Computer Graphics

Line Drawing

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CG Notes

Line Drawing

2D

Geometric Primitives

→ lines / poly lines
→ circles / ellipses
→ curves / splines

2D + texture + lighting (illumination) (shading)

= 3D Models

wireframe / mesh

↓ texture

solid modeling

metallic reflective properties

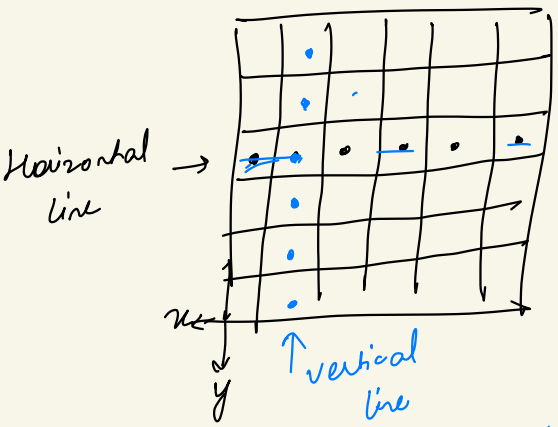
ambient light ⇒ ↓ shading

matte / shiny

Basic Concepts

Raster scan ⇒

pixel / pel ⇒ addressable locations



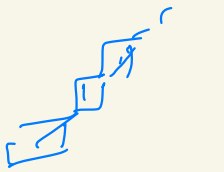
→ They have same width
→ Straight lines
→ More height (effective spacing is not much)

$x_{i+1} = x_{i+1}$
$y_{i+1} = y_i$

$x_{i+1} = x_i$
$y_{i+1} = y_i + 1$

x_i, y_i's whole numbers

No fraction / subpixel location



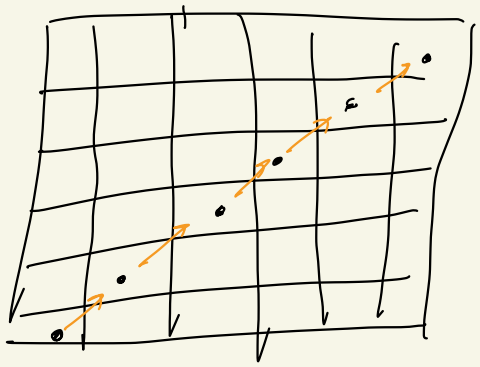
1, 1

↓

1, 1.123

↓

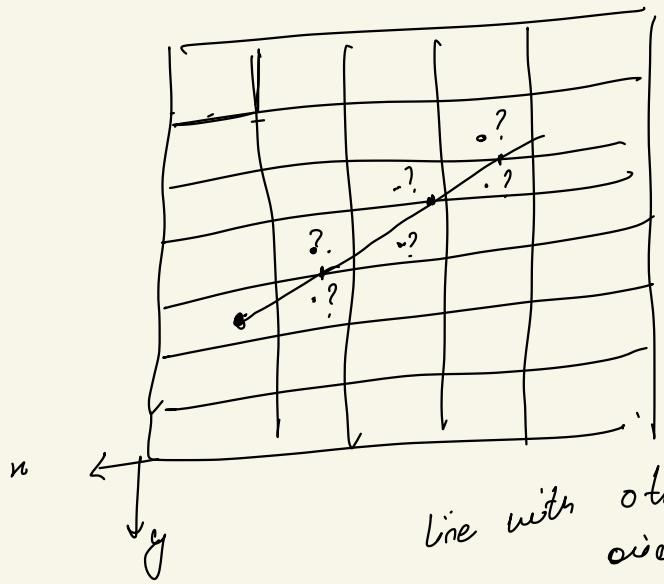
subpixel location



→ diagonal lines
→ width is not constant
→ More spacing is b/w them thus this results in less brightness

(3, 3) → 3, 4.5
3, 4 / 3, 5

⇒ They are of finite resolution & we have to approximate the pixels



Line with other orientation

⇒ Whenever you have such subpixel locations for lines ∉ {0°, 90°, ±45°}

↓

Line Drawing algos.

