DIP Ch-5

Image Restoration

→ Reconstruct or recover an image
→ degraded using prior knowledge

→ Restoration → modelling the degradation

Applying inverse process → recover

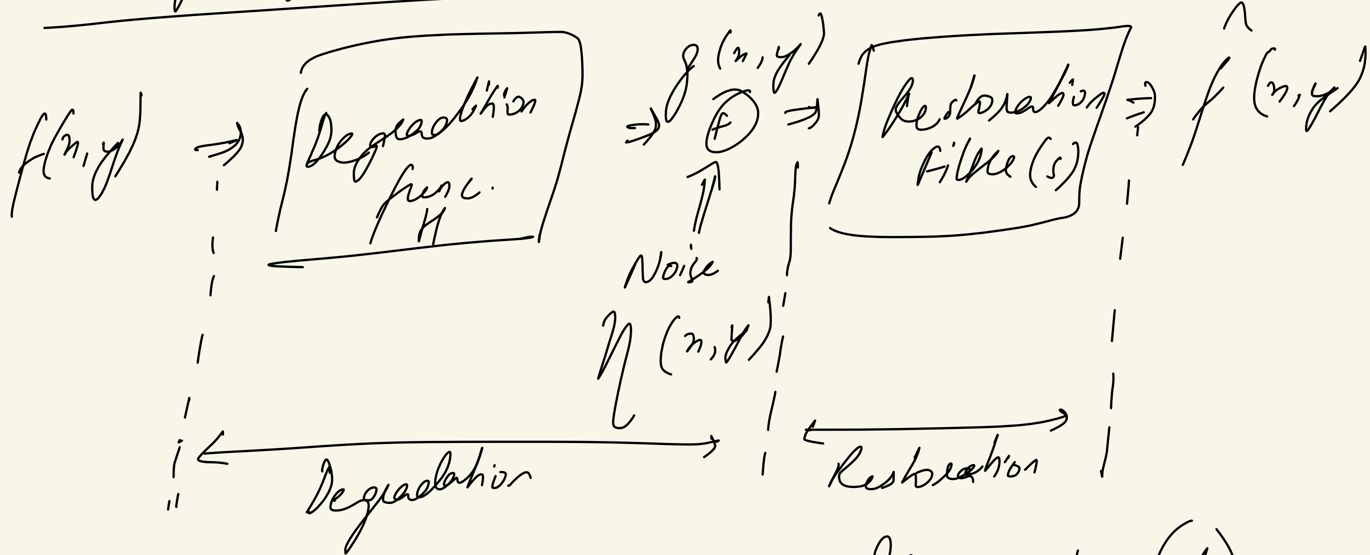
→ Restoration techniques

i) Spatial domain ii) frequency domain

Additive Noise
↳ spatial domain

Image blur ⇒ Degradation process
↳ difficult in spatial thus in freq. domain

"Model of Image Degradation/Restoration":



$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y) \quad \text{— (1)}$$

In freq domain (1) becomes

$$G(u,v) = H(u,v)\,F(u,v) + N(u,v) \quad \text{— (2)}$$

Let H → linear & position invariant process

then

Important Noise probability density fn. (PDF)

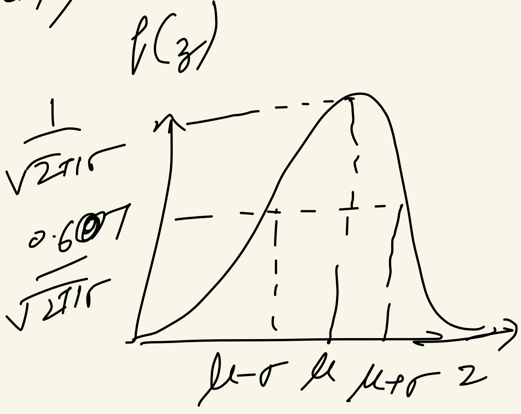
Noise → Image acquisition / transmission

→ Spatial properties of noise ⇒ spatial characteristics of noise

→ freq properties of noise ⇒ freq contents of noise

i) Gaussian noise [Normal noise model] ⇒

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$



z → gray level

μ → mean avg of z

σ → Std deviation

σ^2 → variance

due to electronic circuit noise, sensor noise, high temp.

