

Memory Management

As we know that to enhance the overall performance of computer, several processes must be executed simultaneously. So, several processes must be kept in memory simultaneously. So, memory should be shared and managed.

Volatile memory is one of the important component of a computer system. In terms of memory, the efficiency of a computer system will depend on size, access time and cost of the memory.

So, size should be larger, access time should be smaller and cost should be smaller.

By maintaining a hard disk as secondary memory, size and cost factor can be managed but to reduce the access time, we need maintain primary memory i.e. RAM or random access memory.

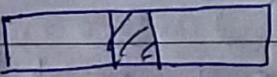
Locality of reference :- A program is a set of instructions. When CPU executes a program, it executes in sequential manner. I.e. if currently CPU is executing instruction number 120 then it is highly probable that next instruction executed by CPU will be 121 until any jump statement is not coming. So, the concept of next instruction getting executed is called Locality of reference.

So, even if size of RAM is smaller than hard disk, due to the concept of locality of reference, reallocation of smaller size can be minimized.

Allocation of main memory: The process of assigning space to a program in RAM
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Memory allocation can be contiguous or non-contiguous. In contiguous allocation, whole program is given space in RAM at a stretch whereas in case of non-contiguous allocation, the program is broken into multiple pieces and each piece is kept in different location of RAM.

Advantages and disadvantages of both methods can be realized with the example of array (contiguous) and linked list (non-contiguous).

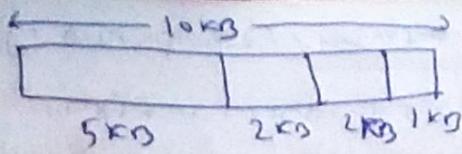
In contiguous allocation, access will be fast but space will be ill-managed. E.g.  A 10KB process cannot be accommodated even if 8KB is vacant. This drawback is called external fragmentation.

In case of non-contiguous allocation, like linked list, random access is not possible, however, space will be well managed.

(Contiguous) memory allocation is of two types:

- (i) Fixed size partitioning (ii) Variable size partitioning

In fixed size partitioning, an entire partition has to be allocated to a program.



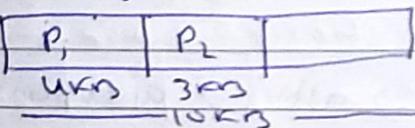
Here, 10KB RAM is divided into 3 types of partitions 5KB, ~~2KB~~ and 1KB.

Now, if a process is of size 3KB comes then whole 5KB partition has to be allocated to the process. So, 2KB will be unused and cannot be allocated to another smaller size process.

This wastage of space is called internal fragmentation.

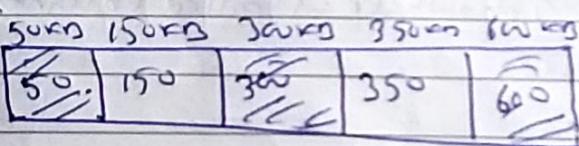
In case of variable size partitioning, there is no pre-defined partitioning.

$$P_1 = 4KB$$



So, no internal fragmentation

Managing the memory in fixed size partitioning is easy.



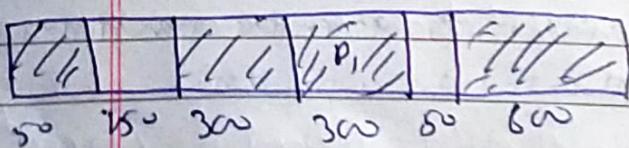
$$P_1 = 300 \text{ KB}, P_2 = 25 \text{ KB}, P_3 = 125 \text{ KB}$$

$$P_4 = 50 \text{ KB}$$

We are following variable size partitioning

First fit allocation - Process will be stored in the 1st capable partition.

So, after allocation of P_1 , status of memory.



After alloc. of P_2 status of memory

$\backslash\backslash\backslash P_2$	$\backslash\backslash C$	$\backslash\backslash P_1$	$\backslash\backslash C_d$
50	25	125	300 300 50 600

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After alloc. of P_3 status of mem. ->

$\backslash\backslash C$	$\backslash\backslash P_2$	$\backslash\backslash P_3$	$\backslash\backslash C$	$\backslash\backslash C_d$
50	25	125	300 300 50 600	

After alloc. of P_4 status of mem. ->

$\backslash\backslash C$	$\backslash\backslash P_2$	$\backslash\backslash P_3$	$\backslash\backslash C$	$\backslash\backslash P_4$	$\backslash\backslash C_d$
50	25	125	300 300 50 600		

Best fit allocation :- process will be allocated stored in the smallest capable partition.

Status of memory after allocation of P_1, P_2, P_3

$\backslash\backslash C$	$\backslash\backslash P_2$	$\backslash\backslash C$	$\backslash\backslash P_1$	$\backslash\backslash P_3$	$\backslash\backslash C_d$
50	125	25	300 300	25 25	600

Here, P_4 cannot be allocated. i.e. case of external fragmentation is there.

Worst fit allocation :- process will be stored in the longest capable partition.

Status of memory after allocation of P_1, P_2, P_3, P_4

$\backslash\backslash C$	$\backslash\backslash P_2$	$\backslash\backslash P_3$	$\backslash\backslash C$	$\backslash\backslash P_1$	$\backslash\backslash P_4$	$\backslash\backslash C_d$
50	125	25	300 300	50	600	

worst fit policy works best in case of variable size partitioning but it is highly probable that left over space can be reused again.

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Now we will analyze first, best, worst fit policies in ~~var~~ fixed size partitioning

Let there are 6 types of partition of size 200, 400, 600, 500, 300 and 250

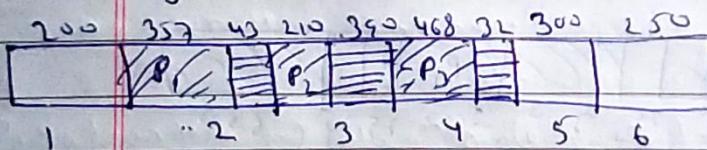
Let the memory size = 2250

Processes arrival order and sizes are

P₁ 357, P₂ 210, P₃ 468, P₄ 491
200 400 600 500 300 250



Status of memory after 1st fit policy



Hence partitions 2, 3, 4 have internal fragmentation.

