Part 1 : Trajectory

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$

$$\ddot{q}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

where,

$$
\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}
=
\begin{bmatrix}
1 & t_0 & t_0^2 & t_0^3 & t_0^4 & t_0^5 \\
0 & 1 & 2t_0 & 3t_0^2 & 4t_0^3 & 5t_0^4 \\
0 & 0 & 2 & 6t_0 & 12t_0^2 & 20t_0^3 \\
1 & t_f & t_f^2 & t_f^3 & t_f^4 & t_f^5 \\
0 & 1 & 2t_f & 3t_f^2 & 4t_f^3 & 5t_f^4 \\
0 & 0 & 2 & 6t_f & 12t_f^2 & 20t_f^3
\end{bmatrix}^{-1}
\begin{bmatrix} q_0 \\ \dot{q}_0 \\ \ddot{q}_0 \\ q_f \\ \dot{q}_f \\ \ddot{q}_f \end{bmatrix}
$$

Example - 1st Waypoint for 'z',

$$t_0 = 0 \quad , \quad t_f = 5 \quad , \quad z_0 = 0 \quad . \quad z_f = 1$$

$$\dot{z}_0 = \dot{z}_f = \ddot{z}_0 = \ddot{z}_f = 0$$

$$
\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 \\
1 & 5 & 25 & 125 & 625 & 3125 \\
0 & 1 & 10 & 75 & 500 & 3125 \\
0 & 0 & 2 & 30 & 300 & 2500
\end{bmatrix}^{-1}
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}
$$

Part 2 : Control Inputs

$$\ddot{q} = f(q,\dot{q}) + g(q,\dot{q})u$$

Let us consider the surface as, $S = \dot{e} + \lambda e$

where, $e = q - q_d$

$\dot{e} = \dot{q} - \dot{q}_d$

Sliding condition: $S\dot{S} \leq -K|s|$

$$\dot{S} = \ddot{e} + \lambda \dot{e}$$

$$= \ddot{q} - \ddot{q}_d + \lambda(\dot{q} - \dot{q}_d)$$

$$= f(q,\dot{q}) + g(q,\dot{q})u - \ddot{q}_d + \lambda(\dot{q} - \dot{q}_d)$$

$$S\dot{S} = S \cdot g(q,\dot{q}) \left[\frac{f(q,\dot{q}) - \ddot{q}_d + \lambda(\dot{q} - \dot{q}_d)}{g(q,\dot{q})} + u \right]$$

Design u such that $S\dot{S} \leq -K|s|$

$$\boxed{u = \frac{-f(q,\dot{q}) + \ddot{q}_d - \lambda(\dot{q} - \dot{q}_d) + u_r}{g(q,\dot{q})}}$$

where $u_r \rightarrow$ control input

$$u_r = -(\beta + K) \, sgn(s)$$

Hence we donot have uncertainities, $\beta = 0$

$$\Rightarrow u_r = -K \cdot sgn(s)$$

Now,

For z, $\qquad \ddot{z} = \frac{\cos\phi \cos\theta}{m} u_1 - g$

$$\Rightarrow f(q,\dot{q}) = -g \quad, \quad g(q,\dot{q}) = \frac{\cos\phi \cos\theta}{m}$$

$$e_z = z - z_d \quad, \quad \dot{e} = \dot{z} - \dot{z}_d$$

$$S_1 = \dot{e}_z + \lambda e_z$$

$$\Rightarrow u_1 = \frac{m}{\cos\phi \cos\theta} \left[g + \ddot{z}_d - \lambda_1(\dot{z} - \dot{z}_d) - K_1 \cdot sgn(s_1) \right]$$

For ϕ,

$$\ddot{\phi} = \dot{\theta}\dot{\psi}\,\frac{I_y - I_z}{I_x} - \frac{I_p}{I_x}\Omega\dot{\theta} + \frac{1}{I_x}u_2$$

$\underbrace{\phantom{\dot{\theta}\dot{\psi}\,\frac{I_y - I_z}{I_x} - \frac{I_p}{I_x}\Omega\dot{\theta}}}_{f(q,\dot{q})}$ $\underbrace{\phantom{\frac{1}{I_x}}}_{g(q,\dot{q})}$

