Homework 2

February 12, 2024

1 Task 2.1

Show that similar matrices have equal determinants: det A = det B if AB
Solution: AS=AB $\rightarrow det(AS) \rightarrow det(SB) \rightarrow det(A)det(S) \rightarrow det(S)det(B) \rightarrow det(A) = det(B)$

2 Task 2.2

AV=LV; AAV=ALV; $A^2V = LAV; A^2V = L(LV) = L^2V; A^2V = AV = LV = L^2V; LV = L^2V; L^2V - LV = 0; V(L^2 - L) = 0;$ *Since V is nonzero, we have* : $L^2 - L = 0; L(L-1) = 0; L1 = 0$ *and* $L2 = 1$

3 Task 2.4

A matrix is orthogonal if: $Q^T = Q^{-}1\ QQ^T = QQ^{-}1\ QQ^T = I$

3.1 Example

A= $\begin{pmatrix} 7 & 2 \\ 2 & 7 \end{pmatrix}$

$A^T = \begin{pmatrix} 7 & 2 \\ 2 & 7 \end{pmatrix}$

$x^T Ax = \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} 7 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 & x_2 \end{pmatrix} * \begin{pmatrix} 7x_1 + 2x_2 \\ 2x_1 + x_2 \end{pmatrix} =$
$x_1(2x_1 + 4x_2) + x_2(3x_1 5x_2) = 2x_1^2 = 4x_1x_2 + 3x_2x_1 - 5x_2^2 = 2x_1^2 + 7x_1x_2 - 5x_2^2$