Homework Assignment 4 – 600.445/645 Fall 2016 (Circle One)

Instructions and Score Sheet (hand in with answers)

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Question	Points (445)	Points (645)		Totals
1A	6	2		
1B	4	2		
1C	4	3		
1D	6	3		
1E	15	10		
1F	15	15		
1G	See Note (15)	15		/ 50
2A	2	2		
2B	3	3		
2C	10	10		
2D	10	10		
2E	10	10		
2F	10	10		
2G	5	5		/ 50
3	See Note (15)	See Note (15)		/ 15
Total	100	100		

Note: Students may attempt any or all these problems for extra credit. We will award up to 15 extra points, but your total score will be limited to 100. I.e., if your total on the remaining problems is S and you score a total of E points on the extra credit problems, your net homework score will be $\min(100, S+\min(E,15))$. They are good problems, and I would urge people to try them.

600.445 Computer Integrated Surgery
Homework #4 Solutions

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1

A

Our goal in this problem is to find an expression for the incremental change in rotation ΔR_{left} and ΔR_{right} such that

$$R_{rob}(\theta + \Delta\theta) = \Delta R_{left} R_{rob} = R_{rob} \Delta R_{right}$$

.

$$\begin{aligned}
R_{rob}(\theta + \Delta\theta) &= R_1(\theta_1 + \Delta\theta_1) R_2(\theta_2 + \Delta\theta_2) R_3(\theta_3 + \Delta\theta_3) R_4(\theta_4 + \Delta\theta_4) \\
&= R_1(\theta_1) Rot(\vec{a}_1, \Delta\theta_1) R_2(\theta_2) Rot(\vec{a}_2, \Delta\theta_2) R_3(\theta_3) Rot(\vec{a}_3, \Delta\theta_3) R_4(\theta_4) Rot(\vec{a}_4, \Delta\theta_4) \\
&= Rot(\vec{a}_1, \Delta\theta_1) R_1(\theta_1) Rot(\vec{a}_2, \Delta\theta_2) R_2(\theta_2) Rot(\vec{a}_3, \Delta\theta_3) R_3(\theta_3) Rot(\vec{a}_4, \Delta\theta_4) R_4(\theta_4) \\
&= Rot(\vec{a}_1, \Delta\theta_1) Rot(R_1\vec{a}_2, \Delta\theta_2) R_1 Rot(R_2\vec{a}_3, \Delta\theta_3) R_2 Rot(R_3\vec{a}_4, \Delta\theta_4) R_3 R_4 \\
&= Rot(\vec{a}_1, \Delta\theta_1) Rot(R_1\vec{a}_2, \Delta\theta_2) Rot(R_1 R_2\vec{a}_3, \Delta\theta_3) R_1 Rot(R_2\vec{a}_4, \Delta\theta_4) R_2 R_3 R_4 \\
&= Rot(\vec{a}_1, \Delta\theta_1) Rot(R_1\vec{a}_2, \Delta\theta_2) Rot(R_1 R_2\vec{a}_3, \Delta\theta_3) Rot(R_1 R_2 R_3\vec{a}_4, \Delta\theta_4) R_1 R_2 R_3 R_4
\end{aligned}$$

Thus,

$$\Delta R_{left} = Rot(\vec{a}_1, \Delta\theta_1) Rot(R_1\vec{a}_2, \Delta\theta_2) Rot(R_1 R_2\vec{a}_3, \Delta\theta_3) Rot(R_1 R_2 R_3\vec{a}_4, \Delta\theta_4)$$

. The same simplification process can be repeated in the other direction to generate

$$\Delta R_{right} = Rot(\vec{a}_4, \Delta\theta_4) Rot(R_4^{-1}\vec{a}_3, \Delta\theta_3) Rot(R_4^{-1} R_3^{-1}\vec{a}_2, \Delta\theta_2) Rot(R_4^{-1} R_3^{-1} R_2^{-1}\vec{a}_1, \Delta\theta_1)$$

.

