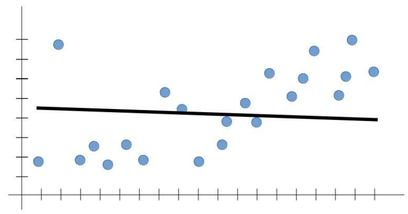
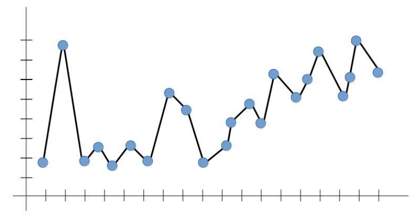
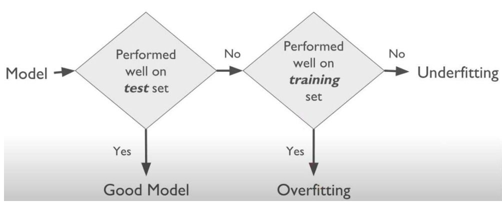
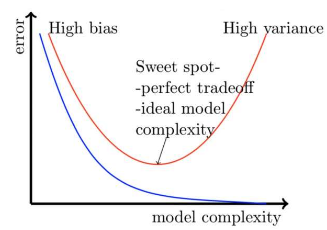
Underfitting



Overfitting







Performance Estimation of a Classifier

- Predictive accuracy works fine, when the classes are balanced
  - That is, every class in the data set are equally important
- In fact, data sets with imbalanced class distributions are quite common in many real life applications
- When the classifier classified a test data set with imbalanced class distributions then, predictive accuracy on its own is not a reliable indicator of a classifier's effectiveness.

Example 1: Effectiveness of Predictive Accuracy

- Given a data set of stock markets, we are to classify them as "good" and "worst". Suppose, in the data set, out of 100 entries, 98 belong to "good" class and only 2 are in "worst" class.
  - With this data set, if classifier's predictive accuracy is 0.98, a very high value!
    - Here, there is a high chance that 2 "worst" stock markets may incorrectly be classified as "good"
  - On the other hand, if the predictive accuracy is 0.02, then none of the stock markets may be classified as "good"

- Thus, when the classifier classified a test data set with imbalanced class distributions, then predictive accuracy on its own is not a reliable indicator of a classifier's effectiveness.
- This necessitates an alternative metrics to judge the classifier.

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Confusion Matrix

- A confusion matrix for a two classes (+, -) is shown below.

Predicted Value

Actual Value		C1	C2
	C1	True positive	False negative
	C2	False positive	True negative

	+	-
+	++	+-
-	-+	--

- There are four quadrants in the confusion matrix, which are symbolized as below.
  - True Positive (TP: f_{++}) : The number of instances that were positive (+) and correctly classified as positive (+v).
  - False Negative (FN: f_{+-}): The number of instances that were positive (+) and incorrectly classified as negative (-). It is also known as **Type 2 Error**.
  - False Positive (FP: f_{-+}): The number of instances that were negative (-) and incorrectly classified as (+). This also known as **Type 1 Error**.
  - True Negative (TN: f_{--}): The number of instances that were negative (-) and correctly classified as (-).

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Confusion Matrix

Note:

- $N_p = TP\,(f_{++}) + FN\,(f_{+-})$
  = is the total number of positive instances.
- $N_n = FP(f_{-+}) + TN(f_{--})$
  = is the total number of negative instances.
- $N = N_p + N_n$
  = is the total number of instances.
- (TP + TN) denotes the number of correct classification
- (FP + FN) denotes the number of errors in classification.
- For a perfect classifier FP = FN = 0, that is, there would be no Type 1 or Type 2 errors.

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Confusion Matrix

Example 2: Confusion matrix

A classifier is built on a dataset regarding Good and Worst classes of stock markets. The model is then tested with a test set of 10000 unseen instances. The result is shown in the form of a confusion matrix. The result is self explanatory.

Class	Good	Worst	Total	Rate(%)
Good	6954	46	7000	99.34
Worst	412	2588	3000	86.27
Total	**7366**	**2634**	**10000**	**95.42**

Predictive accuracy?

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Performance Evaluation Metrics

- Let us now define a number of metrics for the measurement of a classifier.
  - Assume that there are only two classes: + (positive) and – (negative)
  - Nevertheless, the metrics can easily be extended to multi-class classifiers (with some modifications)
- **True Positive Rate (TPR):** It is defined as the fraction of the positive examples predicted correctly by the classifier.

$$TPR = \frac{TP}{P} = \frac{TP}{TP+FN} = \frac{f_{++}}{f_{++}+f_{+-}}$$

  - This metrics is also known as ***Recall***, ***Sensitivity*** or ***Hit rate***.
- **False Positive Rate (FPR):** It is defined as the fraction of negative examples classified as positive class by the classifier.

$$FPR = \frac{FP}{N} = \frac{FP}{FP+TN} = \frac{f_{-+}}{f_{-+}+f_{--}}$$

  - This metric is also known as ***False Alarm Rate***.

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Performance Evaluation Metrics

- **False Negative Rate (FNR):** It is defined as the fraction of positive examples classified as a negative class by the classifier.

$$FNR = \frac{FN}{P} = \frac{FN}{TP+FN} = \frac{f_{+-}}{f_{++}+f_{+-}}$$

- **True Negative Rate (TNR):** It is defined as the fraction of negative examples classified correctly by the classifier

$$TNR = \frac{TN}{N} = \frac{TN}{TN+FP} = \frac{f_{--}}{f_{--}+f_{-+}}$$

  - This metric is also known as ***Specificity***.

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Performance Evaluation Metrics

- **Positive Predictive Value (PPV):** It is defined as the fraction of the positive examples classified as positive that are really positive

$$PPV = \frac{TP}{TP+FP} = \frac{f_{++}}{f_{++}+f_{-+}}$$

  - It is also known as *Precision*.
- **F_1 Score (F_1):** Recall (r) and Precision (p) are two widely used metrics employed in analysis, where detection of one of the classes is considered more significant than the others.
  - It is defined in terms of (r or TPR) and (p or PPV) as follows.

$$F_1 = \frac{2r.p}{r+p} = \frac{2TP}{2TP+FP+FN}$$

$$= \frac{2f_{++}}{2f_{++}+f_{-+}+f_{+-}} = \frac{2}{\frac{1}{r}+\frac{1}{p}}$$

Note

- F_1 represents the harmonic mean between recall and precision
- High value of F_1 score ensures that both Precision and Recall are reasonably high.

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Predictive Accuracy (ε)

- It is defined as the fraction of the number of examples that are correctly classified by the classifier to the total number of instances.

$$\varepsilon = \frac{TP+TN}{P+N}$$

$$= \frac{TP+TN}{TP+FP+FN+TN}$$

$$= \frac{f_{++}+f_{--}}{f_{++}+f_{+-}+f_{-+}+f_{--}}$$

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Error Rate ($\bar{\varepsilon}$)

- The error rate $\bar{\varepsilon}$ is defined as the fraction of the examples that are incorrectly classified.

$$\bar{\varepsilon} = \frac{FP + FN}{P + N}$$

$$= \frac{FP + FN}{TP + TN + FP + FN}$$

$$= \frac{f_{+-} + f_{-+}}{f_{++} + f_{+-} + f_{-+} + f_{--}}$$

Note

$\bar{\varepsilon} = 1 - \varepsilon.$

Analysis with Performance Measurement Metrics

- Based on the various performance metrics, we can characterize a classifier.
- We do it in terms of TPR, FPR, Precision and Recall and Accuracy
- **Case 1: Perfect Classifier**

When every instance is correctly classified, it is called the perfect classifier. In this case, $TP = P$, $TN = N$ and CM is

$TPR = \frac{P}{P} = 1 = \text{Recall}$

$FPR = \frac{0}{N} = 0$

$Precision = \frac{P}{P} = 1$

$F_1\ Score = \frac{2 \times 1}{1+1} = 1$

$Accuracy = \frac{P+N}{P+N} = 1$

		Predicted Class	
		+	-
Actual class	+	P	0
	-	0	N

Analysis with Performance Measurement Metrics

- **Case 2: Worst Classifier**

When every instance is wrongly classified, it is called the worst classifier. In this case, $TP = 0$, $TN = 0$ and the CM is

$TPR = \frac{0}{P} = 0 = \text{Recall}$

$FPR = \frac{N}{N} = 1$

$Precision = \frac{0}{N} = 0$

$F_1\ Score$ = Not applicable as $Recall + Precision = 0$

Accuracy $= \frac{0}{P+N} = 0$

		Predicted Class	
		+	-
Actual class	+	0	P
	-	N	0

Analysis with Performance Measurement Metrics

- **Case 3: Ultra-Liberal Classifier**

The classifier always predicts the + class correctly. Here, the False Negative (FN) and True Negative (TN) are zero. The CM is

$TPR = \frac{P}{P} = 1 = \text{Recall}$

$FPR = \frac{N}{N} = 1$

$Precision = \frac{P}{P+N}$

$F_1\ Score = \frac{2P}{2P+N}$

Accuracy $= \frac{P}{P+N}$

		Predicted Class	
		+	-
Actual class	+	P	0
	-	N	0

Analysis with Performance Measurement Metrics

- **Case 4: Ultra-Conservative Classifier**

This classifier always predicts the - class correctly. Here, the False Negative (FN) and True Negative (TN) are zero. The CM is

$TPR = \frac{0}{P} = 0 = \text{Recall}$

$FPR = \frac{0}{N} = 0$

$Precision$ = Not applicable (as $TP + FP = 0$)

$F_1\ Score$ = Not applicable

Accuracy $= \frac{N}{P+N}$

		Predicted Class	
		+	-
Actual class	+	0	p
	-	0	N

Predictive Accuracy versus TPR and FPR

- One strength of characterizing a classifier by its *TPR* and *FPR* is that they do not depend on the relative size of *P* and *N*.
    - The same is also applicable for *FNR* and *TNR* and others measures from CM.
- In contrast, the *Predictive Accuracy*, *Precision*, *Error Rate*, F_1 *Score*, etc. are affected by the relative size of *P* and *N*.
- *FPR*, *TPR*, *FNR* and *TNR* are calculated from the different rows of the CM.
    - On the other hand Predictive Accuracy, etc. are derived from the values in both rows.
- This suggests that *FPR*, *TPR*, *FNR* and TNR are more effective than *Predictive Accuracy*, etc.