70% of values $[(\mu-\sigma), (\mu+\sigma)]$

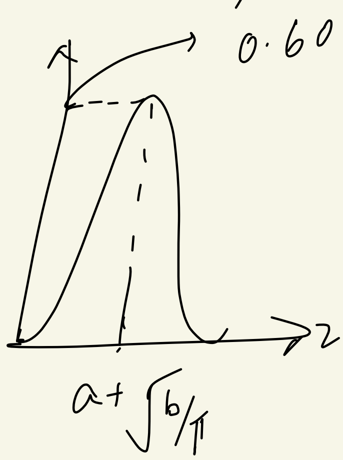
95% of values $[\mu-2\sigma, \mu+2\sigma]$

ii) Rayleigh Noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-(z-a)^2/b} & ; z \geq a \\ 0 & ; z < a \end{cases}$$

$$\mu = a + \sqrt{\pi b/4}$$

$$\sigma^2 = \frac{b(4 + \pi)}{4}$$

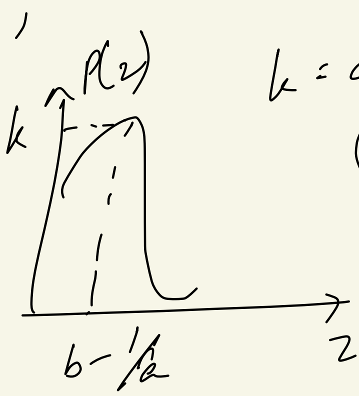


used in range imaging in digital cameras

iii) Erlang Gamma Noise

laser imaging

$$P(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & ; z \geq 0 \\ 0 & ; z < 0 \end{cases}$$



$$k = \frac{a(b-1)^{b-1}}{(b-1)!} e^{-(b-1)}$$

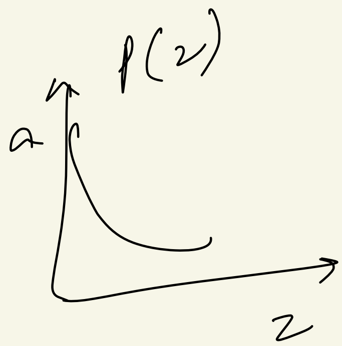
$\mu = b/a$

$\sigma^2 = b/a^2$

used in laser imaging

iv) Exponential noise

$$P(z) = \begin{cases} a e^{-az} & ; z \geq 0 \\ 0 & z < 0 \end{cases}$$



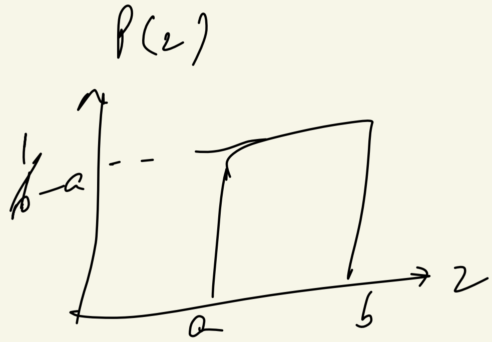
if $b = 0$
Erlang = exponential

$\mu = 1/a$

$\sigma^2 = 1/a^2$

v) Uniform noise

$$P(z) \begin{cases} \dfrac{1}{b-a} & a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

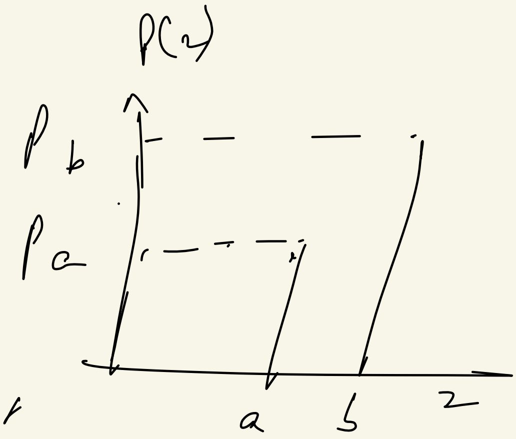


$\mu = \frac{a+b}{2}$

$\sigma^2 = \frac{(b+a)^2}{12}$

random no. generation

vi) Salt & pepper noise [Impulse noise]



$$P(z) = \begin{cases} P_a & ; z = a \\ P_b & ; z = b \\ 0 & ; \text{otherwise} \end{cases}$$

