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Computational Project

Heat Transfer in Aerospace Applications (AE608A)

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Chapter 1

Problem Formulation

1.1 Given Data

$T_i = 300$ K, $T_\infty = 0$ K, T_{max} at backend $= 305$ K, $\epsilon = 0.9$

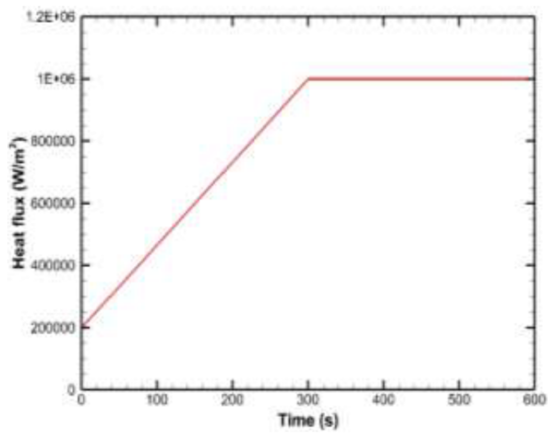


Figure 1.1: Surface heat flux on exposed TPS surface

$$q'' = \begin{cases} \left(\frac{8\tau}{300} + 2\right) * 10^5, & \text{if } 0 \leq \tau \leq 300 \\ 10^6, & 300 \leq \tau \leq 600 \end{cases} \tag{1.1}$$

1.2 Assumptions

1. The mesh is uniform.
2. No internal heat generation.
3. No property variation along Y direction. Essentialy Temperature varies only in X direcction.
4. Temperature at the backend of the TPS is same as that of the base material.

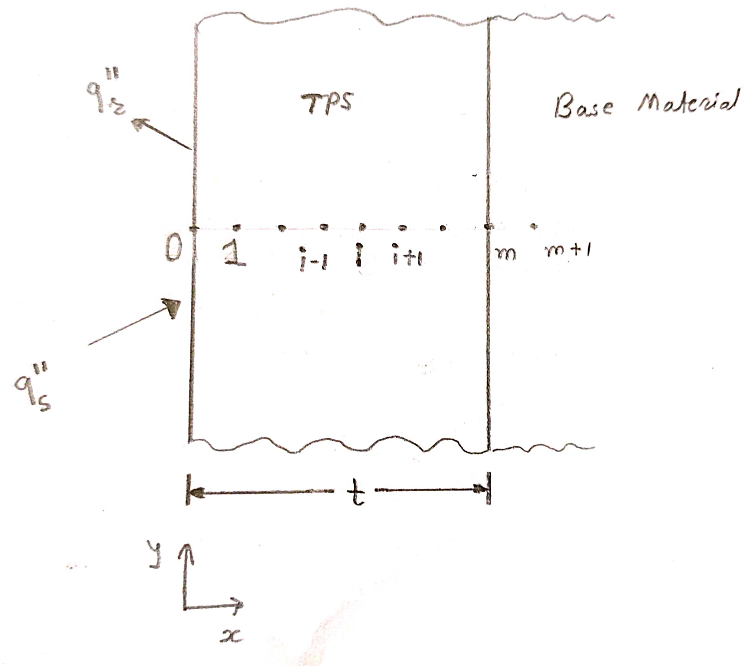


Figure 1.2: Discretized Computational Domain

1.3 Governing Equations

For any interior node i of TPS without heat generation, the governing equation is given as,

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{1.2}$$

Discretizing above equation using **1D Finite Difference method(FDM)** ;

$$\frac{T_i^{p+1} - T_i^p}{\Delta t} = \alpha \frac{T_{i-1}^p - 2T_i^p + T_{i+1}^p}{\Delta x^2}$$

$$\therefore T_i^{p+1} - T_i^p = \frac{\alpha \Delta t}{\Delta x^2}(T_{i-1}^p - 2T_i^p + T_{i+1}^p)$$

$$\boxed{\therefore T_i^{p+1} = (T_{i-1}^p + T_{i+1}^p)F_o + (1 - 2F_o)T_i^p} \tag{1.3}$$

where F_o = Mesh Fourier Number = $\dfrac{\alpha \Delta t}{\Delta x^2}$

Handling of the Boundary Conditions:

For **left hand boundary** of TPS, i.e. at x = 0 or node i = 0; the governing equation is given by,

$$\epsilon\sigma(T_\infty^4 - T_0^4)^p + k\frac{T_1^p - T_0^p}{\Delta x} + q_s'' = \rho c \frac{\Delta x}{2}\left(\frac{T_0^{p+1} - T_0^p}{\Delta t}\right)$$

$$\boxed{\therefore T_o^{p+1} = F_o\left(2T_1^p + \frac{2\sigma\epsilon(\Delta x)}{k}T_\infty^4 + \frac{2q_s''(\Delta x)}{k}\right) + T_0^p\left(1 - 2F_o - 2F_o\frac{\sigma\epsilon(\Delta x)}{k}T_0^{p^3}\right)} \tag{1.4}$$

For **right hand boundary** of TPS, i.e. at x = t or node i = m; the governing equation can be obtained by substituting i = m in Eq. 1.3.

$$T_m^{p+1} = (T_{m-1}^p + T_{m+1}^p)F_o + (1 - 2F_o)T_m^p$$

It is assumed that *Temperature at the backend of the TPS can be considered to be same as that of the base material.* Hence

$$T_{m+1}^{p} = T_{m}^{p}$$

$$\boxed{T_m^{p+1} = T_m^p + F_o \left(T_{m-1}^p - T_m^p\right)} \tag{1.5}$$

1.4 Stability criterion for Explicit Method : Limitation on Δt

The boundary node involving radiation is more restrictive than interior nodes and thus require smaller time step. Therefore, the most restrictive boundary node should be used in determination of maximum allowable time step Δt when a transient problem is solved with the explicit method. From Eq. 1.4, coefficient of T_0 must not be negative.

$$\therefore 1 - 2F_o - 2F_o \frac{\sigma\epsilon(\Delta x)}{k} T_0^{p^3} \geq 0$$

$$\therefore F_o = \frac{\alpha \Delta t}{\Delta x^2} \leq \frac{1}{2\left(1 + \frac{\sigma\epsilon(\Delta x)}{k} T_0^{p^3}\right)} \tag{1.6}$$

The largest allowable value of the time step Δt can be determined from this relation. Note that the value of $T_0^{p^3}$ may be taken as maximum allowable temperature of the TPS material.

$$\boxed{\therefore \Delta t = \frac{\Delta x^2}{2\alpha\left(1 + \frac{\sigma\epsilon(\Delta x)}{k} T_{melt}^3\right)}} \tag{1.7}$$

1.5 Available Materials

Since Ablative Material involve both heat and mass transfer, analysis becomes complicated. Hence only reusable Non-ablative materials are chosen for the analysis. All the properties are evaluated at approx. 0 atm pressure and mean temperature of 300 K and maximum allowable temperature of material.

