Machine Learning

Lect. 6

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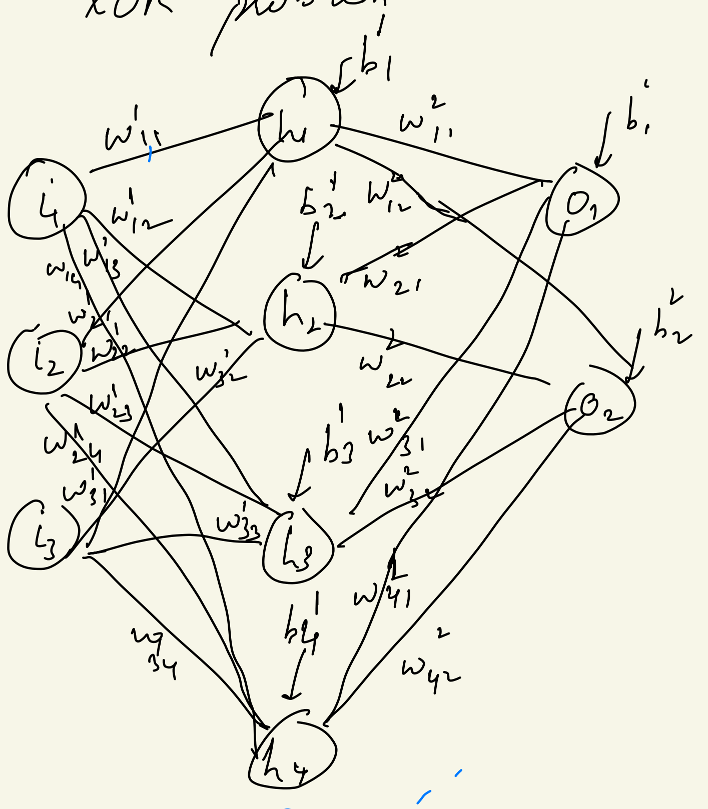
Multi Layer Perceptron

XOR problem

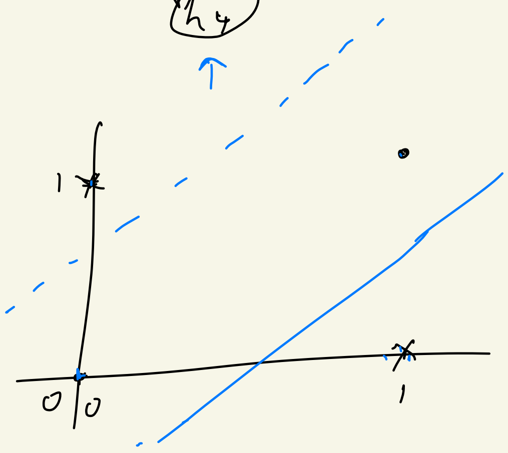


$$h_n = a\left(\sum_{k=1}^{3} i_k w_{kn} + b_n\right)$$

$a(x) = \text{sigmoid}(x) = \dfrac{1}{1+e^{-x}}$

Universal approximation theorem

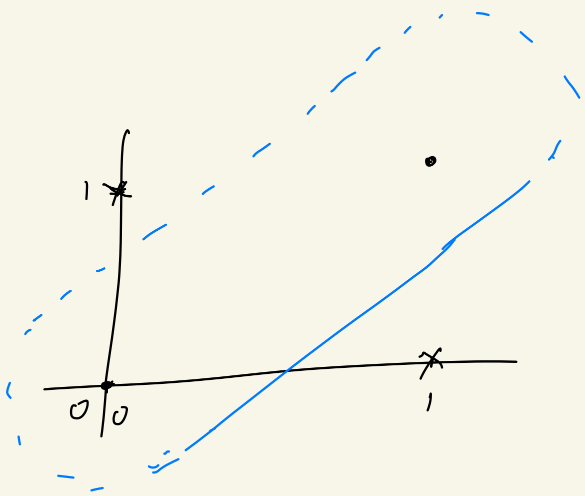
x_1	x_2	O/P y
0	0	0
0	1	1
1	0	1
1	1	0



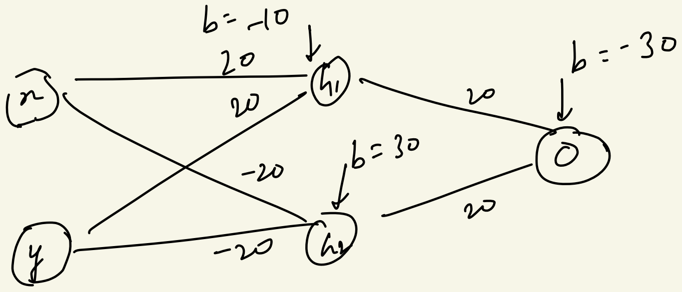
How to seperate this with a linear boundary

Perceptron can't solve this

but MLP can!