$b > a \Rightarrow$ gray level $b \rightarrow$ light dot

gray level $a \rightarrow$ dark dot

$P_a = 0$ or $P_b = 0 \rightarrow$ unipolar

$P_a \approx P_b \Rightarrow$ salt & pepper granules

shot & spike noise

$-$ve impulses $\rightarrow$ black (pepper) point

$+$ve impulses $\rightarrow$ white (salt) point

8 bit'd $a = 0$ (black)

$b = 255$ (white)

Periodic noise

→ Electrical or electromechanical interference
  ↳ Image acquisition

→ Spatially dependent noise

→ Reduced ⇒ Frequency domain filtering

Estimation : → Inspection of Fourier spectrum

→ Automated analysis → knowledge → general location of frequency components

→ If images systems are available → study characteristics of system noise → capture a set of images → "Flat" (uniformly illuminated)

→ use data from image strips → calculating → mean & variance of gray level.

→ Strip (subimage) → S

$$\mu = \sum_{z_i \in S} z_i \, p(z_i) \quad \text{— (1)}$$

$$\& \; \sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 \, p(z_i) \quad \text{— (2)}$$

z_i → gray level values of pixels 'S'

$p(z_i)$ → normalized histogram values

→ Histogram shape → closest PDF match

→ if the slope is Gaussian →
mean & variance

Restoration of image in presence of noise only

Spatial filtering $g(x,y) = f(x,y) \cancel{* h(x,y)} + \eta(x,y)$

Degradation → only due to noise (additive)

$$g(x,y) = f(x,y) + \eta(x,y)$$
$$\& \; G(u,v) = F(u,v) + N(u,v)$$

I. Mean filters

i) Arithmetic Mean filters → Simplest mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t) \quad - (1)$$

S_{xy} → set of coordinates in a rectangular subimage window of size $m \times n$ centered at point (x,y)

→ Avg of corrupted image $g(x,y)$ in the area defined by S_{xy}

→ Smoothens local variation

→ noise is reduced due to blurring

ii) Geometric mean filter :

$$\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{1/mn} \quad \text{--- ii}$$

- product of pixels in the subimage window → raised to power $1/mn$
- smoothing similar to arithmetic mean filter

→ lose less image details

iii) Harmonic mean filter :

$$\boxed{\hat{f}(x,y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}} \quad \text{--- iii}$$

→ works well for salt, but fails for pepper noise

→ also works well for Gaussian noise

iv) Contra harmonic mean filter:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

$Q \rightarrow$ order of the filter

→ Reducing the effect of salt & pepper noise

→ $Q \rightarrow +ve$: eliminates pepper noise

$Q \rightarrow -ve$: eliminates salt noise

→ cannot do both simultaneously

→ Reduces to arithmetic mean filter if $Q = 1$

Order Statistics filter

Restoration in presence of noise only

Spatial filtering:

II. Order statistics filters:

→ Response → ordering (ranking) the pixels

→ determined by the ranking results

i) Median filter:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}} \{g(s,t)\} \quad \text{—(i)}$$

→ replaces the value of pixel by median of gray levels

→ Excellent noise reduction with less blurring

→ effective in presence → bipolar & unipolar impulse noise

ii) Min & Max filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\max} \{g(s,t)\} \quad \text{— ii)}$$

(pepper noise ↓)

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\} \quad \text{(ii) Salt noise}$$

max → finding brightest point

min → finding darkest point

iii) Mid point filter:

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

iv

→ computes the mid point b/w max & min values

→ combines order statistic & averaging

→ works best → randomly distributed noise

iv) Alpha-trimmed mean filter

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

When $d=0$ → Reduced to Arithmetic filter

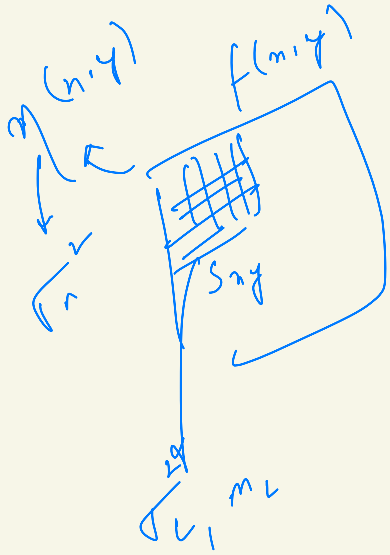
Remove highest $d/2$ pixels & lowest $d/2$ pixels → Remaining pixels $(mn-d)$