1. **LI -900 Rigid Tile**

   - LI-900 is a relatively low density (9 lbs/cu.ft.) and very low thermal conductivity fibrous insulation material baselined for use on all the shuttle orbiters. It is the primary tile insulation flying almost everywhere on all of the orbiters due to its thermal conductivity, thermal shock resistance and efficiency as a TPS.

Table 1.1: LI -900 Properties at Standard Conditions

Property	Value	SI Unit
Density	$1.442 * 10^2$	kg/m^3
Thermal Conductivity (Thru-the-Thickness)	$8.51 * 10^{-2}$	W/m-K
Specific Heat	$1.26 * 10^3$	J/kg-K
Single Use Temperature Limit	$1.76 * 10^3$	K

2. **LI - 2200 Rigid Tile**

   - LI-2200 is a relatively high density (22 lbs/cu.ft.) and thermal conductivity fibrous insulation material with higher strength than the LI-900 baselined for use on all the shuttle orbiters. It is currently flying on all orbiters, but to a very limited extent due to its weight. It was originally developed to provide an enhanced strength RSI tile relative to LI-900.

Table 1.2: LI -2200 Properties at Standard Conditions

Property	Value	SI Unit
Density	$3.52 * 10^2$	kg/m^3
Thermal Conductivity (Thru-the-Thickness)	$9.522 * 10^{-2}$	W/m-K
Specific Heat	$1.26 * 10^3$	J/kg-K
Single Use Temperature Limit	$1.81 * 10^3$	K

3. **FRCI-12 Rigid Tile**

   - FRCI-12 is a relatively low density (12 lbs/cu.ft.), low thermal conductivity fibrous insulation material that is substantially stronger than the original LI-900 and equivalent in strength to LI-2200 at a lower density. It was baselined for use on all of the shuttle orbiters. It has been flying successfully on the shuttle since its adoption shortly after the second orbiter Challenger was built.

Table 1.3: FRCI - 12 Properties at Standard Conditions

Property	Value	SI Unit
Density	$1.92 * 10^2$	kg/m^3
Thermal Conductivity (Thru-the-Thickness)	$6.664 * 10^{-2}$	W/m-K
Specific Heat	$1.26 * 10^3$	J/kg-K
Single Use Temperature Limit	$1.81 * 10^3$	K

4. **HTP - 12**

Table 1.4: HTP - 12 Properties at Standard Conditions

Property	Value	SI Unit
Density	$1.92 * 10^2$	kg/m^3
Thermal Conductivity (Thru-the-Thickness)	$6.635 * 10^{-2}$	W/m-K
Specific Heat	$1.02 * 10^3$	J/kg-K
Single Use Temperature Limit	$1.76 * 10^3$	K

5. **TUFI Composite Surface**

   - TUFI, or Toughened Uni-piece Fibrous Insulation, is a functional gradient material possessing high emissivity, high temperature capability, toughness, and whose properties can be tailored, or adjusted to specific requirements.

Table 1.5: TUFI Properties at Standard Conditions

Property	Value	SI Unit
Density	$1.31 * 10^3$	kg/m^3
Thermal Conductivity (Thru-the-Thickness)	1.38	W/m-K
Specific Heat	$1.32 * 10^3$	J/kg-K
Single Use Temperature Limit	$1.87 * 10^3$	K

6. **C/SiC (Carbon/Silicon-Carbide)**

   - Carbon fiber reinforced Silicon-Carbide is a ceramic matrix composite comprised of carbon fiber reinforcement in a SiC matrix. The SiC is used because of its excellent oxidation resistance at high temperatures. As it passively oxidizes up to about 3100'F, it produces a layer of SiO2. This SiO2 slows the diffusion of oxidizing agents to the SiO2/SiC interface, thus inhibiting further oxidation. Beyond about 3100¡F, a passive to active oxidation transition takes place and the products of oxidation change from the solid SiO2 to all gaseous species. The C/SiC material is produced by depositing a SiC matrix on carbon fibers via Chemical Vapor Deposition (CVD).

Table 1.6: C/SiC Properties at Standard Conditions

Property	Value	SI Unit
Density	$2.08 * 10^3$	kg/m^3
Thermal Conductivity (Thru-the-Thickness)	6.76	W/m-K
Specific Heat	$1.03 * 10^3$	J/kg-K
Single Use Temperature Limit	$2.01 * 10^3$	K

7. **RCC (Reinforced Carbon-Carbon Composite)**

   - Reinforced Carbon-Carbon (RCC) Composite was developed for use on the Shuttle leading edges and nosecap by LTV Corporation. It is comprised of a silicon-carbide coated carbon-carbon composite, fabricated from a low modulus rayon precursor-based fabric with a filled phenolic resin.

Table 1.7: RCC Properties at Standard Conditions

Property	Value	SI Unit
Density	$1.58 * 10^3$	kg/m^3
Thermal Conductivity (Thru-the-Thickness)	7.72	W/m-K
Specific Heat	$1.485 * 10^3$	J/kg-K
Single Use Temperature Limit	$2.03 * 10^3$	K

Other materials and their properties can be obtained from reference [1] and [2].

1.6 Material Selection

- While silicon based composite materials 1 through 5 listed in available materials show excellent insulation properties along with very low density, they are not found suitable when applied to TPS.m code listed at the end of the report.
- The reason was found that if the thickness is taken large, heat accumulates at frontol area and thus increasing the temperature of exposed surface beyond allowable temperature. Eventually, the exposed surface will start melting.
- If the thickness is taken small, the temperature at backend of TPS exceeds beyond allowable temperature of 305 K.
- Hence we have to go for Carbon based reusable composites 6 and 7 listed in available materials which have inferior insulation and higher thermal conductivity compared to silicon based insulators.

