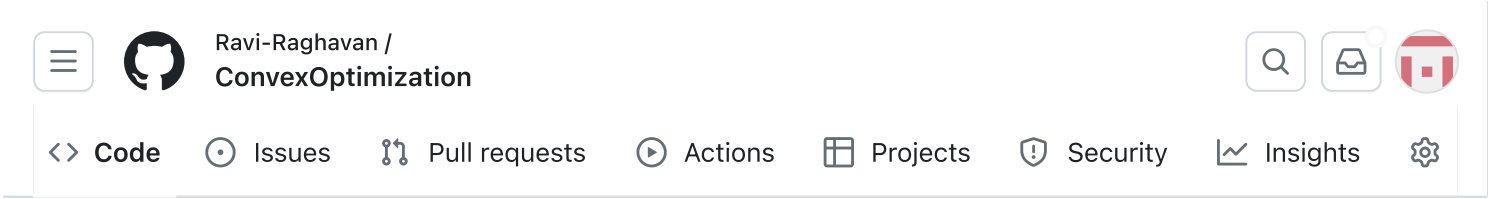


ConvexOptimization / Homework #5 - Part 2.ipynb

Ravi-Raghavan I think Im done with Part 2 35 minutes ago

2.14 MB

Homework #5 - Ravi Raghavan

In [1]:
```
#Import libraries
import numpy as np
import matplotlib.pyplot as plt
```

Theoretical Calculations of Gradient and Hessian

$f(x) = -\sum_{i=1}^{m} \log(1 - a_i^T x) - \sum_{i=1}^{n} \log(1 - x_i^2)$

$x \in \mathbb{R}^n$ and $domf = \{x | a_i^T x < 1, i = 1, 2, \ldots, m, |x_i| < 1, i = 1, 2, \ldots, n\}$

This can also be expressed as

$f(x) = -\sum_{i=1}^{m} \log(1 - a_i^T x) - \sum_{i=1}^{n} \log(1 - x_i) - \sum_{i=1}^{n} \log(1 + x_i)$

$x \in \mathbb{R}^n$ and $domf = \{x | a_i^T x < 1, i = 1, 2, \ldots, m, |x_i| < 1, i = 1, 2, \ldots, n\}$

$\frac{\partial f}{\partial x_j} = -(\sum_{i=1}^{m}(\frac{1}{1-a_i^T x})(-a_{ij})) - \frac{1}{(1-x_j^2)}(-2x_j)$

$\frac{\partial f}{\partial x_j} = \sum_{i=1}^{m}(\frac{a_{ij}}{1-a_i^T x}) + \frac{2x_j}{(1-x_j^2)}$

$\frac{\partial f}{\partial x_k \partial x_j}$:

Case where $j = k$:

$\frac{\partial f}{\partial x_k \partial x_j} = \sum_{i=1}^{m} a_{ij}(\frac{-1}{(1-a_i^T x)^2})(-a_{ik}) + \frac{2(1-x_j^2)-2x_j(-2x_j)}{(1-x_j^2)^2}$

$\frac{\partial f}{\partial x_k \partial x_j} = \sum_{i=1}^{m}(\frac{a_{ij}a_{ik}}{(1-a_i^T x)^2}) + \frac{2(1+x_j^2)}{(1-x_j^2)^2}$

$\frac{\partial f}{\partial x_k \partial x_j} = \sum_{i=1}^{m}(\frac{a_{ij}a_{ik}}{(1-a_i^T x)^2}) + \frac{2(1+x_j^2)}{(1-x_j^2)^2}$

Case where $j \neq k$:

$\frac{\partial f}{\partial x_k \partial x_j} = \sum_{i=1}^{m}(\frac{a_{ij}a_{ik}}{(1-a_i^T x)^2})$

Programmatic Implementations of Descent Methods

In [2]:
```
#Function to Perform Gradient Descent
#f: Compute function value at given point
#gradient: compute gradient of function at given point
#backtracking_algorithm: function used to perform backtracking
#A: matrix of ai vectors where ai are the rows of A
#x0: initial point of gradient descent
#max_iter: maximum number of iterations of Gradient Descent to run
#eta: used in convergence criteria
#alpha: alpha parameter for backtracking
```

```
#beta: beta parameter used for backtracking
def gradient_descent(f, gradient, backtracking_algorithm, A: np.ndarray, x0: np
    #Set initial iterate and function value
    x = x0
    fx = f(A, x)

    #maintain arrays to store iterates and function values throughout gradient
    points = np.array([x])
    function_values = []
    function_values.append(fx)

    #Store step sizes
    step_sizes = []

    #enter for loop of max_iter times
    for iter in range(max_iter):
        grad = gradient(A, x) #compute gradient
        # print(f"Iteration: {iter + 1}, Max of A @ x: {np.max(A @ x)}, Max of

        #if we have satisfied our convergence criteria, break from loop
        if np.linalg.norm(grad) <= eta:
            break

        descent_direction = -1 * grad #our descent direction is the negative gr
        step_size = backtracking_algorithm(A, x, descent_direction, alpha, beta
        x = x + (step_size * descent_direction) #compute next iterate

        points = np.append(points, x[np.newaxis, :, :], axis=0) #store point in
        fx = f(A, x) #calculate updated function value
        function_values.append(fx) #store updated function value
        step_sizes.append(step_size) #Store step size

    #store function value in array
    function_values = np.array(function_values)
    step_sizes = np.array(step_sizes)
    return points, function_values, step_sizes
```

In [3]:

```
#H: Hessian of function at a given point
#g: gradient of function at given point
#This function solves the system H @ x_nt = -g to calculate the newton directio
def solve_newton_system(H: np.ndarray, g: np.ndarray):
    L = np.linalg.cholesky(H) #compute Cholesky Decomposition of H
    n = L.shape[0]

    #forward substitution to solve Lw = b where b = -g
    w = np.zeros(shape = g.shape)
    b = -1 * g

    w[0, 0] = b[0, 0] / L[0, 0]
    for idx in range(1, n):
        w[idx, 0] = (b[idx, 0] - np.dot(L[idx, :idx].flatten(), w[:idx, 0].flat

    #backward substitution to solve L^T @ x_nt = w
    L_transpose = L.T
    x_nt = np.zeros(shape = g.shape)

    x_nt[n - 1, 0] = w[n - 1, 0] / L_transpose[n - 1, n - 1]
    for idx in range(n - 2, -1, -1):
        x_nt[idx, 0] = (w[idx, 0] - np.dot(L_transpose[idx, -1:idx:-1].flatten(

    #Calculate lambda^2
    lambda_squared = np.linalg.norm(w) ** 2
```

```
    ##Sanity Check
    closed_form = -1 * (np.linalg.inv(H) @ g)
    assert np.allclose(x_nt, closed_form)

    return x_nt, lambda_squared
```

In [4]:

```
#Function to Perform Newton's Method
#f: Compute function value at given point
#gradient: compute gradient of function at given point
#hessian: compute hessian of function at given point
#backtracking_algorithm: function used to perform backtracking
#A: matrix of ai vectors where ai are the rows of A
#x0: initial point of gradient descent
#max_iter: maximum number of iterations of Gradient Descent to run
#eta: used in convergence criteria
#alpha: alpha parameter for backtracking
#beta: beta parameter used for backtracking
def newton_descent(f, gradient, hessian, backtracking_algorithm, A: np.ndarray,
    #Set initial iterate and function value
    x = x0
    fx = f(A, x)

    #maintain arrays to store iterates and function values throughout gradient
    points = np.array([x])
    function_values = []
    function_values.append(fx)

    #Store step sizes
    step_sizes = []

    #enter for loop of max_iter times
    for iter in range(max_iter):
        grad = gradient(A, x) #compute gradient
        H = hessian(A, x) #compute hessian
        newton_direction, newton_parameter = solve_newton_system(H, grad) #Solv
        # print(f"Iteration: {iter + 1}, Max of A @ x: {np.max(A @ x)}, Max of

        #if we have satisfied our convergence criteria, break from loop
        if newton_parameter <= eta:
            break

        descent_direction = newton_direction #our descent direction is the newt
        step_size = backtracking_algorithm(A, x, descent_direction, alpha, beta
        x = x + (step_size * descent_direction) #compute next iterate

        points = np.append(points, x[np.newaxis, :, :], axis=0) #store point in
        fx = f(A, x) #calculate updated function value
        function_values.append(fx) #store updated function value
        step_sizes.append(step_size) #Store step size

    #store function value in array
    function_values = np.array(function_values)
    step_sizes = np.array(step_sizes)
    return points, function_values, step_sizes
```

In [5]:

```
#Function to Perform Newton's Method where the Hessian is reused every N iterat
#f: Compute function value at given point
#gradient: compute gradient of function at given point
#hessian: compute hessian of function at given point
#backtracking_algorithm: function used to perform backtracking
#A: matrix of ai vectors where ai are the rows of A
```

```
#x0: initial point of gradient descent
#max_iter: maximum number of iterations of Gradient Descent to run
#eta: used in convergence criteria
#alpha: alpha parameter for backtracking
#beta: beta parameter used for backtracking
#N: number of iterations after which Hessian is evaluated
def reuse_newton_descent(f, gradient, hessian, backtracking_algorithm, A: np.nd
    #Set initial iterate and function value
    x = x0
    fx = f(A, x)

    #maintain arrays to store iterates and function values throughout gradient
    points = np.array([x])
    function_values = []
    function_values.append(fx)

    #Store step sizes
    step_sizes = []

    #Store Hessians
    hessians = []
    H = hessian(A, x) #compute hessian
    hessians.append(H)

    #enter for loop of max_iter times
    for iter in range(max_iter):
        grad = gradient(A, x) #compute gradient

        #Get Hessian
        if (iter > 0) and (iter % N == 0):
            H = hessian(A, x)
            hessians.append(H)
        else:
            H = hessians[-1]

        newton_direction, newton_parameter = solve_newton_system(H, grad) #Solv
        # print(f"Iteration: {iter + 1}, Max of A @ x: {np.max(A @ x)}, Max of

        #if we have satisfied our convergence criteria, break from loop
        if newton_parameter <= eta:
            break

        descent_direction = newton_direction #our descent direction is the newt
        step_size = backtracking_algorithm(A, x, descent_direction, alpha, beta
        x = x + (step_size * descent_direction) #compute next iterate

        points = np.append(points, x[np.newaxis, :, :], axis=0) #store point in
        fx = f(A, x) #calculate updated function value
        function_values.append(fx) #store updated function value
        step_sizes.append(step_size) #Store step size

    #store function value in array
    function_values = np.array(function_values)
    step_sizes = np.array(step_sizes)
    return points, function_values, step_sizes
```

In [6]:

```
#Function to Perform Newton's Method where the Hessian is approximated by its d
#f: Compute function value at given point
#gradient: compute gradient of function at given point
#hessian: compute hessian of function at given point
#backtracking_algorithm: function used to perform backtracking
#A: matrix of ai vectors where ai are the rows of A
#x0: initial point of gradient descent
```

```
#x0: initial point of gradient descent
#max_iter: maximum number of iterations of Gradient Descent to run
#eta: used in convergence criteria
#alpha: alpha parameter for backtracking
#beta: beta parameter used for backtracking
def diagonal_newton_descent(f, gradient, diagonal_hessian, backtracking_algorit
    #Set initial iterate and function value
    x = x0
    fx = f(A, x)

    #maintain arrays to store iterates and function values throughout gradient
    points = np.array([x])
    function_values = []
    function_values.append(fx)

    #Store alphas
    step_sizes = []

    #enter for loop of max_iter times
    for iter in range(max_iter):
        grad = gradient(A, x) #compute gradient
        H = diagonal_hessian(A, x) #compute diagonal hessian
        newton_direction, newton_parameter = solve_newton_system(H, grad) #Solv
        # print(f"Iteration: {iter + 1}, Max of A @ x: {np.max(A @ x)}, Max of

        #if we have satisfied our convergence criteria, break from loop
        if newton_parameter <= eta:
            break

        descent_direction = newton_direction #our descent direction is the newt
        step_size = backtracking_algorithm(A, x, descent_direction, alpha, beta
        x = x + (step_size * descent_direction) #compute next iterate

        points = np.append(points, x[np.newaxis, :, :], axis=0) #store point in
        fx = f(A, x) #calculate updated function value
        function_values.append(fx) #store updated function value
        step_sizes.append(step_size) #Store alpha

    #store function value in array
    function_values = np.array(function_values)
    step_sizes = np.array(step_sizes)
    return points, function_values, step_sizes
```

In [7]:

```
#A: Matrix such that each row of A is ai
#x: x
#This function, given A and x, check to see if x is within the domain of f
#returns True if x is in the domain of f
#returns False is x is NOT in the domain of f
def domain_check(A: np.ndarray, x: np.ndarray):
    B = A @ x
    max_B = np.max(B)
    max_x = np.max(np.abs(x))
    return max_B < 1 and max_x < 1

#A: Matrix such that each row of A is ai
#x: x
#Compute the value of the function at x, given A.
#If x is not in the domain of f, infinity is returned
def f(A: np.ndarray, x: np.ndarray):
    if not domain_check(A, x):
        return np.inf
    B = A @ x
```

```
    log_comp_B = np.log(1 - B)
    log_plus_x = np.log(1 + x)
    log_minus_x = np.log(1 - x)

    return (-1 * np.sum(log_comp_B)) - np.sum(log_plus_x) - np.sum(log_minus_x)

#A: Matrix such that each row of A is ai
#x: x
#Compute the value of the gradient of function at x, given A.
#If x is not in the domain of f, infinity is returned
def f_gradient(A: np.ndarray, x: np.ndarray):
    if not domain_check(A, x):
        return np.inf

    B = A @ x
    gradient_vector = (A.T @ (1 / (1 - B))) + (1 / (1 - x)) - (1 / (1 + x))
    return gradient_vector

#A: Matrix such that each row of A is ai
#x: x
#Compute the value of the Hessian of function at x, given A.
#If x is not in the domain of f, infinity is returned
def f_hessian(A: np.ndarray, x: np.ndarray):
    if not domain_check(A, x):
        return np.inf

    B = A @ x
    C = 1 / (1 - B)
    C_squared = C ** 2
    C_squared = C_squared.flatten()

    plus_x_squared = 1 / ((1 + x) ** 2)
    minus_x_squared = 1 / ((1 - x) ** 2)
    combined = plus_x_squared + minus_x_squared
    combined = combined.flatten()

    H = (A.T @ np.diag(C_squared) @ A) + np.diag(combined)
    return H

#A: Matrix such that each row of A is ai
#x: x
#Compute the value of the Diagonal Approximation of Hessian of function at x, g
#If x is not in the domain of f, infinity is returned
def f_diagonal_hessian(A: np.ndarray, x: np.ndarray):
    if not domain_check(A, x):
        return np.inf

    B = A @ x
    C = 1 / (1 - B)
    C_squared = C ** 2

    A_transpose_squared = ((A.T) ** 2)
    D = A_transpose_squared @ C_squared
    D = D.flatten()

    plus_x_squared = 1 / ((1 + x) ** 2)
    minus_x_squared = 1 / ((1 - x) ** 2)
    combined = plus_x_squared + minus_x_squared
    combined = combined.flatten()

    H = np.diag(D) + np.diag(combined)
    return H

#This function performs backtracking
```

```
#A: Matrix such that each row of A is ai
#x: x
#delta_x: search direction
#alpha: alpha parameter
#beta: beta parameter
def f_backtracking_algorithm(A: np.ndarray, x: np.ndarray, delta_x: np.ndarray,
    t = 1

    #First we must multiply t by Beta until we get within the domain
    while not domain_check(A, x + (t * delta_x)):
        t = beta * t

    #Multiply t by beta until the backtracking stopping condition is met
    while (f(A, x + (t * delta_x))) > (f(A, x) + (alpha * t * (f_gradient(A, x)
        t = beta * t

    return t
```

9.30 (a): Gradient Descent Method with Backtracking Line Search

Experiment #1: n = 100, m = 200, $\alpha = 0.01$, $\beta = 0.1$

In [8]:

```
#Global Variables
n = 100
m = 200
alpha = 0.01
beta = 0.1
eta = 1e-3
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(0)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [9]:

```
## Investigate Matrix A
max_A = np.max(A)
min_A = np.min(A)
print(f"Minimum Element in A: {min_A}")
print(f"Maximum Element in A: {max_A}")
```

```
Minimum Element in A: -4.4466322413592945
Maximum Element in A: 3.8016602149671153
```

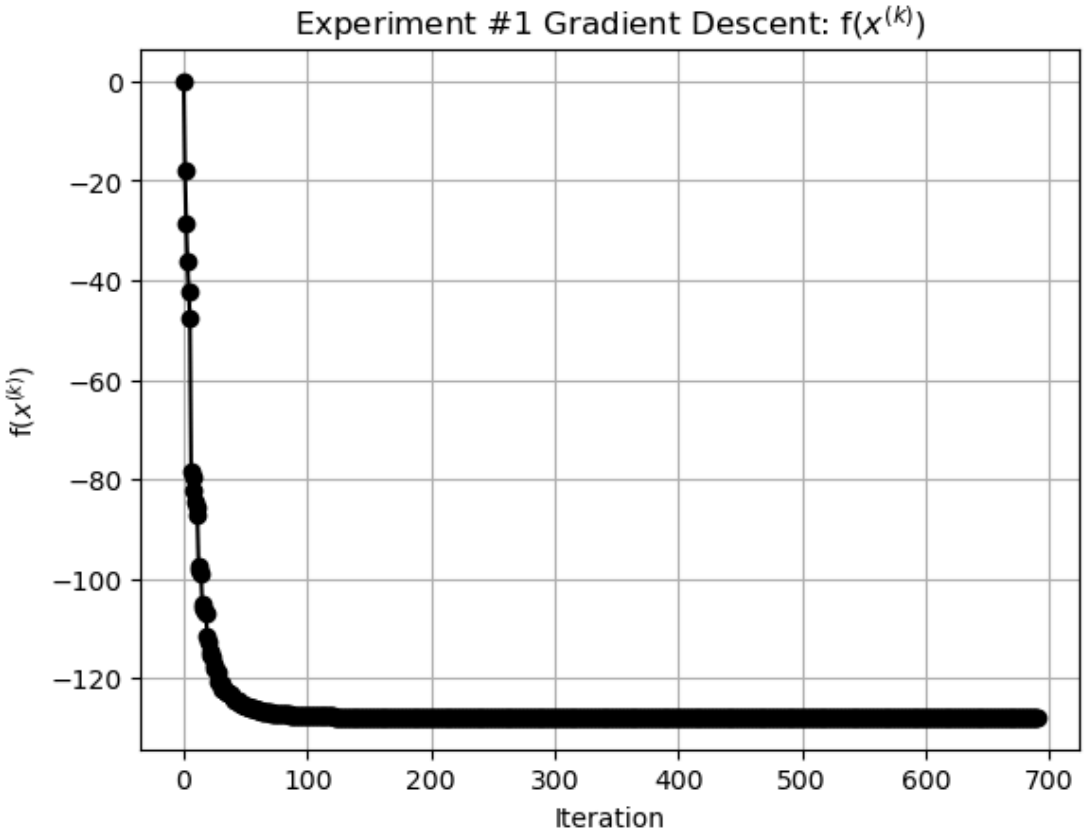
In [10]:

```
#Run Gradient Descent
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
p_star_seed0 = gradient_descent_function_values[-1] #the optimal function value
```

In [11]:

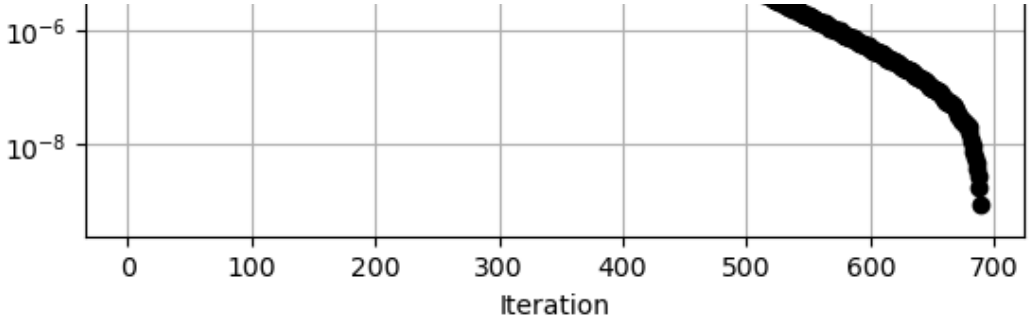
```
#Plot function values throughout gradient descent
fig1 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_function_values))), gradient_descent_f
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #1 Gradient Descent: f($x^{(k)}$)')
```

```
plt.grid(True)
plt.show()
```

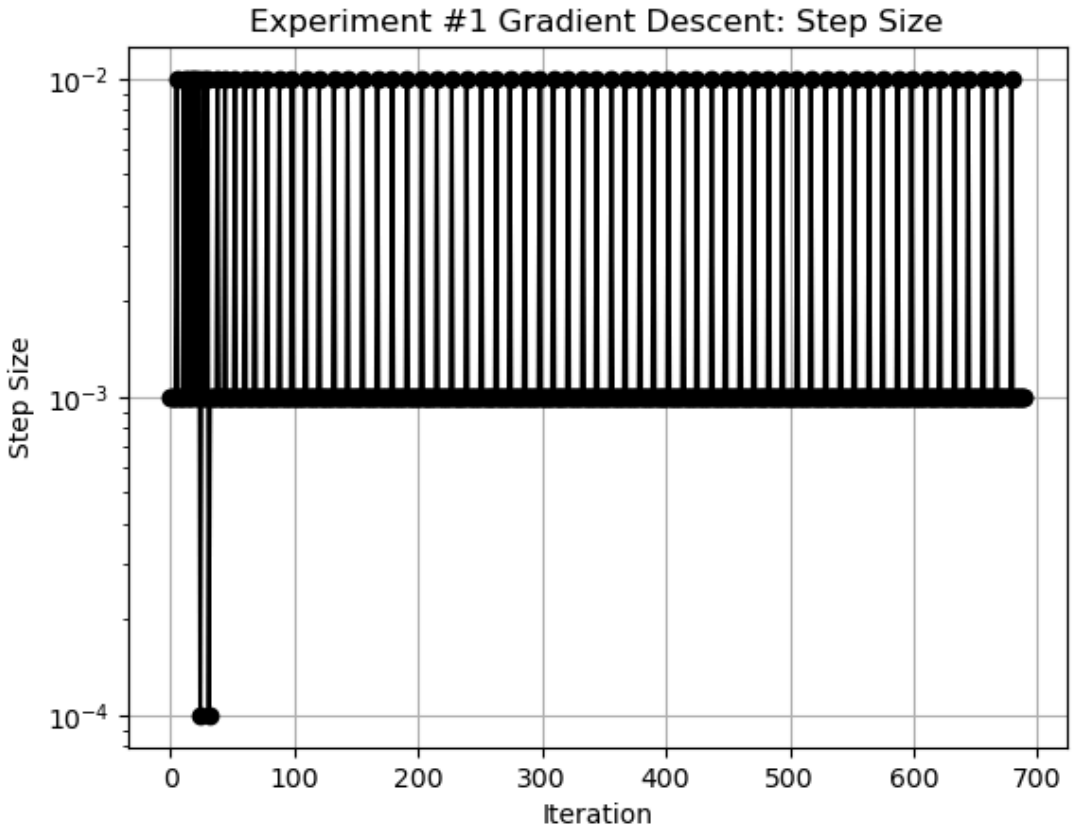


In [12]:

```
#Plot distance to optimal function value throughout gradient descent
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
gradient_descent_gap_to_optimal = gradient_descent_gap_to_optimal[:-1]

fig2 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_descent_ga
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #1 Gradient Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```



In [13]:
```
#Plot step sizes that were used during each iteration of gradient descent
fig3 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_step_sizes))), gradient_descent_step_s
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #1 Gradient Descent: Step Size')
plt.grid(True)
plt.show()
```



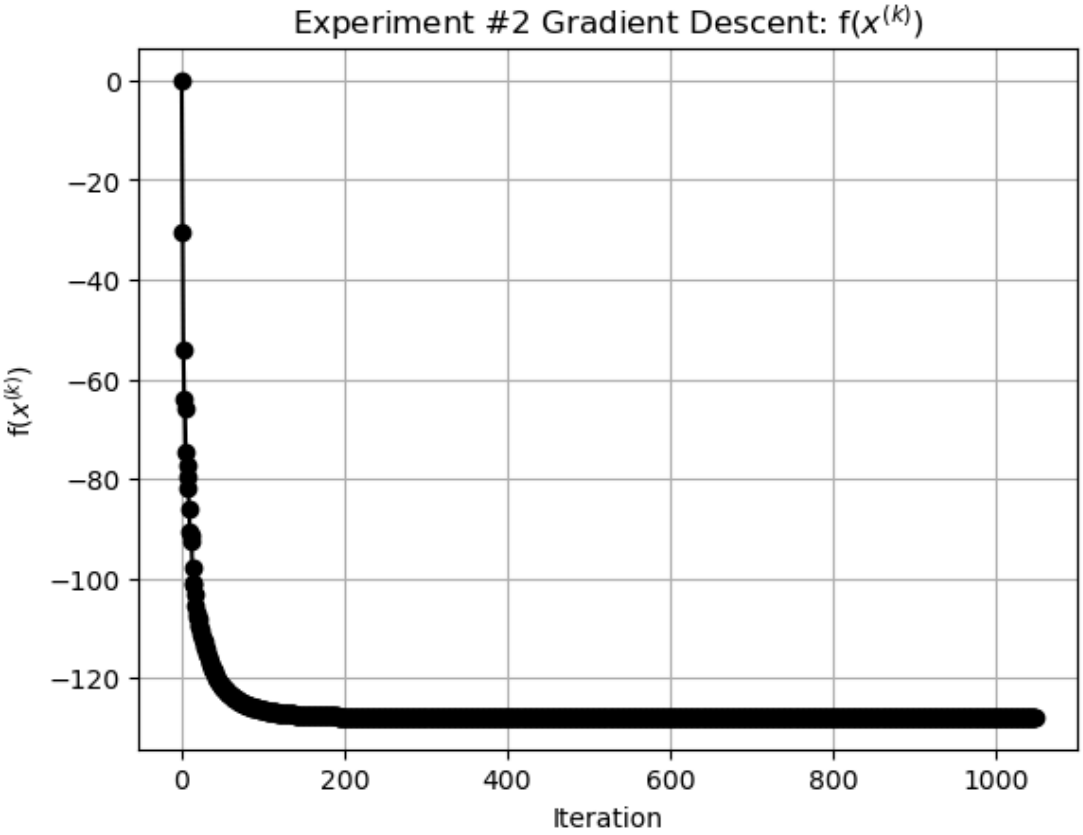
Experiment #2: n = 100, m = 200, $\alpha = 0.01$, $\beta = 0.5$

In [14]:
```
#Global Variables
n = 100
m = 200
alpha = 0.01
beta = 0.5

eta = 1e-3
max_iter = 2000
```