Due to internal fragmentation P₄ cannot be allocated.

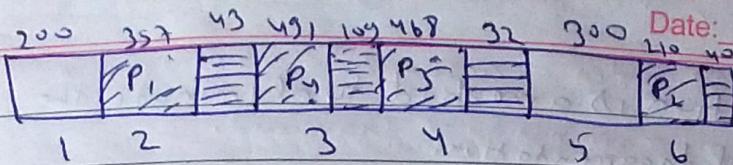
If size of the process is greater than size of available space then it is not called external fragmentation. e.g. if P₄ is of size 800 then no external fragmentation.

If size of the process is less than available space then it is ~~less~~ external fragmentation may occur. In that case, size of the process will be the size of external fragmentation.

Status of memory in case of best fit

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Hence, partitions 2, 3, 4, 6 faces internal fragmentation.

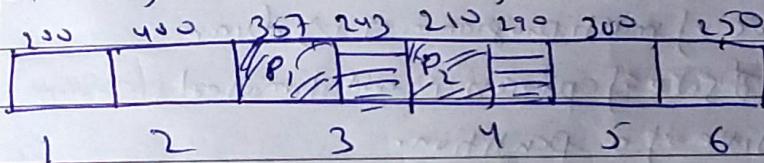
memory wasted in internal fragmentation,

$$= 43 + 109 + 32 + 40$$

memory wasted in external fragmentation

= 0 b/w all processes were allocated.

Status of memory in case of worst fit



Partitions 3 and 4 faces internal fragmentation

Space wasted due to internal frag. = 243 + 290

Since, total space available after allocation of P₁ and P₂ is greater than sum of sizes of P₃ and P₄.
So, external fragmentation exists.

So, space wasted due to external fragmentation
= 468 + 491

Compaction is a method to eliminate the external fragmentation.

Pulling all occupied part together and

-pulling all free part together is called compaction or defragmentation.

However, during the compaction, running processes need to be stopped for sometime to shift the place of storage of the processes.

So due to stopping of processes, the efficiency of the system will go down.

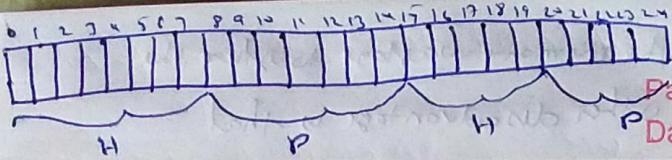
Advantages of Variable size partitioning:

- (1) degree of multiprogramming is dynamic
- (2) size of process is not limited by size of a partition.

Disadvantage - (1) allocation & deallocation of memory
is complex

Data structures to keep tracks of occupied and free space in case of variable size partitioning.

- (1) Bit map :- Let allocation unit is 4^k byte.
So, whole memory will be divided into chunks of 4^k bytes. Now, each allocation unit is represented by a bit. bit will be 0 if the allocation unit is free else 1.



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Solution from notes

Here, memory is divided into 25 allocn units.

from allocn unit 0 to 7, there is a hole i.e.
the memory space is unallocated.

from 8 to 14, a process is there.

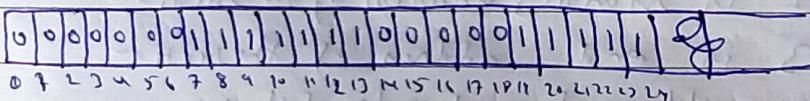
from 15 to 19 a hole

from 19 to 24 occupied by process.

now, to represent the above occupied and
free space of above memory in bit map

we can maintain an array of size equal
to no. of allocn units in memory. If

the value at a particular posn in array is zero, it
means corresponding allocn unit is unoccupied in
memory. In this case



So, if allocn unit is of 4 bytes, i.e. 32 bit
then for each 32 bit of memory, 1 bit of
bit map is wasted.

If allocn unit is of big size then bit map size
will decrease but if due to big allocn unit
memory space will be wasted but allocn will always
start from beginning of allocn unit.

~~Want from the memory wastage to store the bit map, other disadvantage is that~~

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~~to find the holes to accommodate a new process, entire bit map has to be traversed.~~

Advantage of bit map is that when a process is terminated, the corresponding allocation unit or entry in the bitmap is made which is easy. However, due to its disadvantages, bit map is not recommended.

(ii) Linked RM: - Node of the DLL will be

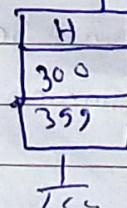
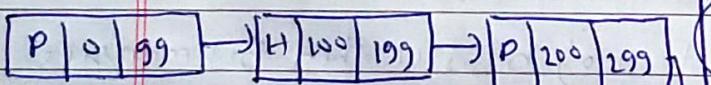
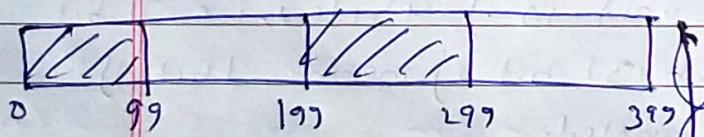
1	2	3	4
---	---	---	---

1 will contain either the section is a process or whole.

