Computer Graphics

3D Transformations

Shearing 3D

Shearing in X direction $\rightarrow Sh_x$

" " Y " $\rightarrow Sh_y$

" " Z " $\rightarrow Sh_z$

$$O(x, y, z) \rightarrow \underset{x_{new}}{x'}, \underset{y_{new}}{y'}, \underset{z_{new}}{z'}$$

Shearing in X axis

$$x_{new} = x$$

$$y_{new} = y + Sh_y\, x$$

$$z_{new} = z + Sh_z\, x$$

3D Shearing matrix $\Rightarrow$ (in X axis)

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ Sh_y & 1 & 0 & 0 \\ Sh_z & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Shearing matrix in Y axis

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & Sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Shearing matrix in Z axis

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & Sh_x & 0 \\ 0 & 1 & Sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3) Shearing matrix

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{bmatrix} 1 & Sh_x & Sh_x & 0 \\ Sh_y & 1 & Sh_y & 0 \\ Sh_z & Sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Q1 $(1, 2, 3) \longrightarrow$ find new coordinates

$R_x = 90^\circ$

$R_y = 90^\circ$

$R_z = 90^\circ$

$R_X = x_{new} = x_{old} = 1$

$y_{new} = \underset{2}{y} \cos 90^\circ - \underset{3}{z} \sin 90^\circ$

$= 2 \times 0 - 3 = -3$

$z_{new} = y \times \sin 90^\circ + 3 \cos 90 = 2$

R_y $x_{new} = 3 \sin 90 + 1 \cos 90 = 3$

$y_{new} = 2$

$z_{new} = 2 \cos 90 - \sin 90 = -1$

$R_z =$ $x_{new} = 1 \cos 90 - 2 \sin 90 = -2$

$y_{new} = 1 \sin 90 + 2 \cos 90 = 1$

$z_{new} = 3$

Q2 Shearing

triangle A 0, 0, 0

B 1, 1, 2

C 1, 2, 3

$Sh_x = 2$ $Sh_y = 2$ $Sh_z = 3$

Rotation about an arbitrary axis in space

Assume we want to perform rotation about an axis in space by θ degrees passing through the point (x_0, y_0, z_0) and direction cosines (c_x, c_y, c_z)

1. Translate by $[T] = (x_0, y_0, z_0)^T$ & bring to origin
2. Next we rotate the axis into one of the principal axes say z axis i.e rotation w.r.t x axis & y-axis

   $z\,([R_x], [R_y])$
3. We rotate by θ degrees in z $(R_z(\theta))$
4. Then we undo the rotations to align the axis
5. We undo translation

