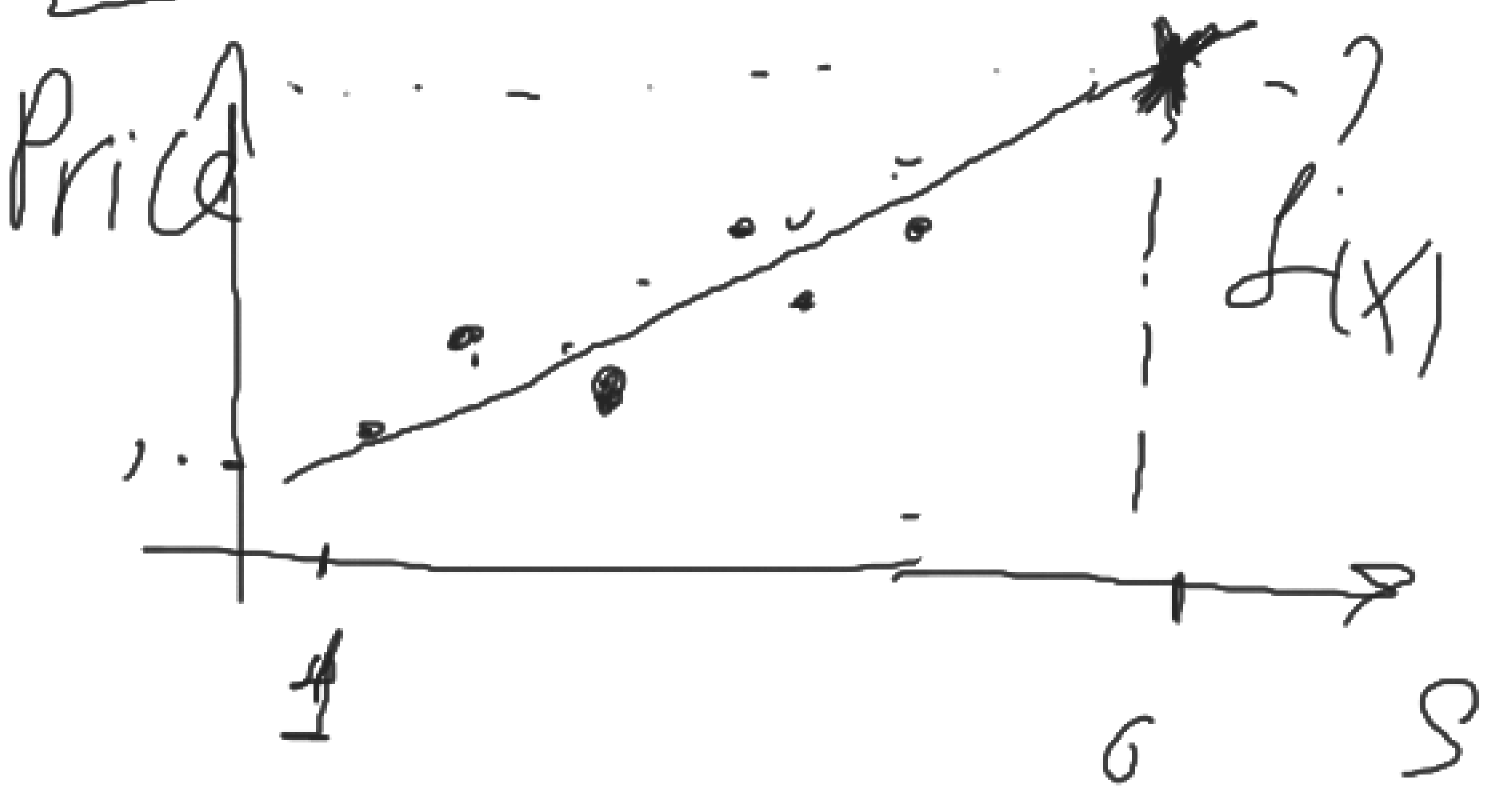
Least Square Method

Square	1	2	3	4	5	6
Price	1.1	3.8	6.5	10.2	13.1	?