Only material 6 and 7 can withstand the boundary condition given in the problem. Since material 7 (RCC) has lower density compared to material 6 (C/SiC), we select RCC as an insulating material.

1.7 Optimising the thickness

Thickness of tps is an important parameter as it dictates the weight of the system. Here, in *TPS.m* code, we start with the thickness of 5 mm and increase it upto 100 cm with the interval of 5 mm. Optimum thickness is otained when both of the following criterions are met.

1. Temperature of TPS material must be less than maximum allowable temperature throughout 600 seconds at any grid point.
2. Temperature at the backend of TPS must not exceed 305 K.

Chapter 2

Results

2.1 At Constant Properties

1. **Material Selected : RCC (Reinforced Carbon-Carbon Composite)**

2. **Material Properties:** Evaluated at approx. 0 atm pressure and $T_{avg} = \dfrac{T_{initial} + T_{max}}{2} = 1165K$

Table 2.1: RCC Properties

Property	Value	SI Unit
Density	$1.58 * 10^3$	kg/m^3
Thermal Conductivity (Thru-the-Thickness)	7.72	W/m-K
Specific Heat	$1.485 * 10^3$	J/kg-K
Thermal Diffusivity	$3.29 * 10^{-6}$	m^2/s
Single Use Temperature Limit	$2.03 * 10^3$	K

3. **Optimum thickness obtained :** $\boxed{\mathbf{t = 18.5\ cm}}$

4. **Weight per square meter of the TPS:**

$$\implies W = \rho * (A * t) * g$$

$$\implies W = 1.58 * 10^3 * (1 * 0.185) * g$$

$$\boxed{W = 292.3g \ \ N/m^2}$$

where g = acceleration due to gravity = 9.81 m/s^2

5. **Grid Independence Study:**

Results are considered to be independent of size of grid when maximum absolute error between two successive results is less than 10^{-2}. By this criterion, optimum grid size is found to be 671 for the thickness of 18.5 cm.

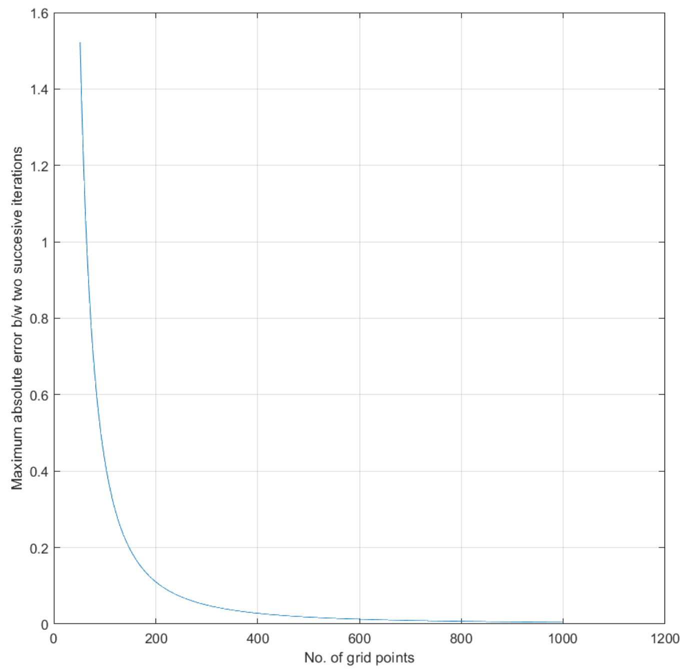


Figure 2.1: Variation of absolute maximum error with no. of grid points at constant thickness

6. **Temperature variation with time at the exposed and back ends at any y location:**

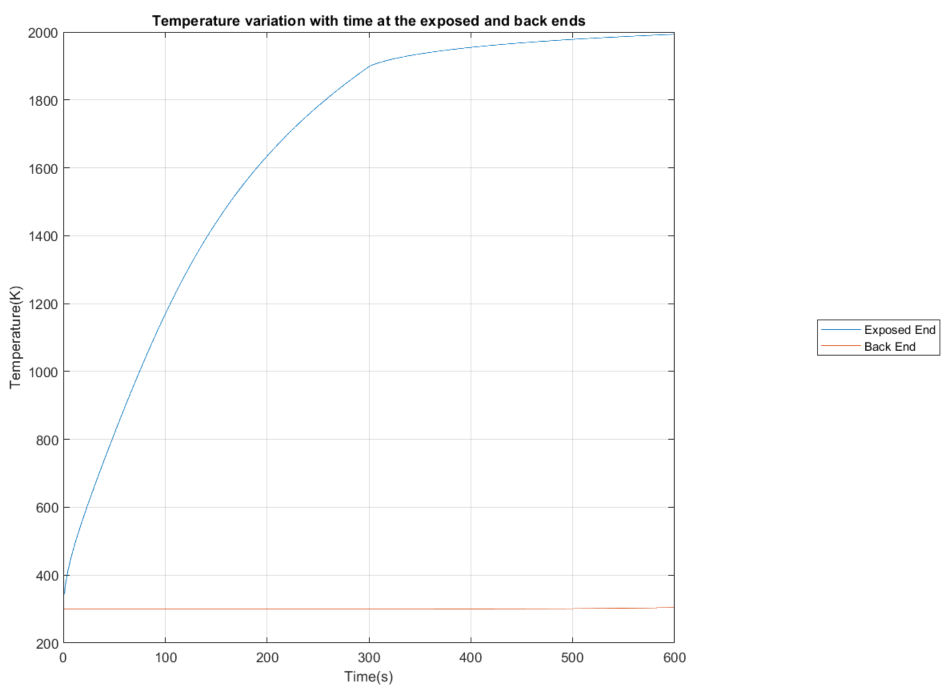


Figure 2.2: Temperature variation with time at the exposed and back ends at any y location

7. **Temperature distribution through the TPS material along x coordinate at any y location:**

