Problem 4 :-

(i) Given bivariate Gaussian random variable

$Y := (Y_1, Y_2)$

Let variance of random variable $Y_1$ is $\sigma_1$

and variance of random variable $Y_2$ is $\sigma_2$

then covariance matrix $C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$

where $c_{11} = cov(Y_1, Y_1) = \sigma_1^2$

$c_{22} = cov(Y_2, Y_2) = \sigma_2^2$

$c_{12} = cov(Y_1, Y_2)$

$c_{21} = cov(Y_2, Y_1)$ $\rightarrow$ equal

Given 'C' is a diagonal matrix

$\therefore c_{12} = c_{21} = 0$

$\Rightarrow cov(Y_1, Y_2) = 0 = cov(Y_2, Y_1)$

$\therefore$ Covariance of $Y_1, Y_2$ is zero

$\Rightarrow$ random variables $Y_1, Y_2$ are uncorrelated.

(ii) Given bivariate random Variable $z := (z_1, z_2)$

So the pair $(z_1, z_2)$ of random Variable has a bivariate normal distribution means that every constant (i.e. not random) linear combination ~~ose~~ $az_1 + bz_2$ of $z_1, z_2$ has a univariate normal distribution.

In this case if $z_1, z_2$ are uncorrelated then they are independent.

If $z_1, z_2$ are not bivariate random Variables then we can't say that they are independent.