Incremental Method (increase)

1) Curr_position = Start
   Step ← increment
2) if |curr_position − End| < Accuracy
   then Goto step 5
   if (Curr_position < End) then Goto Step 3
   if (Curr_position > End) then Goto Step 4
3) Curr_position = Curr_position + Step, Goto step 2
4) Curr_position = Curr_position − Step, Goto step 2
5) Stop

Approximate decimation correct pixel

alg

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   if (curr_position < End) then Goto Step 3
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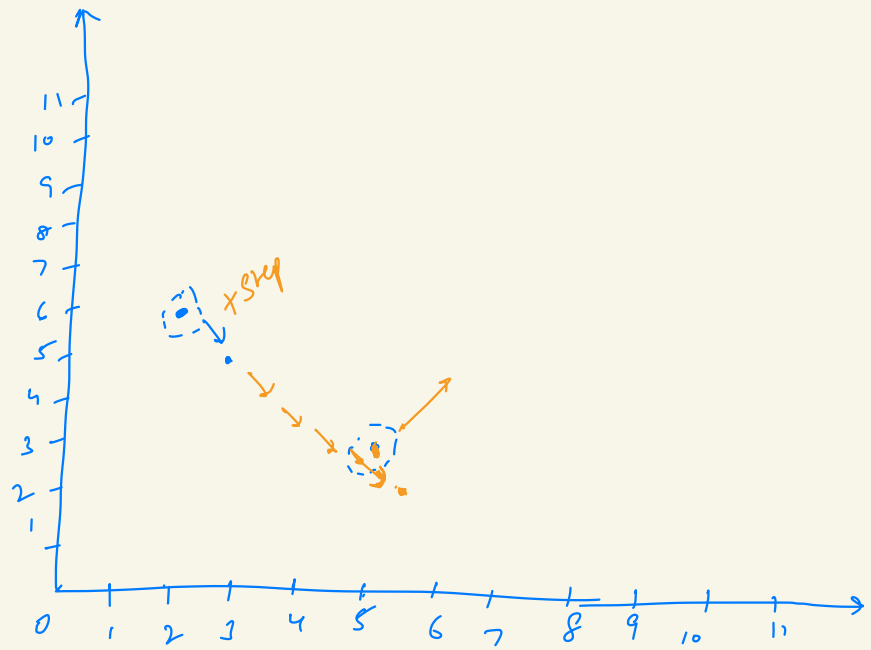
5) Stop

Approximate deviation correct path

(x, 6)
(5, 3)
step : 0.5



$y = mx + b$

$x_i, y_i \rightarrow x_{i+1}, y_{i+1}$

$x_{i+1} = x_i + 1$

$y_{i+1} = mx_{i+1} + b$

Direct method of line drawing

Line equation is given as:

$y = mx + b$

(m: slope, b: intercept)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

$|m| \leq 1$

$x_{i+1} = x_i + 1$

$y_{i+1} = m x_{i+1} + b$

or

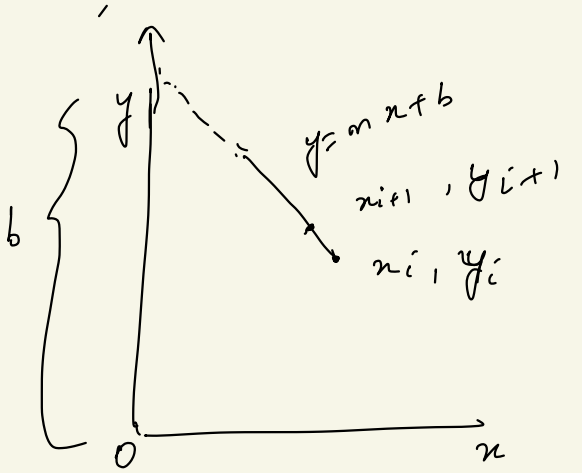
$|m| > 1$

$y_{i+1} = y_i + 1$

x_{i+1}

$y_{i+1} = m x_{i+1} + b$

$$\frac{y_{i+1} - b}{m} = x_{i+1}$$



Numerical (1, 2) (5, 3)

$$m = \frac{3-2}{5-1} = \frac{1}{4}, \quad m \leq 1$$

b = ?

(1,2)

$y = mx + b$

$b = 7/4$

$2 = \frac{1}{4} \times 1 + b$

$b = \frac{7}{4}$

(5, 3)

$3 - \frac{5}{4} = b$

$b = \frac{7}{4}$

x_i	y_i	x_{i+1}	y_{i+1}
1	2	?	?

