Assignment_02

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1.5 Exercise 1

Let's suppose we have a set of observations $x = (x_1, \ldots, x_N)^T$, that are drawn independent and identically distributed (i.i.d) from a Gaussian distribution with unknown mean μ and variance σ^2

For this example, we are going to assume that the unknown parameters are μ=2 and σ^2=25 and the number of samples N=100.

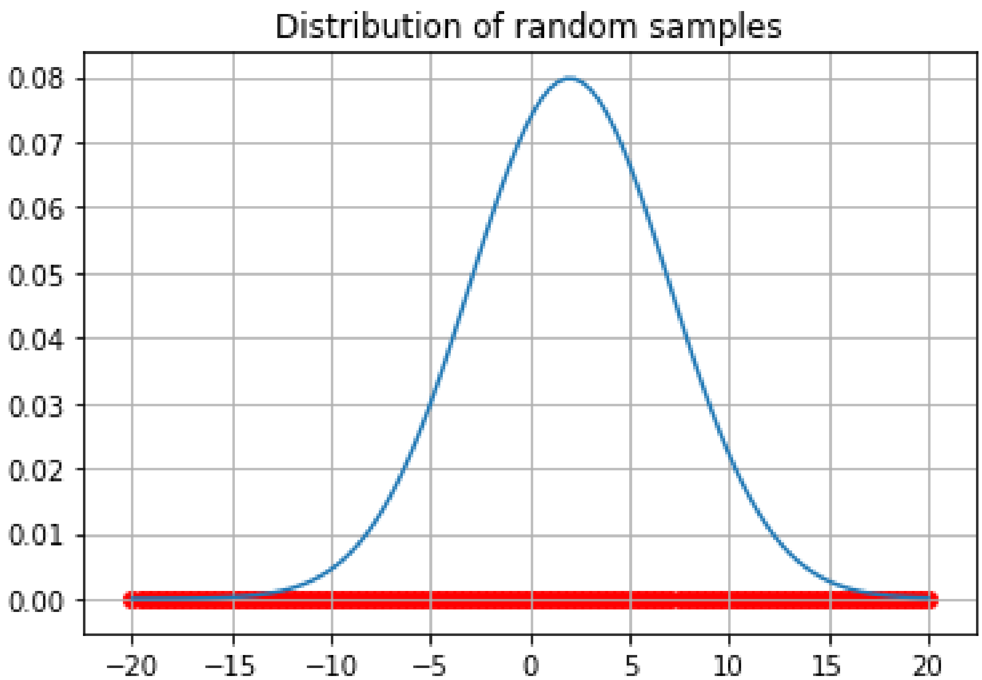
```
In [4]: import numpy as np
        import matplotlib.pyplot as plt
        from mpl_toolkits.mplot3d import Axes3D
        from scipy.stats import norm
        from scipy.stats import multivariate_normal
        import seaborn as sns
        import scipy
        from scipy import stats
        from scipy.optimize import minimize
```

2 Task1:

Plot this (unknown) distribution together with the samples in the range [-20, 20].

```
In [5]: mu = 2.
        sigma = 5.
        N = 100
        x = np.linspace(-20, 20, N)
        fig, ax = plt.subplots(1,1)
        ax.plot(x, norm.pdf(x, mu, sigma))
        ax.scatter(x, np.zeros(N),color="r")
```

```
ax.grid(True)
plt.title('Distribution of random samples')
plt.show()
```



3 Task2:

- Implement the likelihood function in python (you can simply use the existing python implementations)
- Use a general optimization method to find the values for μ and σ^2.

In [6]:
```
mu = 2.
sigma = 5.
N = 100
x = np.linspace(-20, 20, N)
y = np.random.normal(mu,sigma,100)

print "actual mean : " + str(y.mean())
print "actual standard deviation : " + str(y.std())

def likelihood_function(params):
    predicted_mean = params[0]
    std = params[1]
    log_likelihood = -np.sum(norm.logpdf(y, predicted_mean, std))
    return(log_likelihood)
```

```
init_params = [1, 1]
results = minimize(likelihood_function, init_params, method='Powell')
print "predicted mean using likelyhood function: " + str(results.x[0])
print "predicted standard deviation using likelyhood function : " + str(res
```

```
actual mean : 2.596122301769203
actual standard deviation : 4.833335849703718
predicted mean using likelyhood function: 2.5961703683890742
predicted standard deviation using likelyhood function : 4.833325914474777
```