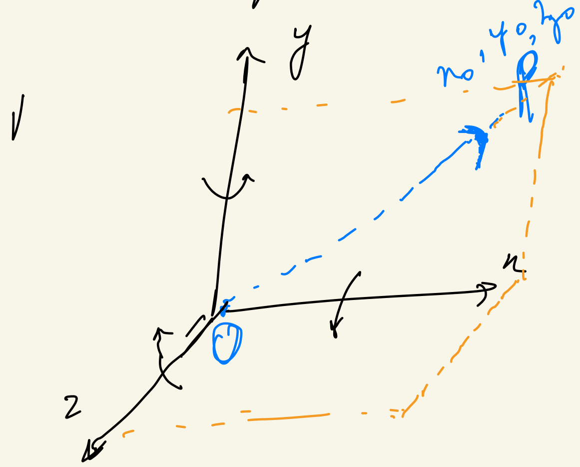
Now step 2 will take 2 rotation

i) about x axis

   to place the axis in xz plane

ii) about y axis

   to place the axis is coincide with z-axis



along z axis

$z' = z$

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

x axis

$x' = x$

$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

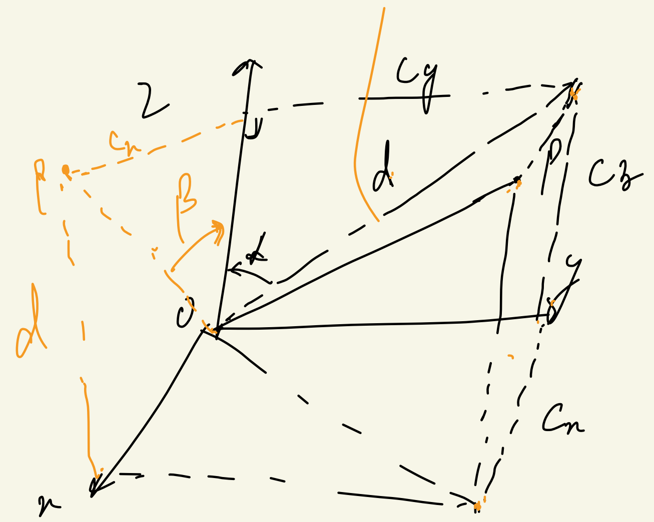
y axis

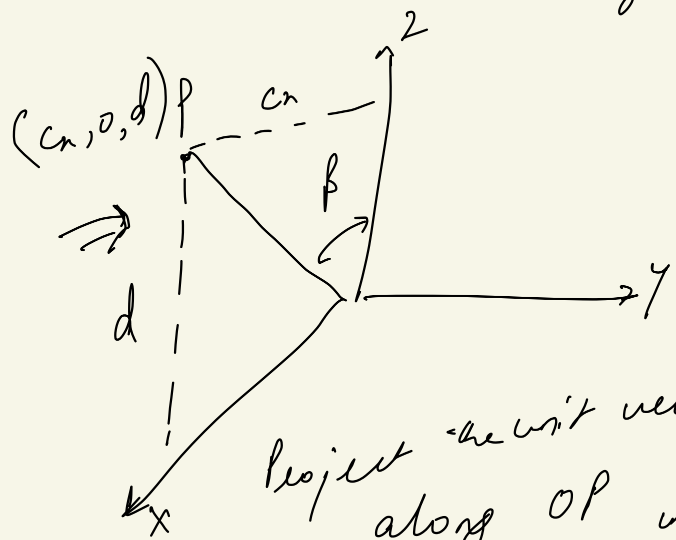
$y' = y$

$x' = \cancel{z}\, x\cos\theta - z\sin\theta$ (written as: $= z\cos\theta - x\sin\theta$)

$z' = z\sin\theta + \cancel{x}\, x\cos\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$





Project the unit vector along OP into YZ plane.

The y & z component c_y & c_z are the directional cosines of the unit vector along the arbitrary axis

$d = \sqrt{c_y^2 + c_z^2}$

$\cos\alpha = \dfrac{c_z}{d}$ $\qquad \alpha = \sin^{-1}\left(\dfrac{c_y}{\sqrt{c_y^2+c_z^2}}\right)$

$\sin\alpha = c_y/d$

Rotate about y axis by angle β

x component is c_x &

z component is d

$$\cos\beta = \frac{d}{\text{length of unit vector}}$$

$$\sin\beta = \frac{c_x}{\text{length of unit vector}}$$

Final transformation for 3D rotation about arbitrary axis

$$M = |T|\,|R_x|\,|R_y|\,|R_z|\,|R_y|^{-1}\,|R_x|^{-1}\,|T|^{-1}$$

$$|T| = \begin{pmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$|R_x| = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_z/d & -c_y/d & 0 \\ 0 & c_y/d & c_z/d & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$|R_y| = \begin{pmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} d & 0 & -c_x & 0 \\ 0 & 1 & 0 & 0 \\ c_x & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$|R_z| = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$|T|^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -x_0 & -y_0 & -z_0 & 1 \end{pmatrix}$$

$$[R_x]^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_z/d & c_y/d & 0 \\ 0 & -c_y/d & c_z/d & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_y^{-1} \begin{pmatrix} d & 0 & c_x & 0 \\ 0 & 1 & 0 & 0 \\ -c_x & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3D viewing pipeline

Modeling coordinate → [Modeling Transformation] → World coordinates → [Viewing Transformation] → View coor. → [Projection Trans.] → Projection coor. → [Viewport Transformation] → Device coor.

Modeling Transformation: making ppl sit for photograph

Viewing Transformation: camera set; acc. to camera

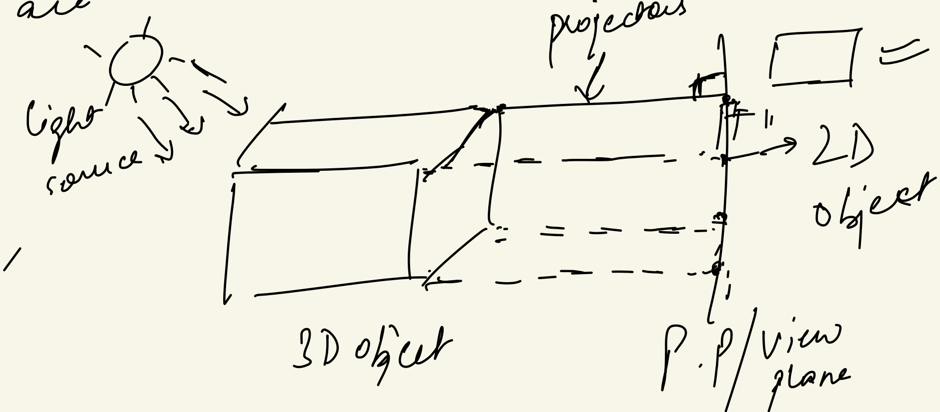
Projection Trans.: focus

Projection

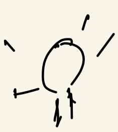
We represent a 3D object on a 2D plane

When geometric objects are formed by the intersection of lines with a plane, the plane is called the projection plane & the lines are called projectors

