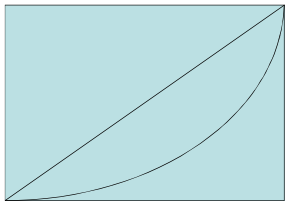
Gini Index

Measure of resource inequality in a population. Varies from 0 to 1, zero means no inequality and 1 maximum possible inequality.

Based on the Lorenz curve, which plots cumulative family income against the number of families from poorest to the richest.



Lorenz Curve

The Gini Index is the ratio of the area between the Lorenz curve and the 45-degree line to the area under the 45-degree line.

Smaller the ratio, the more evenly distributed the wealth.

The example shows wide variation in the wealth distribution in various countries.

Brazil	61
Mexico	53
Malaysia	49
Thailand, USA	41
China, Russia	40
India	38
UK	37
Australia	35
Canada, Indonesia	32
Sweden, Japan	25

Queensland Crime Data

Queensland Government has analysed crime data using the Gini Index to find how different types of crimes are distributed in the state.

Stealing from Homes	0.17
Unlawful entry shops	0.21
Drug offences	0.22
Assault	0.23
Fraud	0.26
Motor Vehicle theft	0.30
Kidnapping	0.33
Armed robbery	0.38
Liquor offences	0.52
Women offences	0.65

Gini Index

Varies from 0 to 1, zero means no inequality and 1 maximum possible inequality.

$$G(T) = 1 - \sum_{j=1}^{n} p_j^2$$

p_j is the relative frequency of class j in S. Consider tossing a coin:

G = 1 – 0.5*0.5 – 0.5*0.5 = 0.5

This comes out to 0.5 and is the maximum value for an event with two possible values.

Gini Index

Consider a loaded coin with 70% chance of a head and another coin with heads on both sides.

G = 1 – 0.7*0.7 – 0.3*0.3 = 0.42
G = 1 – 1.0*1.0 = 0

The Gini Index for a dice with six possible outcomes with equal probability is:

G = 1 - 6 * (1/6) * (1/6) = 5/6 = 0.833

Gini Index

If a loaded dice had much more chance of getting a 6, say 50% or even 75%, does the roll of the dice has lower or higher Gini Index? The Index is given by:

50% G = 1 – 5 * (0.1 * 0.1) – 0.5 * 0.5 = 0.70

75% I = 1 – 5 * (0.05 * 0.05) – 0.75 * 0.75 = 0.425

Clearly the index is largest with the largest uncertainty.

Gini Index

If a data set S is split into two subsets P and N with sizes p and n respectively, the Gini index of the split data contains examples from 2 classes, the Gini Index is defined as

$$G(S) = \frac{p}{s} G(P) + \frac{n}{s} G(N)$$

Split attribute

Once we have computed the Gini Index for all the attributes, how do we select the attribute to split the data?

In the case of information theory we selected the attribute with the highest information gain.

Back to the Example

Sore Throat	Fever	Swollen Glands	Congestion	Headache	Diagnosis
Yes	Yes	Yes	Yes	Yes	Strep throat
No	No	No	Yes	Yes	Allergy
Yes	Yes	No	Yes	No	Cold
Yes	No	Yes	No	No	Strep throat
No	Yes	No	Yes	No	Cold
No	No	No	Yes	No	Allergy
No	No	Yes	No	No	Strep throat
Yes	No	No	Yes	Yes	Allergy
No	Yes	No	Yes	Yes	Cold
Yes	Yes	No	Yes	Yes	Cold

Back to the Example

There are 10 (s = 10) samples and three classes.

Strep throat = t = 3
Cold = c = 4
Allergy = a = 3

G = 1 – 2*(3/10 * 3/10) – (4/10 * 4/10) = 0.66

Let us now consider using the various symptoms to split the sample.

Example

Sore Throat
Yes has t = 2, c = 2, a =1, total 5
No has t = 1, c = 2, a = 2, total 5
G(y) = 1 –2*(2/5 * 2/5) – (1/5 * 1/5) = 0.64
G(n) = G(y) = 0.64
G = 0.5*0.64 + 0.5*0.64 = 0.64

Fever
Yes has t = 1, c = 4, a =0, total 5
No has t = 2, c = 0, a = 3, total 5
G(y) = 1 – 1/5 * 1/5 – 4/5 * 4/5 = 0.32
G(n) = 1 – 2/5 * 2/5 – 3/5 * 3/5 = 0.48
G = 0.5 G(y) + 0.5 G(n) = 0.40

Example

Swollen Glands
Yes has t = 3, c = 0, a =0, total 3
No has t = 0, c = 4, a = 3, total 7
G(y) = 1 – 1 = 0
G(n) = 1 – 4/7 * 4/7 – 3/7 * 3/7 = 0.511
G = 0.3*G(y) + 0.7*G(n) = 0.358

Congestion
Yes has t = 1, c = 4, a =3, total 8
No has t = 2, c = 0, a = 0, total 2
G(y) = 1 –1/8 * 1/8 – 4/8 * 4/8 – 3/8 * 3/8 = 0.594
G(n) = 0
G = 0.8 * G(y) + 0.2 * G(n) = 0.475

Example

Headache
Yes has t = 2, c = 2, a =1, total 5
No has t = 1, c = 2, a = 2, total 5
G(y) = 1 - 2*(2/5 * 2/5) – (1/5 * 1/5) = 0.64
G(n) = G(y)
G = 0.5*0.64 + 0.5*0.64 = 0.64

So the values for the Gini Index are:

Sore Throat	0.64
Fever	0.40
Swollen Glands	0.358
Congestion	0.475
Headache	0.64

Example

We select the attribute with the smallest value of the index as the split attribute. So the attribute is Swollen Glands.