x 6

y 5



$$y' = f(x) = a + bx$$

where x - square

$$E = \frac{1}{n}\sum_{i=1}^{n}(y' - y)^2$$

$$E = \frac{1}{n} \sum_{i=1}^{n} \left((a\cancel{x} + b) - y_{fact} \right)^2 \to \min$$

$$\left((ax + b) - y \right)^2 = 0 \qquad 0$$

$$(ax + b)^2 - 2y(ax + b) + y^2 \Rightarrow$$

$$\boxed{a^2x^2 + 2abx_i + (b)^2 - 2ay_ix - 2by_i + y^2} = 0$$

$$a = 2ax_i^2 + 2bx_i - 2y_ix; \quad b = 2ax_i + 2b - 2y_i;$$

$$a \begin{cases} a x_i^2 + b x_i - y_i x = 0 \\ b \quad a x_i + b n - y_i = 0 \end{cases}$$

$a = 3{,}04$

$b = -2{,}18$

$$b = \frac{y_i - a x_i}{n}$$

$$a = \frac{n \sum x_i y_i - \sum x_i \cdot \sum y_i}{n \sum x_i^2 - \left(\sum x_i\right)^2}$$

$$b = \frac{\sum y_i - a \cdot \sum x_i}{n}$$

$y_{pred} = ax + b$

$$y_{pred_1} = a \cdot x_1 - b$$

$$y_{pred_2} \quad 2 \quad a \cdot x_2 - b \qquad 5$$

$$y_1' \quad y_2' \quad y_3' \quad y_4' \quad y_5'$$

$$y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5$$

$a = 3{,}04$; $b = -2{,}18$

$$y_1' = 3{,}04 \cdot 1 + (-2{,}18) = 0{,}86 - 1{,}1 = 0{,}24^2$$

$$y_2 = 3{,}04 \cdot 2 - 2{,}18 = 3{,}9 - 3{,}8 = 0{,}1^2$$

$$y_3' = 3{,}04 \cdot 3 - 2{,}18 = 6{,}94 - 6{,}5 = 0{,}44^2$$

$$y_4' = 3{,}04 \cdot 4 - 2{,}18 = 9{,}98 - 10{,}2 = 0{,}22^2$$

$$y_5' = 3{,}04 \cdot 5 - 2{,}18 = 13{,}02 - 13{,}1 = 0{,}08^2$$

$$MSE = \frac{1}{n} \sum (y_i' - y_i)^2 = \frac{0{,}316}{5} = 0{,}0632$$

$RMSE = 0{,}24$

$$\begin{bmatrix}2 & 4\\ 6 & 8\end{bmatrix} \overset{?}{\ast} \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} \leftarrow \begin{bmatrix}4 & 8\\ 12 & 16\end{bmatrix}$$

$2X$ $\qquad$ X $\qquad$ $4X$

$\uparrow$

Eigenvalues and Eigenvectors.

$$M = \begin{pmatrix} 7 & 2 & -2 \\ 4 & 5 & -2 \\ 0 & 0 & 3 \end{pmatrix} - 5 \cdot \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$$

$$M \cdot V = \lambda \cdot V$$

eigenvalue

eigenvectors

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M \cdot v - \lambda \cdot v = 0$$

$$v(M - \lambda) = 0$$

$$M - \lambda \cdot I = 0$$

$$\det = \begin{vmatrix} 7-\lambda & 2 & -2 \\ 4 & 5-\lambda & -2 \\ 0 & 0 & 3-\lambda \end{vmatrix}$$

$$\det(M - \lambda \cdot I) = (7-\lambda)\cdot(5-\lambda)\cdot(3-\lambda) -$$

$$(3-\lambda)\cdot 8 \Rightarrow (3-\lambda)\big((7-\lambda)(5-\lambda) - 8\big).$$

