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Why does the power law for stock price hold?

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ABSTRACT

The aim of this paper is to explain why the power law for stock price holds. We first show that the complementary cumulative distributions of stock prices follow a power law using a large database assembled from the balance sheets and stock prices of a number of worldwide companies for the period 2004 through 2013. Secondly, we estimate company fundamentals from a simple cross-sectional regression model using three financial indicators-dividends per share, cash flow per share, and book value per share—as explanatory variables for stock price. Thirdly, we demonstrate that the complementary cumulative distributions of fundamentals follow a power law. We find that the power laws for stock prices and for fundamentals hold for the 10-year period of our study, and that the estimated values of the power law exponents are close to unity. Furthermore, we illustrate that the tail distribution of fundamentals closely matches the tail distribution of stock prices. On these grounds, we conclude that the power law for stock price is caused by the power law behavior of the fundamentals.

1. Introduction

Power laws are generally considered to be a common property characterizing complex systems composed of many interacting units. They can play an important role in understanding the behavior of such systems. Over the past decades, a considerable number of power laws have been found in a variety of complex systems (e.g. [1,8,14,16,18,20,22–24]). During that time, a great deal of effort has been made to explore the theoretical origins of power laws. (For examples, see [17,19].) More recently, Chakraborti and Patriarca [3] proposed a novel theory of power law distributions derived solely from a variational principle based on the Boltzmann entropy.

Recent studies on power laws in Economics and Finance are fully developed in Econophysics, a new scientific field that applies physics to economic and financial phenomena. (For a review of Econophysics, see [4].) Numerous attempts have been made by scholars of Econophysics to show the power law behavior of stock price fluctuations—so-called stock returns-and to explain its origin.[1] In observing the mechanism of stock markets, the heartbeat of a capitalist economic system, it is apparent that these markets can be viewed as complex systems made up of many investors interacting with one another. Thus, it seems that finding power laws in stock markets is a natural result and evidence that stock markets are indeed similar to other complex systems.

In this paper, we analyze a large database gleaned from the balance sheets of nearly 5000 of the world's largest listed companies over a 10-year period (2004–2013). We find that the power law distributions of the top share prices hold robustly over these ten years,[2] and that the

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[1] For examples, see [5–7,11,13,15,21].
[2] Kaizoji and Kaizoi [9], and Kaizoji [10] provided evidence of power laws for share prices in the Japanese stock market.

power law exponents are close to unity . The question we have to ask here is why the power law for share price is generated. We attempt to understand this power law from the point of view of company fundamentals as a measure of business performance.

To estimate company fundamentals, we construct a simple regression model with three financial indicators-dividends per share, cash flow per share, and book value per share—as the explanatory variables for stock price. These financial indicators are representative variables commonly used to evaluate a firm's business performance. We define fundamentals as the theoretical value calculated from the estimated regression model. We find that these fundamentals follow power laws, and that the power law distributions of these fundamentals match, to a very high degree of approximation, the power law distributions of the stock prices. It follows from our findings that the power law behavior of fundamentals is likely reflected in the power law for share price.

This paper is organized as follows: In Section 2, we describe the data used in this study. Section 3 shows the power law for stock prices. Section 4 indicates the power law for fundamentals. Section 5 reports our conclusions.

2. Data

The data for this paper came from the OSIRIS database of company financial statements and reports provided by Bureau Van Djk. OSIRIS includes 80,000 companies listed around the world. It provides information on company financials in a standardized format, and detailed stock data in US dollars. In this paper, we performed a statistical investigation of stock prices and financial indicators per share for more than 4000 companies listed worldwide, over a 10-year period (2004 through 2013).