$e_\phi = \phi - \phi_d$

$\dot{e}_\phi = \dot{\phi} - \dot{\phi}_d$

$\Rightarrow S_2 = \dot{e}_\phi + \lambda e_\phi$

$$u_2 = I_x\left[-\dot{\theta}\dot{\psi}\,\frac{I_y - I_2}{I_x} + \frac{I_p}{I_x}\Omega\dot{\theta} + \ddot{\phi}_d - \lambda_2\left(\dot{\phi} - \dot{\phi}_d\right) - K_2\,sgn\left(s_2\right)\right]$$

$$\Rightarrow u_2 = -\dot{\theta}\dot{\psi}\left(I_y - I_2\right) + I_p\Omega\dot{\theta} + I_x\left[\ddot{\phi}_d - \lambda_2\left(\dot{\phi} - \dot{\phi}_d\right) - K_2\cdot sgn\left(s_2\right)\right]$$

For θ,

$$\ddot{\theta} = \dot{\phi}\dot{\psi}\,\frac{I_2 - I_x}{I_y} + \frac{I_p}{I_y}\Omega\dot{\phi} + \frac{1}{I_y}u_3$$

$\underbrace{\phantom{\dot{\phi}\dot{\psi}\,\frac{I_2 - I_x}{I_y} + \frac{I_p}{I_y}\Omega\dot{\phi}}}_{f(q,\dot{q})}$ $\underbrace{\phantom{\frac{1}{I_y}}}_{g(q,\dot{q})}$

$e_\theta = \theta - \theta_d$

$\dot{e}_\theta = \dot{\theta} - \dot{\theta}_d \qquad \Rightarrow \qquad S_3 = \dot{e}_\theta + \lambda e_\theta$

$$\Rightarrow u_3 = -\dot{\phi}\dot{\psi}\left(I_2 - I_x\right) - I_p\Omega\dot{\phi} + I_y\left[\ddot{\theta}_d - \lambda_3\left(\dot{\theta} - \dot{\theta}_d\right) - K_3\cdot sgn\left(s_3\right)\right]$$

For ψ,

$$\ddot{\psi} = \dot{\phi}\dot{\theta}\,\frac{I_x - I_y}{I_2} + \frac{1}{I_2}u_4$$

$\underbrace{\phantom{\dot{\phi}\dot{\theta}\,\frac{I_x - I_y}{I_2}}}_{f(q,\dot{q})}$ $\underbrace{\phantom{\frac{1}{I_2}}}_{g(q,\dot{q})}$

$e_\psi = \psi - \psi_d$

$\dot{e}_\psi = \dot{\psi} - \dot{\psi}_d \qquad \Rightarrow \qquad S_4 = \dot{e}_\psi + \lambda e_\psi$

$$\Rightarrow u_4 = -\dot{\phi}\dot{\theta}\left(I_x - I_y\right) + I_2\left[\ddot{\psi}_d - \lambda_4\left(\dot{\psi} - \dot{\psi}_d\right) - K_4\cdot sgn\left(s_4\right)\right]$$

Using the boundary condition, $\text{sat}\left(\dfrac{s}{\phi}\right)$

Replace $\text{sgn}(s_i)$ with $\text{sat}\left(\dfrac{s_i}{\phi}\right)$ in u_1, u_2, u_3 & u_4

where $\phi > 0$

This is used to reduce the chattering.

