

# 智慧型汽車導論 Homework 2

## 1. MILP Linearization

Q1.

prove  $\alpha + \beta + \gamma = 2 \Leftrightarrow \alpha + \beta - \gamma \leq 1 \wedge \alpha - \beta + \gamma \leq 1 \wedge -\alpha + \beta + \gamma \leq 1$

| $\alpha$ | $\beta$ | $\gamma$ | LHS | $\alpha + \beta - \gamma \leq 1$ | $\alpha - \beta + \gamma \leq 1$ | $-\alpha + \beta + \gamma \leq 1$ | RHS | LHS=RHS? |
|----------|---------|----------|-----|----------------------------------|----------------------------------|-----------------------------------|-----|----------|
| 0        | 0       | 0        | T   | T                                | T                                | T                                 | T   | T        |
| 0        | 0       | 1        | T   | T                                | T                                | T                                 | T   | T        |
| 0        | 1       | 0        | T   | T                                | T                                | T                                 | T   | T        |
| 0        | 1       | 1        | F   | T                                | T                                | F                                 | F   | T        |
| 1        | 0       | 0        | T   | T                                | T                                | T                                 | T   | T        |
| 1        | 0       | 1        | F   | T                                | F                                | T                                 | F   | T        |
| 1        | 1       | 0        | F   | F                                | T                                | T                                 | F   | T        |
| 1        | 1       | 1        | T   | T                                | T                                | T                                 | T   | T        |

$$\alpha + \beta + \gamma = 0, 1, 1, 2, 1, 2, 2, 3$$

$$\alpha + \beta - \gamma = 0, -1, 1, 0, 1, 0, 2, 1$$

$$\alpha - \beta + \gamma = 0, 1, -1, 0, 1, 2, 0, 1$$

$$-\alpha + \beta + \gamma = 0, 1, 1, 2, -1, 0, 0, 1$$

Q2.

prove  $\alpha\beta = \gamma \Leftrightarrow \alpha + \beta - 1 \leq \gamma \wedge \gamma \leq \alpha \wedge \gamma \leq \beta$

| $\alpha$ | $\beta$ | $\gamma$ | LHS | $\alpha + \beta - 1 \leq \gamma$ | $\gamma \leq \alpha$ | $\gamma \leq \beta$ | RHS | LHS=RHS? |
|----------|---------|----------|-----|----------------------------------|----------------------|---------------------|-----|----------|
| 0        | 0       | 0        | T   | T                                | T                    | T                   | T   | T        |
| 0        | 0       | 1        | F   | T                                | F                    | F                   | F   | T        |
| 0        | 1       | 0        | T   | T                                | T                    | T                   | T   | T        |
| 0        | 1       | 1        | F   | T                                | F                    | T                   | F   | T        |
| 1        | 0       | 0        | T   | T                                | T                    | T                   | T   | T        |
| 1        | 0       | 1        | F   | T                                | T                    | F                   | F   | T        |
| 1        | 1       | 0        | F   | F                                | T                    | T                   | F   | T        |
| 1        | 1       | 1        | T   | T                                | T                    | T                   | T   | T        |

$$\alpha\beta = 0, 0, 0, 0, 0, 0, 1, 1$$

$$\alpha + \beta - 1 = -1, -1, 0, 0, 0, 0, 1, 1$$

Q3.

Given  $\beta$  which is a binary variable,  $x, y$  which are non-negative real variables, and a constraint  $x \leq 2021$ , select a value of  $M$  to guarantee  $\beta x = y \Leftrightarrow 0 \leq y \leq x \wedge x - M(1 - \beta) \leq y \wedge y \leq M\beta$ .

