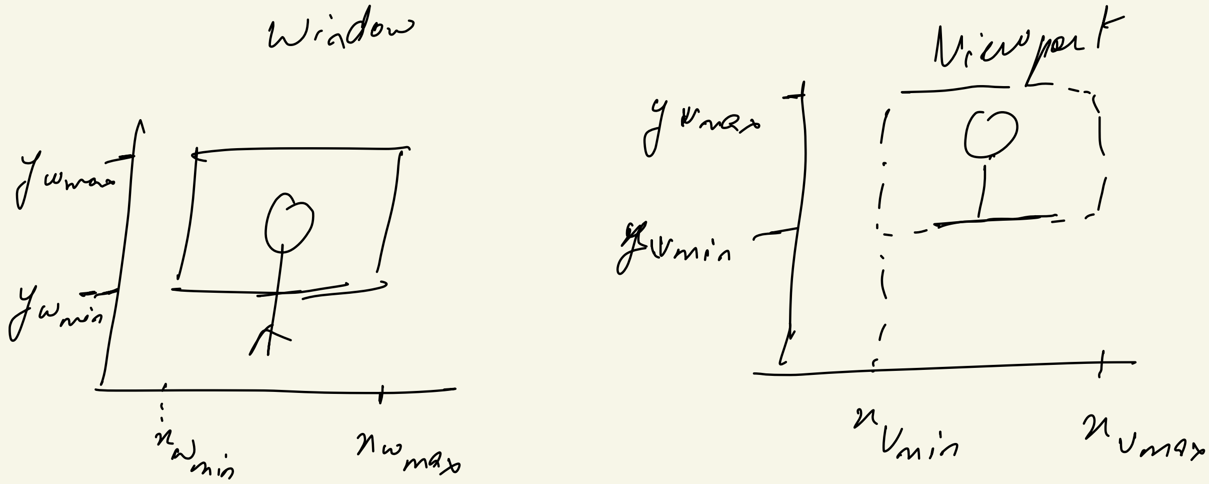
Computer Graphics

2D Transformations

Window to Viewport Transformation in CG

It is the process of transforming a 2D world coordinate objects to device coordinates. Objects inside the world or clipping window are mapped to the viewport which is the area of the screen where world coordinates are mapped to be displayed



window coordinates

$[x_{wmin}, x_{wmax}, y_{wmin}, y_{wmax}]$

Device coordinates

$[x_{vmin}, x_{vmax}, y_{vmin}, y_{vmax}]$

Window $\longrightarrow$ x_w, y_w $\longrightarrow$ Viewport x_v, y_v

Given ??

Normalized point on Window $\Rightarrow$ $\dfrac{x_w - x_{wmin}}{x_{wmax} - x_{wmin}}, \dfrac{y_w - y_{wmin}}{y_{wmax} - y_{wmin}}$

Normalized point on Viewport $\Rightarrow$ $\dfrac{x_v - x_{vmin}}{x_{vmax} - x_{vmin}}, \dfrac{y_v - y_{vmin}}{y_{vmax} - y_{vmin}}$

Assumption $\Rightarrow$ Relative position of object in window & viewport are same

For x coordinate :

$$\frac{x_w - x_{wmin}}{x_{wmax} - x_{wmin}} = \frac{x_v - x_{vmin}}{x_{vmax} - x_{vmin}} \quad \text{— (1)}$$

for y coordinate

$$\frac{y_w - y_{wmin}}{y_{wmax} - y_{wmin}} = \frac{y_v - y_{vmin}}{y_{vmax} - y_{vmin}} \quad -② $$

From ①

$$x_v = \frac{x_w - x_{wmin}}{x_{wmax} - x_{wmin}} (x_{vmax} - x_{vmin}) + x_{vmin}$$

complementation

$$\boxed{x_v = (x_w - x_{wmin}) S_x + x_{vmin}}$$

From ②

$$y_v = y_{vmin} + (y_w - y_{wmin}) S_y$$

Scaling factor of x coordinate $S_x = \frac{x_{vmax} - x_{vmin}}{x_{wmax} - x_{wmin}}$

Scaling factor of y coordinate $S_y = \frac{y_{vmax} - y_{vmin}}{y_{wmax} - y_{wmin}}$

Ques

Window $x_{wmin} = 20$, $x_{wmax} = 80$, $y_{wmin} = 40$, $y_{wmax} = 80$

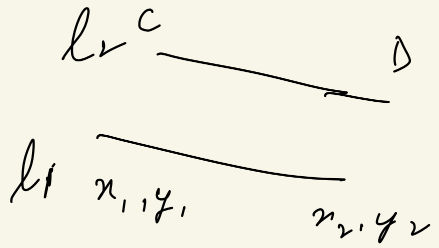
Viewport $x_{vmin} = 30$, $x_{vmax} = 60$, $y_{vmin} = 40$, $y_{vmax} = 60$

$x_w, y_w = 30, 80$

↓

x_v, y_v ??

Transformation of Parallel Lines



$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m' = O\, m$$

$$x' = T x$$

$$= \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}$$

$$= \begin{bmatrix} ax_1 + cy_1 & ax_2 + cy_2 \\ bx_1 + dy_1 & bx_2 + dy_2 \end{bmatrix}$$

$$m' = \frac{(bx_2 + dy_2) - (bx_1 + dy_1)}{(ax_2 + cy_2) - (ax_1 - cy_1)}$$

$$\boxed{m' = \frac{b + dm}{a + cm}}$$

$$= \frac{b(x_2 - x_1) + d(y_2 - y_1)}{a(x_2 - x_1) + c(y_2 - y_1)}$$

$$m' = \frac{b + d\left(\frac{y_2 - y_1}{x_2 - x_1}\right)}{a + c\left(\frac{y_2 - y_1}{x_2 - x_1}\right)} = \frac{b + dm}{a + cm}$$

Rotation about arbitrary plane ??

Ques Show that following transformation matrix gives a pure rotation

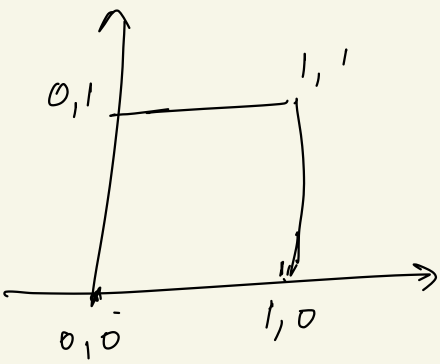
i) det T = 1

ii) $T^{-1} = T'$

$$T = \begin{pmatrix} \dfrac{1-t^2}{1+t^2} & \dfrac{2t}{1+t^2} \\[2ex] \dfrac{-2t}{1+t^2} & \dfrac{1-t^2}{1+t^2} \end{pmatrix} \Rightarrow$$

Ques A unit square is transformed by a 2 × 2 transformation matrix. The resulting position vector ⇒

$$\begin{bmatrix} 0 & 0 \\ 2 & 3 \\ 8 & 4 \\ 6 & 1 \end{bmatrix} = \underset{?}{T} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$



What is T? $\begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$

Ques Show that shear transformation in x & y direction together is not the same as shear along x followed by shear along y.

$$\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix} \neq \begin{bmatrix} 1+bc & b \\ c & 1 \end{bmatrix}$$

Shear in x . Shear in y = Shear along x & y

Ques Consider a triangle whose vertices are (2,2), (4,2) & (4,4). Find the concatenated transformation matrix & transformed vertices for rotation of 90° about origin followed by reflection through the line $y = -x$.

Also comment on sequence on transformations.