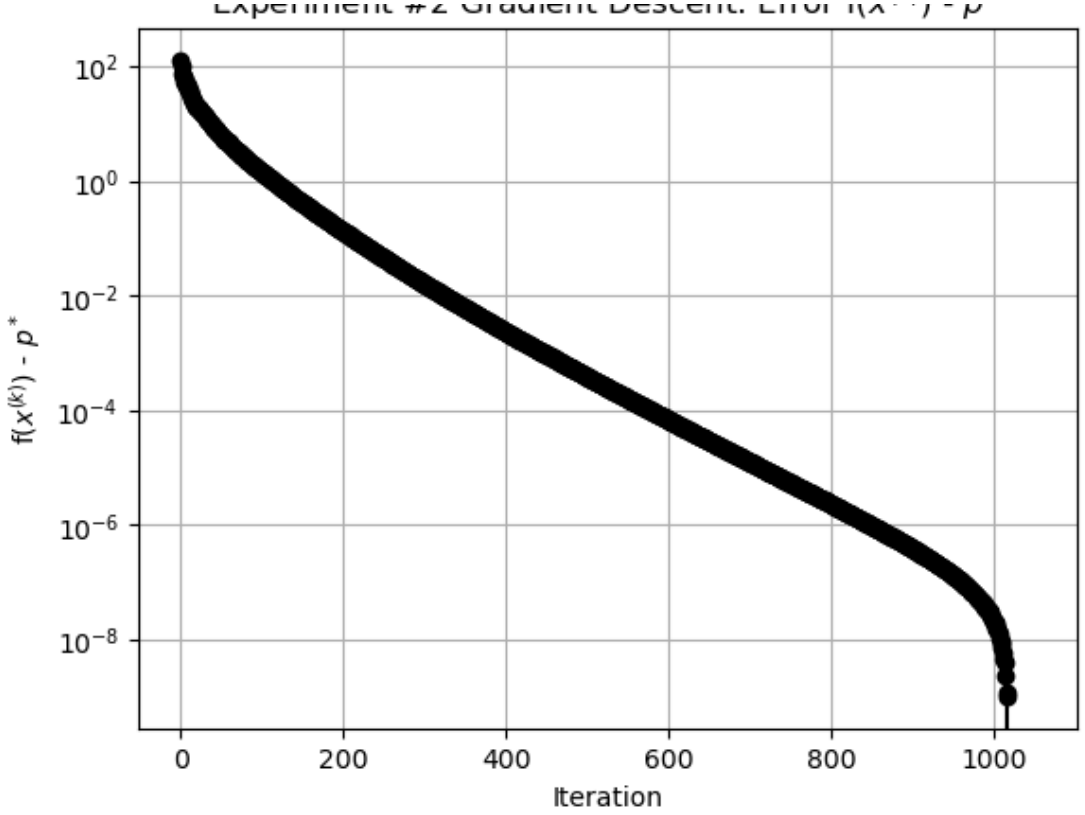
```
x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(0)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [15]:
```
#Run Gradient Descent
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

In [16]:
```
#Plot function values throughout gradient descent
fig4 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_function_values))), gradient_descent_f
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #2 Gradient Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```

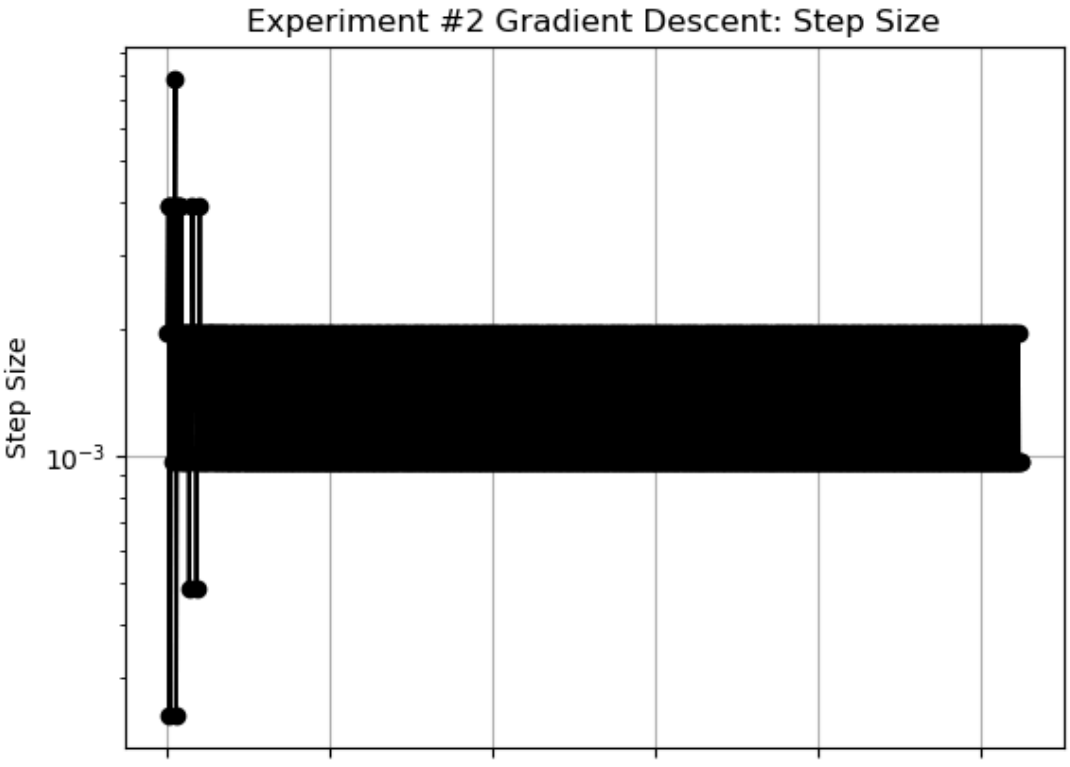


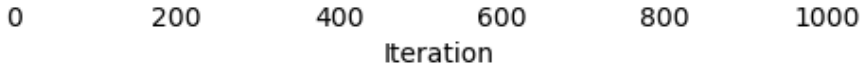
In [17]:
```
#Plot distance to optimal function value throughout gradient descent
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
fig5 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_descent_ga
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #2 Gradient Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```



In [18]:

```
#Plot step sizes that were used during each iteration of gradient descent
fig6 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_step_sizes))), gradient_descent_step_s
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #2 Gradient Descent: Step Size')
plt.grid(True)
plt.show()
```



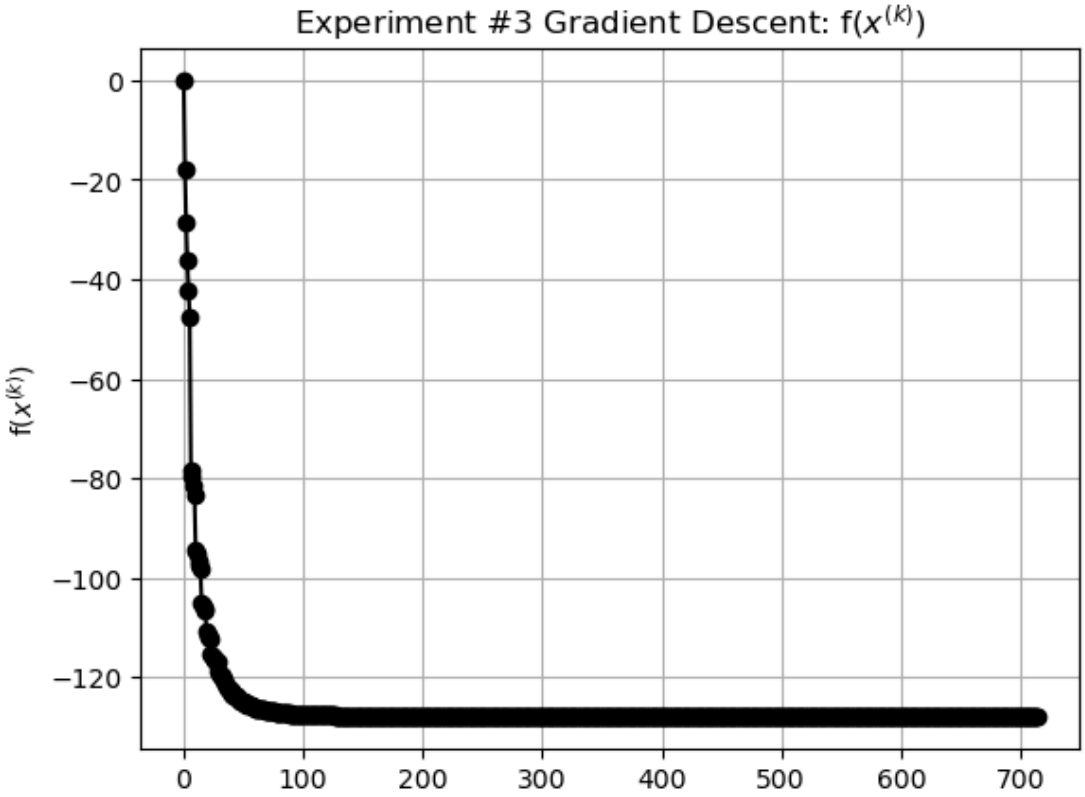


Experiment #3: n = 100, m = 200, $\alpha = 0.15$, $\beta = 0.1$

In [19]:
```
#Global Variables
n = 100
m = 200
alpha = 0.15
beta = 0.1

eta = 1e-3
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(0)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

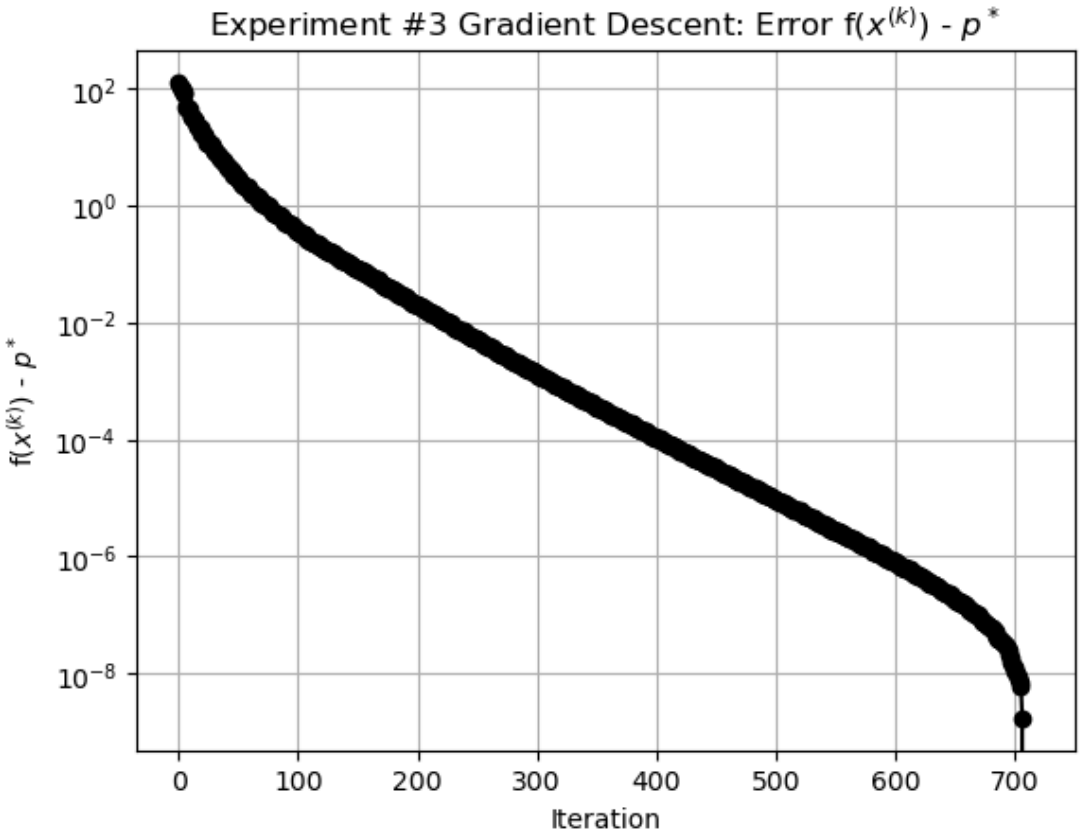
In [20]:
```
#Run Gradient Descent
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

In [21]:
```
#Plot function values throughout gradient descent
fig7 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_function_values))), gradient_descent_f
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #3 Gradient Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```

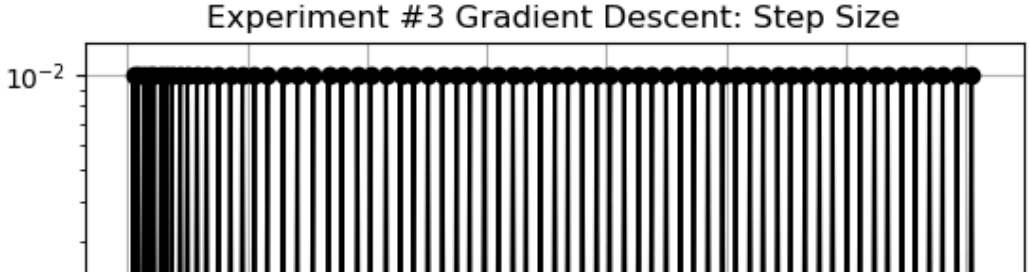


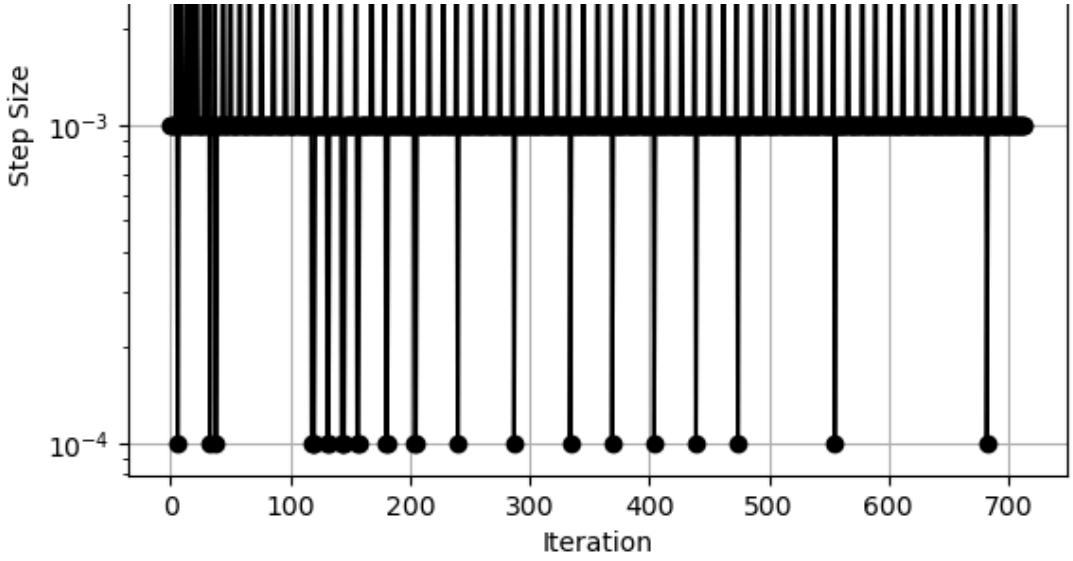
Iteration

In [22]:
```
#Plot distance to optimal function value throughout gradient descent
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
fig8 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_descent_ga
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #3 Gradient Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```



In [23]:
```
#Plot step sizes that were used during each iteration of gradient descent
fig9 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_step_sizes))), gradient_descent_step_s
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #3 Gradient Descent: Step Size')
plt.grid(True)
plt.show()
```



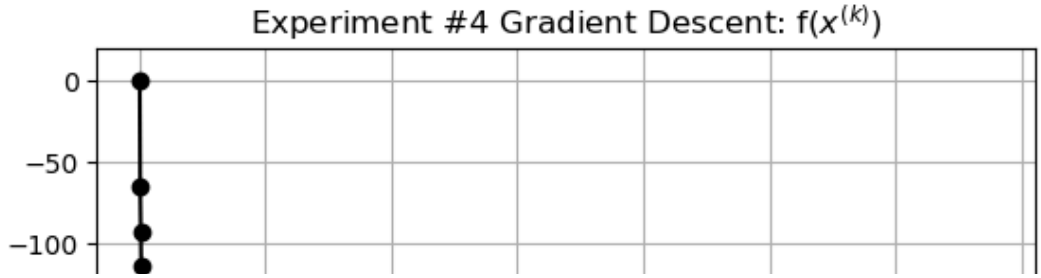


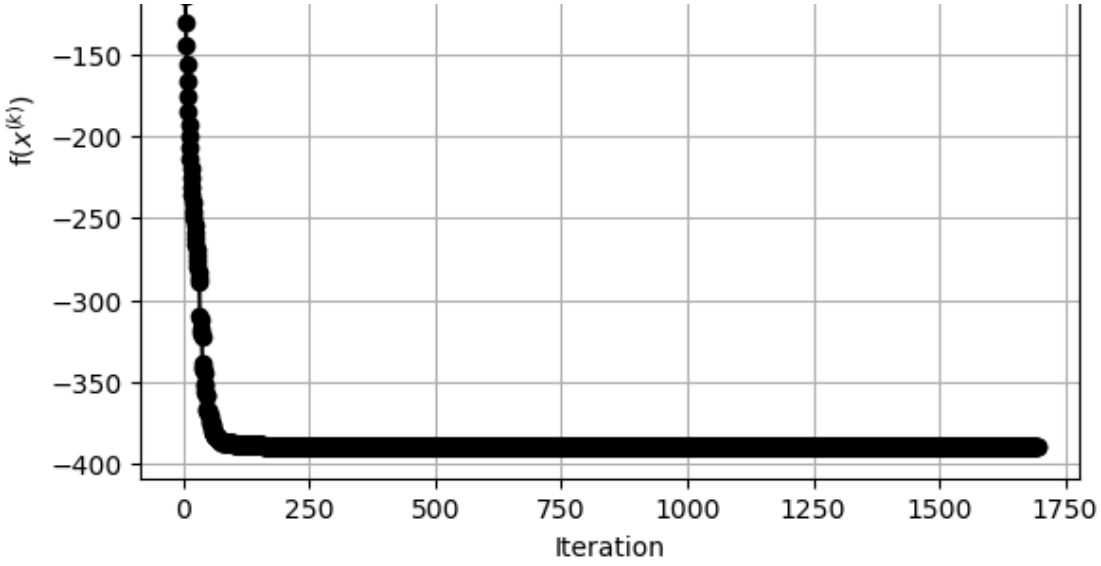
Experiment #4: n = 200, m = 400, $\alpha = 0.01$, $\beta = 0.1$

In [24]:
```
#Global Variables
n = 200
m = 400
alpha = 0.01
beta = 0.1

eta = 1e-3
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(1)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [25]:
```
#Run Gradient Descent
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
p_star_seed1 = gradient_descent_function_values[-1] #the optimal function value
```

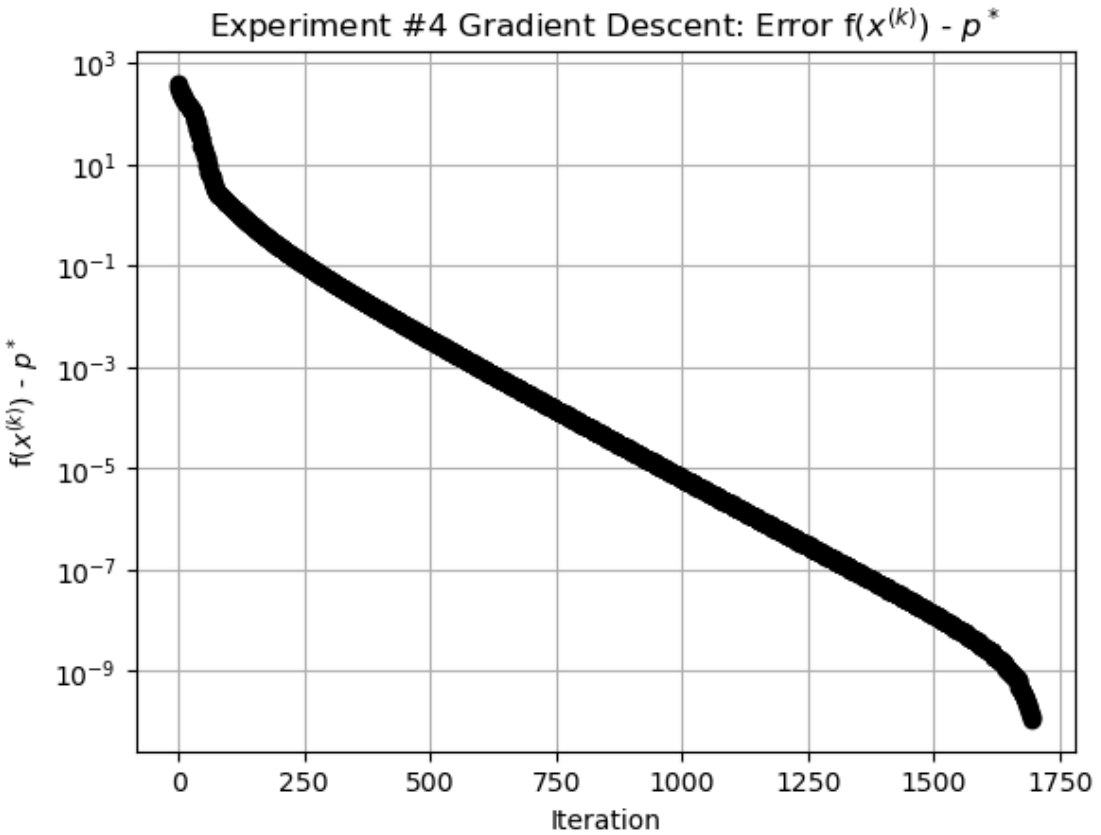
In [26]:
```
#Plot function values throughout gradient descent
fig10 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_function_values))), gradient_descent_f
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #4 Gradient Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```





In [27]:

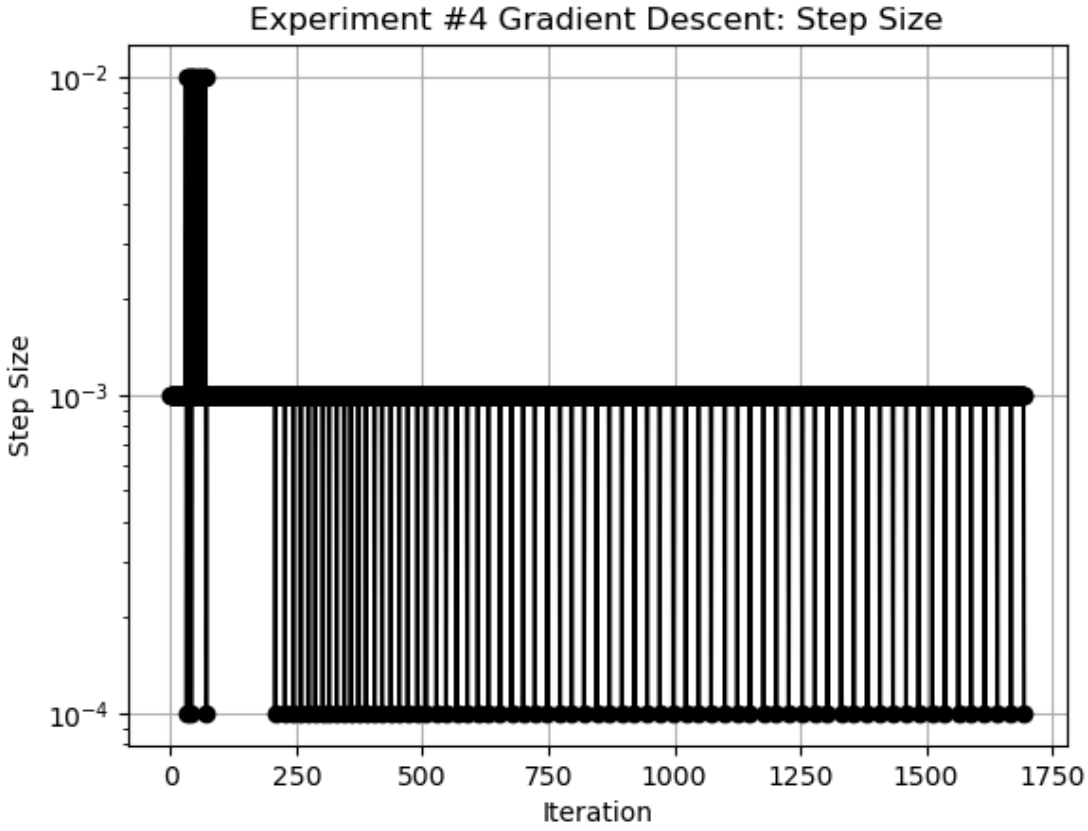
```
#Plot distance to optimal function value throughout gradient descent
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
gradient_descent_gap_to_optimal = gradient_descent_gap_to_optimal[:-1]
fig11 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_descent_ga
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #4 Gradient Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```



In [28]:

```
#Plot step sizes that were used during each iteration of gradient descent
fig12 = plt.figure()  # Create a new figure
```

```
fig12 = plt.figure()  # create a new figure
plt.plot(list(range(len(gradient_descent_step_sizes))), gradient_descent_step_s
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #4 Gradient Descent: Step Size')
plt.grid(True)
plt.show()
```



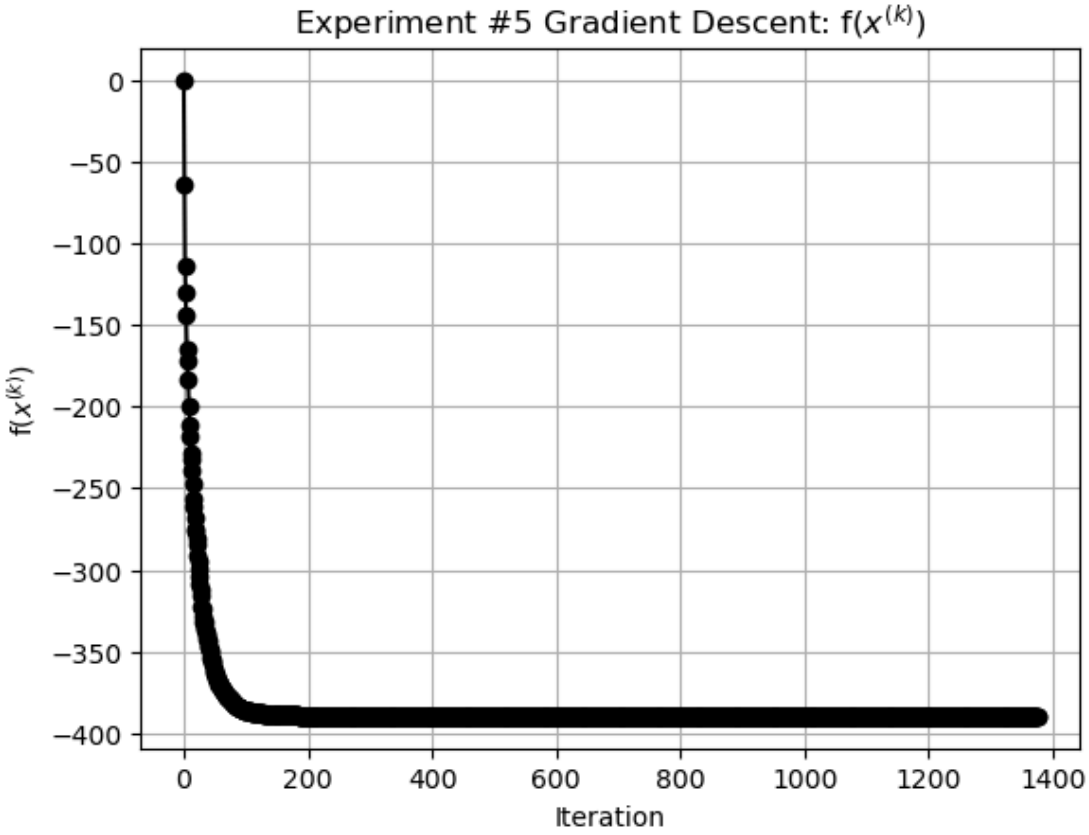
Experiment #5: n = 200, m = 400, $\alpha = 0.01, \beta = 0.5$

In [29]:
```
#Global Variables
n = 200
m = 400
alpha = 0.01
beta = 0.5

eta = 1e-3
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(1)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [30]:
```
#Run Gradient Descent
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

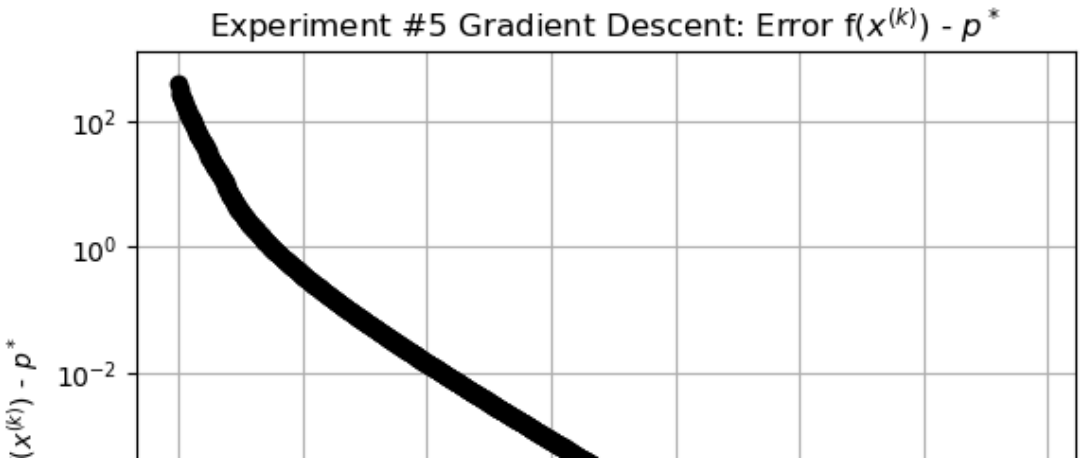
In [31]:
```
#Plot function values throughout gradient descent
fig13 = plt.figure()  # Create a new figure
```

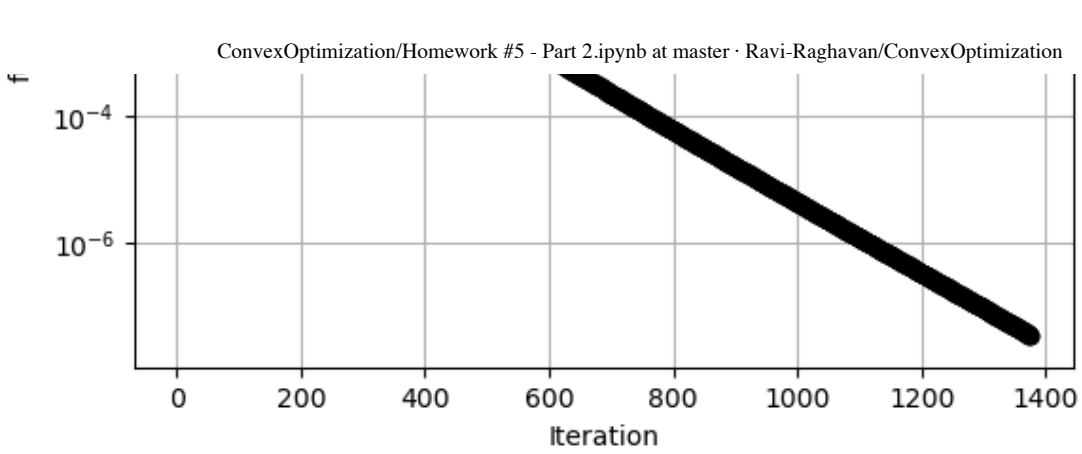
```
plt.plot(list(range(len(gradient_descent_function_values))), gradient_descent_f
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #5 Gradient Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```



