DOCUMENTATION

Part 1 : Trajectory

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$
$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$
$$\ddot{q}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

where,

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 & t_0^4 & t_0^5 \\ 0 & 1 & 2t_0 & 3t_0^2 & 4t_0^3 & 5t_0^4 \\ 0 & 0 & 2 & 6t_0 & 12t_0^2 & 20t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 & t_f^4 & t_f^5 \\ 0 & 1 & 2t_f & 3t_f^2 & 4t_f^3 & 5t_f^4 \\ 0 & 0 & 2 & 6t_f & 12t_f^2 & 20t_f^3 \end{bmatrix}^{-1} \begin{bmatrix} q_0 \\ \dot{q}_0 \\ \ddot{q}_0 \\ q_f \\ \dot{q}_f \\ \ddot{q}_f \end{bmatrix}$$

Example - 1st Waypoint for 'z',

$$t_0 = 0, \quad t_f = 5, \quad z_0 = 0, \quad z_f = 1$$
$$\dot{z}_0 = \dot{z}_f = \ddot{z}_0 = \ddot{z}_f = 0$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & 5 & 25 & 125 & 625 & 3125 \\ 0 & 1 & 10 & 75 & 500 & 3125 \\ 0 & 0 & 2 & 30 & 300 & 2500 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Part 2 : Control Inputs

$$\ddot{q} = f(q, \dot{q}) + g(q, \dot{q})u$$

Let us consider the surface as, $s = \dot{e} + \lambda e$

where, $e = q - q_d$

$\dot{e} = \dot{q} - \dot{q}_d$

Sliding condition: $s\dot{s} \leq -K|s|$

$$\dot{s} = \ddot{e} + \lambda \dot{e}$$

$$= \ddot{q} - \ddot{q}_d + \lambda(\dot{q} - \dot{q}_d)$$

$$= f(q,\dot{q}) + g(q,\dot{q})u - \ddot{q}_d + \lambda(\dot{q} - \dot{q}_d)$$

$$s\dot{s} = s \cdot g(q,\dot{q}) \left[\frac{f(q,\dot{q}) - \ddot{q}_d + \lambda(\dot{q} - \dot{q}_d)}{g(q,\dot{q})} + u \right]$$

Design 'u' such that $s\dot{s} < -K|s|$

$$\boxed{u = \frac{-f(q,\dot{q}) + \ddot{q}_d - \lambda(\dot{q} - \dot{q}_d) + u_r}{g(q,\dot{q})}}$$

where $u_r \rightarrow$ control input

$u_r = -(\rho + K)\, sgn(s)$

Hence we donot have uncertainities, $\rho = 0$

$\rightarrow u_r = -K \cdot sgn(s)$

Now,

<u>For z</u>, $\ddot{z} = \frac{\cos\phi \cos\theta}{m} u_1 - g$

$\Rightarrow f(q,\dot{q}) = -g$, $g(q,\dot{q}) = \frac{\cos\phi\cos\theta}{m}$

$e_z = z - z_d$, $\dot{e} = \dot{z} - \dot{z}_d$

$S_1 = \dot{e}_z + \lambda e_z$

$$\Rightarrow u_1 = \frac{m}{\cos\phi \cos\theta}\left[g + \ddot{z}_d - \lambda_1(\dot{z} - \dot{z}_d) - K_1 \cdot sgn(s_1) \right]$$

For ϕ,

$$\ddot{\phi} = \underbrace{\dot{\theta}\dot{\psi}\frac{I_y - I_z}{I_x} - \frac{I_p}{I_x}\Omega\dot{\theta}}_{f(q,\dot{q})} + \underbrace{\frac{1}{I_x}}_{g(q,\dot{q})} u_2$$

$$e_\phi = \phi - \phi_d$$
$$\dot{e}_\phi = \dot{\phi} - \dot{\phi}_d$$

$$\Rightarrow S_2 = \dot{e}_\phi + \lambda e_\phi$$

$$u_2 = I_x\left[-\dot{\theta}\dot{\psi}\frac{I_y - I_z}{I_x} + \frac{I_p}{I_x}\Omega\dot{\theta} + \ddot{\phi}_d - \lambda_2(\dot{\phi} - \dot{\phi}_d) - K_2\,\mathrm{sgn}(s_2)\right]$$

$$\Rightarrow u_2 = -\dot{\theta}\dot{\psi}(I_y - I_z) + I_p\Omega\dot{\theta} + I_x\left[\ddot{\phi}_d - \lambda_2(\dot{\phi} - \dot{\phi}_d) - K_2 \cdot \mathrm{sgn}(s_2)\right]$$