We can now take "Swollen Glands" out of consideration and take the three instances where the Swollen Glands symptom value was Yes also out of consideration since they have already been classified.

Next we look at the remaining data and remaining attributes and find the next split attribute.

Understanding Decision Trees

- Decision trees may be easily represented by rules.
- One rule is created for each path from the root to a leaf.
- Each internal node is a test and the leaf node gives the class prediction
- Example

  IF swollen glands = yes THEN diagnosis = strep throat

  IF swollen glands = no and fever = yes then ??
- Each class may have a number of rules

Bayesian Classification

- A different approach: Does not require building of a decision tree. Assume hypothesis that given data belongs to a given class. Calculate probabilities for the hypothesis. This is among the most practical approaches for certain types of problems.
- Each training example can incrementally increase/decrease the probability that a hypothesis is correct.

Bayes Theorem

- A very important theorem.
- Hypothesis is that a given data belongs to class C. Without any information about the data, let the probability of any data belonging to C be P(C). This is called apriori probability of the hypothesis.
- Given data *D,* posteriori probability of a hypothesis h, that is D is in class C, is written as P(h | D). It is given by the Bayes theorem

$$P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)}$$

Bayes Theorem

Consider the Bayes theorem again

$$P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)}$$

- P(h | D) is probability of h being true (that is D belongs to class C) given that D is known.
- P(D | h) is probability of obtaining data D if we knew the hypothesis was correct i.e. D belongs to class C.
- P(h) is just the probability of h without any information
- P(D) is just the probability of obtaining D without any information

Back to the Example

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No	No	No	Yes	No	Allergy
No	No	Yes	No	No	Strep throat
Yes	No	No	Yes	Yes	Allergy
No	Yes	No	Yes	Yes	Cold
Yes	Yes	No	Yes	Yes	Cold

Bayes Theorem

Suppose the data D is {yes, no, no, yes, yes}
The hypothesis h is that D has cold.

$$P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)}$$

P(h | D) is probability of D having cold knowing D.
P(D | h) is probability of obtaining data D if we knew D had cold.
P(h) is just the probability of having cold without any information
P(D) is just the probability of obtaining D without any information

Bayes Theorem

What can we compute? P(h | D), P(D | h), P(h) and P(D)?

$$P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)}$$

To compute P(h | D) we need to compute the RHS.
Actually what we really need is to compute P(h | D) for h = cold, for h = strep throat and for h = allergy. We then choose the diagnosis that has the highest probability.

Since we are comparing we don't need to worry about P(D) which we cannot compute.

Bayes Theorem

- Compute the maximum posteriori hypothesis by computing P(D | h)P(h) for each of the three hypothesis.
- There are 10 (s = 10) samples and three classes.
  Strep throat = t = 3
  Cold = c = 4
  Allergy = a = 3
- P(ST) = 0.3, P(C) = 0.4 and P(A) = 0.3
- P(D | h) = P({yes, no, no, yes, yes} | h) = P(sore throat = yes | h)*P(fever=no | h) * P(swollen glands=no | h) * P(congestion=yes | h) * P(headache=yes | h)

Example

- We compute P(D | h)P(h) for each of the three hypothesis given P(ST) = 0.3, P(C) = 0.4 and P(A) = 0.3.
- P(D | h) = P({yes, no, no, yes, yes} | h)
- P(sore throat = yes | h)
- P(fever = no | h)
- P(swollen glands = no | h)
- P(congestion = yes | h)
- P(headache = yes | h)

Example

Sore Throat	Fever	Swollen Glands	Congestion	Headache	Diagnosis
No	No	No	Yes	Yes	Allergy
No	No	No	Yes	No	Allergy
Yes	No	No	Yes	Yes	Allergy
Yes	Yes	No	Yes	No	Cold
No	Yes	No	Yes	No	Cold
No	Yes	No	Yes	Yes	Cold
Yes	Yes	No	Yes	Yes	Cold
Yes	Yes	Yes	Yes	Yes	Strep throat
Yes	No	Yes	No	No	Strep throat
No	No	Yes	No	No	Strep throat

Cold

Sore Throat	Fever	Swollen Glands	Congestion	Headache
Y	Y	N	Y	N
N	Y	N	Y	N
N	Y	N	Y	Y
Y	Y	N	Y	Y
0.5	0.0	1.0	1.0	0.5

Strep Throat

Sore Throat	Fever	Swollen Glands	Congestion	Headache
Y	Y	Y	Y	Y
Y	N	Y	N	N
N	N	Y	N	N
2/3	2/3	0.0	1/3	1/3

Allergy

Sore Throat	Fever	Swollen Glands	Congestion	Headache
N	N	N	Y	Y
N	N	N	Y	N
Y	N	N	Y	Y
1/3	1.0	1.0	1.0	2/3

P(D | h)P(h)

P(D | h) = P({yes, no, no, yes, yes} | h) = P(sore throat = yes | h)*P(fever=no | h) * P(swollen glands=no | h) * P(congestion=yes | h) * P(headache=yes | h)

P(D | Allergy) = 2/9
P(D | Strep Throat) = 0
P(D | Cold) = 0

Values of P(D | h)P(h) are 2/9*0.3, 0 and 0
The diagnosis therefore is Allergy.

Assumptions

- Assuming that attributes are independent
- Assuming that the training sample is a good sample to estimate probabilities.
- These assumptions are often not true in practice, as attributes are often correlated.
- Other techniques have been designed to overcome this limitation.

Summary

- Supervised classification
- Decision tree using information measure and Gini Index
- Pruning and Testing
- Bayes Theorem