UIL Physics Notes

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§1 DC Circuits

We use the following notation for the section:

$\mathcal{E}$ = emf	P = power	R = resistance	W = work
I = current	q = charge	ΔV = voltage	

Author's Note: For this section, we adopt the conventional current standard of circuits (i.e. positive carriers carry charge along a circuit). For equations that determine the direction of current, use discretion to determine the direction of current.

§1.1 Introduction

We begin by defining the electromotive force, also denoted $\mathcal{E}$. It is defined as the work per unit charge:

$$\mathcal{E} = W/q = dW/dq. \tag{1.1}$$

It is important to note that the electromotive force is not an actual force; rather, it is a potential difference that causes charge to move. Thus, it is important to think of the electromotive force as something that *motivates* charge to move rather than an actual force causing charge to move.

§1.2 Voltage Equations

Since the electromotive force can be converted into a voltage potential, to properly perform computations in DC circuits, we need to perform calculations based on voltage. Thus, we introduce the following theorems regarding voltage:

Theorem 1.2 (Kirchoff's Loop Rule). The sum of all differences in potential around a complete circuit loop must be zero.

Theorem 1.3 (Kirchoff's Junction Rule). The sum of the currents entering a junction is equal to the sum of the currents leaving the junction.

Theorem 1.4 (Voltage as a State Function). Voltage is *path-dependent*; that is, voltage difference between two points does not depend on the path that the voltage takes.

We also have one law that essentially acts as our bread and butter when it comes to working with DC circuits, Ohm's Law:

$$\Delta V = IR. \tag{1.5}$$

Now, given that voltage is path-independent, we can combine Theorems 1.2, 1.3, and 1.5 to get the following important theorem for all DC circuits:

Theorem 1.6 (Resistors in Series and Parallel). Consider n resistors with resistances $R_1, R_2, \ldots, R_n$. If they are placed in a series, then the equivalent resistance is

$$R_{\text{eq}} = \sum_n R_n.$$

If they are placed in parallel, then the equivalent resistance satisfies

$$\frac{1}{R_{\text{eq}}} = \sum_n \frac{1}{R_n}.$$

§2 Special Relativity

§2.1 Introduction

Relativity is a modern physics idea that relates space and time together while considering objects with particularly large mass or particularly large speed. It is based off of two basic ideas:

Definition 2.1 (Basic Ideas of Relativity).

1. The laws of physics are invariant in all inertial frames of reference (that is, frames with no acceleration).
2. The speed of light in vacuum is the same for all observers, regardless of the motion of light source or observer.

From these basic ideas, we can derive the theory of relativity. We will begin with special relativity.

§2.2 Special Relativity Equations

Since relativity is inherently related to the speed of light, we need a way to determine how "close" something is to relativistic speeds, or the speed of light. This is the definition of the Lorentz factor:

$$\lambda = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2.2}$$

Notice that we always have the Lorentz factor greater than or equal to 1. It is important to note that many of our basic definitions of energy and time break down when moving at relativistic speeds. Thus, we must rederive many of our existing equations. First, we have the most famous two effects in special relativity: length contraction and time dilation. At relativistic speeds*, distances seem to contract in the direction of the movement of the observer's reference frame, while time seems to dilate during movement. Both length contraction and time dilation are scaled by the Lorentz factor, resulting in the two relations

$$L = \frac{L_0}{\lambda} \tag{2.3}$$

$$t = t_0 \lambda. \tag{2.4}$$

Momentum follows a similar effect at relativistic speeds, similarly scaling by the Lorentz factor:

$$p = \lambda m v. \tag{2.5}$$

We now consider the energy of a particle moving at relativistic speeds. There are two types of energy to consider: the rest energy of a particle and its kinetic energy. The rest energy of a particle is the energy the particle contains within its mass while not moving, while the kinetic energy of a particle is the energy it gains from its movement. The equations for these two values are given respectively by:

*The term *relativistic speeds* will denote any speed in which the effects of special relativity are substantial; generally, this will be at velocities $v > c/10$; however, the equations can be applied to any situation where the effect s of special relativity must be considered.

$$E_0 = mc^2 \tag{2.6}$$

$$K = (\lambda - 1)mc^2. \tag{2.7}$$

Now, we can combine Equations 2.5 and 2.6 by summing the two equations together to get the total energy of the particle:

$$E = E_0 + K = \lambda mc^2. \tag{2.8}$$

Given each of the above relativistic equations, it should now be possible for each of the quantities from classical mechanics to be rederived in terms of special relativity[†]. We will leave these derivations as an exercise to the reader.

[†]Many of these derivations do require calculus (e.g. the derivation of force requires the derivative of relativistic linear momentum with respect to time), so we advise caution in blatantly applying classical mechanics to special relativity.

§3 Nuclear Physics

§3.1 Introduction

Most of our investigation of nuclear physics will arise from analyzing nuclear reactions: specifically, we care about what types of nuclear reactions can occur and how to classify them. We will begin by analyzing basic reaction types.

§3.2 Introductory Reactions

There are two basic flavors of nuclear reactions that are important to know: the alpha decay and the beta decay. Alpha decay occurs at the emission of a Helium-4 nucleus from an atom, as seen in Equation 3.1:

$$ {}^{238}\mathrm{U} \to {}^{234}\mathrm{Th} + {}^{4}\mathrm{He}. \tag{3.1} $$

In contrast, beta decay occurs at the conversion of a neutron to a proton or vice versa, resulting in the creation of an electron or a positron. Beta decay that creates an electron is called β^- decay, while beta decay that creates a positron is called β^+ decay. The basic reaction for the former is shown in Equation 3.2, while the basic reaction for the latter is shown in Equation 3.3:

$$ \mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{v} \tag{3.2} $$

$$ \mathrm{p} \to \mathrm{n} + \mathrm{e}^+ + v \tag{3.3} $$

In both of these equations, either $\bar{v}$ or v showed up as a result of the decay; we wil note that these are the antineutrino and neutrino, respectively. For now, we will not elaborate on the production of this neutrino further.

§3.3 The Four Fundamental Forces

Before we can go deeper into examining nuclear and particle reactions, we must first discuss the forces that mediate such interactions. These are the four fundamental forces, described below:

Definition 3.4 (Four Fundamental Forces). The four fundamental forces are:

1. *Gravity.* Gravity is the weakest out of the four fundamental forces; as such, on a particle by particle level, gravity will be negligible compared to the other three forces. Almost no particle interactions are mediated by the gravitational force. Reactions mediated by this take years.

2. *The weak force.* The weak force is responsible for beta decay and other processes involving fundamental particles. However, due to its low relative strength, it has a range of less than 1 fm and is significantly less powerful than the strong force. Reactions mediated by this take between 10^{-13} and 10^{-8} seconds.

3. *The electromagnetic force.* The electromagnetic force is the interaction of charges within different particles. Interactions can occur due to the electromagnetic force, as it has an infinite range, and many common macroscopic forces such as drag and friction are the result of the electromagnetic force. The electromagnetic force is relatively strong enough to be critical in determining the structure and stability of nuclei. Reactions mediated by this take between 10^{-20} and 10^{-14} seconds.

4. *The strong force.* The strong force is responsible for the binding of nuclei, and is dominant in many of the reactions and decays between fundamental particles. However, its scope is limited both in that some particles can not feel its effects, and that it has a relatively short range of around 1 fm. Reactions mediated by this take around 10^{-23} seconds.

§3.4 Families of Particles

The final step we must perform before we can finally analyze particle reactions in full is to understand families of particles. Particles differ widely in their properties, so we must separate them into some general groups to allow for us to get some grasp on the mechanisms of nuclear reactions.

