Lectures 1-3
Course code:TBA

Digital Image Processing

Gonzalez, R. C. and Woods, R. E., "Digital Image Processing", Prentice Hall, 3rd Ed.
Jain, A. K., "Fundamentals of Digital Image Processing", PHI Learning, 1st Ed.
Bernd, J., "Digital Image Processing", Springer, 6th Ed.
Burger, W. and Burge, M. J., "Principles of Digital Image Processing", Springer
Scherzer, O., " Handbook of Mathematical Methods in Imaging", Springer

Image Acquisition Process

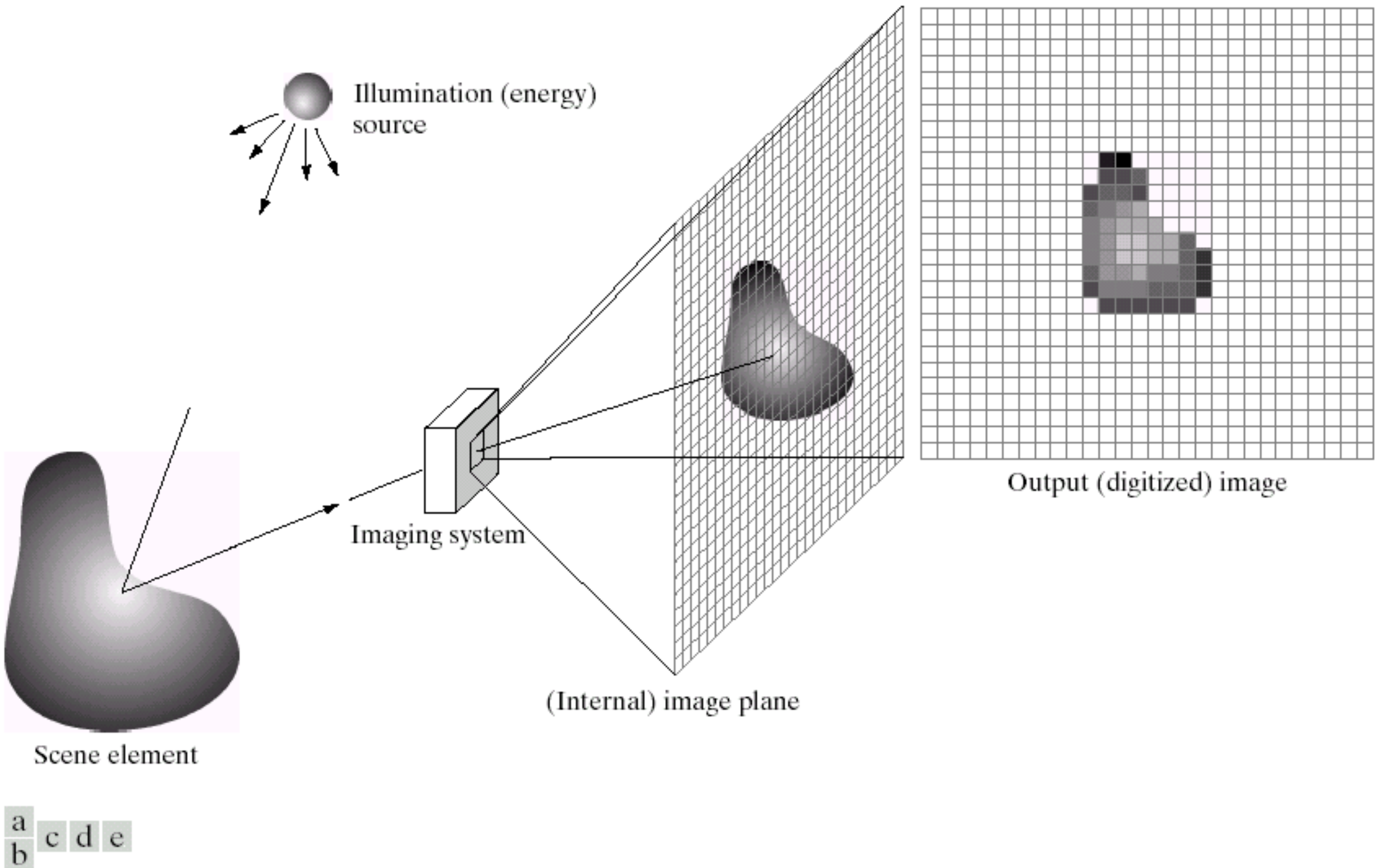


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Introduction

- What is Digital Image Processing?

Digital Image

— a two-dimensional function $f(x, y)$

x and y are spatial coordinates

The amplitude of f is called **intensity** or **gray level** at the point (x, y)

Digital Image Processing

— process digital images by means of computer, it covers low-, mid-, and high-level processes

low-level: inputs and outputs are images

mid-level: outputs are attributes extracted from input images

high-level: an ensemble of recognition of individual objects

Pixel

— the elements of a digital image

A Simple Image Formation Model

$* r(x,y)$

$$f(x,y) = i(x,y)\,\boxed{\cancel{?}}\cancel{(x,y)}$$

$f(x,y)$: intensity at the point (x,y)

$i(x,y)$: illumination at the point (x,y)

(the amount of source illumination incident on the scene)

$r(x,y)$: reflectance/transmissivity at the point (x,y)

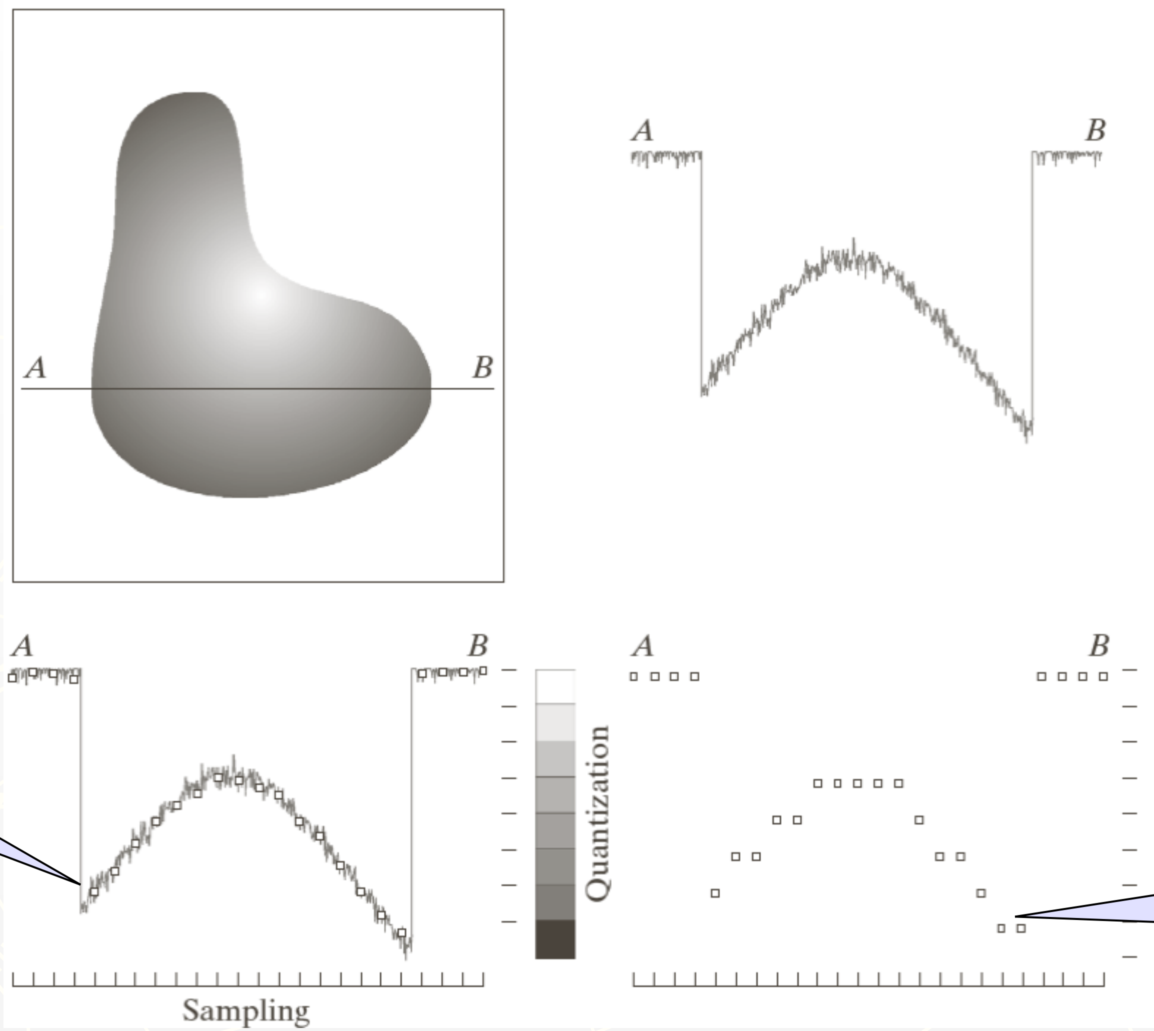
(the amount of illumination reflected/transmitted by the object)

where $0 < i(x,y) < \infty$ and $0 < r(x,y) < 1$

Some Typical Ranges of Reflectance

- Reflectance
  - 0.01 for black velvet
  - 0.65 for stainless steel
  - 0.80 for flat-white wall paint
  - 0.90 for silver-plated metal
  - 0.93 for snow

Image Sampling and Quantization



a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Digitizing the coordinate values

Digitizing the amplitude values

Image Sampling and Quantization

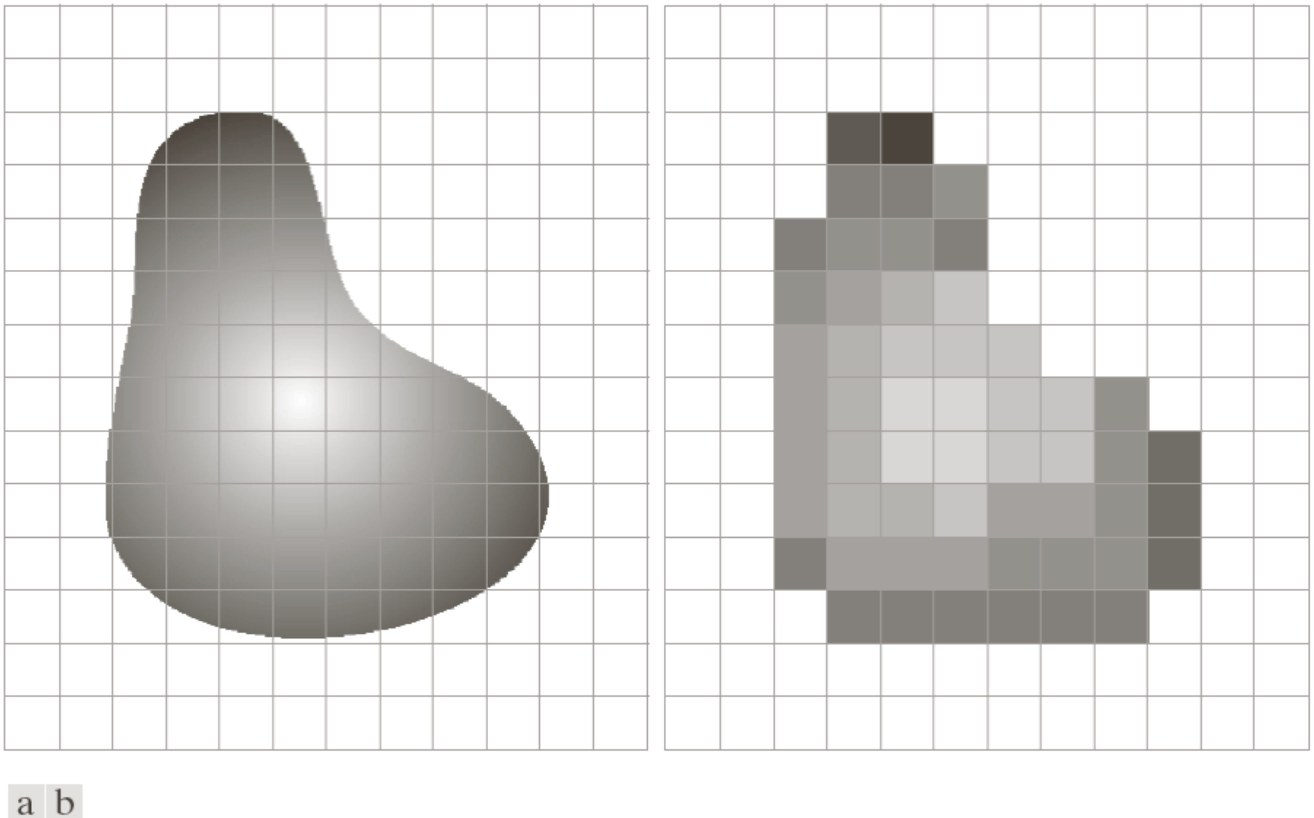


FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Representing Digital Images

- The representation of an M×N numerical array as

$$f(x,y)=\begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \dots & \dots & \dots & \dots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}$$

Representing Digital Images

- The representation of an M×N numerical array as

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \cdots & \cdots & \cdots & \cdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

Representing Digital Images

- The representation of an M×N numerical array in MATLAB

$$f(x,y)=\begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,N) \\ f(2,1) & f(2,2) & \dots & f(2,N) \\ \dots & \dots & \dots & \dots \\ f(M,1) & f(M,2) & \dots & f(M,N) \end{bmatrix}$$

Representing Digital Images

- Discrete intensity interval [0, L-1], $L=2^k$

- The number b of bits required to store a $M \times N$ digitized image

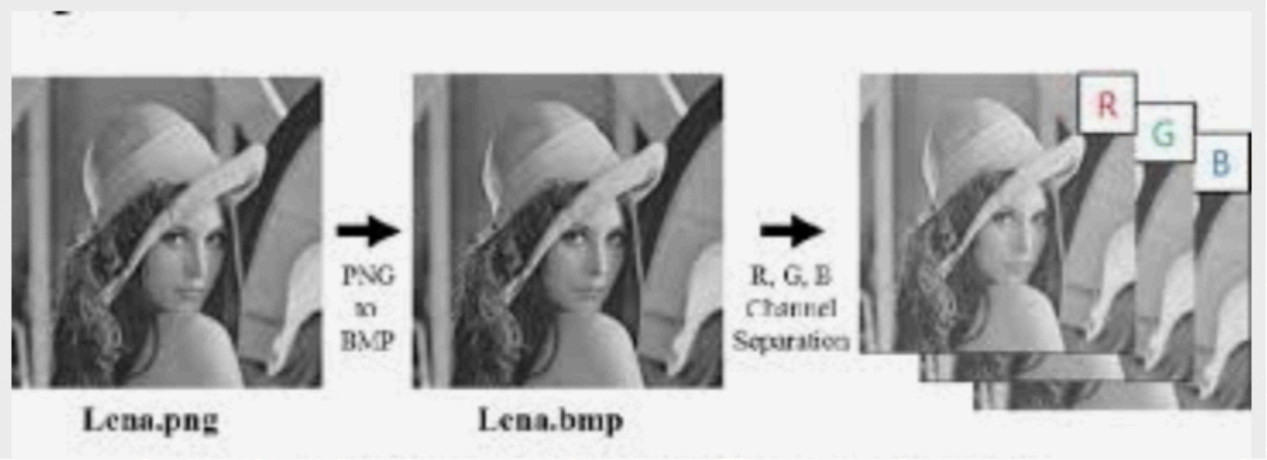
$$b = M \times N \times k$$

Representing Digital Images

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912





PNG
to
BMP
R, G, B
Channel
Separation
R
G
B
Lena.png
Lena.bmp

What is a Digital Image? (cont…)

- Common image formats include:
  - 1 sample per point (B&W or Grayscale)
  - 3 samples per point (Red, Green, and Blue)
  - 4 samples per point (Red, Green, Blue, and "Alpha", a.k.a. Opacity)

- For most of this course we will focus on grey-scale images

Image processing

- An image processing operation typically defines a new image g in terms of an existing image f.
- We can transform either the range of f.

$$g(x, y) = t(f(x, y))$$

- Or the domain of f:

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

- What kinds of operations can each perform?