Center of projection : is an arbitrary point from where the lines are drawn on each point of an object



$3D \to 2D$

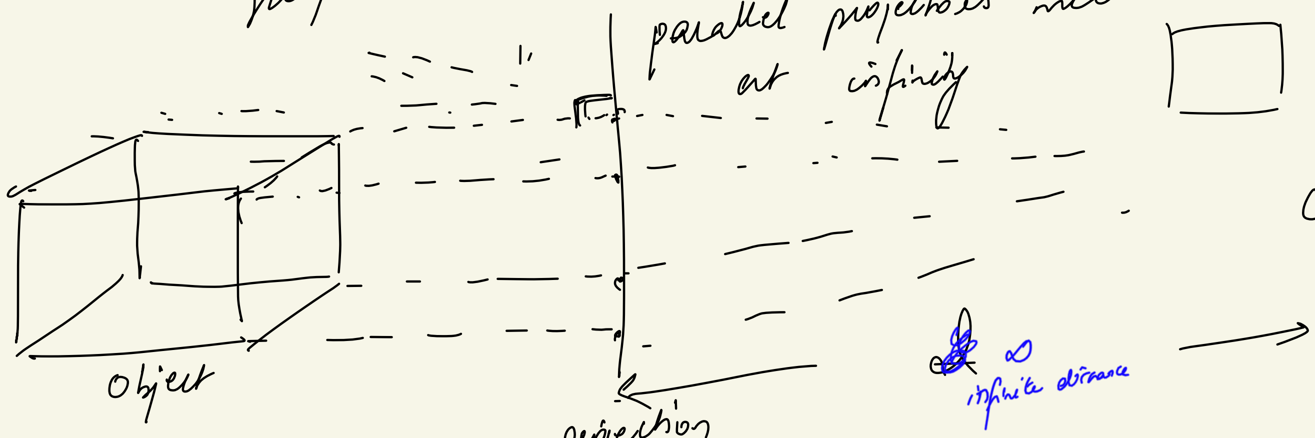


Projection

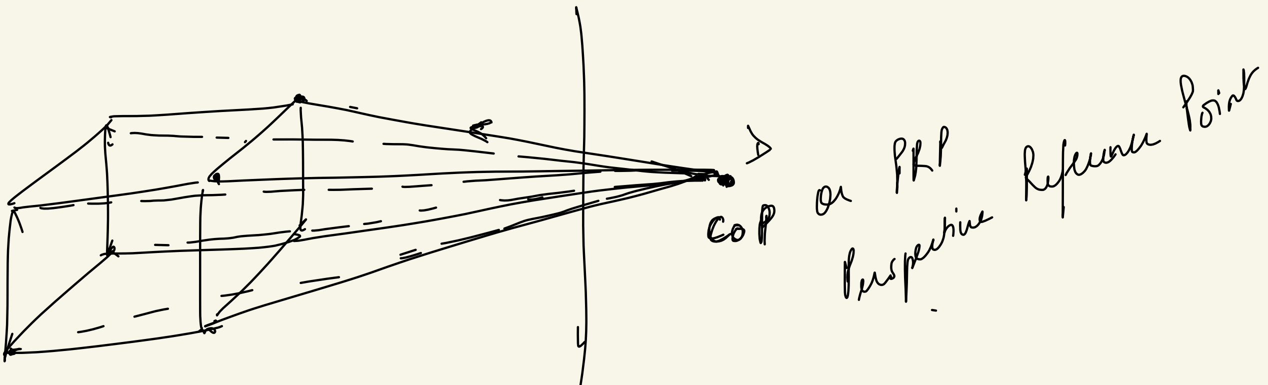
- Parallel
  - Oblique
    - Cavalier
    - Cabinet
  - Orthographic (90° with view plane)
    - Multiview (2D)
      - Top
      - side
      - front view
    - Axonometric
      - Isometric
      - Dimetric
      - Trimetric
- Perspective
  - One point
  - 2 "
  - 3 point

→ if COP is located at a finite point in 3D space perspective projection

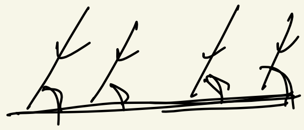
→ if COP is located at an infinite point line are parallel then result is parallel projection.