For θ,

$$\ddot{\theta} = \underbrace{\dot{\phi}\dot{\psi}\frac{I_z - I_x}{I_y} + \frac{I_p}{I_y}\Omega\dot{\phi}}_{f(q,\dot{q})} + \underbrace{\frac{1}{I_y}}_{g(q,\dot{q})} u_3$$

$$e_\theta = \theta - \theta_d$$
$$\dot{e}_\theta = \dot{\theta} - \dot{\theta}_d \quad \Rightarrow \quad S_3 = \dot{e}_\theta + \lambda e_\theta$$

$$\Rightarrow u_3 = -\dot{\phi}\dot{\psi}(I_z - I_x) - I_p\Omega\dot{\phi} + I_y\left[\ddot{\theta}_d - \lambda_3(\dot{\theta} - \dot{\theta}_d) - K_3 \cdot \mathrm{sgn}(s_3)\right]$$

For ψ,

$$\ddot{\psi} = \underbrace{\dot{\phi}\dot{\theta}\frac{I_x - I_y}{I_z}}_{f(q,\dot{q})} + \underbrace{\frac{1}{I_z}}_{g(q,\dot{q})} u_4$$

$$e_\psi = \psi - \psi_d$$
$$\dot{e}_\psi = \dot{\psi} - \dot{\psi}_d \quad \rightarrow \quad S_4 = \dot{e}_\psi + \lambda e_\psi$$

$$\Rightarrow u_4 = -\dot{\phi}\dot{\theta}(I_x - I_y) + I_z\left[\ddot{\psi}_d - \lambda_4(\dot{\psi} - \dot{\psi}_d) - K_4 \cdot \mathrm{sgn}(s_4)\right]$$

Using the boundary condition, $sat\left(\frac{s}{\phi}\right)$

Replace $sgn(s_i)$ with $sat\left(\frac{s_i}{\phi}\right)$ in u_1, u_2, u_3 & u_4

where $\phi > 0$

This is used to reduce the chattering.

Let $\phi = 0.9$

$$\boxed{u_1 = \frac{m}{\cos\phi\cos\theta}\left[g + \ddot{z}_d - \lambda_1(\dot{z} - \dot{z}_d) - K_1 \cdot sat\left(\frac{s_1}{\phi}\right)\right]}$$

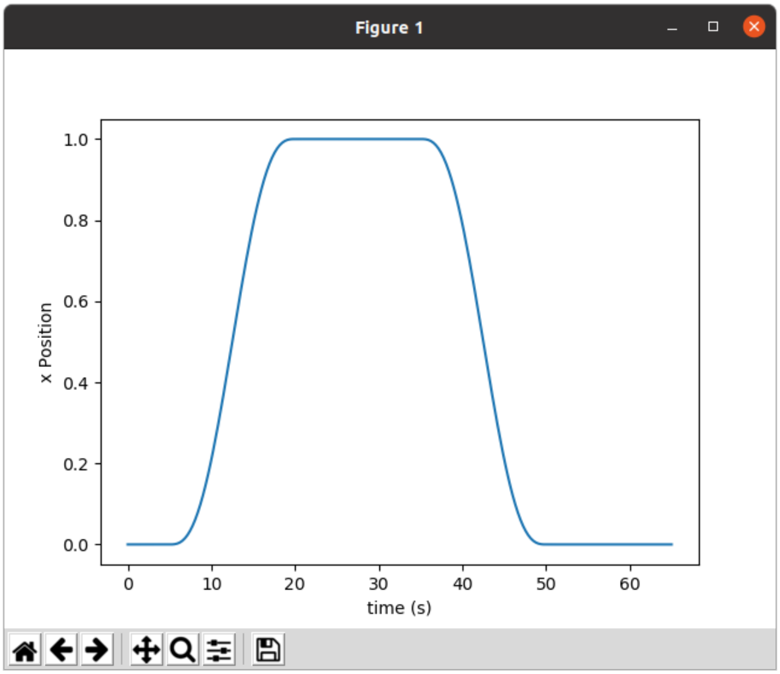
$$\boxed{u_2 = -\dot{\theta}\dot{\psi}(I_y - I_z) + I_p\Omega\dot{\theta} + I_x\left[\ddot{\phi}_d - \lambda_2(\dot{\phi} - \dot{\phi}_d) - K_2 \cdot sgn(s_2)\right]}$$