3. Power-law for stock price

In this section, we investigate the distributions of stock prices. Fig. 1(a) shows the complementary cumulative distribution of the share prices of 4823 of the world's large companies at the end of 2013. We see that the distribution is extremely right-skewed. While most shares have low prices, there are a small number of shares which have very high prices. As a result, the distribution has a long tail to the right. Fig. 2(b) shows the complementary cumulative distribution plotted with logarithmic horizontal and vertical axes. As shown, this plot tends towards becoming a linear function in the high price range, indicating that the right tail of the cumulative probability distribution of stock price is described by a power law distribution,

$$Pr(X_i > x) = cx^{-\alpha} \tag{1}$$

where $Pr(X_i > x)$ denotes the probability that X_i is greater than x, c is a constant, and α is power law exponent. The power law distribution becomes, in natural logs:

$$\ln(Pr(X_i > x)) = \ln(c) - \alpha \ln(x) \tag{2}$$

It is well known that this procedure is strongly biased in small samples. To correct the bias, Gabaix and Ibragimov

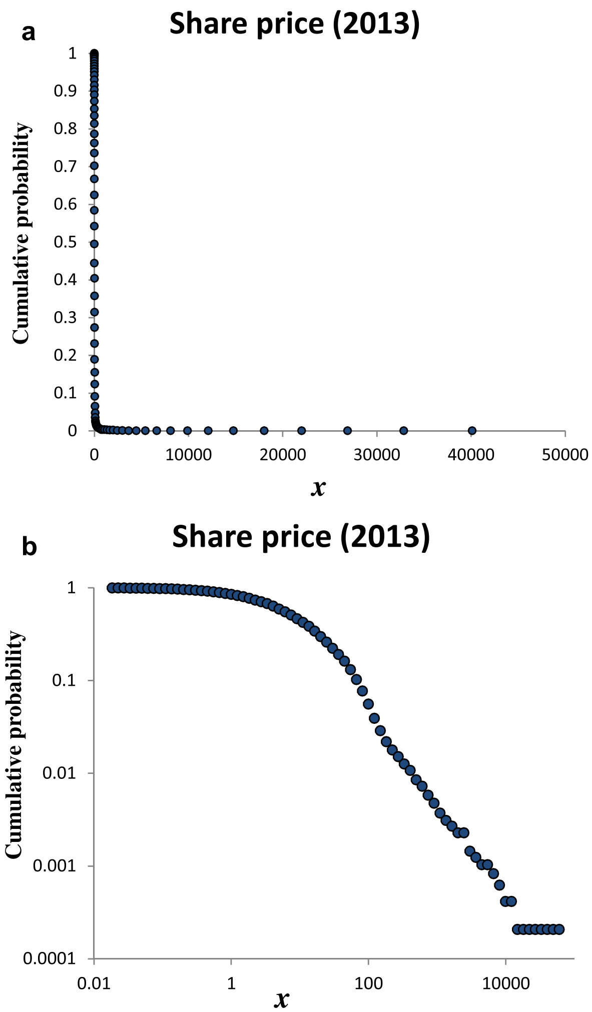

Fig. 1. (a) The complementary cumulative distribution of the share prices of 4823 of the world's largest companies at the end of 2013. (b) The complementary cumulative distribution plotted with logarithmic horizontal and vertical axes.

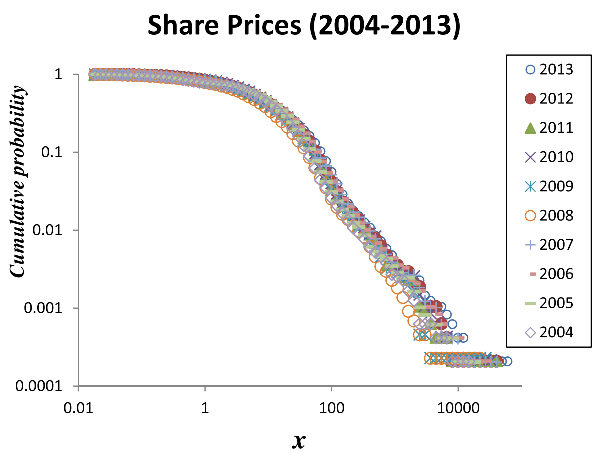

Fig. 2. Log-log plots of the complementary cumulative distribution plotted with logarithmic horizontal and vertical axes for each year from 2004 to 2013.

Table 1
The estimation of the power law exponents, α, of stock prices for each year from 2004 to 2013. The exponents are estimated from the regression model (3) using the method proposed by Gabaix and Ibragimov [12].

