Verifying Boolean Algebra

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For verifying whether the given set(B, I+,I.,+,.) forms a Boolean algebra, we need to verify:

1. Commutativity of the two operations "." and "+" which means a.b=b.a and a+b=b+a
2. Distributive Property of "." over "+" and "+" over "." which means a.(b+c)=a.b+a.c and a+(b.c)=(a+b).(a+c)
3. Associativity Property of "+" and "." which means a+(b+c)=(a+b)+c and a.(b.c)=(a.b).c
4. Identity Property which means a+(I+)=a and a.(I.)=a(Since, 1 is identity on + and N is identity on .)
5. Complement Property which means a+a'=(I.) and a.a'=(I+)

Proving commutativity and associativity are straightforward as

1. LCM(a,b)=LCM(b,a)
2. GCD(a,b)=GCD(b,a)
3. LCM(LCM(a,b),c)=LCM(a,b,c)
4. GCD(GCD(a,b),c)=GCD(a,b,c)

Since a$\in$ B is divisor of N,we get GCD(a,N)=a and LCM(a,1)=a proving identity property.For proving distributive property we have to make use of facts of fundamental theorem of arithmetic about prime-factorization and laws of following logical operations:

1. min(a,max(b,c))=max(min(a,b),min(a,c))
2. max(a,min(b,c))=min(max(a,b),max(a,c))

For complement property to be true,we need to show that the number is square free,i.e, no perfect square other than 1 divides N. My entry number is 2019CS10369. Hence in my case N=69 i.e, N=3*23. So there is no perfect square other than 1 that divides 69. Hence the set (B,+,.,I+,I.) forms a valid Boolean-Algebra when N=69.