Let $\phi = 0.9$

$$u_1 = \frac{m}{\cos\phi\cos\theta}\left[g + \ddot{Z}_d - \lambda_1(\dot{Z} - \dot{z}_d) - K_1\cdot\text{sat}\left(\frac{s_1}{\phi}\right)\right]$$
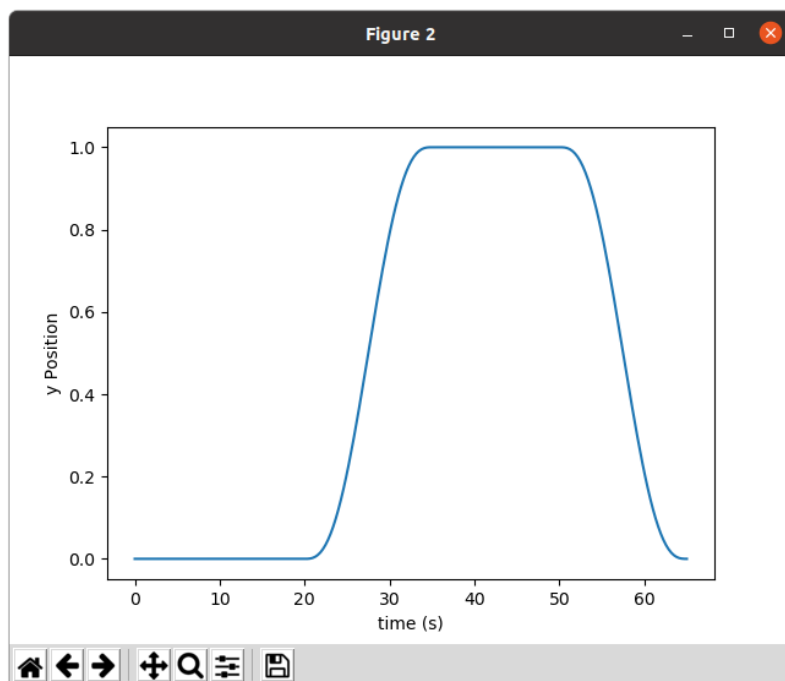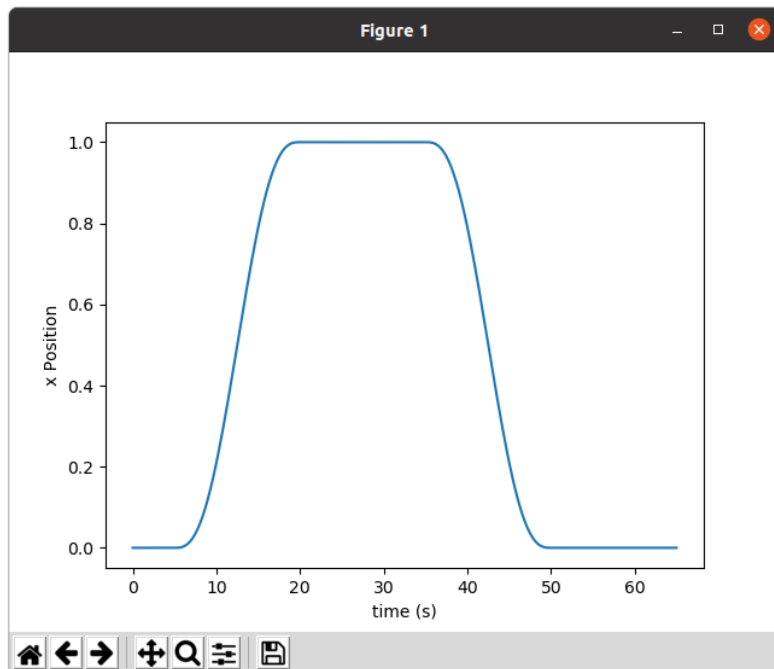