Electrostatic force Vs Gravitational Force (Universal Law of Gravitation)

Electrostatic force (F_e)	Gravitational Force (F_G)
$\vec{F_e}\ \alpha\ \frac{q_1 q_2}{r^2}$ $\lvert\vec{F_e}\rvert = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q_1 q_2}{r^2}$ Where ϵ_0 *is the permittivity of free space* $\left(\frac{1}{4\pi\epsilon_0}\right) = 9x10^9 Nm^2C^{-2}$	As per Universal Law of Gravitation, there will always be a force of attraction between tow bodeis. The force of attraction is given by Newton as $\vec{F_G}\ \alpha\ \frac{m_1 m_2}{r^2}$ $\vec{F_G} = -G\frac{m_1 m_2}{r^2}$ ➢ Where G is universal constant, does not depend on the medium ; G = 6.67 x 10^{-11} Nm^2kg^{-2} ➢ $m_1 m_2$ is the masses of the two bodies ➢ r is the distance between the two bodies
➢ Attractive or repulsive	➢ Attractive always
➢ Charge is of 2 types (positive and negative)	➢ Mass is only one type
➢ Depends on medium	➢ F_G does not depend on medium. G is universal constant
➢ Stronger	➢ Weaker
➢ Charge is quantized	➢ For mass, not established yet

Electrostatic force between 2 protons	Gravitational Force between 2 protons
(say r = 1 $\dot{A} = 10^{-10}m$) $\lvert\vec{F_e}\rvert = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{r^2}$, Where $\left(\frac{1}{4\pi\epsilon_0}\right) = 9x10^9 Nm^2C^{-2}$ e = 1.6 x 10^{-19} C	**(say r = 1 $\dot{A} = 10^{-10}m$)** $\lvert\vec{F_G}\rvert = G\frac{m_p^2}{r^2}$; where G = 6.67 x 10^{-11} Nm^2kg^{-2} $m_p \approx$ 1.6 x 10^{-27} Kg

$$\left|\frac{\vec{F_e}}{\vec{F_G}}\right| = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{\cancel{r^2}}\ x\ \frac{\cancel{r^2}}{Gm_p^2} = \frac{1.6\ x\ 1.6\ x\ 10^{-19}x10^{-19}\ x\ 9\ x\ 10^9}{6.67\ x\ 10^{-11}x\ 1.6\ x\ 1.6\ x\ 10^{-27}x10^{-27}} = 1.3\ x\ \frac{10^{-29}}{10^{-65}} = 1.3\ x\ 10^{36} \approx 10^{36}$$

$F_e \approx 10^{36} x\ F_G$

Electrostatic force between 2 protons is 10^{36} times stronger than the gravitational force between them.
Electrostatic force is repulsive whereas gravitational force is attractive force.
Note that the ratio is independent of the distance between the two bodies (as distance cancels out in the ratio calculation)

Electrostatic force between 2 electrons	Gravitational Force between 2 electrons
(say r = 1 $\dot{A} = 10^{-10}m$) $\lvert\vec{F_e}\rvert = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{r^2}$, Where $\left(\frac{1}{4\pi\epsilon_0}\right) = 9x10^9 Nm^2C^{-2}$ e = 1.6 x 10^{-19} C	**(say r = 1 $\dot{A} = 10^{-10}m$)** $\lvert\vec{F_G}\rvert = G\frac{m_e^2}{r^2}$; where G = 6.67 x 10^{-11} Nm^2kg^{-2} m_e = 9.11 x 10^{-31} Kg ≈ 9 x 10^{-31} Kg

$$\left|\frac{\vec{F_e}}{\vec{F_G}}\right| = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{\cancel{r^2}}\ x\ \frac{\cancel{r^2}}{Gm_e^2} = \frac{1.6\ x\ 1.6\ x\ 10^{-19}x10^{-19}\ x\ 9\ x\ 10^9}{6.67\ x\ 10^{-11}x\ 9\ x\ 9\ x\ 10^{-31}x10^{-31}} \approx 10^{43}$$

$F_e \approx 10^{43}\ x\ F_G$

Electrostatic force between 2 electrons is 10^{43} times stronger than the gravitational force between them.
Electrostatic force is repulsive whereas gravitational force is attractive force.
Note that the ratio is independent of the distance between the two bodies (as distance cancels out in the ratio calculation)

Electrostatic force between an electron and a proton	Gravitational Force between an electron and a proton
(say r = 1 $\dot{A} = 10^{-10}m$) $\lvert\vec{F_e}\rvert = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{r^2}$, Where $\left(\frac{1}{4\pi\epsilon_0}\right) = 9x10^9 Nm^2C^{-2}$ e = 1.6 x 10^{-19} C	**(say r = 1 $\dot{A} = 10^{-10}m$)** $\lvert\vec{F_G}\rvert = G\frac{m_e m_p}{r^2}$; where G = 6.67 x 10^{-11} Nm^2kg^{-2} $m_e \approx$ 9 x 10^{-31} Kg $m_p \approx$ 1.6 x 10^{-27} Kg

$$\left|\frac{\vec{F_e}}{\vec{F_G}}\right| = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{\cancel{r^2}}\ x\ \frac{\cancel{r^2}}{Gm_e m_p} = \frac{1.6\ x\ 1.6\ x\ 10^{-19}x10^{-19}\ x\ 9\ x\ 10^9}{6.67\ x\ 10^{-11}x\ 9\ \ x\ 10^{-31}x\ 1.6\ x\ 10^{-27}} \approx 2.4\ x\ 10^{39}$$

$F_e \approx 2.4\ x\ 10^{39}\ x\ F_G$

Electrostatic force between an electron and a proton is $2.4\ x\ 10^{39}$ times stronger than the gravitational force between them.
Both Electrostatic force and gravitational force is attractive forces.
Note that the ratio is independent of the distance between the two bodies (as distance cancels out in the ratio calculation)

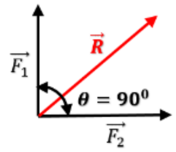
In summary,

- ➢ Between 2 protons, $F_e \approx 10^{36} x\ F_G$
- ➢ Between 2 electrons, $F_e \approx 10^{43}\ x\ F_G$
- ➢ Between an electron and a proton, $F_e \approx 2.4\ x\ 10^{39}\ x\ F_G$

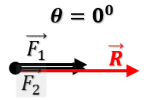
Resultant vector

$|\vec{R}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2cos\theta}\,; where\ \theta\ is\ the\ angle\ between\ the\ two\ vectors\ F_1\ and\ F_2$

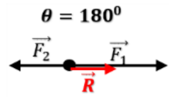
- If θ = 90^0, cos90^0 = 0 ; $|\vec{R}| = \sqrt{F_1^2 + F_2^2}$; If $F_1 = F_2$ = F, then $|\vec{R}| = \sqrt{2}\ |F|$
- If F_1 = 3N, F_2 = 4N and θ=90^0, then R = $\sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5\ N$



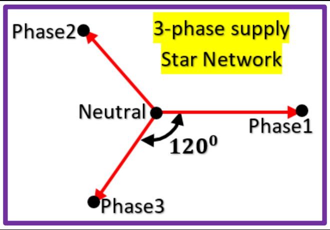
- If θ = 0^0, cos0^0 = 1 ; $|\vec{R}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2}$; If $F_1 = F_2$ = F, then $|\vec{R}| = 2\ |F|$
- If F_1 = 3N, F_2 = 4N and θ=0^0, then R = $\sqrt{3^2+4^2+24} = \sqrt{9+16+24} = \sqrt{49} = 7\ N$
- If F_1 = 4N, F_2 = 4N and θ=0^0, then R = $\sqrt{4^2+4^2+32} = \sqrt{32+32} = \sqrt{64} = 8\ N$
- From the above 2 examples, if θ = 0^0 between 2 vectors, then simply add the magnitude of 2 vectors to get the resultant vector magnitude. Direction of resultant vector is same as the two vectors.



- If θ = 180^0, cos180^0 = -1 ; $|\vec{R}| = \sqrt{F_1^2 + F_2^2 - 2F_1F_2}$; If $F_1 = F_2$ = F, then $|\vec{R}| = 0$
- If F_1 = 3N, F_2 = 4N and θ=180^0, then R = $\sqrt{3^2+4^2-24} = \sqrt{9+16-24} = \sqrt{1} = 1\ N$
- If F_1 = 4N, F_2 = 4N and θ=180^0, then R = $\sqrt{4^2+4^2-32} = \sqrt{32-32} = 0\ N$
- From the above 2 examples, if θ = 0^0 between 2 vectors, then simply subtract the magnitude of 2 vectors to get the resultant vector magnitude. Direction of resultant vector is same as the bigger vector (See figure).



Info: 3-Phase supply voltage
- Phase difference between two phases = 1200
- Voltage between phase and neutral is called phase voltage
- Voltage between two phases is called line voltage.
- Formula : Line voltage = 1.73 x Phase voltage
- In India phase voltage = 220V (nominal)
- Therefore, line voltage (phase to phase) = 1.73 x 220 = 381 V



Problem : Can a body has charge $Q\ = +\sqrt{2}\ \mu C$?