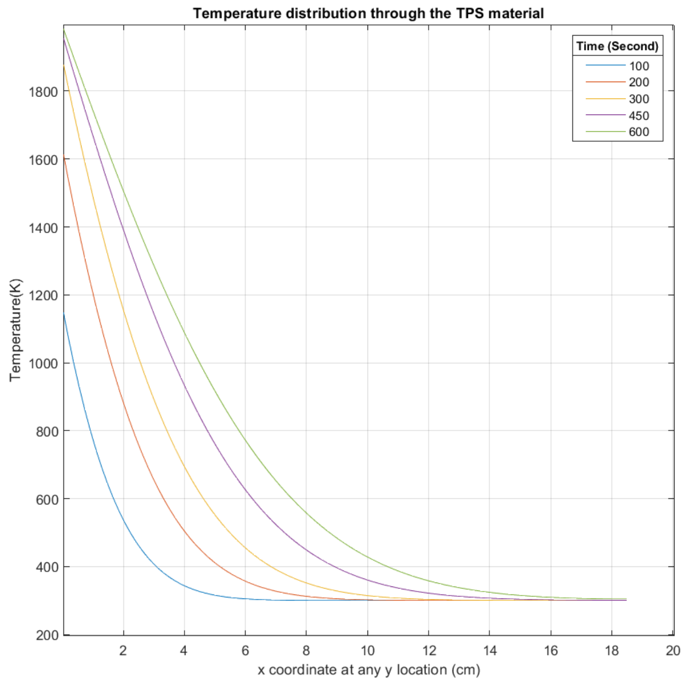


Figure 2.3: Temperature variation along the thickness of TPS at different time interval

8. **Effect of Surface Radiation:**
Effects of surface radiation can be seen by changing values of ϵ. As seen from the following plots, surface temperature exceeds melting temperature of TPS by almost 3 times whereas tempearture at backend is almost same due to lower value of thermal diffusivity(α). hence radiation plays important role in preventing the material to melt and hence the thickness and material of TPS.

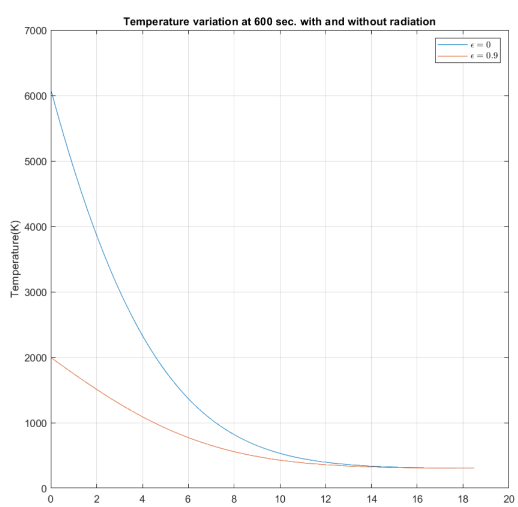


Figure 2.4: Temperature variation through thickness with and without radiation

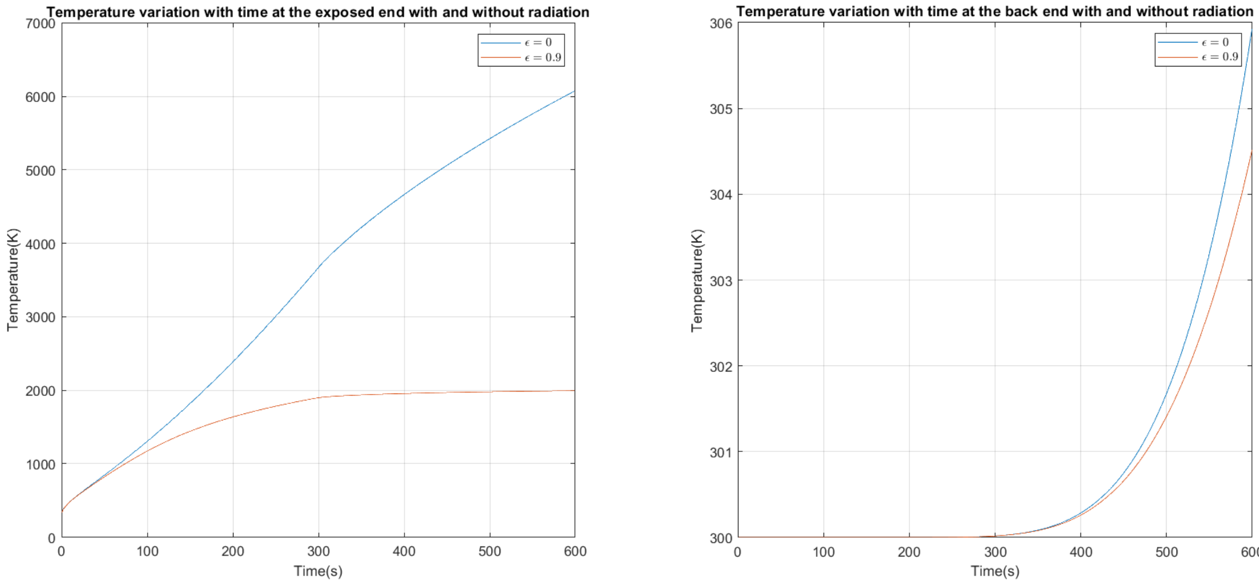


Figure 2.5: Temperature variation with time at the exposed and back ends with and without radiation

2.2 At Variable Properties

Till now, constant values of k and c were assumed for analysis. But both k and c varies with Temperature and Pressure. Let us consider k = f(T) and c = f(T). Since T = f(x,t), k = c = f(x,t). To understand the effect of variation of k and c, we have to discretize the governing equations accordingly;

- **For any interior node i of TPS :**

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) \tag{2.1}$$

Discretization

$$(\rho c)_i \frac{\partial T_i}{\partial t} = k_i\left(\frac{\partial^2 T}{\partial x^2}\right)_i + \left(\frac{\partial T}{\partial x}\right)_i\left(\frac{\partial k}{\partial x}\right)_i$$

$$\therefore (\rho c)_i \frac{\partial T_i}{\partial t} = \left[k_i\frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta x)^2} + \left(\frac{T_{i+1} - T_{i-1}}{2\Delta x}\right)\left(\frac{k_{i+1} - k_{i-1}}{2\Delta x}\right)\right]$$

$$\boxed{T_i^{p+1} = \frac{\Delta t}{(\Delta x)^2(\rho c)_i}\left[\left(k_i + \frac{k_{i+1}}{4} - \frac{k_{i-1}}{4}\right)^p T_{i+1}^p + \left(k_i - \frac{k_{i+1}}{4} + \frac{k_{i-1}}{4}\right)^p T_{i-1}^p\right] + T_i^p\left(1 - \frac{2k_i^p\Delta t}{(\Delta x)^2(\rho c)_i}\right)}$$