$d = \frac{mn-1}{2}$ → median filter

Other values → useful in ~~mul~~ multiple types of noise reduction

Adaptive filters

i) Adaptive, local noise reduction



mean → measure of avg gray level

& variance → measure of avg. contrast

filter → based on 4 quantities

a) $g(x,y)$ → value of noisy image (x,y)

b) σ_n^2 → variance of noise corrupting $f(x,y)$ to form $g(x,y)$

c) m_L → local mean of the pixels in S_{xy}

d) σ_L^2 → local variance of the pixels in S_{xy}

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_n^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

ii) Adaptive median filter

→ median filter works well only when the impulse noise is not large i.e P_a & P_b is less than 0.2

→ Adaptive median - can handle impulse noise of larger value

→ preserve details → smoothing non impulse noise

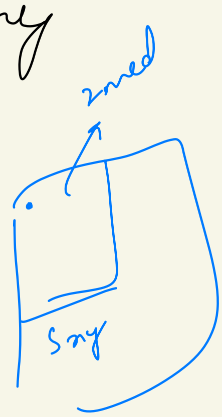
→ Let z_{min} = min value of gray level in S_{xy}

z_{max} = max

z_{med} = median

S_{max} = max allowed size of S_{xy}

→ Algorithm works in 2 levels



<u>Level A</u> : $A1 = z_{med} - z_{min}$

$A2 = z_{med} - z_{max}$

if $A_1 > 0$ & $A2 < 0$, go to level B

else increase window size

if window $\leq S_{max}$ repeat level A

else O/P z_{xy}

<u>Level B</u> : $B1 = z_{xy} - z_{min}$

$B2 = z_{xy} - z_{max}$

if $B1 > 0$ & $B2 < 0$; O/P z_{xy}

else O/P z_{med}

Advantages → remove salt & pepper noise

→ provide smoothing → reduce distortion

Periodic noise reduction using freq. domain filtering

i) Band reject filters :

→ Attenuate a band of frequencies above the origin of the F.T

Ideal

$$H(u,v) = \begin{cases} 1 \; ; & D(u,v) < D_0 - W/2 \\ 0 \; ; & D_0 - W/2 \le D(u,v) \le D_0 + W/2 \\ 1 \; ; & D(u,v) > D_0 + W/2 \end{cases} \quad \text{—} (1)$$

region over which band reject filter works

$D(u,v)$ → distance from the origin of the centered freq. band

W → width of the band

D_0 → radial center / cut off freq.

Similarly Butterworth : —(2)

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)\,W}{D^2(u,v) - D_0^2} \right]^{2n}}$$

$2n$ ⇓ order of filter

Gaussian

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)\,W}\right]^2} \quad (3)$$

ii) Bandpass filter

→ opposite operation of a bandreject

$$H_{bp}(u,v) = 1 - H_{br}(u,v) \quad \text{— (1)}$$

iii) Notch filters :

→ passes frequencies in predefined neighborhoods

Ideal notch Reject

radius D_0 & centres at (u_0, v_0) with symmetry at $(-u_0, -v_0)$ is

$$H(u,v) = \begin{cases} 0 ; & D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0 \\ 1 ; & \text{Otherwise} \end{cases} \quad \text{— (1)}$$

$$\text{where } D_1(u,v) = \left[\left(u - M/2 - u_0\right)^2 + \left(v - N/2 - v_0\right)^2\right]^{1/2}$$

$$\& \; D_2(u,v) = \left[\left(u - M/2 + u_0\right)^2 + \left(v - N/2 + v_0\right)^2\right]^{1/2}$$

Butterworth

$$H(u,v) = \frac{1}{1 + \left[\dfrac{D_0^2}{D_1(u,v)\, D_2(u,v)}\right]^n} \quad \text{— (2)}$$

Gaussian

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D_1(u,v)\, D_2(u,v)}{D_0^2}\right]} \rightarrow 3$$

High pass filter $\rightarrow$ $u_0 = v_0 = 0$

$$H_{np}(u,v) = 1 - H_{nr}(u,v) \rightarrow 4$$

np $\rightarrow$ notch pass

nr $\rightarrow$ notch reject