Formula q = ne ; n = q/e = $\frac{+\sqrt{2}\ x\ 10^{-6}}{1.6\ x\ 10^{-19}} = \frac{\sqrt{2}}{1.6}\ x 10^{13}$ = 0.8838834764831844 x 10^{13} = 8838834764831.844 which is not an integer. So the body having charge $+\sqrt{2}\ \mu C$ cannot exist as per the rule quantization of charge.

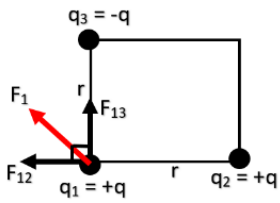
Problem : What is the net force on charge q_1 ?

$|\vec{F_{12}}| = K\ \frac{q^2}{r^2}$; $|\vec{F_{13}}| = K\ \frac{q^2}{r^2}$;

So, $|\vec{F_{12}}| = |\vec{F_{13}}| = |\vec{F}|$ (say) ------------------- (1)

$F_1 = \sqrt{F_{12}^2 + F_{13}^2 + 2F_{12}F_{13}\cos 90^0} = \sqrt{F^2+F^2} = \sqrt{2}F$; since (cos90^0) = 0 and using (1)

Resultant force on q1 = $F_1 = \sqrt{2}\ K\ \frac{q^2}{r^2} = \sqrt{2}\ \left(\frac{1}{4\pi\epsilon_0}\right)\left[\frac{q^2}{r^2}\right]$ and the direction is as shown in the figure.



Problem : What is the net force on charge q_1 ?

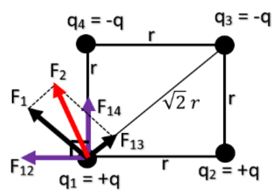
$|\vec{F_{12}}| = K\ \frac{q^2}{r^2}$; $|\vec{F_{14}}| = K\ \frac{q^2}{r^2}$;

So, $|\vec{F_{12}}| = |\vec{F_{14}}| = |\vec{F}|$ (say) --------------------- (1)

$|\vec{F_{13}}| = K\ \frac{q^2}{2r^2} = \frac{1}{2}|\vec{F}|$ ---------- (2)

$F_1 = \sqrt{F_{12}^2 + F_{14}^2 + 2F_{12}F_{14}\cos 90^0} = \sqrt{F^2+F^2} = \sqrt{2}F$; since (cos90^0) = 0 and using (1)

Resultant force on q1 = $F_1 = \sqrt{2}\ K\ \frac{q^2}{r^2} = \sqrt{2}\ \left(\frac{1}{4\pi\epsilon_0}\right)\left[\frac{q^2}{r^2}\right]$ and the direction is as shown in the figure.



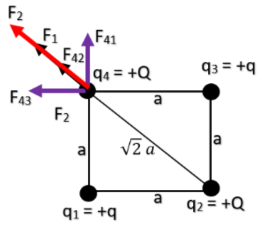
Net force between F_1 and F_{13} = F_2 = $\sqrt{F_1^2 + F_{13}^2 + 2F_1F_{13}\cos 90^0} = \sqrt{F_1^2 + F_{13}^2} = \sqrt{2F^2 + \frac{1}{4}F^2} = F\sqrt{2.25} = 1.5F$

(Angle between F_1 and F_{13} = 90^0)

∴ the net force on q_1 = $\vec{F_2}$ = 1.5 $\left(\frac{1}{4\pi\epsilon_0}\right)\left[\frac{q^2}{r^2}\right]$. The direction is as shown in the figure.

- We can calculate the exact direction of F_2.
- The angle between F_1 and F_2 at q_1 = θ. Sinθ = 0.5/1.5 ; θ ≈ 20^0
- Angle between F_2 and North = 45^0-20^0 = 25^0
- ∴ F_2 is 25^0 west of north

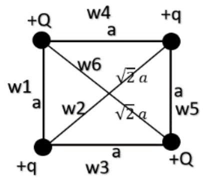
Problem : Four point charges Q, q, Q & q are placed at the corners of a square of side 'a' as shown in figure. Find the net electric force on Q (left top corner charge)



Applying coulomb's law, we have

- Force on q_4 due to q_1 = $\left|\overrightarrow{F_{41}}\right| = K\,\frac{qQ}{a^2}$; direction is as shown in the figure
- Force on q_4 due to q_3 = $\left|\overrightarrow{F_{43}}\right| = K\,\frac{qQ}{a^2}$; direction is as shown in the figure
- So, $\left|\overrightarrow{F_{41}}\right| = \left|\overrightarrow{F_{43}}\right| = \left|\vec{F}\right|$ (say) and the angle between $\overrightarrow{F_{41}}$ and $\overrightarrow{F_{43}}$ is 90^0 and $\cos 90^0 = 0$
- Net force due to $\overrightarrow{F_{41}}$ and $\overrightarrow{F_{43}} = \overrightarrow{F_1}$; $\left|\overrightarrow{F_1}\right| = \sqrt{F_{41}^2 + F_{43}^2 + 2F_{41}F_{43}\cos 90^0}$
- $\left|\overrightarrow{F_1}\right| = \sqrt{F^2 + F^2 + 0} = \sqrt{2}\,F$; direction of $\overrightarrow{F_1}$ is as shown in the figure ; it is 45^0 west of north
- Force on q_4 due to q_2 = $\left|\overrightarrow{F_{42}}\right| = K\,\frac{Q^2}{2a^2}$; direction is as shown in the figure; it is 45^0 west of north
- $\overrightarrow{F_1}$ *and* $\overrightarrow{F_{42}}$ *vectors are in the same direction*. Hence the magnitudes are added to get the overall net force $\overrightarrow{F_2}$
- $\left|\overrightarrow{F_2}\right| = \left|\overrightarrow{F_1}\right| + \left|\overrightarrow{F_{42}}\right| = \sqrt{2}\,F + K\,\frac{Q^2}{2a^2} = \sqrt{2}\,K\,\frac{qQ}{a^2} + K\,\frac{Q^2}{2a^2}$ where $K = \frac{1}{4\pi\epsilon_0}$
- Net force on q4 (+Q) is $\left|\overrightarrow{F_2}\right| = \sqrt{2}\,K\,\frac{qQ}{a^2} + K\,\frac{Q^2}{2a^2} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q}{a^2}\left[\sqrt{2}\,q + \frac{Q}{2}\right]$
- Direction is as shown in the figure. It is 45^0 west of north.

Extra info on the above problem : Potential energy (PE) of the system



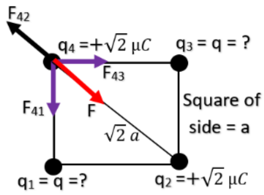
w1 = w3 = w4 = w5 = $K\,\frac{qQ}{a}$

w2 = $K\,\frac{q^2}{\sqrt{2}a}$

w6 = $K\,\frac{Q^2}{\sqrt{2}a}$

w = w1 + w2 + w3 + w4 + w5 + w6

Problem: Two equal charges of value $+\sqrt{2}\mu C$ are placed at two opposite corners of a square (see fig). And an equal two point charges are placed at other two corners(see fig). What must be the value of q so that the resultant force on "charge on top left corner" is zero ?



Applying coulomb's law, we have

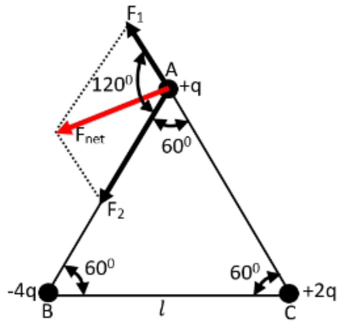
Force on q_4 due to q_2 = $\left|\overrightarrow{F_{42}}\right| = K\,\frac{q_2 q_4}{2a^2}$;

direction is as shown in the figure and *where* $K = \frac{1}{4\pi\epsilon_0}$

$\left|\overrightarrow{F_{42}}\right| = K\,\frac{2\,x\,10^{-12}}{2a^2} = K\,\frac{10^{-12}}{a^2}$ ------------------------------ (1)

- In order to make net force on q_4 = 0, the resultant of the other 2 forces F_{41} and F_{43} on q_4 due to charges q_1 and q_3 respectively should be equal and opposite in direction to F_{42}. Therefore, F_{41} and F_{43} should be directed as shown in the figure. Since it should be an attractive force, both q_1 and q_3 must be negative.
- F is the resultant vector of F_{41} and F_{43}
- $\left|\overrightarrow{F_{41}}\right| = K\,\frac{\sqrt{2}\,x\,10^{-6}\,q}{a^2}$; $\left|\overrightarrow{F_{43}}\right| = K\,\frac{\sqrt{2}\,x\,10^{-6}\,q}{a^2}$
- $\therefore\ \left|\overrightarrow{F_{41}}\right| = \left|\overrightarrow{F_{43}}\right|$ and the angle between these two vectors = 90^0. ($\cos 90^0 = 0$)
- $\left|\vec{F}\right| = \sqrt{F_{41}^2 + F_{43}^2 + 2F_{41}F_{43}\cos 90^0} = \sqrt{2\left[K\,\frac{\sqrt{2}\,x\,10^{-6}\,q}{a^2}\right]^2} = \frac{2K\,x\,10^{-6}\,q}{a^2}$ -------- (2)
- For equilibrium of q_4, we must have $\vec{F} + \overrightarrow{F_{42}} = 0 \implies \vec{F} = -\overrightarrow{F_{42}}$. From (1) and (2), we have
- $\frac{2K\,x\,10^{-6}\,q}{a^2} = -\frac{K\,x\,10^{-12}}{a^2} \implies q = -\frac{1}{2}x\,10^{-6} = -0.5\mu C$
- $\therefore\ q = -0.5\mu C$

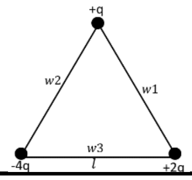
Problem : Three point charges q, -4q and 2q are placed at the vertices of an equilateral triangle ABC of side 'l' as shown in figure.
(a) Obtain the expression for the magnitude of the resultant electric force acting on the charge "q".
(b) Find out the amount of work to be done to separate the charges to infinite distance.