B

See work on attached sheets of notebook paper. Final result:

$$M_{left} = \begin{bmatrix} a_1 & R_1 a_2 & R_1 R_2 a_3 & R_1 R_2 R_3 a_4 \end{bmatrix} \begin{bmatrix} \Delta\Theta_1 \\ \Delta\Theta_2 \\ \Delta\Theta_3 \\ \Delta\Theta_4 \end{bmatrix}$$

.

$$M_{right} = \begin{bmatrix} R_4^{-1}R_3^{-1}R_2^{-1}a_1 & R_4^{-1}R_3^{-1}a_2 & R_4^{-1}a_3 & a_4 \end{bmatrix} \begin{bmatrix} \Delta\Theta_1 \\ \Delta\Theta_2 \\ \Delta\Theta_3 \\ \Delta\Theta_4 \end{bmatrix}$$

.

C

See work on attached sheets of notebook paper. Final result

$$\Delta F_{left} = [\Delta R_{left}, (I - \Delta R_{left})p_{cart} + \Delta p_{cart}]$$

$$\Delta F_{right} = [\Delta R_{right}, (\Delta R_{right} - I)p_{cr} + R_{rob}^{-1}\Delta p_{cart}]$$

where $\Delta R_{left}, \Delta R_{right}$ are the transformations computed in part B.

D

See work on attached sheets of notebook paper. Final result

$$J_{left} = \begin{bmatrix} M_{left} & 0 \\ skew(p_{cart})M_{left} & I \end{bmatrix}$$

.

E

To compute the correct $\vec{q}^{goal}$, we use the least squares solver available in the system. To use this, we define a cost function and constraints for the system. We define the distance from the current pose to the goal pose as

$$D(\Delta q) = F_{rob}(q)\dot{F}_{rob}(\Delta q)\dot{n} = F_{rob}^{goal}\dot{n}$$

where n is a unit vector. We then use the least squares solver to find

$$\Delta q = \mathit{argmin}||F_{rob}(q)\dot{F}_{rob}(\Delta q)\dot{n} - F_{rob}^{goal}\dot{n}||_2$$

We define the constraints of the robot joints:

$$\begin{aligned} \vec{\alpha} &= J_\alpha(\vec{q})\Delta q \\ \vec{\epsilon} &= J_\epsilon(\vec{q})\Delta q \\ q_L - q \leq \Delta q &\leq q_U - q \end{aligned}$$

where q_L is the lower limits and q_U is the upper limits of the robot movement.

F

Currently the joint positions are limited by the constraint given above. However there is equal weight given to any valid position. To keep the joints near the mid point, we introduce an additional term in the minimization problem, where we penalize for joint positions away from the mid point.

$$\Delta q = \mathit{argmin}||F_{rob}(q) \cdot F_{rob}(\Delta q) \cdot n - F_{rob}^{goal} \cdot n||^2 + \eta||q_L + (q_U - q_L)/2 - (q + \Delta q)||^2$$

where η is a small factor to prevent an impact on accuracy of the target pose.

G

We assume that there are no disturbances to the arm as it is moving to the target position, and that the target position remains constant throughout the time needed to move the arm. We want the arm to travel at a uniform velocity. We divide the movement into several small motions. The total number of incremental updates is given by

$$n = \frac{||p_{rob}^{goal} - p_{rob}||}{v\Delta t}$$

To compute Δq for each step, we divide the the linear transformation to get

$$\Delta x_{d,p} = \frac{p_{rob}^{goal} - p_{rob}}{n}$$

Additionally, we are able to get the total rotational change as a rotation around a single axis a, as describe in part A. The rotation around this axis is also split into n parts to obtain $\Delta x_{d,R}$. Here $\Delta x_d = [\Delta x_{d,p} \quad \Delta x_{d,R}]$ represents the incremental pose updates to α and ϵ. Our optimization problem is now

$$\underset{\Delta q}{\arg\min}\, ||\Delta x - \Delta x_d||^2 + \eta||q_L + (q_U - q_L)/2 - (q + \Delta q)||^2$$

, where $\Delta x = J_{left}\Delta q$ and η continues to be a weight less than one.

2

A

$$F_{rc} = F_{2c}^{-1} F_2^{-1} F_{r1}^{-1}$$

B

$$\begin{aligned} F_{rc}^{-1} F_{rob}^{goal} F_{rt} &= F_{tool}^{goal} \\ F_{rob}^{goal} F_{rt} &= F_{rc} F_{tool}^{goal} \\ F_{rob}^{goal} &= F_{rc} F_{tool}^{goal} F_{rt}^{-1} \end{aligned}$$

C

For this approach, we can simply use the same approach as in 1G.