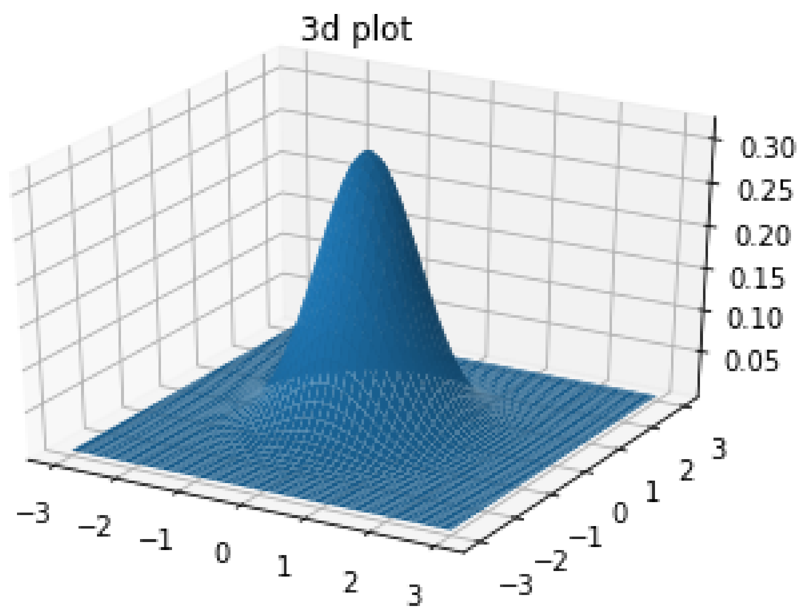
4 Task3:

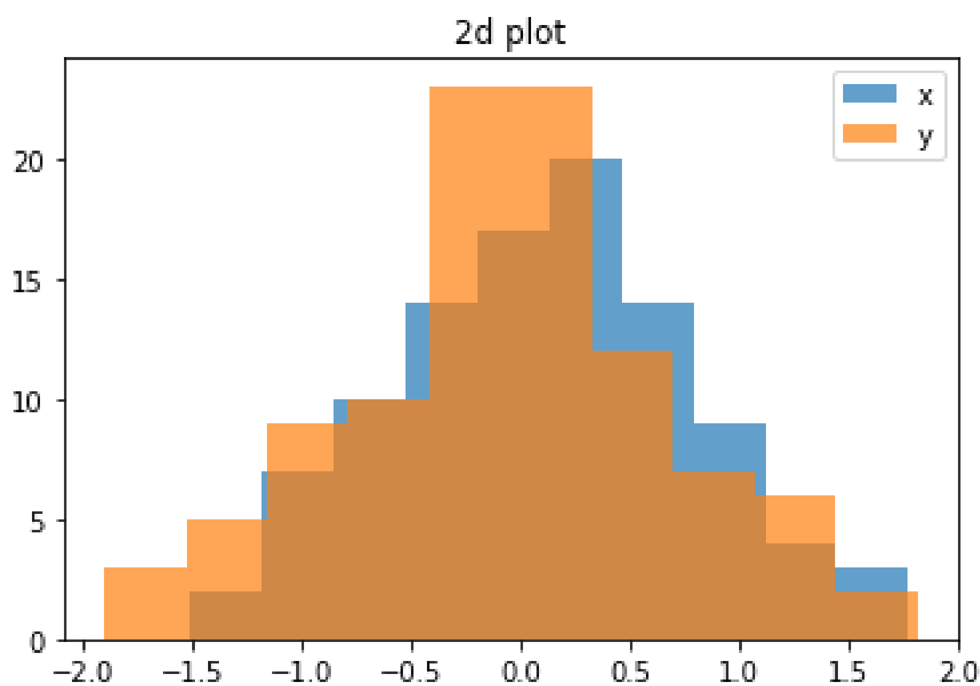
Given: $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ 1. Visualise a Gaussian with the given parameters. 2. Visualise a marginal Gaussian. 3. Visualise a slice of Gaussian.

