Predicates   Quantifier

Ram is a good boy.
$\underline{\text{Ram}}_{S}$ is a $\underline{\text{good boy}}_{P}$

B(r).

Quantifier → Universal, Existential

SFO3

x is a living thing

	Universal	Existential
1. Defn:	"for all x" (or) "for every x"	"There exists some x" or "for some x"
2. Denoted:	$(\forall x)$ or (x)	$(\exists x)$
3. Symbolic rep.:	' $\longrightarrow$ '	' $\wedge$ '
4. Phrases are	(i) for all x (ii) for every x (iii) for each x (iv) everything x such that (v) for each thing x is such that	(i) there exists x such that (ii) for some x (iii) Some x such that (iv) There is atleast some x such that
5. Example:	Every apple is red A(x) : x is an apple R(x) : x is red $(\forall x)\ [A(x) \rightarrow R(x)]$ For all x, if x is an apple, then x is red	Some men are genius M(x) : x is a man G(x) : x is genius $(\exists x)\ [M(x) \wedge G(x)]$ There exists some x, such that x is a man and x is a

Univer. Exis.

$(\forall x)$ (or) (x) $(\exists x)$

$\rightarrow$ $\wedge$

$(\forall x) \, (P(x) \rightarrow Q(x)) \vee R(x)$

1^{st} x $\rightarrow$ Bound

Scope

last x $\rightarrow$ Free.

Universe of Discourse:

eg. 1 $(x)\,[P(x) \to Q(x)] \wedge R(y)$

Sol:

Bound variable : x

Free variable : y

Scope : $P(x) \to Q(x)$

Sol:

with UD:

Universe of discourse : Set of all std in the cl.

$D(x)$: x Tak DM.

$(\forall x)\ D(x)$

without UD:

$C(x)$: x is Std. in cl.

$D(x)$: x is tak DM

$(\forall x)\ [C(x) \rightarrow D(x)]$

$P(z)$: z is the pr.

$F(x, z)$: x is fath of z

$M(z, y)$: z is mom of y

$$
\begin{tikzcd}
x \arrow[d, "\text{Dad}"] & \\
z \arrow[r, "\text{Mom}"] & y
\end{tikzcd}
$$

$$(\exists z)\left[P(z) \land F(x,z) \land M(z,y)\right]$$

$Q(x)$: x less than 5

(i) $(\forall x)\, Q(x)$ (ii) $(\exists x)\, Q(x)$.

1. $\{-1, 0, 1, 2, 3\}$ 2. $\{3, -2, 7, 8, -3\}$ 3. $\{15, 20, 25\}$.

Sol:

given $Q(x)$:

Let UD1 : $\{-1, 0, 1, 2, 3\}$

UD2 :

UD3 :

(i) $(\forall x)\, Q(x)$	(ii) $(\exists x)\, Q(x)$
T	T
F	T
F	F

Negation of Quantified Statement:

1. $\neg((\forall x)P(x)) \equiv (\exists x)\ \neg P(x)$
2. $\neg((\exists x)\ P(x)) \equiv (\forall x)\ \neg P(x)$

Logical Expressions & Implications:

1. $(\forall x)\ A(x) \vee (\forall x)\ B(x) \Rightarrow (\forall x)\ [A(x) \vee B(x)]$
2. $(\exists x)\ (A(x) \wedge B(x)) \Rightarrow (\exists x)\ A(x) \wedge (\exists x)\ B(x)$
3. $(\exists x)\ (A(x) \vee B(x)) \equiv (\exists x)\ A(x) \vee (\exists x)\ B(x)$
4. $(\forall x)\ (A(x) \wedge B(x)) \equiv (\forall x) A(x) \wedge (\forall x) B(x)$
5. $\neg [(\exists x) A(x)] \equiv (\forall x)\ \neg A(x)$
6. $\neg [(\forall x)\ A(x)] \equiv (\exists x)\ \neg A(x)$

(i) Universal Specification (US)

$$(\forall x) P(x) \Rightarrow P(y) \text{ or } P(a)$$

(ii) Existential Specification (ES)

$$(\exists x) P(x) \Rightarrow P(y) \text{ or } P(a)$$

Delete

(iii) Universal Generalisation (UG)

$$P(y) \Rightarrow (\forall x) P(x)$$

(iv) Existential Generalisation (EG)

$$P(y) \Rightarrow (\exists x) P(x)$$

Add

1. $(\forall x)[P(x) \to Q(x)], (\forall x)(Q(x) \to R(x)) \Rightarrow (\forall x)[P(x) \to R(x)]$

Step. No.	Derivation / Premise	Rules	Reason
1.	$(\forall x)[P(x) \to Q(x)]$	P	
2.	$(\forall x)[Q(x) \to R(x)]$	P	,
3.	$P(y) \to Q(y)$	T(1)	US
4	$Q(y) \to R(y)$	T(2)	US
5	$P(y) \to R(y)$	T(3, 4)	$P \to Q$, $Q \to R$ $\Rightarrow P \to R$
6.	$(\forall x)(P(x) \to R(x))$	T(5)	UG