D

In this case, we want to define a virtual forbidden space surrounding the target path in space. The task-based optimization problem is:

$$\underset{\Delta q}{\arg\min}\, ||A\vec{f} - J\Delta q||^2$$

with the constraint based on the virtual forbidden space around the line in space. For accuracy's sake, we would likely implement a hard boundary in order to truly maintain the desired trajectory.

We define $\Delta x = J\Delta q$. This hard boundary is defined by finding the updated point $x + \Delta x$ in space and comparing it to the goal trajectory. We will define the goal trajectory as a line $L = \vec{l} * s$ where $\vec{l}$ is a unit vector which defines the line. We then determine the closest point on this line to the updated point in space and find the vector $\vec{n}$ between them. Then, we project this vector onto a plane whose norm is defined by $\vec{l}$. Our goal is to have a hard boundary around the line, thus we constrain this projection by an acceptable error of ϵ_n. This defines another constraint for our minimization problem.

E

Instead of a hard boundary on the line directly around the line, we would likely implement a soft boundary around the axis of the cylinder between 0.95ρ and ρ. This means that the surgeon would have full freedom of operation within this range but start feeling resistance as he/she gets too close to the walls of the cylinder. These constraints can be defined by:

$$A_i(\Delta q) - s_i \leq 0.95$$

$$0 \leq s_i \leq 0.05$$

Where A_i represents the function that computes the length of the projection of the updated point onto the plane defined by the target line. And we can add another term to our minimization problem ηs_i^2.

To ensure that the probe is within 30 degrees of the central axis of the cylinder, we can treat each step of the motion as a "maintain direction" problem with a virtual boundary of 30 degrees of offset from the central axis (target orientation).

F

Once we know the area we need to cut, we plan a tool path for the known cutter. We take an approach similar to a fully automated cutter. The tool path can be defined in some way that fills the entire area, such as a spiral, concentric circles, or a zig-zag pattern. One the tool path is plotted, we can define a curve tracing the outline of the tool path as a rigid boundary. When the surgeon enters, the cutter will be pushed into the tool path, or the tool path can be generated once cutting begins. As the surgeon moves the cutter, the rigid constraint will force the surgeon to follow the pre-planned tool path (forwards or backwards). This ensures that the surgeon is guided into cutting all of the necessary bone. The outer rigid constraint still exists, such that there is no over-cutting.

G

We would simply at a soft boundary plane starting at $z = -\psi + \eta$ which then becomes a hard boundary plane at $z = \psi$. This creates the resistance desired to alert the surgeon. The location of the tip in CT coordinates is $F_{rc}^{-1} F_{rob} F_{rt}$. To get the z-component, we simply multiply by $\vec{z} = [0 \quad 0 \quad 1]$.

3

We explored failure modes related to the effective detection of problems, as well as operation. Since there is a lot that happens in software and hardware unseen to the user, verification of proper and correct operation is important and is where most of the failures we explored are.

	Ranking Definitions				
	5	**4**	**3**	**2**	1
Severity	Permanent damage	Immediate complication	Non-permanent trauma	Surgical ineffectiveness	Prolonged surgery
Detectability	No method to detect problem	Detectable with equipment	Not directly known	Indirect detection	Immediately obvious

				Risk			**Risk after mitigation**	
Process element/component	Failure Mode	Causes	Current Detection	Severity	Detectability	Mitigation	Severity	Detectability
Pose tracking	Line of sight interruption	Users blocking the line of sight	Tracker keeps a status of number of markers in frame	1	1	Audible feedback to user about LOS blocking	1	1
Calibration of F_rt	Improper tool tip positioning	User error during calibration	None	4	2	Correlate expected tool force vs. actual (are you cutting when the system thinks your in free space?)	3	1
Mid-level controller	Total Failure / Comm failure	Electrical issues, software crash	None	3	4	Tool shutoff without consistant control, live status indicator	3	1
Registration	Improper control	Mixing robot and patient reference markers	User noticies misbehavior	3	2	Use unique calibration markers for each, robot markers are fixed	3	1
Procedure	Robot cannot reach needed location	DOF limitations	User notices reach issues	1	2	Robot awareness of total DOF, verify working region during registration	1	1
Robot	Robot joint sensor error	Sensor lifetime, defect, failure	None	4	4	Have local robot registration, e.g. points on base to touch to verify sensors	1	1