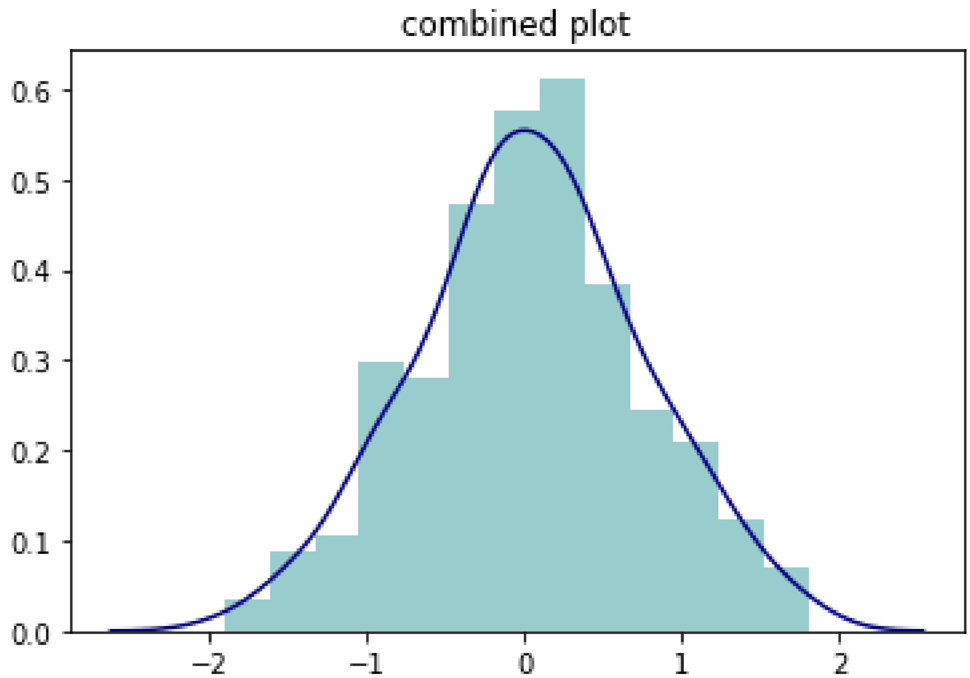
```
In [7]: mu = np.array([0,0])
        cov = np.array([[0.5, 0],[0, 0.5]])
```

```
In [14]: # 1. Visualise a Gaussian with the given parameters.
         #3d plot
         x = np.linspace(-3, 3, 100)
         y = np.linspace(-3, 3, 100)
         X, Y = np.meshgrid(x, y)
         pos = np.dstack((X, Y))
         rv = multivariate_normal(mu, cov)
         Z = rv.pdf(pos)
         fig = plt.figure()
         ax = fig.add_subplot(111, projection='3d')
         ax.plot_surface(X, Y, Z)
         plt.title('3d plot')
         fig.show()

         #2d plot
         x, y = np.random.multivariate_normal(mu, cov, 100).T
         plt.figure()
         plt.hist(x, alpha=0.7, bins=10, label='x')
         plt.hist(y, alpha=0.7, bins=10, label='y')
         plt.title('2d plot')
         plt.legend()

         #Combine plot
         combined = np.concatenate((x,y))
         plt.figure()
         plt.title('combined plot')
         sns.distplot(combined, hist_kws={'color': 'Teal'}, kde_kws={'color' : 'Nav
```

