Normal forms

Disjunctive Normal form (DNF)

$\downarrow$

$\wedge$

AND

Product

$\downarrow$

Elementary product

$p \wedge \sim p \wedge q$, $r \wedge \sim r \wedge q$

$\downarrow$

Sum of elem. product

$(p \wedge \sim p \wedge q) \vee (r \wedge \sim r \wedge q)$

$\downarrow$

Min terms

Two Sta. p, q : $p, q, \sim p, \sim q$

2^2 $\quad p \wedge q$, $\sim p \wedge q$, $p \wedge \sim q$, $\sim p \wedge \sim q$

Three Stat p, q & r :

2^3 $\quad p, q, r, \sim p, \sim q, \sim r$

$p \wedge q \wedge r$, $\sim p \wedge q \wedge r$, $p \wedge \sim q \wedge r$,

$p \wedge q \wedge \sim r$, $\sim p \wedge q \wedge \sim r$,

$p \wedge \sim q \wedge \sim r$, $\sim p \wedge \sim q \wedge r$,

$\sim p \wedge \sim q \wedge \sim r$

Principal Disjunctive Normal form. (PDNF)

PDNF = (Minterms) $\vee \ldots \vee$ (Minterms)

Conjunctive normal form (CNF)

$\downarrow$

$\vee$

OR

Sum

Elem. Sum

$p \vee \sim p \vee q$, $r \vee \sim r \vee q$

$\downarrow$

Product of el. sum

$(p \vee \sim p \vee q) \wedge (r \vee \sim r \vee q)$

$\downarrow$

Max terms

$p \vee q$, $\sim p \vee q$,

$p \vee \sim q$, $\sim p \vee \sim q$

Principal Conjunctive Normal form. (PCNF)

PCNF = (Maxterms) $\wedge \ldots \wedge$ (Maxterms)

$(p \to (q \wedge r)) \wedge (\neg p \to (\neg q \wedge \neg r))$

p	q	r	$\neg p$	$\neg q$	$\neg r$	$q \wedge r \equiv a$	$p \to a \equiv b$	$\neg q \wedge \neg r \equiv c$	$\neg p \to c \equiv d$	$b \wedge d$
T	T	T								T
T	T	F								F
T	F	T								F
F	T	T								F
T	F	F								F
F	T	F								F
F	F	T								F
F	F	F								T

$(\sim p \vee \sim q \vee r)$

$PDNF = (p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)$

$PCNF = \neg$ (Missing terms in PDNF)

$= \neg ((\sim p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (p \wedge q \wedge \sim r)$
$\vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge \sim r))$

$\therefore PCNF = (p \vee \sim q \vee \sim r) \wedge (\sim p \vee q \vee \sim r) \vee (\sim p \vee \sim q \vee r)$
$\wedge (p \vee q \vee \sim r) \wedge (p \vee \sim q \vee r) \wedge (\sim p \vee q \vee r)$

2) $p \vee (\neg P \to (Q \vee (\neg Q \to R)))$

Sol:

$$(\neg P \to R) \wedge (Q \leftrightarrow P) \equiv (\neg P \to R) \wedge ((Q \to P) \wedge (P \to Q)) \quad (\because \text{biconditional})$$

$$\equiv (\neg(\neg P) \vee R) \wedge (\neg Q \vee P) \wedge (\neg P \vee Q) \quad (\because \text{condi})$$

$\vee F$

$\wedge T$

$100 = 100 \times 1$

$= 100(2-1)$

$$\equiv (P \vee R) \wedge (\neg Q \vee P) \wedge (\neg P \vee Q) \quad (\because \text{double Neg.})$$

$$\equiv ((P \vee R) \vee F) \wedge ((\neg Q \vee P) \vee F) \wedge ((\neg P \vee Q) \vee F)$$

$$\equiv ((P \vee R) \vee (Q \wedge \sim Q)) \wedge ((\neg Q \vee P) \vee (R \wedge \sim R)) \wedge ((\neg P \vee Q) \vee (R \wedge \sim R))$$

$$= (P \vee R \vee Q) \wedge (P \vee R \vee \sim Q) \wedge (\neg Q \vee P \vee R) \wedge (\neg Q \vee P \vee \sim R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \sim R)$$

$$\equiv (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \sim R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \sim R)$$

$$\equiv (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (\neg P \vee Q \vee \sim R)$$

$$\text{PDNF} = \neg(\text{Missing terms in PCNF})$$

$$= \neg((P \vee Q \vee \sim R) \wedge (\sim P \vee \sim Q \vee R) \wedge (\sim P \vee \sim Q \vee \sim R))$$

$$\equiv (\sim P \wedge \sim Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge Q \wedge R)$$