In [32]:

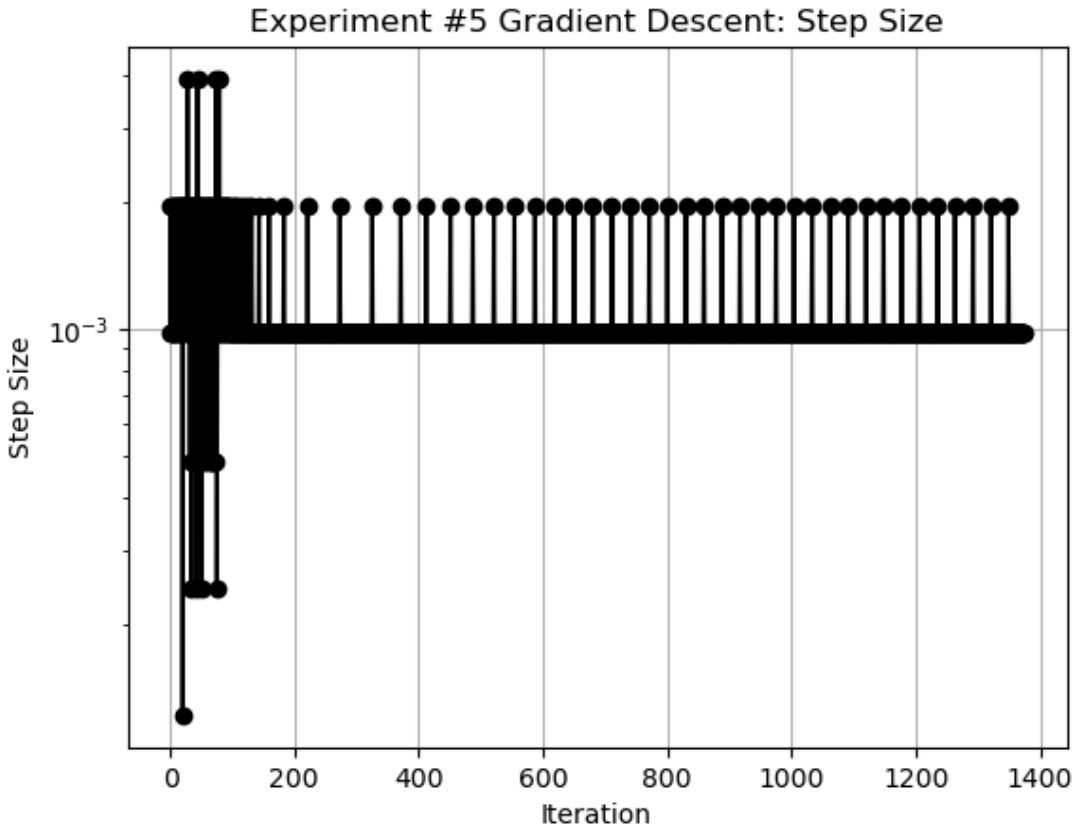
```
#Plot distance to optimal function value throughout gradient descent
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
fig14 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_descent_ga
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #5 Gradient Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```





In [33]:

```
#Plot step sizes that were used during each iteration of gradient descent
fig15 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_step_sizes))), gradient_descent_step_s
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #5 Gradient Descent: Step Size')
plt.grid(True)
plt.show()
```



Experiment #6: n = 200, m = 400, $\alpha = 0.15$, $\beta = 0.1$

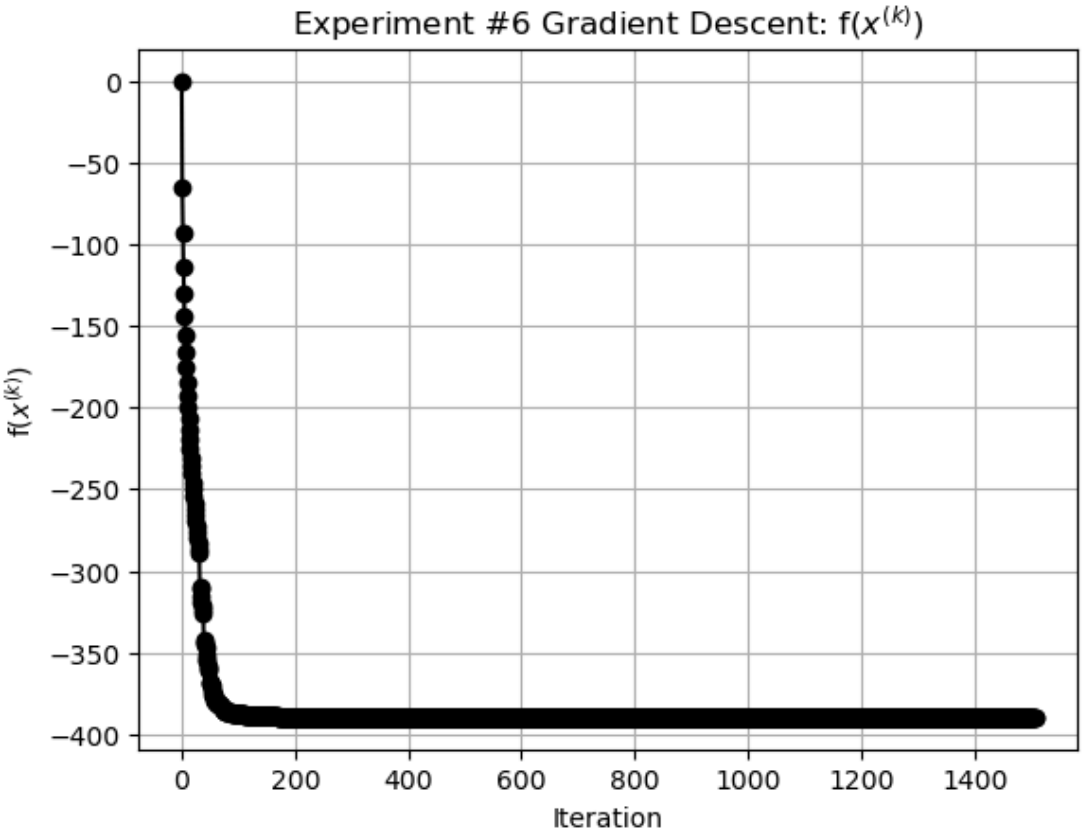
In [34]:

```
#Global Variables
n = 200
m = 400
alpha = 0.15
beta = 0.1

eta = 10.3
```

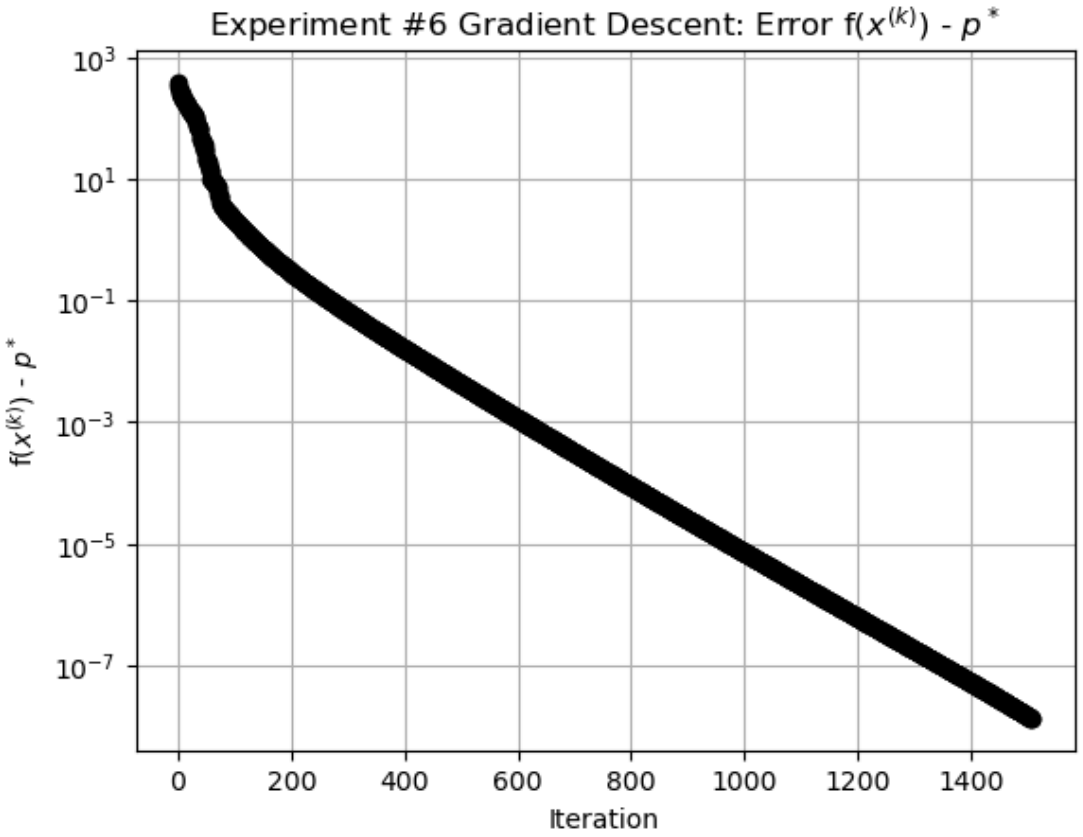
```
eta = 1e-5
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(1)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [35]:
```
#Run Gradient Descent
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

In [36]:
```
#Plot function values throughout gradient descent
fig16 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_function_values))), gradient_descent_f
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #6 Gradient Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```

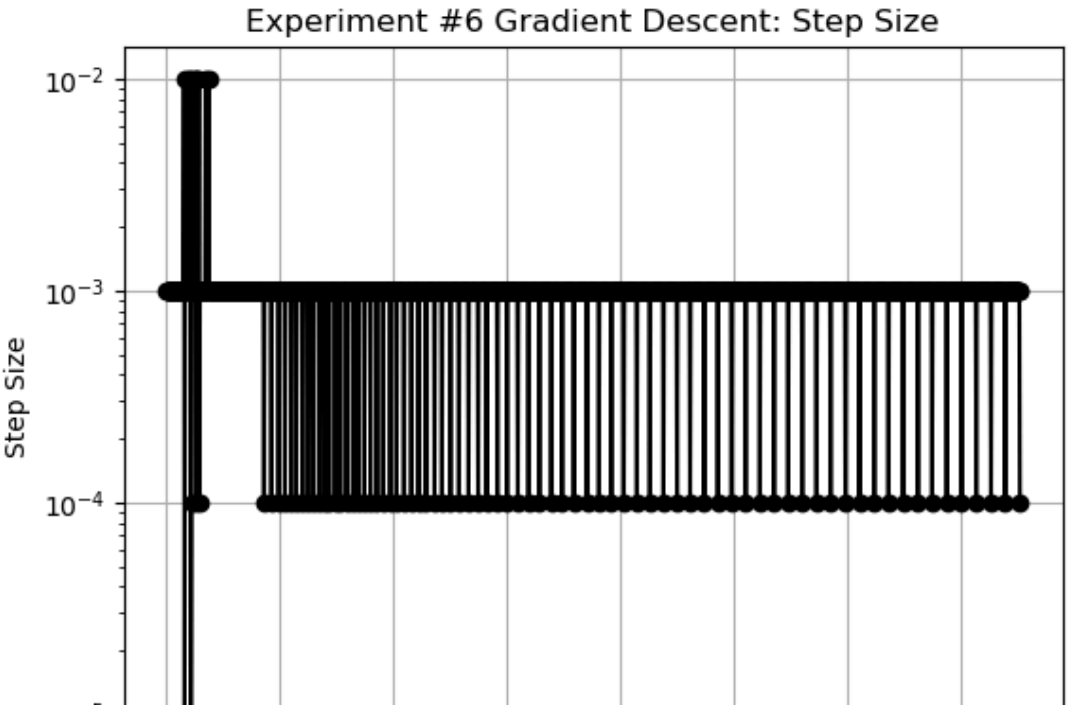


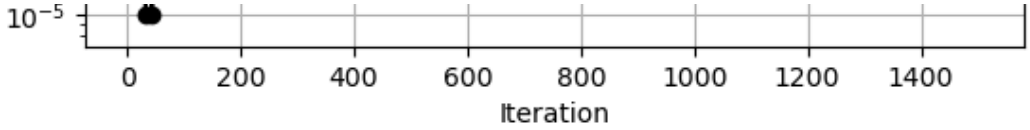
In [37]:
```
#Plot distance to optimal function value throughout gradient descent
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
fig17 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_descent_ga
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #6 Gradient Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```



In [38]:

```
#Plot step sizes that were used during each iteration of gradient descent
fig18 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_step_sizes))), gradient_descent_step_s
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #6 Gradient Descent: Step Size')
plt.grid(True)
plt.show()
```



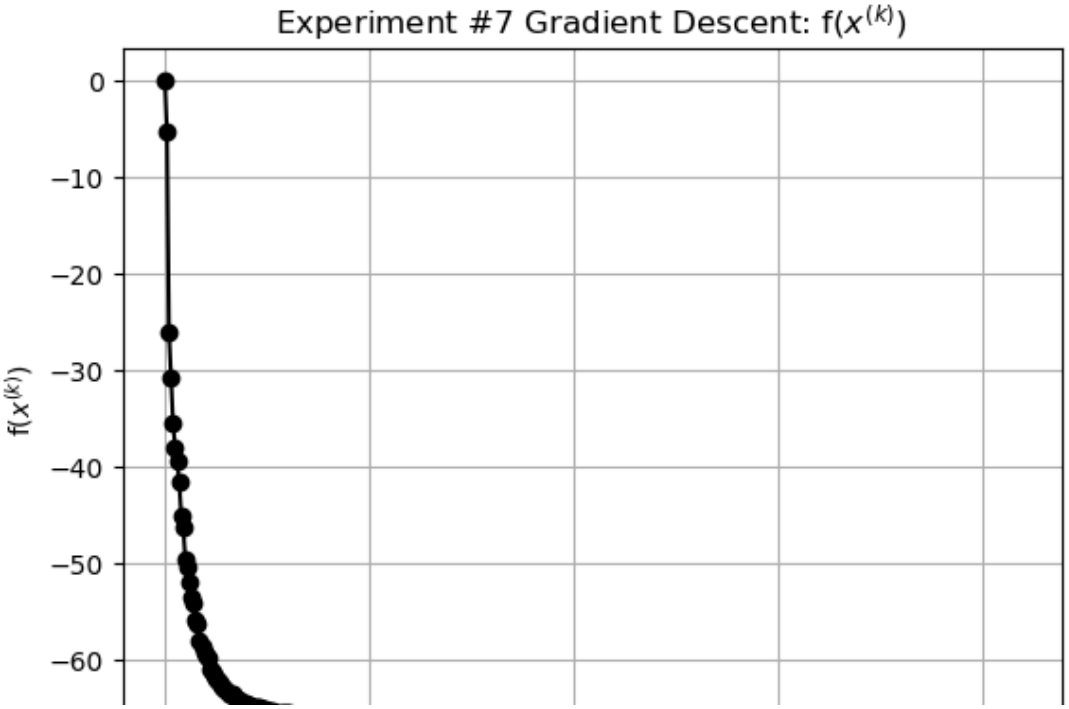


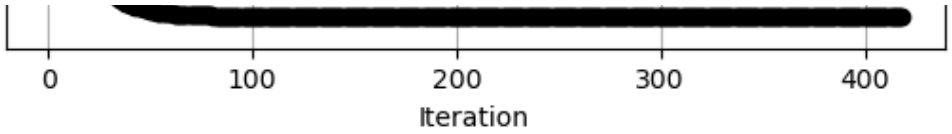
Experiment #7: n = 50, m = 100, $\alpha = 0.01$, $\beta = 0.1$

In [39]:
```
#Global Variables
n = 50
m = 100
alpha = 0.01
beta = 0.1

eta = 1e-3
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(2)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [40]:
```
#Run Gradient Descent
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
p_star_seed2 = gradient_descent_function_values[-1] #the optimal function value
```

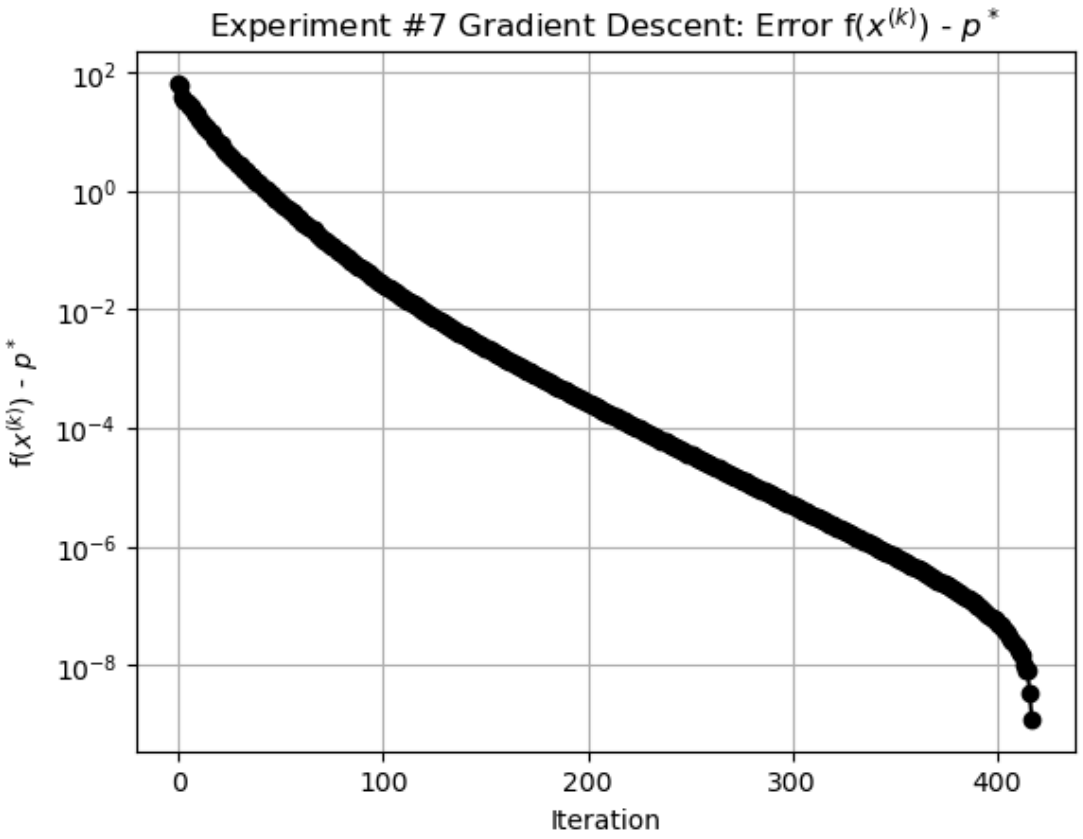
In [41]:
```
#Plot function values throughout gradient descent
fig19 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_function_values))), gradient_descent_f
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #7 Gradient Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```





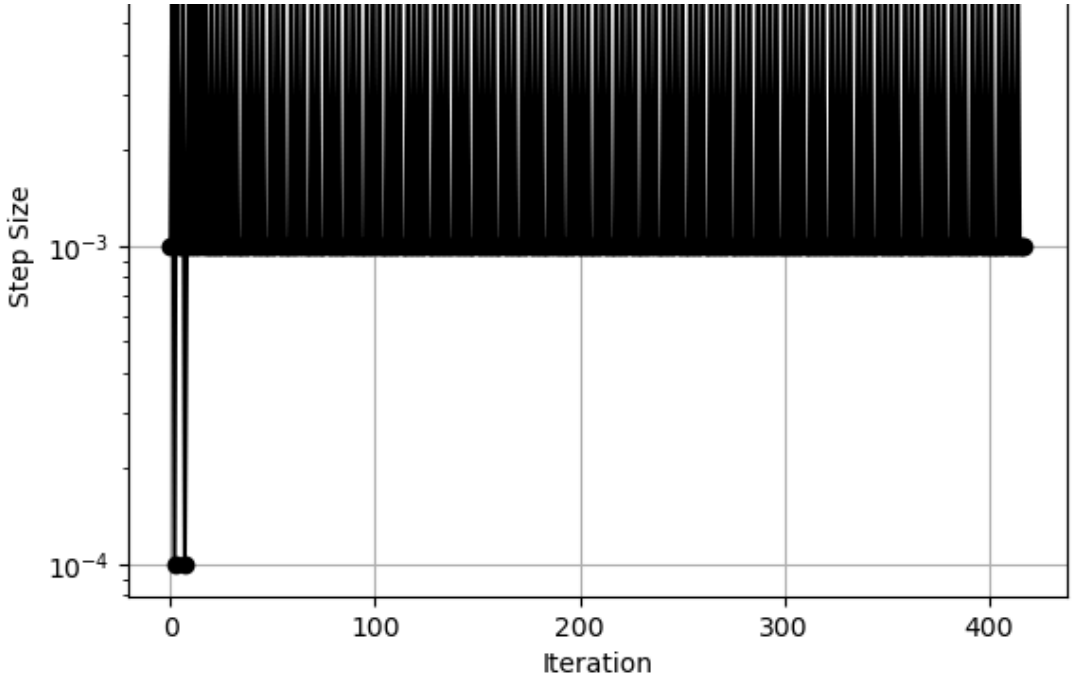
In [42]:

```
#Plot distance to optimal function value throughout gradient descent
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
gradient_descent_gap_to_optimal = gradient_descent_gap_to_optimal[:-1]
fig20 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_descent_ga
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #7 Gradient Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```



In [43]:

```
#Plot step sizes that were used during each iteration of gradient descent
fig21 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_step_sizes))), gradient_descent_step_s
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #7 Gradient Descent: Step Size')
plt.grid(True)
plt.show()
```

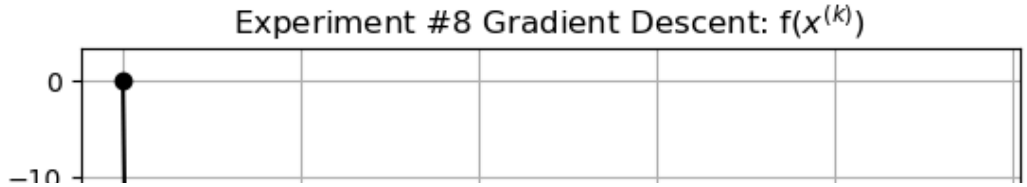


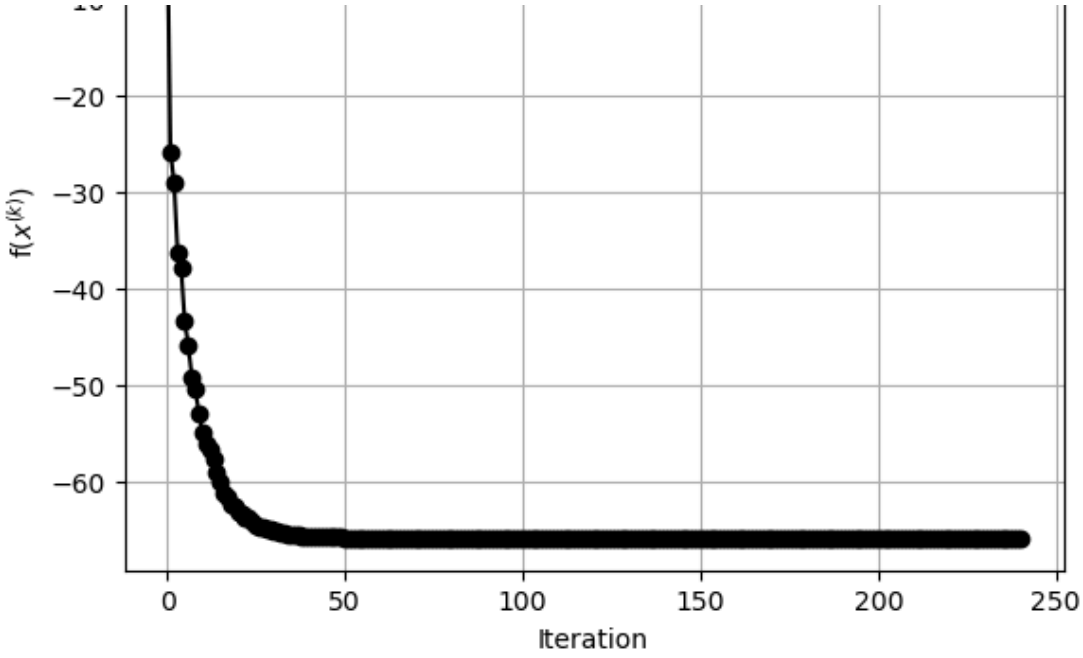
Experiment #8: n = 50, m = 100, $\alpha = 0.01$, $\beta = 0.5$

In [44]:
```
#Global Variables
n = 50
m = 100
alpha = 0.01
beta = 0.5

eta = 1e-3
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(2)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [45]:
```
#Run Gradient Descent
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

