Problem1.

Since normally distributed random numbers are independent of each other, their expected values add up to the sum of their respective expected values.

I assume that price(P_0) = 1, and r_t is between -1 and +1, so $\sigma = 0.3$. And $r_t \sim N(0, \sigma^2)$ with simulating 10000

(1) Classical Brownian Motion

In theory:

$P_t = P_{t-1} + r$

$E(P_t) = E(P_{t-1} + r_t) = E(P_{t-1}) + E(r_t) = E(P_0) + 0 = 1$

$\sigma(P_t) = \sigma(P_{t-1} + r_t) = \sigma(r_t) = 0.3$

In PyCharm:

```
Classical Brownian Motion
1.0010948337463685
0.29880439720537566
```

Result is close to my expectation.

(2) Arithmetic Return System

In theory:

$P_t = P_{t-1}*(r_t + 1)$

$E(P_t) = E(P_0 * \prod (r_i+1)_{i=0}^{t}) = E(r_t) = 1$

$\sigma(P_t) = \sigma(P_0 * \prod (r_i+1)_{i=0}^{t}) = \sigma(r_t) = 0.3$

In PyCharm:

```
Arithmetic Return System
1.0010948337463685
0.29880439720537566
```

Result is close to my expectation.

(3) Log Return or Geometric Brownian Motion

In theory:

$P_t = P_{t-1}*e^{rt}$

$E(P) = E(P_0 * \prod e^{ri}{}_{i=0}^{t}) = P_0 * E(\prod e^{ri}{}_{i=0}^{t}) = e^{\sigma\wedge 2/2} = 1.046$

$\sigma(P) = \sigma(P_0 * \prod e^{ri}{}_{i=0}^{t}) = P_0 * \sigma(\prod e^{ri}{}_{i=0}^{t}) = (e^{\sigma\wedge 2} - 1)*e^{\sigma\wedge 2} = 0.3203$

```
Log Return or Geometric Brownian Motion
1.0468933909689206
0.32108778116766784
```

Result is close to my expectation.

Problem2:

Comparing these VaR values, we can observe the following order from the lowest to highest VaR:

Exponentially Weighted Variance VaR (2.84%) <- lambda = 0.94 (main reason)

Historic Simulation VaR (3.95%)

MLE Fitted T Distribution VaR (4.53%)

Normal Distribution VaR (5.35%)

Fitted AR (1) Model VaR (5.56%)

Problem 3:

I utilized discrete returns for calculating Value at Risk (VaR) using two different methods: the normal distribution with an Exponentially Weighted variance (EW) with $\lambda = 0.94$ as instructed, and the historical VaR method. Here are the specific assumptions and steps for both approaches:

Discrete Returns and Data Handling: I treated the holdings as lots, considering 1 low as 100 units. For instance, 58 lots were treated as 5800 units. The present price was taken as the last day's price from the DailyPrices.csv dataset.

Normal Distribution with EW Variance: For this method, I used the normal distribution assumption with an Exponentially Weighted variance ($\lambda = 0.94$). The portfolio VaR was computed using the formula:

$$\text{VaR}_{\text{portfolio}} = \text{Portfolio Value} \times \text{z-score} \times \text{Portfolio Standard Deviation}$$

where the z-score was set to 1.645 based on the Delta normal assumption.

The resulting VaR values for the individual portfolios and the total portfolio were:

- VaR for Portfolio A: $1,542,834.08
- VaR for Portfolio B: $808,329.17
- VaR for Portfolio C: $1,816,490.79
- Total Portfolio VaR: $4,167,654.04

Historical VaR: I chose the historical VaR method to address the non-normality of returns and the non-linearity of asset prices. This method utilizes historical price data to estimate VaR. The resulting VaR values were as follows:

- VaR for Portfolio A: $1,274,857.34
- VaR for Portfolio B: $726,386.76
- VaR for Portfolio C: $1,608,305.36

- Total Portfolio VaR: $3,609,549.46

The historical VaR method considers a broader range of price data, including older data points, potentially influencing the results due to the inclusion of more historical price information.