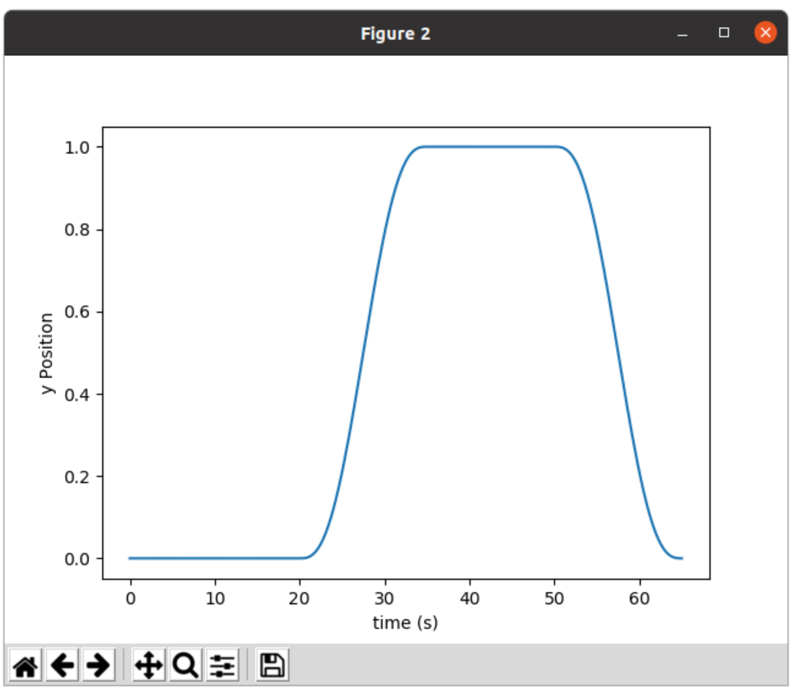
$$\boxed{u_3 = -\dot{\phi}\dot{\psi}(I_z - I_x) - I_p\Omega\dot{\phi} + I_y\left[\ddot{\theta}_d - \lambda_3(\dot{\theta} - \dot{\theta}_d) - K_3 \cdot sgn(s_3)\right]}$$

$$\boxed{u_4 = -\dot{\phi}\dot{\theta}(I_x - I_y) + I_z\left[\ddot{\psi}_d - \lambda_4(\dot{\psi} - \dot{\psi}_d) - K_4 \cdot sgn(s_4)\right]}$$