Year	$\ln(c)$	α	Std. Error	R^2
2004	4.4	0.993	0.126	0.991
2005	4.4	1.003	0.115	0.992
2006	4.6	1.017	0.115	0.994
2007	4.6	1.043	0.106	0.993
2008	4.4	1.069	0.144	0.984
2009	4.6	1.044	0.139	0.981
2010	4.4	1.006	0.104	0.992
2011	4.4	1.003	0.109	0.993
2012	4.4	1.007	0.101	0.993
2003	4.4	1.020	0.088	0.989

[12] propose the following OLS regression:

$$\ln(Rank_i - 1/2) = \ln(c) - \alpha \ln(x_i) \tag{3}$$

where i indexes company, and α is the estimator of the exponent. Gabaix and Ibragimov [12] prove that the estimated exponent, α, has a standard error of $\alpha(2/N)^{1/2}$, where N is the number of companies. The estimated power law exponent, α, is 1.02 ± 0.007 for 2013. The coefficient of determination is $R^2 = 0.989$. The result shows that the distribution fits the power law in the high price range.

We repeated the same regression for the stock price data for each year from 2004 to 2013. The power law exponents α are in the narrow range of 0.993 to 1.069. The results are shown in Table 1 and Fig. 2. Overall, the results suggest that the power law distribution of share prices is robust and the power law exponents are close to unity.

4. Power law for fundamentals

In the preceding section, it was found that the distributions of stock price follow a power law. In this section, we consider a reason why the power law for share price holds. We pay close attention to the so-called company fundamentals, the fair value of a share estimated from the company's business performance, assuming that investors buy a company's share based on company fundamentals. In fact, various methods are developed to justify share prices. (For a detailed explanation of stock valuation models, see, for example, [2].) We develop a simple empirical model of fundamentals using the data from the balance sheets of the companies in the study. We use dividends per share, cash flow per share, and book value per share as financial indicators to determine company fundamentals. Dividends per share and cash flow per share are commonly used to evaluate fundamentals since most basic stock valuation methods are based on company profits available to shareholders. We add book value per share as a third indicator. Book value per share is the amount of money that a shareholder would receive if a company were to liquidate. We propose a regression model of stock price, written as

$$Y_i = a + b_1 lnX_{1,i} + b_2 lnX_{2,i} + b_3 lnX_{3,i} + \varepsilon_i \tag{4}$$

where $a =$ a constant value,
$b_1 =$ the effect on stock price of dividends per share in a given year
$b_2 =$ the effect on stock price of cash flow per share in a given year
$b_3 =$ the effect on stock price of book value per share in a given year and
$\varepsilon_i =$ the error term reflecting other factors that have an impact on stock price.

The estimated coefficients for the linear regression Eq. (4) are summarized in Table 2. All of the estimated coefficients have a positive sign. The value in parenthesis is the p-value for a test of the null hypothesis that the coefficient is equal to zero. The p-values for all terms are very low (close to zero), indicating that the null hypothesis can be rejected. The R^2 statistic has a high value for all of the regression models, indicating that the regression model explains the variation in stock prices very well. More concretely, it means that the theoretical value explains more than 90% of the total variation in the stock prices about the average. The theoretical value of share price can be written as

$$\ln \hat{Y}_i = a + b_1 lnX_{1,i} + b_2 lnX_{2,i} + b_3 lnX_{3,i} \tag{5}$$

Since the theoretical value is modeled by only three financial indicators representing a company's business performance, we regard the theoretical value, $\hat{Y}_i$ calculated from regression Eq. (5) as the fundamentals. More precisely, we define fundamentals as

$$\hat{Y}_i = A(X_{1,i}^{b_1} X_{2,i}^{b_2} X_{3,i}^{b3}). \tag{6}$$

where $A = exp(a)$.

Table 2
The estimation of the coefficients of the linear regression Eq. (4). The table shows the estimated coefficients for each year from 2004 to 2013. Within the parenthesis indicates p-value.