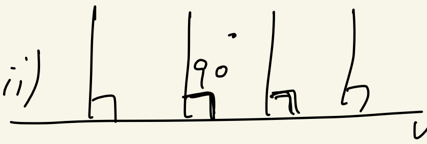
Parallel projection



Perspective projection

i)  view plane

Oblique ≡

ii) 

view plane

≡ orthographic

Perspective projection : Distance from the COP to projection plane is finite. The projectors are not parallel and we specify a centre of projection COP.

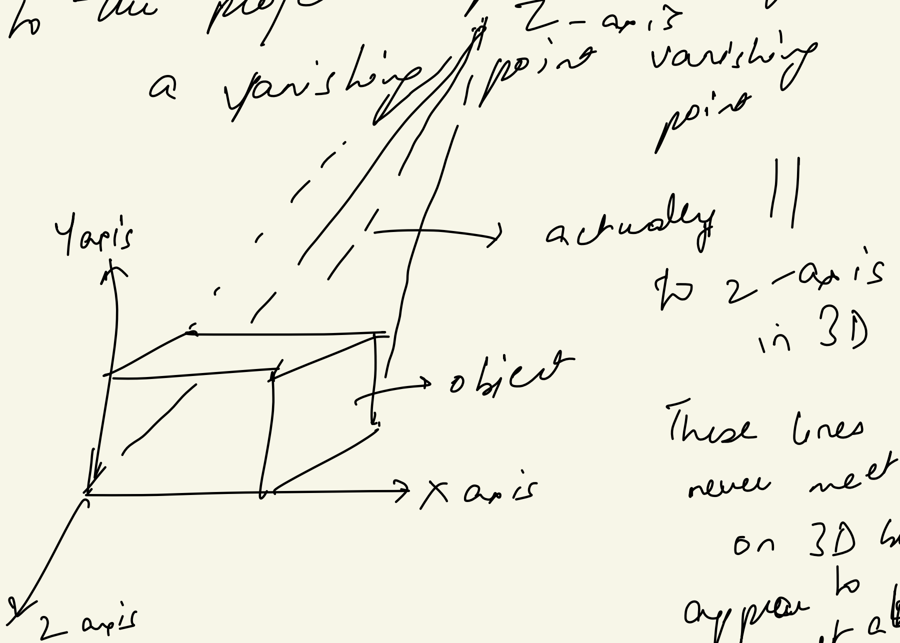
i) One point Perspective : In this, exactly one principal axis has a finite vanishing point.

ii) Two point : exactly 2 principal axes have the vanishing points

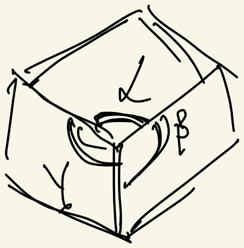
iii) Three point : exactly 3 principal axes have finite vanishing point.

* Perspective foreshortening : The size of the perspective projection of the object varies inversely with distance of the object from the centre of projection

Vanishing point : The perspective projections of any set of parallel lines that are not parallel to the projection plane converge to a vanishing point



These lines never meet on 3D but appear to meet at a point



$\alpha = \beta = \gamma$ Isometric

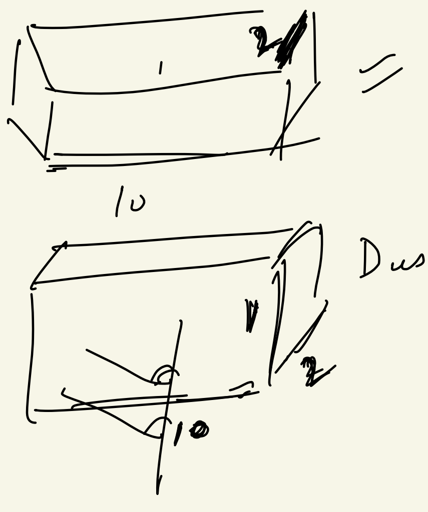
$\alpha = \beta \neq \gamma$] Dimetric
$\alpha \neq \beta = \gamma$

$\alpha \neq \beta \neq \gamma$] Trimetric

Oblique projections used in curved / spherical surfaces to
obtain most fea

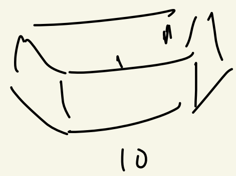
Cavalier

[30-45°], actual
shape/ratio
retained



Cabinet

63.4° → 1/2



Duster