⇒ Non linear seperation

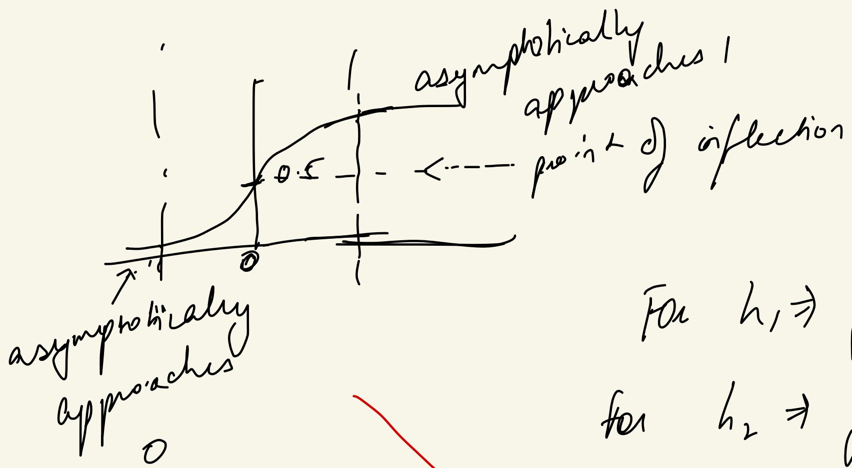


x_1	x_2	O/P $\hat{y}$
0	0	0.000045
0	1	0.99995
1	0	0.9995
1	1	0.000045

$h_1 = \text{sigmoid}(20x + 20y - 10) = 0.5$

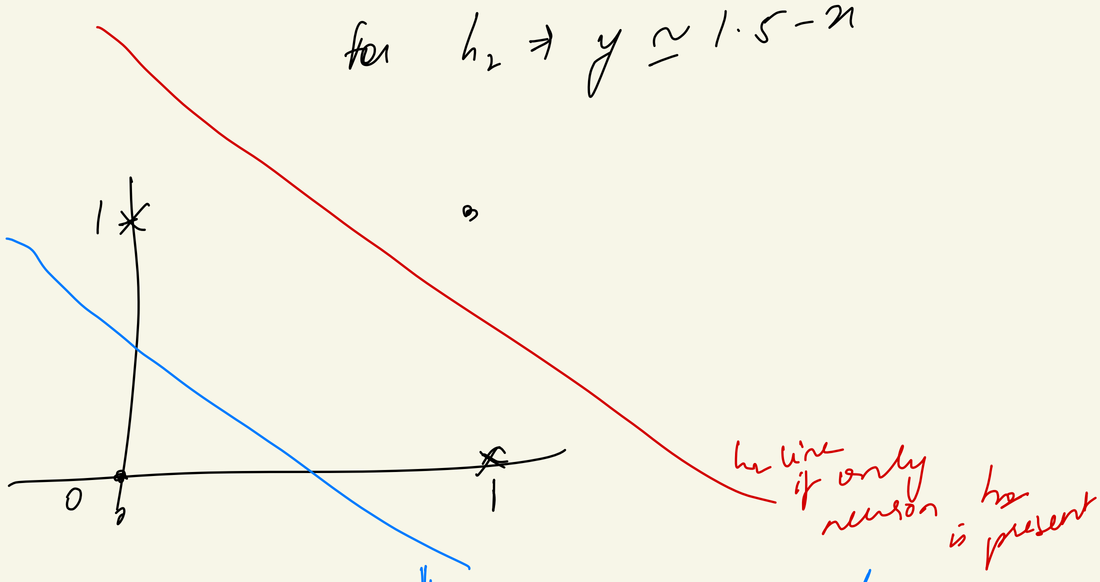
$h_2 = \text{sigmoid}(-20x - 20y + 30) = 0.5$

$O = 20h_1 + 20h_2 - 30$



For $h_1 \Rightarrow y \simeq 0.5 - x$

for $h_2 \Rightarrow y \simeq 1.5 - x$



then 2 class division

with only these given set of weights you can learn such complex XOR functions

But how to learn these weights?

↓

Soln: Backpropagation

$$\sigma(20*0+20*0-10) \approx 0$$

$$\sigma(20*1+20*1-10) \approx 1$$

$$\sigma(20*0+20*1-10) \approx 1$$

$$\sigma(20*1+20*0-10) \approx 1$$

$$\sigma(-20*0-20*0+30) \approx 1$$

$$\sigma(-20*1-20*1+30) \approx 0$$

$$\sigma(-20*0-20*1+30) \approx 1$$

$$\sigma(-20*1-20*0+30) \approx 1$$

$$\sigma(20*0+20*1-30) \approx 0$$

$$\sigma(20*1+20*0-30) \approx 0$$

$$\sigma(20*1+20*1-30) \approx 1$$

$$\sigma(20*1+20*1-30) \approx 1$$