2 is starting address of the section

3 is end address of the section

4 - address of next node of the LL

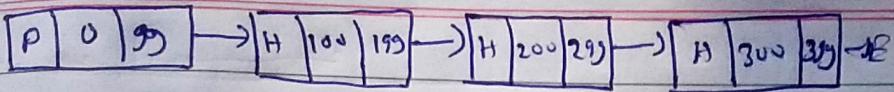


if the chunk b/w 199 to 299 becomes

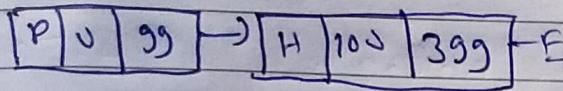
valant then LL will become:

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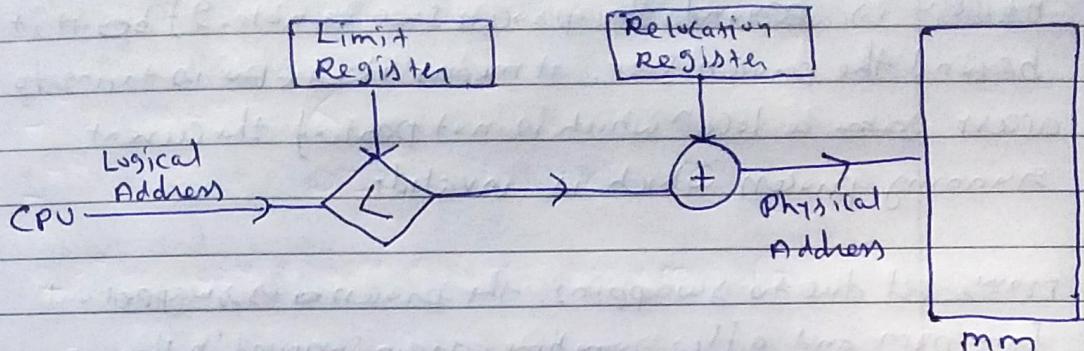


If in place of singly LL, if we use double LL
then 2nd, 3rd and 4th nodes can be merged

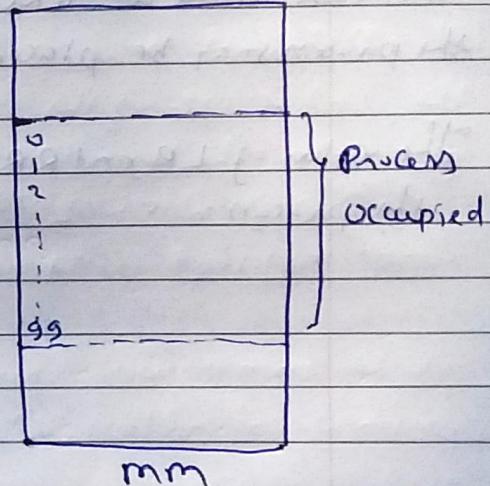


- ⇒ one of the soln of external fragmentation is
non-contiguous allocat. ⇒
non-contiguous allocat is of two type:
① paging ② Segmentation

Relocation in terms case of contiguous memory allocation



Let a process is of 100 byte is in the mm. Let it has occupied the space in mm from 100 byte starting from 100Kth byte of the mm.



In order to fetch any instruction of the process from mm, the CPU will generate logical address, i.e., any no. b/w 0 to 99 in this case. However, in reality, in mm the instruction at 5th byte of the process is actually located at (100K + 5)th byte of the mm, which is the physical address of the instruction.
The conversion of logical address to physical address is done by memory management unit (MMU).

In this case, value at limit register will be 100
Relocation register = 100K.

Limit register is to check that logical address generated by CPU is beyond the process size or not. If beyond it is beyond the process size, it means the CPU is trying to access ~~some~~ a location which is not part of the current running process, which is invalid.

Now, let due to swapping, the process is swapped out from MM and after sometime, again swapped in then in this case value at relevant register will change but the process may be placed at some different location in MM.

The value of LR and RR is set by MMU and not by the process.

Non-Contiguous Allocation

In case of contiguous allocation, external fragmentation is a major drawback.

In case of variable size partitioning, keeping track of occupied and unoccupied spaces in MM is troublesome.

As a soln, concept of paging is proposed.

It is a soln for non-contiguous allocation with fixed partition.

In paging, a process is divided into multiple fixed size pages. Similarly, MM is also divided into ~~some~~ multiple same size page. In case of MM, these pages are called frames.

Let, page size = 1 KB

Program size = 1 MB

MM size = 1 GB

So, we can say that Program size is of $\frac{1}{1K}$ Pages.
MM size is of 1 M pages.

Let a process has size is of 3 pages. In case of non-contiguous allocation,

Here, the pages P_1, P_2, P_3 of the processes are stored in 3 different places in MM.

So, efficient use of spaces in MM.

P_1	frame no. 1
	2
	3
	4
	5

Since, before coming to MM, a process space is already divided into pages. In other words, the hard disk is actually divided into pages.

In case of paging, now the logical address generated by CPU will be in terms of page no. and offset.

Let, CPU want to access the 4th instruction of the 2nd page of the process then it will generate the logical address as

Page no. - 2

Offset - 4

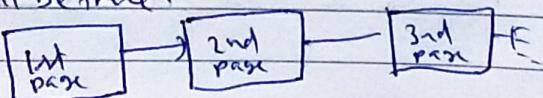
Now, MMU will convert this logical address (LA) to physical address (PA), i.e., actual address in MM.

Since, page size and frame size of MM are same. So, offset no. will be same in both LA and PA.

Now, MMU will have to find frame no. corresponding to the page no. 2 of the process.

One of the method to find the corresponding frame no. is can be to keep the address of next page in the previous page, like a linked list.

e.g. Let we know the address of 1st page of process in MM then at the end of 1st page, address of 2nd page will be there and at the end of 2nd page, address of 3rd page will be there.



But, this method will result into slow access of the required page.

The second method which is followed is to maintain a data structure, called as page table, corresponding to each process. This page table will contain frame no. cor in mm corresponding to each page of the process.

Let, there are 3 pages of a process. P_1, P_2, P_3 .

Let P_1 is stored at 1001 th frame of the mm.