LHS:  $0 = y$   
RHS:  $x - M \leq y = 0 \leq x$   
constraint:  $0 \leq x \leq 2021$   
 $x - M \leq 0 \rightarrow x \leq M \rightarrow 2021 \leq M$

LHS:  $x = y$   
RHS:  $0 \leq y = x \leq M$   
constraint:  $0 \leq x \leq 2021$   
 $0 \leq y = x \leq M \rightarrow 0 \leq x \leq M \rightarrow M = 2021$

**ANS:  $M = 2021$**

## 2. Signal Packing

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**Q1.**

**[Original design]**

length of  $\mu_0$ :  $8+44+3 = 55$   
length of  $\mu_1$ :  $16+44+3 = 63$   
Total length:  $55+63 = 118$

**[Redesign]**

length of  $\mu'_0$ :  $16+44+3 = 63$   
Total length: 63

The new design is better, since the number of bits that need to be transmitted are reduced by 55 bits. New design cuts down the duplicate transmission of headers and other fields.

**Q2.**

No, the  $\mu_2$  and  $\mu'_0$  are on different ECUs. Messages from different senders can not be merged together.

**Q3.**

According to Q2,  $\mu_2$  can not be merged into  $\mu'_0$ .  
 $\mu_3$  has the same sender ECU as  $\mu'_0$ , but their periods are not the same. If  $\mu_3$  is to be merged into  $\mu'_0$ , the period will be 50ms, because it is not allowed to have less frequent (period=100ms) messages.

**[Original design]**

length of  $\mu_3$ :  $16+44+3 = 63$   
Every 100ms,  $\mu'_0$  and  $\mu_3$  transmit  $(63 \times 2) + 63 = 189$  bits

**[Redesign]**

length of new  $\mu'_0$ :  $32+44+3 = 79$   
Every 100ms, new  $\mu'_0$  transmits  $79 \times 2 = 158$  bits

By merging the  $\mu'_0$  and  $\mu_3$  and changing the period to 50ms, the number of transmitted bits can be reduced by 31 bit every 100ms.

## 3. Simulated Annealing for Priority Assignment

---

**Result**

```

[(base) pattyde-MacBook-Pro:Homework2 patty$ python3 hw2.py
11
5
2
4
3
6
9
1
7
8
14
15
13
0
16
10
12
204.12

```

## Code

```

hw2.py
1 import math
2 import random
3 import sys
4 import copy
5
6 # Read the input.dat file. Get the total number of the messages, tau, and lines contains the priority (Pi), the transmission time (Ci),
7 def ImportData(filename):
8     n = open(filename).readlines()[1]
9     tau = open(filename).readlines()[1:2]
10    n = int(''.join(n).strip())
11    tau = float(''.join(tau).strip())
12    dataList = [i.strip().split() for i in open(filename).readlines()[2:]] # read data to a 2D list
13    dataList = [list(map(float, data)) for data in dataList] # turn type string to float
14
15    return n, tau, dataList
16
17 # Find the message's index based on the priority
18 def PriorityFindMessageLocation(prior, localDataList, n):
19     idx = 0
20     for x in range(n):
21         if localDataList[x][0] == prior:
22             idx = x
23     return idx
24
25 def CalculateResponse(index, n, tau, localDataList):
26     Q, B, R = 0, 0, 0
27     isViolate = False
28
29     # Find the B value (blocking time of the longest lower or same priority message).
30     for blockIndex in (lower for lower in [firstColumn[0] for firstColumn in localDataList] if lower >= localDataList[index][0]):
31         location = PriorityFindMessageLocation(int(blockIndex), localDataList, n)
32         if B < localDataList[location][1]:
33             B = localDataList[location][1]
34
35     Q = B