Sol:

$$P\wedge(Q\wedge R)\vee(P\to Q) \equiv (P\wedge Q\wedge R)\vee(\neg P\vee Q) \quad (\because \text{cond: L})$$

$$\equiv \overbrace{(P\wedge Q\wedge R)}^{PDNF}\vee\big((\neg P\vee Q)\wedge T\big)$$

$$\equiv (P\wedge Q\wedge R)\vee\big((\neg P\vee Q)\wedge(R\vee\neg R)\big)$$

$$\equiv (P\wedge Q\wedge R)\vee\big((\neg P\vee Q)\wedge R\big)\vee\big((\neg P\vee Q)\wedge\neg R\big)$$

$$\equiv (P\wedge Q\wedge R)\vee(\neg P\wedge R)\vee(Q\wedge R)\vee(\neg P\wedge\neg R)\vee(Q\wedge\neg R)$$

$$\equiv (P\wedge Q\wedge R)\vee\big((\neg P\wedge R)\wedge T\big)\vee\big((Q\wedge R)\wedge T\big)\vee\big((\neg P\wedge\neg R)\wedge T\big)\vee\big((Q\wedge\neg R)\wedge T\big)$$

$$\equiv (P\wedge Q\wedge R)\vee\big((\neg P\wedge R)\wedge(Q\vee\neg Q)\big)\vee\big((Q\wedge R)\wedge(P\vee\neg P)\big)\vee\big((\neg P\wedge\neg R)\wedge(Q\vee\neg Q)\big)\vee\big((Q\wedge\neg R)\wedge(P\vee\neg P)\big)$$

$$\equiv (P\wedge Q\wedge R)\vee(\neg P\wedge R\wedge Q)\vee(\neg P\wedge R\wedge\neg Q)\vee(Q\wedge R\wedge P)\vee(Q\wedge R\wedge\neg P)\vee(\neg P\wedge\neg R\wedge Q)\vee(\neg P\wedge\neg R\wedge\neg Q)$$
$$\vee(Q\wedge\neg R\wedge P)\vee(Q\wedge\neg R\wedge\neg P)$$

$$\equiv (P\wedge Q\wedge R)\vee(\neg P\wedge Q\wedge R)\vee(\neg P\wedge\neg Q\wedge R)\vee(P\wedge Q\wedge R)\vee(\neg P\wedge Q\wedge R)\vee(\neg P\wedge\neg Q\wedge R)\vee(\neg P\wedge\neg Q\wedge\neg R)\vee(P\wedge Q\wedge\neg R)\vee(\neg P\wedge Q\wedge\neg R)$$

$$PDNF \equiv (P\wedge Q\wedge R)\vee(\neg P\wedge Q\wedge R)\vee(\neg P\wedge\neg Q\wedge\neg R)\vee(P\wedge Q\wedge\neg R)\vee(\neg P\wedge\neg Q\wedge\neg R)\vee(\neg P\wedge Q\wedge\neg R)$$

$$PCNF \equiv \neg\Big[(P\wedge\neg Q\wedge R)\vee(P\wedge\neg Q\wedge\neg R)\Big]$$

$$\equiv (\neg P\vee Q\vee\neg R)\wedge(\neg P\vee Q\vee R)$$

Module - I

Tutorial - 1

1. Prove $(P \vee Q) \wedge (\neg P \wedge (\neg P \wedge Q) \equiv (\neg P \wedge Q)$ without using the truth table.

2. Without using the truth table obtain the PDNF and PCNF of the formula

$$(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$$

H.W: Obtain PDNF and PCNF for

(i) $\neg P \vee Q$

(ii) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

$2(x+y) = 2x+2y.$

$$(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge R))$$

$$\equiv (\neg P \vee (Q \wedge R)) \wedge (\neg(\neg P) \vee (\neg Q \wedge R))$$

$$\equiv (\neg P \vee (Q \wedge R)) \wedge (P \vee (\neg Q \wedge R))$$

$$\equiv (\neg P \vee Q) \wedge (\neg P \vee R) \wedge (P \vee \neg Q) \wedge (P \vee R)$$

$$\equiv ((\neg P \vee Q) \vee F) \wedge ((\neg P \vee R) \vee F) \wedge ((P \vee \neg Q) \vee F) \wedge ((P \vee R) \vee F)$$