- **For left hand boundary:**

$$\epsilon\sigma(T_\infty^4 - T_0^4)^p + \left(\frac{k_1 + k_0}{2}\right)\frac{T_1^p - T_0^p}{\Delta x} + q_s'' = \rho c_0\frac{\Delta x}{2}\left(\frac{T_0^{p+1} - T_0^p}{\Delta t}\right)$$

$$\boxed{T_0^{p+1} = \left(\frac{2\Delta t}{\rho c_0\Delta x}\right)\left(\epsilon\sigma(T_\infty^4 - T_0^4)^p + \left(\frac{k_1 + k_0}{2}\right)\frac{T_1^p - T_0^p}{\Delta x} + q_s''\right) + T_0^p}$$

- **For right hand boundary:**

$$\boxed{T_m^{p+1} = T_m^p + \left(\frac{\left(\frac{k_m + k_{m-1}}{2}\right)\Delta t}{\rho c_m(\Delta x)^2}\right)(T_{m-1}^p - T_m^p)}$$

- **Stability Criterion:** Similar to constant property case;

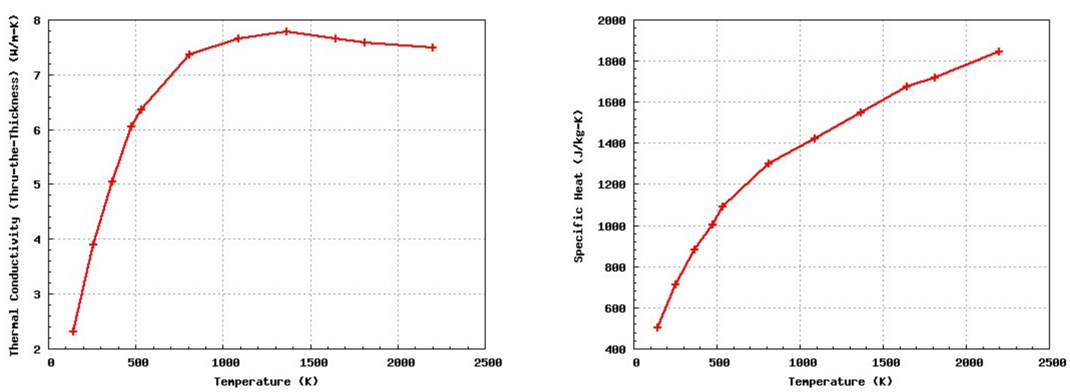
$$\boxed{\Delta t \leq \frac{\Delta x^2}{2\left(\frac{k_i}{\rho c_i}\right)\left(1 + \frac{\sigma\epsilon(\Delta x)}{k_i}T_i^{p^3}\right)}} \tag{2.2}$$

Due to variation of properties, we can not assume constant time step for computation. Before the computation of the next time step, the stability criterion in Eq. 2.2 must be checked because k_i^p and c_i^p will change from one time step to the next.
For each iteration, Δt is calculated at each grid points and minimum value of Δt is selected for the calculation of that iteration.

- **Variation of k and c:** Following data is available for RCC at atmospheric pressure;

Table 2.2: RCC Properties variation with temperature

Temperature(T)	Thermal Conductivity(k)	Specific heat(c)
K	$W/m.K$	$J/Kg.K$
144.4	2.30	$5.02 * 10^2$
255.6	3.89	$7.12 * 10^2$
366.7	5.05	$8.79 * 10^2$
477.8	6.06	$1.00 * 10^3$
533.3	6.35	$1.09 * 10^3$
811.1	7.36	$1.30 * 10^3$
1088.9	7.65	$1.42 * 10^3$
1366.7	7.79	$1.55 * 10^3$
1644.4	7.65	$1.67 * 10^3$
1811.1	7.58	$1.72 * 10^3$
2200.0	7.49	$1.84 * 10^3$



(a) k vs. T

(b) c vs T

Figure 2.6: RCC Property variation with Temperature

Assume

$$k = a_1 + b_1 T + c_1 T^2 + d_1 T^3$$

$$c = a_2 + b_2 T + c_2 T^2 + d_2 T^3$$

Fitting above data with **least square method**,

$$\boxed{k = 0.3884 + 0.0162T - 1.1164 * 10^{-5}T^2 + 2.3977 * 10^{-9}T^3}$$

$$\boxed{c = 257.4370 + 2.0196T - 0.0011T^2 + 2.2643 * 10^{-7}T^3}$$

- **Variation of T w.r.t. x and temperature profile at exposed and back ends at variable properties**