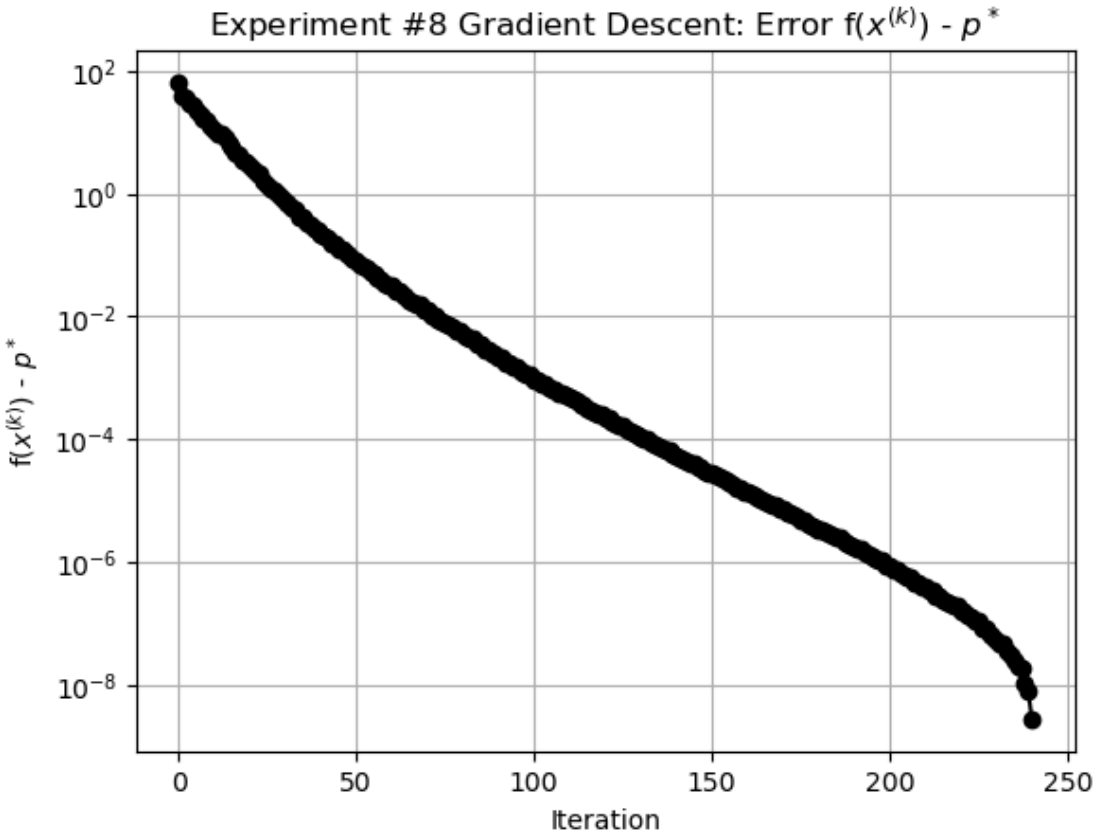
In [46]:
```
#Plot function values throughout gradient descent
fig22 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_function_values))), gradient_descent_f
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #8 Gradient Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```



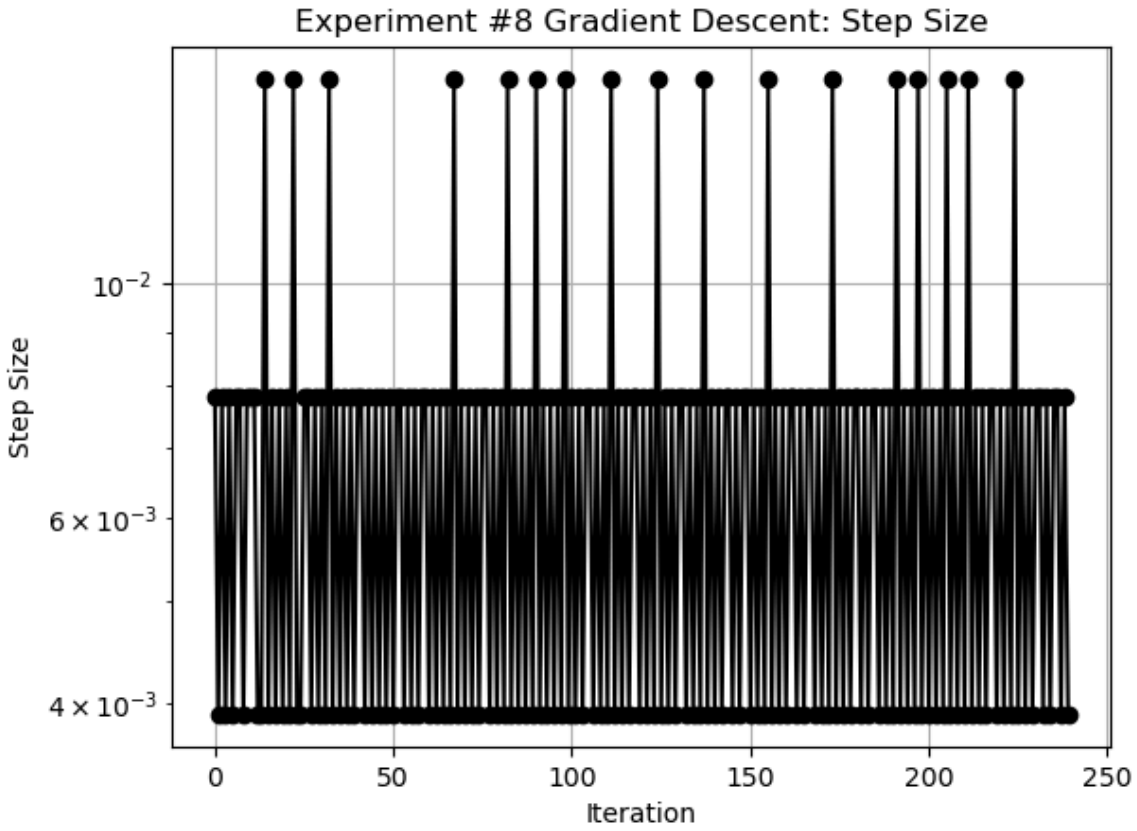


In [47]:

```
#Plot distance to optimal function value throughout gradient descent
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
fig23 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_descent_ga
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #8 Gradient Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```



In [48]:
```
#Plot step sizes that were used during each iteration of gradient descent
fig24 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_step_sizes))), gradient_descent_step_s
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #8 Gradient Descent: Step Size')
plt.grid(True)
plt.show()
```

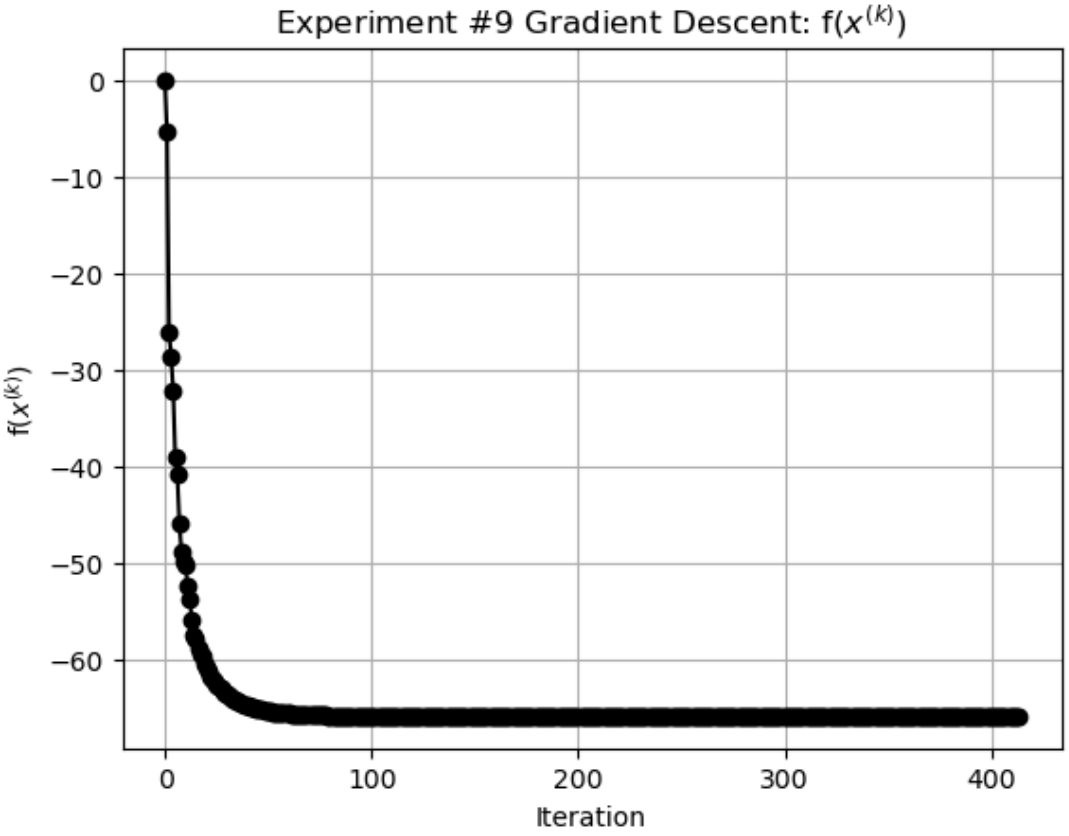


Experiment #9: n = 50, m = 100, $\alpha = 0.15$, $\beta = 0.1$

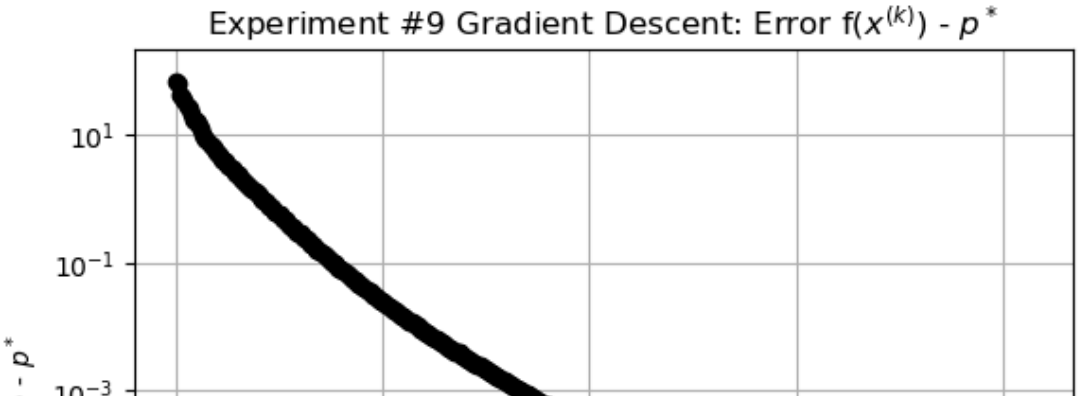
In [49]:
```
#Global Variables
n = 50
m = 100
alpha = 0.15
beta = 0.1

eta = 1e-3
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(2)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

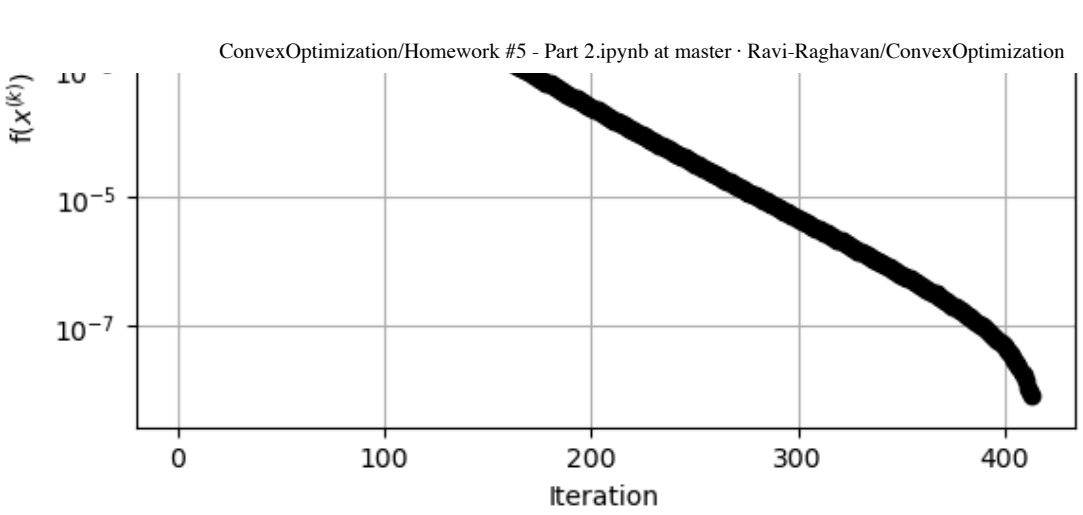
In [50]:
```
#Run Gradient Descent
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

In [51]:
```
#Plot function values throughout gradient descent
fig25 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_function_values))), gradient_descent_f
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #9 Gradient Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```



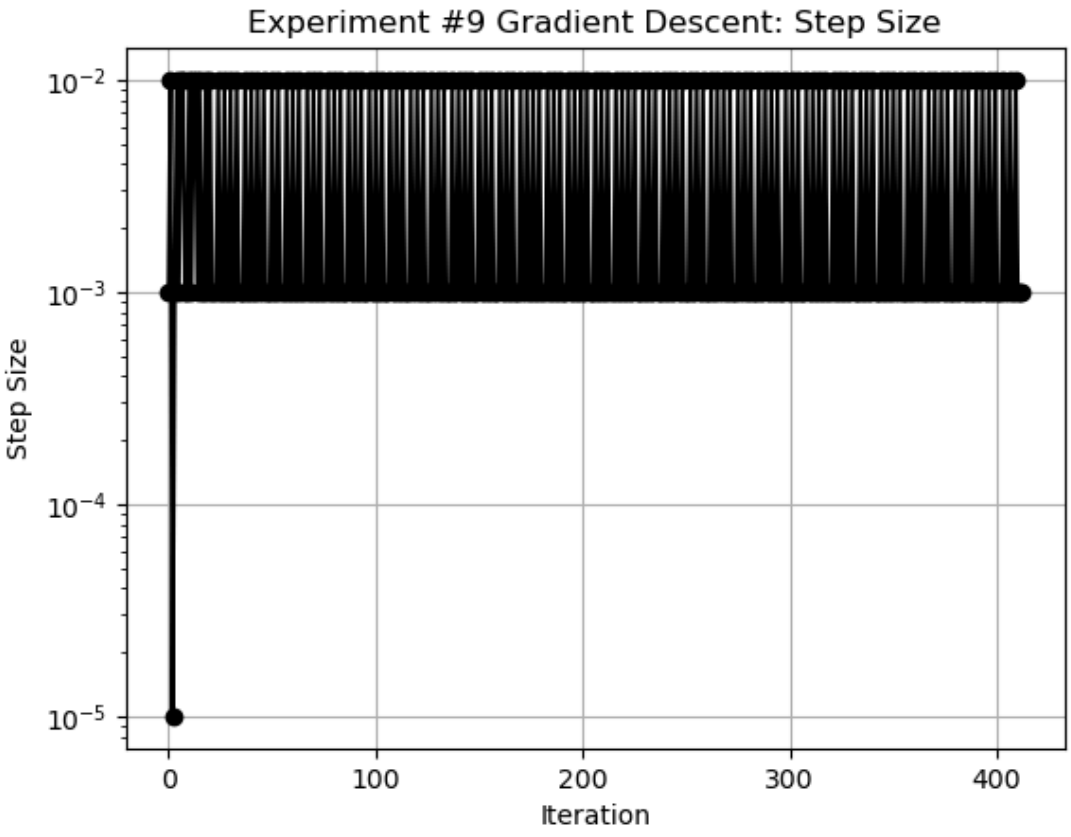
In [52]:
```
#Plot distance to optimal function value throughout gradient descent
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
fig26 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_descent_ga
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #9 Gradient Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```





In [53]:

```
#Plot step sizes that were used during each iteration of gradient descent
fig27 = plt.figure()  # Create a new figure
plt.plot(list(range(len(gradient_descent_step_sizes))), gradient_descent_step_s
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #9 Gradient Descent: Step Size')
plt.grid(True)
plt.show()
```



Observations:

- Backtracking Parameters α and β affect the performance of Gradient Descent Algorithm
- Performance does not really scale well with problem size

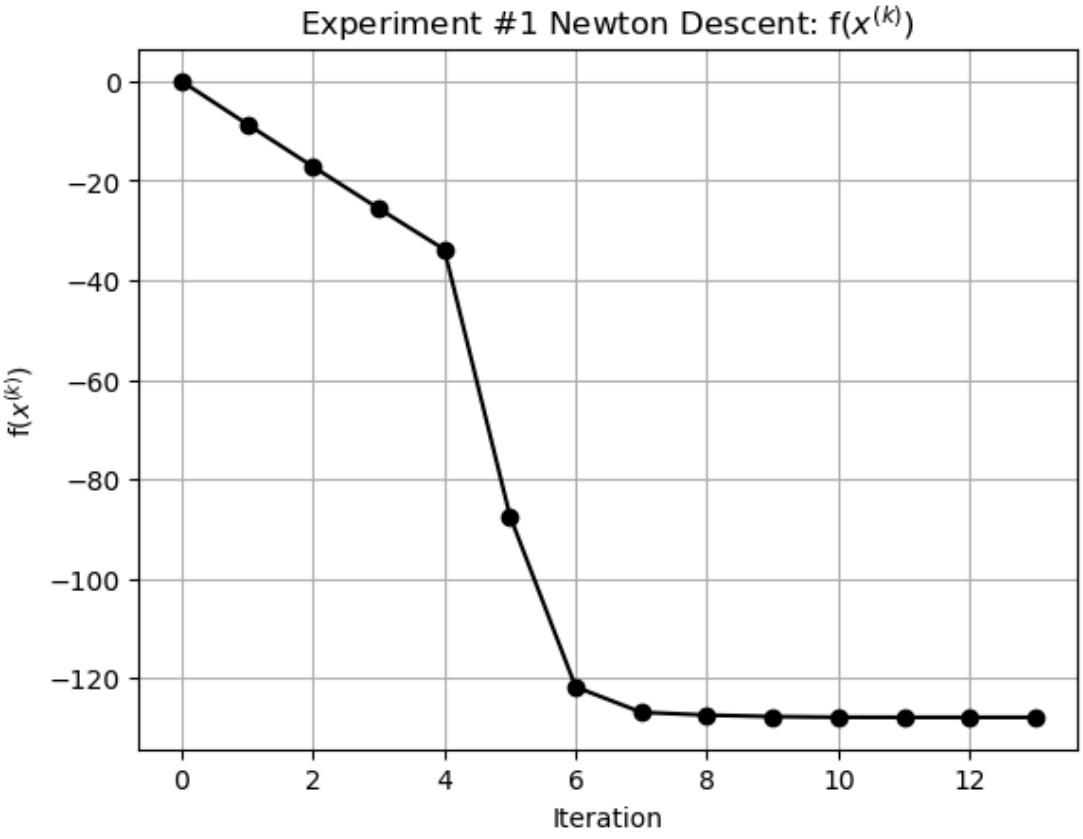
9.30 (b)

Experiment #1: n = 100, m = 200, $\alpha = 0.01$, $\beta = 0.1$

In [54]:
```
#Global Variables
n = 100
m = 200
alpha = 0.01
beta = 0.1
eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(0)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

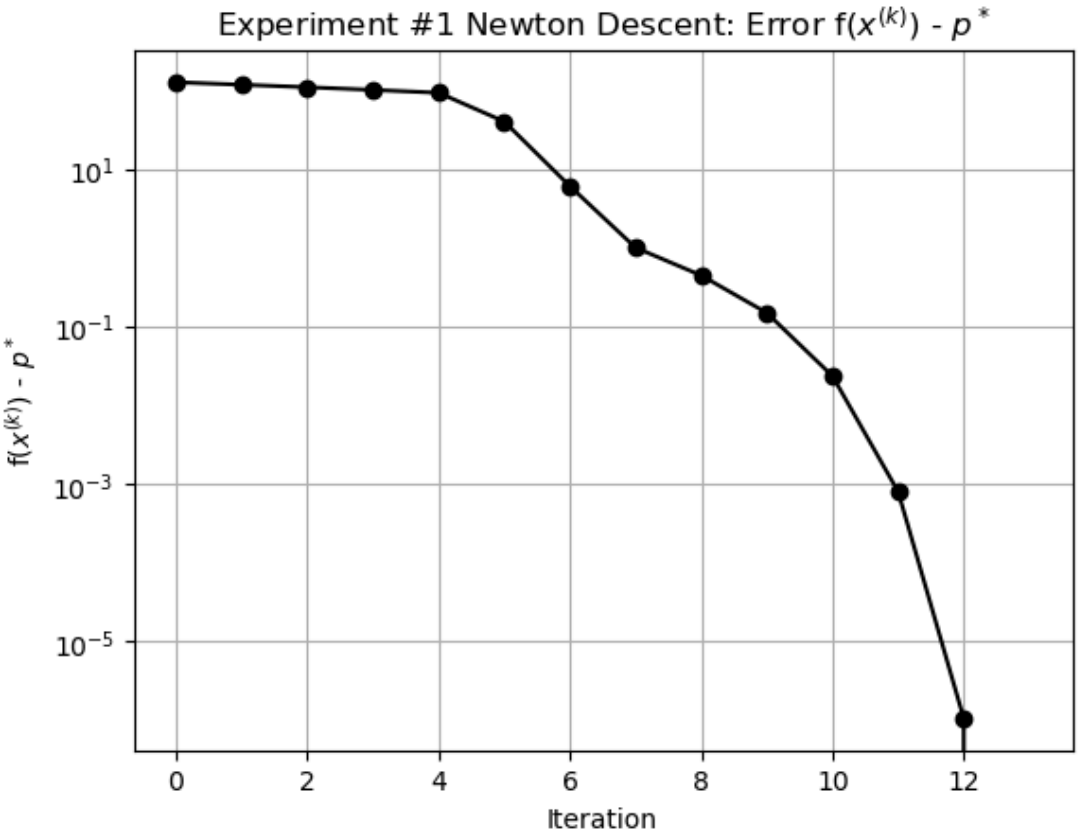
In [55]:
```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

In [56]:
```
#Plot function values throughout Newton's Method
fig28 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_function_values))), newton_descent_funct
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #1 Newton Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```



In [57]:

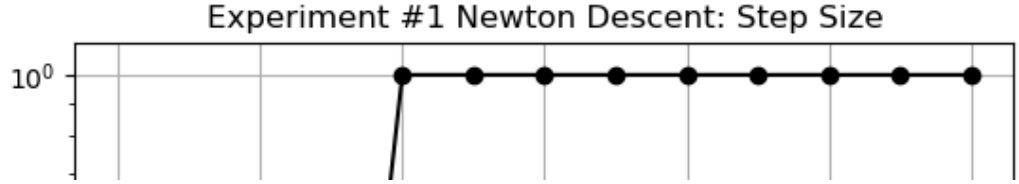
In [57]:
```
#Plot distance to optimal function value throughout Newton's Method
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed0
fig29 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_descent_gap_to
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #1 Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```

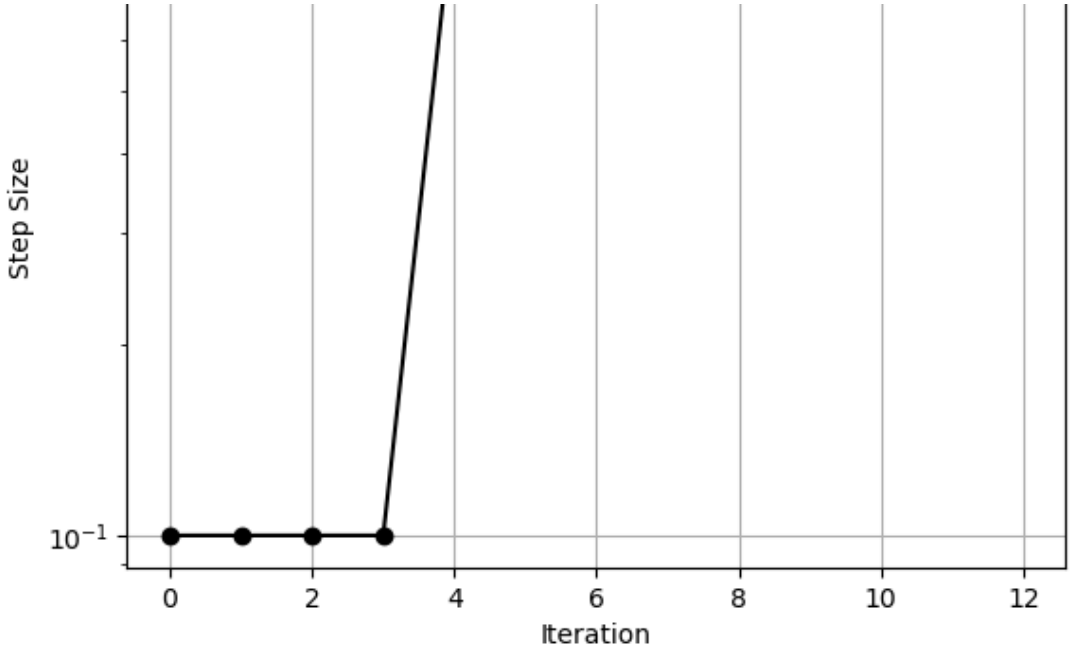


In [58]:
```
#Print step sizes during Newton's Method
newton_descent_step_sizes
```

Out[58]:
```
array([0.1, 0.1, 0.1, 0.1, 1. , 1. , 1. , 1. , 1. , 1. , 1. , 1. , 1. ])
```

In [59]:
```
#Plot step sizes during Newton's Method
fig30 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_step_sizes))), newton_descent_step_sizes
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #1 Newton Descent: Step Size')
plt.grid(True)
plt.show()
```



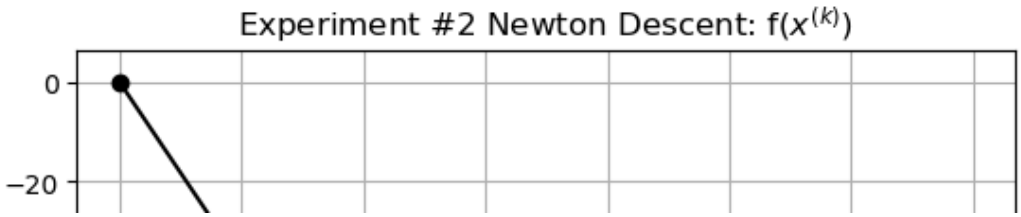


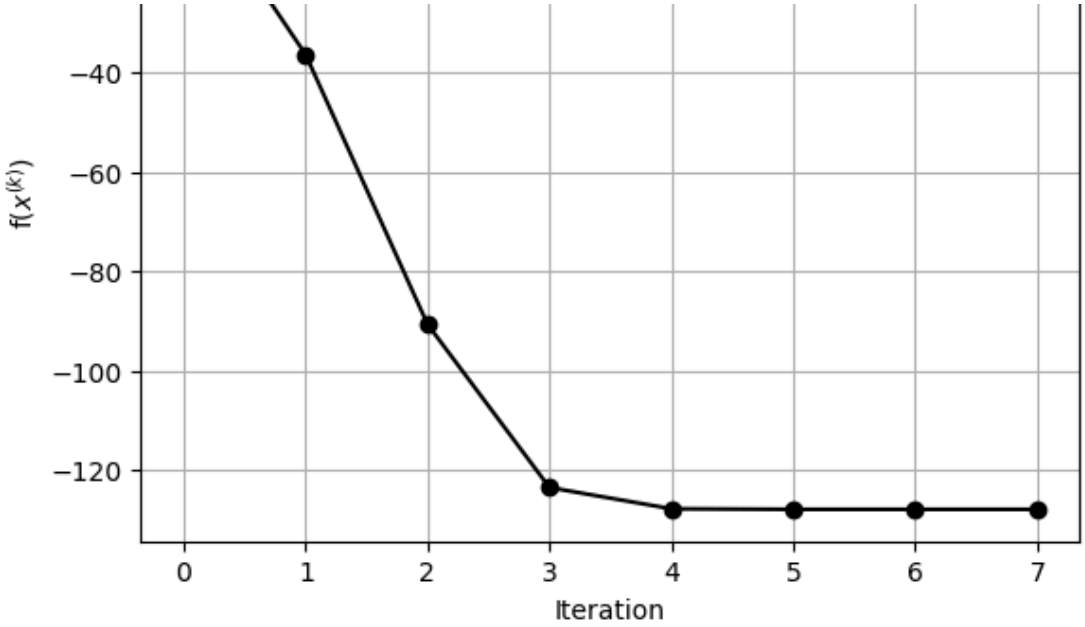
Experiment #2: n = 100, m = 200, $\alpha = 0.01$, $\beta = 0.5$

In [60]:
```
#Global Variables
n = 100
m = 200
alpha = 0.01
beta = 0.5

eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(0)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [61]:
```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