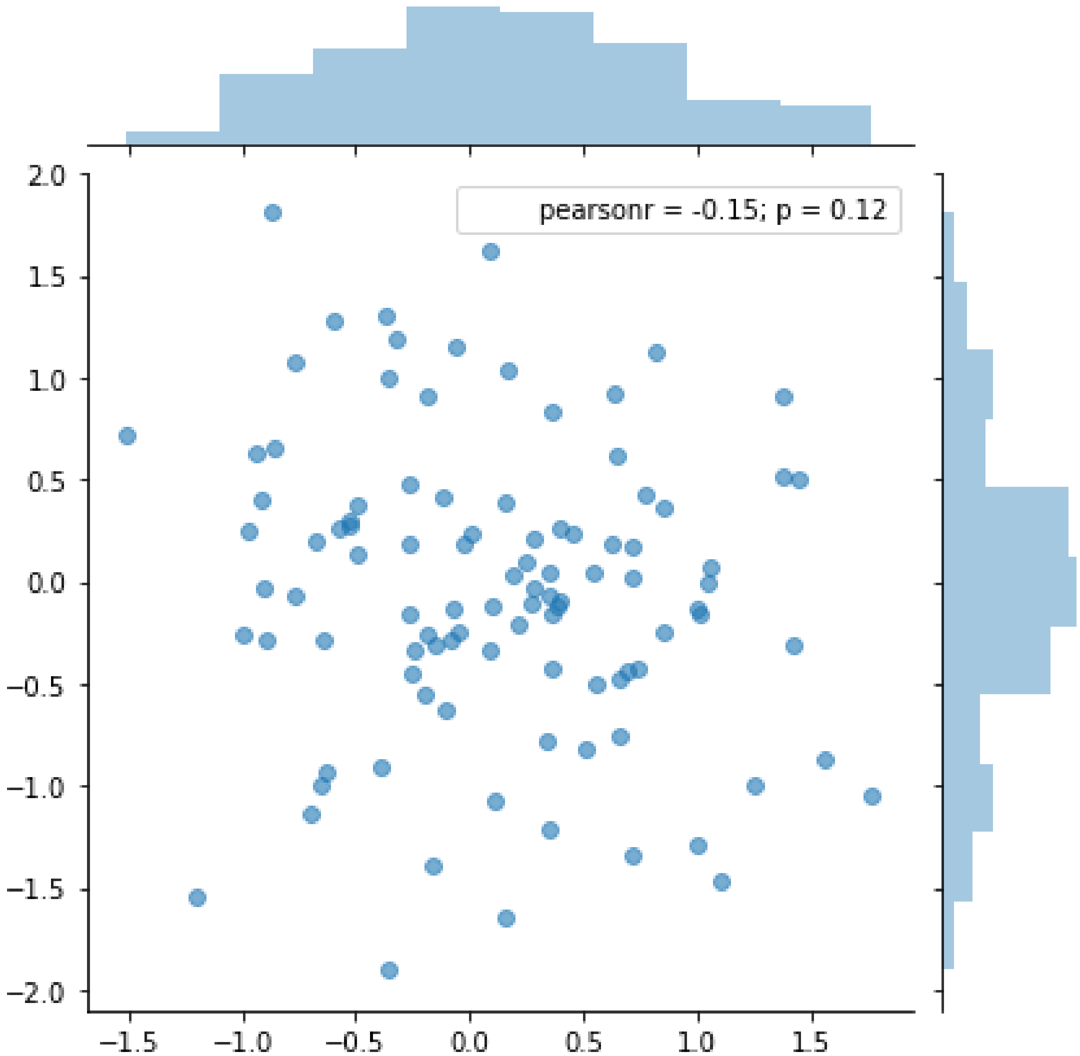
Out[14]: <matplotlib.axes._subplots.AxesSubplot at 0x7f52e4712490>