The first category of particles we analyze is the leptons. Leptons are characterized by all known leptons having a spin of 1/2. Interesting, this entire group of particles can only interact through weak and electromagnetic interactions, as none of them are affected by the strong force. The complete list of leptons and their antiparticles can be seen in the below table:

Particle	Antiparticle
e^-	e^+
v_e	$\bar{v}_\mathrm{e}$
μ^-	μ^+
v_μ	$\bar{v}_\mu$
τ^-	τ^+
v_τ	$\bar{v}_\tau$

Note that there are three charged particles paired with an uncharged neutrino. Although the electron is a stable particle, the meson and tau decay into other leptons:

$$\mu \to \mathrm{e}^- + \bar{v}_\mathrm{e} + v_\mu \tag{3.5}$$

$$\tau \rightarrow \mu^- + \bar{v}_\mu + v_\tau \tag{3.6}$$

We should note that both of the above reactions must be mediated by the weak force, as any reaction involving neutrinos will be mediated by the weak interaction.

The second family of particles we will analyze is the mesons. mesons are strongly interacting particles with integral spin. The most common examples of mesons are the pion and Kaon, which are denoted π and K, respectively. Generally, mesons are produced in reactions mediated by the strong interaction and will decay into other mesons and leptons in reactions mediated through the strong, electromagnetic, or weak interaction. meson production can be seen in Equation 3.7, while meson decay can be seen mediated by the weak interaction and electromagnetic interaction, respectively, in Equations 3.8 and 3.9.

$$\mathrm{p} + \mathrm{n} \rightarrow \mathrm{p} + \mathrm{p} + \pi^- \tag{3.7}$$

$$\pi^- \rightarrow \mu^- + \bar{v}_\mu \tag{3.8}$$

$$\pi^0 \rightarrow \gamma + \gamma \tag{3.9}$$

The third class of particles that we will analyze are the baryons. Baryons have half-integral spins. Notable examples of baryons are the proton and neutron. Baryons will have distinct antiparticles; for example, the antiproton ($\bar{p}$) and the antineutron ($\bar{n}$). Interactions between nucleons (protons and neutrons) can produce heavier baryons, such as the reaction shown in Equation 3.10:

$$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \Lambda^0 + \mathrm{K}^+ \tag{3.10}$$

In the above equation, Λ^0 is the desired heavy baryon.

The final class of particles we will analyze are the field particles, also known as the exchange particles. These particles essentially act as the mediators for the forces that govern each reaction; these particles establish the fields the forces are generated from. Each force has its characteristic field particles: the weak bosons, $W^+, W^- Z^0$, mediate the weak force, while the photon γ, mediates the electromagnetic force, and the gluon g, mediates the strong force. The presence of any of these particles determines the type of interaction a reaction is mediated by, which can provide useful information regarding what can and can not occur.

We also note that field particles have a slight contradiction to them: Each of the particles are emitted by the reactants in a nuclear reaction to form the force, yet each of the particles also has a nonzero mass, so how can conservation of mass be satisfied? This comes down to the idea of uncertainty: essentially, if we measure the energy of a reaction over some given time interval Δt, we are met with a corresponding uncertainty in the energy of reaction by the Heisenberg Uncertainty Principle, shown in Equation 3.11:

$$\Delta E = \frac{h}{2\pi \Delta t}. \tag{3.11}$$

Thus, if we measure for a small enough interval of time, we are able to neglect the energy loss from the emission of a field particle due to uncertainty. Interestingly, this is what defines the range of each of the fundamental forces; because we can only observe a reaction for a given amount of time, a field particle will only be able to move a finite distance during the course of the reaction, so the reaction has a maximum range.