Year	a	b_1	b_2	b_3	R^2	No. sample
2004	1.818 (0.00)	0.198 (0.00)	0.395 (0.00)	0.391 (0.00)	0.902	4807
2005	1.833 (0.00)	0.190 (0.00)	0.410 (0.00)	0.431 (0.00)	0.895	4884
2006	2.086 (0.00)	0.268 (0.00)	0.404 (0.00)	0.338 (0.00)	0.898	4822
2007	2.180 (0.00)	0.332 (0.00)	0.353 (0.00)	0274 (0.00)	0.872	4914
2008	1.613 (0.00)	0.344 (0.00)	0.277 (0.00)	0.392 (0.00)	0.891	4383
2009	1.917 (0.00)	0.338 (0.00)	0.265 (0.00)	0.376 (0.00)	0.888	4364
2010	2.060 (0.00)	0.339 (0.00)	0.326 (0.00)	0.296 (0.00)	0.890	4675
2011	1.956 (0.00)	0.345 (0.00)	0.333 (0.00)	0.308 (0.00)	0.895	4719
2012	2.011 (0.00)	0.348 (0.00)	0.317 (0.00)	0.311 (0.00)	0.891	4770
2013	2.119 (0.00)	0.322 (0.00)	0.377 (0.00)	0.286 (0.00)	0.888	4823

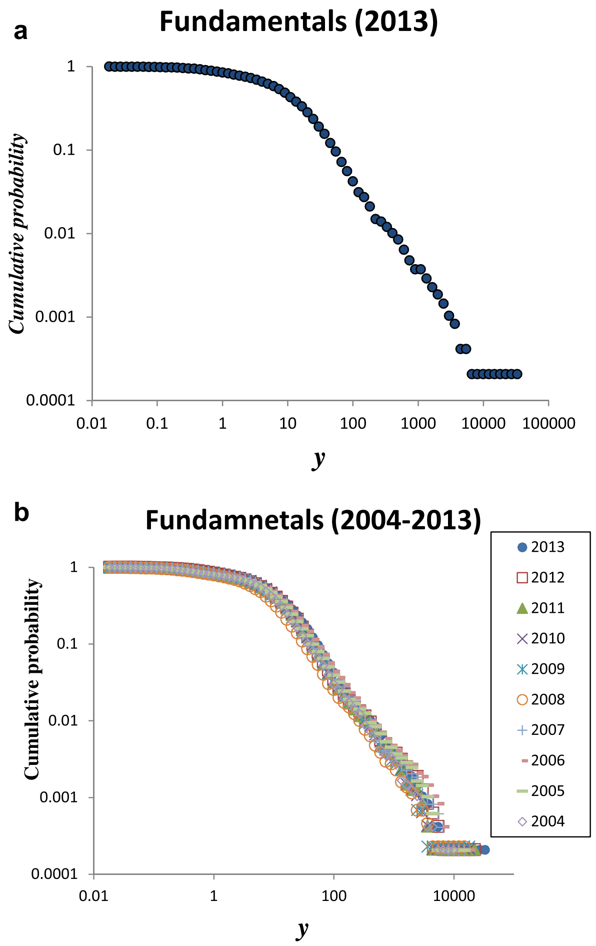

Fig. 3. (a) A log-log plot of the complementary cumulative distribution of the fundamentals of the world's large companies at the end of 2013. (b) Log-log plots of the complementary cumulative distributions of the fundamentals for each year from 2004 to 2013.

Fig. 3 (a) indicates the complementary cumulative distribution of fundamentals in 2013 and Fig. 3(b) shows the complementary cumulative distributions of fundamentals (6) for the years 2004 through 2013, plotted on log-log coordinates. As seen in the figures, the complementary cumulative distributions of fundamentals can be expressed as power law distributions,

$$Pr(\hat{Y}_i > y) = By^{-\beta}. \tag{7}$$

The power law exponent of fundamentals, β, for 2013, estimated from the OLS regression method proposed by Gabaix and Ibragimov [12], is 1.033 ± 0.008, ($R^2 = 0.991$), while the power law exponent of the stock price, α, in 2013 is 1.02 ± 0.007, ($R^2 = 0.989$). The results are summarized in Table 3. The exponents are all in the narrow range of 1.002 to 1.096. The standard errors of the exponents are sufficiently low.

The scatter plot in Fig. 4(a) reveals relationships between year-end stock prices in 2013 and the fundamentals for that year. Fig. 4(b) plots the complementary cumulative distribution of year-end stock prices for 2013, and the distribution of fundamentals in the same year. As the figures show, the distribution of fundamentals matches substantially the distribution of stock price.