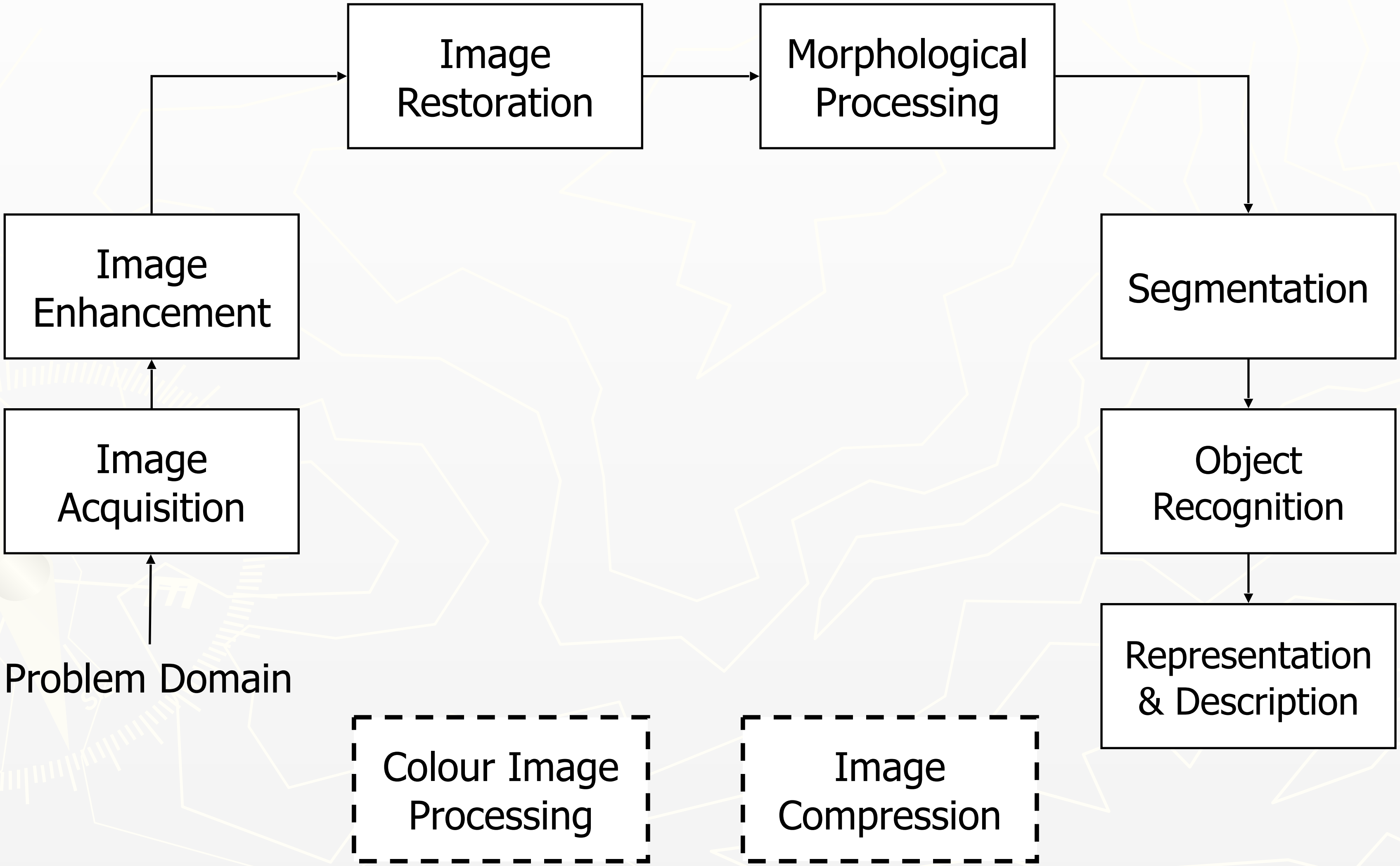
What is DIP? (cont…)

- The continuum from image processing to computer vision can be broken up into low-, mid- and high-level processes

Low Level Process	Mid Level Process	High Level Process
Input: Image Output: Image	Input: Image Output: Attributes	Input: Attributes Output: Understanding
Examples: Noise removal, image sharpening	Examples: Object recognition, segmentation	Examples: Scene understanding, autonomous navigation

In this course we will stop here

Key Stages in Digital Image Processing



Key Stages in Digital Image Processing: Image Aquisition

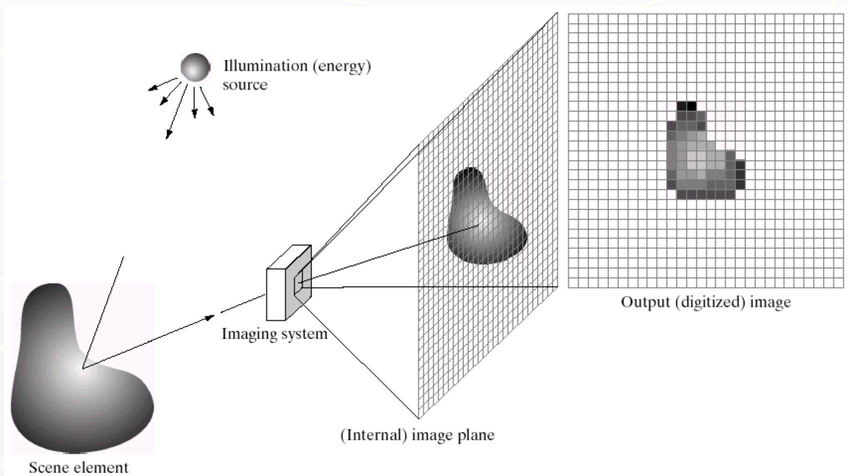
Image Restoration

Morphological Processing

Image Enhancement

Image Acquisition

Problem Domain



Segmentation

Object Recognition

Representation & Description

Colour Image Processing

Image Compression

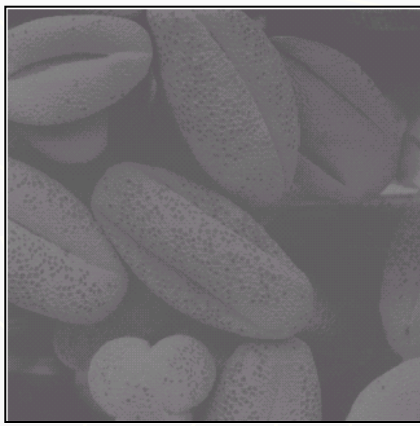
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

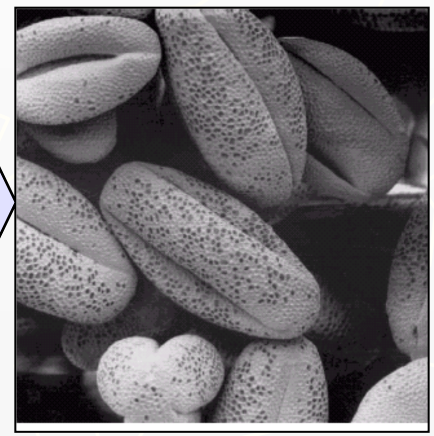
Key Stages in Digital Image Processing: Image Enhancement

Image Restoration

Morphological Processing

Image Enhancement





Segmentation

Image Acquisition

Object Recognition

Problem Domain

Representation & Description

Colour Image Processing

Image Compression

Key Stages in Digital Image Processing: Image Restoration

Image Restoration

Morphological Processing

Image Enhancement

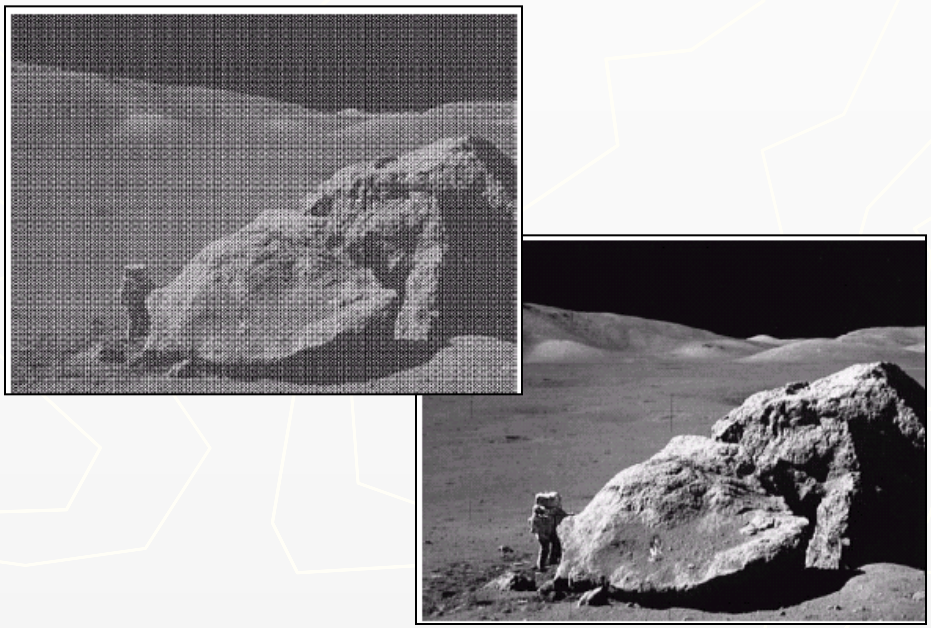
Segmentation

Image Acquisition

Object Recognition

Problem Domain

Representation & Description



Colour Image Processing

Image Compression

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Key Stages in Digital Image Processing: Morphological Processing

Image Restoration

Morphological Processing

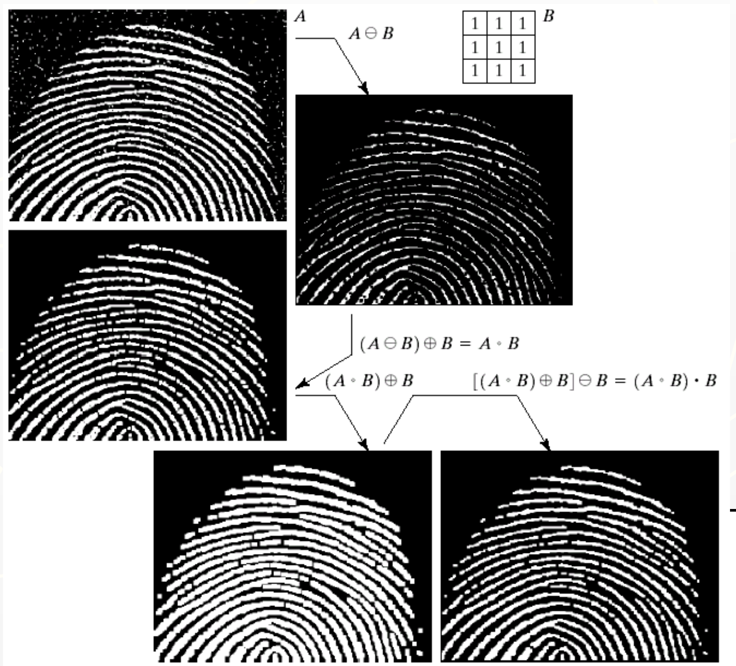
Image Enhancement

Segmentation

Image Acquisition

Object Recognition

Problem Domain



Colour Image Processing

Image Compression

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Key Stages in Digital Image Processing: Segmentation

Image Restoration

Morphological Processing

Image Enhancement

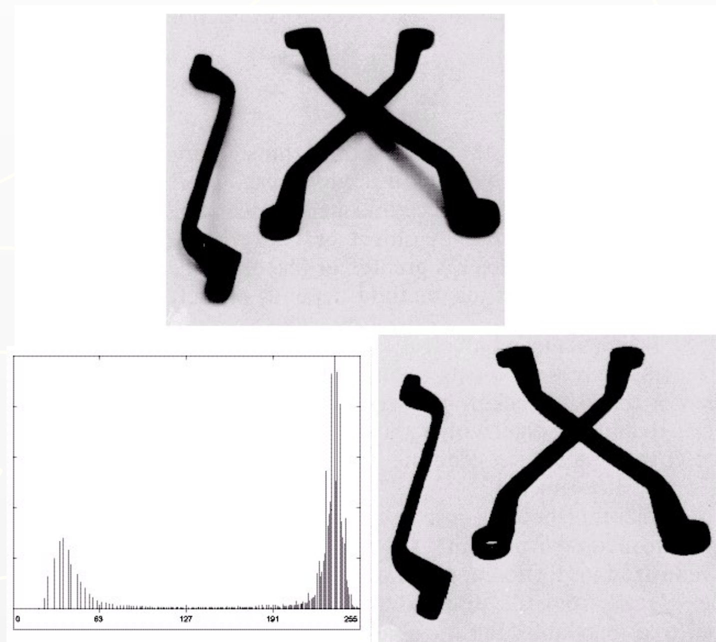
Segmentation

Image Acquisition

Object Recognition

Problem Domain

Representation & Description



Colour Image Processing

Image Compression

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Key Stages in Digital Image Processing: Object Recognition