Problem 1A

$$R_{rob}(\theta + \Delta\theta) = R_1(\theta_1 + \Delta\theta_1) R_2(\theta_2 + \Delta\theta_2) R_3(\theta_3 + \Delta\theta_3) R_4(\theta_4 + \Delta\theta_4)$$

$$= R_1(\theta_1) R_1(\Delta\theta_1) R_2(\theta_2) R_2(\Delta\theta_2) R_3(\theta_3) R_3(\Delta\theta_3) R_4(\theta_4) R_4(\Delta\theta_4)$$

$$= R_1(\theta)\left(I\cos(\Delta\theta_1) + \text{skew}(\vec{a}_1)\sin(\Delta\theta_1) + \vec{a}_1 \cdot a_1^T(1-\cos\Delta\theta_1)\right)$$
$$R_2(\theta)\left(I\cos(\Delta\theta_2) + \text{skew}(\vec{a}_2)\sin(\Delta\theta_2) + \vec{a}_2 \cdot a_2^T(1-\cos(\Delta\theta_2)\right)$$
$$R_3(\theta)\left(I\cos(\Delta\theta_3) + \text{skew}(\vec{a}_3)\sin(\Delta\theta_3) + \vec{a}_3 \cdot a_3^T(1-\cos(\Delta\theta_3)\right)$$
$$R_4(\theta)\left(I\cos(\Delta\theta_4) + \text{skew}(\vec{a}_4)\sin(\Delta\theta_4) + \vec{a}_4 \cdot a_4^T(1-\cos(\Delta\theta_4)\right)$$

$$= \left(R_1(\theta_1)\cos(\Delta\theta_1) + R_1(\theta_1)\text{skew}(\vec{a}_1)\sin(\Delta\theta_1) + R_1(\theta_1)\vec{a}_1 \cdot a_1^T(1-\cos\Delta\theta_1)\right)$$
$$\left(R_2(\theta_2)\cos(\Delta\theta_2) + R_2(\theta_2)\text{skew}(\vec{a}_2)\sin(\Delta\theta_2) + R_2(\theta_2) a_2 a_2^T(1-\cos(\Delta\theta_2))\right)$$
$$\left(R_3(\theta_3)\cos(\Delta\theta_3) + R_3(\theta_3)\text{skew}(\vec{a}_3)\sin(\Delta\theta_3) + R_3(\theta_3) a_3 a_3^T(1-\cos(\Delta\theta_3))\right)$$
$$\left(R_4(\theta_4)\cos(\Delta\theta_4) + R_4(\theta_4)\text{skew}(\vec{a}_4)\sin(\Delta\theta_4) + R_4(\theta_4) a_4 a_4^T(1-\cos(\Delta\theta_4))\right)$$

$$= R_1(\theta_1) R_2(\theta_2) R_3(\theta_3) R_4(\theta_4) \begin{pmatrix} \cos(\Delta\theta_1) + \sin(\Delta\theta_1)\text{skew}(a_1) \\ -A_1(\cos(\Delta\theta_1) - 1) \end{pmatrix}$$

$$= R_1(\theta_1)\,\text{Rot}(a_1, \Delta\theta_1)\, R_2(\theta_2)\,\text{Rot}(a_2, \Delta\theta_2)\, R_3(\theta_3)\,\text{Rot}(a_3, \Delta\theta_3) \ldots$$

$$= \text{Rot}(\vec{a}_1, \Delta\theta_1) \cdot R_1(\theta_1)\,\text{Rot}(a_2, \Delta\theta_2)\, R_2(\theta_2)\,\text{Rot}(a_3, \Delta\theta_3)\, R_3(\theta_3)\,\text{Rot}(a_4, \Delta\theta_4) R_4$$

$$= \text{Rot}(\vec{a}_1, \Delta\theta_1)\,\text{Rot}(R_1(\theta)\vec{a}_2, \Delta\theta_2)\, R_1(\theta)\,\text{Rot}(R_2(\theta)a_3, \Delta\theta_3) R_2(\theta)\,\text{Rot}(R_3(\theta_3)a_4, \Delta\theta_4)$$
$$R_3(\theta_3) R_4 \theta_4$$