$$\equiv ((\neg P \vee Q) \vee (R \wedge \neg R)) \wedge ((\neg P \vee R) \vee (Q \wedge \neg Q)) \wedge ((P \vee \neg Q) \vee (R \wedge \neg R))$$

$$\wedge ((P \vee R) \vee (Q \wedge \neg Q))$$

$$\equiv (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee R \vee Q) \wedge (\neg P \vee R \vee \neg Q) \wedge (P \vee \neg Q \vee R)$$

$$\wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q)$$

$$p \to \left[(p \to q) \wedge \left(\neg(\neg q \vee \neg p)\right)\right]$$

$$\equiv p \to \left[(\neg p \vee q) \wedge (q \wedge p)\right]$$

$$\equiv \neg p \vee \left[\overset{P}{(\neg p} \vee \overset{Q}{q)} \wedge \overbrace{(q \wedge p)}^{R}\right]$$

$$\equiv \neg p \vee \left[\left(\neg p \wedge (q \wedge p)\right) \vee \left(q \wedge (q \wedge p)\right)\right]$$

$$\equiv \neg p \vee \left[\left(\neg p \wedge (p \wedge q)\right) \vee \left(q \wedge (q \wedge p)\right)\right]$$

$$\equiv \neg p \vee \left[\left((\neg p \wedge p) \wedge q\right) \vee \left((q \wedge q) \wedge p\right)\right]$$

$$\equiv \neg p \vee \left((F \wedge q) \vee (q \wedge p)\right)$$

$$\equiv \neg p \vee \left(F \vee (q \wedge p)\right)$$

$$\equiv (\neg p) \vee (q \wedge p)$$

$$\equiv (\neg p \wedge T) \vee (p \wedge q)$$

$$\equiv \left(\neg p \wedge (q \vee \neg q)\right) \vee (p \wedge q)$$

$$PDNF \equiv (\neg p \wedge q) \vee (\neg p \wedge \neg q) \vee (p \wedge q)$$

$$PCNF = \neg(\text{Min. } PDNF)$$

$$= \neg(p \wedge \neg q)$$

$$PCNF = \neg p \vee q$$

Theory of Inference:

C

premises H_1 and H_2

1. $H_1 : \neg p$, $H_2 : p \vee q$, $C : q$

Sol:

p	q (C)	$\neg p$ (H_1)	$p \vee q$ (H_2)
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

2) $H_1 : p \to q$, $H_2 : \neg p$, $C : q$.

p	q (C)	$\neg p$ (H_2)	$p \to q$ (H_1)
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

H.W:

1. $H_1 : p \to q$, $H_2 : p$, $C : q$

2. $H_1 : p \to q$, $H_2 : \neg(p \wedge q)$, $C : \neg p$

3. $H_1 : \neg p$, $H_2 : p \leftrightarrow q$, $C : \neg(p \wedge q)$

4. $H_1 : p \to q$, $H_2 : q$, $C : p$

$P \to Q$, $Q \to R$ and $P \Rightarrow R$.

Step No.	Den.	Rule	Re.
1.	$P \to Q$	P	1. Rule P
2.	$Q \to R$	P	2. Rule T
3.	$P \to R$	T(1,2)	3. Rule CP
4	P	P	
5.	R	T(3,4)	$P, P \to Q \equiv Q$
6	$P \to R$	CP	

$R \to S$

Rules of inference:

Rule P:

premise

$R \to S$

Type - 1: Direct Method

Type - 2: Based on Rule CP:

(Write Rule CP)

(i). If ——, then ——. form

LHS

(ii) $(P_1 \wedge P_2 \wedge \ldots \wedge P_n) \to F$

$P \Rightarrow Q$

$P \to Q$ is ta..

P, Q, R

$P \vee Q$

$(P \wedge Q) \to R$

$P \to Q$

(iii) $\left[\left((P_1 \wedge P_2 \wedge \ldots \wedge P_n) \wedge \neg C\right)\right] \to F$

$P \wedge \sim P \equiv F$

Rule T:

$P, P \to Q \equiv Q$

$P \to Q \Rightarrow \neg Q \to \neg P$

$P \to Q,\ Q \to R \Rightarrow P \to R$

$P, Q \Rightarrow P \wedge Q$

$P \wedge Q \Rightarrow P, Q$

$\neg Q, P \to Q \Rightarrow \neg P$

$P \to Q \equiv \neg P \vee Q$

$(P \vee Q) \wedge \neg P \Rightarrow Q$

$(P \to Q) \wedge (P \to R) \Rightarrow$

$\neg P \vee \neg P \Rightarrow \neg P$

$Q, P \to Q \Rightarrow \neg P$

$P \wedge \neg P \equiv F$

$\neg(P \wedge Q) \equiv \neg P \vee \neg$

$P \wedge Q \Rightarrow Q \wedge$

$C \vee D$, $(C \vee D) \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$ and $(A \wedge \neg B) \rightarrow (R \vee S) \Rightarrow R \vee S$.