$$35 - 7\lambda - 5\lambda + \lambda^2 - 8$$

$$\lambda^2 - 12\lambda + 27 = 0$$

$$\boxed{\lambda_1 = 3, \quad \lambda_2 = 9}$$

$\lambda = 3.$

$$\begin{vmatrix} 7-3 & 2 & -2 \\ 4 & 5-3 & -2 \\ 0 & 0 & 3-3 \end{vmatrix}$$

2

$$\begin{array}{cccc} & x_1 & x_2 & x_3 \\ 2 & 4 & 2 & -2 \\ & 4 & 2 & -2 \\ & 0 & 0 & 0 \end{array} = 0$$

$$\Rightarrow \quad 4x_1 + 2x_2 - 2x_3 = 0 \quad | :2$$

$$2x_1 + x_2 - x_3 = 0$$

$$x_3 = 2x_1 + x_2$$

$$\left(x_1 ; x_2 ; 2x_1 + x_2\right)$$

Where $x_1 ; x_2 \in \mathbb{R}$

$$x_1 = 1$$

$$x_2 = -1$$

$$x_3 = 2 - 1 = 1$$

$x_1 = 1$ $\lambda = 3.$

$x_2 = -1$

$x_3 = 1$

$$\begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$$

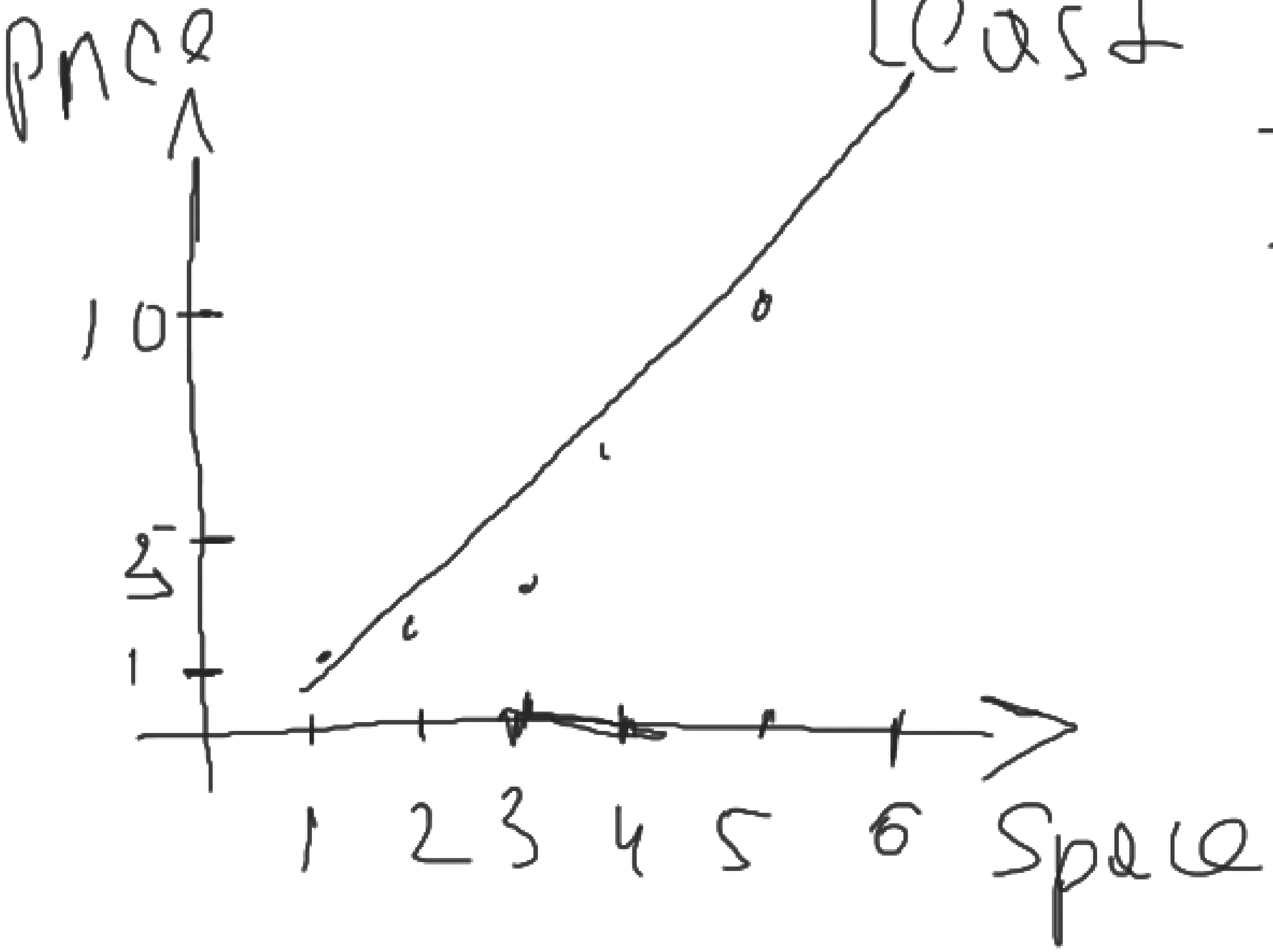
$$M \cdot v = \lambda \cdot v$$

$$\begin{pmatrix} 7 & 2 & -2 \\ 4 & 5 & -2 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 7 & -2 & -2 \\ 4 & -5 & -2 \\ 0 & 0 & 3 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

SVD → singullor value decomposition

$$100\left\{\overbrace{\begin{bmatrix} | & | & | \\ x_1 & x_2 & x_3 \\ | & | & | \end{bmatrix}}^{50}\right. \Rightarrow U \cdot \Sigma \cdot V^{T} = r\left\{\underbrace{\begin{bmatrix} | & | & | \\ u & u & u \\ | & | & | \end{bmatrix}}_{100}\right. \cdot \begin{bmatrix} G & & \\ & G & \\ & & G \end{bmatrix} \cdot \underbrace{\begin{bmatrix} v_1 & v & v \end{bmatrix}}_{50}$$

