P Q.

Negotation $\sim P$, $\neg P$

AND $\wedge$

OR $\vee$

P	Q	$P \wedge Q$	$P \vee Q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

If

Let

$P \rightarrow Q$

$P \leftrightarrow Q$

Module - I

Propositional Calculus

Truth Table:

Sub-propositions

$p \vee q$

atomic (or) primary or primitive

1. Negation

" " " $\sim p$

"it

$\neg p$ (or) p' (or) $\bar{p}$ (or) $\sim p$ not p

Truth Table of Negation.

2) p: $x < y$.

$\sim p$: $x \geq y$

p	$\sim p$
T	F
F	T

Less $<$

Conjunction (AND):

" p and q "

$p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	f	F

If p & q is,

Let p & q be

p :

q :

$p \wedge q$:

Disjunction (OR):

P:

Q:

P ∨ Q:

" if P, then Q "

$P \rightarrow Q$ (P then Q)

T ⟵ P : Water 100°c

F ⟵ Q : Water vap. → hot

want to ✗ Water vap. ⟵

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

key:

F ⟵ P : If Tiger wings, then.

T ⟵ Q : Earth rotate.

T ⇒ Tiger wig ✗

F ⇒ Earth rotate ✗

Biconditional

"p if and only if q" (or)

"p iff q"

$p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Eg:

p: $x = 5$

Q: $x + 5 = 10$

$P \leftrightarrow Q$: $x = 5$ iff $x + 5 = 10$.

Note: (i) $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

1. $\sim(p \wedge q) \longleftrightarrow (\sim p) \vee (\sim q)$

p	q	$p \wedge q$	$\sim(p \wedge q)$ $\equiv a$	$\sim p$	$\sim q$	$(\sim p) \vee (\sim q)$ $\equiv b$	$a \leftrightarrow b$
T	T						
T	F						
F	T						
F	F						

2. $p \vee (\sim q)$

3. $p \wedge (\sim p)$

4. $\neg(P \vee Q) \wedge (P \vee R)$

P	Q	R
T	T	T
T	T	F
T	F	T
F	T	T
T	F	F
F	F	T
F	T	F
F	F	F

Tautology:

eg: $P \vee \sim P$

P	$\sim P$	$P \vee \sim P$	$P \wedge \sim P$
T	F	T	F
F	T	T	F

Contradictions:

eg: $P \wedge (\sim P)$

Contingency:

1. $P \vee Q \leftrightarrow Q \vee P$ is Taut

2. $P \rightarrow (P \vee Q)$ is Taut.

Fallacy:

3. $(P \wedge Q) \wedge \neg(P \vee Q)$ is cont.

H.W:

1. $(P \leftrightarrow Q) \leftrightarrow (R \leftrightarrow S)$ TT

2. $q \wedge (p \rightarrow q) \rightarrow p$

P : Rich

Q : Hap.

$\sim P$: Poor

$\sim Q$: Unh.

Symbolic form

(i) $\sim P \wedge Q$

(ii) $P \vee \sim Q$

$p \to q$: Converse, Inverse and Contrapositive:

$q \to p$ $\sim p \to \sim q$ $\sim q \to \sim p$

Converse:

$q \to p$

Logical equivalence:

$p \leftrightarrow q$ $\quad p \Longleftrightarrow q$ (or) $p \equiv q$ $\quad = \quad \equiv$

Equivalence laws:

Laws	Name
1. $\sim\sim p \equiv p$	Double Negation
2. $P \wedge Q \equiv Q \wedge P$ $P \vee Q \equiv Q \vee P$	Commutative law
3. $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$ $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$	Associative law
4. $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$ $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$	Distributive law
5. $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$ $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$	De Morgan's law
6. $P \wedge P \equiv P$ $P \vee P \equiv P$	Idempotent law $\quad 1^2 = 1$
7. $P \wedge T \equiv P$ $P \vee F \equiv P$	Identity law $\quad a \cdot 1 = a$
8. $P \vee T \equiv T$ $P \wedge F \equiv F$	Dominance law
9. $P \wedge \sim P \equiv F$ $P \vee \sim P \equiv T$	Negation law
10. $P \vee (P \wedge Q) \equiv P$ $P \wedge (P \vee Q) \equiv P$	Absorption law.