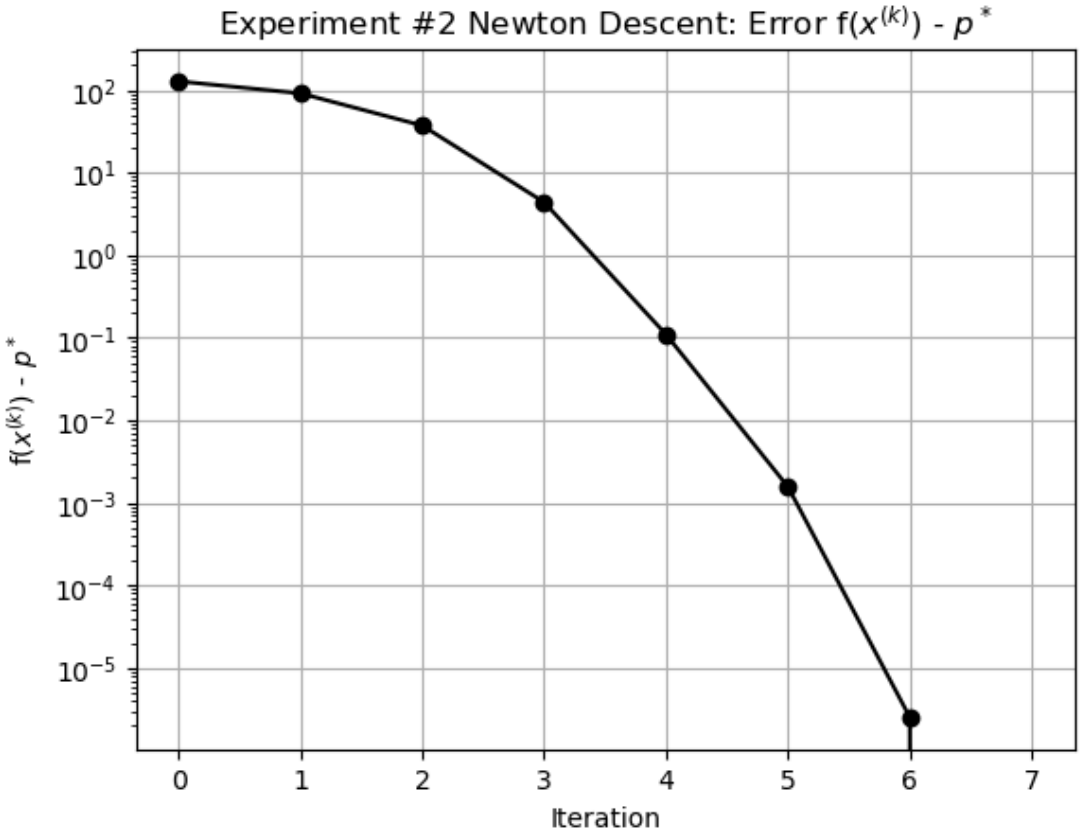
In [62]:
```
#Plot function values throughout Newton's Method
fig31 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_function_values))), newton_descent_funct
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #2 Newton Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```





In [63]:

```
#Plot distance to optimal function value throughout Newton's Method
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed0
fig32 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_descent_gap_to
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #2 Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```



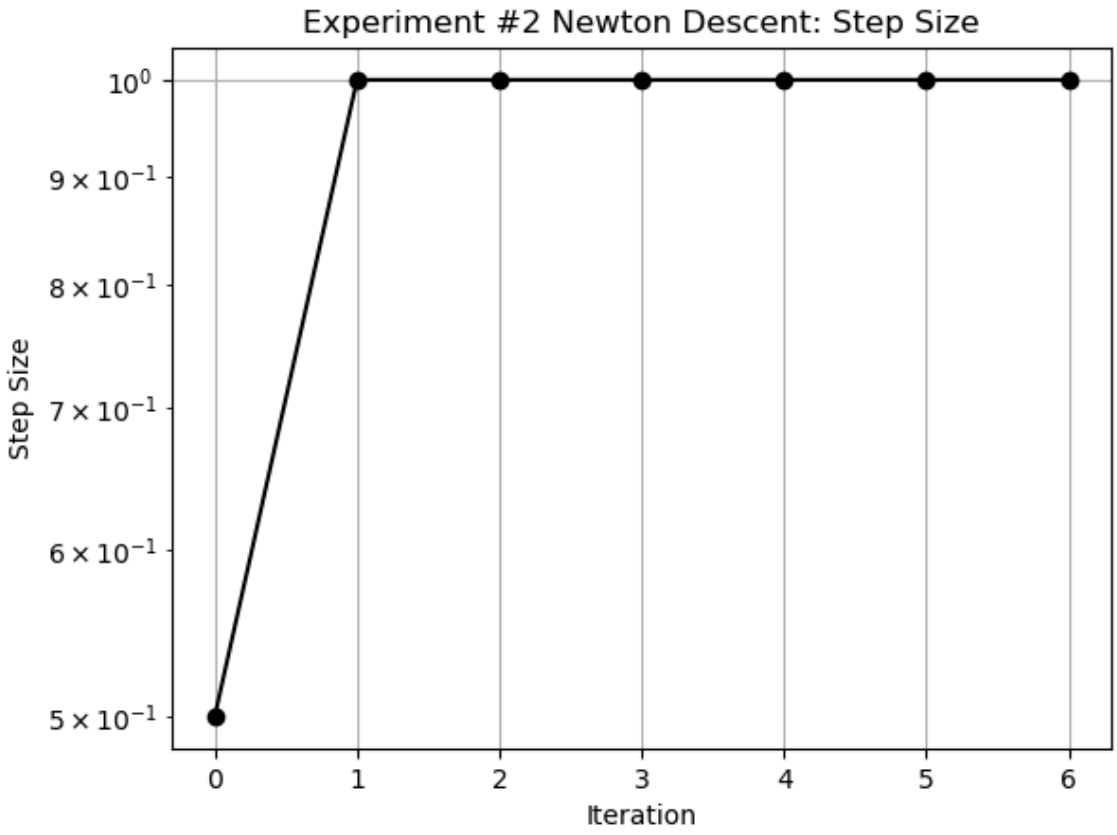
In [64]:

```
#Print step sizes during Newton's Method
```

```
#Print step sizes during Newton's Method
newton_descent_step_sizes
```

Out[64]:
```
array([0.5, 1. , 1. , 1. , 1. , 1. , 1. ])
```

In [65]:
```
#Plot step sizes during Newton's Method
fig33 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_step_sizes))), newton_descent_step_sizes
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #2 Newton Descent: Step Size')
plt.grid(True)
plt.show()
```

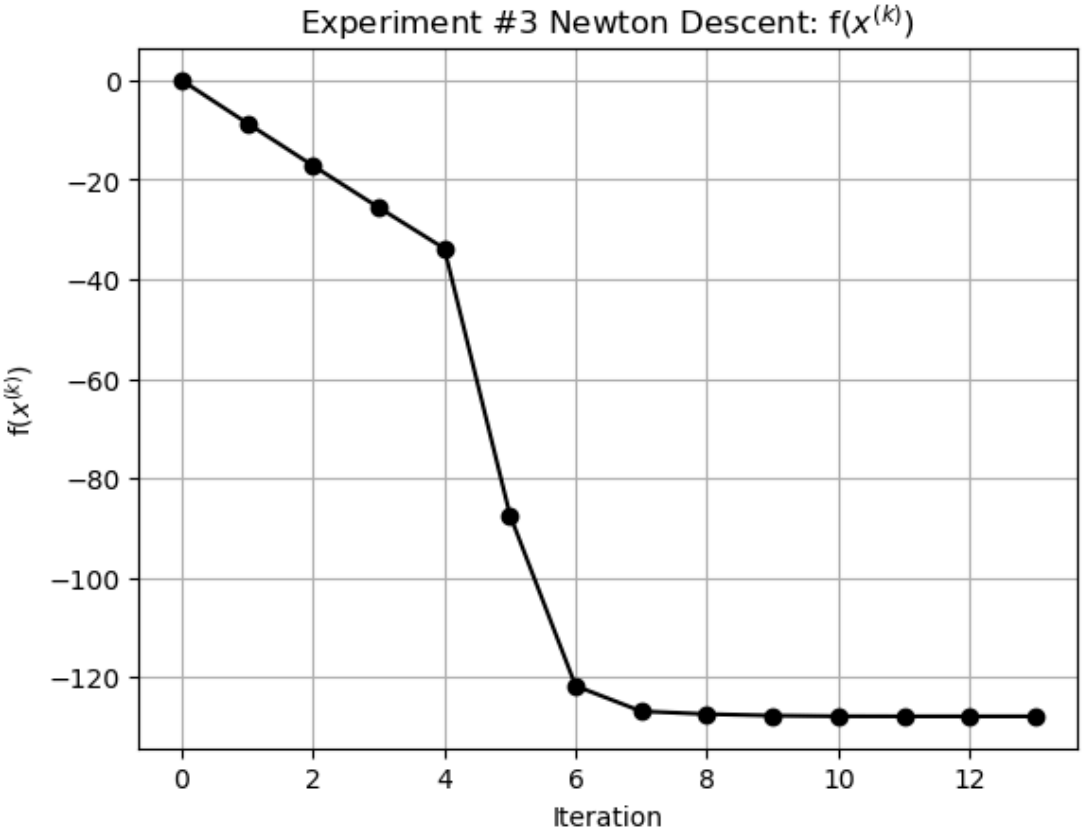


Experiment #3: n = 100, m = 200, $\alpha = 0.15$, $\beta = 0.1$

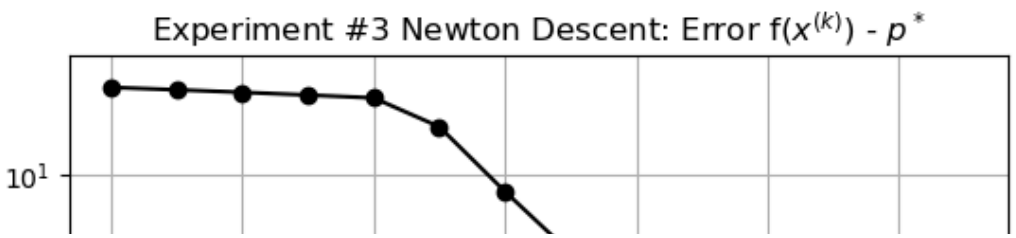
In [66]:
```
#Global Variables
n = 100
m = 200
alpha = 0.15
beta = 0.1

eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(0)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

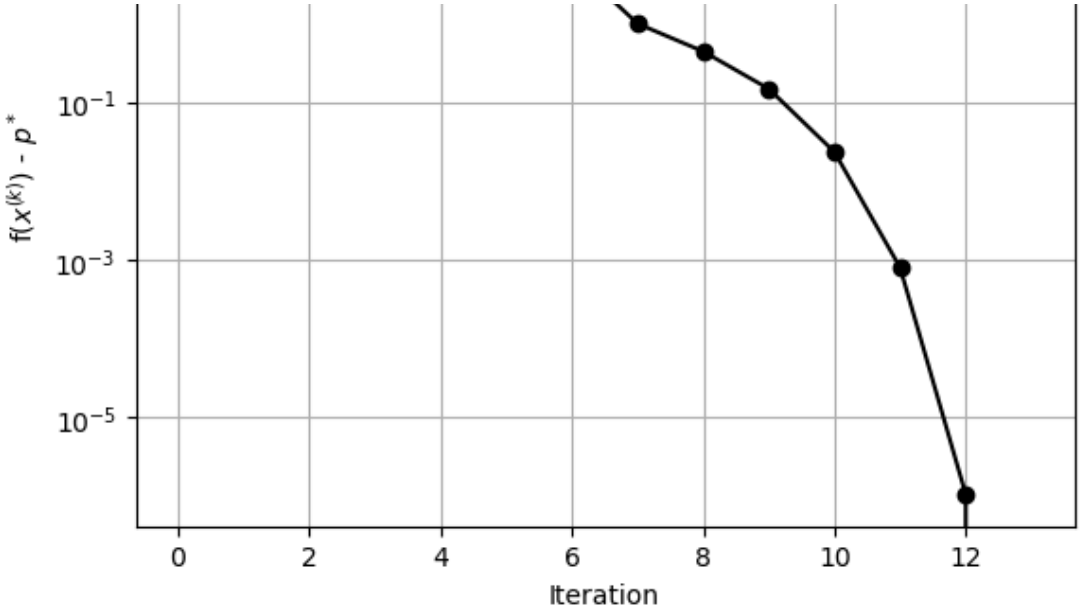
In [67]:
```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

In [68]:
```
#Plot function values throughout Newton's Method
fig34 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_function_values))), newton_descent_funct
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #3 Newton Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```



In [69]:
```
#Plot distance to optimal function value throughout Newton's Method
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed0
fig35 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_descent_gap_to
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #3 Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```

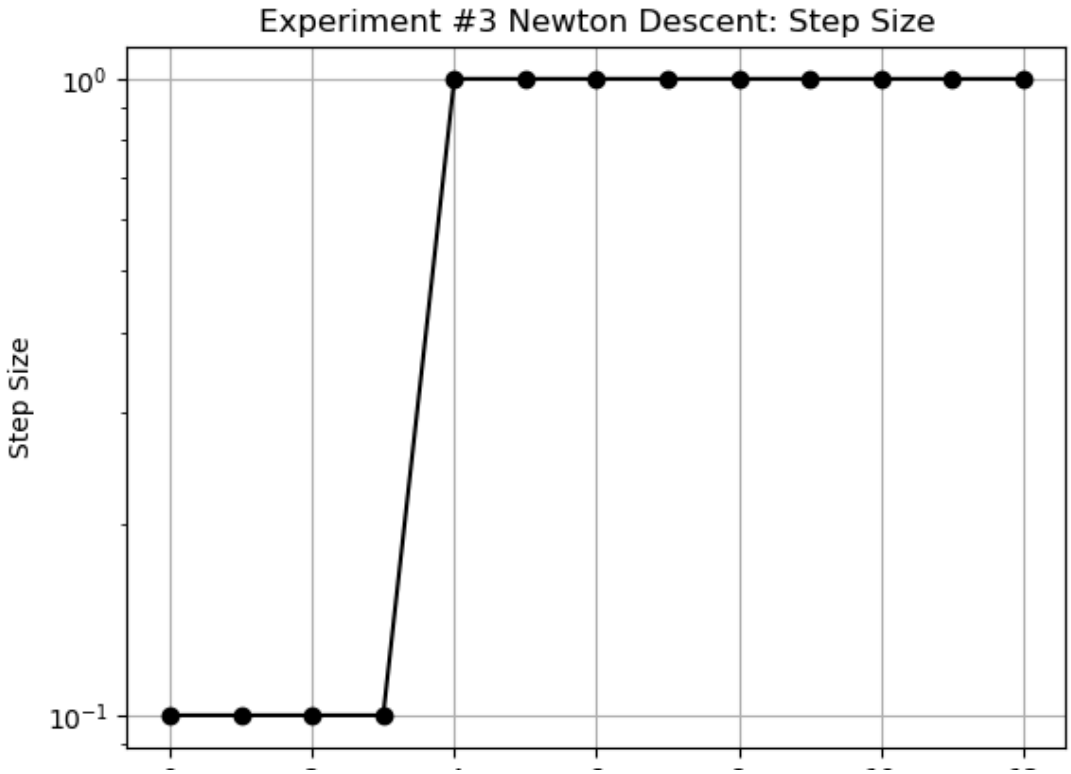


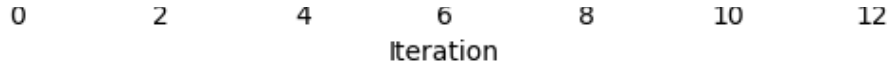


In [70]:
```
#Print step sizes during Newton's Method
newton_descent_step_sizes
```

Out[70]:
```
array([0.1, 0.1, 0.1, 0.1, 1. , 1. , 1. , 1. , 1. , 1. , 1. , 1. , 1. ])
```

In [71]:
```
#Plot step sizes during Newton's Method
fig36 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_step_sizes))), newton_descent_step_sizes
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #3 Newton Descent: Step Size')
plt.grid(True)
plt.show()
```



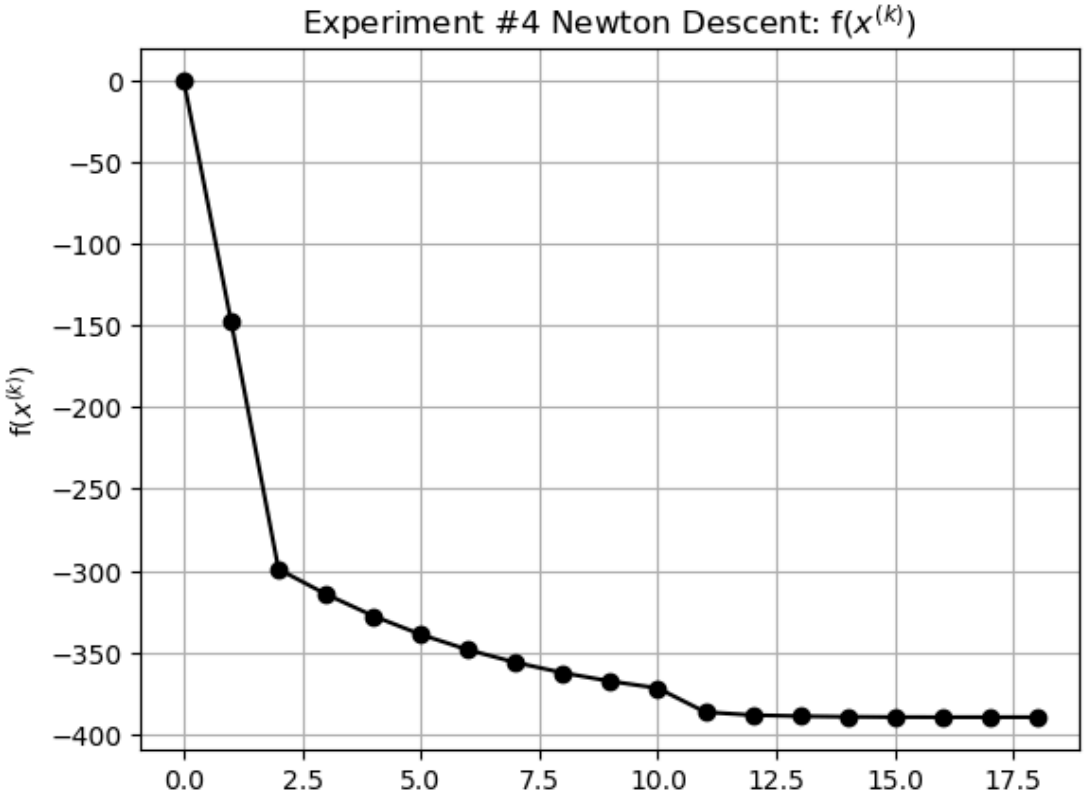


Experiment #4: n = 200, m = 400, $\alpha = 0.01$, $\beta = 0.1$

In [72]:
```
#Global Variables
n = 200
m = 400
alpha = 0.01
beta = 0.1

eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(1)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

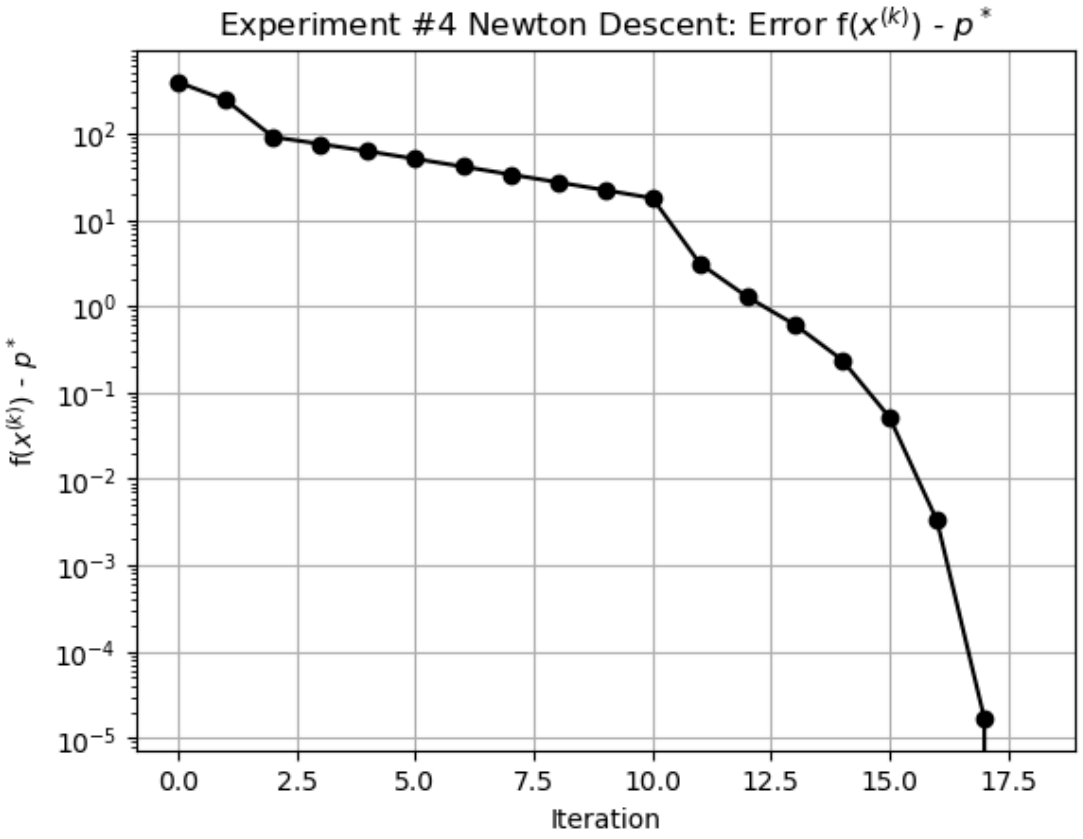
In [73]:
```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

In [74]:
```
#Plot function values throughout Newton's Method
fig37 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_function_values))), newton_descent_funct
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #4 Newton Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```



Iteration

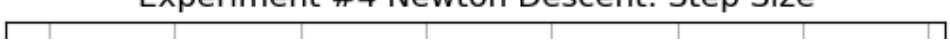
In [75]:
```
#Plot distance to optimal function value throughout Newton's Method
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed1
fig38 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_descent_gap_to
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #4 Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```

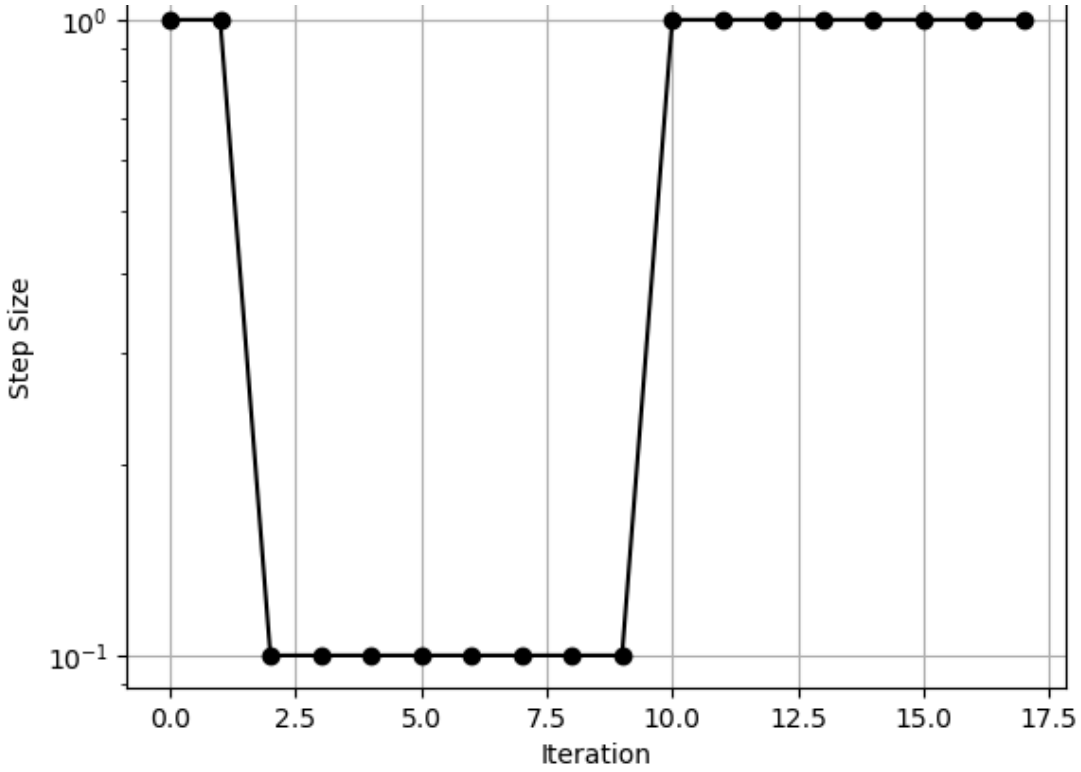


In [76]:
```
#Print step sizes during Newton's Method
newton_descent_step_sizes
```

Out[76]:
```
array([1. , 1. , 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 1. , 1. , 1. ,
       1. , 1. , 1. , 1. , 1. ])
```

In [77]:
```
#Plot step sizes during Newton's Method
fig39 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_step_sizes))), newton_descent_step_sizes
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #4 Newton Descent: Step Size')
plt.grid(True)
plt.show()
```



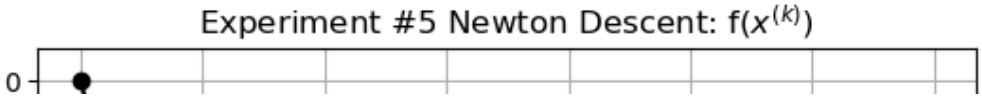


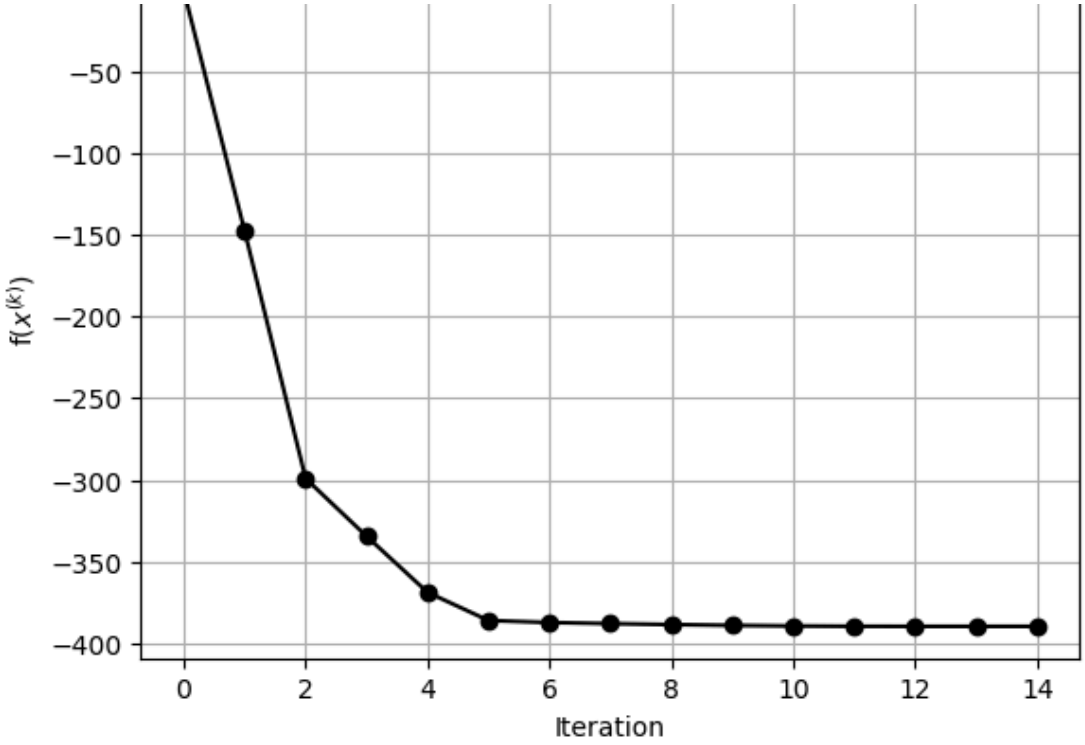
Experiment #5: n = 200, m = 400, $\alpha = 0.01, \beta = 0.5$