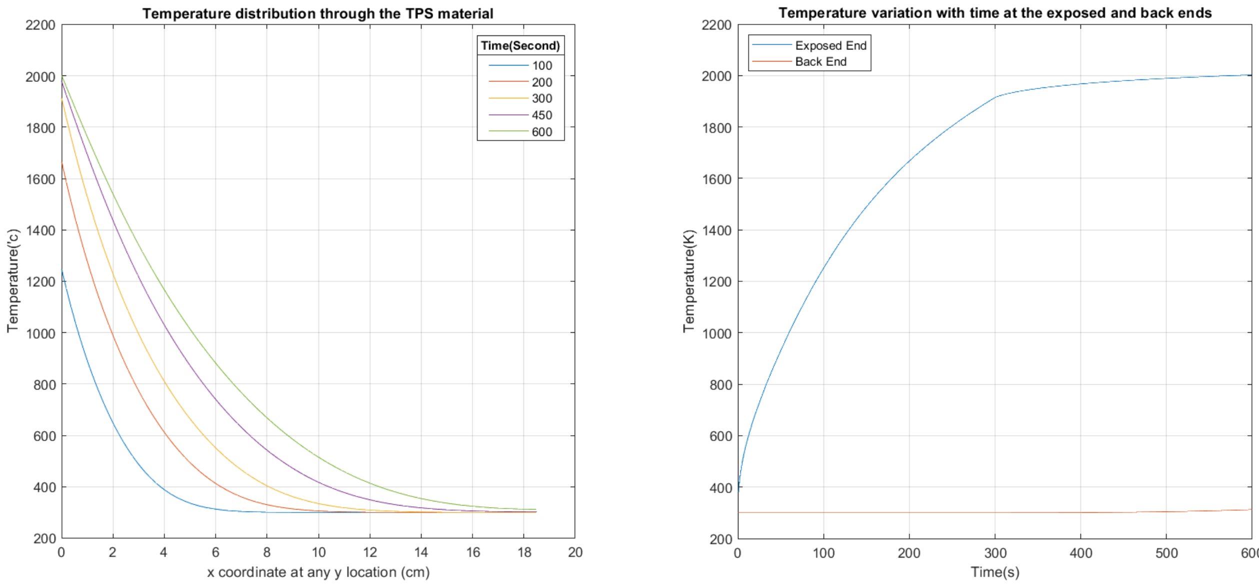


Figure 2.7: Results with variable properties

- **Comparison of results at constant and variable properties**

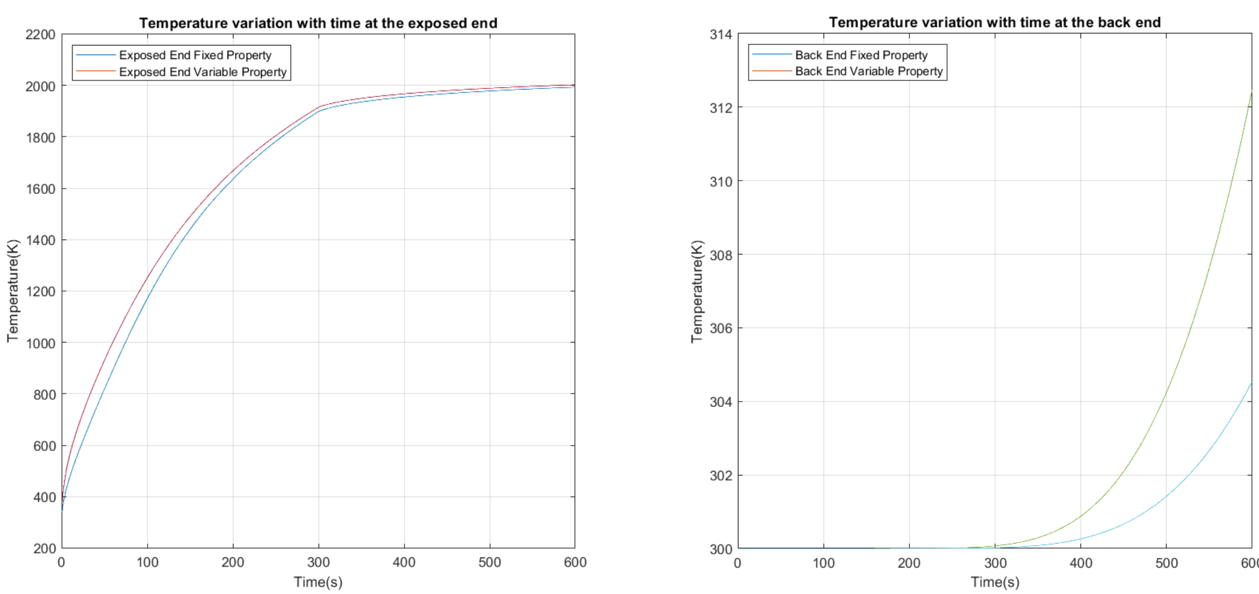


Figure 2.8: Temperature variation with time at the exposed and back ends with variable properties

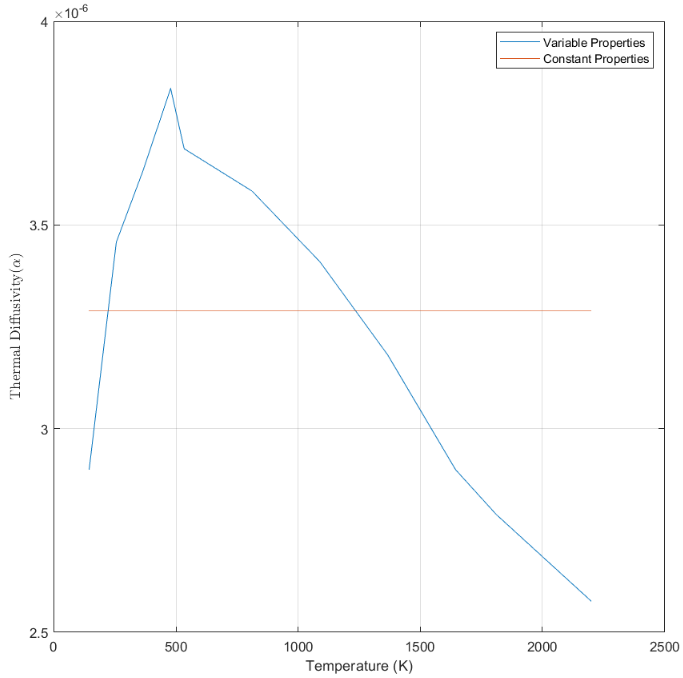


Figure 2.9: Variation of thermal diffusivity due to Temperature

- From Fig 2.8, the exposed end has very less effect due to variation of k and c. However, at the backend, temperature exceeds the limit of 305 K.
- From Fig 2.9, it is clear that the value ot *Thermal Diffusivity (α)* in the range of 300 to 500 K is larger than constant value assumed for calculation. *Thermal diffusivity is a measure of the rate at which heat disperses throughout the body.* Since α is larger, we get higher temperature at the backend of TPS.
- Hence, $\boxed{\textbf{Optimum thickness = 20.4 cm}}$ is obtained by *TPS.m* code when variable properties are considered.

Chapter 3

Summary

For the given problem, following materials are tested for TPS design.