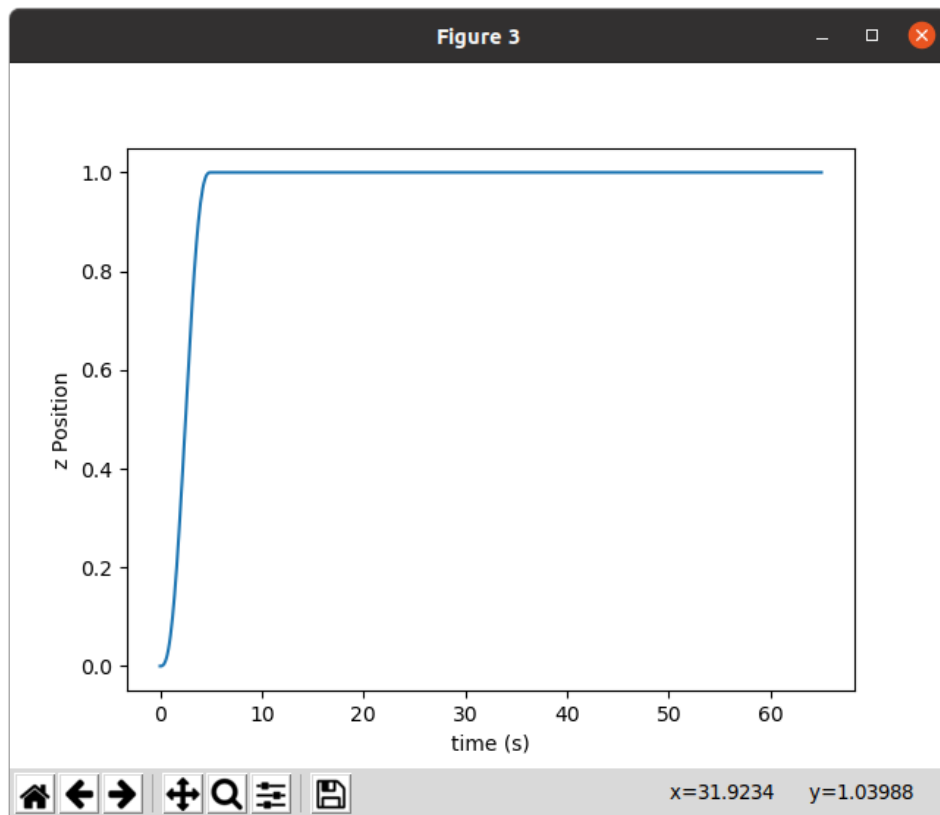
$$u_2 = -\dot{\theta}\dot{\psi}\left(I_y - I_z\right) + I_p\Omega\dot{\theta} + I_x\left[\ddot{\phi}_d - \lambda_2(\dot{\phi} - \dot{\phi}_d) - K_2\cdot\text{sgn}(s_2)\right]$$

