$\boxed{SVD}$

$$\Sigma = G_n \cdot I$$

$$G_i = \sqrt{\lambda_i}$$

$$A = U_i \cdot \Sigma \cdot V_i^T$$

U - unitary orthogonal matrices

Σ - rectangular diagonal matrices of singular value

$$V = \{U_1; \dots U_n\} = \left[\frac{1}{G_1} \cdot AV_1; \dots \frac{1}{G_n} \cdot AV_n\right]$$

$$\Sigma = \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_n \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

$$M \cdot v = \lambda \cdot v$$

the same

$$A \cdot A^T = M$$

$$\boxed{A \cdot A^T} \cdot v = \lambda \cdot v$$

$$A \cdot A^T = \begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 4+1 & 4-1 \\ 4-1 & 4+1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$$

$$A \cdot A^T - \lambda \cdot I = \begin{vmatrix} 5-\lambda & 3 \\ 3 & 5-\lambda \end{vmatrix} = (5-\lambda)^2 - 9 =$$

$$= 25 - 10\lambda + \lambda^2 - 9 = \lambda^2 - 10\lambda + 16$$

$$\lambda_1 = 8$$

$$\lambda_2 = 2$$

$$G_1 = \sqrt{\lambda_1} = \sqrt{8}$$
$$G_2 = \sqrt{\lambda_2} = \sqrt{2}$$

Orthogonal Transformation

$$\boxed{x_1^2 + x_2^2 = 1}$$

Eigenvectors: $\lambda_1 = 8$

$$\begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 5-8 & 3 \\ 3 & 5-8 \end{bmatrix} = \begin{array}{c} \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix} \end{array}$$

$$\begin{cases} -3x_1 + 3x_2 = 0 \\ 3x_1 - 3x_2 = 0 \end{cases} \qquad x_1 = x_2 \qquad V_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Transform: $x_1^2 + x_1^2 = 1$

$2x^2 = 1$

$$\boxed{x_1 = \frac{1}{\sqrt{2}}} \; ; \; \boxed{x_2 = \frac{1}{\sqrt{2}}}$$

$\lambda_2 = 2$

$$\begin{bmatrix} 5-\lambda & 3 \\ 3 & 5-\lambda \end{bmatrix} \Rightarrow \begin{array}{cc} x_1 & x_2 \\ \end{array} \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \begin{cases} 3x_1 + 3x_2 = 0 \\ 3x_1 + 3x_2 = 0 \end{cases}$$

$$x_1 = -x_2$$

$$x_1^2 + x_2^2 = 1 \qquad x_2 = -x_1$$

$$x_1 = \frac{1}{\sqrt{2}} \; ; \; x_2 = -\frac{1}{\sqrt{2}} \Rightarrow \boxed{\begin{bmatrix} -\frac{1}{\sqrt{2}} \\ +\frac{1}{\sqrt{2}} \end{bmatrix}}$$

V_2

$$\boxed{V = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}}$$

$$U = \{ u_1, \dots u_n \}$$

$$G_1 = \sqrt{8}$$

$$G_2 = \sqrt{2}$$

$$u_i = \frac{1}{G_i} \cdot A \cdot V_i$$

$$u_1 = \frac{1}{\sqrt{8}} \cdot \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{8}} \begin{pmatrix} 2 \cdot \frac{1}{\sqrt{2}} + 2 \cdot \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{1}{\cancel{2} \cdot \sqrt{2}} \begin{bmatrix} \frac{\cancel{4}}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = u_1$$

$$u_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ +\frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -2 \cdot \frac{1}{\sqrt{2}} + 2 \cdot \frac{1}{\sqrt{2}} \\ +\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \end{bmatrix} =$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{0}{\sqrt{2}} \\ \sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = u_2 .$$

$$\boxed{U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

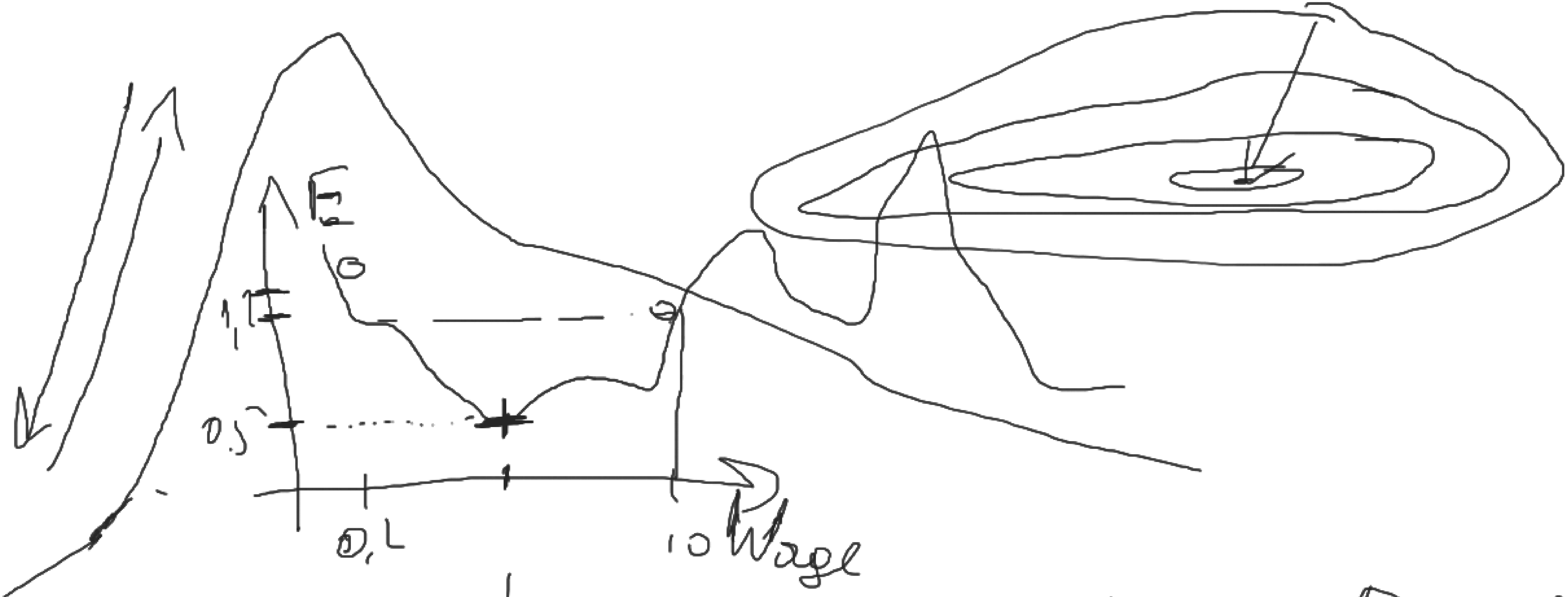
$$A = U \cdot \Sigma \cdot V^T$$

$$A = U \Sigma V^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{8} & 0 \\ 0 & \sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\boxed{V^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \end{bmatrix}^T \Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}$$

$$= \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

Intro to Gradients



anti gradient ⇄ Gradient Descent