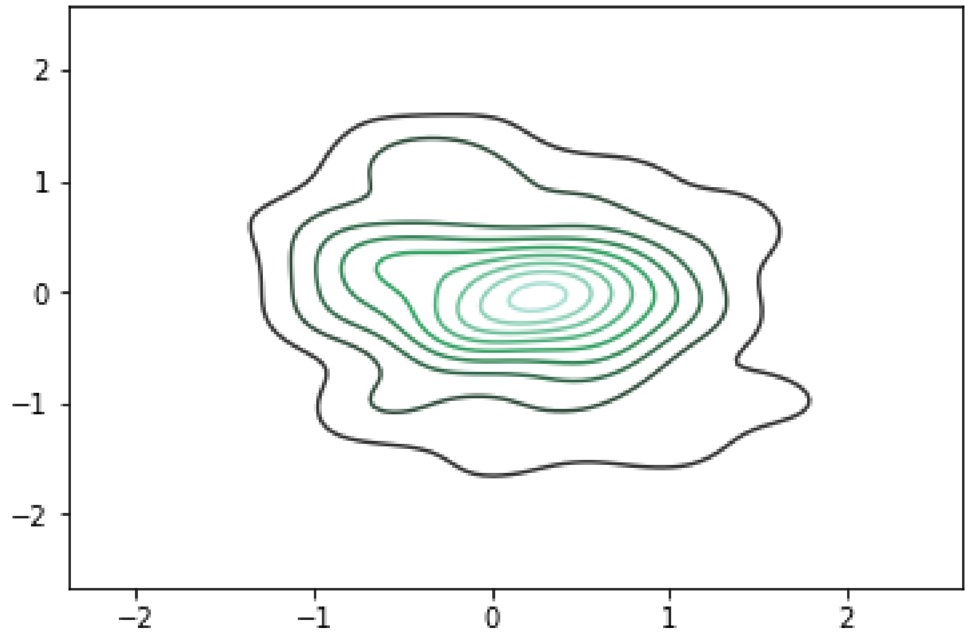





```
In [15]: #2. Visualise a marginal Gaussian.
         sns.jointplot(x,y, alpha=0.6)

Out[15]: <seaborn.axisgrid.JointGrid at 0x7f52e4233550>
```




```
In [16]: #3.Visualise a slice of Gaussian.
         sns.kdeplot(x, y)

Out[16]: <matplotlib.axes._subplots.AxesSubplot at 0x7f52e446e990>
```



5 Task4:

Given: Number of samples is 1000 from them 330 samples are labeled as class A and 670 samples are labeled as class B. There are 2 features $X1$ and $X2$. It is observed that $p(A, X1)$=248, $p(A, X2)$=82, $p(B, X1)$=168, $p(B, X2)$=502 Compute: Prior $p(A)$, $p(B)$ Likelihood $p(X1|A)$, $p(X1|B)$ Posterior $p(A|X1)$

```
In [10]: p_A = 330.0/1000.0
         p_B = 670.0/1000.0
         print "Prior"
         print "probability of A: " + str(p_A)
         print "probability of B: " + str(p_B)
```

```
Prior
probability of A: 0.33
probability of B: 0.67
```

```
In [19]: p_A_X1 = 248
         p_B_X1 = 168
         p_X1_A = p_A_X1 / p_A
         p_X1_B = p_B_X1 / p_B
         print "Likelihood"
         print "p(X1|A): " + str(p_X1_A)
         print "p(X2|A): " + str(p_X1_B)
```

```
Likelihood
p(X1|A): 751.515151515
p(X2|A): 250.746268657
```

```
In [22]: posterior_p_A_X1 = (p_X1_A*p_A) / ((p_X1_A*p_A) + (p_X1_B*p_B))
         print "Posterior"
         print "p(A|X1): " + str(posterior_p_A_X1)
```

```
Posterior
p(A|X1): 0.596153846154
```

5.1 Exercise 2

5.2 Task1

Implement in Python (you can use SciPy library) the Maximum Likelihood Estimator to estimate the parameters for example mean and variance of some data. Your steps are: * Create a data set: - Set x-values for example: *x = np.linspace(0, 100, num=100)*, - Set observed y-values using a known slope (1.4), intercept (4), and sd (3), for example *y = 4 + 1.4x + np.random.normal(0, 3, 100)* * Create a likelihood function which arguments is a list of initial parameters * Test this function on various data sets (Hint: you can use minimize from scipy.optimize and scipy.stats to compute the negative log-likelihood)