P_1	59	..	-	-	-
P_2	-	-	5036	-	-	-	..
P_3	-	-					

So, page table corresponding to this process will be

1001	P_1
59	P_2
5036	P_3

Since, each process will have their personal page table and these page tables are also stored in mm.

The addresses of beginning to these page tables will be stored in page table base register (PTBR) which is located in CPU.

So, when CPU generates a logical address in terms of page no. and offset, the MMU will first find the address of beginning of the page table corresponding to the process running in CPU from PTB & using PTBR. Now, MMU will go to the page table located in mm. MMU, from the pagetable, the MMU will extract the corresponding frame no. and then again MMU will access the mm to get the entire A after extracting the frame no., MMU

will generate physical address (PA), which is frame no. and offset.

Here, offset will be same as that of LA bcoz page size and frame size are same.

So, if LA is page no. - 2

^{frame}
offset - 4

then PA will be frame no. 59
offset - 4

After generating the PA, the mmu will assign to mmu to access the generated frame no.

In this whole process, we can see that to access a memory instruction, two times memory access is needed. First time for to search in the page table and 2nd time for actual access of the instruction.

If a page size is 1 KB and Psize is 3.5 KB

so, here fourth page will face internal fragmentation.

$$2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32, 2^6 = 64$$

$$2^7 = 128, 2^8 = 256, 2^9 = 512, 2^{10}$$

$$2^{10} = 1024 = K$$

$$2^{20} = 1024 \times 1024 = 1024K = M$$

$$2^{30} = 1024 \times 1024 \times 1024 = 1024M = G$$

$$2^{40} = 1024 \times 1024 \times 1024 \times 1024 = 1024G = 1T$$

$$128GB = 2^7 \times 2^{30} = 2^{37}$$

$$2^{28} = 2^8 \times 2^{20} = 256MB$$

From 2 bit we can generate 4 combinations

00 01 10 11

Similarly for 3 bits, we can generate 8 combinations

000 001 010 011 100 101 110 111

∴ for n bits, we can generate 2^n combinations

If MM is of size 8 byte then to give an unique address to each byte, we need 3 bits. $\leftarrow 1 \text{ bit} \rightarrow$

In other words, if it is given that 3 bits are used to address for addressing the memory location and each location is of 1 byte ~~low~~ size the total size of the memory is $2^3 = 8$ byte.

000	
001	.
010	.
011	.
100	.
101	.
110	.
111	.

If 14 bits are used for addressing and each memory location is of size 1 byte then size of the MM
 $= 2^{14} \text{ byte} = 2^{10} \times 2^4 \text{ byte} = 16 \text{ KB}$

If no. of bits used is 22 and each memory location is of size 2 byte then size of MM

$$= 2^{22} \times 2^3 \text{ byte} = 2^9 \times 2^3 \text{ MB} = 8 \text{ MB}$$

\Rightarrow If size of mm = 64 KB and size of memory location is \rightarrow 1B then no. of bits required for addressing

$$\therefore \frac{64 \text{ KB}}{1 \text{ B}} = 64 \text{ K} = 2^6 \times 2^{10} \\ = 2^{16} = 16 \text{ bits}$$

\Rightarrow If size of mm = 32 KB and size of memory location is 1 B then no. of bits required for addressing

$$\therefore 32 \text{ K} = 8 \text{ S} \times 2^{10} = 15$$

$$\Rightarrow 256 \text{ mrs} = 256 \text{ M} = 2^8 \times 2^{20} = 28 \text{ bits}$$

$$\Rightarrow 16 \text{ mrs} = 2^4 \times 2^{30} = 34 \text{ bits}$$

\Rightarrow Let a sector in a city has 16 houses and 4 streets. Each street has 4 houses.

So, addressing of houses can be done either by giving any no. between 0 to 15 to each house. (i.e. 4 bit needed)

In this method, searching a house will be difficult.

So, in the 2nd method, if each street is given a no. and a house is given a no. in that street

So, 1st 2 bits can be used to give street no. and last 2 bits can be used to give house no. in a particular street. So, address of house no. 2 of 3rd street will be 10'01

/ \\\
Street no. house no.

In this method, searching a house will be easy.

Similar method can be used to generate logical and physical address.

Let, hard disk or ~~SM~~ is secondary memory (SM) is of size 16 MB and it is byte addressable.

So, $2^4 \times 2^{20} = 2^{24}$ bits will be needed to ~~give address~~ ^{give} each byte in addressing of ~~er~~ SM.

So, logical address (LA) generated by CPU will be of 24 bits.

But this i.e. to access an instruction from RAM, CPU will generate 24 bit of LA (which will be later converted into PA) but in this method searching will take time. So, we can divide this 24 bits in page no. and instruction within a page.

Let page size = 1 KB

So, 1K instructions in a page. So, 10 bits are needed to address each instruction in a page.

$$\text{No. of pages in SM} = \frac{16 \text{ MB}}{\frac{1 \text{ KB}}{2^{10}}} = \frac{2^4 \times 2^{20}}{2^{10}} = 2^{14}$$

So, 14 bits needs to give numbering to each page.

So, now 24 bits in 24 bits of page LA, 14 bits will be used to number a page and last 10 bits will be used to number an instruction in a page.

Now, let RAM or MM is of size 32 KB

$$\text{No. of pages} = \frac{32 \text{ KB}}{1 \text{ KB}} = 32$$

\therefore 5 bits will be used to number each page frame of MM.

10 bits will be used to number each instruction of a page.

So, physical address will be of ~~15 bytes~~. 15 bits where 14 bits will be used to number pages and last 10 bits will be used to number instructions.

Hence, LA = 33 bits long.