```

```

hw2.py
36 Q = B
37 while True:
38     sum = 0
39     for blockIndex in (lower for lower in [firstColumn[0] for firstColumn in localDataList] if lower < localDataList[index][0]):
40         location = PriorityFindMessageLocation(int(blockIndex), localDataList, n)
41         sum += math.ceil((Q + tau) / localDataList[location][2]) * localDataList[location][1]
42
43     if (B+sum)+localDataList[index][1] > localDataList[index][2] and not isViolate:
44         # print("Constraint violation")
45         isViolate = True
46
47     if Q == B+sum:
48         # worst-case R
49         R = round((B+sum)+localDataList[index][1],2)
50         # print(f'{ R }')
51         break
52     else:
53         Q = B+sum
54
55     return float(R), isViolate
56
57
58 # Swap two messages' priority
59 def SwapPriority(n, localDataList):
60     tempDataList = copy.deepcopy(localDataList)
61     # Random select two priority to swap
62     x, y = random.sample(range(n), 2)
63     tempDataList[x][0], tempDataList[y][0] = tempDataList[y][0], tempDataList[x][0] # Swap two messages' priority
64     # print(f'Swap {x} and {y}')
65     return tempDataList
66
67
68
69 if __name__ == '__main__':
70     # Read in the data set
71     n, tau, dataList = ImportData("input.dat")

```

```

hw2.py
69 if __name__ == '__main__':
70     # Read in the data set
71     n, tau, dataList = ImportData("input.dat")
72     priorityList = [int(dataListPriority[0]) for dataListPriority in dataList]
73
74     # Summation of the worst-case response times of all messages. The objective is to minimize it.
75     summation, newSummation = 0, 0
76     T = 1000000
77     reduceRate = 0.97
78     isResponseViolate = False
79     finalOutput = 0
80
81     # Calculate the response time of each message
82     for i in range(int(n)):
83         idx = PriorityFindMessageLocation(i, dataList, n)
84         temp_summation, isResponseViolate = CalculateResponse(idx, n, tau, dataList)
85         summation += temp_summation
86         if isResponseViolate:
87             print("Constraint violation")
88             sys.exit()
89     finalOutput = summation
90     # print(f'Origin sum = {summation}\n')
91
92
93     ### ----- Simulated Annealing -----
94     while T > 0.0001:
95         newDataList = SwapPriority(n, dataList)
96         tempR = 0
97         newSummation = 0
98         isResponseViolate = False
99
100        # Calculate the response time of each message
101        for i in range(int(n)):
102            idx = PriorityFindMessageLocation(i, newDataList, n)
103            tempR, temp_isViolate = CalculateResponse(idx, n, tau, newDataList)
104            if temp_isViolate:

```

```

hw2.py
104         if temp_isViolate:
105             isResponseViolate = True
106             newSummation += tempR
107
108         newSummation = round(newSummation, 2)
109
110         ### ----- "down-hill" move -----
111         if newSummation <= summation:
112             # Violated the constraint, make it harder to go down hill in this direction
113             if isResponseViolate: # Constraint violation
114                 prob = min(math.exp(-(summation - newSummation + 1400000)/T), 1)
115                 if random.random() <= prob: # go
116                     print(f'\tNO Sum = {summation} New Sum = {newSummation} / Prob = {prob}')
117                 else: # don't go
118                     T = T * reduceRate
119                     continue
120             else:
121                 print(f'\tOK Sum = {summation} New Sum = {newSummation}')
122
123             dataList = copy.deepcopy(newDataList)
124             summation = newSummation
125
126             if isResponseViolate == False and finalOutput > summation:
127                 finalOutput = summation
128
129         ### ----- "up-hill" move -----
130         else:
131             if isResponseViolate == False:
132                 prob = min(math.exp(-(newSummation - summation)/T), 1)
133             else:
134                 prob = min(math.exp(-(newSummation - summation + 1400000)/T), 1)
135
136             # Take the chance to go up hill
137             if random.random() <= prob: # go
138                 if isResponseViolate: # Constraint violation
139                     print(f'\tV Sum = {summation} New Sum = {newSummation} / Prob = {prob}')