Image Restoration

Morphological Processing

Image Enhancement

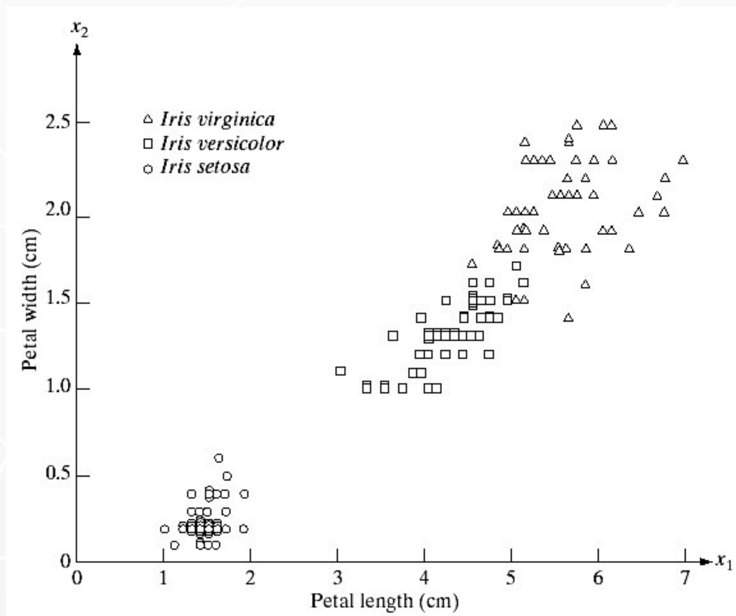
Segmentation

Image Acquisition

Object Recognition

Problem Domain

Representation & Description

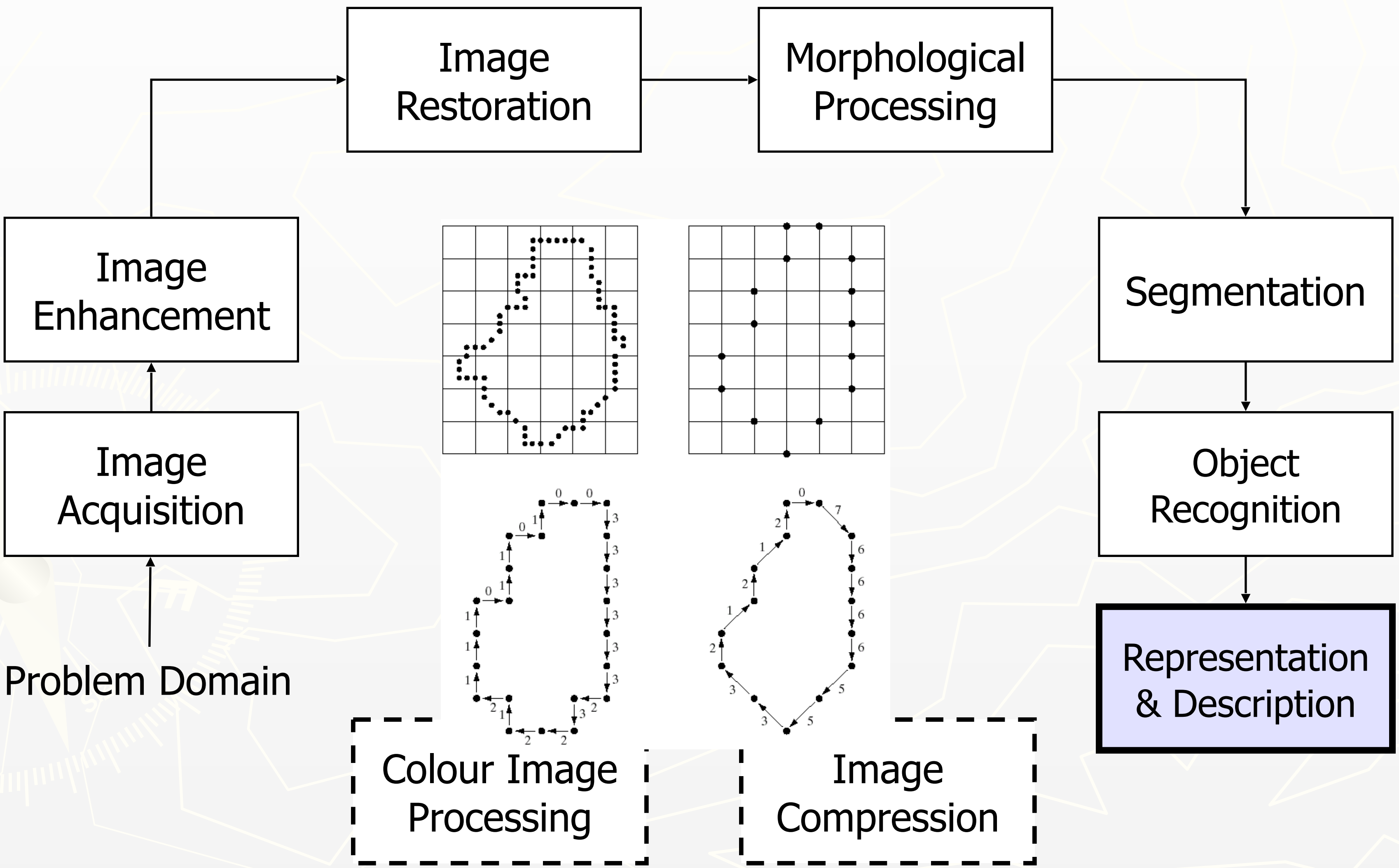


Colour Image Processing

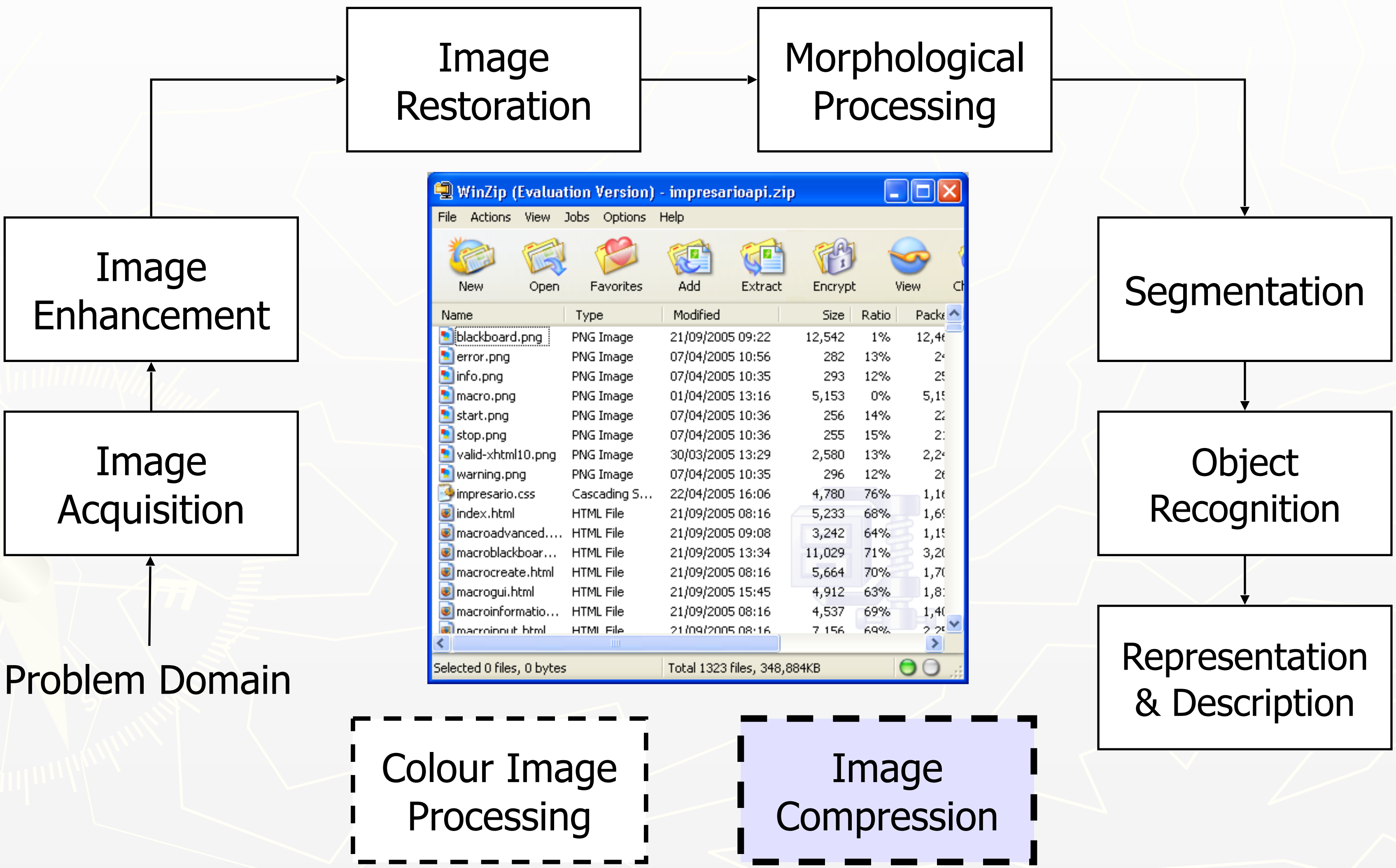
Image Compression

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Key Stages in Digital Image Processing: Representation & Description



Key Stages in Digital Image Processing: Image Compression

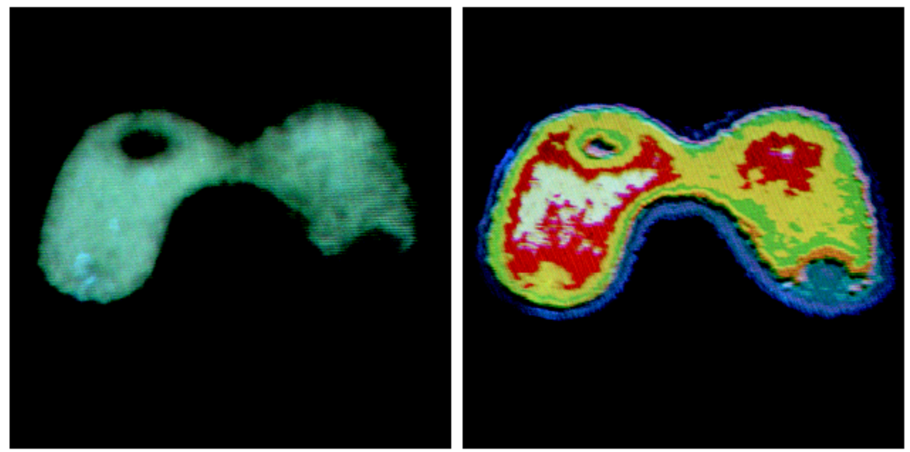


Key Stages in Digital Image Processing: Colour Image Processing

Image Restoration

Morphological Processing

Image Enhancement



Segmentation

Image Acquisition

Object Recognition

Representation & Description

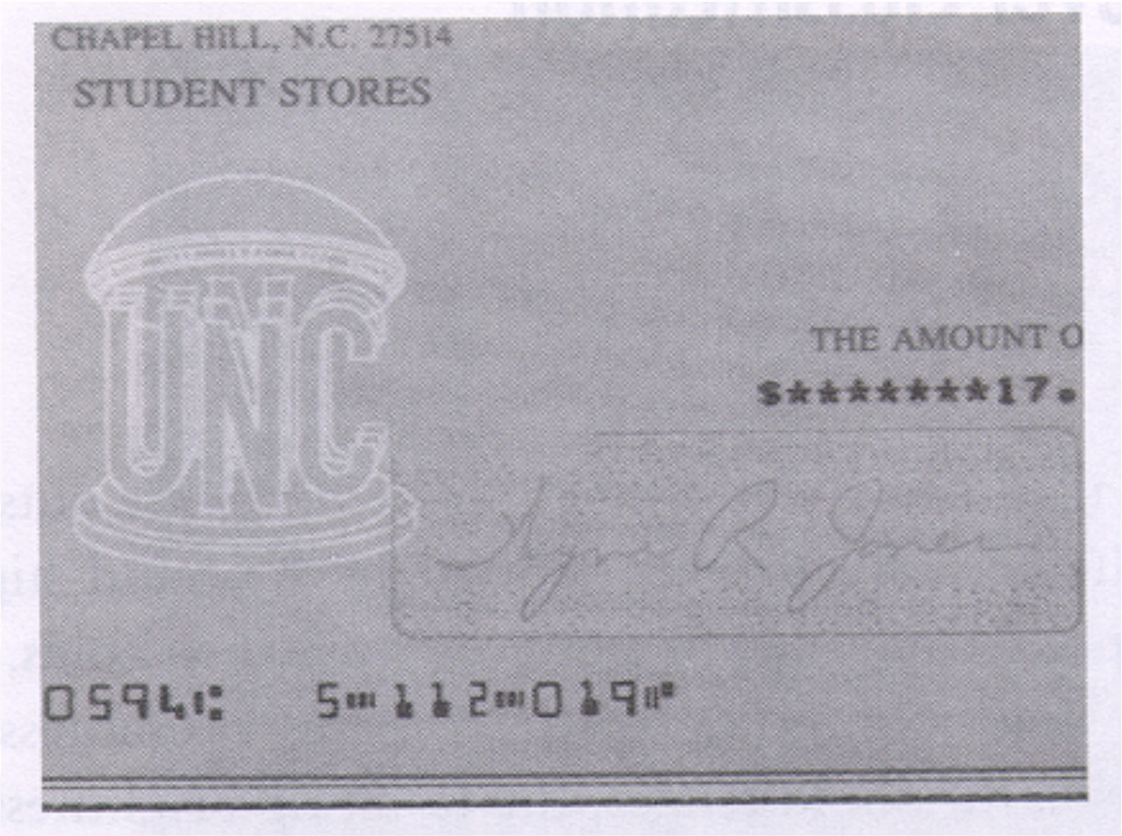
Problem Domain

Colour Image Processing

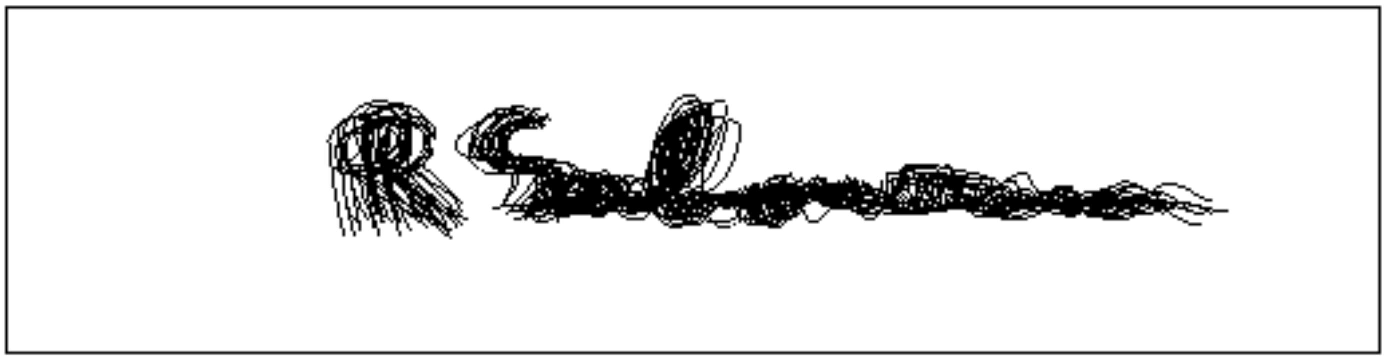
Image Compression

Applications
&
Research Topics

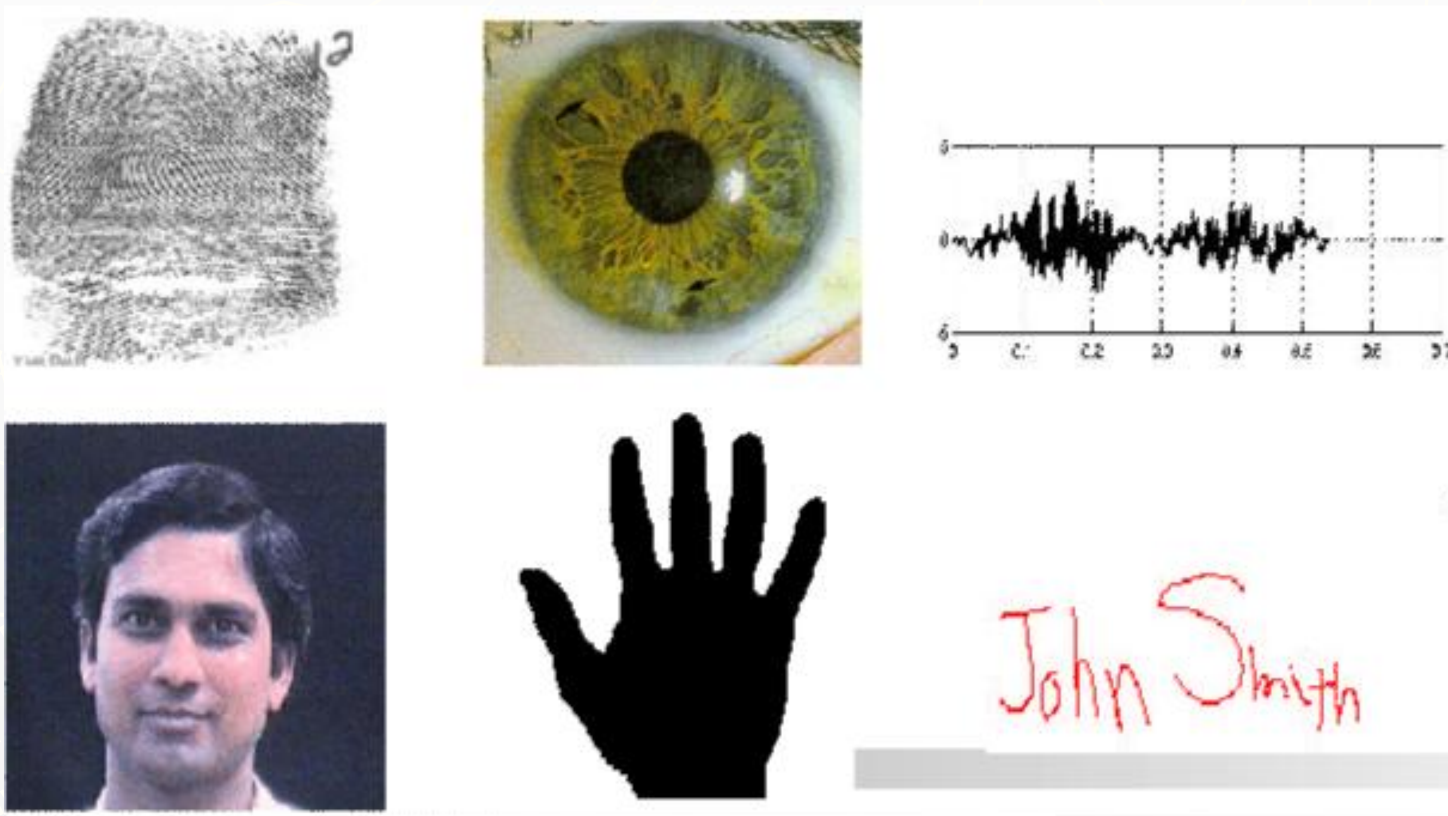
Document Handling



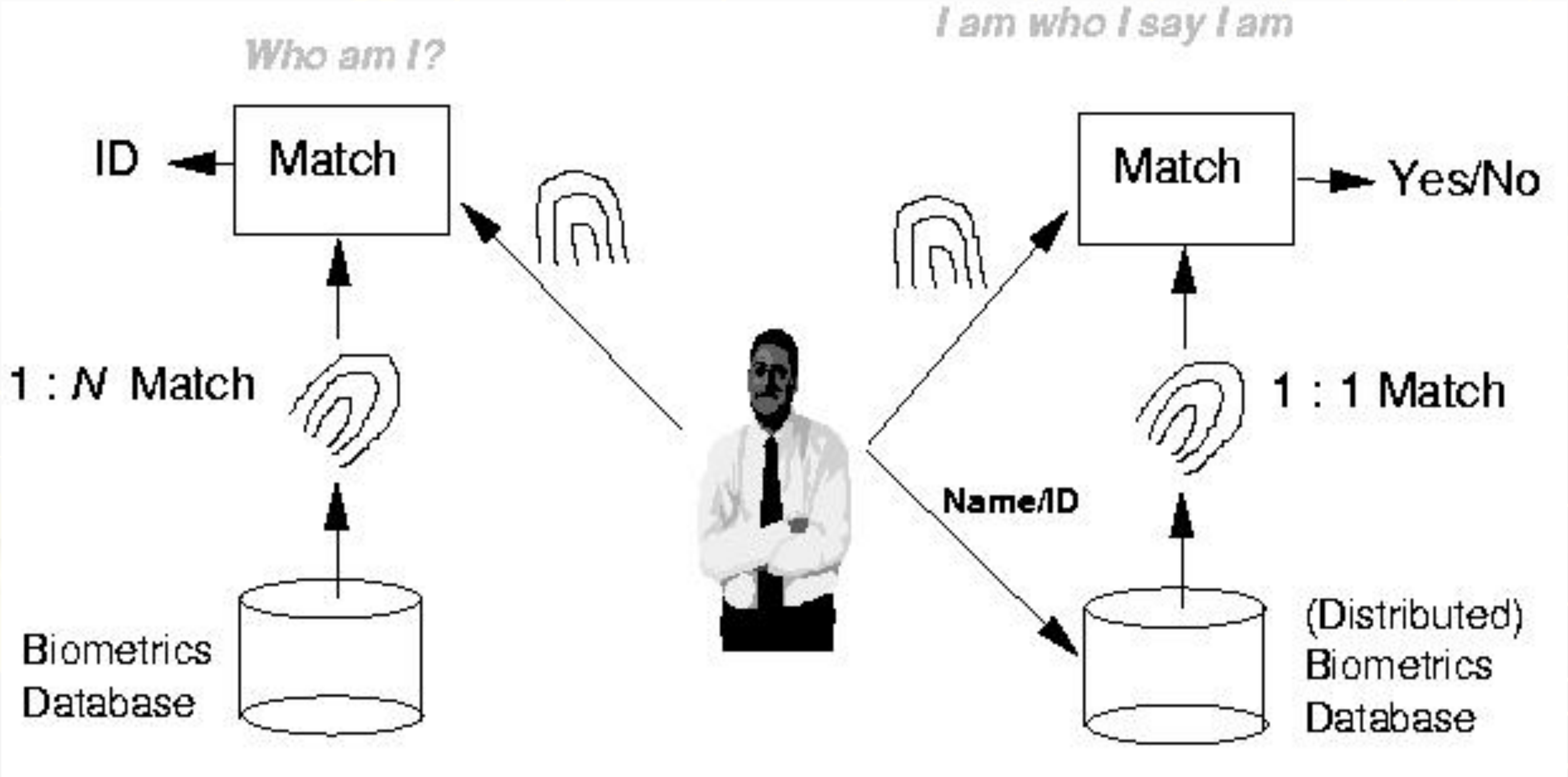
Signature Verification



Biometrics

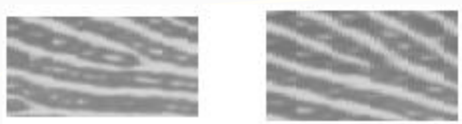


Fingerprint Verification / Identification



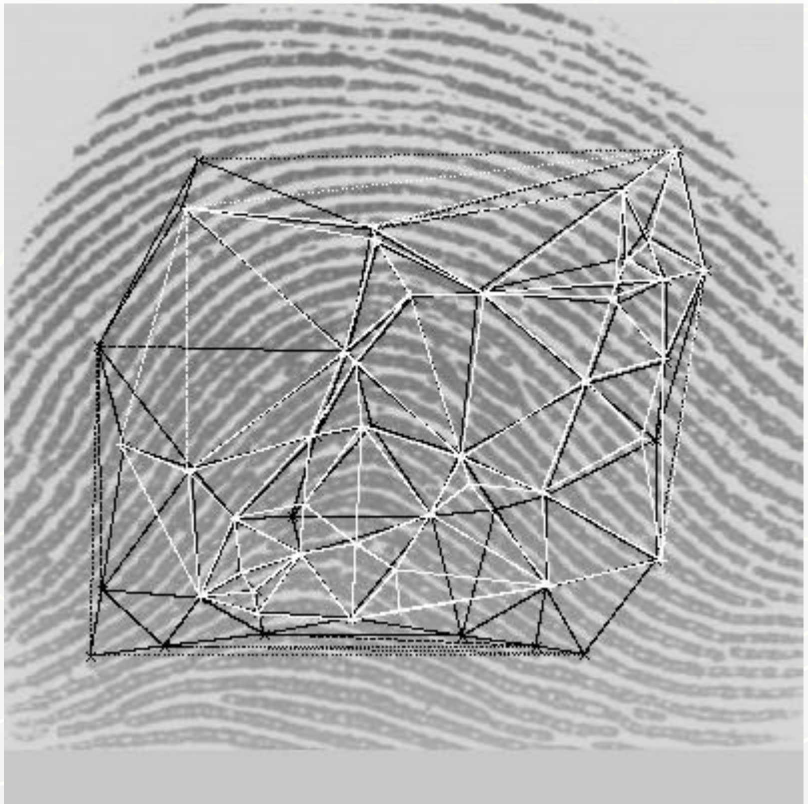
Fingerprint Identification Research at UNR

Minutiae

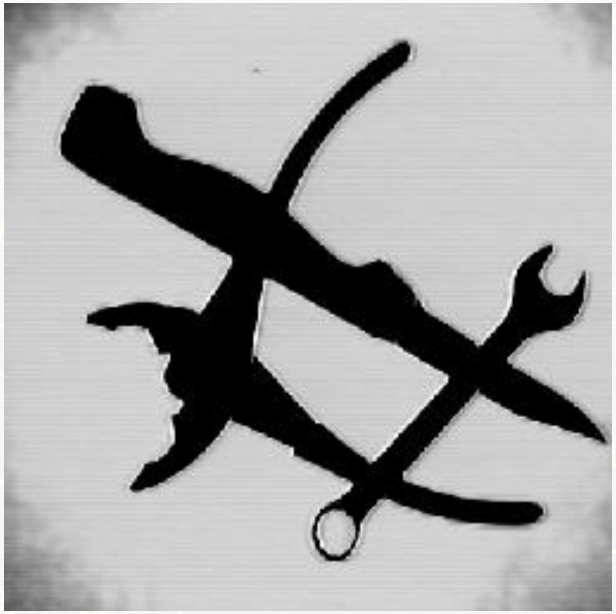


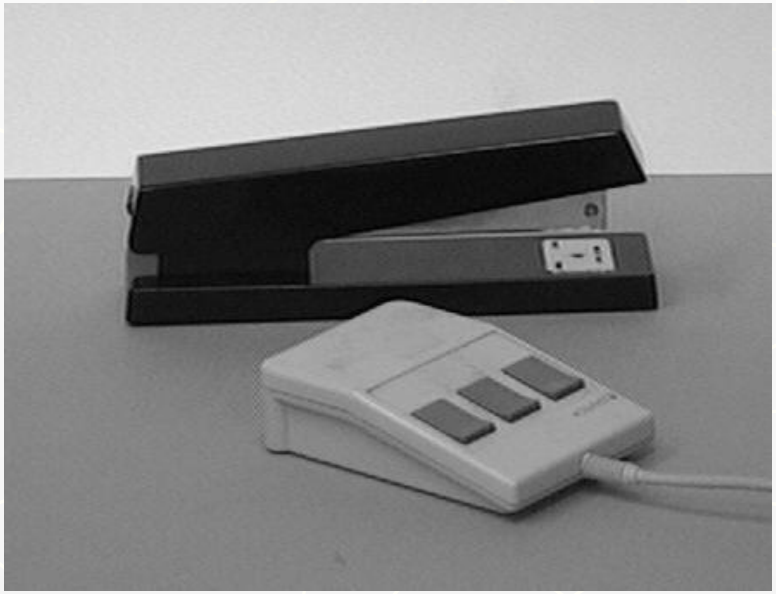
Delaunay Triangulation

Matching



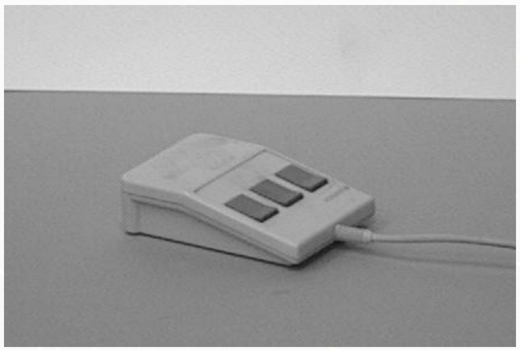
Object Recognition



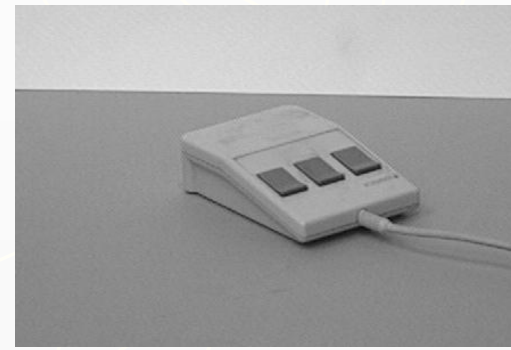


Object Recognition Research

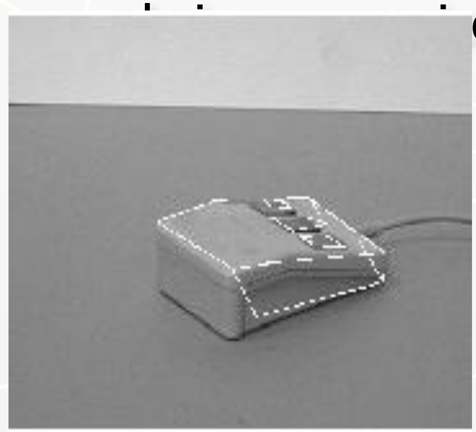
reference view 1



reference view 2

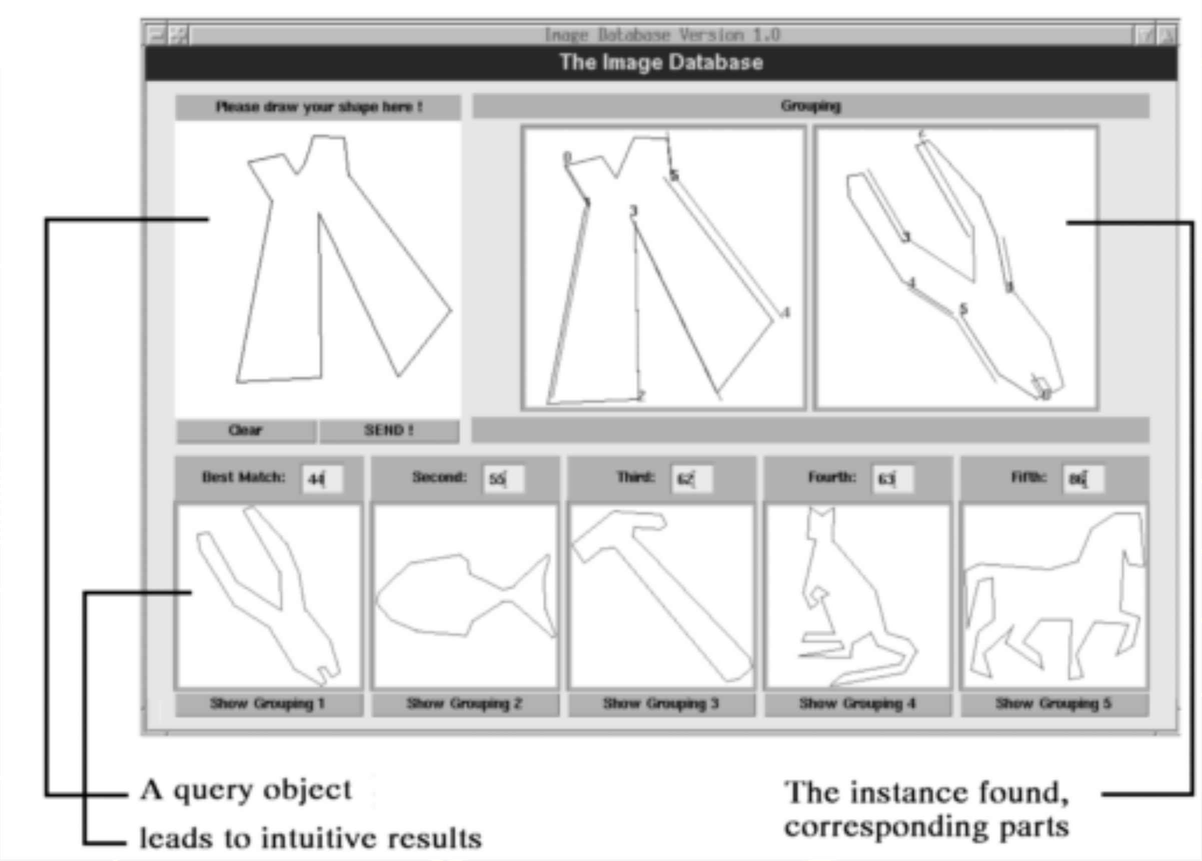


ed



Indexing into Databases

- Shape content



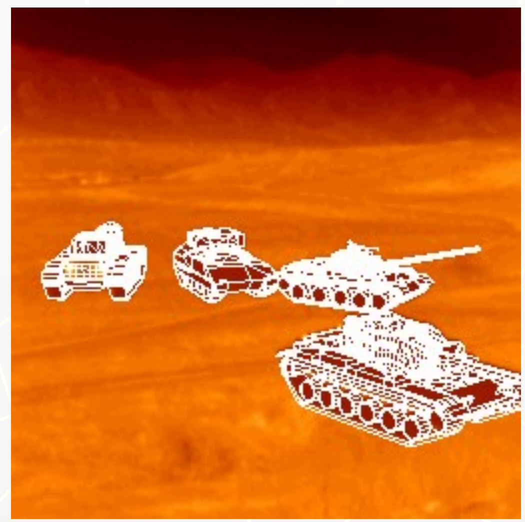
Indexing into Databases (cont'd)

- Color, texture

$T = 33.6$s, found 2 of 2

Target Recognition

- Department of Defense (Army, Airforce, Navy)

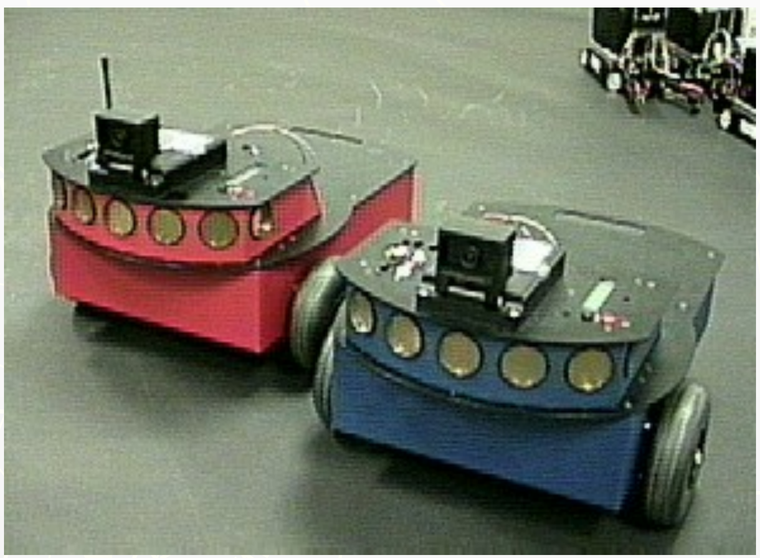


Interpretation of Aerial Photography

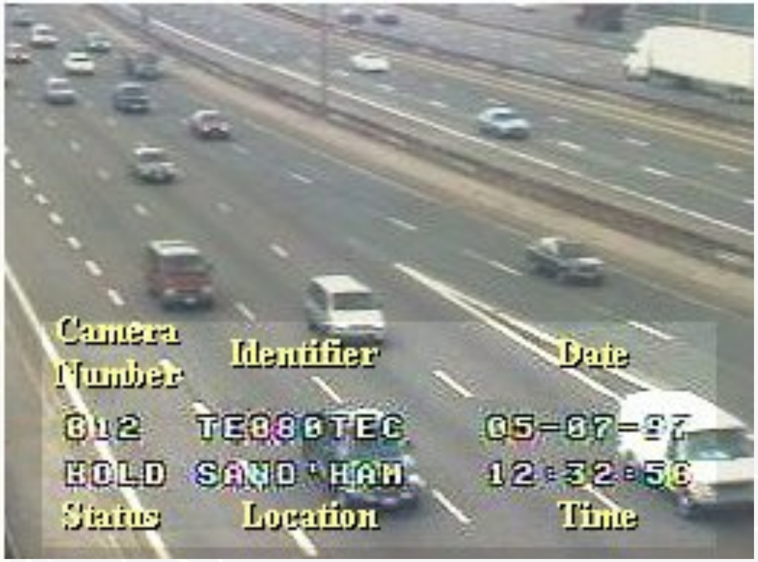
Interpretation of aerial photography is a problem domain in both computer vision and registration.

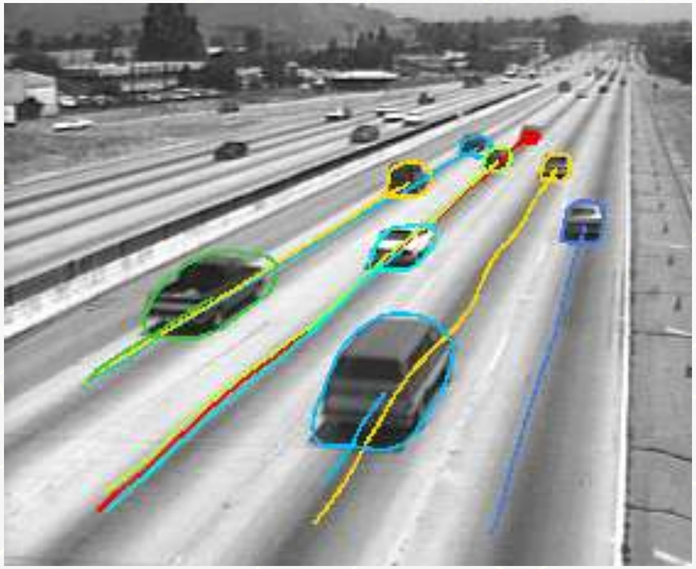
Autonomous Vehicles

- Land, Underwater, Space

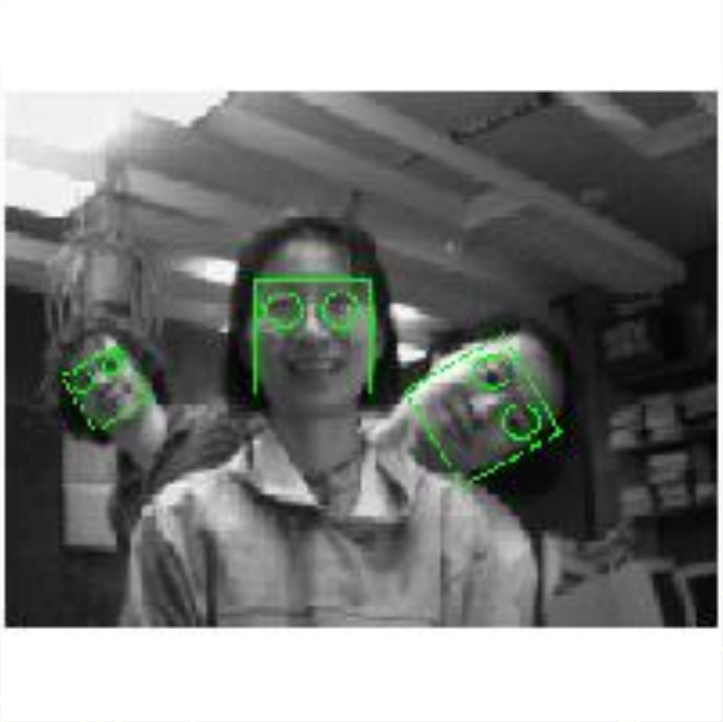


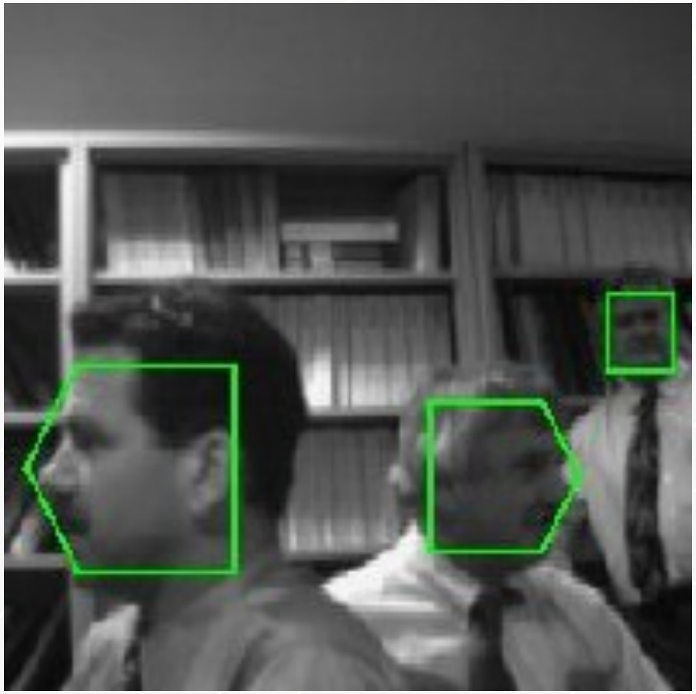
Traffic Monitoring





Face Detection

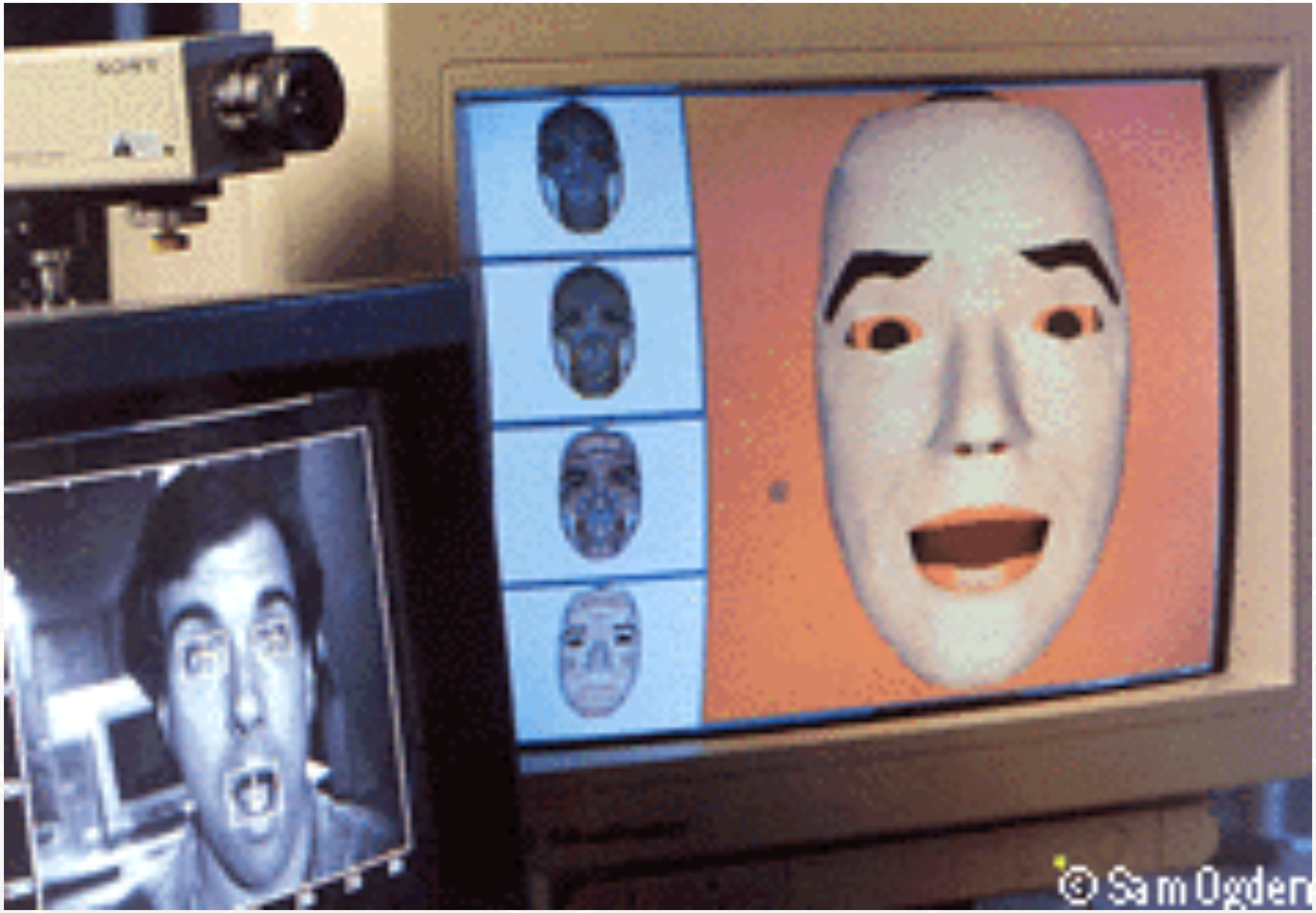




Face Recognition

Face Detection/Recognition Research at UNR

Facial Expression Recognition



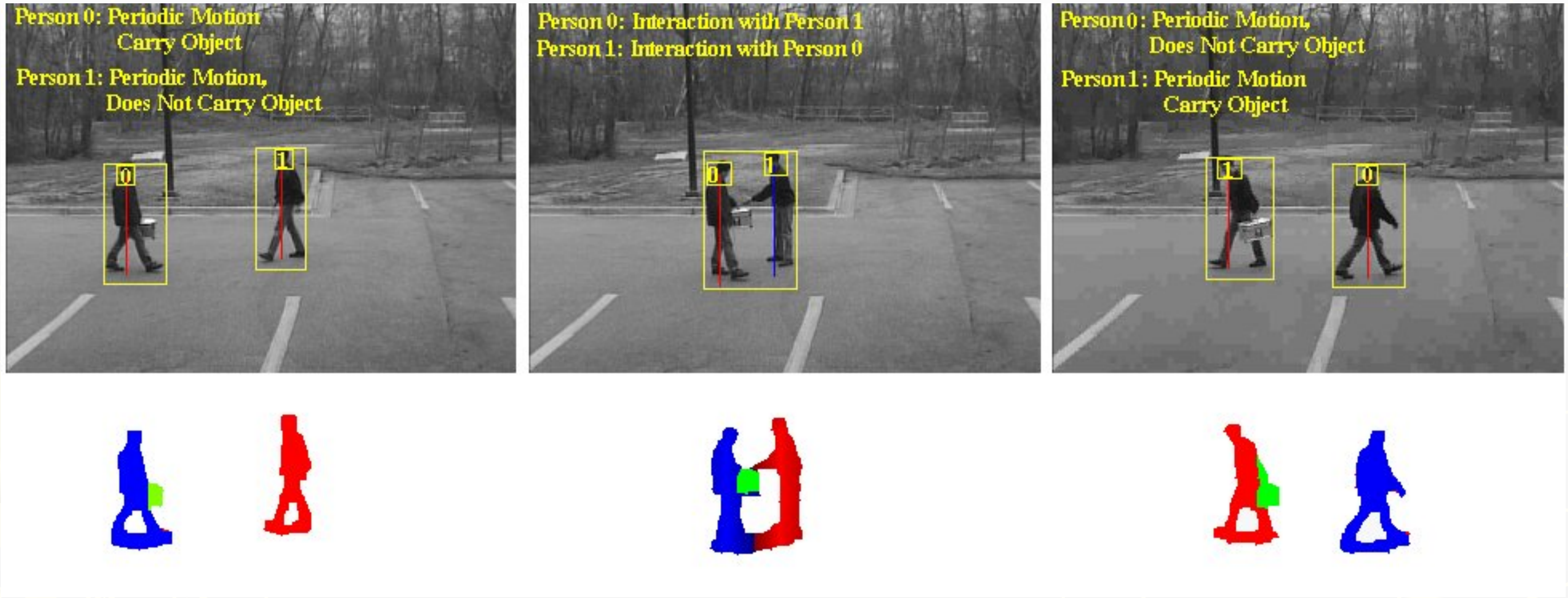
Face Tracking

Face Tracking (cont'd)

Hand Gesture Recognition

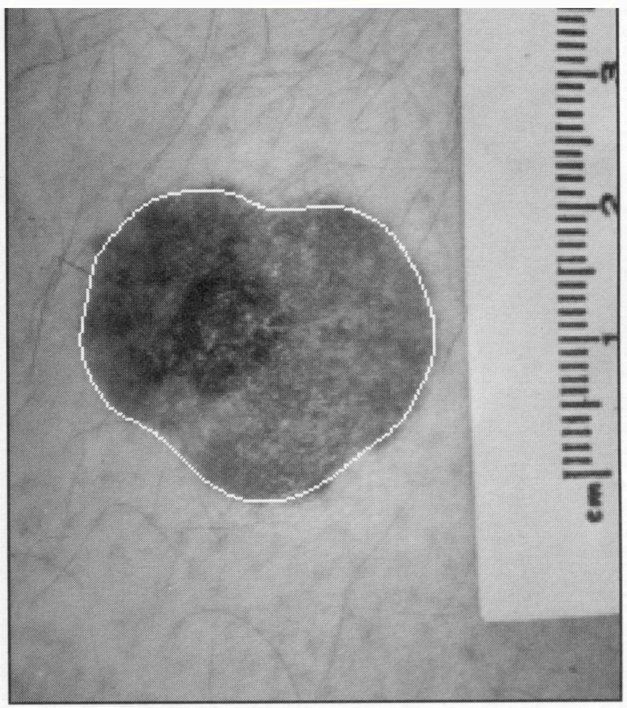
- Smart Human-Computer User Interfaces
- Sign Language Recognition

Human Activity Recognition



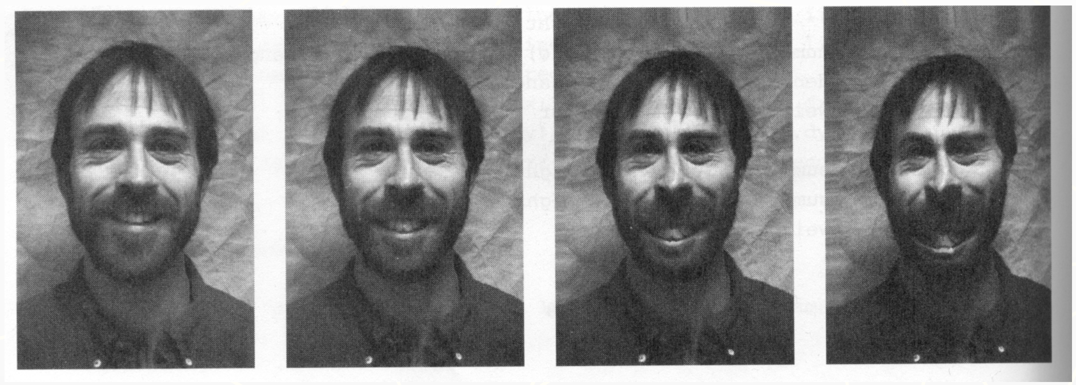
Medical Applications

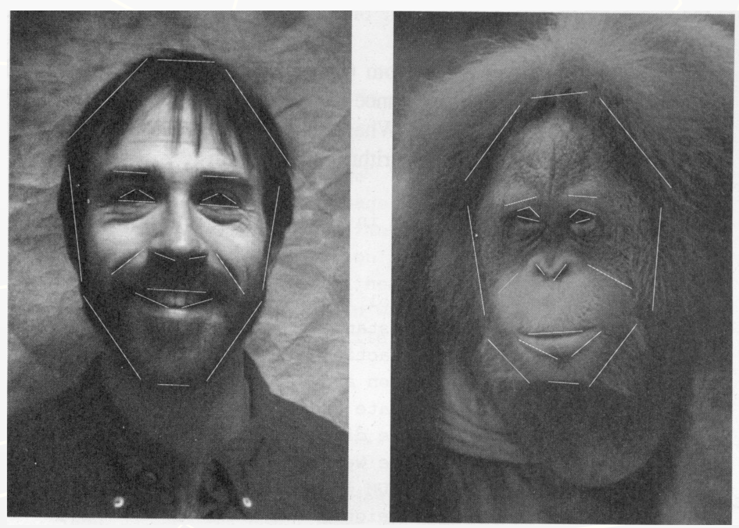
- skin cancer



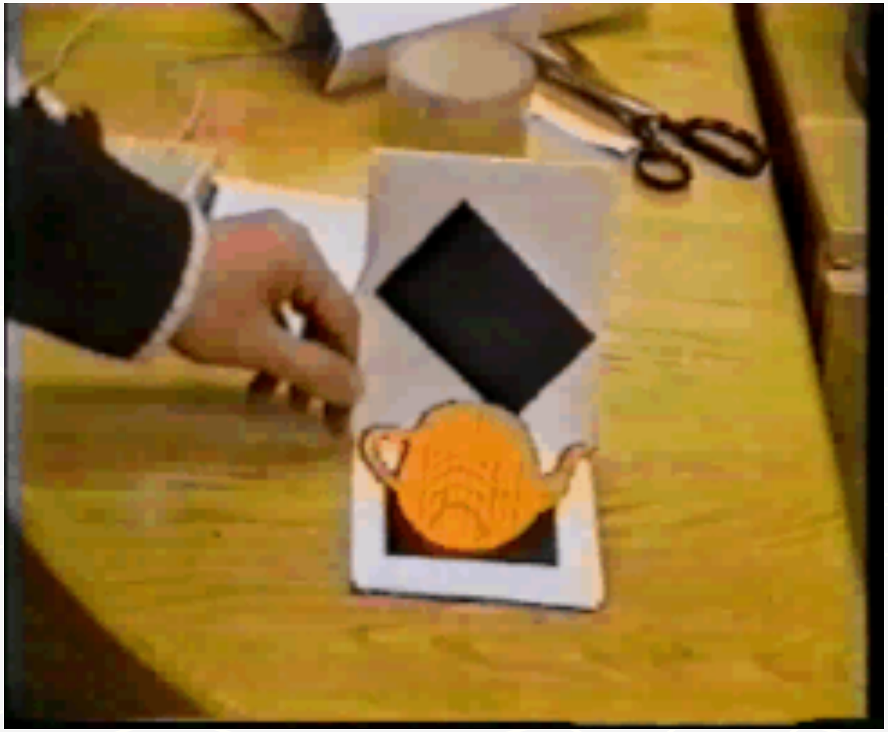
breast cancer

Morphing





Inserting Artificial Objects into a Scene



Companies In this Field In India

- Sarnoff Corporation
- Kritikal Solutions
- National Instruments
- GE Laboratories
- Ittiam, Bangalore
- Interra Systems, Noida
- Yahoo India (Multimedia Searching)
- nVidia Graphics, Pune (have high requirements)
- Microsoft research
- DRDO labs
- ISRO labs
- …

Neighborhood Operations in Images

Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries

Basic Relationships Between Pixels

- Neighbors of a pixel p at coordinates (x,y)

  - 4-neighbors of p, denoted by $N_4(p)$:
    (x-1, y), (x+1, y), (x,y-1), and (x, y+1).

  - 4 diagonal neighbors of p, denoted by $N_D(p)$:
    (x-1, y-1), (x+1, y+1), (x+1,y-1), and (x-1, y+1).

  - 8 neighbors of p, denoted $N_8(p)$
    $N_8(p) = N_4(p) \cup N_D(p)$

Basic Relationships Between Pixels

- Adjacency

  Let V be the set of intensity values

- 4-adjacency: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

- 8-adjacency: Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

Basic Relationships Between Pixels

- Adjacency

  Let V be the set of intensity values

- m-adjacency: Two pixels p and q with values from V are m-adjacent if

  (i) q is in the set $N_4(p)$, or

  (ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

Basic Relationships Between Pixels

- Path
- A (digital) path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$.

- Here n is the length of the path.
- If $(x_0, y_0) = (x_n, y_n)$, the path is closed path.
- We can define 4-, 8-, and m-paths based on the type of adjacency used.

Examples: Adjacency and Path

$V = \{1, 2\}$

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Examples: Adjacency and Path

V = {1, 2}

0	1	1		0	1	1		0	1	1
0	2	0		0	2	0		0	2	0
0	0	1		0	0	1		0	0	1

8-adjacent

Examples: Adjacency and Path

$V = \{1, 2\}$

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacent

m-adjacent

Examples: Adjacency and Path

$V = \{1, 2\}$

$$\begin{matrix} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 2_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacent

m-adjacent

The 8-path from (1,3) to (3,3):
(i) (1,3), (1,2), (2,2), (3,3)
(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):
(1,3), (1,2), (2,2), (3,3)

Basic Relationships Between Pixels

- Connected in S

  Let S represent a subset of pixels in an image. Two pixels p with coordinates (x_0, y_0) and q with coordinates (x_n, y_n) are said to be connected in S if there exists a path

  $$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

  Where $\forall i, 0 \le i \le n, (x_i, y_i) \in S$

Basic Relationships Between Pixels

Let S represent a subset of pixels in an image

- For every pixel p in S, the set of pixels in S that are connected to p is called a connected component of S.
- If S has only one connected component, then S is called Connected Set.
- We call R a region of the image if R is a connected set
- Two regions, R_i and R_j are said to be adjacent if their union forms a connected set.
- Regions that are not to be adjacent are said to be disjoint.

Basic Relationships Between Pixels

```
BW = imread('text.png');
imshow(BW);
CC = bwconncomp(BW);
numPixels =
cellfun(@numel,CC.PixelIdxList);
[biggest,idx] = max(numPixels);
BW(CC.PixelIdxList{idx}) = 0;
figure, imshow(BW);
```

Basic Relationships Between Pixels

- Boundary (or border)

  - The boundary of the region R is the set of pixels in the region that have one or more neighbors that are not in R.
  - If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

- Foreground and background

  - An image contains K disjoint regions, R_k, k = 1, 2, …, K. Let R_u denote the union of all the K regions, and let $(R_u)^c$ denote its complement.

  All the points in R_u is called foreground;

  All the points in $(R_u)^c$ is called background.

Question 1

- In the following arrangement of pixels, are the two regions (of 1s) adjacent? (if 8-adjacency is used)

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

Question 2

- In the following arrangement of pixels, are the two parts (of 1s) adjacent? (if 4-adjacency is used)

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Part 1

Part 2

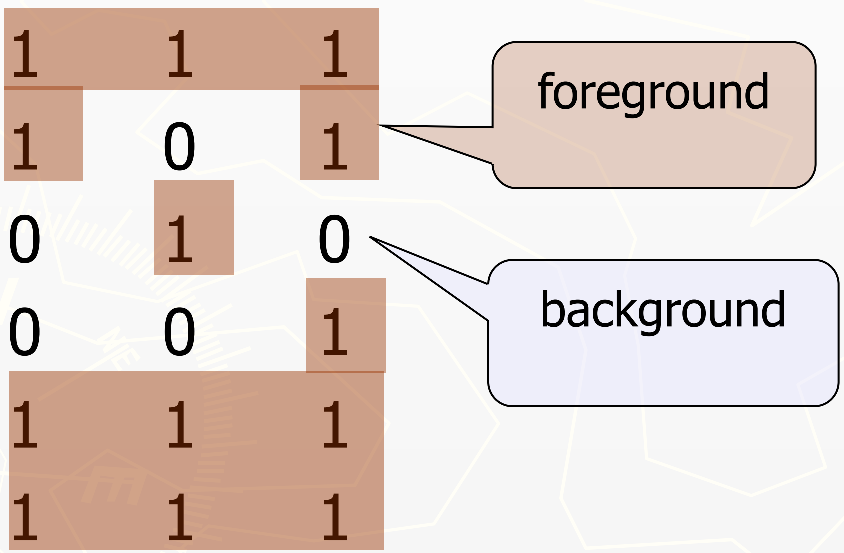
- In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

- In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)



Question 3

- In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 8-adjacency is used, true or false?

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Question 4

- In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 4-adjacency is used, true or false?

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Distance Measures

- Given pixels p, q and z with coordinates (x, y), (s, t), (u, v) respectively, the distance function D has following properties:

a. $D(p, q) \geq 0 \quad [D(p, q) = 0, \text{ iff } p = q]$

b. $D(p, q) = D(q, p)$

c. $D(p, z) \leq D(p, q) + D(q, z)$

Distance Measures

The following are the different Distance measures:

a. Euclidean Distance :

$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$

b. City Block Distance:

$D_4(p, q) = |x-s| + |y-t|$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

c. Chess Board Distance:

$D_8(p, q) = \max(|x-s|, |y-t|)$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Question 5

- In the following arrangement of pixels, what's the value of the chessboard distance between the circled two points?

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 6

- In the following arrangement of pixels, what's the value of the city-block distance between the circled two points?

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Introduction to Mathematical Operations in DIP

- Array vs. Matrix Operation

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Array product operator

$$A \;.*\; B = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Array product

Matrix product operator

$$A * B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Matrix product

Introduction to Mathematical Operations in DIP

- Linear vs. Nonlinear Operation

$$H\left[f(x,y)\right]=g(x,y)$$

$$H\left[a_i f_i(x,y)+a_j f_j(x,y)\right]$$

$$=H\left[a_i f_i(x,y)\right]+H\left[a_j f_j(x,y)\right]$$

Additivity

$$=a_i H\left[f_i(x,y)\right]+a_j H\left[f_j(x,y)\right]$$

Homogeneity

$$=a_i g_i(x,y)+a_j g_j(x,y)$$

H is said to be a linear operator;

H is said to be a nonlinear operator if it does not meet the above qualification.

Arithmetic Operations

- Arithmetic operations between images are array operations. The four arithmetic operations are denoted as

$$s(x,y) = f(x,y) + g(x,y)$$
$$d(x,y) = f(x,y) - g(x,y)$$
$$p(x,y) = f(x,y) \times g(x,y)$$
$$v(x,y) = f(x,y) \div g(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

Noiseless image: $f(x,y)$

Noise: $n(x,y)$ (at every pair of coordinates (x,y), the noise is uncorrelated and has zero average value)

Corrupted image: $g(x,y)$

$$g(x,y) = f(x,y) + n(x,y)$$

Reducing the noise by adding a set of noisy images, $\{g_i(x,y)\}$

$$\overline{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

$$E\{\bar{g}(x,y)\} = E\left\{\frac{1}{K}\sum_{i=1}^{K} g_i(x,y)\right\}$$

$$= E\left\{\frac{1}{K}\sum_{i=1}^{K}\left[f(x,y)+n_i(x,y)\right]\right\}$$

$$= f(x,y) + E\left\{\frac{1}{K}\sum_{i=1}^{K} n_i(x,y)\right\}$$

$$= f(x,y)$$

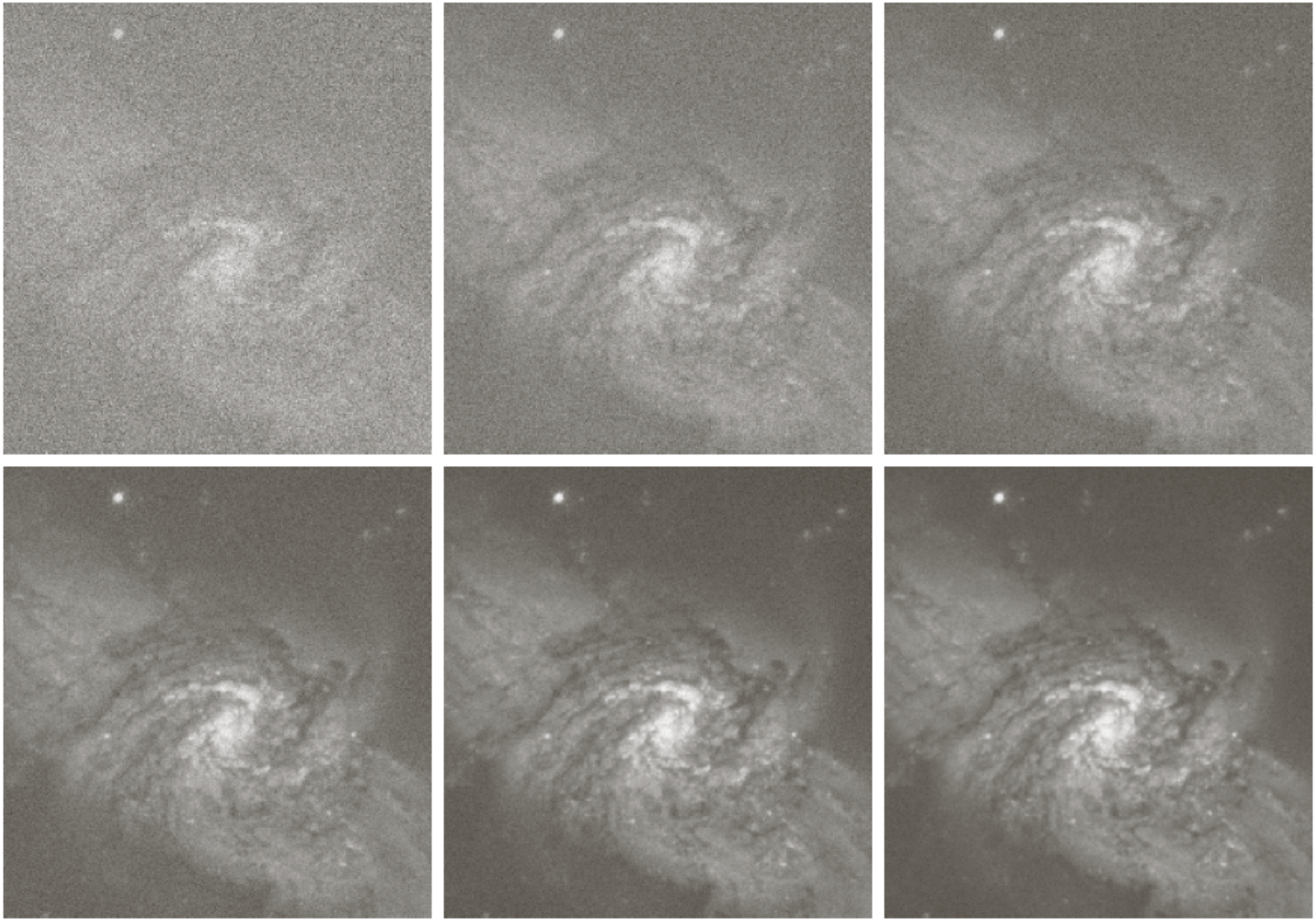
$$\sigma^2_{\bar{g}(x,y)} = \sigma^2_{\frac{1}{K}\sum_{i=1}^{K} g_i(x,y)}$$

$$= \sigma^2_{\frac{1}{K}\sum_{i=1}^{K} n_i(x,y)} \qquad = \frac{1}{K}\sigma^2_{n(x,y)}$$

Example: Addition of Noisy Images for Noise Reduction

- In astronomy, imaging under very low light levels frequently causes sensor noise to render single images virtually useless for analysis.

- In astronomical observations, similar sensors for noise reduction by observing the same scene over long periods of time. Image averaging is then used to reduce the noise.



a	b	c
d	e	f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

An Example of Image Subtraction: Mask Mode Radiography

Mask $h(x,y)$: an X-ray image of a region of a patient's body

Live images $f(x,y)$: X-ray images captured at TV rates after injection of the contrast medium

Enhanced detail $g(x,y)$

$$g(x,y) = f(x,y) - h(x,y)$$

The procedure gives a movie showing how the contrast medium propagates through the various arteries in the area being observed.

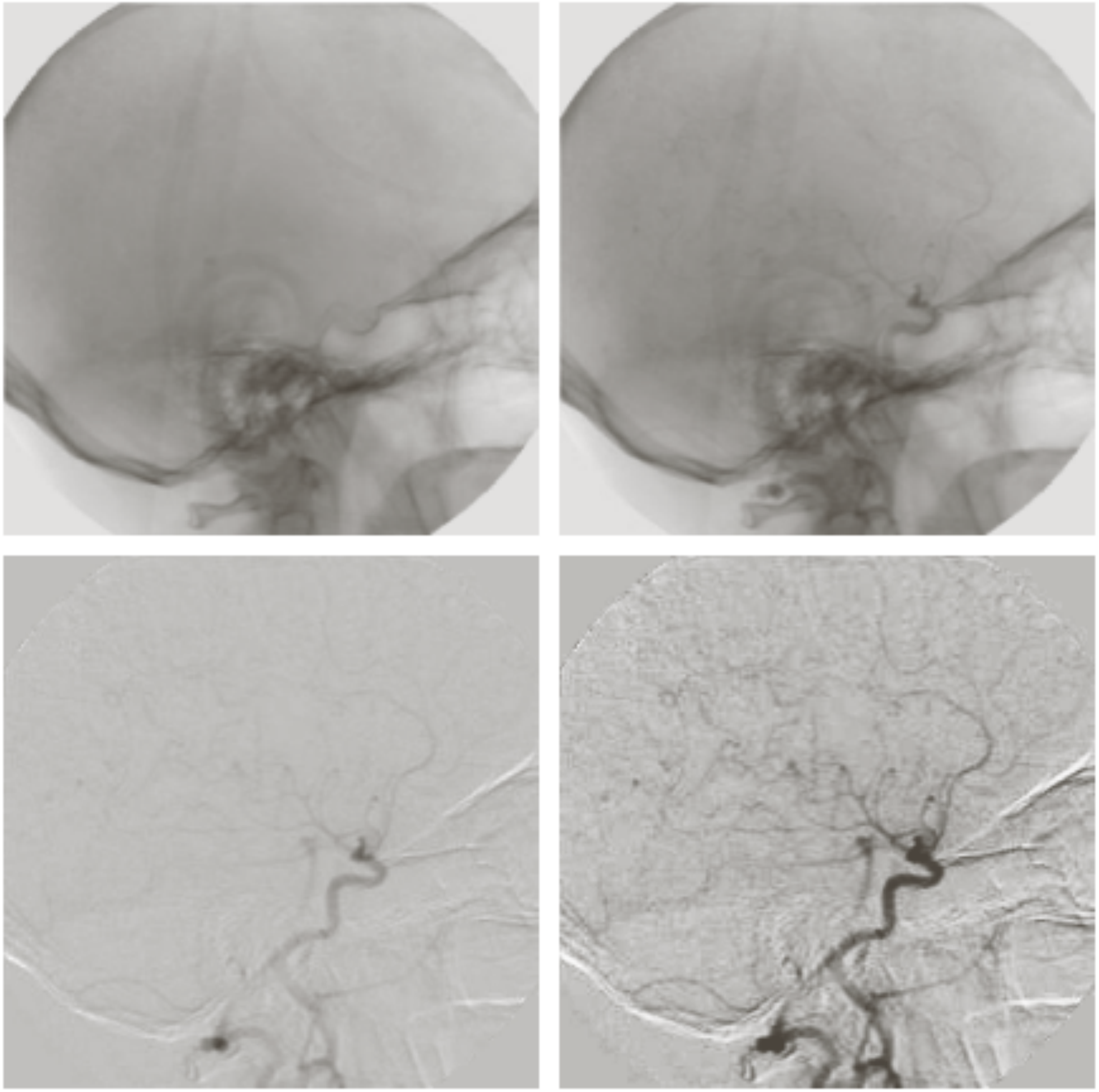
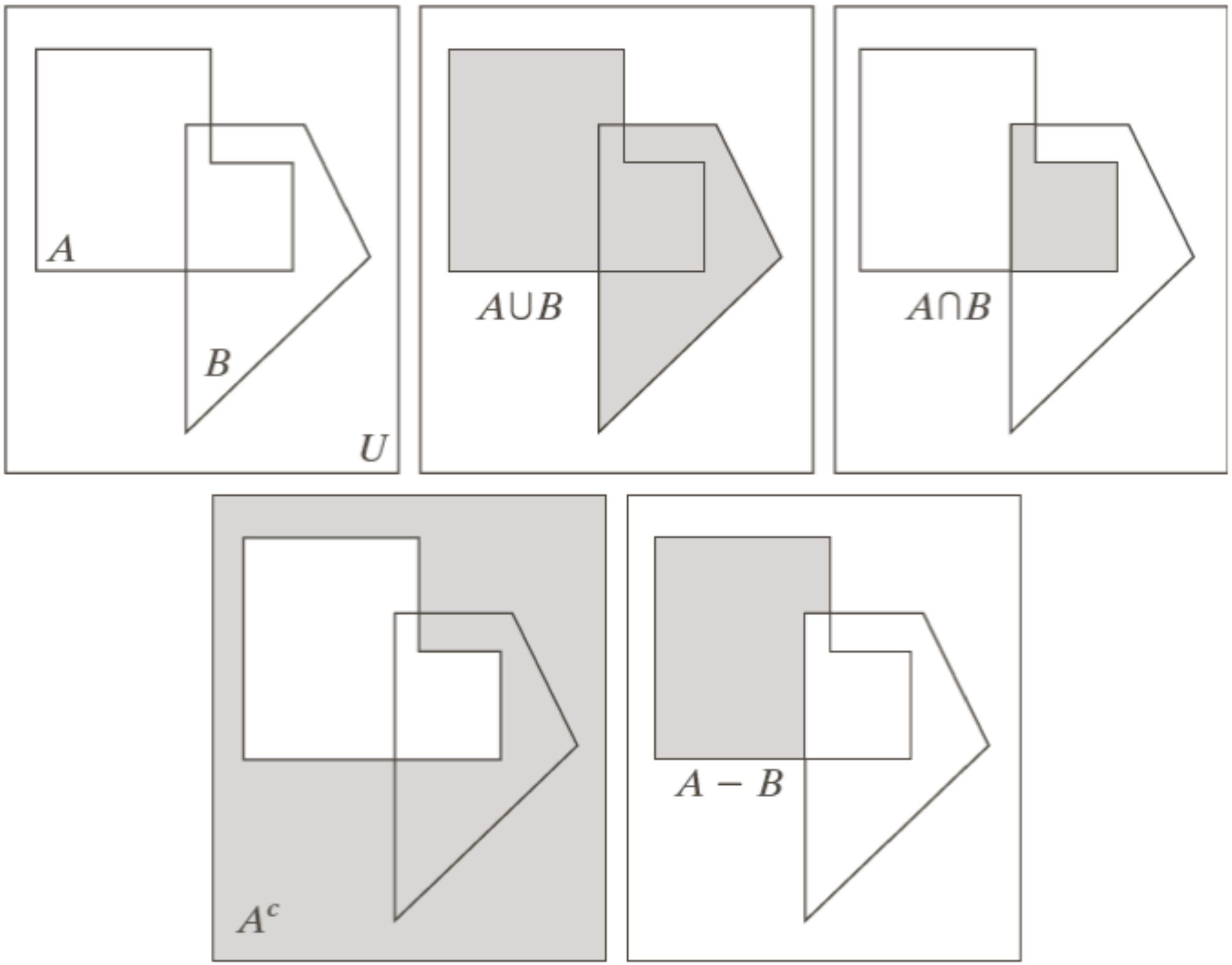


FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

An Example of Image Multiplication

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Set and Logical Operations



a b c
d e

FIGURE 2.31 (a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

Set and Logical Operations

- Let A be the elements of a gray-scale image

  The elements of A are triplets of the form (x, y, z), where x and y are spatial coordinates and z denotes the intensity at the point (x, y).

$$A = \{(x, y, z) \mid z = f(x, y)\}$$

- The complement of A is denoted A^c

$$A^c = \{(x, y, K - z) \mid (x, y, z) \in A\}$$

$K = 2^k - 1; k$ is the number of intensity bits used to represent z

Set and Logical Operations

- The union of two gray-scale images (sets) A and B is defined as the set

$$A \cup B = \{\max_z(a,b) \mid a \in A, b \in B\}$$

Set and Logical Operations

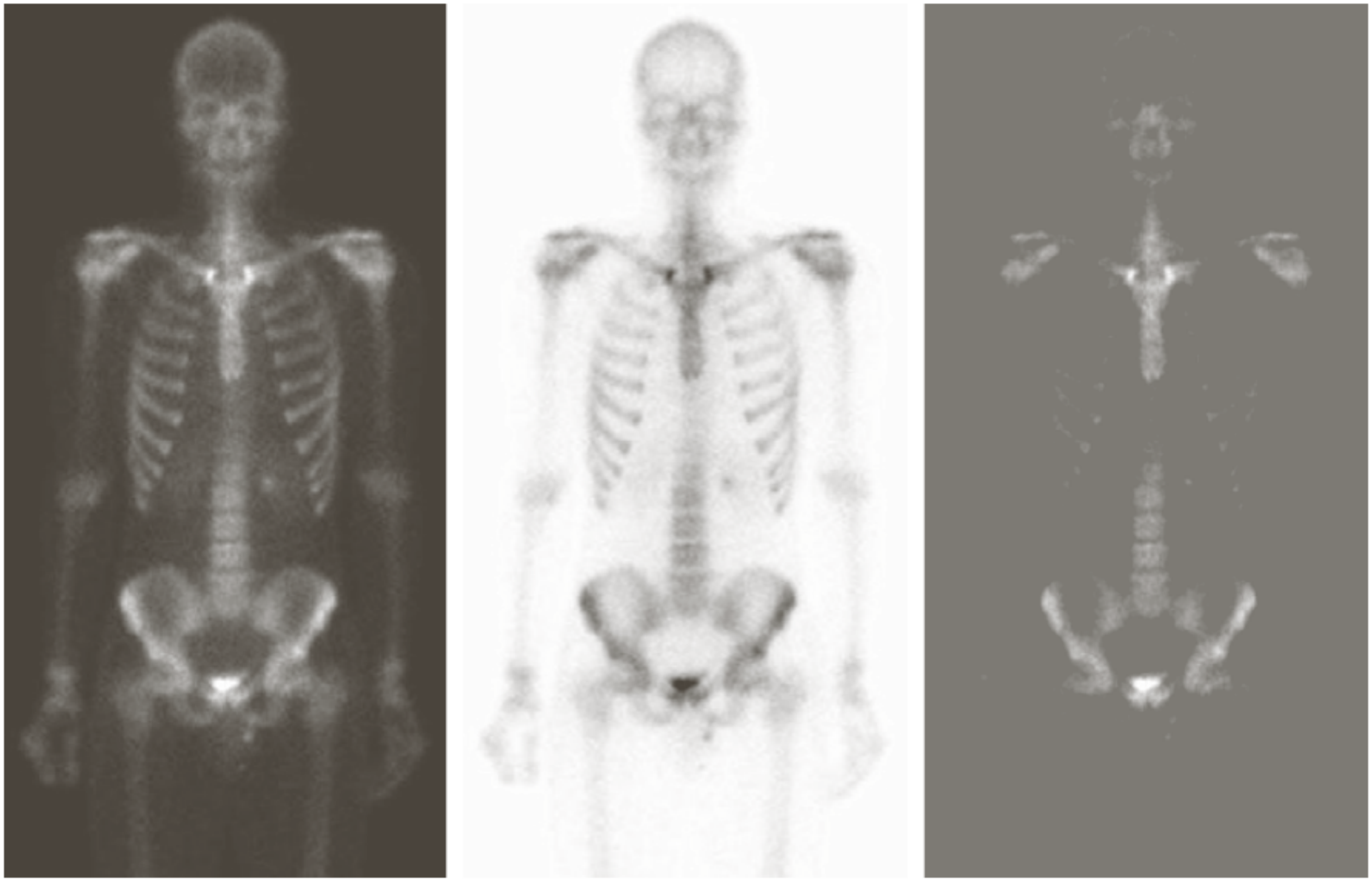


FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

Set and Logical Operations

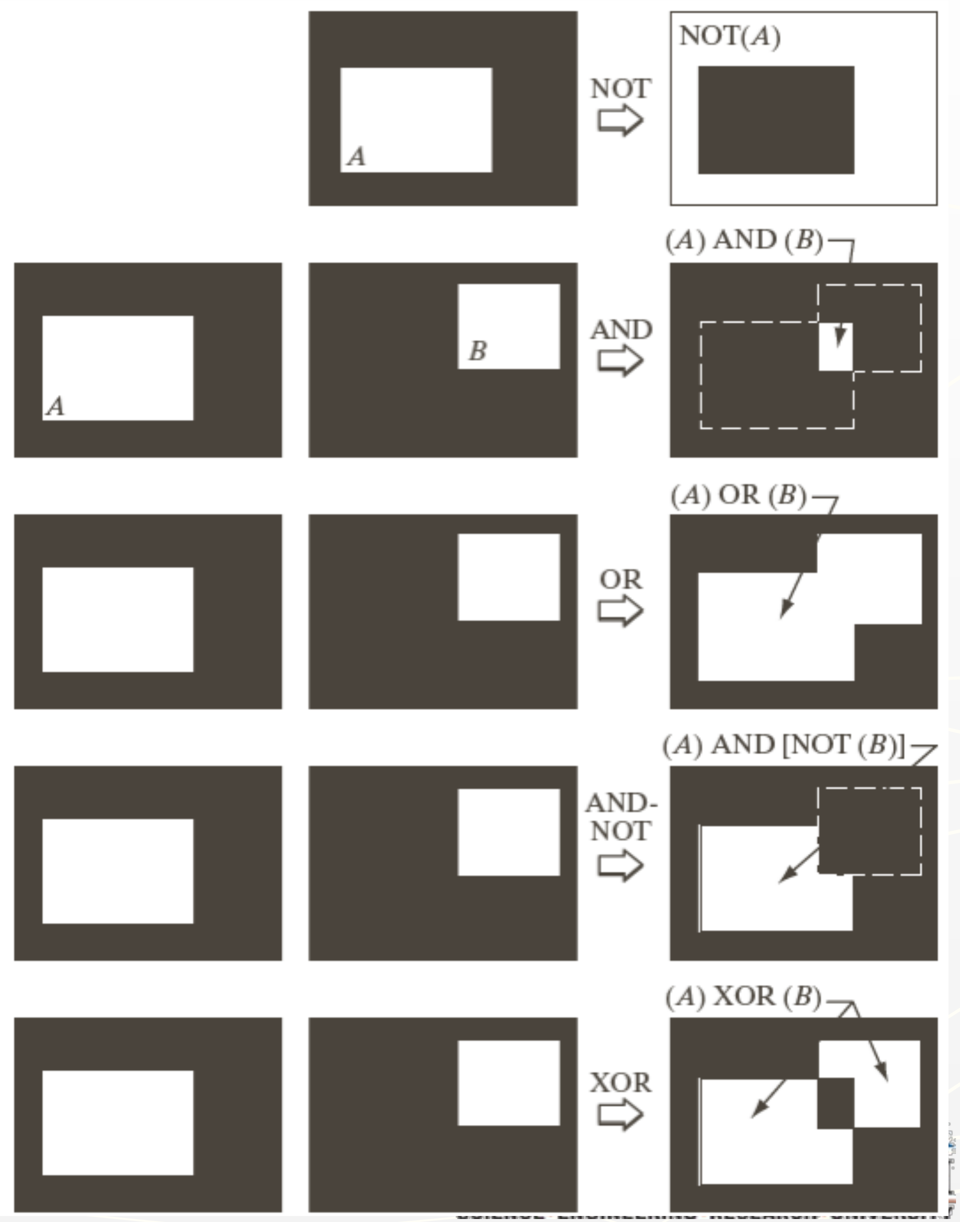


FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

Spatial Operations

- Single-pixel operations

Alter the values of an image's pixels based on the intensity.

$$s = T(z)$$

e.g.,

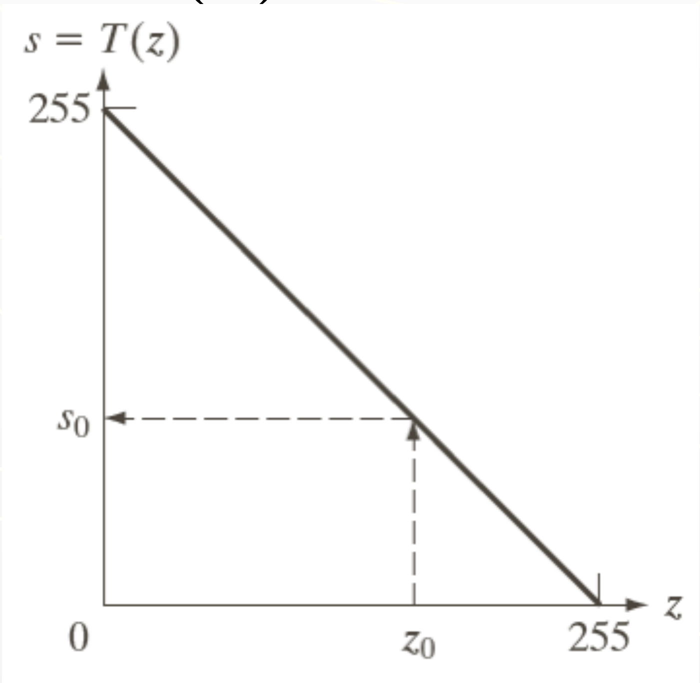
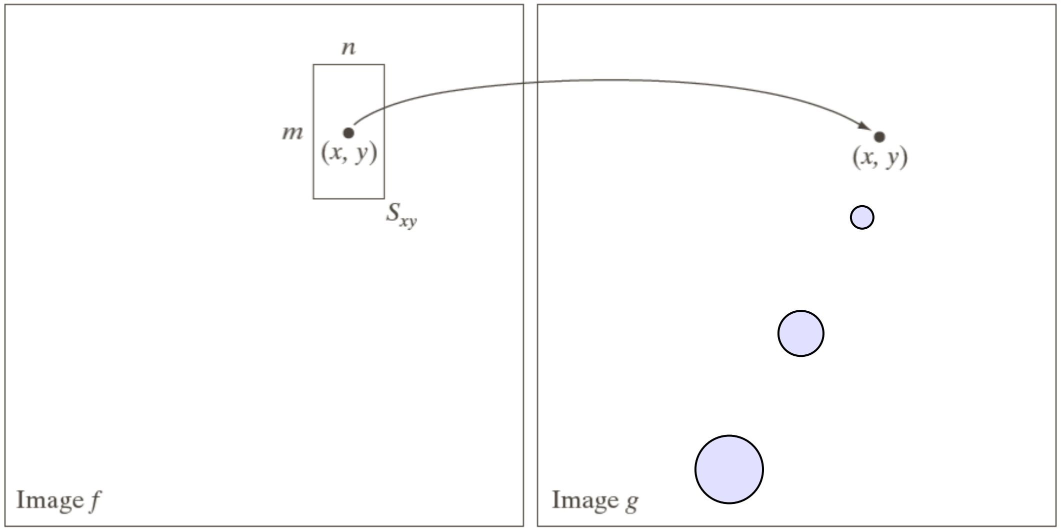


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0.

Spatial Operations

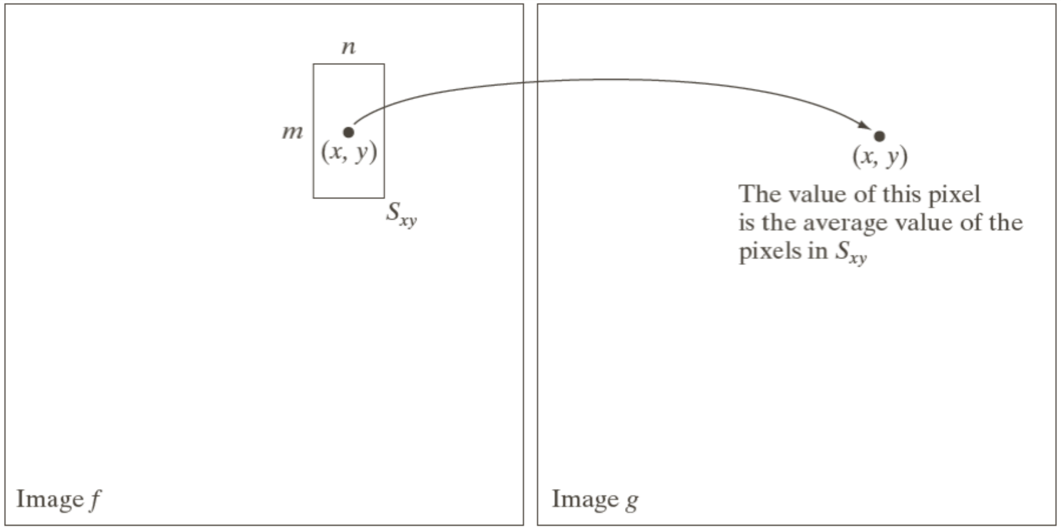
- Neighborhood operations

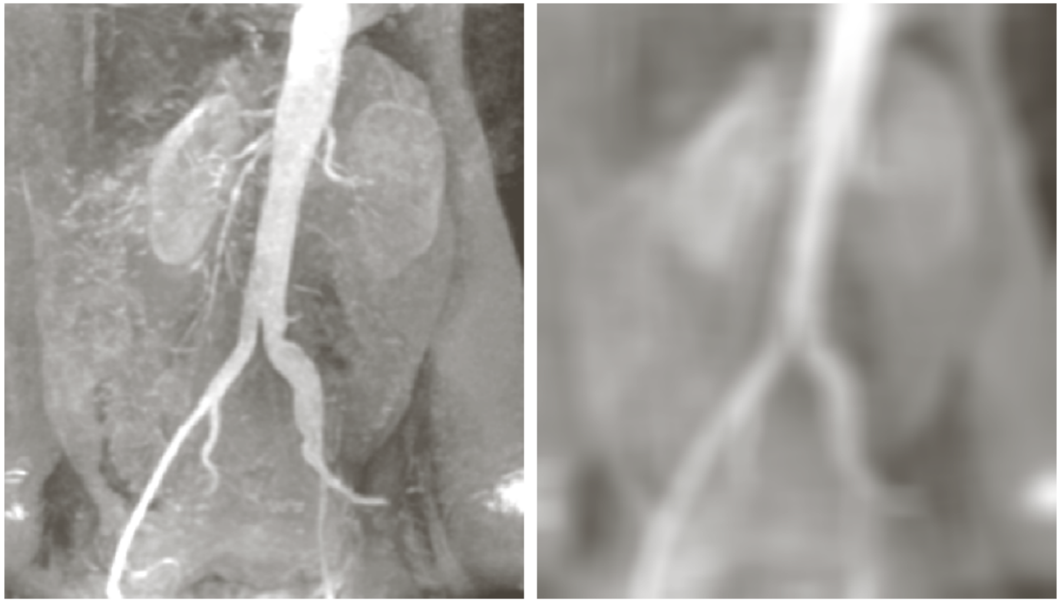


The value of this pixel is determined by a specified operation involving the pixels in the input image with coordinates in S_{xy}

Spatial Operations

- Neighborhood operations





Geometric Spatial Transformations

- Geometric transformation (rubber-sheet transformation)

  — A spatial transformation of coordinates

$$(x, y) = T\{(v, w)\}$$

  — intensity interpolation that assigns intensity values to the spatially transformed pixels.

- Affine transform

$$\begin{bmatrix} x & y & 1 \end{bmatrix} = \begin{bmatrix} v & w & 1 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

TABLE 2.2
Affine transformations based on Eq. (2.6.–23).

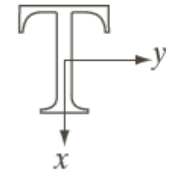
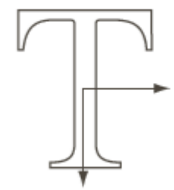
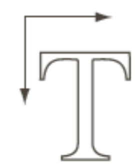
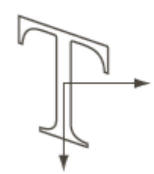
Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Image Registration

- Input and output images are available but the transformation function is unknown.

  Goal: estimate the transformation function and use it to register the two images.

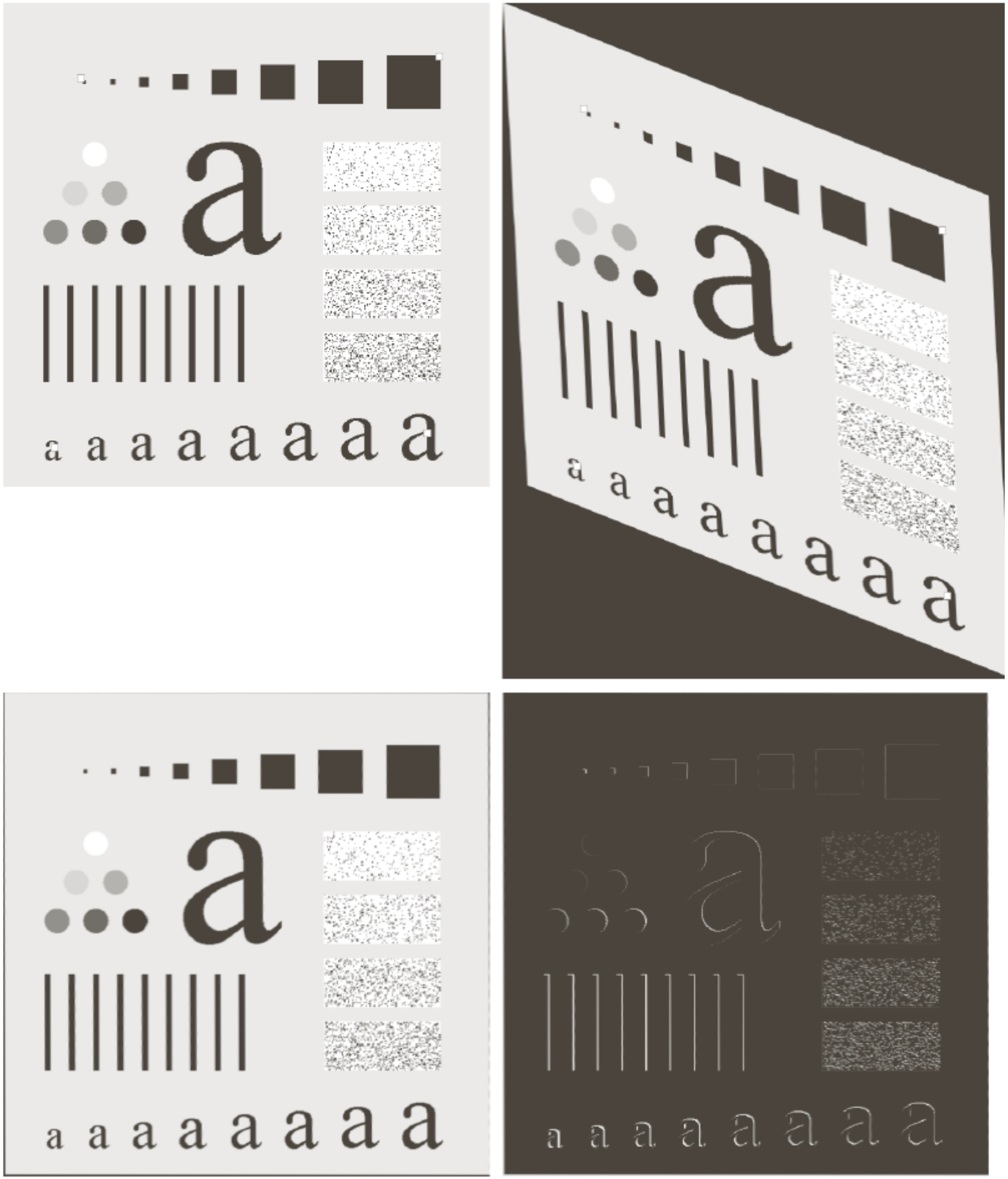
- One of the principal approaches for image registration is to use tie points (also called control points)
  - The corresponding points are known precisely in the input and output (reference) images.

Image Registration

- A simple model based on bilinear approximation:

$$\begin{cases} x = c_1 v + c_2 w + c_3 vw + c_4 \\ y = c_5 v + c_6 w + c_7 vw + c_8 \end{cases}$$

Where (v, w) and (x, y) are the coordinates of tie points in the input and reference images.



a	b
c	d

FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the borders). (d) Difference between (a) and (c), showing more registration errors.

Image Transform

- A particularly important class of 2-D linear transforms, denoted T(u, v)

$$T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) r(x,y,u,v)$$

where $f(x,y)$ is the input image,

$r(x,y,u,v)$ is the *forward transformation kernel*,

variables u and v are the transform variables,

u = 0, 1, 2, ..., M-1 and v = 0, 1, ..., N-1.

Image Transform

- Given T(u, v), the original image f(x, y) can be recoverd using the inverse tranformation of T(u, v).

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u,v) s(x,y,u,v)$$

where $s(x,y,u,v)$ is the *inverse transformation kernel*,

x = 0, 1, 2, ..., M-1 and y = 0, 1, ..., N-1.

Image Transform

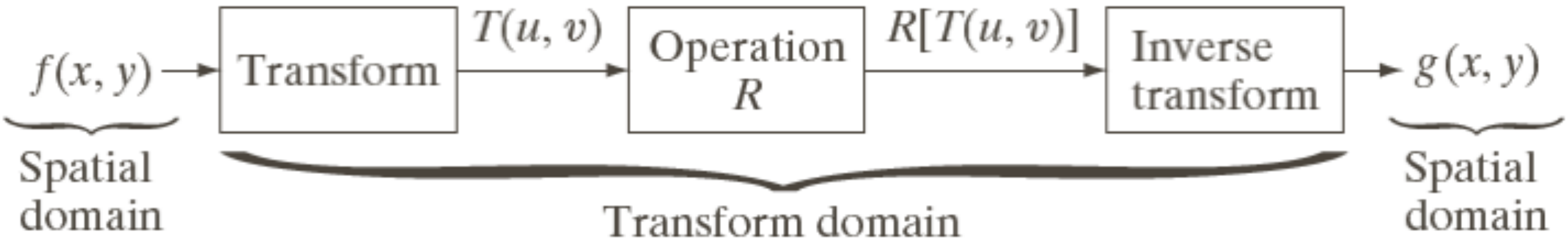
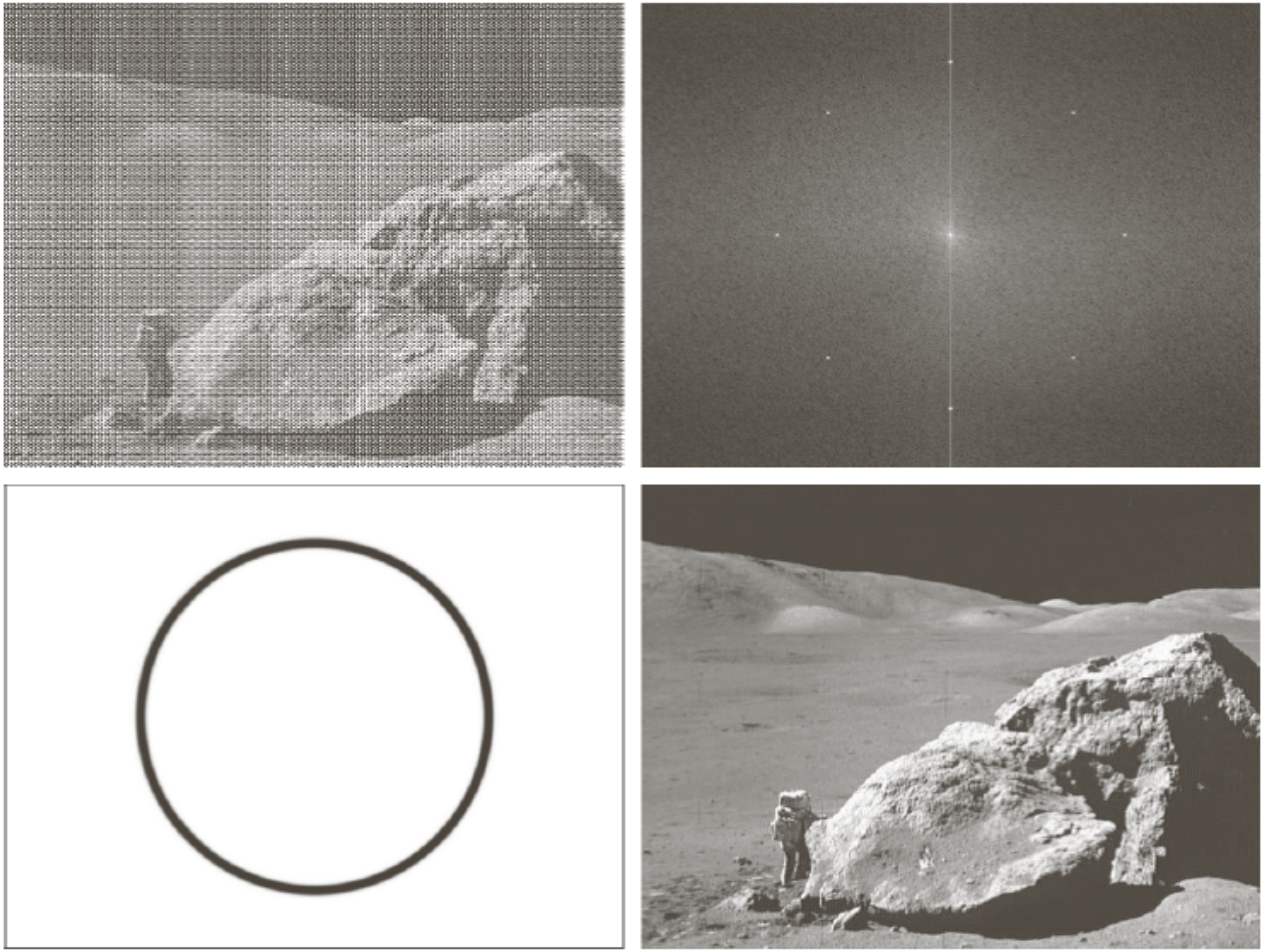


FIGURE 2.39
General approach for operating in the linear transform domain.

Example: Image Denoising by Using DCT Transform



a	b
c	d

FIGURE 2.40
(a) Image corrupted by sinusoidal interference. (b) Magnitude of the Fourier transform showing the bursts of energy responsible for the interference. (c) Mask used to eliminate the energy bursts. (d) Result of computing the inverse of the modified Fourier transform. (Original image courtesy of NASA.)

Forward Transform Kernel

$$T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) r(x,y,u,v)$$

The kernel $r(x,y,u,v)$ is said to be SEPERABLE if

$$r(x,y,u,v) = r_1(x,u) r_2(y,v)$$

In addition, the kernel is said to be SYMMETRIC if

$r_1(x,u)$ is functionally equal to $r_2(y,v)$, so that

$$r(x,y,u,v) = r_1(x,u) r_1(y,u)$$

The Kernels for 2-D Fourier Transform

The *forward* kernel

$$r(x, y, u, v) = e^{-j2\pi(ux/M + vy/N)}$$

Where $j = \sqrt{-1}$

The *inverse* kernel

$$s(x, y, u, v) = \frac{1}{MN} e^{j2\pi(ux/M + vy/N)}$$

2-D Fourier Transform

$$T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u,v) e^{j2\pi(ux/M + vy/N)}$$

Probabilistic Methods

Let z_i, $i = 0, 1, 2, ..., L-1$, denote the values of all possible intensities in an $M \times N$ digital image. The probability, $p(z_k)$, of intensity level z_k occurring in a given image is estimated as

$$p(z_k) = \frac{n_k}{MN},$$

where n_k is the number of times that intensity z_k occurs in the image.

$$\sum_{k=0}^{L-1} p(z_k) = 1$$

The mean (average) intensity is given by

$$m = \sum_{k=0}^{L-1} z_k p(z_k)$$

Probabilistic Methods

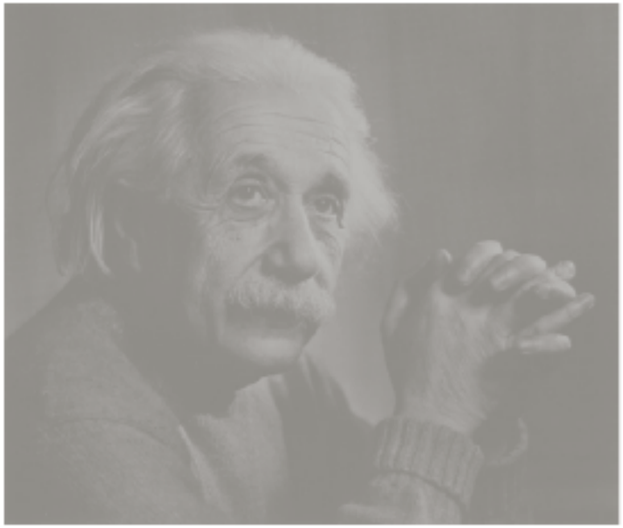
The variance of the intensities is given by

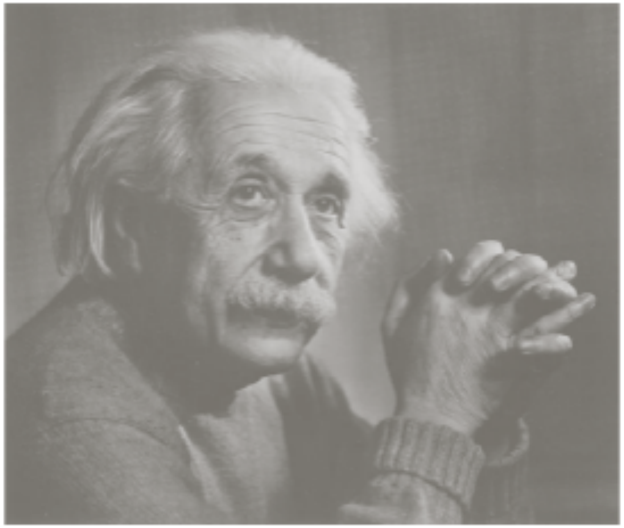
$$\sigma^2 = \sum_{k=0}^{L-1} (z_k - m)^2 p(z_k)$$

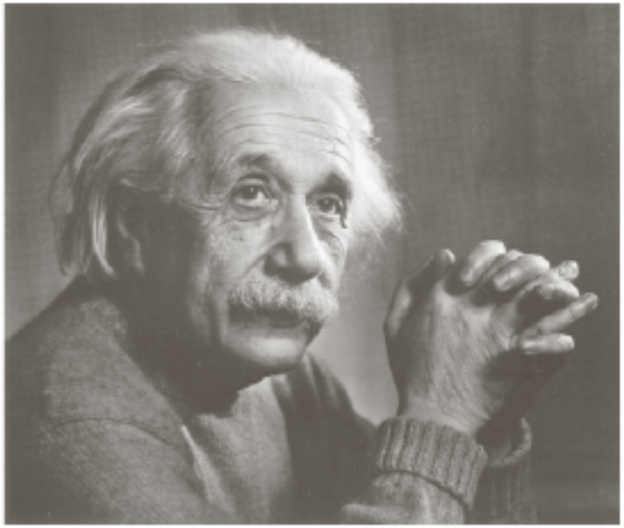
The n^{th} moment of the intensity variable z is

$$u_n(z) = \sum_{k=0}^{L-1} (z_k - m)^n p(z_k)$$

Example: Comparison of Standard Deviation Values







$\sigma = 14.3$ $\sigma = 31.6$ $\sigma = 49.2$