PA = 24 bit long

Page size = 2KB

$$\text{So, } 2^{33} \text{ bits} = 2^3 \times 2^{20} = 8 \text{ GB}$$

$$\text{Page size} = 2 \text{ KB} = 2^{11} \text{ B}$$

Ex - 11 bits needs to number indexions in a page.

$$\text{So, } 33 - 11 = 22 \text{ bits needs to number pages.}$$

PA = 24 bit long

$$= 2^4 = 2^4 \times 2^{20} = 16 \text{ MB}$$

- bits needed to number the frames = 24 - 11

$$= 13$$

In case of accessing an instruction from MM, MMU needs 1st to access the MM to find the page frame no. in the page table of the running process. Second time to actual access of the instruction. Accessing two times the MM is actually slows down the operation.

CPU generates LA which contains page no. and offset. During execution of a process, it since, execution of instructions of a program is sequential, it is highly probable that page no. in the LA generated by CPU for current instruction will be same as that of previous instruction. So, each time searching the same page no. in the page table is not needed can be avoided if a register hardware is maintained in the CPU which contains the page no. of and its corresponding frame no. (same as PageTable) for the previous instruction. This hardware is called translational lookaside buffer (TLB).

$$\Rightarrow \text{MM access time} = 400 \text{ ns}$$

$$\text{TLB} = 50 \text{ ns}$$

$$\text{Hit ratio of TLB} = 90\%$$

$$\therefore \text{Average memory access time} = ? \\ (\text{AMAT})$$

$$\text{In case of no TLB, AMAT} = 2 \times 400 = 800 \text{ ns}$$

$$\text{In case of TLB} = 0.9 [50 + 400] + 0.1 [50 + 2 \times 400]$$

$$= 0.9 \times 450 + 0.1 \times 850$$

$$= 490 \text{ ns}$$

$$S_1 \rightarrow \text{AMAT} = h [TLB + mm]$$

$$+ \\ (1-h) [TLB + 2 \cdot mm]$$

TLBs can hold data of the information of pages of one process at a time. In case of context switch, all TLB data will be erased and new page information of new process will start filling in the TLB. However, it will take some time to fill all the places of TLB with the ~~information~~ page information of new process. So, in case of frequent context switch, TLB miss will increase, so, AMAT will increase. As a side, multiple TLBs can be maintained so that in case of context switch, page information of old process will not be erased. In that case, when old process will again come to the running, no need to wait for TLB to fill, so, TLB miss will be reduced. However, multiple TLBs will increase the cost of the system.

Content switch also occurs when kernel goes to user mode to kernel mode. Here, again TLB miss will increase. So, in general, OS ~~task~~ goes to kernel mode for less time and again control is given to user mode. To reduce the TLB miss, smaller portion of the TLB can be reserved for OS so that TLB erase is not needed in case of switching from user to kernel mode.

$$\Rightarrow \text{mm} = 64 \text{ mB}$$

$$\text{LA} = 32 \text{ bits}$$

$$\text{Page Size} = 4 \text{ KB}$$

find total space wasted in maintaining page tables^{maximum}

Ans:- total no. of frames in mm = $\frac{\text{mm size}}{\text{Page size}}$

$$= \frac{64 \text{ mB}}{4 \text{ KB}}$$

$$= \frac{2^6 \times 2^{20}}{2^2 \times 2^{10}} = 2^{14}$$

∴ 14 bits required to address per frame no.

$$\text{Page Size} = 4 \text{ KB} = 2^2 \times 2^{10} = 2^{12} \text{ B}$$

∴ 12 bits required to address offsets
of a page.

A page table will look like

Page no. 0	frame no
page no 1	
Page no 2	
:	
:	

so, an entry in Page table will need 14 bits
 $\approx 2 \text{ byte}$

$$\text{total no. of pages in hard disk} = \frac{\text{LA}}{\text{Page size}}$$

$$= \frac{2^{32}}{2^2 \times 2^{10}} = 2^{20}$$

i.e. 20 bits required to address all pages.

Page No. _____	_____	_____
Date	_____	_____

∴ total no. of entries in all page tables = 2^{20}

total space wasted due to all page tables

$$= \text{total no. of entries in all page tables} \times \\ \text{size of an entry in a page table}$$

$$= 2^{20} \times 14 \text{ bits} = 2^{20} \times 2 \text{ bytes} \\ = 2 \text{ MB}$$

$$\Rightarrow MM = 256 \text{ MB}$$

$$\text{Page size} = 4 \text{ KB}$$

$$LA = 40 \text{ bits}$$

$$\text{Process size} = 4 \text{ MB}$$

Find total spaces wasted to maintain the page tables of this process?

$$\text{Ans. : no. of frames in MM} = \frac{256 \text{ MB}}{4 \text{ KB}} \\ = \frac{2^8 \times 2^{20}}{2^2 \times 2^{10}} = 2^{16}$$

∴ 16 bits are needed to address each frame.

$$\text{no. of Process Pages in Process} = \frac{4 \text{ MB}}{4 \text{ KB}} \\ = \frac{2^2 \times 2^{20}}{2^2 \times 2^{10}} = 2^{10}$$

∴ total wasted due to the given process = no. of pages in the process \times bits required for each page table entry

$$= 2^{10} \times 16 \text{ bits} = 2 \text{ KB}$$

\Rightarrow TLB access time = t

main memory access time = m

TLB miss rate = P

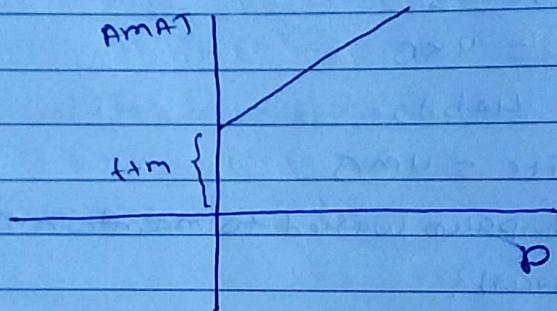
$$\text{AMAT} = P(t+m+t) + (1-P)(t+m)$$

$$= t + m + Pm$$

$$\text{AMAT} - Pm = t + m$$

$$Pm - \text{AMAT} = -(t+m)$$

$$\frac{P}{-(t+m)} + \frac{\text{AMAT}}{t+m} = 1$$



So, if P increases, AMAT increases.

minimum value of AMAT = $t+m$

Page table entries

As we know that an entry in page table contains a corresponding frame no. in mm. However, apart from frame no., some entries also exist in the more fields also exists in a particular entry of the page table.