Trajectory Plots

Positions





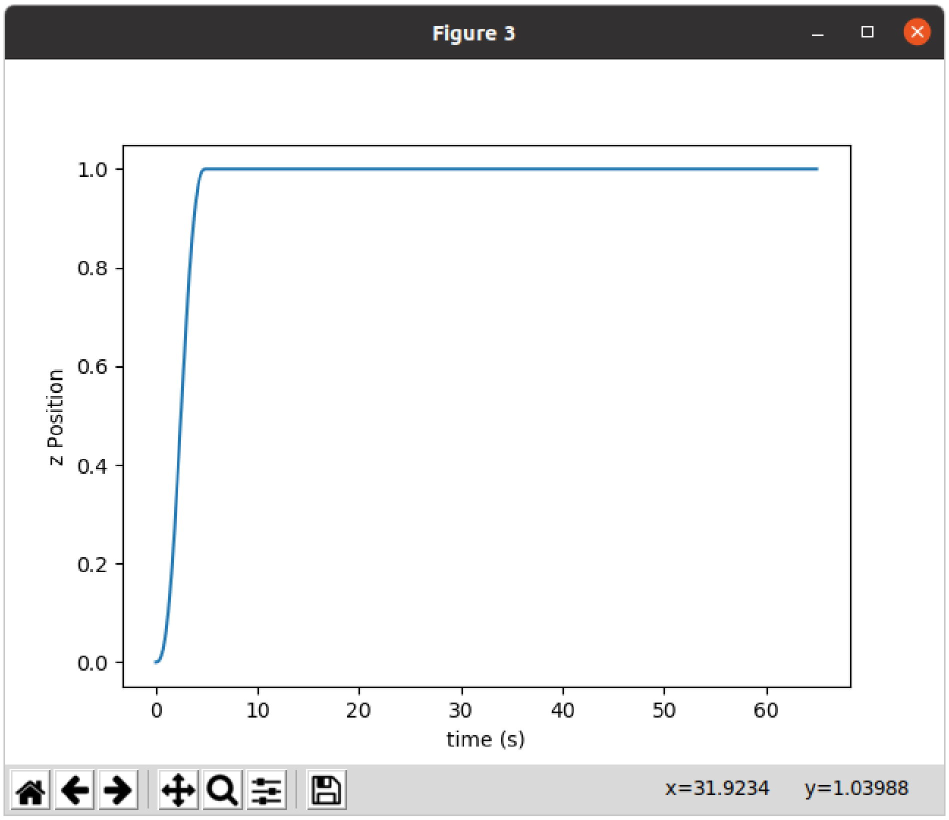


Figure 3
1.0
0.8
0.6
0.4
0.2
0.0
z Position
0
10
20
30
40
50
60
time (s)
x=31.9234
y=1.03988

Velocities

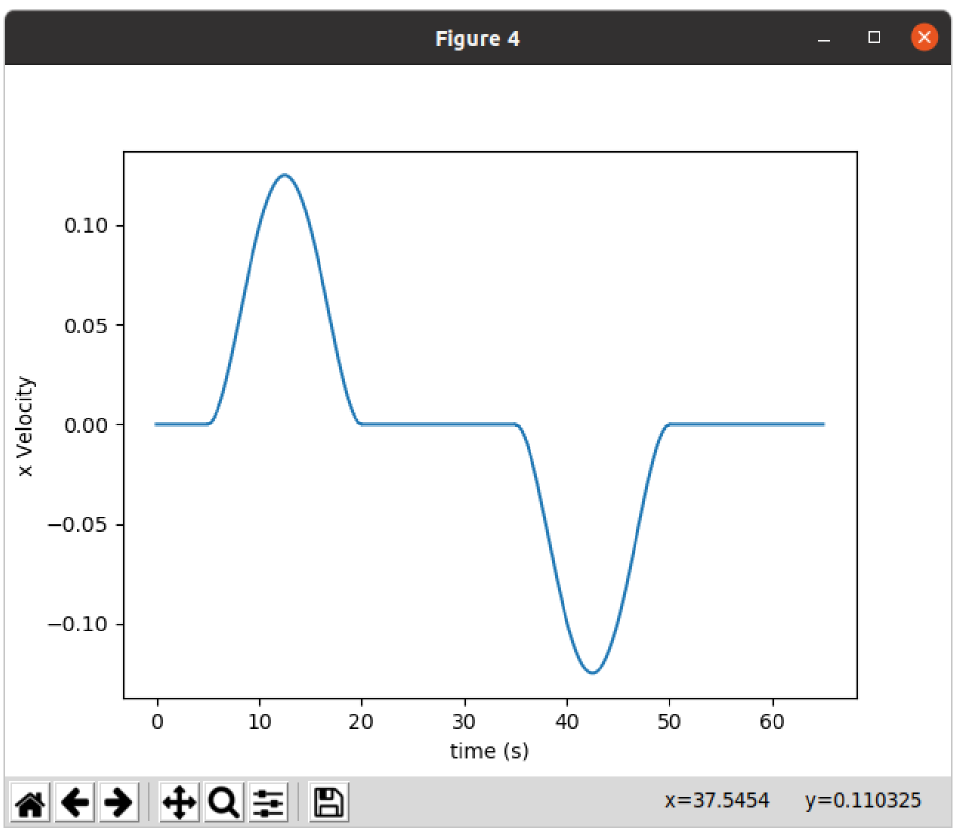


Figure 4
0.10
0.05
0.00
−0.05
−0.10
x Velocity
0
10
20
30
40
50
60
time (s)
x=37.5454
y=0.110325

