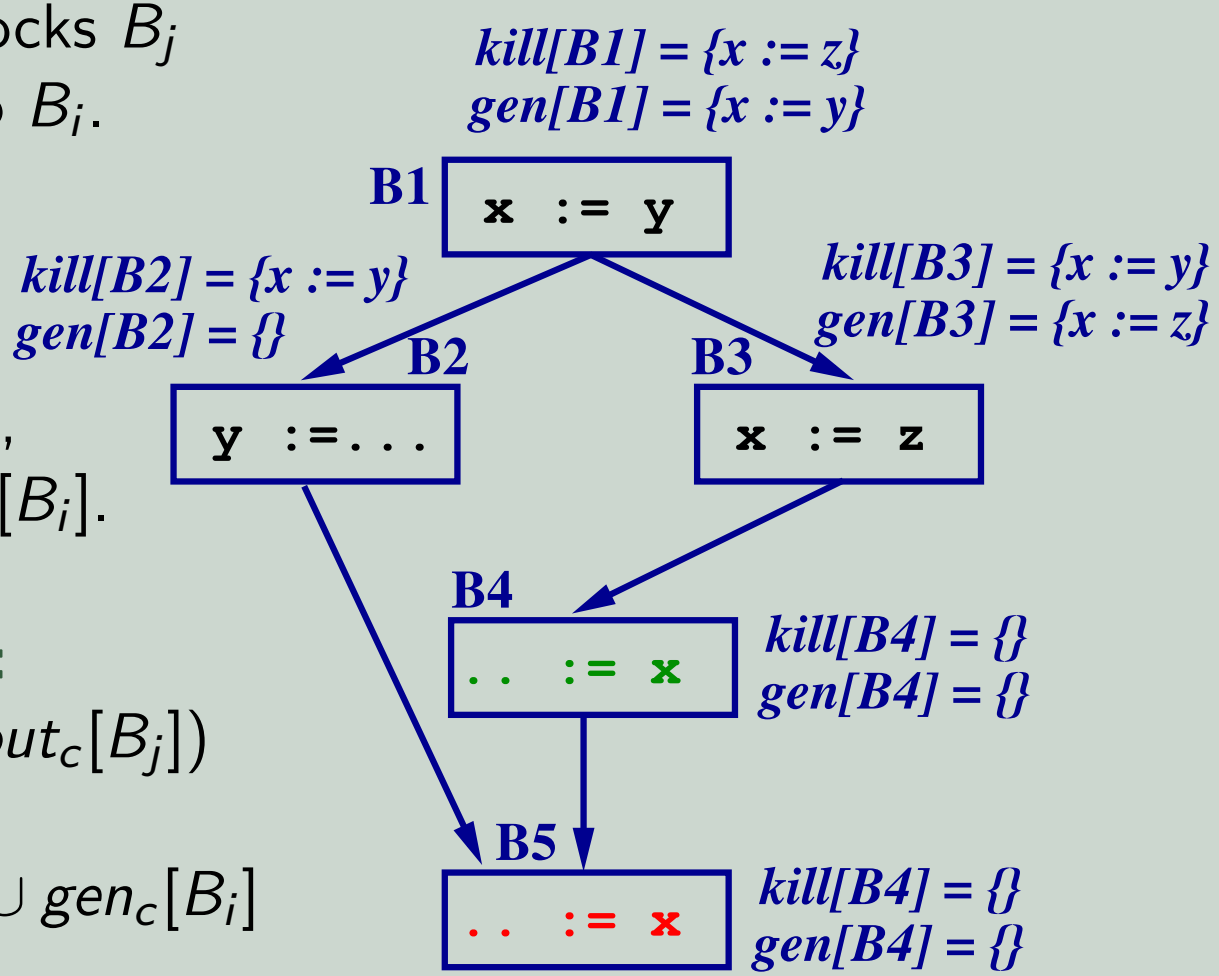
$out_c[B_i] = C - kill_c[B_i]$.

Data-Flow Equations:

$in_c[B_i] = \cap_{B_j \in pred(B_i)}(out_c[B_j])$

$out_c[B_i] =$

$(in_c[B_i] - kill_c[B_i]) \cup gen_c[B_i]$



Solve it $\Rightarrow$

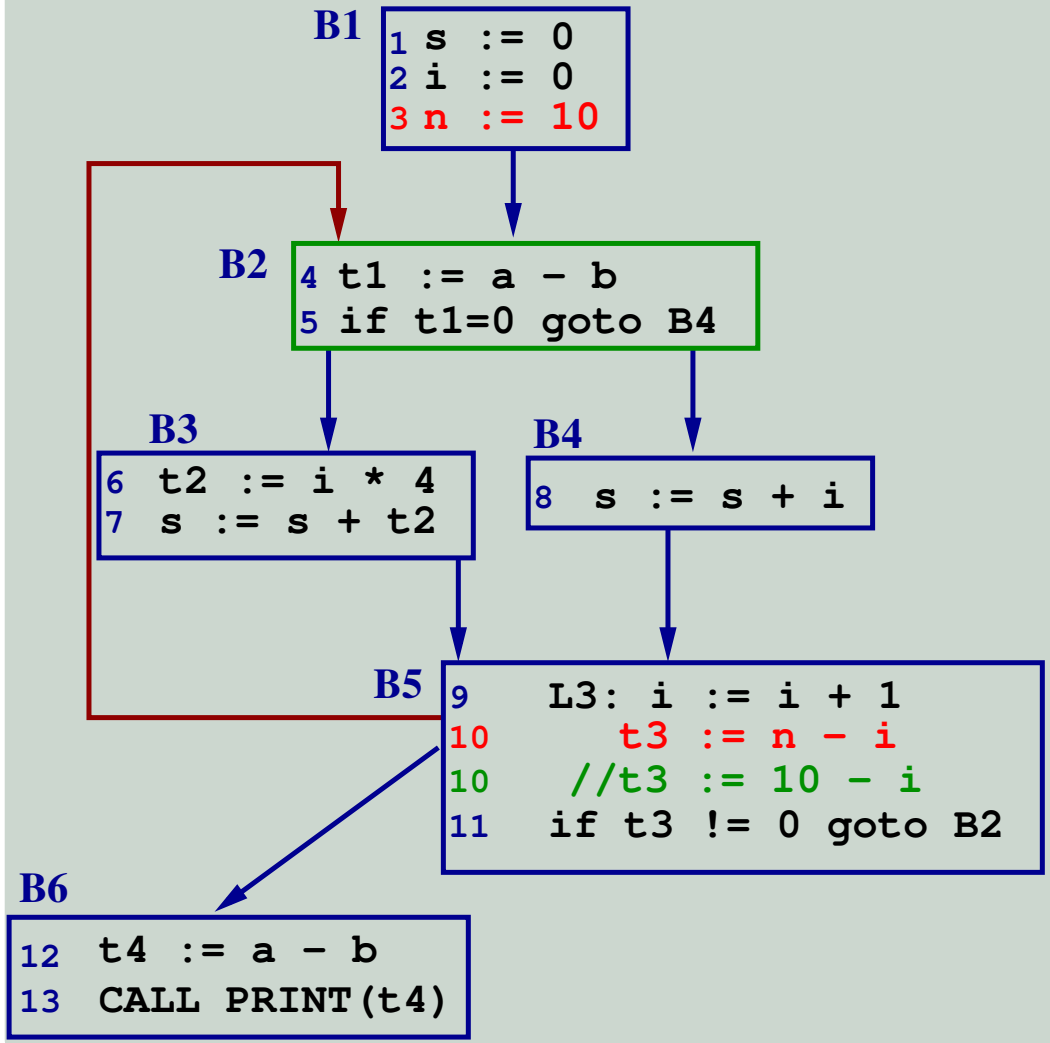
Global Copy Propagation Algorithm

Algorithm. For each copy stmt γ, of shape b := c in program do:

- Find all uses of b that are reached by γ, i.e., references to b in block B such that $\gamma \in in_r[B]$ and B does not contain any earlier redefinition of b or c,
- For each such use, if furthermore $\gamma \in in_c[B]$ then replace b by c,
- If replacement succeeds in all the above cases, then remove γ.

Demonstrating Copy Propagation

Copy-Propagation Example



Copy stmt $1 \in in_r[B_3]$ and $1 \in in_r[B_4]$. However $1 \notin in_c[B_3]$ and $1 \notin in_c[B_4]$, hence replacement fails!

Similar for copy stmt 2.

Copy stmt $3 \in in_r[B_5] \cap in_c[B_5]$ and reaches the reference of `n` in stmt 10.
Hence it is safe to delete stmt 3 and to replace `n` with 10 in stmt 10!

Common-Subexpression Elimination (CSE)

We wish to find two expressions that would always yield the same value, and to replace the second one with the result of the first!

We say that `b op c` is an available expression at stmt S iff:

- all routes from the start of the program to S must pass through the evaluation of `b op c`, and
- after the **last** eval of `b op c` there are no redefinitions of `b` or `c`.

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- after the **last** eval of `b op c` there are no redefinitions of `b` or `c`.

This means that in S we can replace `b op c` with the result of its previous evaluation. **Looks very similar to Copy Propagation!**

Intuitively, we can think again of killing and generating expressions:

- `b op c` is generated by the statement where it is evaluated,
- `b op c` is killed by a statement that redefines `b` or `c`.

CSE at Basic-Block and Global Level

Basic-Block (B) Level: Initially set $gen_e[B] = \emptyset$, then

- Go through B's stmts in sequence, and for each `a := b op c`
  - add `b op c` to $gen_e[B]$
  - delete any expression containing `a` from $gen_e[B]$
- $kill_e[B] =$ the set of all expressions anywhere in the program that are killed in B by a redefinition of expression's operands.

Init: $in_e[B_1] = \emptyset$, $out_e[B_1] = gen_e[B_1]$, $out_e[B_i] = E - kill_e[B_i]$, where E is the set of all available expressions in the program.