In [78]:
```
#Global Variables
n = 200
m = 400
alpha = 0.01
beta = 0.5

eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(1)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [79]:
```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

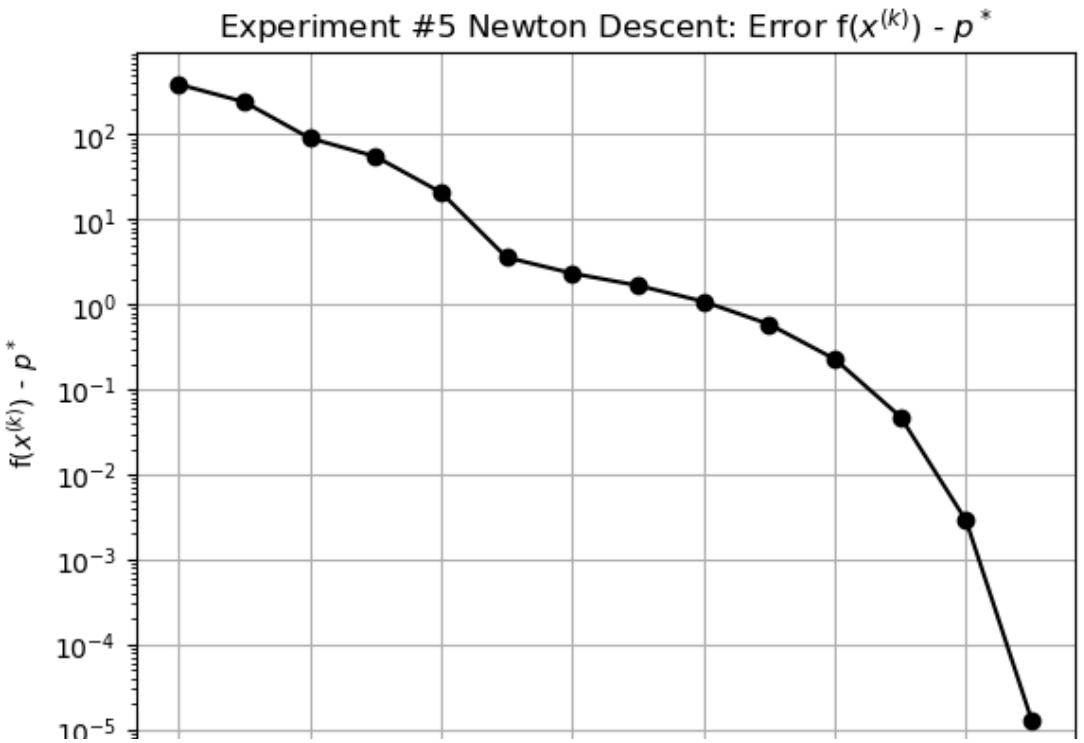
In [80]:
```
#Plot function values throughout Newton's Method
fig40 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_function_values))), newton_descent_funct
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #5 Newton Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```

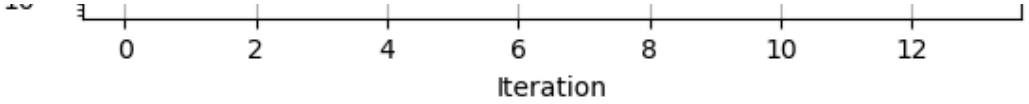




In [81]:

```
#Plot distance to optimal function value throughout Newton's Method
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed1
newton_descent_gap_to_optimal = newton_descent_gap_to_optimal[:-1]

fig41 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_descent_gap_to
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #5 Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```

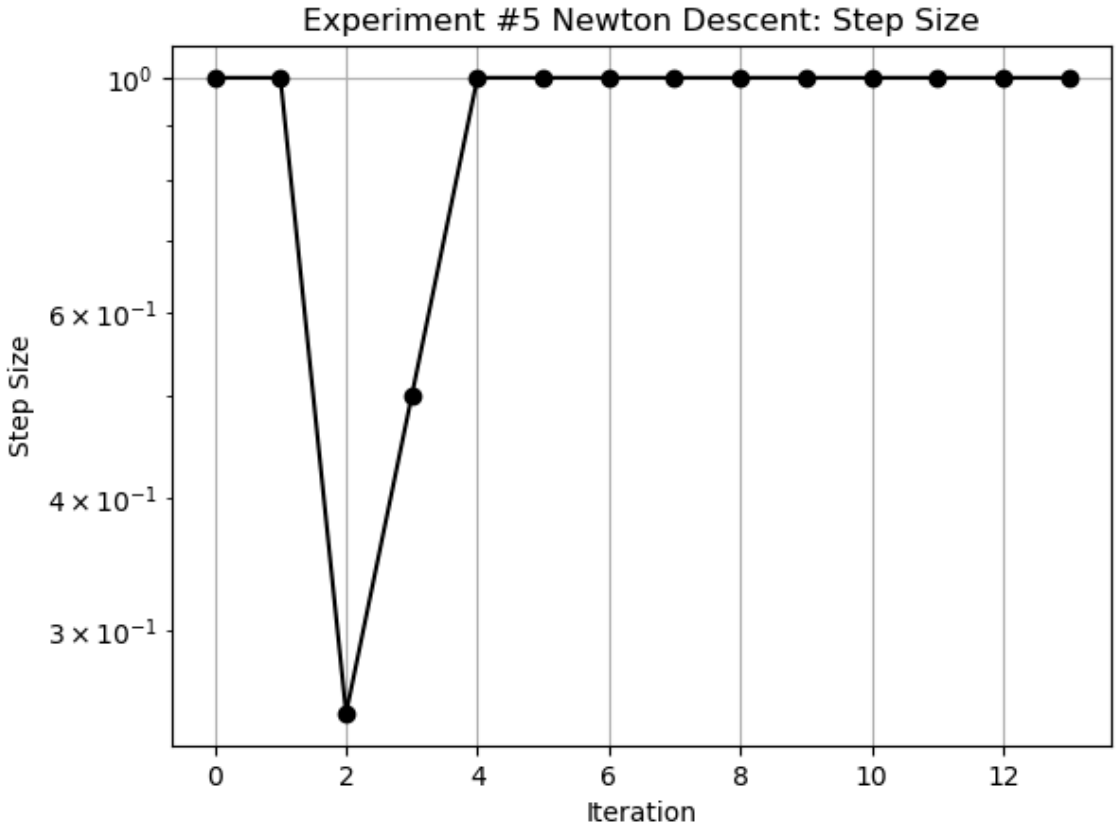




In [82]:
```
#Print step sizes during Newton's Method
newton_descent_step_sizes
```

Out[82]:
```
array([1.  , 1.  , 0.25, 0.5 , 1.  , 1.  , 1.  , 1.  , 1.  , 1.  , 1.  ,
       1.  , 1.  , 1.  ])
```

In [83]:
```
#Plot step sizes during Newton's Method
fig42 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_step_sizes))), newton_descent_step_sizes
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #5 Newton Descent: Step Size')
plt.grid(True)
plt.show()
```



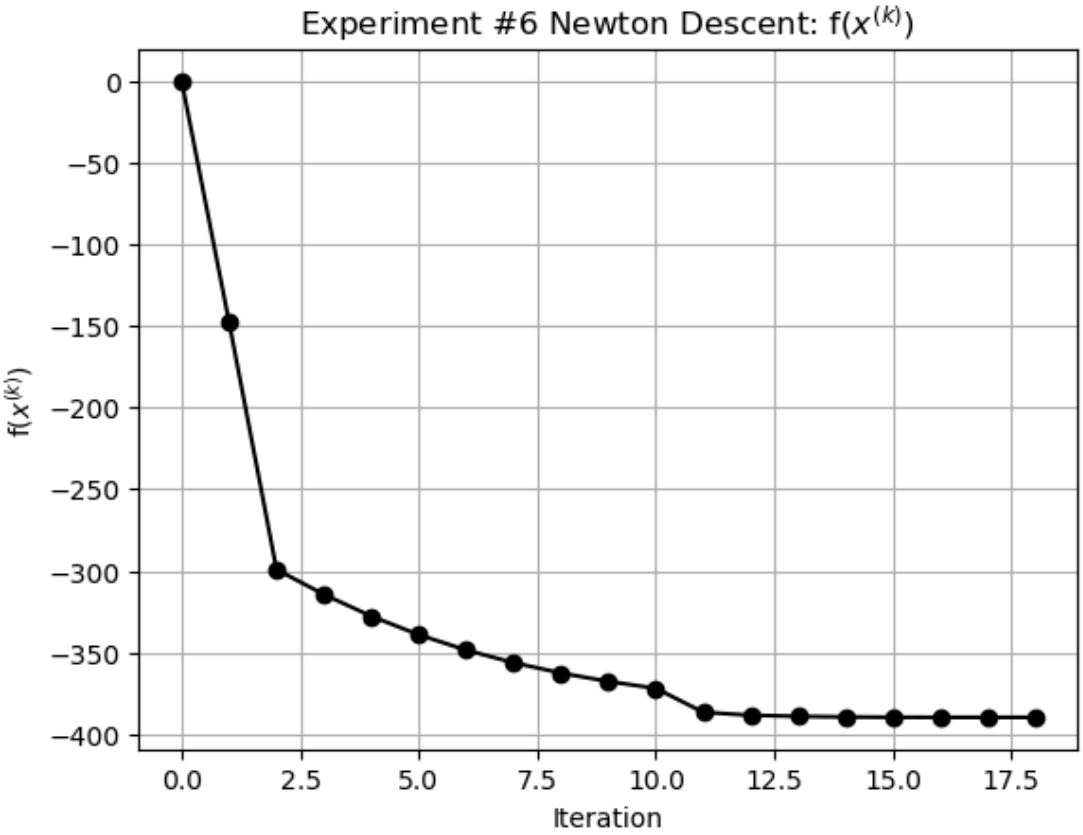
Experiment #6: n = 200, m = 400, $\alpha = 0.15$, $\beta = 0.1$

In [84]:
```
#Global Variables
n = 200
m = 400
alpha = 0.15
beta = 0.1

eta = 1e-8
max_iter = 2000
```

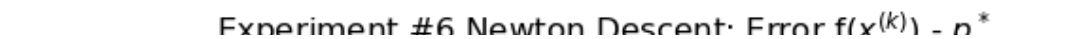
```
x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(1)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

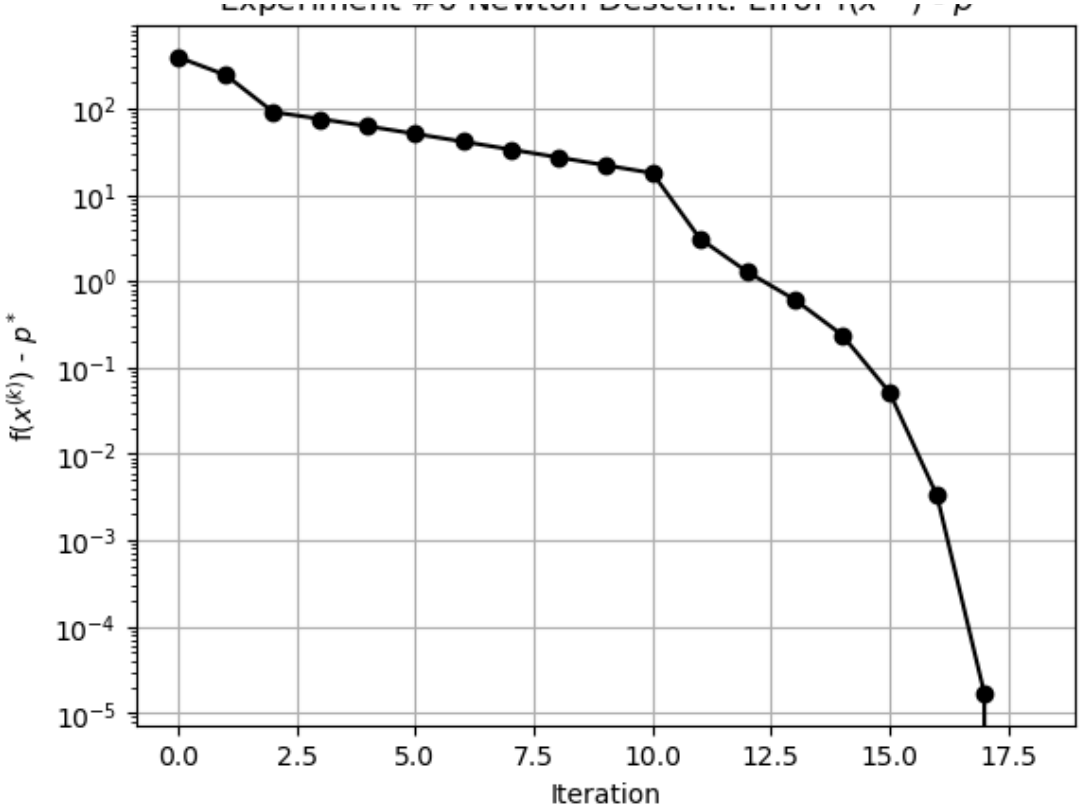
In [85]:
```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

In [86]:
```
#Plot function values throughout Newton's Method
fig43 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_function_values))), newton_descent_funct
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #6 Newton Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```



In [87]:
```
#Plot distance to optimal function value throughout Newton's Method
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed1
fig44 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_descent_gap_to
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #6 Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```

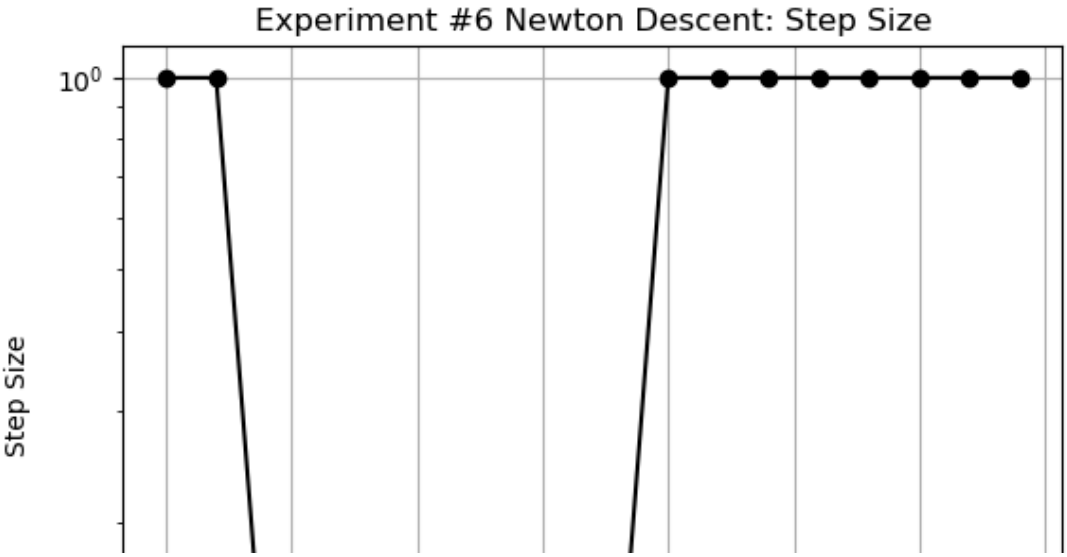


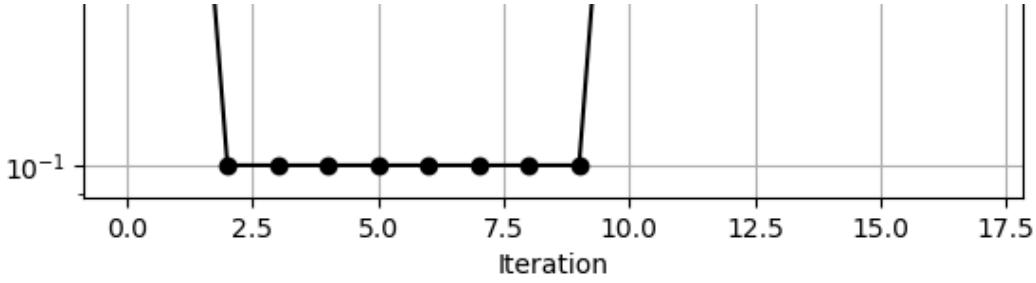


In [88]:
```
#Print step sizes during Newton's Method
newton_descent_step_sizes
```

Out[88]:
```
array([1. , 1. , 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 1. , 1. , 1. ,
       1. , 1. , 1. , 1. , 1. ])
```

In [89]:
```
#Plot step sizes during Newton's Method
fig45 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_step_sizes))), newton_descent_step_sizes
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #6 Newton Descent: Step Size')
plt.grid(True)
plt.show()
```



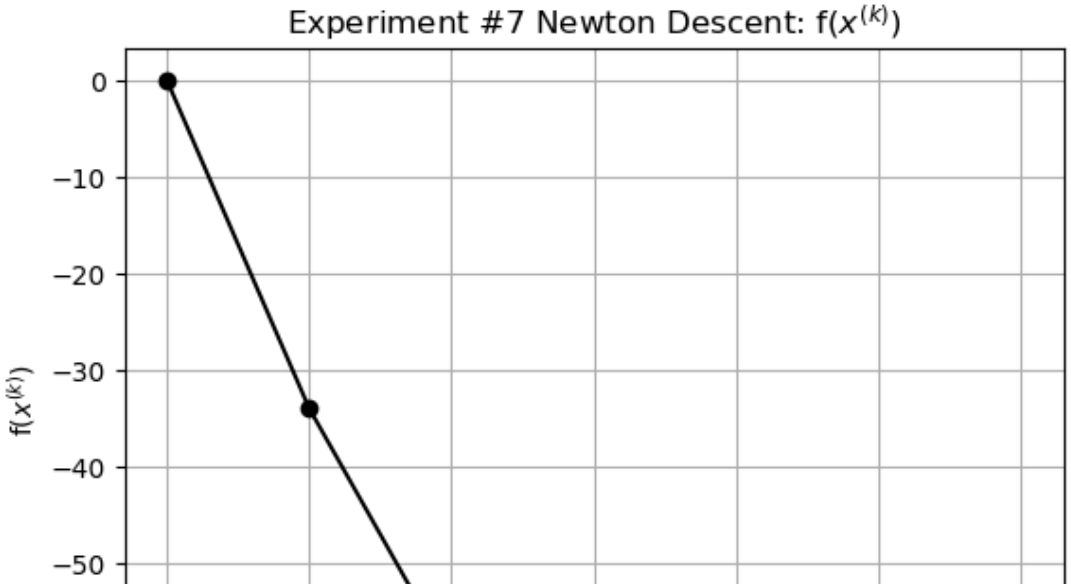


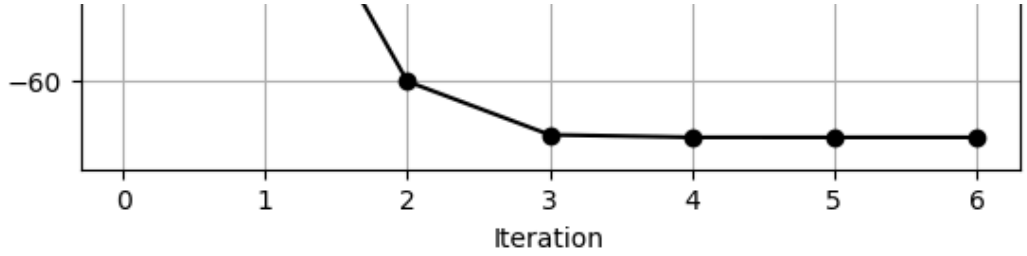
Experiment #7: n = 50, m = 100, $\alpha = 0.01, \beta = 0.1$

In [90]:
```
#Global Variables
n = 50
m = 100
alpha = 0.01
beta = 0.1

eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(2)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

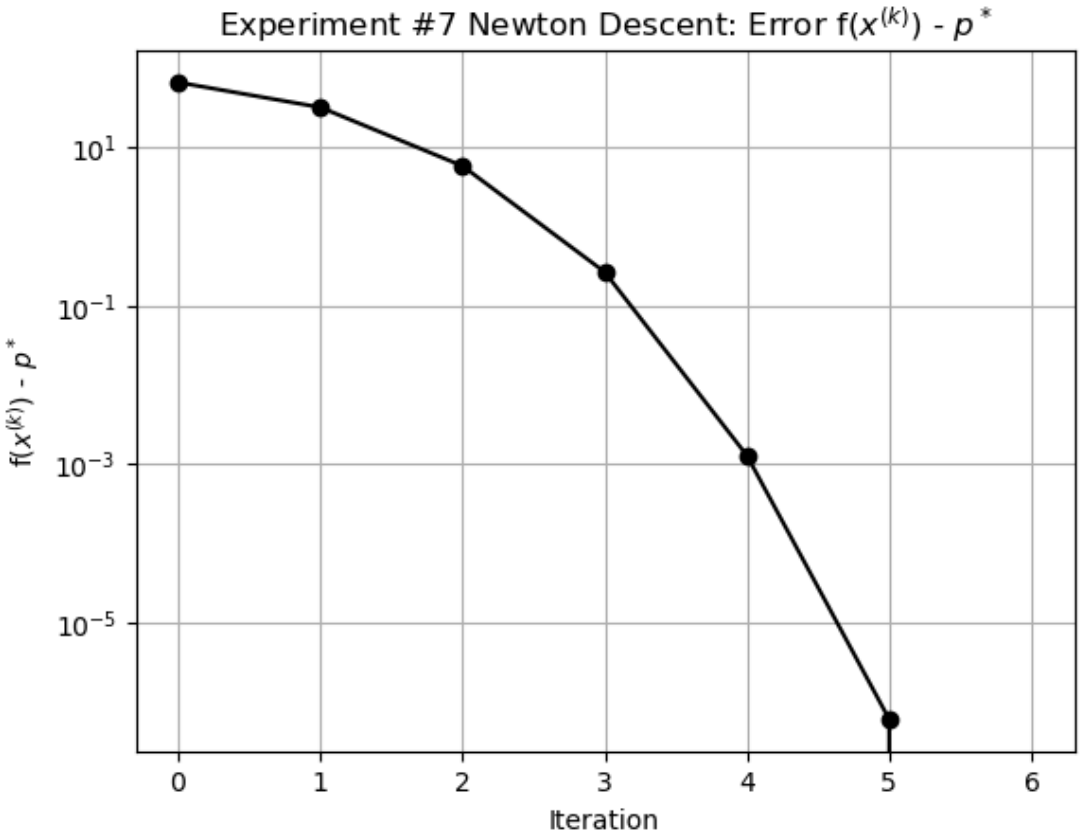
In [91]:
```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

In [92]:
```
#Plot function values throughout Newton's Method
fig46 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_function_values))), newton_descent_funct
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #7 Newton Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```





In [93]:
```
#Plot distance to optimal function value throughout Newton's Method
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed2
fig47 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_descent_gap_to
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #7 Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```

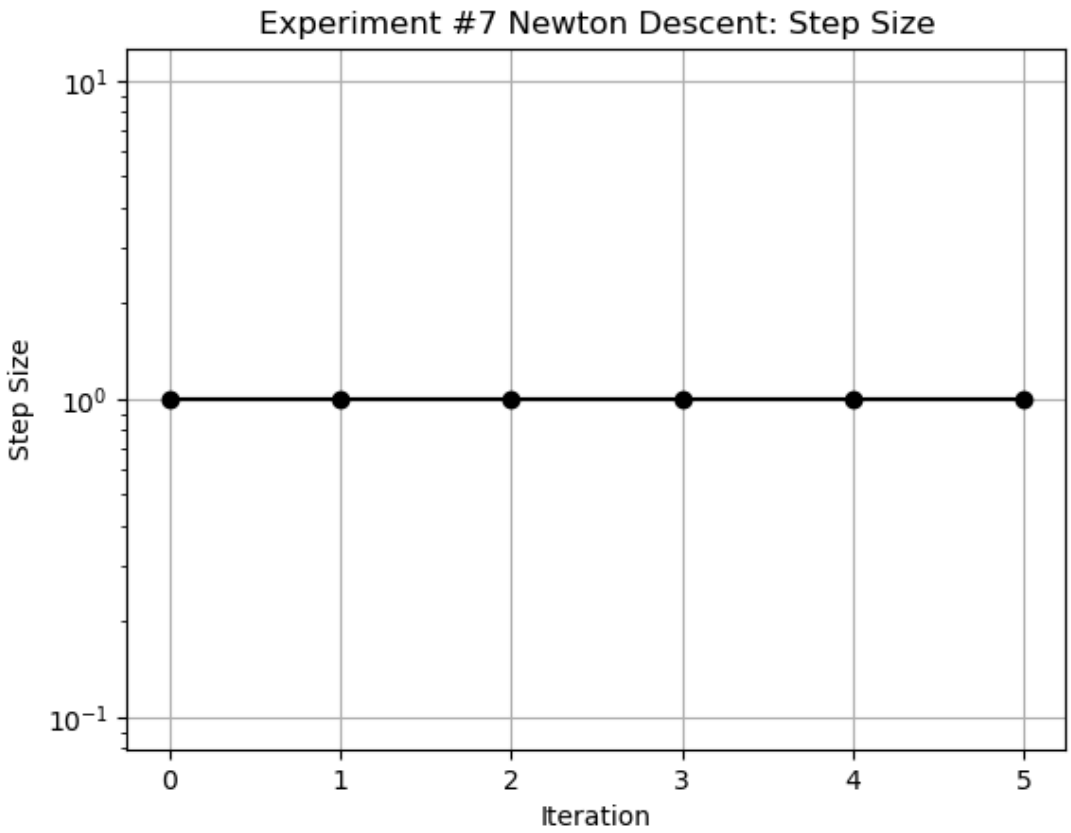


In [94]:
```
#Print step sizes during Newton's Method
newton_descent_step_sizes
```

Out[94]:
```
array([1, 1, 1, 1, 1, 1])
```

In [95]:
```
#Plot step sizes during Newton's Method
fig48 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_step_sizes))), newton_descent_step_sizes
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
```

```
plt.title(r'Experiment #7 Newton Descent: Step Size')
plt.grid(True)
plt.show()
```



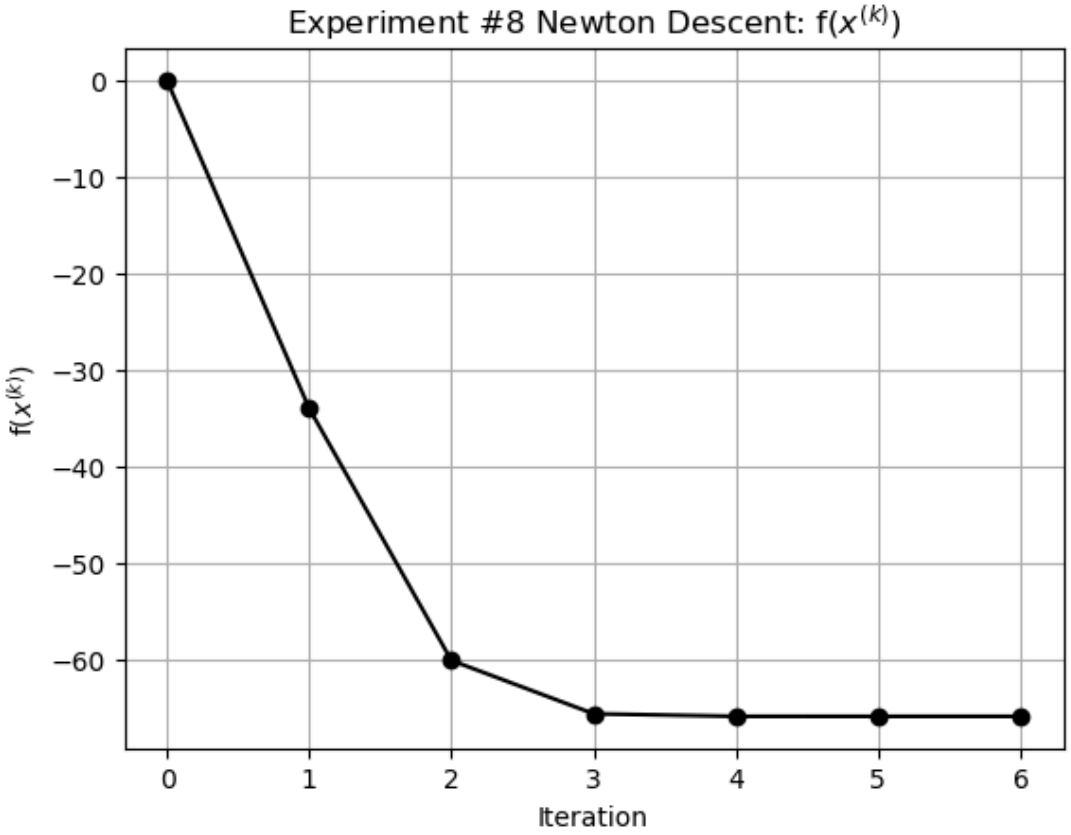
Experiment #8: n = 50, m = 100, $\alpha = 0.01, \beta = 0.5$