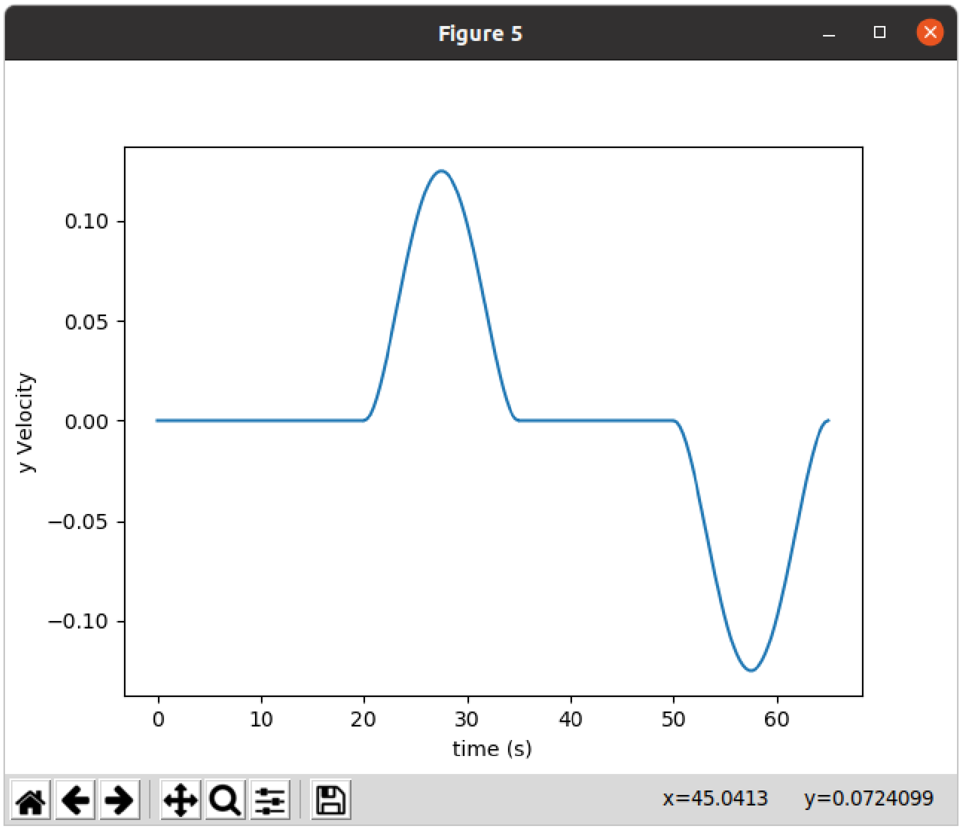


Figure 5
0.10
0.05
0.00
−0.05
−0.10
y Velocity
0
10
20
30
40
50
60
time (s)
x=45.0413
y=0.0724099