(a) Applying coulomb's law, we have

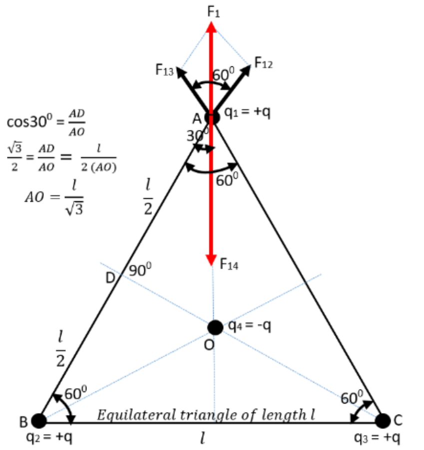
- Force on +q due to +2q = $\left|\vec{F_1}\right| = K\,\frac{2q^2}{l^2}$ $where\ K = \frac{1}{4\pi\epsilon_0}$
- Force on +q due to -4q = $\left|\vec{F_2}\right| = K\,\frac{4q^2}{l^2} = 2\left|\vec{F_1}\right|$
- From figure, note that the angle between $\vec{F_1}$ & $\vec{F_2}$ is 120^0 and cos120^0 = -1/2
- Net force due to $\vec{F_1}\ and\ \vec{F_2} = \vec{F_{net}} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\,cos120^0}$
- $\left|\vec{F_{net}}\right| = \sqrt{F_1^2 + 4F_1^2 - 4F_1^2 x\frac{1}{2}} = \sqrt{3F_1^2} = \sqrt{3}\,F_1$
- $\left|\vec{F_{net}}\right| = \frac{2\sqrt{3}}{4\pi\epsilon_0}\frac{q^2}{l^2}$

(b) The amount of work required to separate the charges to infinite distance = electric PE of the system



- $w1 = K\,\frac{q(2q)}{l};\ w2 = K\,\frac{(-4q)(q)}{l};w3 = K\,\frac{(-4q)(2q)}{l}$ $where\ K = \frac{1}{4\pi\epsilon_0}$
- $w1 = K\,\frac{2q^2}{l};\ w2 = -K\,\frac{4q^2}{l};w3 = -K\,\frac{8q^2}{l}$
- $\therefore w = w1 + w2 + w3 = -\,\frac{10Kq^2}{l}$

Problem : Three charges each +q are placed at the vertices of an equilateral triangle ABC of side 'l' as shown in figure. A 4th charge Q is placed at the centre of the triangle.
(a) If Q = -q, will the charges at the corners move towards the centre or fly away from it ?
(b) For what value of Q will all the 4 charges remain stationary ?
(c) How much work will be done in removing the charges to infinity in situation (b)



(a) Charge q_1 experiences an electrical force due to 3 charges (q_2, q_3 , q_4)

- $\left|\vec{F_{12}}\right| = K\,\frac{q^2}{l^2}$ $where\ K = \frac{1}{4\pi\epsilon_0}$
- $\left|\vec{F_{13}}\right| = K\,\frac{q^2}{l^2} = \left|\vec{F_{12}}\right| = F\ (say)\ ;$
- Angle between F_{12} and F_{13} = 60^0 and cos60^0 = 1/2
- Net force due to $\vec{F_{12}}\ and\ \vec{F_{13}} = \vec{F_1} = \sqrt{F^2 + F^2 + 2FF\,cos60^0} = \sqrt{3}F$
- $\left|\vec{F_{14}}\right| = K\,\frac{q^2}{\left(\frac{l}{\sqrt{3}}\right)^2} = 3K\,\frac{q^2}{l^2} = 3F$
- So, $\left|\vec{F_{14}}\right| = 3F\ and\ \left|\vec{F_1}\right| = \sqrt{3}F,$
- $\therefore \left|\vec{F_{14}}\right| > \left|\vec{F_1}\right| \Longrightarrow$ the resultant of these 2 vectors is towards the centre ; the charge q_1 is pulled towards the centre "O" of the circle (that is towards q_4)
- This is also true for q_2 and q_3 → they are also pulled towards q_4

(b)

- Since all 3 charges q_1, q_2 and q_3 move towards q_4, the resultant force on charge q_4 will be zero. If we want all charges to be stationary, then we must have $\vec{F_{14}} = -\,\vec{F_1}$
- $\vec{F_1} = \sqrt{3}\,K\,\frac{q^2}{l^2}\ \ and\ \vec{F_{14}} = 3\,K\,\frac{qQ}{l^2}$
- $3\,K\,\frac{qQ}{l^2} = -\sqrt{3}\,K\,\frac{q^2}{l^2}$
- $Q = -\frac{q}{\sqrt{3}}$

(c)

- $If\ Q = -\frac{q}{\sqrt{3}}\ ,all\ 4\ charges\ are\ stationary$. The resultant force on each charge is zero and hence no work will be done in removing the charges to infinity.

-- 17 -- Important problem

Problem: Two point charges q_1 and q_2 are 3m apart and their combined sum charge = 20 µC. If one repels the other with a force = 0.075 N. What are the values of two charges?

First method (easier one)

Given:

- Force of **repulsion** F = 0.075 N = $75\ x\ 10^{-3}\ N$ ($\Longrightarrow$ charges are of same type, either both +ve or both –ve)
- $q_1 + q_2$ = +20 µC $\Longrightarrow$ both charges must be positive {we must get both charges positive as answers}
- Distance between the charges = r = 3 m
- We know that $\frac{1}{4\pi\epsilon_0} = 9\ x\ 10^9\ Nm^2C^{-2}$

Formula (considering only magnitude): $|\vec{F}_{12}| = \left[\frac{1}{4\pi\epsilon_0}\right]\left[\frac{q_1q_2}{r^2}\right] \Longrightarrow 75 \text{ x } 10^{-3} = 9\ x\ 10^9 \text{ x } \frac{q_1q_2}{9}$ $\therefore q_1q_2 = 75\ x\ 10^{-12}\ C^2$

We know that $(x-y)^2 = (x+y)^2 - 4xy$; using this….

$(\boldsymbol{q_1} - \boldsymbol{q_2})^2 = (q_1+q_2)^2 - 4q_1q_2 = (400\ \ x\ 10^{-12})C^2 - (4\ x\ 75\ x\ 10^{-12}\ )C^2 = \mathbf{100\ x10^{-12}\ C^2}$

$\therefore q_1 - q_2 = \pm\ 10\ x10^{-6}\ C$

- Taking $[q_1 - q_2 = +10\ \mu C]$ and we have $[q_1 + q_2 = +20\ \mu C]$; adding these two, we get
  - $2q_1 = 30\ \mu C$ *or* $q_1 = +15\mu C$; Using $q_1 + q_2 = +20\ \mu C,\quad \therefore q_2 = +5\ \mu C$
- Taking $[q_1 - q_2 = -10\ \mu C]$ and we have $[q_1 + q_2 = +20\ \mu C]$; adding these two, we get
  - $2q_1 = 10\ \mu C$ *or* $q_1 = +5\mu C$; Using $q_1 + q_2 = +20\ \mu C,\quad \therefore q_2 = +15\ \mu C$

$\therefore q_1 = +15\mu C\ , q_2 = +5\ \mu C$ OR $q_1 = +5\mu C\ , q_2 = +15\ \mu C$; Since both are +ve charges, they repel each other

Second method : using quadratic formula

Given:

- Force of **repulsion** F = 0.075 N = $75\ x\ 10^{-3}\ N$ ($\Longrightarrow$ charges are of same type, either both +ve or both –ve)
- $q_1 + q_2$ = +20 µC $\Longrightarrow$ both charges must be positive {we must get both charges as positive as answers}
- Distance between the charges = r = 3 m
- We know that $\frac{1}{4\pi\epsilon_0} = 9\ x\ 10^9\ Nm^2C^{-2}$

Using quadratic formula (see next row of the table)

Formula (considering only magnitude): $|\vec{F}_{12}| = \left[\frac{1}{4\pi\epsilon_0}\right]\left[\frac{q_1q_2}{r^2}\right] \Longrightarrow 75 \text{ x } 10^{-3} = 9\ x\ 10^9 \text{ x } \frac{q_1q_2}{9}$ $\therefore q_1q_2 = 75\ x\ 10^{-12}\ C^2$

$q_1 + q_2 = +20\ \mu C\ \therefore\ q_2 = [20\ \mu C - q_1]$

$q_1q_2 = 75\ x\ 10^{-12}\ C^2$ becomes $q_1(20\ \mu C - q_1) = 75\ x\ 10^{-12}$

$(20\ x\ 10^{-6})\ q_1 - q_1^2 - 75\ x\ 10^{-12} = 0$

$q_1^2 - (20\ x\ 10^{-6})\ q_1 + 75\ x\ 10^{-12} = 0$ (this is similar to $ax^2 + bx + c = 0$) ; $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$\therefore\ q_1 = \frac{20\ x\ 10^{-6}\ \pm\sqrt{(400\ x\ 10^{-12} - 4\ x\ 75\ x\ 10^{-12})}}{2} = 10\ x\ 10^{-6}\ \pm\sqrt{(100\ x\ 10^{-12} - 75\ x\ 10^{-12})}$$

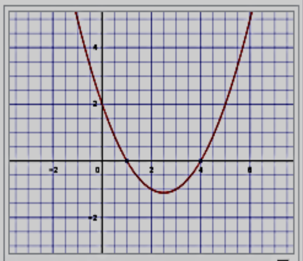
$\therefore\ q_1 = 10\ x\ 10^{-6}\ \pm\sqrt{(25\ x\ 10^{-12})} = (10\ x\ 10^{-6}) \pm (5\ x\ 10^{-6}) = (10 \pm 5)\ \mu C$

$\therefore\ q_1 = 15\mu C\ \ or\ 5\mu C\ ; so\ correspondingly\ q_2 = 5\mu C\ or\ 15\mu C$

$\therefore q_1 = +15\mu C\ , q_2 = +5\ \mu C$ OR $q_1 = +5\mu C\ , q_2 = +15\ \mu C$; Since both are +ve charges, they repel each other

Quadratic formula

In elementary algebra, the **quadratic formula** is a formula that provides the solution(s) to a quadratic equation. There are other ways of solving a quadratic equation instead of using the quadratic formula, such as factoring (direct factoring, grouping, AC method), completing the square, graphing and others.[1]



A quadratic function with roots $x = 1$ and $x = 4$.

Given a general quadratic equation of the form

$$ax^2 + bx + c = 0$$

with x representing an unknown, a, b and c representing constants with $a \neq 0$, the quadratic formula is:

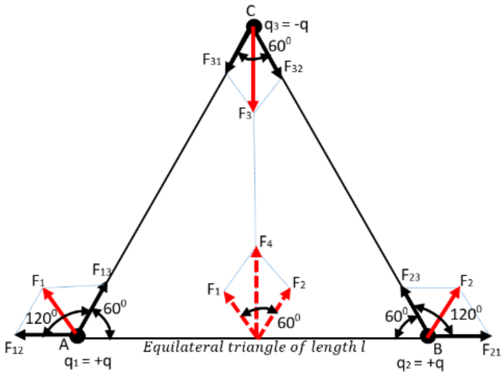
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where the plus-minus symbol "±" indicates that the quadratic equation has two solutions.[2] Written separately, they become:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Each of these two solutions is also called a root (or zero) of the quadratic equation. Geometrically, these roots represent the x-values at which *any* parabola, explicitly given as $y = ax^2 + bx + c$, crosses the x-axis.[3]

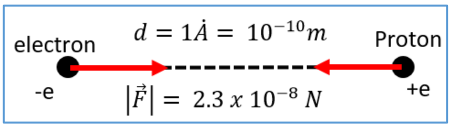
<u>Problem</u> : Consider the charges q, q and –q placed at the vertices of an equilateral triangle as shown below. What is the force on each charge.



Applying coulomb's law, we have

- Force on q_1 due to q_2 = $|\overrightarrow{F_{12}}| = K\,\frac{q^2}{l^2}$ $where\ K = \frac{1}{4\pi\epsilon_0}$
- Force on q_1 due to q_3 = $|\overrightarrow{F_{13}}| = K\,\frac{q^2}{l^2};\ |\overrightarrow{F_{12}}| = |\overrightarrow{F_{13}}| = |\vec{F}|\ say$
- From fig, note that the angle between $\overrightarrow{F_{12}}$ & $\overrightarrow{F_{13}}$ is 120^0 & $\cos 120^0 = -1/2$
- Net force due to $\overrightarrow{F_{12}}\ and\ \overrightarrow{F_{13}} = \overrightarrow{F_1} = \sqrt{F^2 + F^2 + 2FF\cos 120^0}$
- $|\overrightarrow{F_1}| = \sqrt{2F^2 - 2F^2 x \frac{1}{2}} = \sqrt{F^2} = F$; direction along BC
- Similarly, $|\overrightarrow{F_2}| = K\,\frac{q^2}{l^2}$ = F ; direction along AC
- Similarly, $\overrightarrow{F_3} = \sqrt{\overrightarrow{F_{31}} + \overrightarrow{F_{32}} + 2F_{31}F_{32}\cos 60^0}$ [cos 60^0 = 1/2]
- Since $\overrightarrow{F_{31}} = |\overrightarrow{F_{32}}| = F$; $\overrightarrow{F_3} = \sqrt{F^2 + F^2 + 2FF\cos 60^0} = \sqrt{3F^2} = \sqrt{3}F$; along the direction bisecting $A\hat{C}B$
- $\therefore\ \overrightarrow{F_1} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2}\ along\ BC\ ;\ \overrightarrow{F_2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2}\ along\ AC\ \ and\ \overrightarrow{F_1} = \frac{\sqrt{3}}{4\pi\epsilon_0}\frac{q^2}{l^2}\ along\ the\ direction\ bisecting\ A\hat{C}B$
- Extra : Since $\overrightarrow{F_1}$ is along BC and $\overrightarrow{F_2}$ is along AC, therefore $\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = 0$
- Reason: Resultant of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ = $\overrightarrow{F_4} = \sqrt{F^2 + F^2 + 2FF\cos 60^0} = \sqrt{3F^2} = \sqrt{3}F$ along the direction opposite to $\overrightarrow{F_3}$
- $\overrightarrow{F_3}\ and\ \overrightarrow{F_4}$ have the same magnitude and opposite in direction ; $\overrightarrow{F_4} = -\overrightarrow{F_3}$
- $But\ \overrightarrow{F_4} = \overrightarrow{F_1} + \overrightarrow{F_2}\ [vector\ sum];\ \therefore\ \overrightarrow{F_1} + \overrightarrow{F_2} = -\overrightarrow{F_3}$
- $\therefore\ \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = 0\ ;Vector\ sum\ of\ \overrightarrow{F_1}, \overrightarrow{F_2}\ and\ \overrightarrow{F_3} = 0$

Info:



- The force with which the electron is getting attracted to the proton = the force with which the proton is getting attracted to the electron. This is because the magnitude of charge on a proton and an electron is same.
- For example, when the distance d between them is 1 $\mathring{A}$, force = 2.3 x 10^{-8} N
  - We will get the above result using coulomb's force formula $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{d^2}$
  - $Substituting\ \frac{1}{4\pi\epsilon_0} = 9\ x\ 10^9 Nm^2C^{-2}\ ; d = 10^{-10}m\ \ and\ \ e = 1.6\ x\ 10^{-19}C$
- Direction is as shown in the figure (the line joining the two particles)

Question: How fast the electron accelerates (moves) towards the proton OR how fast the proton accelerates (moves) towards the electron. Are they same ?

No, they are different due to their different masses. (we know that F = ma)

For electron $F_e = m_e a_e$ and for the proton $F_p = m_p a_p$ and we have $F_e = F_p$

For electron $a_e = \frac{F_e}{m_e} = \frac{2.3\ x\ 10^{-8}N}{9.11\ x\ 10^{-31}kg} = 2.5\ x\ 10^{22}ms^{-2}$

For proton $a_p = \frac{F_p}{m_p} = \frac{2.3\ x\ 10^{-8}N}{1.67\ x\ 10^{-27}kg} = 1.4\ x\ 10^{19}ms^{-2}$

Comparing the above results, electron accelerates 1800 times more than that of a proton.

Comparing this acceleration with acceleration due to gravity (=9.8 ms^{-2}), we can conclude that the effect of gravitational field is negligible on the motion of electron and it undergoes very large acceleration under the action of coulomb's force due to a proton.

Info: Let $a_e = 2\ x\ 10^{22}ms^{-2}$ and $a_p = 2\ x\ 10^{19}ms^{-2}$

<u>Electron</u> : At the end of 1s, the electron has accelerated to a velocity = $2\ x\ 10^{22}ms^{-1}$. To cover 1 $\mathring{A}$ distance, the electron just requires → $2\ x\ 10^{22}m \longrightarrow in\ 1s$

10^{-10} m → $\frac{1s\ x 10^{-10}}{2\ x\ 10^{22}} = 0.5\ x\ 10^{-32}s$!!!

∴ When the distance between proton and electron = 1 $\mathring{A}$, the electron is pulled towards proton in just $0.5\ x\ 10^{-32}s$

<u>Proton</u>: At the end of 1s, the proton has accelerated to a velocity = $2\ x\ 10^{19}ms^{-1}$. To cover 1 $\mathring{A}$ distance, the proton just requires → $2\ x\ 10^{19}m \longrightarrow in\ 1s$

10^{-10} m → $\frac{1s\ x 10^{-10}}{2\ x\ 10^{19}} = 0.5\ x\ 10^{-29}s$!!!

∴ When the distance between proton and electron = 1 $\mathring{A}$, the proton is pulled towards electron in just $0.5\ x\ 10^{-29}s$

Therefore, electron is accelerated towards proton in less time as compared to the proton getting accelerated towards electron.

Electric Field: Physical significance of electric field

When we talk about force between charges, we are not giving due credit to a charge in the sense that why it has an influence (force) on the other charge. If the other charge is removed, does it mean that the first charge loses its influence ; or in other words does a charge gets the power only when there is another charge. This concept is not very well characterized when we talk about only force. Another concept would be convenient to define a charge influence and give its due credit.
A term "electric field" hence being introduced to fill the gap between the charge and it effect (force) on the other charge. The concept of the term "electric field" in a way very well characterizes that there is a "strength" at each and every point around the "charged particle" in 3-dimensional space and if any charge is placed in the vicinity of the field, it experiences a force ; If there is no charge, still the strength around the source charge is present in 3-dimensional space. The concept of "electric field" characterises in a very elegant way the action on any particle placed at any point around a charge.

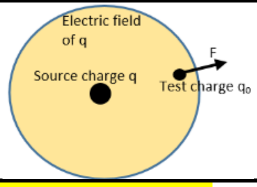
Why a new term or concept "electric field" is introduced when for a point charge or system of charges the measurable quantity is the FORCE (determined by coulomb's law and superposition principle) ; why introduce intermediate quantity called "electric field".
For electrostatics, the concept of electric field is convenient but not really necessary. Electric field is an elegant way of characterising the electrical environment of a point charge or system of charges.
In physics, the term "field" refers to a quantity that is defined at every point in space and my vary from point to point without bothering about whether a test charge place in that field experiences a force or not. It refers to a concept that there is a field strength around a point charge or system of charges.

The true physical significance of the concept of electric field emerges only when we go beyond electrostatics and deal with time-dependent electromagnetic phenomena. It could give indication about time delay between the effect (force on q_2) and the cause (motion of q_1).
The field picture is as follows : the accelerated motion of charge q_1 produces e-m, waves when they propagate with the speed of light and reach q_2 and cause a force on q_2. This implies the motion of field elegantly accounts for time delay. So, field is not just a mathematical construct but is a physical entity.

Another view point is that we talk about magnetic field in a space around a bar magnet; in a similar way we can speak about an electric field around a charged particle or rod.

- The charge q "sets up" an electric field in the space around itself.
- This field acts on test charge q_0 resulting in a force F that q_0 experiences

The field plays an intermediary role in the forces between charges.



Definition of Electric field strength E (aka "strength of electric field" or "intensity of electric field" or "electric field intensity").

A small test body carrying a charge q_0 is placed in an electric field generated by a charge q. We "measure" the electrical force F that q_0 experiences due to the source charge q. This force is the coulomb's force and is given by

$\left|\vec{F}\right| = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$ *where r is the distance between q and* q_0. Electric field is defined as

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

We must use test charge as small as possible. The field due to q_0 should be negligible in comparison with the field due to q. The field due to q_0 should not disturb the field due to q. We can define more appropriately the electric field as

$$\vec{E} = \lim_{q_0 \to 0}\frac{\vec{F}}{q_0}$$

Since F is a vector, E is also vector. **The direction of E is same as that of F when q_0 is positive. The direction is E is opposite to that of F if q_0 is negative. Unit of E is NC^{-1}**

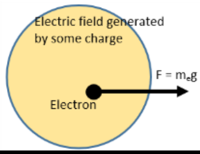
Analogy between $\vec{g}$ and $\vec{E}$

$\vec{g} = \frac{\vec{F}}{m}$ **and** $\vec{E} = \frac{\vec{F}}{q_0}$ **; where m and q_0 are the properties of the test body**

The SI unit of g is N Kg^{-1} or ms^{-2} and the unit of E is NC^{-1}. Thus both g and E are expressed as a force divided by the a property (mass or charge) of the test body.

Example : What is the magnitude of the electric field strength E such that an electron placed in the field would experience an electrical force equal to its weight ?

- electron would experience an electrical force equal to its weight means F = $m_e g$ where m_e is the mass of electron
- We know that $\vec{E} = \frac{\vec{F}}{q_0} = \frac{m_e g}{e} = \frac{(9.1\,x\,10^{-31}\,kg)(9.8\,Nkg^{-1})}{1.6\,x\,10^{-19}C} = 5.6\,x\,10^{-11}\,NC^{-1}$ which is a very weak electric field.



Electric field lines

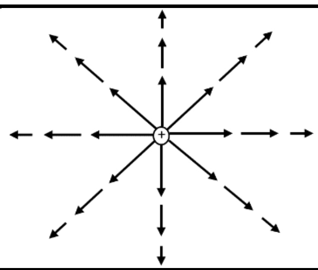
The concept of electric field as a vector was not appreciated by Michael Faraday, who always thought in terms of "lines of force". This term is somewhat misleading, especially in case of magnetic fields. The more appropriate term is "field lines" that is adapted in the NCERT book.

Properties of electric field lines

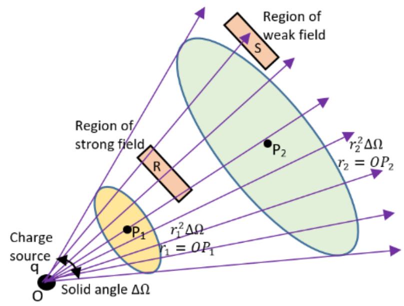
- Field lines start from positive charges and end at negative charges. If there is a single charge, they may start or end at infinity.
- In a charge-free region, electric field lines can be taken to continuous curves without any breaks.
- Two field lines can never cross each other; if they did, the field at the point of intersection will not have a unique direction, which is absurd.
- Electrostatic field lines do not form any closed loops . This follows from the conservative nature of electric field.
- **Direction of E: The tangent to a field line at any point gives the direction of E at that point.**
- **Magnitude of E : The field lines are drawn so that the number of lines per unit cross-sectional area is proportional to the magnitude of E. When the field lines are close together , E is large and where they are far apart E is small.**

Representation of Electric field lines

- Electric field E is a vector. See figure for pictorial representation of field lines.
- Since $E \ \alpha \ \frac{1}{r^2}$, vectors lengths (arrows) decrease as we move away from the charge
- Does the arrows represent the strength of E. In a way, yes. Decrease in length of arrows represents the strength of electric field.
- However, it is the relative density of “field lines” that will represent the strength of E. See the next point.

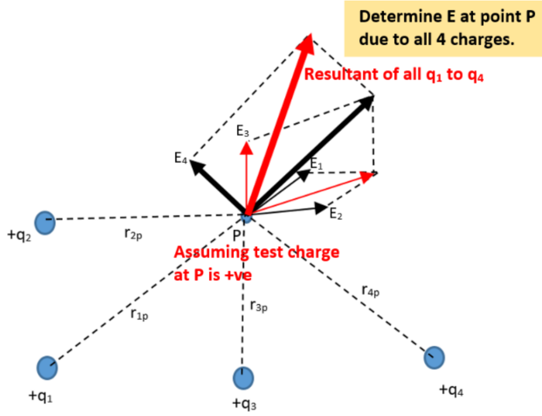


- Consider the following figure. Field lines in area R > field lines in area S
- Therefore, density of field lines in area R > the density of field lines in area S
- Hence, area R represents region of strong field and area S represents region of weak field.
- We can use solid angle $\Delta\Omega \ = \frac{\Delta S}{r^2}$ (this is definition of solid angle)
- For points P_1 and P_2, the solid angle subtended by area around P_1 and P_2 are same.
- Even the field lines cutting those areas are same.
- Areas are given by : $@P_1 -\!\!\rightarrow \ r_1^2\Delta\Omega \quad and \quad @P_2 -\!\!\rightarrow \ r_2^2\Delta\Omega$
- Since ΔΩ and the number of field lines in these areas are same, the strength of the field clearly has a $\frac{1}{r^2}$ dependency.



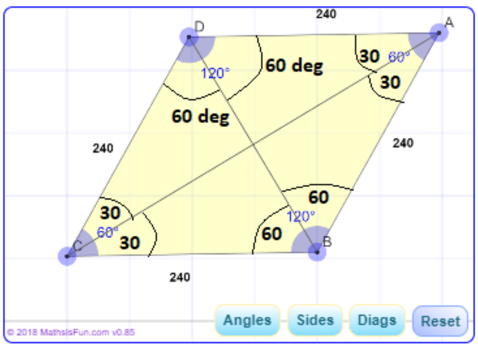
Electric field due to a system of charges

- Consider a system of charges q_1, q_2, …..q_n with position vectors $\vec{r_1}, \vec{r_2} \dots \dots \vec{r_n}$ relative to some origin O
- Like the electric field at a point in space due to a single charge, electric field at a point in space due to system of charges is defined to be the force experienced by a unit test charge placed at that point without disturbing the original position of charges q_1, q_2, …..q_n. This means that the test charge will not disturb the electric fields set up by q_1, q_2, …..q_n and also will not influence any force on them.
- We can use coulomb’s law and superposition principle to determine this field at a point P denoted by position vector $\vec{r}$
- For example, consider 4 charges. q_1 sets-up its own electric field . Similarly q_2 , q_3 , q_4 also sets-up their own electric fields. If we want to know the combined field effect due to fields set up by q_1…q_4 at a point P in space, place a small test charge at P and measure the FORCE that the test charge experiences when placed at point P.
- By knowing force, we can calculate “electric field” at P as
- $E_{net} = \frac{F_{expericed\ by\ test\ charge}}{charge\ on\ test\ body}$
- $\vec{E_1} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{1p}^2}\widehat{r_{1p}}$ *where* $\widehat{r_{1p}}$ *is the unit vector from* q_1 *to* P and $|\vec{r_{1p}}|$ is the distance between q_1 and P
- $\vec{E_2} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_{2p}^2}\widehat{r_{2p}}$
- $\vec{E_3} = \frac{1}{4\pi\epsilon_0}\frac{q_3}{r_{3p}^2}\widehat{r_{3p}}$
- $\vec{E_4} = \frac{1}{4\pi\epsilon_0}\frac{q_4}{r_{4p}^2}\widehat{r_{4p}}$
- By superposition principle, the electric field $\vec{E}$ at $\vec{r}$ due to system of charges is (and extending it to n charges)
- $E(\vec{r}) = E_1(\vec{r}) + E_2(\vec{r}) + \dots\dots\dots E_n(\vec{r})$
- $E(r) = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_{1p}^2}\widehat{r_{1p}} + \frac{q_2}{r_{2p}^2}\widehat{r_{2p}} + \cdots \dots \dots \frac{q_n}{r_{np}^2}\widehat{r_{np}}\right]$
- $E(r) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{ip}^2}\widehat{r_{ip}}$



Info:

- If 4 sides of a parallelogram are equal, then square & rhombus can be the solution. But if diagonals are of different lengths, then it is rhombus and if diagonals are of equal lengths, it is square.
  - If a parallelogram has 4 equal sides and the four angles are each, 90^0 then it is a square. Then both its diagonals will be equal.
- If a parallelogram has one pair of opposite angles acute and the other pair of opposite angles obtuse, it is a rhombus. The diagonals will be of different lengths. The shorter diagonal will be opposite the acute angles and the longer diagonal, opposite the obtuse angles.



Characterizations

A simple (non-self-intersecting) quadrilateral is a rhombus if and only if it is any one of the following:

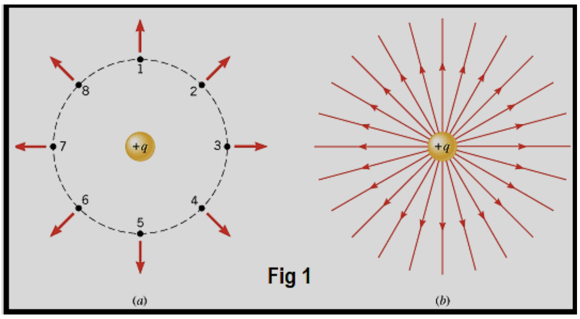
- **a parallelogram in which a diagonal bisects an interior angle**
- a parallelogram in which at least two consecutive sides are equal in length
- a parallelogram in which the diagonals are perpendicular (an orthodiagonal parallelogram)
- a quadrilateral with four sides of equal length (by definition)
- a quadrilateral in which the diagonals are perpendicular and bisect each other
- **a quadrilateral in which each diagonal bisects two opposite interior angles**
- a quadrilateral ABCD possessing a point P in its plane such that the four triangles ABP, BCP, CDP, and DAP are all congruent
- a quadrilateral ABCD in which the incircles in triangles ABC, BCD, CDA and DAB have a common point

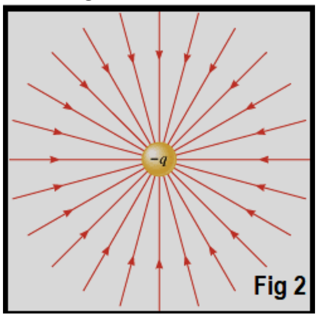
As we have seen, electric charges create an electric field in the space surrounding them. It is useful to have a kind of "map" that gives the direction and indicates the strength of the field at various places. The great English physicist Michael Faraday (1791–1867) proposed an idea that provides such a "map," the idea of ***electric field lines.*** Since the electric field is the electric force per unit charge, the electric field lines are sometimes called ***lines of force.***

To introduce the electric field line concept, Figure 1*a* shows a positive point charge +*q*. At the locations numbered 1–8, a positive test charge would experience a repulsive force, as the arrows in the drawing indicate. Therefore, the electric field created by the charge +*q* is directed radially outward. The electric field lines are lines drawn to show this direction, as part *b* of the drawing illustrates. They begin on the charge +*q* and point radially *outward.* Figure 2 shows the field lines in the vicinity of a negative charge –*q*. In this case they are directed radially *inward* because the force on a positive test charge is one of attraction, indicating that the electric field points inward. In general, ***electric field lines are always directed away from positive charges and toward negative charges.***

Figure 1 (a) At any of the eight marked spots around a positive point charge + q, a positive test charge would experience a repulsive force directed radially outward. (b) The electric field lines are directed radially outward from a positive point charge + q.

Figure 2 The electric field lines are directed radially inward toward a negative point charge – *q*.



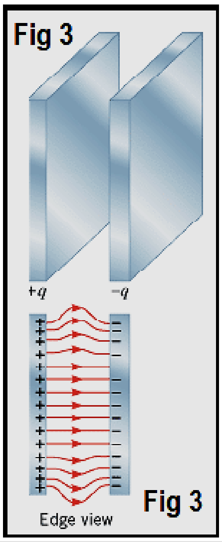


The electric field lines in Figures 1 & 2 are drawn in only two dimensions, as a matter of convenience. Field lines radiate from the charges in three dimensions, and an infinite number of lines could be drawn. However, for clarity only a small number is ever included in pictures. The number is chosen to be proportional to the magnitude of the charge; thus, five times as many lines would emerge from a +5*q* charge as from a +*q* charge.

The pattern of electric field lines also provides information about the magnitude or strength of the field. Notice in Figures 1 & 2 that near the charges, where the electric field is stronger, the lines are closer together. At distances far from the charges, where the electric field is weaker, the lines are more spread out. It is true in general that the electric field is stronger in regions where the field lines are closer together. In fact, no matter how many charges are present, the number of lines per unit area passing perpendicularly through a surface is proportional to the magnitude of the electric field.

In regions where the electric field lines are equally spaced, there is the same number of lines per unit area everywhere, and the electric field has the same strength at all points. For example, Figure 3 shows that the field lines between the plates of a parallel plate capacitor are parallel and equally spaced, except near the edges where they bulge outward. The equally spaced, parallel lines indicate that the electric field has the same magnitude and direction at all points in the central region of the capacitor.

Figure 3: In the central region of a parallel plate capacitor, the electric field lines are parallel and evenly spaced, indicating that the electric field there has the same magnitude and direction at all points --→

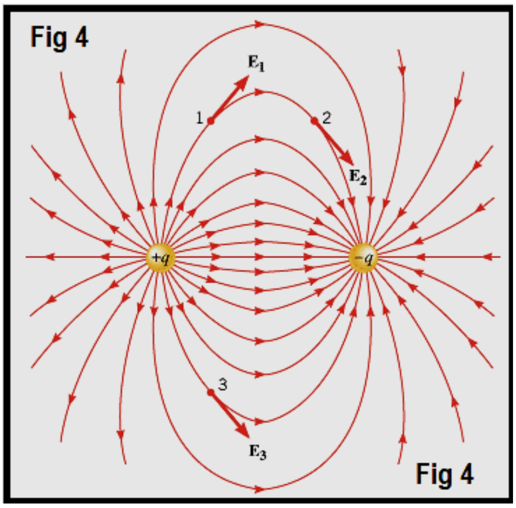


Often, electric field lines are curved, as in the case of an ***electric dipole***. An electric dipole consists of two separated point charges that have the same magnitude but opposite signs. The electric field of a dipole is proportional to the product of the magnitude of one of the charges and the distance between the charges. This product is called the ***dipole moment.*** Many molecules, such as H_2O and HCl, have dipole moments. Figure 4 depicts the field lines in the vicinity of a dipole. For a curved field line, the electric field vector at a point is *tangent* to the line at that point (see points 1, 2, and 3 in the drawing). The pattern of the lines for the dipole indicates that the electric field is greatest in the region between and immediately surrounding the two charges, since the lines are closest together there.

Figure 4: The electric field lines of an electric dipole are curved and extend from the positive to the negative charge. At any point, such as 1, 2, or 3, the field created by the dipole is tangent to the line through the point.

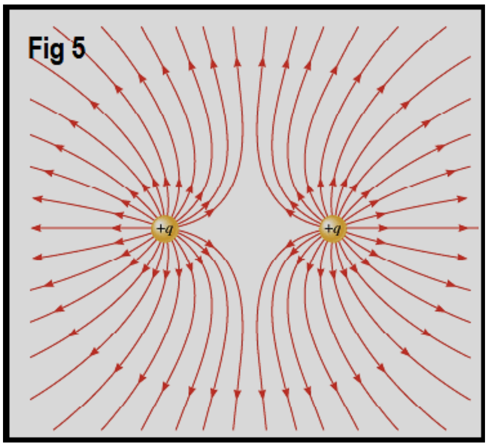
Notice in Figure 4 that any given field line starts on the positive charge and ends on the negative charge. In general, ***electric field lines always begin on a positive charge and end on a negative charge and do not start or stop in mid-space.***

Furthermore, the number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge. This means, for example, that if 100 lines are drawn leaving a +4 *m*C charge, then 75 lines would have to end on a –3 *m*C charge and 25 lines on a –1 *m*C charge. Thus, 100 lines leave the charge of +4 *m*C and end on a total charge of –4 *m*C, so the lines begin and end on equal amounts of total charge.



The electric field lines are also curved in the vicinity of two identical charges. Figure 5 shows the pattern associated with two positive point charges and reveals that there is an absence of lines in the region between the charges. The absence of lines indicates that the electric field is relatively weak between the charges.

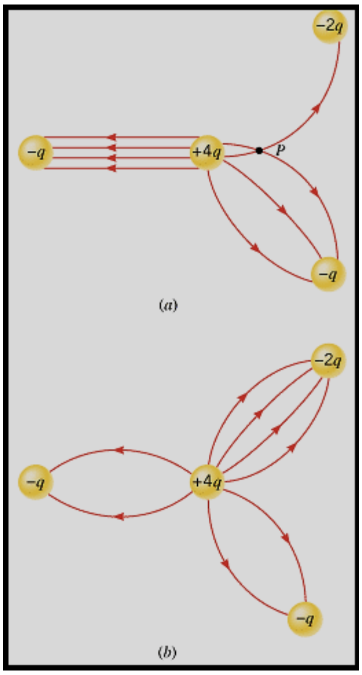
Figure 5 : The electric field lines for two identical positive point charges. If the charges were both negative, the directions of the lines would be reversed.



Some of the important properties of electric field lines are re-examined in below:

Figure shows 3 negative point charges ($-q$, $-q$, and $-2q$) and one positive point charge ($+4q$), along with some electric field lines drawn between the charges. There are three things wrong with this drawing. What are they?

Figure on the right side: *a*) Incorrectly (*b*) correctly drawn electric field lines.



Reasoning and Solution:

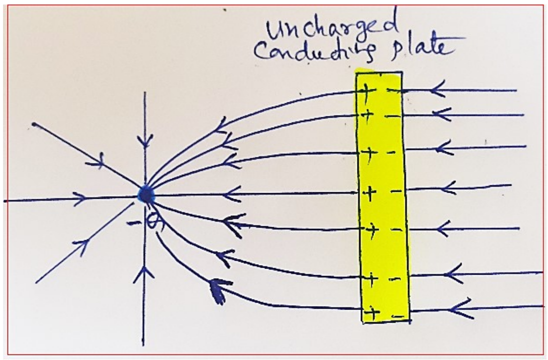
- One aspect of Figure "a" that is incorrect is that electric field lines cross at point *P*. Field lines can never cross, and here's why. An electric charge placed at *P* experiences a single net force due to the presence of the other charges in its environment. Therefore, there is only one value for the electric field (which is the force per unit charge) at that point. If two field lines intersected, there would be two electric fields at the point of intersection, one associated with each line. Since there can be only one value of the electric field at any point, there can be only one electric field line passing through that point.
- Another mistake in Figure "a" is the number of electric field lines that end on the negative charges. Remember that the number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge. The $-2q$ charge has half the magnitude of the $+4q$ charge. Therefore, since 8 lines leave the $+4q$ charge, 4 of them (one-half of them) must enter the $-2q$ charge. Of the remaining 4 lines that leave the positive charge, 2 enter each of the $-q$ charges, according to a similar line of reasoning.
- The third error in Figure "a" is the way in which the electric field lines are drawn between the $+4q$ charge and the $-q$ charge **at the left of the drawing**. As drawn, the lines are parallel and evenly spaced. This would indicate that the electric field everywhere in this region has a constant magnitude and direction, as is the case in the central region of a parallel plate capacitor. But the electric field between the $+4q$ and $-q$ charges is not constant everywhere. It certainly is stronger in places close to the $+4q$ or $-q$ charge than it is midway between them. The field lines, therefore, should be drawn with a curved nature, similar (but not identical) to those that surround a dipole.

Figure "b" shows more nearly correct representations of the field lines for the four charges.

****

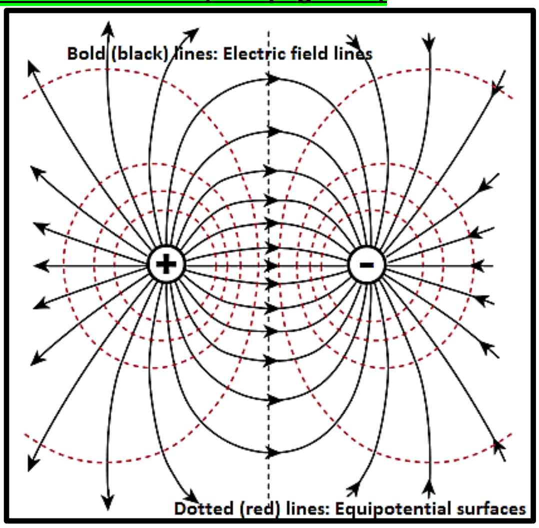
Problem: Draw the pattern of electric field lines, when a point charge –Q is kept near an uncharged conducting plate

Due to process of induction of charges, the uncharged conducting plate facing the –Q charge will get positively charged and the other side of plate will be negatively charged. The electric field line pattern is as shown in the figure.



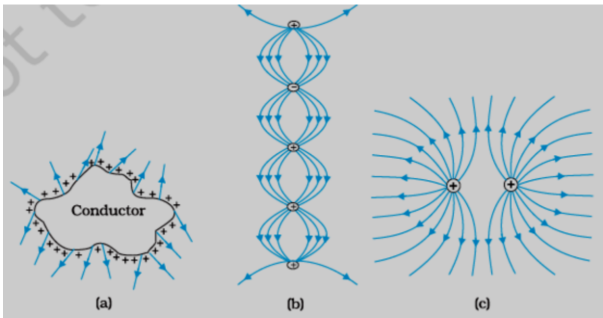
Problem: Draw the equipotential surface for an "electric dipole" **(See next chapter for more information, see page 26a)**

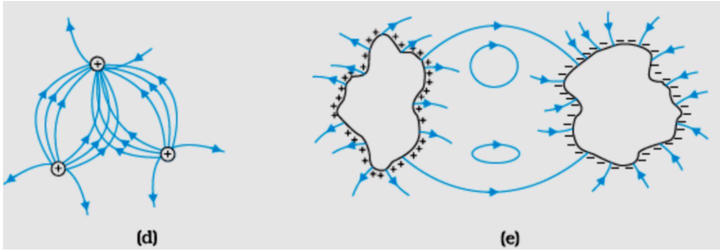
- In figure, the dotted (red) lines represent equipotential surfaces for an electric dipole.
- Note that when drawing equipotential surfaces, make sure that the lines are closer (denser) in the region of strong electric field and are farther apart in the region of weak electric field. In figure, the equipotential lines (surfaces) are closer inside the dipole (where E is stronger inside the dipole) and are farther apart outside the dipole (where E is weaker).
- Note that a dotted line surface will represent the same potential (or voltage).
  - Note that for the same potential surface (or for a given dotted loop), the potential line is closer to the charge q inside the dipole and away outside the dipole. See the explanation below.
- We know that $E = -\frac{dV}{dr}$ $or\ dr = -\frac{dV}{E}$; since dV is constant on the equipotential surface, so $dr \propto \frac{1}{E}$; If E is stronger (or large), dr will be small, that is separation between equipotential surfaces will be smaller.
  - Or $E = -\frac{V}{r}$ $or\ r = -\frac{V}{E};$ $so\ r \propto \frac{1}{E}$; this means for given potential surface, the distance between the charge and the potential surface is less inside the dipole (E is stronger) and more outside the dipole (E is weaker)



- Thus, the spacing between the equipotential surfaces will be less where E is strong and vice-versa. Thus, equipotential surfaces can be used to give a general description of electric field in a certain region of space.
- Therefore, both electric field lines and the equipotential surfaces can be used to depict electric field in space. The advantage of using equipotential surfaces over the electric field lines is that they give a visual picture of both magnitude and direction of the electric field.

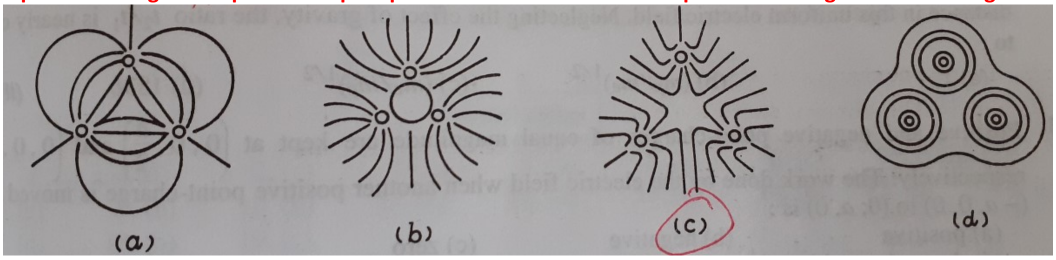
Which among the curves shown below cannot possibly represent electrostatic field lines?





(a) is wrong because field lines must be normal to a conductor,
(b) is wrong because field lines cannot start from a negative charge,
(c) Only (c) is right
(d) is wrong because field lines cannot intersect each other
(e) is wrong because electrostatic field lines cannot form closed loops.