$$u_3 = -\dot{\phi}\dot{\psi}\left(I_z - I_x\right) - I_p\Omega\dot{\phi} + I_y\left[\ddot{\theta}_d - \lambda_3(\dot{\theta} - \dot{\theta}_d) - K_3\cdot\text{sgn}(s_3)\right]$$

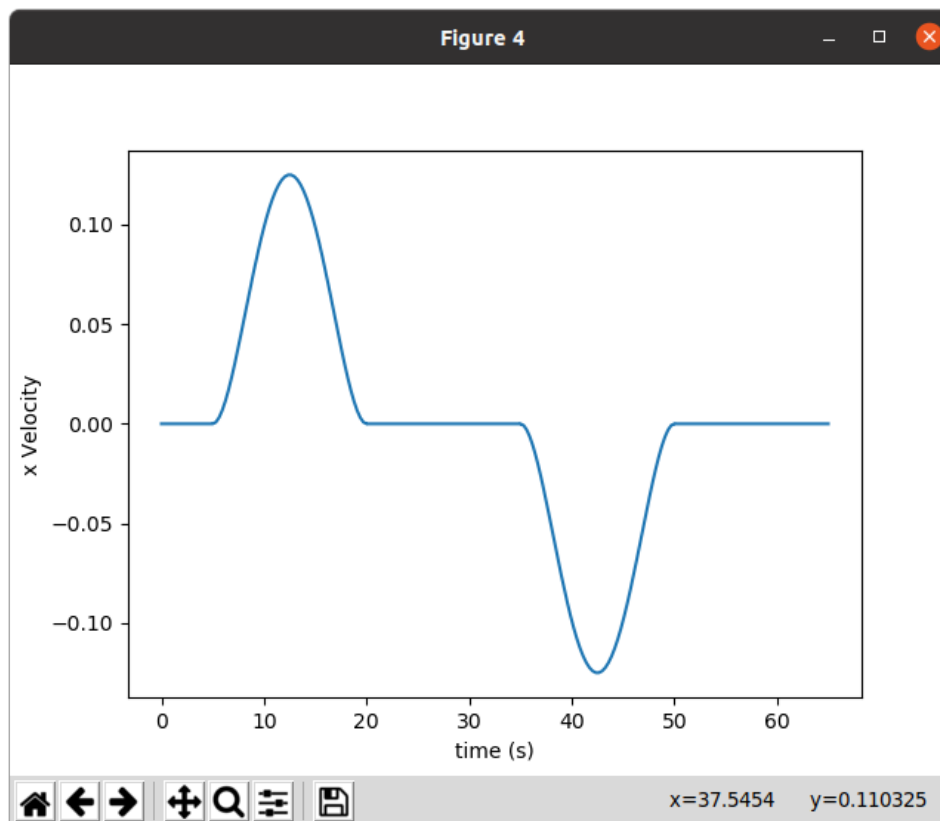$$u_4 = -\dot{\phi}\dot{\theta}\left(I_x - I_y\right) + I_z\left[\ddot{\psi}_d - \lambda_4(\dot{\psi} - \dot{\psi}_d) - K_4\cdot\text{sgn}(s_4)\right]$$
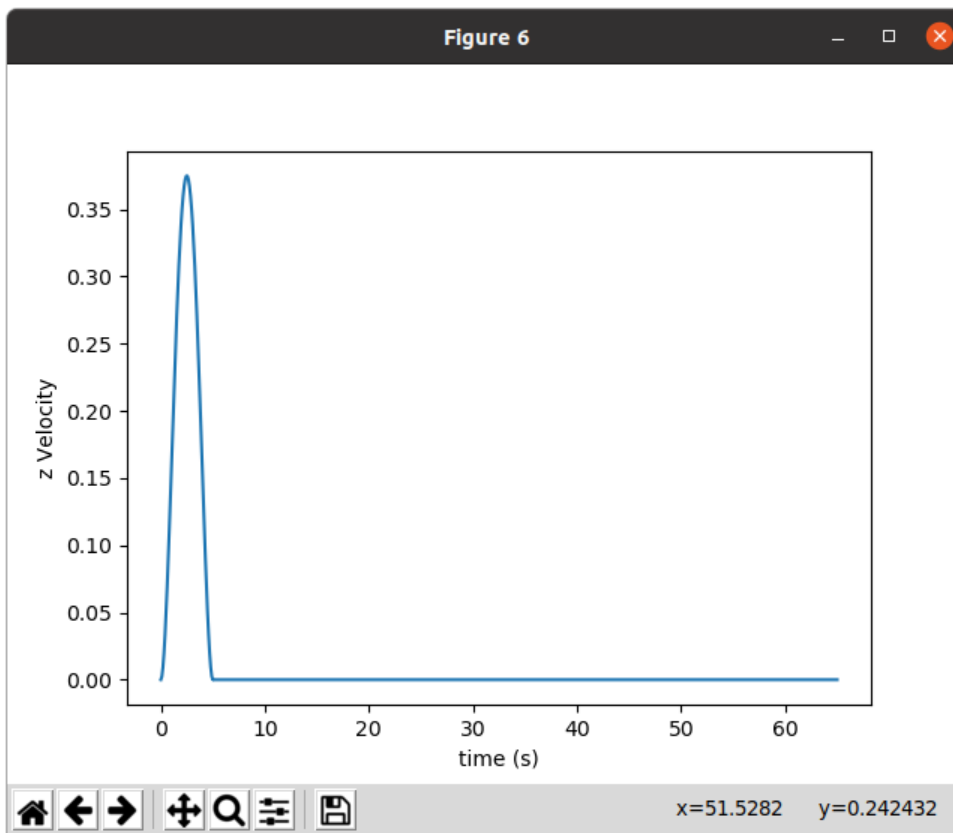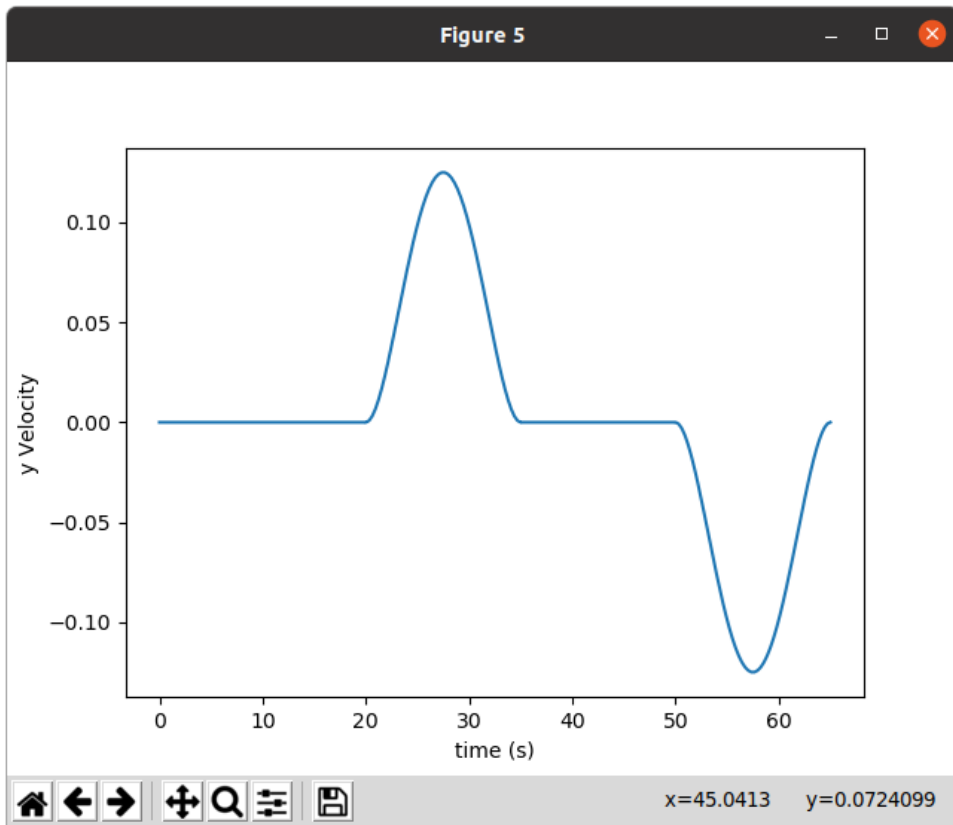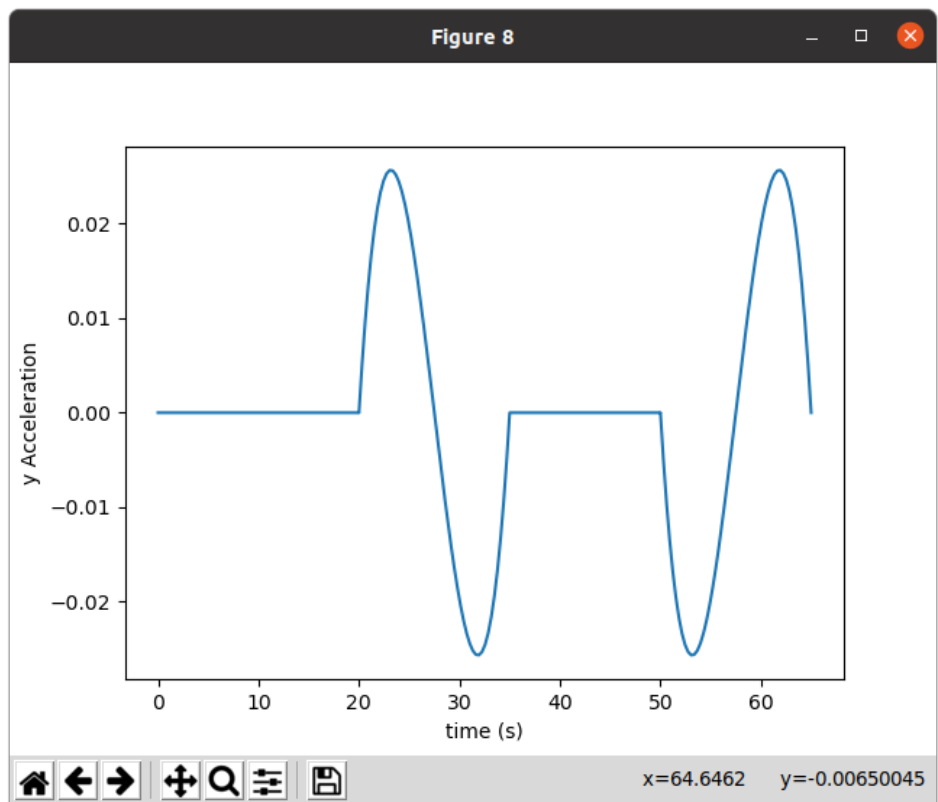
Trajectory Plots