Data-Flow Equations: for each basic block B_i, with $i \neq 1$:

- $in_e[B_i] = \cap_{B_j \in pred(B_i)}(out_e[B_j])$,
- $out_e[B_i] = (in_e[B_i] - kill_e[B_i]) \cup gen_e[B_i]$.

CSE Algorithm

For each basic block B do:

(1) Find all statements γ of shape `a := b op c` in B, such that `b op c` $\in in_e[B]$, and B contains no earlier definitions of `b` or `c`.

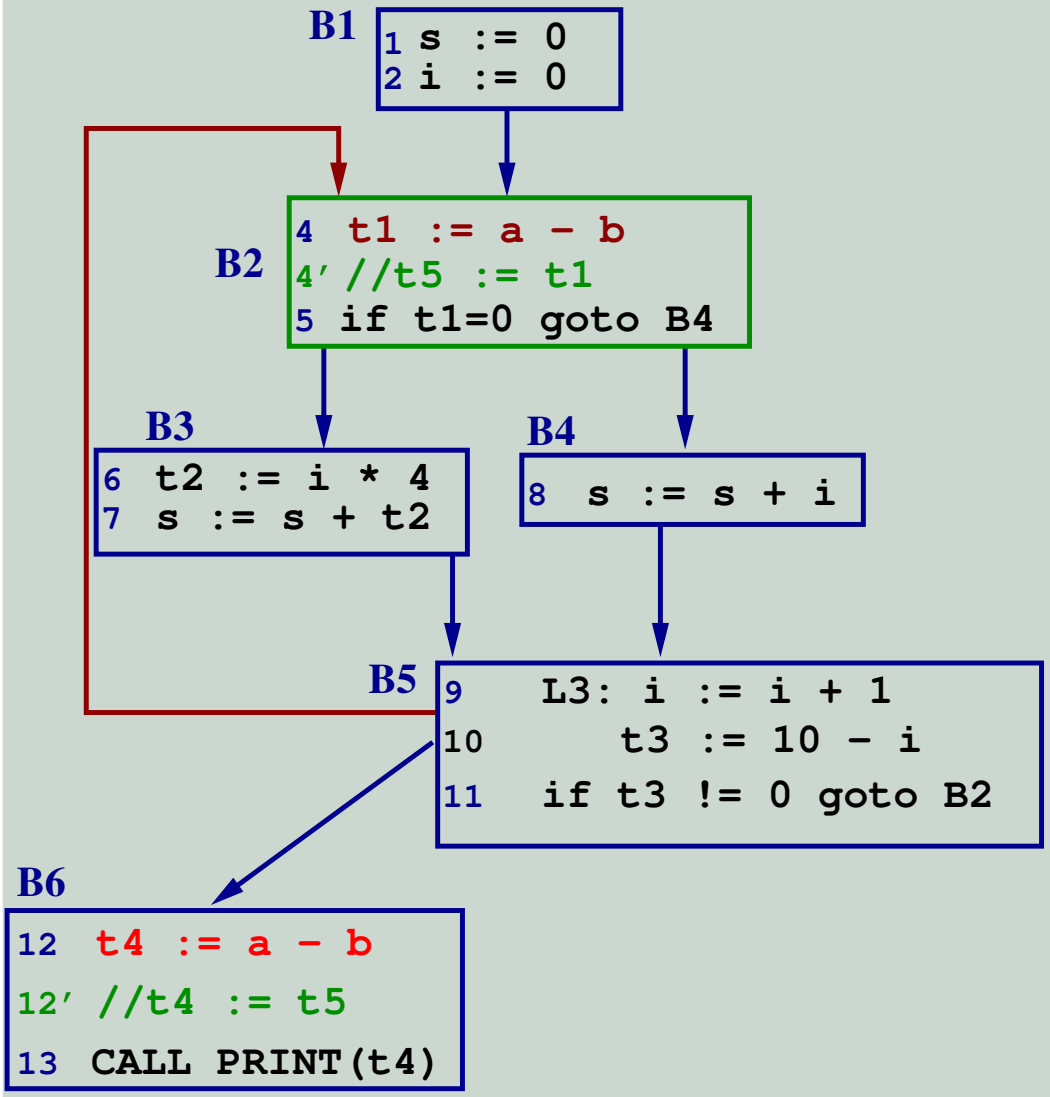
(2) For each stmt γ found in (1), allocate fresh temporary `t`.

(3) From B, search backwards along all possible paths all previous statements `d := b op c`, and immediately after them insert `t := d`.

(4) Replace statement γ by `a := t`.

Demonstrating CSE

Common-Subexpression-Elimination Example



Only stmt 12, i.e., `t4 := a-b` is found in stage (1) of Alg.

Searching backwards, the unique previous evaluation is in stmt 4, hence insert `t5 := t1` as stmt 4'.

Stage (4) of Alg replaces stmt 12 by `t4 := t5`

Subsequent copy propagation and dead-code elimination will eliminate `t4` and `t5`, i.e., both will be replaced by `t1`!