$$= \text{Rot}(\vec{a}_1, \Delta\theta_1)\text{Rot}(R_1 a_2, \Delta\theta_2)\text{Rot}(R_1 R_2 a_3, \Delta\theta_3) R_1\,\text{Rot}(R_2 R_3 a_4, \Delta\theta_4)\, R_2 R_3 R_4$$

$$\boxed{= \text{Rot}(\vec{a}_1, \Delta\theta_1)\,\text{Rot}(R_1 a_2, \Delta\theta_2)\,\text{Rot}(R_1 R_2 a_3, \Delta\theta_3)\,\text{Rot}(R_1 R_2 R_3 a_4, \Delta\theta_4) R_1 R_2 R_3 R_4}$$

$$= R_1 R_2\,\text{Rot}(R_2^{-1} a_1, \Delta\theta_1) R_3\,\text{Rot}(R_3^{-1} a_2, \Delta\theta_2) R_4\,\text{Rot}(R_4^{-1} a_3, \Delta\theta_3)\,\text{Rot}(a_4, \Delta\theta_4)$$

$$= \boxed{R_1 R_2 R_3 R_4}\,\text{Rot}(R_4^{-1} R_3^{-1} R_2^{-1} a, \Delta\theta_1)\,\text{Rot}(R_4^{-1} R_3^{-1} a_2, \Delta\theta_2)\,\text{Rot}(R_4^{-1} a_3, \Delta\theta_3)$$
$$\text{Rot}(a_4, \Delta\theta_4)$$

1B on next page

Problem 1C

$$\Delta q = [\Delta\theta, \Delta p_{cart}]$$

$$F_{rob} = [R_{rob}, \vec{p}_{rob}]$$
$$= R_{rob} \cdot p_{cart} + R_{rob}\vec{p}_{cr}$$

$$\Delta F_{left} F_{rob} = \begin{bmatrix}\Delta R_{left}, & \Delta p_{cart}\\ 0 & 1\end{bmatrix}\begin{bmatrix} R_{rob}, & \vec{p}_{cart} + R_{rob}\vec{p}_{cr} \\ 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}\Delta R_{left} R_{rob} & \Delta R_{left}\vec{p}_{cart} + \Delta R_{left} R_{rob}\vec{p}_{cr} + \Delta p_{cart}\end{bmatrix}$$

$$F_{rob}(q+\Delta q) = [R_{rob}(\theta+\Delta\theta), p_{cart} + R_{rob}(\theta+\Delta\theta)\vec{p}_{cr} + \Delta p_{cart}]$$

$$\Delta F_{left} F_{rob} = [\Delta R_{left} R_{rob}(\theta), p_{cart} + \Delta R_{left} R_{rob}(\theta)\vec{p}_{cr} + \Delta p_{cart}]$$

$$\begin{bmatrix}\Delta R_\ell & \Delta p_\ell \\ 0 & 1\end{bmatrix}\begin{bmatrix} R_{rob}, & p_{cart} + R_{rob}p_{cr}\end{bmatrix}$$

$$[\Delta R_\ell R_{rob}, \Delta R_\ell p_{cart} + \Delta R_\ell R_{rob} p_{cr} + \Delta p_\ell] = [\Delta R_{left} R_{rob}, p_{cart} + \Delta R_{left} R_{rob} p_{cr} + \Delta p_{cart}]$$

$$\Delta R_{left} p_{cart} + \cancel{\Delta R_{left} R_{rob} p_{cr}} + \Delta p_\ell = \boxed{p_{cart}} + \cancel{\Delta R_{left} R_{rob} p_{cr}} + \Delta p_{cart}$$

$$\boxed{\Delta p_\ell} = (I - \Delta R_{left}) p_{cart} + \Delta p_{cart}$$

$$\boxed{\Delta F_{left} = \begin{bmatrix}\Delta R_{left}, & (I - \Delta R_{left}) p_{cart} + \Delta p_{cart} \\ & 1\end{bmatrix}}$$