1, 2 5, 3

1) As $m \leq 1$

$x_{i+1} = x_i + 1$

$= 1 + 1 = 2$

$y_{i+1} = \frac{1}{4} \times 2 + \frac{7}{4} = 2.25$

2) $x_{i+1} = x_i + 1 = 2 + 1 = 3$

$y_{i+1} = \frac{1}{4} \times 3 + \frac{7}{4} = 2.5$

3) $x_{i+1} = x_i + 1 = 3 + 1 = 4$

$y_{i+1} = \frac{1}{4} \times 4 + \frac{7}{4} = 2.75$

4) $x_{i+1} = x_i + 1 = 5$

$y_{i+1} = 5 \times \frac{1}{4} + \frac{7}{4} = 3.0$

x_i	y_i	x_{i+1}	y_{i+1}
1	2	2	2.25
2	2.25	3	2.5
3	2.5	4	2.75
4	2.75	5	3 ← End point

Problem: We get subpixel locations (decimal)

↓

Soln: DDA

Digital Differential Analyzer.

DDA Digital Differential Analyzer

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Delta y = m \cdot \Delta x = \frac{y_2 - y_1}{x_2 - x_1} \cdot \Delta x$$

$$\Delta x = \frac{\Delta y}{m} = \Delta y \cdot \frac{x_2 - x_1}{y_2 - y_1}$$

if $|\Delta x| \geq |\Delta y|$

Then: $\Delta x = 1$

Then: $x_{i+1} = x_i + \Delta x$

$x_{i+1} = x_i + 1$

$y_{i+1} = y_i + \Delta y$

$= y_i + m \cdot \Delta x$

$y_{i+1} = y_i + m$

if $|\Delta x| < |\Delta y|$

Then: $\Delta y = 1$

Then: $x_{i+1} = x_i + \Delta x$

$= x_i + \frac{\Delta y}{m}$

$x_{i+1} = x_i + \frac{1}{m}$

$y_{i+1} = y_i + \Delta y$

$y_{i+1} = y_i + 1$

Ques $(1, 1)$ & $(4, 3)$

Start End

$m = \frac{3-1}{4-1} = \frac{2}{3} = 0.67$ $\quad \Delta x = 3$ $\quad \Delta y = 2$

x_i	y_i	x_{i+1}	y_{i+1}
1	1	2	1.67
2	1.67	3	2.34
3	2.34	4	3.01

Check $|\Delta x| \geq |\Delta y|$

$\therefore \Delta x = 1$

Then $x_{i+1} = x_i + \Delta x = 1 + 1 = 2$

1) $y_{i+1} = y_i + m = 1 + m = 1.67$

2) $x_{i+1} = x_i + 1 = 2 + 1 = 3$

$y_{i+1} = y_i + m = 1.67 + 0.67 = 2.34$

3) $x_{i+1} = x_i + 1 = 3 + 1 = 4$

$y_{i+1} = y_i + m = 2.34 + 0.67 = 3.01$

subpixel → steps jagged → approximate multiplication value of 0.5

1) Read line end points x_1, y_1 & x_2, y_2

2) $\Delta x = |x_2 - x_1|$ $\Delta y = |y_2 - y_1|$

3) if $\Delta x \geq \Delta y$ then
- length $= \Delta x$
else length $= \Delta y$

4) Select raster unit

$$\Delta x = \frac{x_2 - x_1}{\text{length}}$$

$$\Delta y = \frac{y_2 - y_1}{\text{length}}$$

This will make to $\Delta x = 1$ or $\Delta y = 1$

5) $x_{i+1} = x_i + 0.5 \times \text{sign}(\Delta x)$

$y_{i+1} = y_i + 0.5 \times \text{sign}(\Delta y)$

6) Plot points

Ques 0, 0 4, 6

$\Delta x = 4 - 0 = 4$

$\Delta y = 6 - 0 = 6$

$\Delta y > \Delta x$

length $= \Delta y = 6$

x_i	y_i	x_{i+1}	y_{i+1}
0	0	0.5	0.5
0.5	0.5	1.167	1.5
		1	

(0, 0)

$\Rightarrow (1, 1)$

$\Rightarrow (2, 2)$

4 6

Raster unit : $\Delta x = \frac{4}{6}$ $\Delta y = \frac{6}{6} = 1$

$x_{i+1} = x_i + 0.5 \times \text{sign}(\Delta x)$

$= 0 + 0.5 = 0.5$

$y_{i+1} = y_i + 0.5 \times \text{sign}(\Delta y)$

$= 0 + 0.5 \times \text{sign}(1) = 0.5$

Plot integer (x_{i+1}) = integer $(0.5) = 1$] $(1, 1)$

integer $(0.5) = 1$