Sol:

Step No.	Derivation	Rules	Reasons
1.	$C \vee D$	Rule P	
2.	$(C \vee D) \rightarrow \neg H$	P	
3.	$\neg H$	T (1,2)	$P, P \rightarrow Q \Rightarrow Q$
4.	$\neg H \rightarrow (A \wedge \neg B)$	P	
5.	$A \wedge \neg B$	T (3,4)	$P, P \rightarrow Q \Rightarrow Q$
6.	$(A \wedge \neg B) \rightarrow (R \vee S)$	P	
7.	$R \vee S$	T (5,6)	$P, P \rightarrow Q \Rightarrow Q$

⋮

$P \to (Q \to S), \neg R \vee P, Q \Rightarrow R \to S.$

Sol:

Step No.	Derivation	Rules	Reason.
1.	$P \to (Q \to S)$	P	
2.	$\neg R \vee P$	P	
3.	Q	P	
4.	R	P	assumed as additional prem
5.	$R \to P$	T(2)	$P \to Q \equiv \neg P \vee Q$
6.	P	T(4,5)	$P, P \to Q \Rightarrow Q$
7.	$Q \to S$	T(1,6)	$P, P \to Q \Rightarrow Q$
8.	S	T(3,7)	$P, P \to Q \Rightarrow Q$
9.	$R \to S$	CP(4,8)	

Sol:

P : There was a ball game.

Q : Travelling was diff.

R : They arrived on time.

$\neg Q, P \to Q \Rightarrow \neg P$

$P \to Q$, $R \to \neg Q$, R and the cond. is $\neg P$

H.W:

1. $P \to Q$, $Q \to R$ and $P \Rightarrow R$

2. $P \to Q$, $Q \to (R \wedge S)$, $\neg R \vee (\neg T \vee U)$, $P \wedge T \Rightarrow U$

3. $\neg P \vee Q$, $\neg Q \vee R$, $R \to S \Rightarrow P \to S$.

$P \wedge \neg P \equiv F$

P : Jack misses

Q :

$$\underbrace{P \to Q,\ Q \to R,\ S \to \neg R}_{\Downarrow},\ P \wedge S$$

$$\neg(P \wedge S)$$

Step No.	Derivation	Rules	Reasons
1.	$P \to Q$	P	
2	$Q \to R$	P	
3	$S \to \neg R$	P	
4	$P \wedge S$	P	
5.	$P \to R$	T(1,2)	$P \to Q,\ Q \to R \Rightarrow P \to R$
6	$\neg R \to \neg P$	T(5)	$P \to Q \Rightarrow \neg Q \to \neg P$
7.	$S \to \neg P$	T(3,6)	$P \to Q,\ Q \to R \Rightarrow P \to R$
8.	$\neg S \vee \neg P$	T(7)	Condi ln.
9.	$\neg(S \wedge P)$	T(8)	Demorg.
10.	$\neg(P \wedge S)$	T(9)	Comm.
11.	$(P \wedge S) \wedge \neg(P \wedge S) \equiv F$	T(4,10)	Negat ln.

$\therefore$

$P \to Q,\ Q \to R,\ P \vee R \Rightarrow R$

Sol:

Let $H_1 : P \to Q,\ H_2 : Q \to R,\ H_3 : P \vee R,\ C : R$

Assume an additional premes $\neg C : \neg R$

IPT:

Step No.	Derivation	Rule	Rem.
1	$P \to Q$	P	
2	$Q \to R$	P	
3	$P \vee R$	P	
4	$\neg R$	P	Assume an additional Premise
5	$P \to R$	T(1,2)	
6.	$\neg P \to R$	T(3)	$P \to Q \equiv \neg P \vee Q$
7.	$\neg R \to P$	T(6)	$\neg P \to Q \equiv P \vee Q$
8	P	T(4,7)	
9	R	T(5,8)	
10	F	T(4,9)	$P \wedge \sim P \equiv F$

H.W:

1. ST: $P \to Q$, $P \to R$, $Q \to \neg R$ and P are inconsistent.

2. ST: $R \to \neg Q$, $R \vee S$, $S \to \neg Q$, $P \to Q \Rightarrow \neg P$ using indirect method.