In [96]:
```
#Global Variables
n = 50
m = 100
alpha = 0.01
beta = 0.5

eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(2)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [97]:
```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

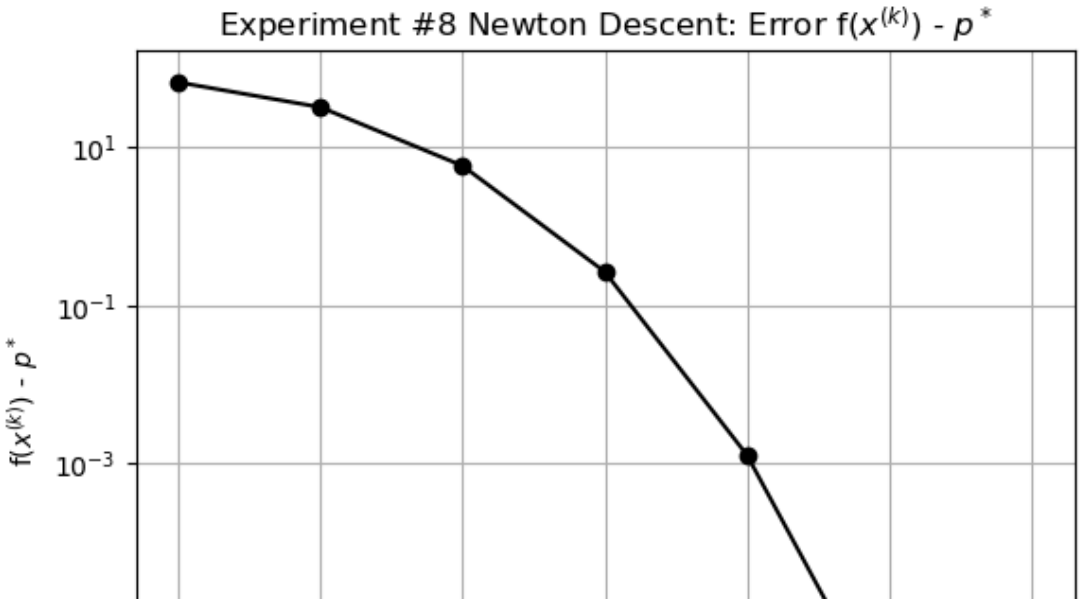
In [98]:
```
#Plot function values throughout Newton's Method
fig49 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_function_values))), newton_descent_funct
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #8 Newton Descent: f($x^{(k)}$)')
```

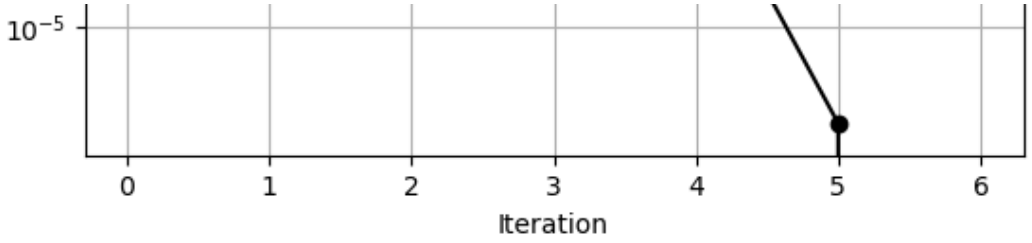
```
plt.grid(True)
plt.show()
```



In [99]:

```
#Plot distance to optimal function value throughout Newton's Method
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed2
fig50 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_descent_gap_to
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #8 Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
plt.show()
```

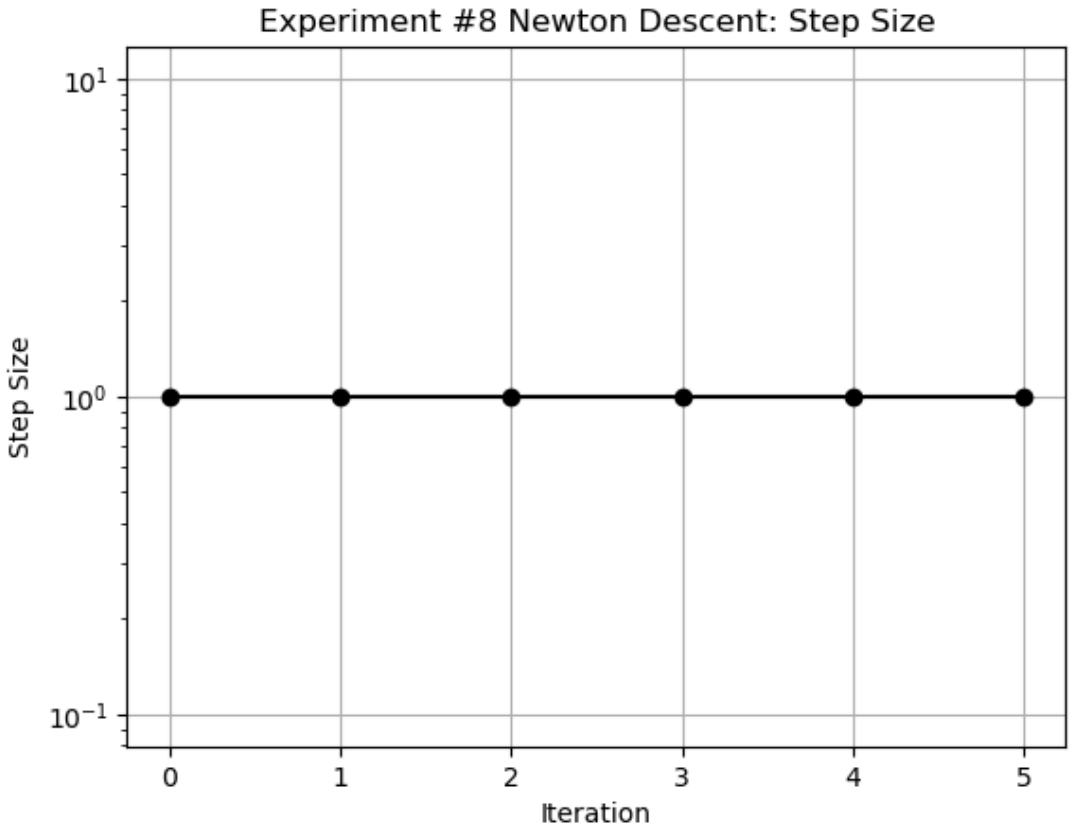




In [100…
```
#Print step sizes during Newton's Method
newton_descent_step_sizes
```

Out[100… array([1, 1, 1, 1, 1, 1])

In [101…
```
#Plot step sizes during Newton's Method
fig51 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_step_sizes))), newton_descent_step_sizes
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #8 Newton Descent: Step Size')
plt.grid(True)
plt.show()
```



Experiment #9: n = 50, m = 100, $\alpha = 0.15, \beta = 0.1$

In [102…
```
#Global Variables
n = 50
m = 100
alpha = 0.15
beta = 0.1
```

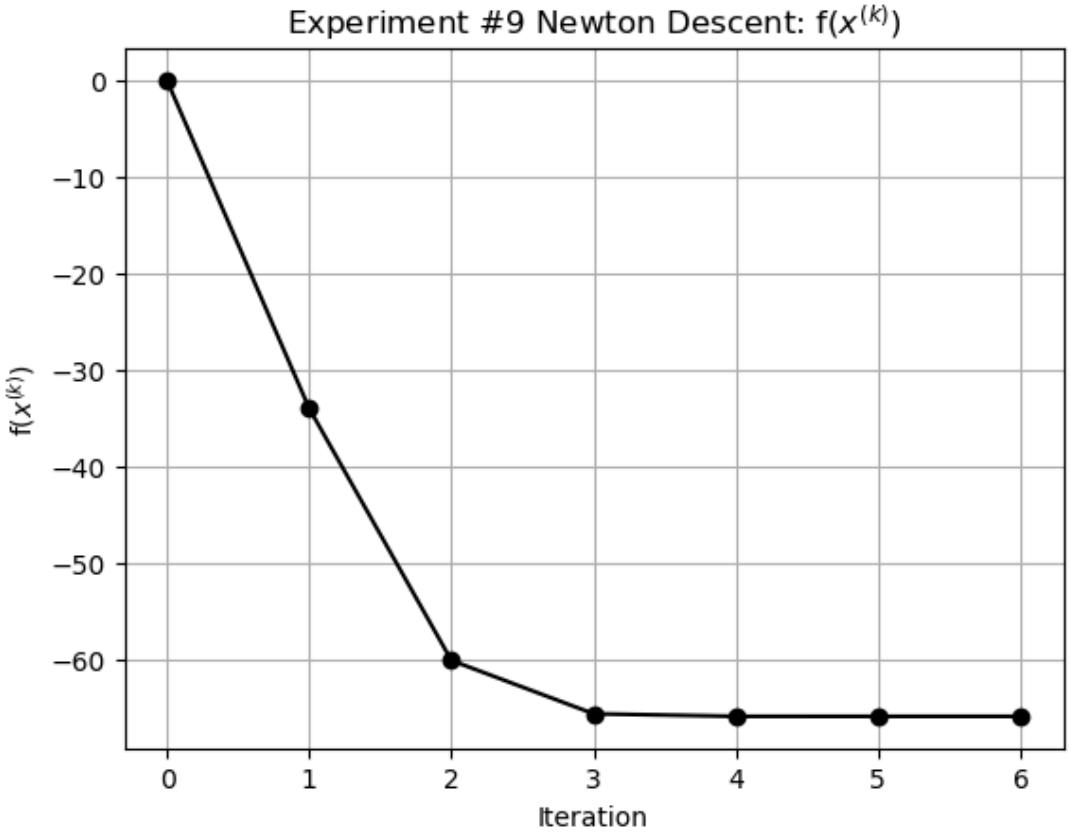
```
beta = 0.1

eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(2)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [103…

```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

In [104…

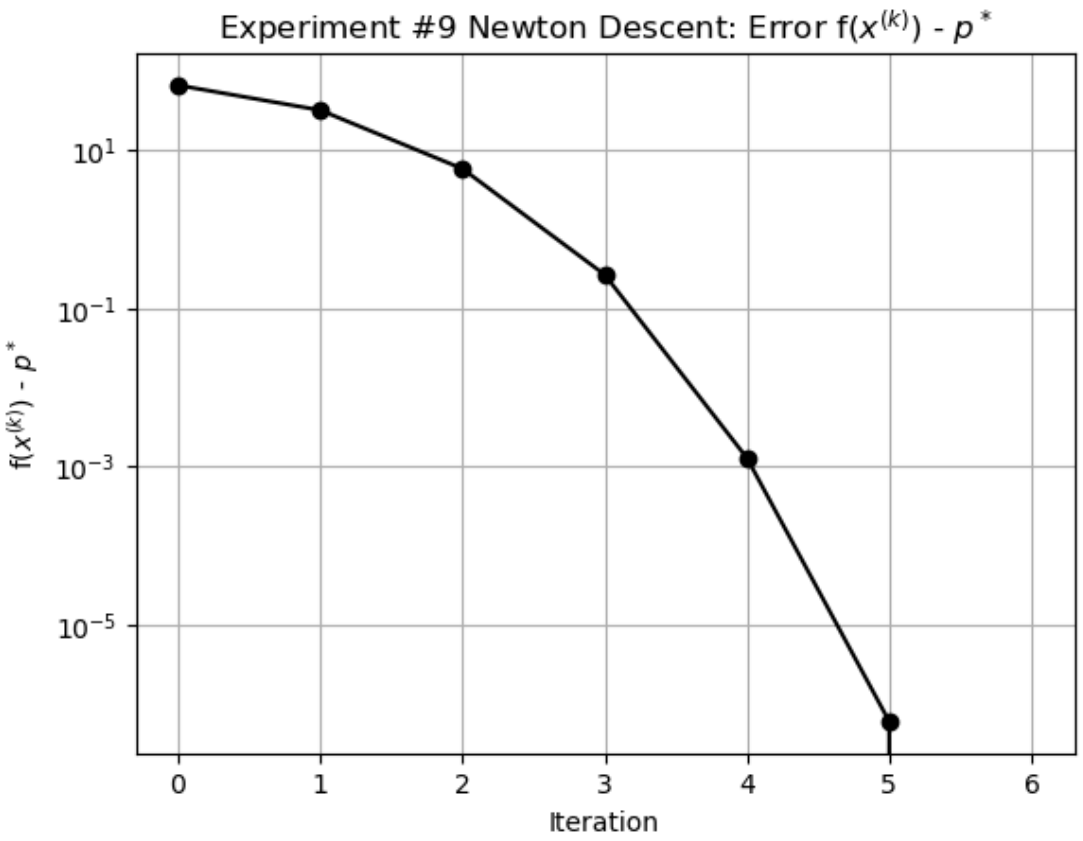
```
#Plot function values throughout Newton's Method
fig52 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_function_values))), newton_descent_funct
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #9 Newton Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```



In [105…

```
#Plot distance to optimal function value throughout Newton's Method
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed2
fig53 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_descent_gap_to
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #9 Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.grid(True)
```

```
plt.show()
```



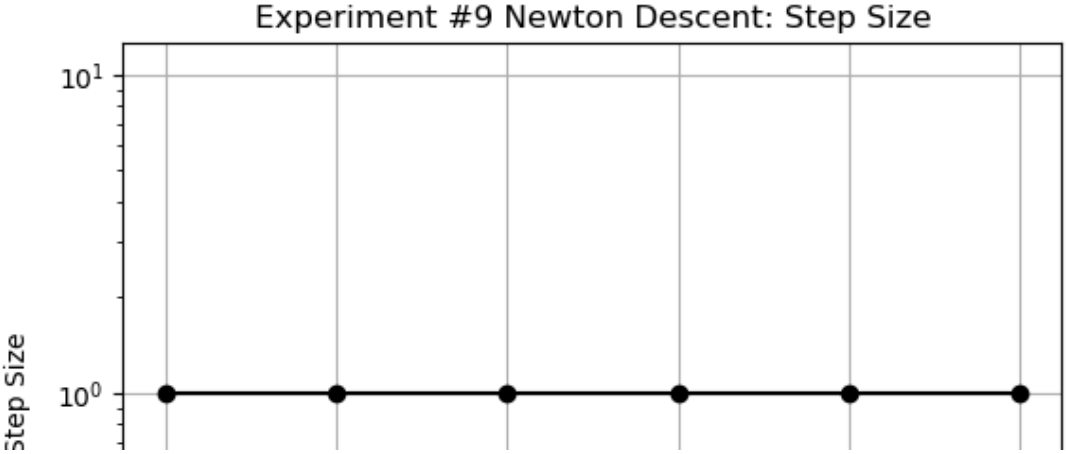
In [106…

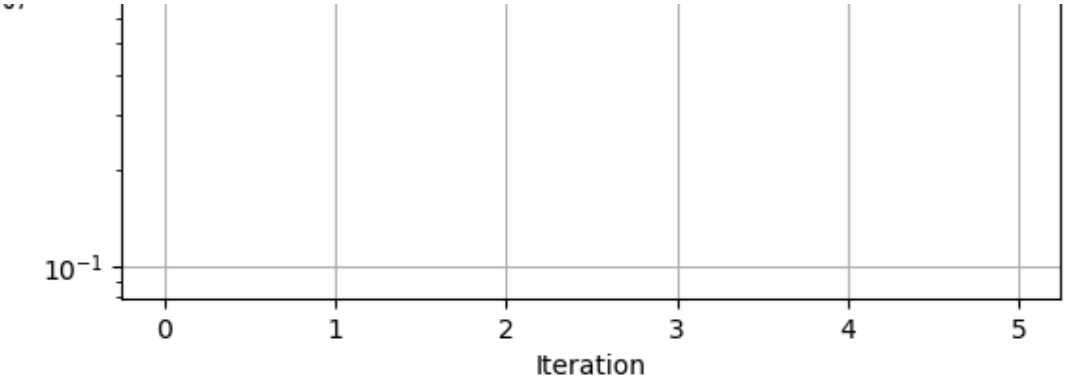
```
#Print step sizes during Newton's Method
newton_descent_step_sizes
```

Out[106… array([1, 1, 1, 1, 1, 1])

In [107…

```
#Plot step sizes during Newton's Method
fig54 = plt.figure()  # Create a new figure
plt.plot(list(range(len(newton_descent_step_sizes))), newton_descent_step_sizes
plt.xlabel('Iteration')
plt.ylabel(r'Step Size')
plt.yscale('log')
plt.title(r'Experiment #9 Newton Descent: Step Size')
plt.grid(True)
plt.show()
```





Observations:

- Backtracking Parameters α and β do not affect the performance of Newton's Method that much when compared to Gradient Descent
- Performance is scaling well with problem size

9.31 (a)

Experiment #1

In [108…
```
#Global Variables
n = 100
m = 200
alpha = 0.01
beta = 0.5
eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(0)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [109…
```
#Run Gradient Descent
gradient_descent_eta = 1e-3
gradient_descent_max_iter = 80
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

In [110…
```
#Run Newton's Method where Hessian is reused every 1 iteration
reused_newton_descent_iterates_N1, reused_newton_descent_function_values_N1, re
```

In [111…
```
#Run Newton's Method where Hessian is reused every 2 iteration
reused_newton_descent_iterates_N2, reused_newton_descent_function_values_N2, re
```

In [112…
```
#Run Newton's Method where Hessian is reused every 5 iteration
reused_newton_descent_iterates_N5, reused_newton_descent_function_values_N5, re
```

In [113…
```
#Run Newton's Method where Hessian is reused every 10 iteration
```

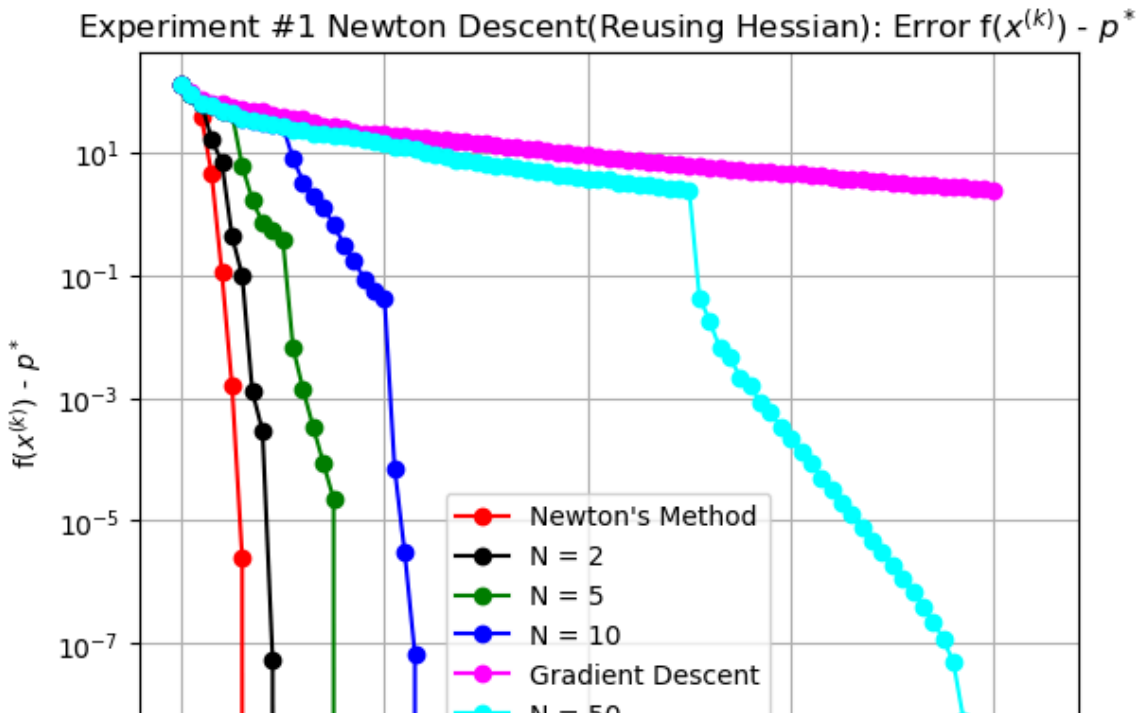
```
reused_newton_descent_iterates_N10, reused_newton_descent_function_values_N10,
```

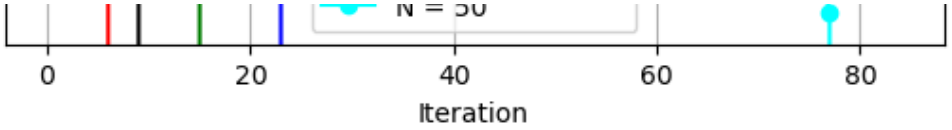
In [114…

```
#Run Newton's Method where Hessian is reused every 50 iteration
reused_newton_descent_iterates_N50, reused_newton_descent_function_values_N50,
```

In [115…

```
#Plot distance to optimal function value throughout Newton's Method for N = 1,
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
reused_newton_descent_gap_to_optimal_N1 = reused_newton_descent_function_values
reused_newton_descent_gap_to_optimal_N2 = reused_newton_descent_function_values
reused_newton_descent_gap_to_optimal_N5 = reused_newton_descent_function_values
reused_newton_descent_gap_to_optimal_N10 = reused_newton_descent_function_value
reused_newton_descent_gap_to_optimal_N50 = reused_newton_descent_function_value


# print(f"Gap to Optimal for N = 1: {reused_newton_descent_gap_to_optimal_N1}")
# print(f"Gap to Optimal for N = 2: {reused_newton_descent_gap_to_optimal_N2}")
# print(f"Gap to Optimal for N = 5: {reused_newton_descent_gap_to_optimal_N5}")
# print(f"Gap to Optimal for N = 10: {reused_newton_descent_gap_to_optimal_N10}

fig55 = plt.figure()  # Create a new figure


line1, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N1))), re
line2, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N2))), re
line3, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N5))), re
line4, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N10))), r
line5, = plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_d
line6, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N50))), r

plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #1 Newton Descent(Reusing Hessian): Error f($x^{(k)}$) -
plt.legend(handles=[line1, line2, line3, line4, line5, line6], labels=['Newton\
plt.grid(True)
plt.show()
```





Experiment #2

In [116…

```
#Global Variables
n = 200
m = 400
alpha = 0.01
beta = 0.5
eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(1)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [117…

```
#Run Gradient Descent
gradient_descent_eta = 1e-3
gradient_descent_max_iter = 60
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

In [118…

```
#Run Newton's Method where Hessian is reused every 1 iteration
reused_newton_descent_iterates_N1, reused_newton_descent_function_values_N1, re
```

In [119…

```
#Run Newton's Method where Hessian is reused every 2 iteration
reused_newton_descent_iterates_N2, reused_newton_descent_function_values_N2, re
```

In [120…

```
#Run Newton's Method where Hessian is reused every 5 iteration
reused_newton_descent_iterates_N5, reused_newton_descent_function_values_N5, re
```

In [121…

```
#Run Newton's Method where Hessian is reused every 10 iteration
reused_newton_descent_iterates_N10, reused_newton_descent_function_values_N10,
```

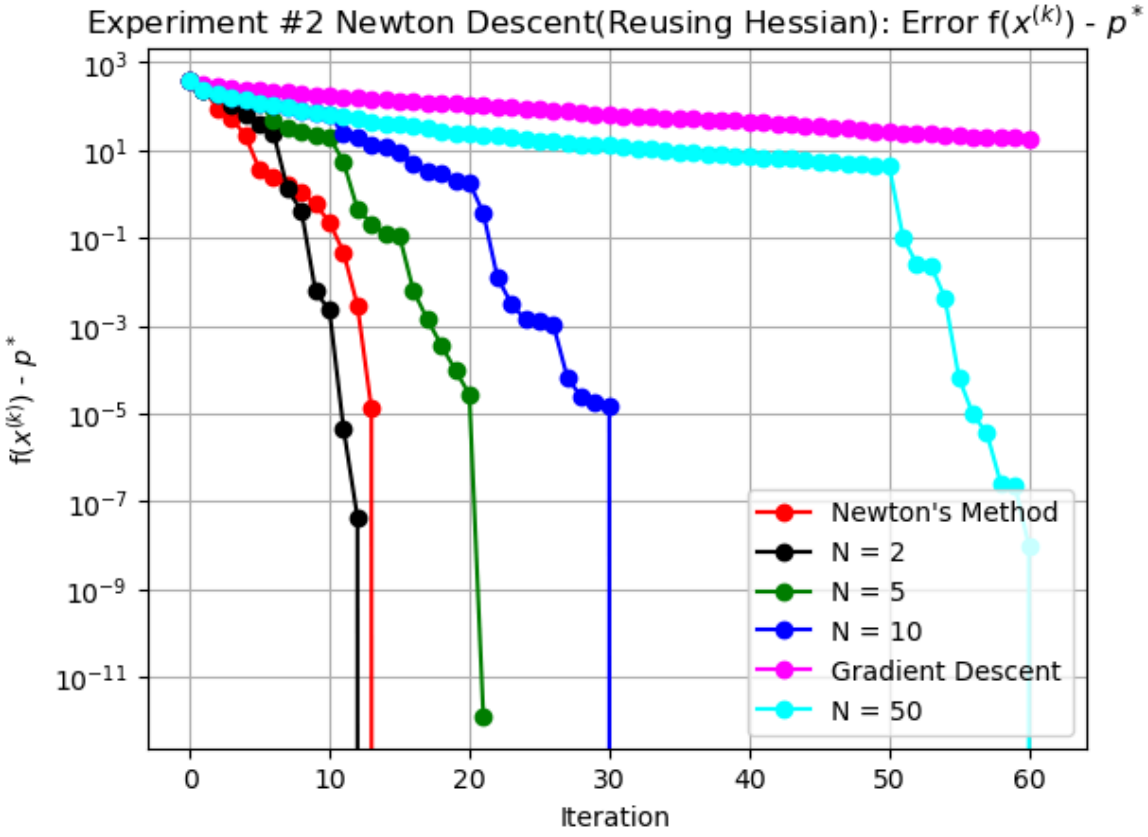
In [122…

```
#Run Newton's Method where Hessian is reused every 50 iteration
reused_newton_descent_iterates_N50, reused_newton_descent_function_values_N50,
```

In [123…

```
#Plot distance to optimal function value throughout Newton's Method for N = 1,
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
reused_newton_descent_gap_to_optimal_N1 = reused_newton_descent_function_values
reused_newton_descent_gap_to_optimal_N2 = reused_newton_descent_function_values
reused_newton_descent_gap_to_optimal_N5 = reused_newton_descent_function_values
reused_newton_descent_gap_to_optimal_N10 = reused_newton_descent_function_value
reused_newton_descent_gap_to_optimal_N50 = reused_newton_descent_function_value

fig56 = plt.figure()  # Create a new figure
```

```
line1, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N1))), re
line2, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N2))), re
line3, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N5))), re
line4, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N10))), r
line5, = plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_d
line6, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N50))), r

plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #2 Newton Descent(Reusing Hessian): Error f($x^{(k)}$) -
plt.legend(handles=[line1, line2, line3, line4, line5, line6], labels=['Newton\
plt.grid(True)
plt.show()
```



Experiment #3

In [124…

```
#Global Variables
n = 50
m = 100
alpha = 0.01
beta = 0.5
eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(2)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [125…

```
#Run Gradient Descent
```

```
gradient_descent_eta = 1e-3
gradient_descent_max_iter = 50
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

In [126…