$$\begin{bmatrix}R_{rob}, & p_{cart} + R_{rob}p_{cr}\\ & 1\end{bmatrix}\begin{bmatrix}\Delta R_r & \Delta p_r\end{bmatrix} = [R_{rob}\Delta R_{right}, p_{cart} + R_{rob}\Delta R_{right}\vec{p}_{cr} + \Delta p_{cart}]$$

$$[R_{rob}\Delta R_r, R_{rob}\Delta p_r + p_{cart} + R_{rob}p_{cr}] = R_{rob}\Delta R_{right}, p_{cart} + R_{rob}\Delta R_{right}\vec{p}_{cr} + \Delta p_{cart}$$

$$\boxed{\Delta R_r = \Delta R_{right}}$$

$$R_{rob}\Delta p_r = R_{rob}(\Delta R_{right} - I)\vec{p}_{cr} + \Delta p_{cart}$$

$$\boxed{\Delta p_r = (\Delta R_{right} - I)\vec{p}_{cr} + R_{rob}^{-1}\Delta p_{cart}}$$

Problem 1B

$$\Delta R_{left} = Rot(a_1, \Delta\theta_1)\, Rot(R_1 a_2, \Delta\theta_2)\, Rot(R_1 R_2 a_3, \Delta\theta_3)\, Rot(R_1 R_2 R_3 a_4, \Delta\theta_4)$$

$$= (I + skew(\Delta\theta_1 a_1))(I + skew(\Delta\theta_2 R_1 a_2))(I + skew(\Delta\theta_3 R_1 R_2 a_3))(I + skew(\Delta\theta_4 R_1 R_2 R_3 a_4))$$

$$= (I + skew(\Delta\theta_2 R_1 a_2) + skew(\Delta\theta_1 a_1))(I + skew(\Delta\theta_4 R_1 R_2 R_3 a_4) + skew(\Delta\theta_3 R_1 R_2 a_3))$$

$$= I + skew(\Delta\theta_4 R_1 R_2 R_3 a_4) + skew(\underline{\Delta\theta_3 R_1 R_2 a_3}) + skew(\Delta\theta_2 R_1 a_2) + skew(\Delta\theta_1 a_1)$$

$$= I + skew(\Delta\theta_4 R_1 R_2 R_3 a_4 + \Delta\theta_3 R_1 R_2 a_3 + \Delta\theta_2 R_1 a_2 + \Delta\theta_1 a_1)$$

$$= I + skew(M_{left} \cdot \Delta\theta) \qquad \boxed{M_{left} = \begin{bmatrix} a_1 & R_1 a_2 & R_1 R_2 a_3 & R_1 R_2 R_3 a_4 \end{bmatrix}} \begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \Delta\theta_3 \\ \Delta\theta_4 \end{bmatrix}$$

$$\Delta R_{right} = Rot(R_4^{-1} R_3^{-1} R_2^{-1} a_1, \Delta\theta_1)\, Rot(R_4^{-1} R_3^{-1} a_2, \Delta\theta_2)\, Rot(R_4^{-1} a_3, \Delta\theta_3)\, Rot(a_4, \Delta\theta_4)$$

$$= (I + skew(\Delta\theta_1 R_4^{-1} R_3^{-1} R_2^{-1} a_1))(I + skew(\Delta\theta_2 R_4^{-1} R_3^{-1} a_2))(I + skew(\Delta\theta_3 R_4^{-1} a_3))(I + skew(\Delta\theta_4 a_4))$$

$$= (I + skew(\Delta\theta_1 R_4^{-1} R_3^{-1} R_2^{-1} a_1) + skew(\Delta\theta_2 R_4^{-1} R_3^{-1} a_2) + skew(\Delta\theta_3 R_4^{-1} a_3) + skew(\Delta\theta_4 a_4))$$

$$= I + skew(\Delta\theta_1 R_4^{-1} R_3^{-1} R_2^{-1} a_1 + \Delta\theta_2 R_4^{-1} R_3^{-1} a_2 + \Delta\theta_3 R_4^{-1} a_3 + \Delta\theta_4 a_4)$$

$$\boxed{M_{right} = \begin{bmatrix} R_4^{-1} R_3^{-1} R_2^{-1} a_1 & R_4^{-1} R_3^{-1} a_2 & R_4^{-1} a_3 & a_4 \end{bmatrix}} \begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \Delta\theta_3 \\ \Delta\theta_4 \end{bmatrix}$$