Table 3.1: TPS materials and results

Material	Comment
LI -900 Rigid Tile	Not Applicable at Any Thickness
LI - 2200 Rigid Tile	Not Applicable at Any Thickness
FRCI-12 Rigid Tile	Not Applicable at Any Thickness
HTP - 12	Not Applicable at Any Thickness
TUFI Composite Surface	Not Applicable at Any Thickness
C/SiC (Carbon/Silicon-Carbide)	18.25 cm thick TPS at constant properties
RCC (Reinforced Carbon-Carbon Composite)	18.5 cm thick TPS at constant properties

Out of these, RCC (Reinforced Carbon-Carbon Composite) material is found suitable with 18.5 cm thickness at constant thermophysical properties and with 20.4 cm thickness at variable thermophysical properties. The maximum temperature of 1993 K is obtained at the exposed end of the TPS. For the given thickness, TPS weighs 292.3 kg per square meter of frontal surface, which is extremely high for space application. Following remedies are suggested for TPS design:

- Application of lighter silicone materials at the locations of lower heat flux.
- Application of Ablative TPS material instead of reusable material. Analysis of Ablative material is not carried out due to involved phenomena of combined heat and mass transfer.
- Application of composite TPS system with face plate of Ultra High Temperature Ceramics (ex. Zirconium Diboride) followed by light insulating materials such as Saffil. Ultra High Temperature Ceramics have very high melting temperature of order of 3000 K whereas insulating material has low conductivity and density. Example of such system is shown in Fig.3.1.

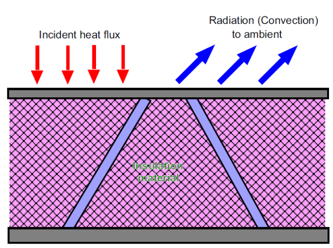


Figure 3.1: TPS with different materials from ref.[4]

Chapter 4

Computer Program

4.1 *TPS.m* Matlab code

```
clc;
clear;
close all;


%% Calculation For constant properties

for m =671%:1001; %optimum no. of grid points is obtained here

T_i = 300; % Temperature of TPS at 0 seconds
T_s = 300; % Temperature of face plate at 0 second
T_back = 305; % Max temp. at bac face of TPS
T_inf = 0; % Space temperature
sigma = 5.67 * 10^-8; % Stefan Boltzmann constant
epsl = 0.9; % Emmisivity can be changed here


for x = 0.185  % x is varied here to obtain optimum thickness of TPS

T(1:m) = T_i; % Initial Temperatures
T_old = T;

% Reinforced Carbon Carbon (RCC) Composite properties

rho = 1580;      % Constant Density
k(1:m) = 7.72;   %constant k & c
c(1:m) = 1485;
alpha = k(1)/(rho*c(1));
T_max = 2030; % Melting/max allowable Temperature of RCC

dx = x/(m-1); % Grid size
for j = 1:m
tvar(j) = ((dx)^2)/((2*(k(j)/(rho*c(j))))*(1 + (sigma*epsl*dx*T(j)^3)/k(j))); ...
    % calculation of variable time step due to variable properties
end
count = 1;
dt = min(tvar); % Smallest time step is taken for each iteration
t = dt; % calculation of cumulative time