Least Square Method



Space	1	2	3	4	5	6
Price	1,1	3,8	6,5	10,2	13,1	?

$$y' = ax_i + b$$

$$E = \frac{1}{n}\sum_{i=1}^{n}(y' - y)^2$$

$$((ax_i + b) - y_i)^2 = (ax_i + b)^2 - 2y_i(ax_i + b) + y_i^2$$

$$= a^2 x_i^2 + 2ax_i b + b^2 - 2y_i ax_i - 2y_i b + y^2 \to \min$$

$$a^2x^2 + 2ax_ib + b^2 - 2y_iax_i - 2y_ib + y^2$$

$$\frac{\partial(a^2x^2)}{\partial(a)} + \frac{\partial(2ax_ib)}{\partial(a)} + \cancel{\frac{\partial(b^2)}{\partial(a)}} - \frac{\partial(2y_iax_i)}{\partial(a)} - \cancel{\frac{\partial(2y_ib)}{\partial(a)}} + \cancel{\frac{\partial(y^2)}{\partial(a)}} = \underbrace{ax_i^2 + x_ib - y_ix_i}_{\partial(a)}$$

$$b = \frac{ax_i + nb - y_i}{} \Rightarrow 0$$

$$\boxed{b = \frac{ax_i - y_i}{n}}$$

$$a x_i^2 + b x_i - x_i y_i$$

$$b = \frac{-a x_i + y_i}{n}$$

$$a x^2 + \frac{y_i - a x_i}{n} \cdot x_i - x_i y = 0$$

$$a = \frac{n \cdot x_i y_i - \frac{x_i y_i - x_i^2 a}{n}}{x^2}$$

$$a = \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{m} y_i}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x \right)^2}$$

$$b = \frac{\sum_{i=1}^{n} y_i - a \sum_{i=1}^{n} x_i}{n}$$

$y_1' = a \cdot x_1 + б$ $\qquad a = 3{,}04;\ б = -2{,}18$

$y_1' = 3{,}04 \cdot 1 - 2{,}18 \Rightarrow 0{,}86 - 1{,}1 = 0{,}24 \xrightarrow{x^2}$

