Two sample t test

Two sample t test (Snedecor and Cochran 1989) is used to determine if two population means are equal. A common application is to test if a new treatment or approach or process is yielding better results than the current treatment or approach or process.

Data is paired - For example, a group of students are given coaching classes and effect of coaching on the marks scored is determined.

Data is not paired - For example, find out whether the miles per gallon of cars of Japanese make is superior to cars of Indian make.

In [2]:
```
import numpy                    as     np
import pandas                   as     pd
from   scipy.stats              import ttest_1samp, ttest_ind
import matplotlib.pyplot        as     plt
import matplotlib
import seaborn as sns
import scipy.stats as stats
import statsmodels.stats.api as sm
```

Example 1 - Independent Two Sample T-Test

A hotel manager looks to enhance the initial impressions that hotel guests have when they check in. Contributing to initial impressions is the time it takes to deliver a guest's luggage to the room after check-in. A random sample of 20 deliveries on a particular day were selected in Wing A of the hotel, and a random sample of 20 deliveries were selected in Wing B. The results are stored in Luggage . Analyze the data and determine whether there is a difference between the mean delivery times in the two wings of the hotel. (Use α = 0.05) Problem 10.83 from the Textbook adapted for Classroom Discussion(Chapter 10-page 387)

In [4]:
```
mydata = pd.read_csv('Luggage+%281%29.csv')
mydata.head()
```

Out[4]:

	WingA	WingB
0	10.70	7.20
1	9.89	6.68
2	11.83	9.29
3	9.04	8.95
4	9.37	6.61

Step 1: Define null and alternative hypotheses

In testing whether the mean time of deliveries of the luggages are same in both the wings of the hotel, the null hypothesis states that the mean time to deliver the luggages are the same, μA equals μB . The alternative hypothesis states that the mean time to deliver the luggages are different, μA is not equal to μB .

$H0$: μA - μB = 0 i.e μA = μB

HA : μA - μB ≠ 0 i.e μA ≠ μB

Step 2: Decide the significance level

Here we select α = 0.05 and the population standard deviation is not known.

Step 3: Identify the test statistic

We have two samples and we do not know the population standard deviation.

Sample sizes for both samples are same.

The sample is not a large sample, n < 30. So you use the t distribution and the $tSTAT$ test

statistic for two sample unpaired test.

Step 4: Calculate the p - value and test statistic

** We use the scipy.stats.ttest_ind to calculate the t-test for the means of TWO INDEPENDENT samples of scores given the two sample observations. This function returns t statistic and two-tailed p value.**

** This is a two-sided test for the null hypothesis that 2 independent samples have identical average (expected) values. This test assumes that the populations have identical variances.**

For this exercise, we are going to first assume that the variance is equal and then compute the necessary statistical values.

In [5]:
```
t_statistic, p_value  = ttest_ind(mydata['WingA'],mydata['WingB'])
print('tstat',t_statistic)
print('P Value',p_value)
```

```
tstat 5.16151166403543
P Value 8.007988032535588e-06
```

Step 5: Decide to reject or accept null hypothesis

In [6]:
```
# p_value < 0.05 => alternative hypothesis:
# they don't have the same mean at the 5% significance level
print ("two-sample t-test p-value=", p_value)

alpha_level = 0.05

if p_value < alpha_level:
    print('We have enough evidence to reject the null hypothesis in favour of alternative hypothesis')
    print('We conclude that the mean time to deliver luggages in of both the wings of the hotel are not
else:
    print('We do not have enough evidence to reject the null hypothesis in favour of alternative hypoth
    print('We conclude that mean time to deliver luggages in of both the wings of the hotel are same.'
```

```
two-sample t-test p-value= 8.007988032535588e-06
We have enough evidence to reject the null hypothesis in favour of alternative hypothesis
We conclude that the mean time to deliver luggages in of both the wings of the hotel are not same.
```

Let us now go ahead and check the confidence intervals at a specific α value.

Example 2 - Paired T-Test

The file Concrete contains the compressive strength, in thousands of pounds per square inch (psi), of 40 samples of concrete taken two and seven days after pouring. (Data extracted from O. Carrillo-Gamboa and R. F. Gunst, “Measurement-Error-Model Collinearities,” Technometrics, 34 (1992): 454–464.)

At the 0.01 level of significance, is there evidence that the mean strength is lower at two days than at seven days?

Problem 10.26 from the Textbook adapted for Classroom Discussion(Chapter 10-page 353)

In [9]:
```
mydata = pd.read_csv('Concrete.csv')
mydata.head()
```

Out[9]:

	Sample	Two Days	Seven Days
0	1	2.830	3.505
1	2	3.295	3.430
2	3	2.710	3.670
3	4	2.855	3.355
4	5	2.980	3.985

Step 1: Define null and alternative hypotheses

In testing whether the number of days has any effect on the lowering the compressive strength of the concrete,

the null hypothesis states that the compressive strength of the cement is not lower at 2 days than at 7 days, $\mu 2 \geq \mu 7$.

The alternative hypthesis states that the compressive strength of the cement is lower at 2 days than at 7 days, $\mu 2 < \mu 7$

$H0 : \mu 2 - \mu 7 \geq 0$

$HA : \mu 2 - \mu 7 < 0$

Here, $\mu 2$ denotes the mean compressive strenght of the cement after two days and $\mu 7$ denotes the mean compressive strength of the cement after seven days.

Step 2: Decide the significance level

Here we select α = 0.01 as given in the question

Step 3: Identify the test statistic

Sample sizes for both samples are same.

We have two paired samples and we do not know the population standard deviation.

The sample is not a large sample, n < 30. So you use the t distribution and the $tSTAT$ test statistic for two sample paired test.

Step 4: Calculate the p - value and test statistic

We use the scipy.stats.ttest_rel to calculate the T-test on TWO RELATED samples of scores. This is a two-sided test for the null hypothesis that 2 related or repeated samples have identical average (expected) values. Here we give the two sample observations as input. This function returns t statistic and two-tailed p value.

In [16]:
```
# paired t-test: doing two measurments on the same experimental unit
# e.g., before and after a treatment
t_statistic, p_value  =  stats.ttest_rel(mydata['Two Days'],mydata['Seven Days'])
print('tstat  %1.3f' % t_statistic)
print("p-value for one-tail:", p_value/2)
```

```
tstat  -9.372
p-value for one-tail: 7.768158524368871e-12
```

Step 5: Decide to reject or accept null hypothesis

In [17]:
```
# p_value < 0.05 => alternative hypothesis:
# they don't have the same mean at the 5% significance level
print ("Paired two-sample t-test p-value=", p_value/2)

alpha_level = 0.01

if (p_value/2) < alpha_level:
    print('We have enough evidence to reject the null hypothesis in favour of alternative hypothesis')

else:
    print('We do not have enough evidence to reject the null hypothesis in favour of alternative hypoth
```

```
Paired two-sample t-test p-value= 7.768158524368871e-12
We have enough evidence to reject the null hypothesis in favour of alternative hypothesis
```

In [ ]: