Suppose that $F \cong_q G$ and $G \cong_q H$. Then by Thm 5.4. (p. 17) there exists two sets of projectors

$$\{ P_{fg} \, ; \, f \in V(F) \text{ and } g \in V(G) \}$$

and

$$\{ Q_{gh} \, ; \, g \in V(G) \text{ and } h \in V(H) \}$$

satisfying certain conditions.

Now define

$$\{ R_{fh} \, ; \, f \in V(F) \text{ and } h \in V(H) \}$$

where $R_{fh} = \sum_{g \in V(G)} P_{fg} \otimes Q_{gh}$.

We then have:

(1) $R_{fh}^2 = R_{fh}^+ = R_{fh}$ so that all the $R_{fh}$ are projectors.

(2) $\sum_{f \in V(F)} R_{fh} = I$

(3) $\sum_{h \in V(H)} R_{fh} = I$

(4) $R_{fh} R_{f'h'} = 0$ if $\operatorname{rel}(f, f') \neq \operatorname{rel}(h, h')$

For properties (1) – (4) you will need to use the fact that your projectors satisfy the conditions given in Thm 5.4. Presumably (4) will be the hardest.