Problem 1D pg. 1

$$\Delta F_{left} = \vec{\delta}_{left} = [\alpha_{left}, \varepsilon_{left}] = \left[(I + skew(\vec{\alpha}_{left})), \vec{\varepsilon}_{left} \right]$$

$$I + skew(\alpha_{left}) = \Delta R_{left}$$
$$= I + skew(M_{left} \cdot \Delta\theta)$$
$$\alpha_{left} = M_{left}\Delta\theta + 0\Delta p_{cart} = \left[M_{left} \mid 0 \right] \begin{bmatrix} \Delta\theta \\ \Delta p_{cart} \end{bmatrix}$$

$$\varepsilon_{left} = (I - (I + skew(M_{left}\Delta\theta)))p_{cart} + \Delta p_{cart}$$
$$= skew(-M_{left}\Delta\theta)\, p_{cart} + \Delta p_{cart}$$

$$= skew(-p_{cart}) - M_{left}\Delta\theta + -\Delta p_{cart}$$
$$= skew(p_{cart}) M_{left}\Delta\theta + \Delta p_{cart}$$
$$= \left[skew(p_{cart}) M_{left} \mid I \right] \begin{bmatrix} \Delta\theta \\ \Delta p_{cart} \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} M_{left} & 0 \\ skew(p_{cart}) M_{left} & I \end{bmatrix}}_{J_{left}} \begin{bmatrix} \Delta\theta \\ \Delta p_{cart} \end{bmatrix} = \begin{bmatrix} \alpha \\ \varepsilon \end{bmatrix}$$

☆ Finish J_{right}

$\cancel{Ax} = b$ $\qquad J_{left}\,\Delta q_{,} \leq \begin{bmatrix} \alpha_{left} \\ \varepsilon_{left} \end{bmatrix}$

1E

$$F_{rob}^{gd} = F(q + \Delta q)$$
$$= \Delta F_{left}\, F_{rob}$$

$$\Delta F_{left} = [I + skew(M\Delta\theta), \varepsilon_{left}]$$

$$= [I + skew(\alpha_{left}), \varepsilon_{left}][R_{rob}, p_{cart} + R_{rob}p_{cr}]$$

$$= R_{rob} + skew(\alpha_{left})R_{rob}, p_{cart} + R_{rob}p_{cr} + skew(\alpha_{left})p_{c}$$
$$+ skew(\alpha_{left})R_{rob}p_{cr} + \varepsilon_{left}$$

Problem 1D pg. 2

J_{right}

$$\Delta F_{right} = \delta_{right} = [\alpha_{right}, \varepsilon_{right}] = [I + skew(\alpha_{right}), \varepsilon_{right}$$

$$I + skew(\alpha_{right}) = \Delta R_{right}$$
$$= I + skew(M_{right} \cdot \Delta\Theta)$$

$$\alpha_{right} = M_{right} \Delta\Theta + 0 \Delta p_{cart}$$

$$\begin{bmatrix} M_{right} & 0 \end{bmatrix} \begin{bmatrix} \Delta\Theta \\ \Delta p_{cart} \end{bmatrix} = \alpha_{right}$$

$$\varepsilon_{right} = (\Delta R_{right} - I)\vec{p}_{cr} + R_{rob}^{-1} \Delta p_{cart}$$
$$= (I + skew(M_{right}(\Delta\Theta) - I)\vec{p}_{cr} + R_{rob}^{-1} \Delta p_{cart}$$
$$= skew(M_{right} \Delta\Theta)\vec{p}_{cr} + R_{rob}^{-1} \Delta p_{cart}$$
$$= skew(-\vec{p}_{cr}) M_{right} \Delta\Theta + R_{rob}^{-1} \Delta p_{cart}$$
$$= \begin{bmatrix} skew(-\vec{p}_{cr}) M_{right} & R_{rob}^{-1} \end{bmatrix} \begin{bmatrix} \Delta\Theta \\ \Delta p_{cart} \end{bmatrix}$$

$$\boxed{J_{right} = \begin{bmatrix} M_{right} & 0 \\ skew(-\vec{p}_{cr}) M_{right} & R_{rob}^{-1} \end{bmatrix}}$$