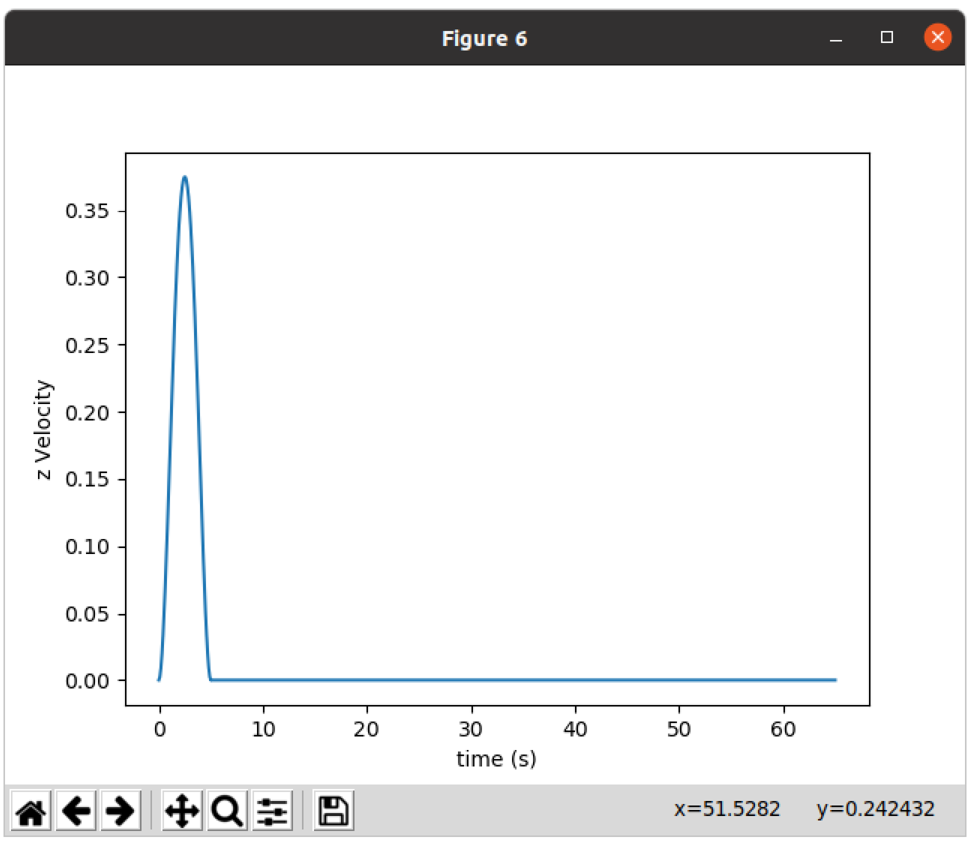
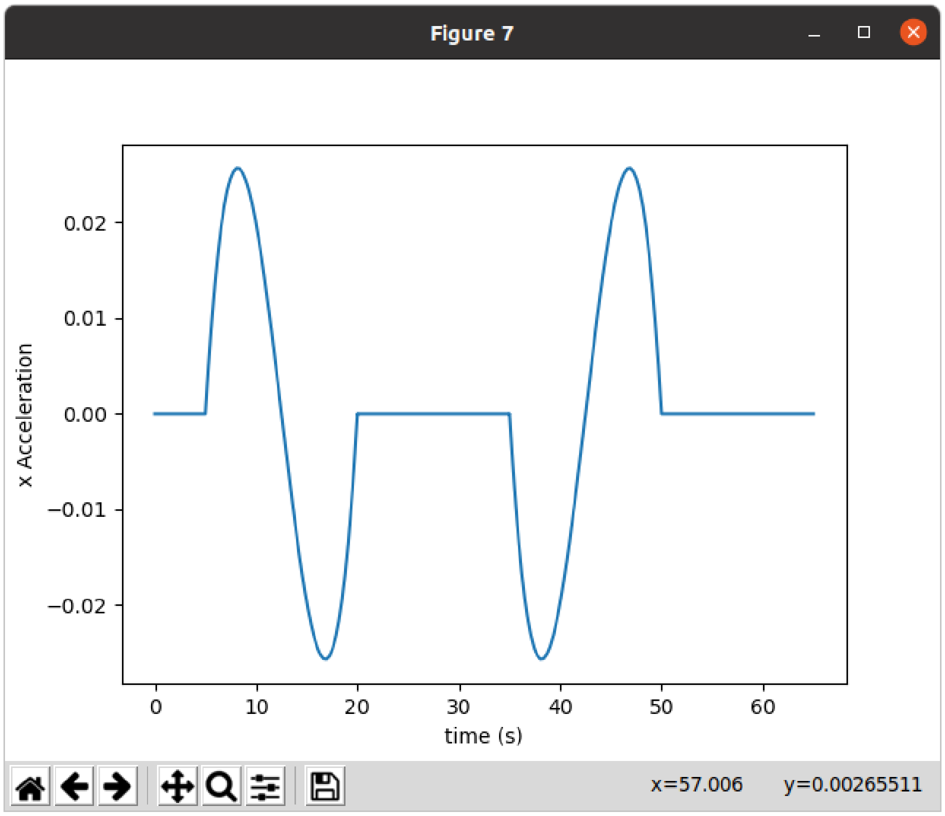
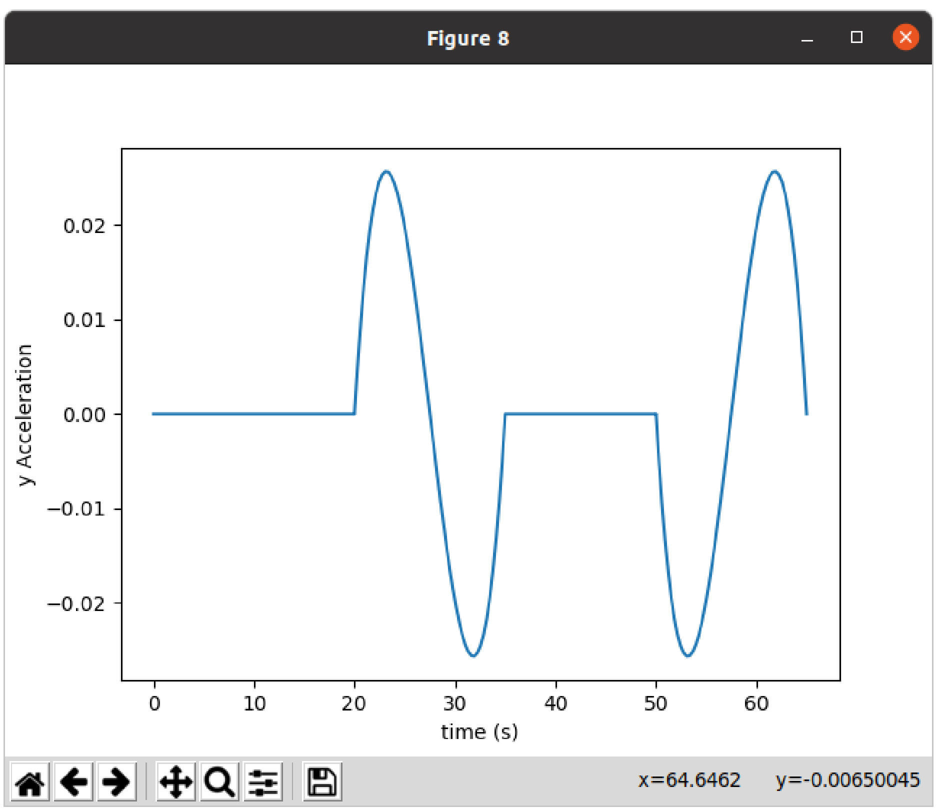
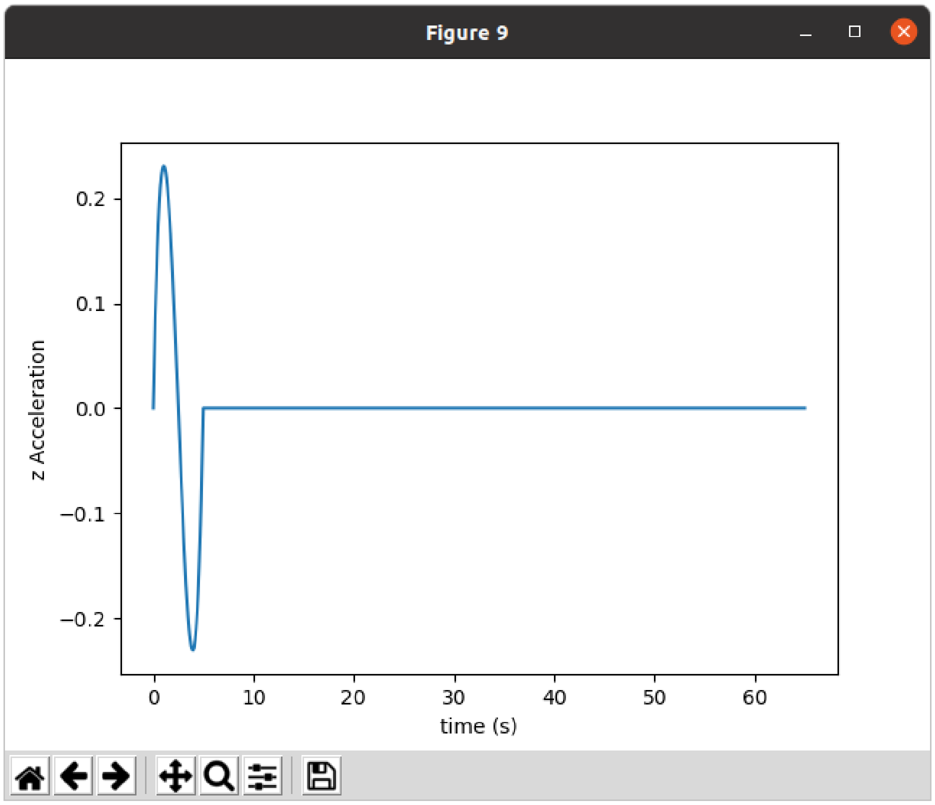


Figure 6
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
z Velocity
0
10
20
30
40
50
60
time (s)
x=51.5282
y=0.242432

Accelerations







The quintic trajectory is a polynomial of order 5 where we can plot the non linear plots for the accelerations of the desired trajectories also.

LIST OF DESIGN PARAMETERS

Kp = 20
Kd = -5
lambda1=0.5
k1=1
lambda2=10
k2=150
lambda3=15
k3=200
lambda4=10
k4=5

ACTUAL TRAJECTORY (vs DESIRED TRAJECTORY)