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        %matplotlib inline
        import scipy
        from scipy import stats
        from scipy.optimize import minimize
```

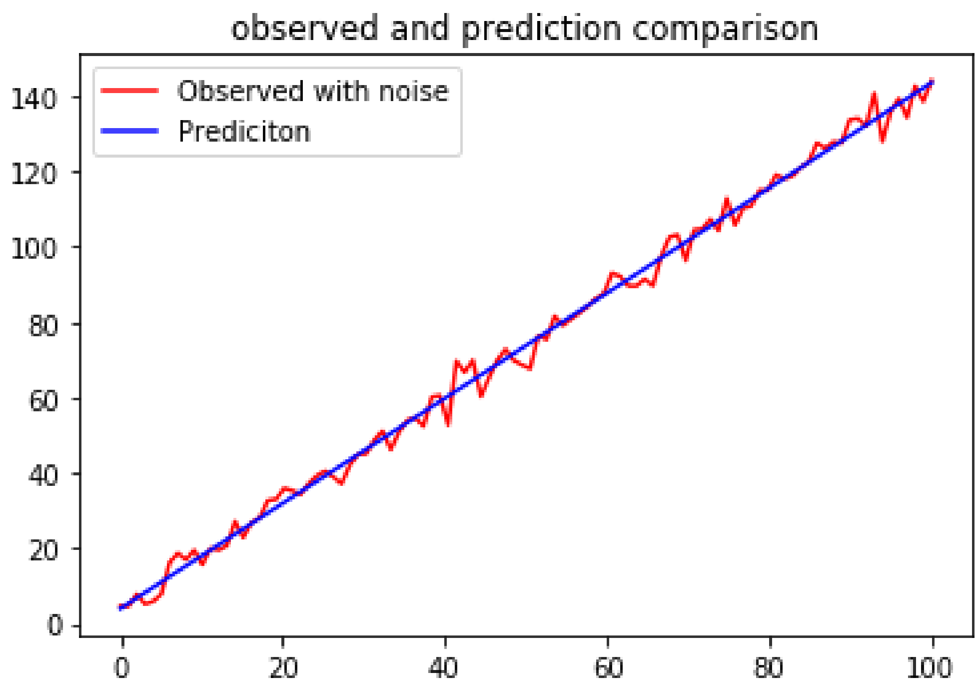
```
In [2]: x = np.linspace(0, 100, num=100)
        y = 4 + 1.4*x + np.random.normal(0, 3, 100)

        print "Given slope 1.4"
        print "Given intercept 4"
        print "Given standard deviation 3"
        print "---------------------------------"
        def likelihood_function(params):
            intercept = params[0]
            slope = params[1]
            predicted_mean = intercept + slope*x
            std = params[2]
            log_likelihood = -np.sum( stats.norm.logpdf(y, predicted_mean, std) )
            return(log_likelihood)

        init_params = [1, 1, 1]
        results = minimize(likelihood_function, init_params, method='nelder-mead')
```

```
        print "Prredicted slope using likelihood function " + str(results.x[1])
        print "Prredicted intercept likelihood function  " + str(results.x[0])
        print "Prredicted standard deviation likelihood function " + str(results.x[
        print "---------------------------------"
```

```
Given slope 1.4
Given intercept 4
Given standard deviation 3
---------------------------------
Prredicted slope using likelihood function 1.39573528115
Prredicted intercept likelihood function  3.972723153
Prredicted standard deviation likelihood function 3.0008105456
---------------------------------
```

```
In [3]: y_prediction = results.x[0] + results.x[1]*x
```

```
In [4]: plt.title("observed and prediction comparison")
        plt.plot(x, y, color='red', label="Observed with noise")
        plt.plot(x, y_prediction, color='blue', label="Prediciton")
        plt.legend()
        plt.show()
```



5.3 Exercise 3

GIVEN: Samples $0, 1, 0, 0, 1, 0$ from a binomial distribution which has the form: P(x=0)=(1-μ), P(x=1)=μ

REQUESTED: What is the maximum likelihood estimate of μ Hint: you can use SymPy to compute the derivities symboliclly

```
In [5]: import sympy as sp
        sp.init_printing("use=latex")
```

```
In [6]: # initializing symbols
        x,p,n = sp.symbols("x,p,n")
```

```
In [7]: #likelihood function
        log_likelihood = (x*sp.log(p)) + ((n-x)*sp.log(1-p))
        log_likelihood
```

Out[7]:

$$x \log{(p)} + (n - x) \log{(-p + 1)}$$

```
In [8]: # liklihood diff
        diff_logL = log_likelihood.diff(p)
        diff_logL
```

Out[8]:

$$-\frac{n - x}{-p + 1} + \frac{x}{p}$$

```
In [9]: mu = sp.solve(diff_logL,p)
        mu = mu[0]
        mu
```

Out[9]:

$$\frac{x}{n}$$

```
In [10]: #P(x=0)
         mu.subs([(x,4),(n,6)])
```

Out[10]:

$$\frac{2}{3}$$

```
In [11]: #P(x=1)
         mu.subs([(x,2),(n,6)])
```

Out[11]:

$$\frac{1}{3}$$