
Tutorial Problem Set 1

CS 152 – Abstractions and Paradigms in Programming

Date - 11 Jan 2017

1. Write a function (`has-solution a b c`) which returns `#t` if the diophantine equation $ax + by = c$ has integer solutions for x and y .
2. Write a function (`sub x y`) which subtracts y from x . You can assume that x is larger than y . Define `sub` in terms of an appropriate function (`sub-single x y z`) with constraints on the values of x , y , and z . The function should check these constraints. You can also use the function `convert` to left-shift and add.
3. Write a function (`ak-mult x y`) to multiply two numbers using the Al-Khwarizmi method. The function should never run out of memory, however large x and y may be. Look at section 1.2.1 of SICP for inspiration.
4. Define a function (`div x y`), $y \neq 0$, which will return a pair of numbers (q, r) such that $x = yq + r$ and $0 \leq r < y$. Like Al-kharizmi's method, this should also work by repeated halvings of x . For this problem and some of the later ones, use the magic function `cons` to pack a pair of numbers and the functions `car` and `cdr` to extract the first and second component of the pair.
5. Given two numbers a and b , write a function (`coeffs a b`) which will return a pair of integers x and y such that $ax + by = \gcd(a, b)$.
6. Given two integers x and n and an integer exponent y , write a function (`modexp x y n`) which will output: $x^y \bmod n$.
7. A Carmichael number is a composite number p , such that for every co-prime in the range $1 \leq a < p$, $a^{(p-1)} \equiv_p 1$. Define a function (`carmichael n`) which will give the n th Carmichael number.
8. Write a function (`inverse e n`) which will return the inverse of e , modulo n .
9. Write a function (`is-prime n`) to implement the Fermats little theorem based probabilistic algorithm to test whether n is a prime. Assume that n is not a Carmichael number. Use the function (`random k`) to generate a random number in the range $0 \dots k - 1$.
10. Goldbachs conjecture says that every positive even number greater than 2 is the sum of two prime numbers. Example: $28 = 5 + 23$. It is one of the most famous seeming fact in number theory that has not been proved to be correct in the general case. It has been numerically confirmed up to very large numbers. Write a function (`goldbach m`) to find the two prime numbers that sum up to a given number m .