Positions

Velocities

Figure 5

Figure 6
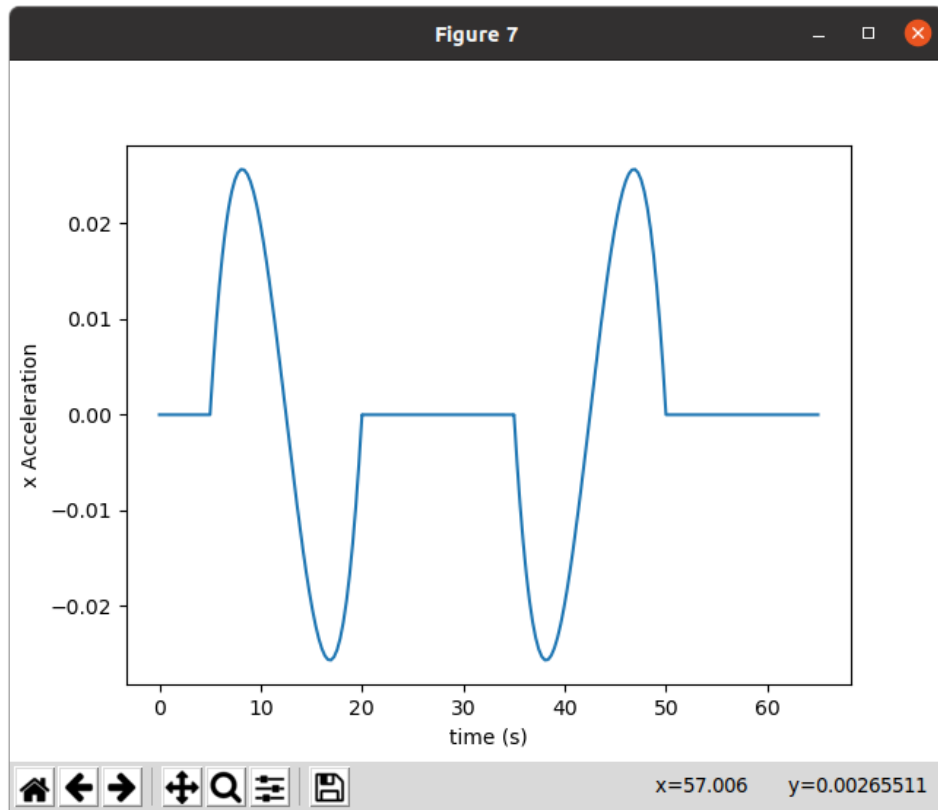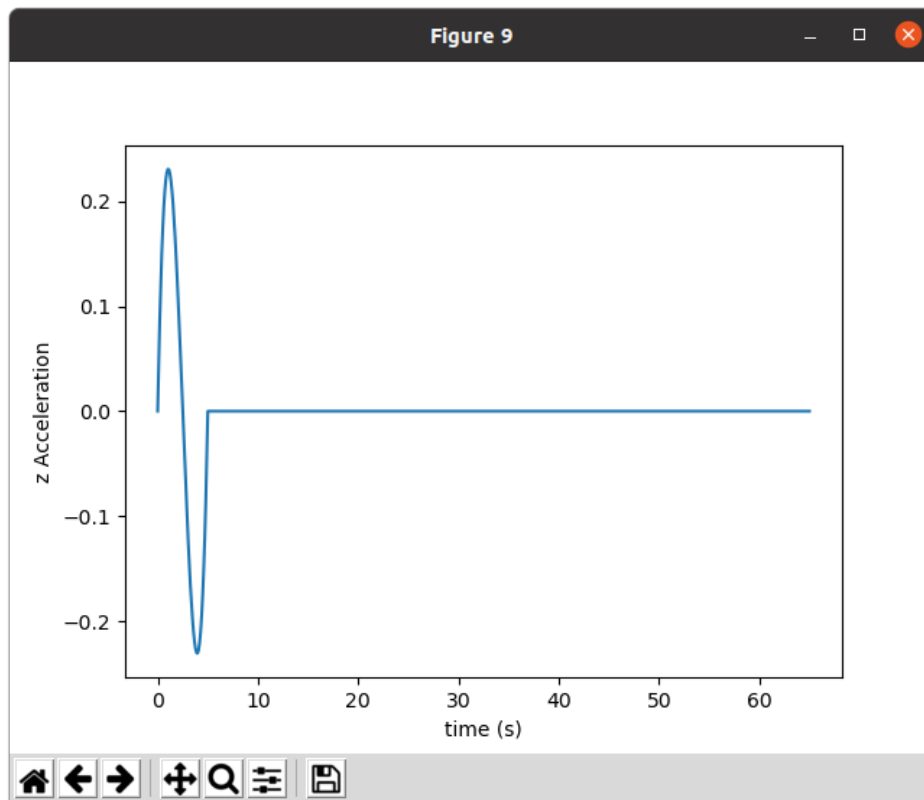
Accelerations

The quintic trajectory is a polynomial of order 5 where we can plot the non linear plots for the accelerations of the desired trajectories also.

LIST OF DESIGN PARAMETERS

Kp = 20
Kd = -5
lambda1=0.5
k1=1
lambda2=10
k2=150
lambda3=15
k3=200
lambda4=10
k4=5