```
#Run Newton's Method where Hessian is reused every 1 iteration
reused_newton_descent_iterates_N1, reused_newton_descent_function_values_N1, re
```

In [127…

```
#Run Newton's Method where Hessian is reused every 2 iteration
reused_newton_descent_iterates_N2, reused_newton_descent_function_values_N2, re
```

In [128…

```
#Run Newton's Method where Hessian is reused every 5 iteration
reused_newton_descent_iterates_N5, reused_newton_descent_function_values_N5, re
```

In [129…

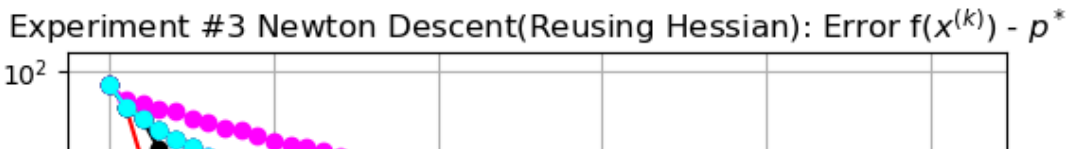
```
#Run Newton's Method where Hessian is reused every 10 iteration
reused_newton_descent_iterates_N10, reused_newton_descent_function_values_N10,
```

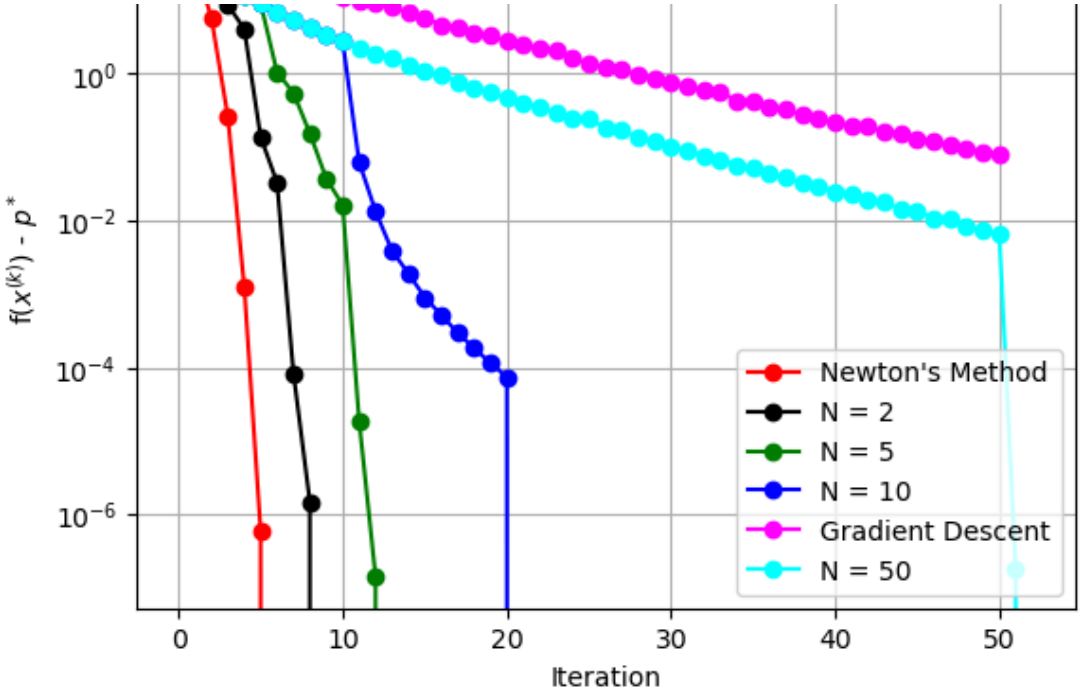
In [130…

```
#Run Newton's Method where Hessian is reused every 50 iteration
reused_newton_descent_iterates_N50, reused_newton_descent_function_values_N50,
```

In [131…

```
#Plot distance to optimal function value throughout Newton's Method for N = 1,
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
reused_newton_descent_gap_to_optimal_N1 = reused_newton_descent_function_values
reused_newton_descent_gap_to_optimal_N2 = reused_newton_descent_function_values
reused_newton_descent_gap_to_optimal_N5 = reused_newton_descent_function_values
reused_newton_descent_gap_to_optimal_N10 = reused_newton_descent_function_value
reused_newton_descent_gap_to_optimal_N50 = reused_newton_descent_function_value


# print(f"Gap to Optimal for N = 1: {reused_newton_descent_gap_to_optimal_N1}")
# print(f"Gap to Optimal for N = 2: {reused_newton_descent_gap_to_optimal_N2}")
# print(f"Gap to Optimal for N = 5: {reused_newton_descent_gap_to_optimal_N5}")
# print(f"Gap to Optimal for N = 10: {reused_newton_descent_gap_to_optimal_N10}

fig57 = plt.figure()  # Create a new figure

line1, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N1))), re
line2, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N2))), re
line3, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N5))), re
line4, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N10))), r
line5, = plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_d
line6, = plt.plot(list(range(len(reused_newton_descent_gap_to_optimal_N50))), r

plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #3 Newton Descent(Reusing Hessian): Error f($x^{(k)}$) -
plt.legend(handles=[line1, line2, line3, line4, line5, line6], labels=['Newton\
plt.grid(True)
plt.show()
```





Observations:

- As N increases, the speed of convergence decreases
- As N increases, the convergence behavior begins to resemble the gradient descent convergence behavior more and more.

9.31 (b)

Experiment #1

In [132…

```
#Global Variables
n = 100
m = 200
alpha = 0.01
beta = 0.5
eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(0)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [133…

```
#Run Gradient Descent
gradient_descent_eta = 1e-3
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

In [134…

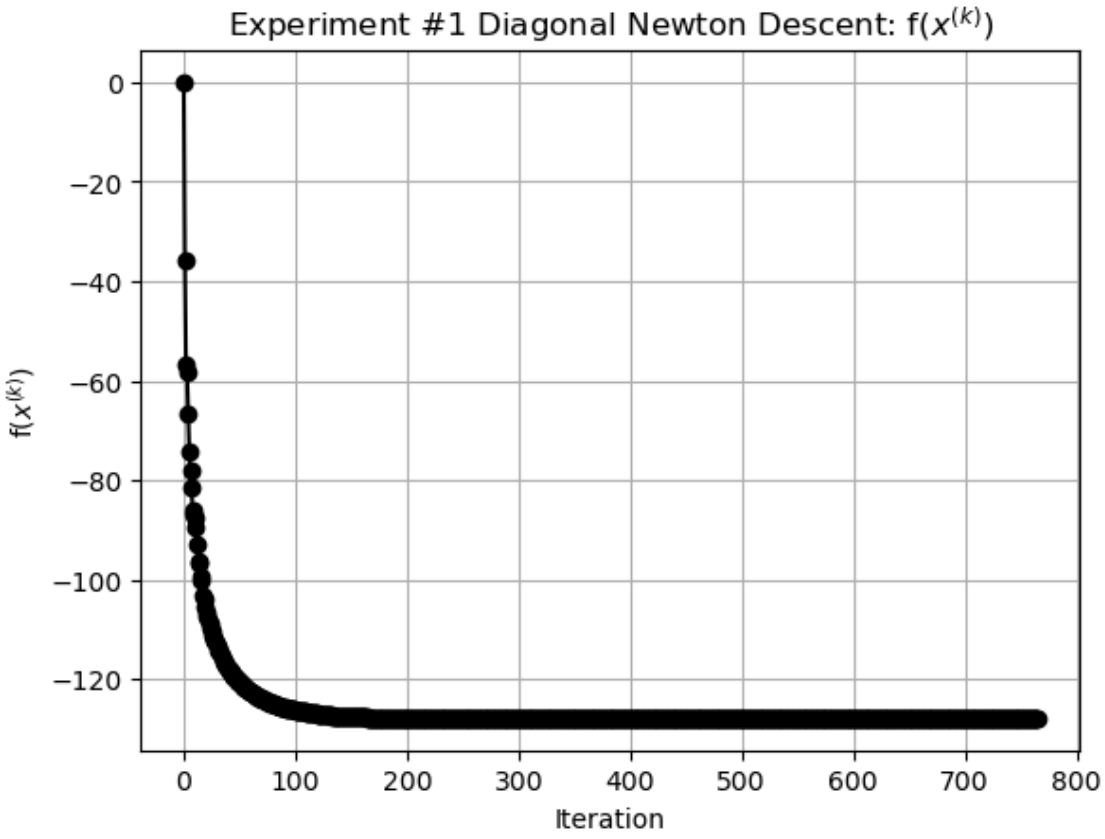
```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

In [135

```
#Run Newton's Method where Hessian is approximated by its diagonal
diagonal_newton_descent_iterates, diagonal_newton_descent_function_values, diag
```

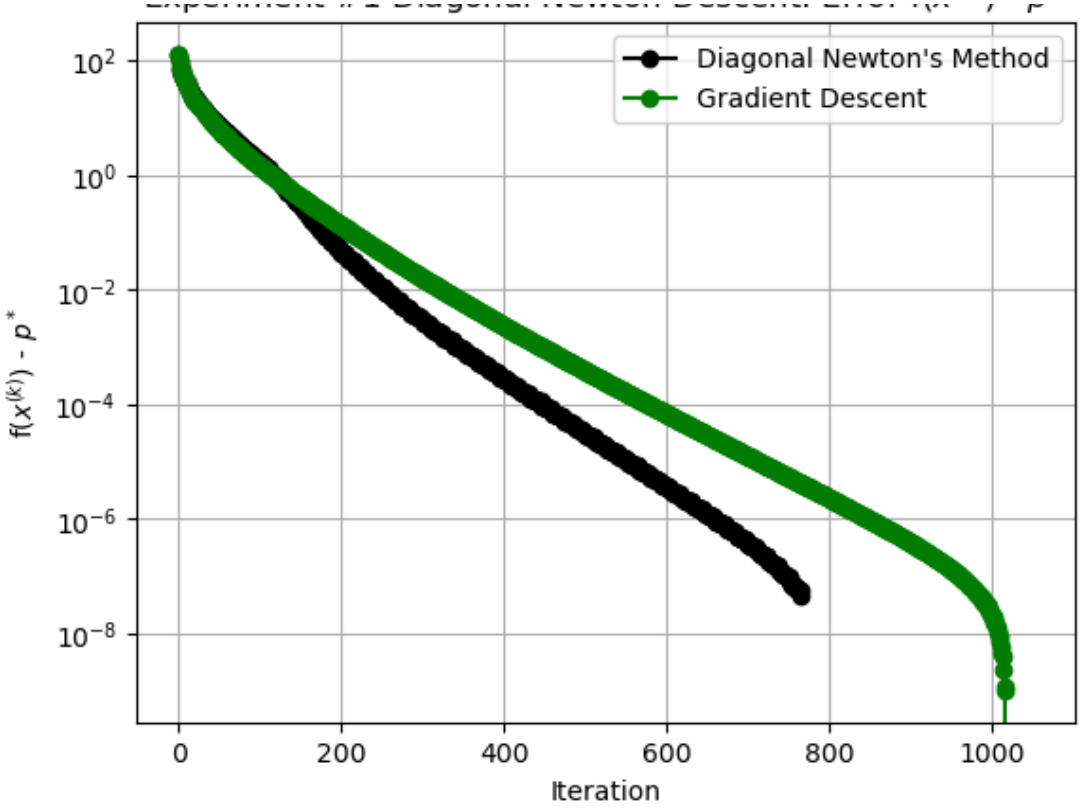
In [136…

```
#Plot function values throughout Newton's Method where the Hessian is replaced
fig58 = plt.figure()  # Create a new figure
plt.plot(list(range(len(diagonal_newton_descent_function_values))), diagonal_ne
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #1 Diagonal Newton Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```



In [137…

```
#Plot distance to optimal function value throughout Newton's Method where the H
#Also do this for regular newton's method and gradient descent for comparison
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed0
diagonal_newton_descent_gap_to_optimal = diagonal_newton_descent_function_value
fig59 = plt.figure()  # Create a new figure

line1, = plt.plot(list(range(len(diagonal_newton_descent_gap_to_optimal))), dia
# line2, = plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_des
line3, = plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_d

plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #1 Diagonal Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.legend(handles=[line1, line3], labels=['Diagonal Newton\'s Method', 'Gradie
plt.grid(True)
plt.show()
```



In [138…

```
#Just a print for debugging purposes
print(len(gradient_descent_gap_to_optimal))
print(len(diagonal_newton_descent_gap_to_optimal))
```

```
1050
766
```

Experiment #2

In [139…

```
#Global Variables
n = 200
m = 400
alpha = 0.01
beta = 0.5
eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(1)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [140…

```
#Run Gradient Descent
gradient_descent_eta = 1e-3
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

In [141…

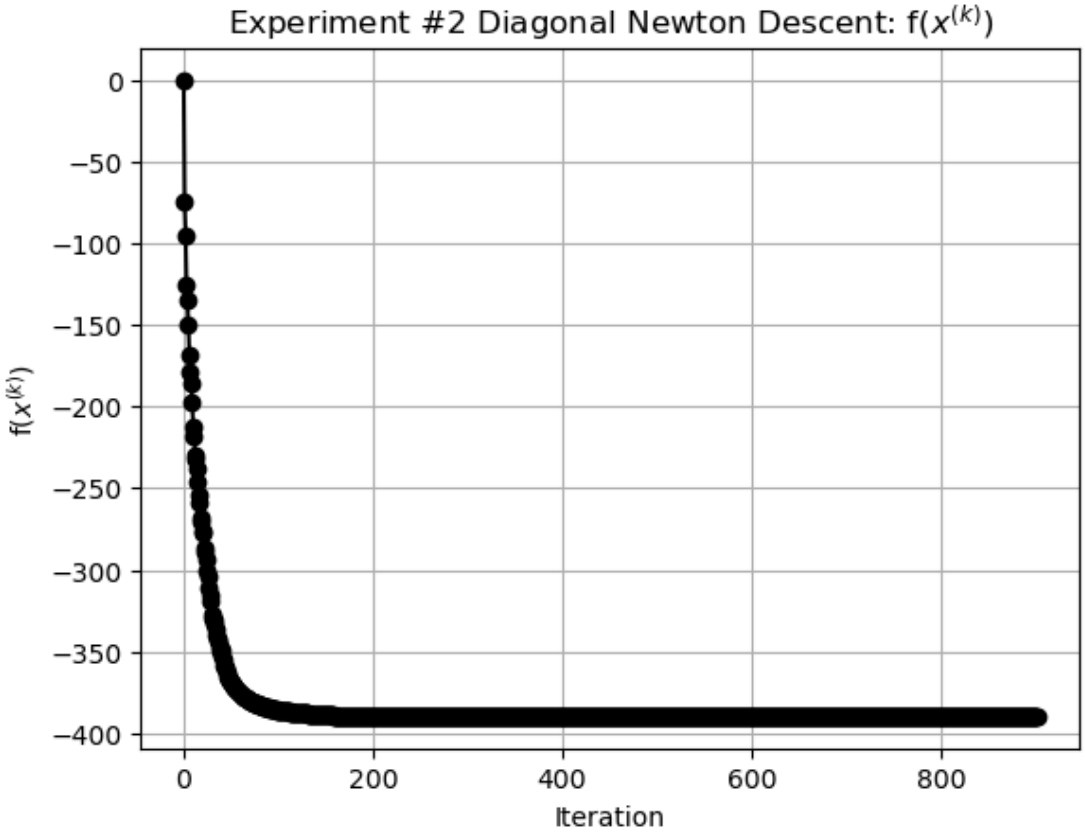
```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

In [142…

```
#Run Newton's Method where Hessian is approximated by its diagonal
diagonal_newton_descent_iterates, diagonal_newton_descent_function_values, diag
```

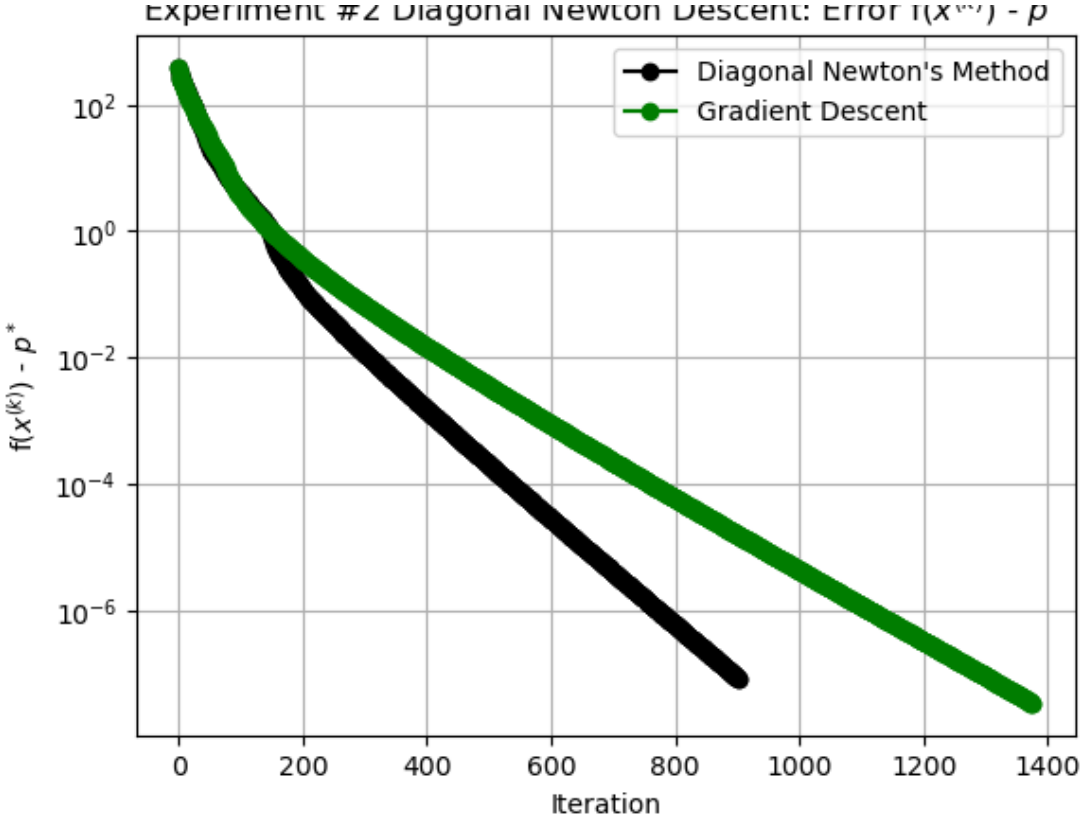
In [143…

```
#Plot function values throughout Newton's Method where the Hessian is replaced
fig60 = plt.figure()  # Create a new figure
plt.plot(list(range(len(diagonal_newton_descent_function_values))), diagonal_ne
plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$)')
plt.title(r'Experiment #2 Diagonal Newton Descent: f($x^{(k)}$)')
plt.grid(True)
plt.show()
```



In [144…

```
#Plot distance to optimal function value throughout Newton's Method where the H
#Also do this for regular newton's method and gradient descent for comparison
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed1
diagonal_newton_descent_gap_to_optimal = diagonal_newton_descent_function_value

fig61 = plt.figure()  # Create a new figure
line1, = plt.plot(list(range(len(diagonal_newton_descent_gap_to_optimal))), dia
# line2, = plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_des
line3, = plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_d

plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #2 Diagonal Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.legend(handles=[line1, line3], labels=['Diagonal Newton\'s Method', 'Gradie
plt.grid(True)
plt.show()
```



Experiment #3

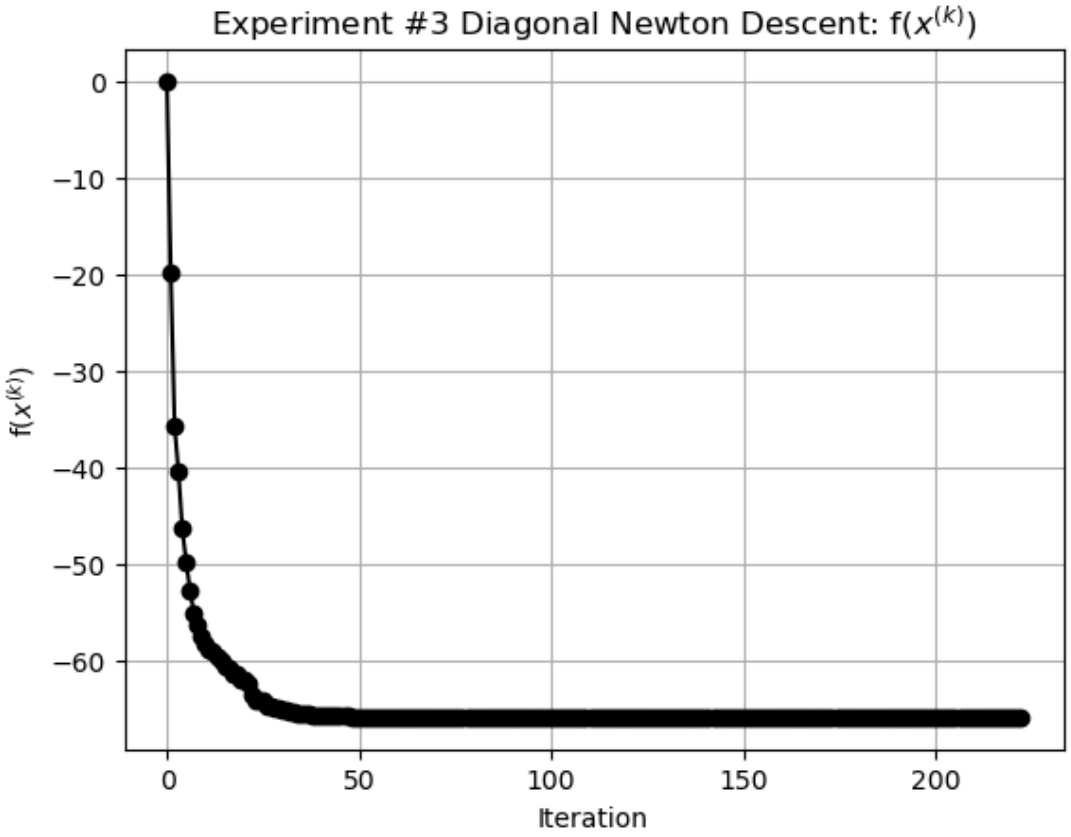
In [145…
```
#Global Variables
n = 50
m = 100
alpha = 0.01
beta = 0.5
eta = 1e-8
max_iter = 2000

x0 = np.zeros(shape = (n, 1)) #initial point will always start at 0 for all exp

#randomly generate A from normal distribution
np.random.seed(2)
A = np.random.normal(loc = 0, scale = 1, size = (m, n))
```

In [146…
```
#Run Gradient Descent
gradient_descent_eta = 1e-3
gradient_descent_iterates, gradient_descent_function_values, gradient_descent_s
```

In [147…
```
#Run Newton's Method
newton_descent_iterates, newton_descent_function_values, newton_descent_step_si
```

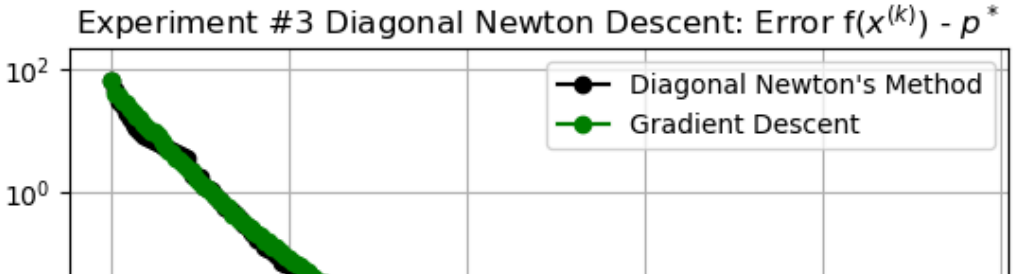
In [148…
```
#Run Newton's Method where Hessian is approximated by its diagonal
diagonal_newton_descent_iterates, diagonal_newton_descent_function_values, diag
```

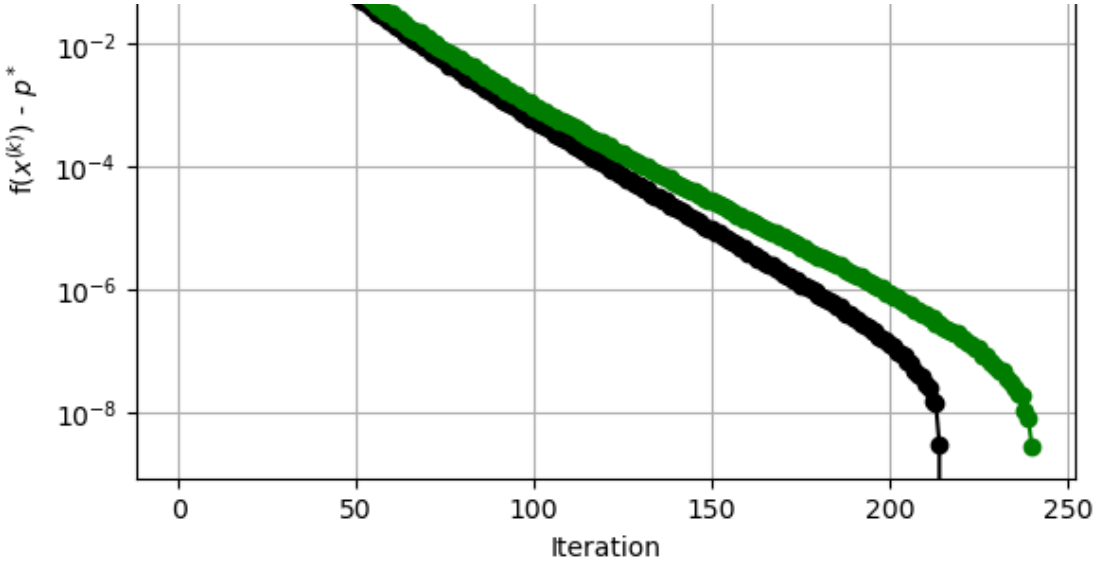
In [149…
```
#Plot function values throughout Newton's Method where the Hessian is replaced
fig62 = plt.figure()  # Create a new figure
plt.plot(list(range(len(diagonal_newton_descent_function_values))), diagonal_ne
```



In [150…

```
#Plot distance to optimal function value throughout Newton's Method where the H
#Also do this for regular newton's method and gradient descent for comparison
gradient_descent_gap_to_optimal = gradient_descent_function_values - p_star_see
newton_descent_gap_to_optimal = newton_descent_function_values - p_star_seed2
diagonal_newton_descent_gap_to_optimal = diagonal_newton_descent_function_value
fig63 = plt.figure()  # Create a new figure
line1, = plt.plot(list(range(len(diagonal_newton_descent_gap_to_optimal))), dia
# line2, = plt.plot(list(range(len(newton_descent_gap_to_optimal))), newton_des
line3, = plt.plot(list(range(len(gradient_descent_gap_to_optimal))), gradient_d

plt.xlabel('Iteration')
plt.ylabel(r'f($x^{(k)}$) - $p^*$')
plt.yscale('log')
plt.title(r'Experiment #3 Diagonal Newton Descent: Error f($x^{(k)}$) - $p^*$')
plt.legend(handles=[line1, line3], labels=['Diagonal Newton\'s Method', 'Gradie
plt.grid(True)
plt.show()
```





In [151…

```
#Just a print for debugging purposes
print(len(gradient_descent_gap_to_optimal))
print(len(diagonal_newton_descent_gap_to_optimal))
```

```
241
223
```

Observations:

- When we are using the Diagonal Approximation of the Hessian, the convergence behavior strongly resembles that of the Gradient Descent Method

In [152…

```
import matplotlib.backends.backend_pdf
import matplotlib.pyplot as plt

pdf = matplotlib.backends.backend_pdf.PdfPages("Homework #5 Plots.pdf")

# Iterate through all the figures in the notebook
for i in range(1, 64):
    # Save each figure to the PDF
    pdf.savefig(globals()[f"fig{i}"])

# Close the PDF file
pdf.close()
```