- (a) Caching disabled :- Sometime, mm contains ~~more~~^{old} fresh information than Cache memory. e.g. if user is providing input in runtime, so, this input will go to mm, i.e. mm has ~~old~~^{more} fresh page in comparison to cache memory. So, in this case, Cache memory should be bypassed and directly mm should be accessed. So, in this case, caching for this particular page should be disabled.
- (b) referenced bit :- whether this page is referred in last clock cycle or not. It is used in case of demand paging (next chapter).
- (c) modified bit / dirty bit :- whether the page has been modified or not. If a page has been modified then the changes should also reflect in hard disk.
- (d) Present / absent bit :- whether the page looking in mm is present there or not. If not present then it is the case of page fault. In that case, the page will be brought from virtual memory to mm called as demand paging. During the time of loading the page from VM to mm, the process will be in wait state.
- (e) Protection :- page should be used in read-only or ~~rw~~^{ro}, or execute etc.
- (f) Page frame no. :-

for simplicity, we used 8 bit frame no. as a fixed entry field in page table.

The fields in Page table however may vary from OS to OS.

$$\Rightarrow \text{Logical address space} = 4 \text{ MB}$$

page size = 4 KB

then what should be page table entry size
so that page table will fit in one page

$$\text{Ans:-- Logical address space} = 4 \text{ MB}$$

$$= 2^2 \times 2^{20} \text{ B}$$

$$= 2^{22} \text{ B}$$

$$\text{Page size} = 4 \text{ KB}$$

$$= 2^2 \times 2^{10} \text{ B}$$

$$\therefore \text{no. of pages} = 2^{\frac{(22-12)}{2}} = 2^{10}$$

$$= 1 \text{ K}$$

Let the page table entry size is n

$$\therefore 1 \text{ K} * n \leq \text{pagesize}$$

$$n \leq \frac{4 \text{ KB}}{1 \text{ K}}$$

$$n \leq 4 \text{ Byte.}$$

- 5) i) Logical address space = 256 MB
 Page table entry size = 40

What should be page size so that page table should fit in one page or frame.

Ans: Let $n \rightarrow$ page size = P

$$\text{Pages} = \frac{256 \text{ MB}}{P} = \frac{2^8}{P}$$

$$\therefore \text{Page table size} = \frac{2^8 + 4}{P}$$

Page size \geq , Page table size

$$P \geq \frac{2^8 + 4}{P}$$

$$P^2 \geq 2^8 + 4$$

$$P^2 \geq 2^{10}$$

Multilevel Paging

Let Logical address = 32 bits

Page size = 4 KB

Physical address = 2^{44} bits

$$\text{no. of frames in RAM} = \frac{2^{32}}{4 \text{ KB}}$$

$$= \frac{2^{32}}{2^2 \times 2^{10}} = 2^{20}$$

here, out of 32 bits of LA, 12 bits are dedicated to address each byte of a page and 20 bits are dedicated to ~~2 bits~~ address each ~~frame~~ page.

Let Physical address = 2^{44} bits

$$\therefore \text{no. of frames in RAM} = \frac{2^{44}}{4 \text{ KB}}$$

$$= 2^{32}$$

∴ out of 44 bits, 12 bits for addressing bytes in a page (~~12 bits of offset~~) and 32 bits for addressing each frame in MM.

No. of entries in the page table is equal to no. of pages in the process or hard disk. $\approx 2^{22}$

$$= 2^{20}$$

g) we consider only frame no. as an entry in the page table then size of an entry in a page table
 $= \text{size bit } \log_2 (\text{no. of frames})$
 $= 32 \log_2 2^{32} = 32 \text{ bits} = 4 \text{ bytes.}$

Since, PT (page table) is also stored in the MM i.e. in pages. So, no. of pages required to store the PT will depend on PT Site.

$$\text{PT Site} = \text{no. of entries in PT} * \text{entry size}$$

$$= \text{no. of pages in the process} * 4 \text{ bytes}$$

$$= 2^{20} * 4 \text{ bytes} = 2^{20} * 2^2 \text{ bytes}$$

$$= 2^{22} \text{ bytes}$$

\therefore no. of ~~pages~~ ^{frames} needed to store the PT

$$= \frac{\text{PT Site}}{\text{Page Size}} = \frac{2^{22}}{4 \text{ KB}} = \frac{2^{22}}{2^2 \times 2^{10}} = 2^10$$

So, ~~1K~~ ^{frames} ~~Pages~~ are needed to store the page PT.

So, we again need a PPT to identify the page frame which is containing the portion of the actual PT where which will give the exact frame no. of the required byte.

~~so,~~ Since, no. of ~~frames~~ ^{pages} needed to store the PT is 2^{10} , so, we need a new PPT where in each entry, ~~it is the frame~~ frame no. corresponding to these pages will be given.

Since, ~~PT~~. Page size = 4 KB

So, in the new PT, 2^10 entries will be there.