§3.5 Conservation within Reactions

With the fundamentals of nuclear physics established, we are finally able to describe which nuclear reactions are possible to occur:

Theorem 3.12 (Reaction Mechanisms). In any particle or nuclear reaction, the following three properties must be satisfied:

1. *Conservation of Momentum:* If a single particle is decaying, then it must decay into at least two other particles.
2. *Conservation of Charge:* The total charge of the reactants must equal the total charge of the products.
3. *Conservation of Baryon Count*: The baryon counts in the reactants and products must be the same.
4. *Conservation of Lepton Count*: The three lepton counts in the reactants and products must be the same.
5. *Conservation of Strangeness*: In any reaction mediated by the electromagnetic or strong interactions, total strangeness must be conserved between products and reactants. In any reaction mediated by the weak interaction, strangeness is allowed to change by at most 1 between products and reactants.
6. *Conservation of Angular Momentum:* Spin must be conserved between the products and reactants.

Points 1-3 of Theorem 3.12 are clearly explanatory[‡]; we will focus on Points 4-6. We will begin with strangeness. The *strangeness* of a particle is a property of that particle that determines its behavior within different particle reactions. Unfortunately, unlike baryon or lepton count, the strangeness of a particle is something intrinsic to each particle. We will provide some of the most common particles and their strangenesses in a later table.

We now look at the idea of lepton count in Theorem 3.12. The point references three lepton numbers. These are L_e, L_μ, and L_τ, which count the number of leptons of each of the three flavors of lepton. **Each** of these must be conserved (e.g. a μ lepton can not be converted to a τ lepton). We note that importantly, lepton count is *decreased* by their antiparticles; thus, $\bar{v}_e$ contributes -1 to the L_e lepton count.

Finally, the last element of particle reactions is up to the idea of angular momentum and spin. We know that angular momentum is conserved in a nuclear reaction, which means that the intrinsic property of spin must be conserved as well. However, this is all we will say on the matter, as actual spin calculations seen to be out of the scope of a typical high school course.

[‡]We should note that the baryon count is decreased by baryon antiparticles; the theorem is slightly deceiving in that regard.

§3.6 Quarks

On top of our discussion of nuclear particle reactions, we must discuss the interactions between particles even more fundamental than the baryons and mesons: the quarks. The quarks are fundamental particles that come in 6 "flavors": up (u), down (d), strange (s), charm (c), top (t), and bottom (b).
The key properties of the six quarks are shown in the below table:[§]

Quark	Charge	Spin	Baryon Number	Strangeness
Up (u)	2/3	1/2	1/3	0
Down (d)	-1/3	1/2	1/3	0
Charm (c)	2/3	1/2	1/3	0
Strange (s)	-1/3	1/2	1/3	-1
Top (t)	2/3	1/2	1/3	0
Bottom (b)	-1/3	1/2	1/3	0

Quarks are found in baryons and mesons, where they serve as the fundamental particles that make baryons and mesons. In fact, the properties of many baryons and mesons are determined by the quark configuration, as simply adding up the values for the spin, baryon number, and charge of the constituent quarks for any given particle will result in the spin, baryon number, and charge of the particle itself. (In this vein, it is important to note that each quark has an antiquark with opposing charge and baryon number, which is how mesons have a baryon number of 0.) The quark configurations of several important particles are shown below, from which this property can be verified.

Particle	Quark Configuration
p	uud
n	udd
π^+	$u\bar{d}$
π^0	$\bar{u}u$, $\bar{d}d$
K^0	$d\bar{s}$
K^-	$\bar{u}s$

In nuclear reactions, the number of each flavor of quark is conserved *if the reaction is mediated by the electromagnetic or strong forces.* This in fact gives us a method to identify reactions mediated by the weak interaction; if a nuclear reaction has a flavor change in one of its quarks between products and reactants, then it must be mediated by the weak interaction. (Note that neither the converse nor the inverse of this statement are necessarily true.) We will call such reactions *flavor-changing reactions.*

[§]TODO: I should probably redo this table using the easytable package.