$y_2' = 3{,}04 \cdot 2 - 2{,}18 \Rightarrow 3{,}9 - 3{,}8 = 0{,}1 \xrightarrow{x^2}$

$y_3' = 3{,}04 \cdot 3 - 2{,}18 \Rightarrow 6{,}94 - 6{,}5 = 0{,}44 \xrightarrow{x^2}$

$y_4' = 3{,}04 \cdot 4 - 2{,}18 \Rightarrow 9{,}98 - 10{,}2 = 0{,}04 \xrightarrow{x^2}$

$y_5' = 3{,}04 \cdot 5 - 2{,}18 \Rightarrow 13{,}02 - 13{,}1 = 0{,}08 \xrightarrow{x^2}$

$\xrightarrow{MSE}$ $0{,}0576 + 0{,}01 + 0{,}1936 + 0{,}0096 + 0{,}0064 =$

$= 0{,}2692 / 5 = 0{,}0538$ (MSE) $\rightarrow 0{,}23$

$x_6 = 6 \cdot 3{,}04 - 2{,}18 \rightarrow \underline{\underline{16{,}06}}$

0,23

$\boxed{16{,}06 \pm 0{,}23}$

15,83 $\rightarrow$ 16,29

0,46

Eigenvalue / Eigenvector

$$\begin{bmatrix} 0 & 4 \\ 2 & 6 \end{bmatrix} \quad \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 0 & 16 \\ 4 & 36 \end{bmatrix}$$

$2x$ $\quad\quad$ $\boxed{1\cancel{x}}$ $\quad\quad$ $(2\cdot x)^2$

a $\quad\quad\quad\quad$ b

$x \cdot 2$

a ⟶

$$M = \begin{pmatrix} 7 & 2 & -2 \\ 4 & 5 & -2 \\ 0 & 0 & 3 \end{pmatrix}$$

Eigenvallies ?

Eigenvectors -?

$$M \cdot V - \lambda \cdot V = 0$$

$$V(M - \lambda) = 0$$

$$V(M - \lambda \cdot I) = 0$$

$$M \cdot V = \lambda \cdot V$$

eigenvalue

eigenvector

$$\det \begin{vmatrix} 7-\lambda & 2 & -2 \\ 4 & 5-\lambda & -2 \\ 0 & 0 & 3-\lambda \end{vmatrix} = (7-\lambda) \cdot (5-\lambda)(3-\lambda) - (8 \cdot (3-\lambda))$$

$$(3-\lambda)(35-7\lambda-5\lambda+\lambda^2-8)$$

$$(3-\lambda)(\lambda^2 - 12\lambda + 27)$$

$$\boxed{2x_1 + x_2 - x_3 \qquad x_3 = 2x_1 + x_2}$$

where $x_1, x_2 \in \mathbb{R}$

$$\lambda = 3 \; ; \quad \lambda_1 = 3 \quad \lambda_2 = 9$$

λ_1

$$\begin{pmatrix} 7-3 & 2 & -2 \\ 4 & 5-3 & -2 \\ 0 & 0 & 0 \end{pmatrix} = \begin{array}{c} \begin{pmatrix} 4 & 2 & -2 \\ 4 & 2 & -2 \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{matrix} x_1 & x_2 & x_3 \end{matrix} \end{array}$$

$$\begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 7-2-2 \\ 4-5-2 \\ 0 \quad 0 \quad +3 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix} \qquad = 3 \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 & 2 & -2 \\ 4 & 5 & -2 \\ 0 & 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$