Page No.		
Date		

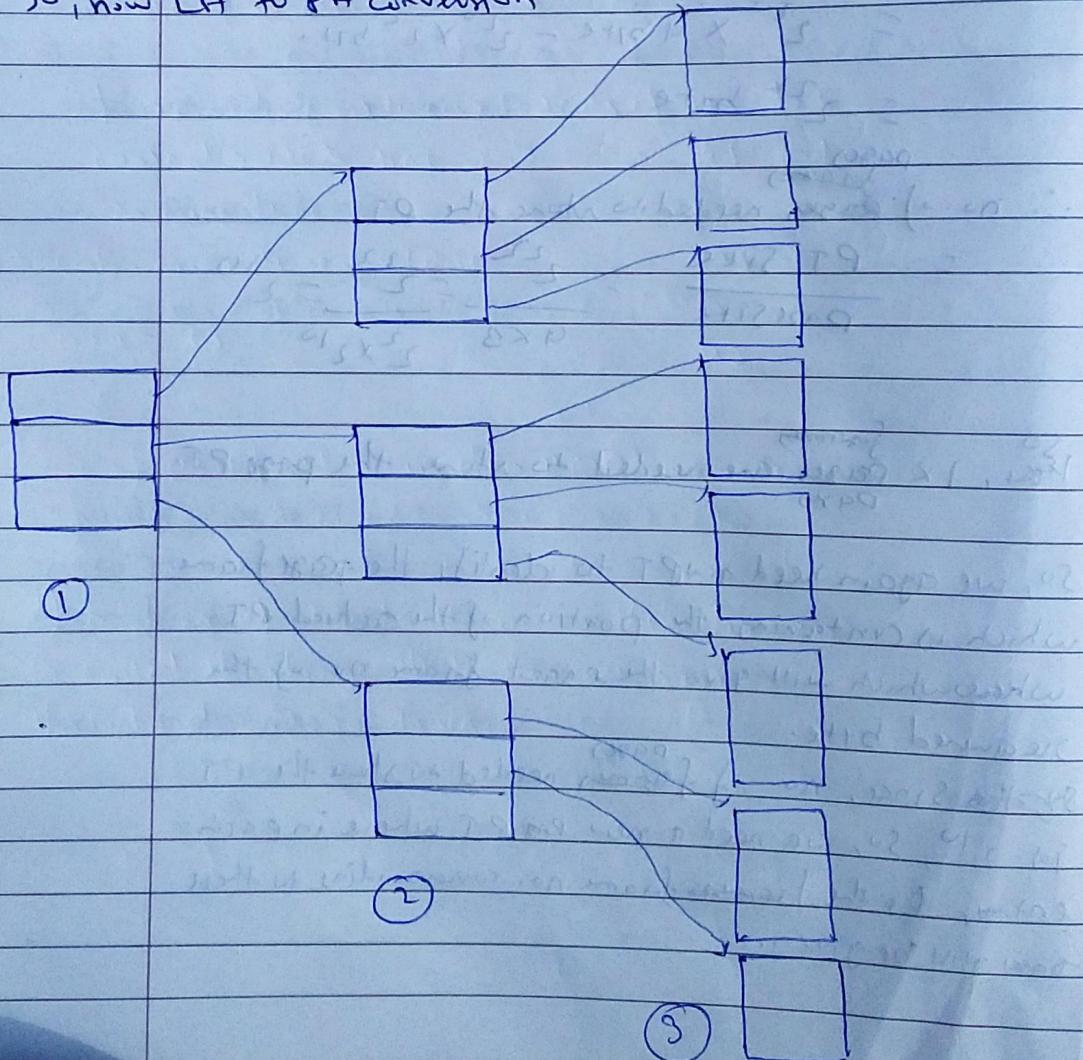
Each entry is of size 4 byte (Same as original PT)

$$\therefore \text{Space needed to store the new PT} = 2^{10} \times 4 \\ = 4 \text{ KB}$$

\therefore the new PT can be stored in 1 page itself.

If space needed to store the new PT is more than the Page size then we will need again one more level of pages i.e. one more PT.

So, now LA to PA conversion



So, page at level ① will give the page no. at level ②. It should be searched. At level ② actual page table PT is there. Now, page at level ② will give page no. at level ③ to reach to the actual required byte.

Since, no. of entries at page table in level ① is 2^{10} .

So, 10 bits are needed to search address each entry of the page at level ①.

Similarly, no. of entries at page in level ② is 2^{10} .

So, 10 bits are needed to address each entry of the page at level ②.

No. of entries stored in a page at level ③ is 2^{12} , where pages contain the actual instructions needed to be fetched by the CPU. 1-12 bits needed. So, 32 bit of L1A will be converted into $(10 + 10 + 12)$.

Here, no. of 44 bits comes after page to decide ~~the size of~~ the size of the entry in a page table i.e. size of an entry in Level ③ is a page at level ① and ②.

Address of the page at level ① will be stored in PTBR register.

Page fault

In the PS, one entry is that whether the required page is in the mm or not. If the bit is present/absent bit

If the required page is not in mm then it is called page fault.

As soon as Page fault occurs, a context switch will take place but now OS kernel will take the control to bring the required page from hard disk to mm. Now, when the required page is brought to the mm by OS, the control will be again given to the user program.

If memory access time is in terms of ns then context switch time is in - - ms and bringing these

time in bringing the required page - - - ms

again CS - - - ms

So, if page fault occurs then AMAT is used
- } ms.

If high no. - } page faults occurs then this situation is called thrashing. In this case, system throughput will go down.