```

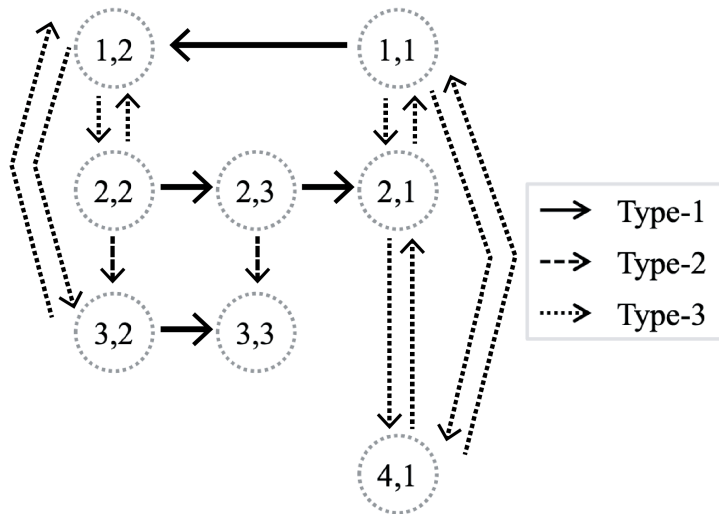
```

hw2.py
141         print(f'\tOK Sum = {summation} New Sum = {newSummation} / Prob = {prob}')
142
143         dataList = copy.deepcopy(newDataList)
144         summation = newSummation
145
146         if isResponseViolate == False and finalOutput > summation:
147             finalOutput = summation
148
149         ### ----- "up-hill" move -----
150         else:
151             if isResponseViolate == False:
152                 prob = min(math.exp(-(newSummation - summation)/T), 1)
153             else:
154                 prob = min(math.exp(-(newSummation - summation + 1400000)/T), 1)
155
156             # Take the chance to go up hill
157             if random.random() <= prob: # go
158                 if isResponseViolate: # Constraint violation
159                     print(f'\tV Sum = {summation} New Sum = {newSummation} / Prob = {prob}')
160                 else:
161                     print(f'\tG Sum = {summation} New Sum = {newSummation} / Prob = {prob}')
162                     dataList = copy.deepcopy(newDataList)
163                     summation = newSummation
164
165             if isResponseViolate == False and finalOutput > summation:
166                 finalOutput = summation
167
168         # print()
169
170         T = T * reduceRate
171
172     for i in range(int(n)):
173         print(int(dataList[i][0]))
174     print(finalOutput)

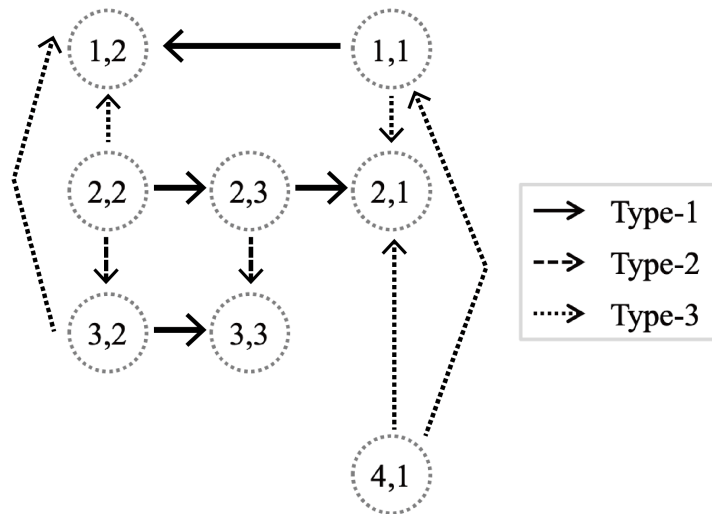
```

## 4. Intersection Management

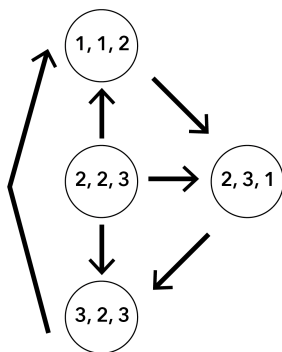
Q1.



Q2.



Explain why there is a deadlock:



Resource conflict graph shows that there will be a cycle between Vehicles 1, 2, and 3. 當 Vehicle 1 停在 1 區，Vehicle 2 停在 3 區，Vehicle 3 停在 2 區，他們會互相等待，造成 deadlock。