§4 AC Circuits

§4.1 Introduction

In this section, we introduce the concept of alternating current. In circuits with alternating current, the voltage-providing element does not provide a constant voltage, but rather a voltage that varies sinusoidally. We will investigate some of the properties of such circuits in this section.

§4.2 Voltage Producing Elements

We begin by focusing on the voltage producing element. We define the *peak voltage* of the circuit as the maximum voltage produced by the voltage producer. We also define the *rms voltage* of the circuit, which is the average voltage of the circuit (computed by essentially taking a quadratic mean). The peak voltage and the rms voltage are related by the equation

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}. \tag{4.1}$$

We note that we can define current similarly as well. Using the same definition of peak current and rms current, we can relate the two values by the equation

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}. \tag{4.2}$$

In an AC circuit, we can assume that the voltage produced gives rise to a current

$$I = I_0 \cos 2\pi f t, \tag{4.3}$$

where f is the frequency of the sinusoidal voltage and t is the time elapsed.

§4.3 Circuit Elements

We now need to consider what occurs when our source of alternating voltage is placed with other circuit elements. We will begin with resistors. Since resistors have a constant resistance, we can use Ohm's law to derive the voltage across the resistor:

$$V = IR = I_0 \cos 2\pi f t. \tag{4.4}$$

As a result, we can say that a resistor is *in phase* with the voltage-producing element, as the graphs of the current and the voltage across the resistor have the same shift at any given time. Additionally, because of this, the power equations for resistors can hold up if we consider the average power dissipated and the rms current and voltages:

$$\bar{P} = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}. \tag{4.5}$$

We now consider inductors within AC circuits. We recall that the counter-emf produced by an inductor has value

$$V = L\frac{dI}{dt}. \tag{4.6}$$

By applying the differential, we can derive the equation across an inductor to be

$$V = -V_0 \sin 2\pi f t. \tag{4.7}$$

As we see above, we see that since the voltage and current are governed by sine and cosine waves, respectively, the current lags behind the voltage by 90° in an inductor. We note that since an inductor impedes the flow of charge similarly to a resistor, we can use an equation similar to that of a resistor. However, since peak voltage and peak current are not reached at the same time, **such an equation can only be used for rms or peak voltages and currents.** This equation is the reactance equation, where we have

$$V = IX_L, \tag{4.8}$$

where X_L is defined as the *inductive reactance* of the circuit. The inductive reactance of a circuit can be found using

$$X_L = \omega L = 2\pi f L. \tag{4.9}$$

We note that in circuits containing inductors along with other circuit elements, the reactance equation should be used carefully: specifically, because the inductor is no longer the only component impeding voltage within the circuit, the reactance equation should only be used when the reactance is large compared to any other resistance.

We now finish the section on circuit elements with considerations regarding capacitors. Using the fact that in a capacitor, $V = Q/C$ and that the capacitor acts like an open switch when it has charge and a wire when it does not, we can derive the fact that when a capacitor has charge, it has a high voltage and a low current. Thus, we can state that current leads voltage by 90° in a capacitor. Equivalently, in an AC circuit with only a capacitor, we write

$$V = V_0 \sin 2\pi f t. \tag{4.10}$$

We can also derive a capacitive reactance equation, where we have

$$V = IX_C, \tag{4.11}$$

where V and I are only defined at rms or peak values and the reactance X_C is defined by

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}. \tag{4.12}$$

We can define the impedance of a circuit. Impedance affects LRC circuits. Essentially, it is a way of calculating the peak or rms voltage and currents across the circuit. Thus, for rms or peak voltages and currents, we have

$$V = IZ, \tag{4.13}$$

where Z is the impedance of the circuit. The impedance Z can be defined by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}. \tag{4.14}$$

Notably, the impedance of a circuit is incredibly helpful, as it allows us to find the peak or rms current. This can all be combined using the three equations 4.4, 4.8, and 4.11 to find the individual voltages across each of the inductor, capacitor, and resistor. This creates a good way of finishing an LRC circuit.