9) If P is page fault rate

m is memory access time

t is service time or page fault time

then $A_{M A T} = P + t + (1-P)m$

$$A_{M A T} = P(t+m) + (1-P)m$$

$$\approx Pt + (1-P)m$$

Multi-level example

LA Page Page
size table
entry

48b 16KB 4B

No. of pages =

$$\text{Pagesize} = 16\text{KB} = 2^4 \times 2^{10}$$

$$\text{No. of Pages} = \frac{2^{48}}{2^{14}} = 2^{34}$$

$$\therefore \text{Size of Pagetables} = 2^{34} \times 2^2 \\ = 2^{36} \text{ B}$$

So, no. of pages required to store pagetables

$$= \frac{2^{36}}{2^{14}} = 2^{22}$$

$$\text{So, Size of Pagetable 2} = 2^{22} \times 4\text{B} \\ = 2^{22} \times 2^2 \\ = 2^{24} \text{ B}$$

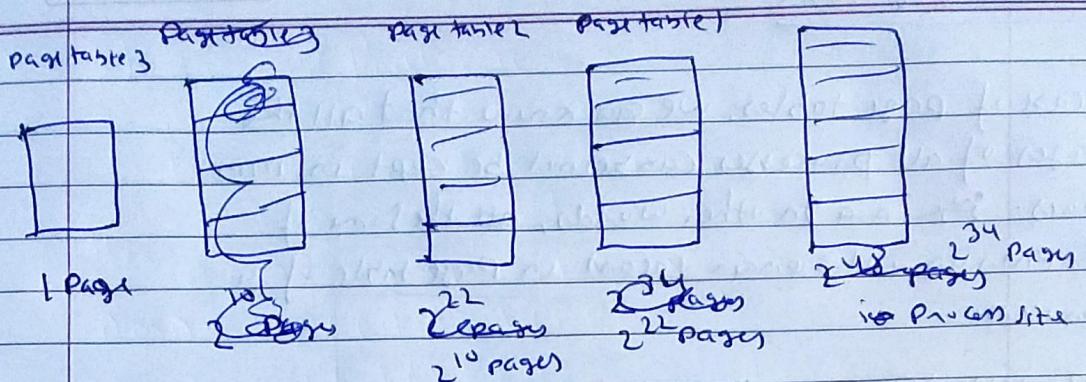
No. of pages required to store pagetables

$$= \frac{2^{24}}{2^{14}} = 2^{10}$$

$$\text{So, Size of Pagetable 3} = 2^{10} \times 4\text{B} \\ = 2^{13}$$

\therefore No. of pages required to store pagetables

$$= \frac{2^{13}}{2^{14}} = 1$$



In case of actual instruction, no. of entries in a page = 16 KB

$$\begin{aligned}
 &= 2^4 \times 2^{10} \\
 &= 2^{14} \\
 &= 14 \text{ bits}
 \end{aligned}$$

(1)

In case of storing page tables in page, no. of entries in a page = $\frac{16 \times 3}{4 \text{ byte}} = \frac{2^4 \times 2^{10}}{2^2} = 2^12 = 12 \text{ bits}$

No. of pages in the page table 2 is 2^{10} . So but in a page, no. of possible entries are 2^{12} . But since we only 2^{10} entries, so, we need 10 bits to address all pages of Page Table 2 in Page Table 3.

No. of pages in Page Table 1 are 2^2 . So, all each page of Page Table 2 will have 2^12 entries. So, 12 bits will be needed to address each page of Page Table 1 in Page Table 2.

Similarly, 2^{34} pages are there in Page Table 1. So, 12 bits will be needed to address each page of Page Table 3 in Page Table 2.

Finally, 14 bits are needed to address each byte in case of actual instruction in (1). So, 48 bits of LA will be distributed as follows:

10	12	12	14
----	----	----	----

Inverted Page Table

In case of page tables, we can know that all the pages of all processes cannot be kept in mm always. Meaning in other words, all the some of the entries will remain vacant in Page table of a process.

Page table of P_1

0	X
1	f_1
2	X
3	f_2
4	X
5	f_3

Page table of P_2

0	
1	
2	f_4
3	f_5
4	f_6
5	

So, Process P_1 , Pages no. 1, 3, 5 are present in mm.

P_1 - 2, 3, 4 - - - mm

So, here we can see that lots of space is being wasted in each page table.

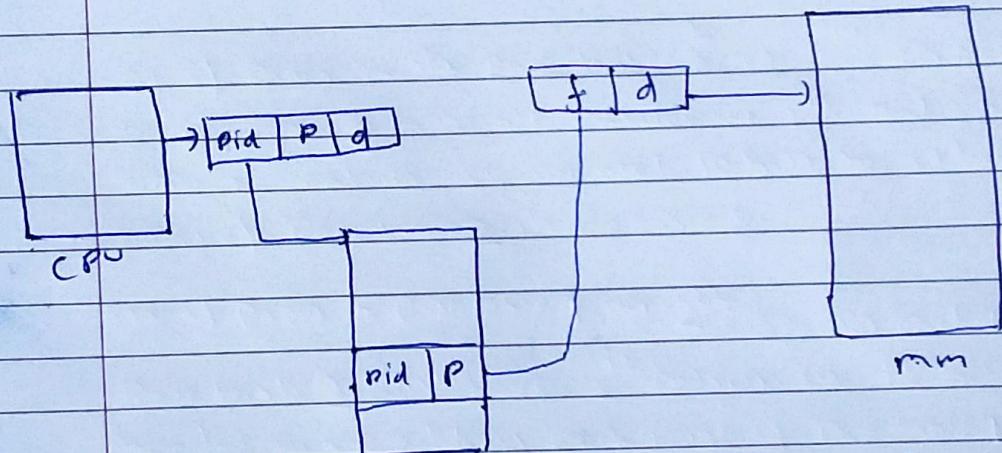
So, in place of having giving allotting 1 page table to each process, let us have only one page table for all the processes. No. of entries in this page table will be no. of frames in the mm.

Inverted page table

frame
no.

	frame no.	page no., Pid
1	1	P ₁
2	3	P ₁
3	5	P ₁
4	2	P ₂
5	3	P ₂
6	4	P ₂
7		
8		
9		

So, in inverted page table, for each frame no. there will be an entry in inverted page table which will contain Page no. and Process id.



- Although, this scheme decreases the amount of memory needed to store each page table, it increases the amount of time needed to search the table when a page reference occurs.
- Because the inverted page table is sorted by frame no., so, a linear search will be needed to search a (PId, P) in the table.