Why does the distribution of stock price coincide with that of the company fundamentals? It is obvious that investors carefully evaluates company fundamentals, and invest their money in the stocks they select. Therefore, it

Table 3
The estimation of the power-law exponents of fundamentals, β, of each year end from 2004 to 2013. The exponents are estimated by the method proposed by Gabaix and Ibragimov [12].

Year	$\ln(B)$	β	Std. Error	R^2
2004	4.2	1.096	0.112	0.994
2005	4.6	1.004	0.114	0.985
2006	4.6	1.017	0.109	0.991
2007	4.4	1.071	0.106	0.993
2008	4.0	1.053	0.112	0.995
2009	4.2	1.073	0.114	0.992
2010	4.2	1.078	0.108	0.994
2011	4.4	1.036	0.119	0.987
2012	4.2	1.002	0.096	0.992
2013	4.6	1.033	0.119	0.991

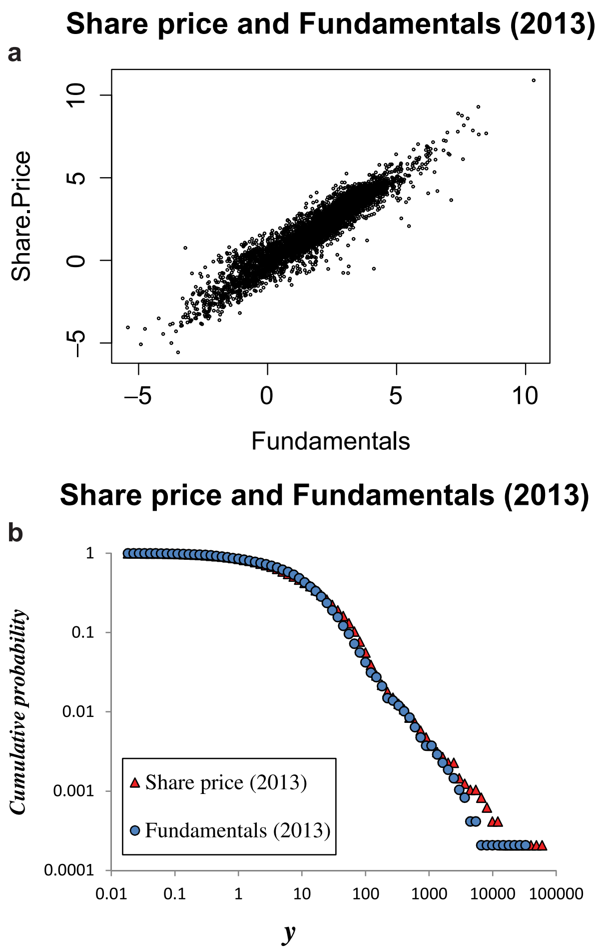

Fig. 4. (a) The scatter plot reveals relationships between year-end stock prices in 2013 and the fundamentals for that year. (b) Log-log plots of the complementary cumulative distribution of year-end stock prices for 2013, and the distribution of fundamentals in the same year.

seems reasonable to suppose that the power law behavior in fundamentals is reflected in the power law for share price.

5. Conclusion

It can be said from our analysis of a comprehensive financial data set that the power law distribution of stock prices and the power law distribution of fundamentals are significantly robust and stable over time. There is no disagreement that financial analysts and smart investors analyze company fundamentals to determine whether the shares they are considering merit their investment. Therefore, a credible hypothesis is that the power law for fundamentals gives rise to the power law for stock price. In our short study, these results can be described only summarily. We intend to discuss the empirical results in detail in future studies, specifically addressing two important issues: (i) Given that the estimated exponents of the power law distributions are close to unity, we will test statistically whether Zipf's law holds for the distributions of share price and company fundamentals; and (ii) even if the power law for fundamentals gives rise to the power law for stock price, this does not mean that stock markets necessarily evaluate fundamentals correctly. To investigate the deviation of share price from fundamentals, we will develop a model for more accurate estimation of company fundamentals using a panel regression model.

Finally, our empirical findings suggest that company values involve very large inequalities in a capitalist economy. Why does capitalism produce such disparities? Do these extreme inter-company disparities have any meaning for our society? So far there is no theory that sheds light on either the origin of the relevant power law. We also leave these questions as research challenges for the future.

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