```

```
while t < 600 && max(T) < T_max && min(T) < T_back % requirements of given ...
    TPS can be changed here
    if t < 300
        qs = (((8*t)/300) + 2) * 10^5; %variable heat flux
    else
        qs = 10^6;
    end
    k(1:m) = 7.72;  % constant k & c
    c(1:m) = 1485;

    for j = 1:m
        tvar(j) = ((dx)^2)/((2*(k(j)/(rho*c(j))))*(1 + ...
            (sigma*epsl*dx*T(j)^3)/k(j)));% time step is calculated at each ...
            time step
    end
    dt = min(tvar);
    for i = 2:m-1 % Governing Eqn. for interior points
        T_new(i) = (((k(i) + 0.25*k(i+1) - 0.25*k(i-1))*T_old(i+1)) + ((k(i) - ...
            0.25*k(i+1) + 0.25*k(i-1))*T_old(i-1)))*(dt/(dx*dx*rho*c(i))) + (1 ...
            - ((2*k(i)*dt)/(dx*dx*rho*c(i))))*T_old(i);
    end
    T_new(m) = (T_old(m-1) - T_old(m))*(((k(m) + ...
        k(m-1))*0.5*dt)/(rho*c(m)*dx*dx)) + T_old(m);% Governing eqn at backend

    T_new(1) = (( epsl*sigma*(T_inf^4 - T_old(1)^4) + ((((k(1) + ...
        k(2))/2)/dx)*(T_old(2) - T_old(1))) + qs ) * ((2*dt)/(rho*c(1)*dx))) + ...
        T_old(1);
    %Governing Eqn at exposed end

    T_old = T_new; %update temperature at each time step
    T = T_new;
    time(count) = t;
    Texp(count) = T(1);
    Tback(count) = T(end);
    count = count + 1;
    t = t + dt;

    if int32(t) == 100  % At 100 sec
       fig1 = T;
    end
    if int32(t) == 200  % At 200 sec
        fig2 = T;
    end
    if int32(t) == 300  % At 300 sec
        fig3 = T;
    end
    if int32(t) == 450  % At 450 sec
        fig4 = T;
    end
    if int32(t) == 600  % At 600 sec
        fig5 = T;
    end


end
end
end

% Results at constant properties
figure(1) %plotting temp. variation at different times
```

```
subplot(1,2,1)
sgt1 = sgtitle('Results at Fixed Property for 18.5 cm TPS');
sgt1.FontSize = 20;
plot(100*(0:dx:x),fig1)
axis square
grid on
title('Temperature distribution through the TPS material')
ylabel("Temperature('c)")
xlabel("x coordinate at any y location (cm)")
hold on
grid on
plot(100*(0:dx:x),fig2)
plot(100*(0:dx:x),fig3)
plot(100*(0:dx:x),fig4)
plot(100*(0:dx:x),fig5)
legend('$\tau$ = 100 s','$\tau$ = 200 s','$\tau$ = 300 s','$\tau$ = 450 ...
    s','$\tau$ = 600 s','Interpreter','latex')

%plotting temperature at exposed and back end during 10 minutes
subplot(1,2,2)
plot(time,Texp)
hold on
plot(time,Tback)
axis square
grid on
title("Temperature variation with time at the exposed and back ends")
ylabel("Temperature(K)")
xlabel("Time(s)")
legend("Exposed End","Back End")

fprintf('\nMaterial Selected : RCC (reinforced Carbon Carbon Composite)\n \n');
fprintf('Thermal conductivity :%6.2f W/mK\n',k(1));
fprintf('Density :%6.2f Kg/m^3\n',rho);
fprintf('Specific heat :%6.2f J/Kg.k\n',c(1));
fprintf('Average Thermal diffusivity :%.9f m^2/s\n',alpha);
fprintf('Maximum allowable temperature :%6.3f K\n',T_max);
fprintf('Thickness :%6.2f cm\n',100*x);
fprintf('Weight :%6.2f kg/m^2\n',rho*x);


%% Calculations for variable properties
clear
% Least square curve fitting using 3rd order polynomial
X = [144.4 255.6 366.7 477.8 533.3 811.1 1088.9 1366.7 1644.4 1811.1 2200.0 ...
    ]; %Temperature in kalvin
Y =  [2.30  3.89  5.05  6.06  6.35  7.36 7.65 7.79  7.65  7.58  7.49 ]; ...
    %Thermal conductivity in W/mK
Z =  [5.02 * 10^2 7.12 * 10^2 8.79 * 10^2 1.00 *10^3 1.09 * 10^3 1.30 * 10^3 ...
    1.42 * 10^3 1.55 * 10^3 1.67 * 10^3 1.72 * 10^3 1.84 * 10^3]; %Specific ...
    heat in J/KgK
p1 = polyfit(X,Y,3); %Polyfit is inbuilt matlab command to approximate ...
    polynomial of given degree to fit data using least square method
p2 = polyfit(X,Z,3); % cubic polynomial is approximated

for m =671%:1001; %optimum no. of grid points is obtained here

T_i = 300; % Temperature of TPS at 0 seconds
T_s = 300; % Temperature of base material at 0 second
T_back = 305; % Max allowable temp. at back face of TPS
T_inf = 0; % Space temperature
```

```
sigma = 5.67 * 10^-8; % Stefan Boltzmann constant
epsl = 0.9; % Emmisivity can be changed here


for x = 0.204  % x is varied here to obtain optimum thickness of TPS

T(1:m) = T_i; % Initial Temperatures
T_old = T;

% Reinforced Carbon Carbon (RCC) Composite properties

rho = 1580;      % Constant Density

k = p1(1)*T.^3 + p1(2)*T.^2 + p1(3)*T + p1(4);    %variable k & c
c = p2(1)*T.^3 + p2(2)*T.^2 + p2(3)*T + p2(4);

T_max = 2030; % Melting/max allowable Temperature of RCC

dx = x/(m-1); % Grid size
for j = 1:m
tvar(j) = ((dx)^2)/((2*(k(j)/(rho*c(j))))*(1 + (sigma*epsl*dx*T(j)^3)/k(j))); ...
    % calculation of variable time step due to variable properties
end
count = 1;
dt = min(tvar); % Smallest time step is taken for each iteration
t = dt; % calculation of cumulative time

while t < 600 && max(T) < T_max && min(T) < T_back % requirements of given ...
    TPS can be changed here
    if t < 300
        qs = (((8*t)/300) + 2) * 10^5; %variable heat flux
    else
        qs = 10^6;
    end

    k = p1(1)*T.^3 + p1(2)*T.^2 + p1(3)*T + p1(4);% k & c are calculated at ...
        each iteration
    c = p2(1)*T.^3 + p2(2)*T.^2 + p2(3)*T + p2(4);
    for j = 1:m
        tvar(j) = ((dx)^2)/((2*(k(j)/(rho*c(j))))*(1 + ...
            (sigma*epsl*dx*T(j)^3)/k(j)));% time step is calculated at each ...
            time step
    end
    dt = min(tvar);
    for i = 2:m-1 % Governing Eqn. for interior points
        T_new(i) = (((k(i) + 0.25*k(i+1) - 0.25*k(i-1))*T_old(i+1)) + ((k(i) - ...
            0.25*k(i+1) + 0.25*k(i-1))*T_old(i-1)))*(dt/(dx*dx*rho*c(i))) + (1 ...
            - ((2*k(i)*dt)/(dx*dx*rho*c(i))))*T_old(i);
    end
    T_new(m) = (T_old(m-1) - T_old(m))*(((k(m) + ...
        k(m-1))*0.5*dt)/(rho*c(m)*dx*dx)) + T_old(m);% Governing eqn at backend

    T_new(1) = (( epsl*sigma*(T_inf^4 - T_old(1)^4) + ((((k(1) + ...
        k(2))/2)/dx)*(T_old(2) - T_old(1))) + qs ) * ((2*dt)/(rho*c(1)*dx))) + ...
        T_old(1);
    %Governing Eqn at exposed end
    T_old = T_new; %update temperature at each time step
    T = T_new;
    time(count) = t;
    Texp(count) = T(1);
```

```
    Tback(count) = T(end);
    count = count + 1;
    t = t + dt;

    if int32(t) == 100  % At 100 sec
       fig1 = T;
    end
    if int32(t) == 200  % At 200 sec
        fig2 = T;
    end
    if int32(t) == 300  % At 300 sec
        fig3 = T;
    end
    if int32(t) == 450  % At 450 sec
        fig4 = T;
    end
    if int32(t) == 600  % At 600 sec
        fig5 = T;
    end


end
end
end

% Results at variable properties
figure(2) %plotting temp. variation at different times
subplot(1,2,1)
sgt2 = sgtitle('Results at Variable Property for 20.4 cm TPS');
sgt2.FontSize = 20;
plot(100*(0:dx:x),fig1)
axis square
grid on
title('Temperature distribution through the TPS material')
ylabel("Temperature('c)")
xlabel("x coordinate at any y location (cm)")
hold on
grid on
plot(100*(0:dx:x),fig2)
plot(100*(0:dx:x),fig3)
plot(100*(0:dx:x),fig4)
plot(100*(0:dx:x),fig5)
legend('$\tau$ = 100 s','$\tau$ = 200 s','$\tau$ = 300 s','$\tau$ = 450 ...
    s','$\tau$ = 600 s','Interpreter','latex')

%plotting temperature at exposed and back end during 10 minutes
subplot(1,2,2)
plot(time,Texp)
hold on
plot(time,Tback)
axis square
grid on
title("Temperature variation with time at the exposed and back ends")
ylabel("Temperature(K)")
xlabel("Time(s)")
legend("Exposed End","Back End")
```

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