1. $P \rightarrow Q \equiv \neg P \vee Q$

2. $P \rightarrow Q \equiv \neg Q \rightarrow \neg P$

3. $P \vee Q \equiv \sim P \rightarrow Q$

4. $P \wedge Q \equiv \sim (P \rightarrow \sim Q)$

5. $\neg (P \rightarrow Q) \equiv P \wedge \neg Q$.

6. $(P \rightarrow Q) \wedge (P \rightarrow R) \equiv P \rightarrow (Q \wedge R)$

7. $(P \rightarrow Q) \vee (P \rightarrow R) \equiv P \rightarrow (Q \vee R)$

8. $(P \rightarrow R) \wedge (Q \rightarrow R) \equiv (P \vee Q) \rightarrow R$

9. $(P \rightarrow R) \vee (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$

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1. $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

2. $P \leftrightarrow Q \equiv \sim P \leftrightarrow \sim Q$

3. $P \leftrightarrow Q \equiv (P \wedge Q) \vee (\sim P \wedge \sim Q)$

4. $\sim (P \leftrightarrow Q) \equiv P \leftrightarrow \sim Q$.

1. Prove that

$$\neg(p \wedge q) \rightarrow (\neg p \vee (\neg p \vee q)) \equiv (\neg p \vee q)$$

H.W (i) using Truth table

$p \rightarrow q \equiv \neg p \vee q$

(ii). using equivalence law

Sol:

$$\neg(p \wedge q) \rightarrow (\neg p \vee (\neg p \vee q))$$

$$\equiv \neg(\neg(p \wedge q)) \vee (\neg p \vee (\neg p \vee q)) \quad (\because \text{conditional law } p \rightarrow q \equiv \neg p \vee q)$$

$$\equiv (p \wedge q) \vee (\neg p \vee (\neg p \vee q)) \quad (\because \text{Double neg. } \sim\sim p \equiv p)$$

$$\equiv (p \wedge q) \vee ((\neg p \vee \neg p) \vee q) \quad (\because \text{Asso. law})$$

$$\equiv (p \wedge q) \vee (\neg p \vee q) \quad (\because \text{idemp. law})$$

$$\equiv ((p \wedge q) \vee \neg p) \vee q \quad (\because \text{Assoc.})$$

$$\equiv ((p \vee \neg p) \wedge (q \vee \neg p)) \vee q \quad (\because \text{Distr.})$$

$$\equiv (T \wedge (q \vee \neg p)) \vee q \quad (\because \text{Negat.})$$

$$\equiv (q \vee \neg p) \vee q \quad (\because \text{identity})$$

$$\equiv (\neg p \vee q) \vee q \quad (\because \text{comm.})$$

$$\equiv \neg p \vee (q \vee q) \quad (\because \text{Asso.})$$

$$\equiv \neg p \vee q \quad (\because \text{idemp.})$$

//

$$p \to (q \to r) \equiv p \to (\neg q \vee r) \equiv (p \wedge q) \to r$$

Sol:

$$p \to (q \to r) \equiv p \to (\neg q \vee r)$$

$$\equiv \neg p \vee (\neg q \vee r)$$

$$\equiv (\neg p \vee \neg q) \vee r$$

$$\equiv \neg (p \wedge q) \vee r$$

$$\equiv (p \wedge q) \to r$$

3) SI: $\big(\neg p \wedge (\neg q \wedge r)\big) \vee (q \wedge r) \vee (p \wedge r) \equiv r.$

NOT

AND

OR

p :

P :

p	q	r
T	T	T
T	F	
F	T	
F	F	

$p \to q$

$p \leftrightarrow q$

$\downarrow$

iff

If $x = 3$, then $x + 2 = 5$