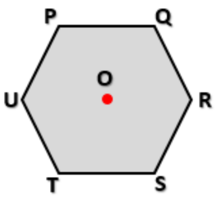
3 positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting electric field lines must as sketched as



(a) Is wrong since field lines cannot end on positive charge and also electrostatic field lines cannot form closed loops.
(b) Is wrong since electrostatic field lines cannot form closed loops (see the centre of picture (b)
(c) Is right since the charges are equal & +ve, the field lines flow radially out of each charge. There are no closed loops & there is no intersecting of field lines
(d) Is wrong electrostatic field lines cannot form closed loops

Six charges of equal magnitudes, 3 positive and 3 negative, are to be placed at PQRSTU corners of a regular hexagon, such that the field at the centre O is double that of what it would have been if only one positive charge is placed at R

a) + + + - - -
b) - + + + - -
c) - + + - + -
d) + - + - + -

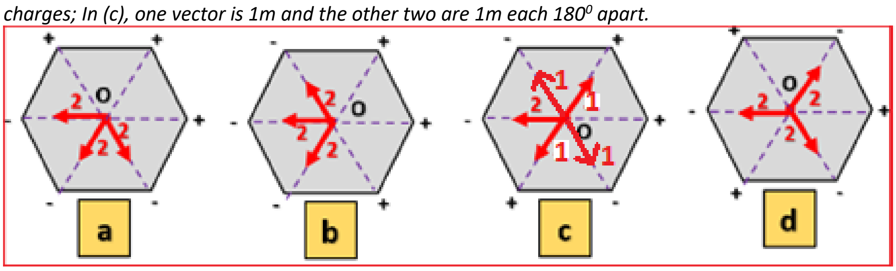


Ans: Let q = 1/9 nC, diagonal distance through centre O in hexagon r = 2m ; $\left[\frac{1}{4\pi\epsilon_0}\right] = k = 9 \times 10^9; E = \frac{kq}{r^2}$

That means distance PS through O = 2m ; distance QT through O = 2m ; distance RU through O = 2 m

$\therefore |E|$ *at centre 0 from each corner charges* $\{r = 1\,m\} = \left[\frac{1}{4\pi\epsilon_0}\right]\frac{q}{r^2} = \frac{1}{r^2} = 1\,NC^{-1}$ (taking magnitude)

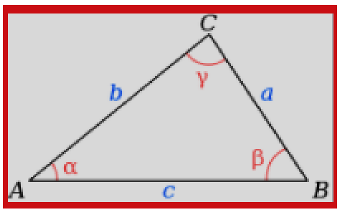
In figure below, (a), (b) and (d), all 3 vectors shown in red are of length = 2m due to different polarity of the charges; In (c), one vector is 1m and the other two are 1m each 180⁰ apart.



Given: If only one +ve charge is placed at R (instead of hexagon), |E| at O = 1 NC^{-1} ; We now need to find the E = 2 NC^{-1} value in 4 different options of the given question. Considering each case,

a) In (a), the resultant E is not equal to 2 NC^{-1}. The net E vector in this case = 4 NC^{-1} **(see next row)**
b) In (b), the resultant E is not equal to 2 NC^{-1}. The net E vector in this case = 4 NC^{-1}
c) In (c), E @ O due to P & S and also due to Q & T add-up to zero; E at O due to R & U will add-up to 2NC^{-1}
d) In (d), the resultant E is not equal to 2 NC^{-1}. The net E vector in this case = 0 NC^{-1}

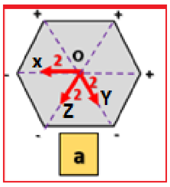
Cosine law of triangle:



$$c = \sqrt{a^2 + b^2 - 2ab\cos(\gamma)}$$

Note: cos60⁰ = ½ ; cos120⁰ = -½ ; cos(π-θ) = -cosθ

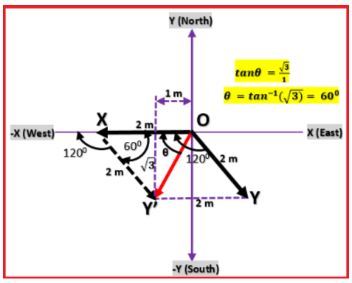
- Consider diagram (a). OX & OY are vectors 120⁰ apart. Shift OY to XY' (tail of second vector OY coincides with the head of first vector OX). Using cosine law of triangle XOY',
  - $OY' = \sqrt{OX^2 + OY^2 - 2(OX)(OY)\cos 60^0} = \sqrt{4 + 4 - 8(0.5)} = \sqrt{8-4} = \sqrt{4} = 2m$
    - In fact, XOY' is an equilateral triangle, so all sides and angles are equal
    - So, the magnitude of resultant vector = 2 m ; what about direction?
    - Since OX = OY, the parallelogram OXY'Y is a rhombus, where the diagonal bisects the interior angle. Since $X\hat{O}Y = 120^0, then\ X\hat{O}Y' = 60^0$
    - ∴ the direction of resultant vector OY' = 60⁰ south of west
  - ∴ Resultant vector OY' is in the same direction as the 3rd vector Z and Z = 2 m
    - therefore overall resultant magnitude = 2 + 2 = 4 NC^{-1} ; direction 60⁰ south of west



- In (b), it is same as in (a), resultant magnitude = 4 NC^{-1}
- In (d), resultant magnitude = 2 – 2 = 0 NC^{-1}
- In (c), E @ O due to P & S and also due to Q & T add-up to zero; E at O due to R & U will add-up to 2NC^{-1}

Note :

- SOUTH of WEST: From WEST axis if you are moving towards SOUTH, it is called SOUTH of WEST ; means "towards south from west"
- NORTH of EAST: From EAST axis if you are moving towards NORTH, it is called NORTH of EAST.
  - NORTH of EAST means "towards North from East"
- EAST of NORTH: From NORTH axis if you are moving towards EAST, it is called EAST of NORTH.



22-6 A POINT CHARGE IN AN ELECTRIC FIELD

Learning Objectives

After reading this module, you should be able to . . .

22.22 For a charged particle placed in an external electric field (a field due to other charged objects), apply the relationship between the electric field $\vec{E}$ at that point, the particle's charge q, and the electrostatic force $\vec{F}$ that acts on the particle, and identify the relative directions of the force and the field when the particle is positively charged and negatively charged.

22.23 Explain Millikan's procedure of measuring the elementary charge.

22.24 Explain the general mechanism of ink-jet printing.

Key Ideas

- If a particle with charge q is placed in an external electric field $\vec{E}$, an electrostatic force $\vec{F}$ acts on the particle:

$$\vec{F} = q\vec{E}.$$

- If charge q is positive, the force vector is in the same direction as the field vector. If charge q is negative, the force vector is in the opposite direction (the minus sign in the equation reverses the force vector from the field vector).

A Point Charge in an Electric Field

In the preceding four modules we worked at the first of our two tasks: given a charge distribution, to find the electric field it produces in the surrounding space. Here we begin the second task: to determine what happens to a charged particle when it is in an electric field set up by other stationary or slowly moving charges.

What happens is that an electrostatic force acts on the particle, as given by

$$\vec{F} = q\vec{E}, \qquad (22\text{-}28)$$

in which q is the charge of the particle (including its sign) and $\vec{E}$ is the electric field that other charges have produced at the location of the particle. (The field is *not* the field set up by the particle itself; to distinguish the two fields, the field acting on the particle in Eq. 22-28 is often called the *external field.* A charged particle or object is not affected by its own electric field.) Equation 22-28 tells us

> The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.

Measuring the Elementary Charge

Equation 22-28 played a role in the measurement of the elementary charge e by American physicist Robert A. Millikan in 1910–1913. Figure 22-16 is a representation of his apparatus. When tiny oil drops are sprayed into chamber A, some of them become charged, either positively or negatively, in the process. Consider a drop that drifts downward through the small hole in plate P_1 and into chamber C. Let us assume that this drop has a negative charge q.

If switch S in Fig. 22-16 is open as shown, battery B has no electrical effect on chamber C. If the switch is closed (the connection between chamber C and the positive terminal of the battery is then complete), the battery causes an excess positive charge on conducting plate P_1 and an excess negative charge on conducting plate P_2. The charged plates set up a downward-directed electric field $\vec{E}$ in chamber C. According to Eq. 22-28, this field exerts an electrostatic force on any charged drop that happens to be in the chamber and affects its motion. In particular, our negatively charged drop will tend to drift upward.

By timing the motion of oil drops with the switch opened and with it closed and thus determining the effect of the charge q, Millikan discovered that the values of q were always given by

$q = ne$, **for** $n = 0, \pm 1, \pm 2, \pm 3, \ldots$ (22-29)

in which e turned out to be the fundamental constant we call the *elementary charge,* 1.60×10^{-19} C.

Millikan's experiment is convincing proof that charge is quantized, and he earned the 1923 Nobel Prize in physics in part for this work. Modern measurements of the elementary charge rely on a variety of interlocking experiments, all more precise than the pioneering experiment of Millikan.

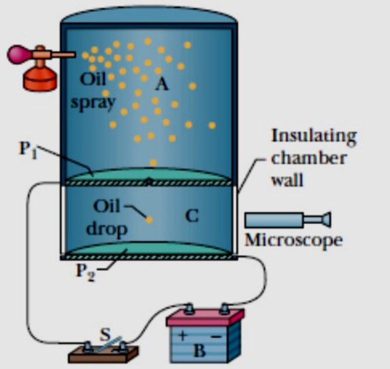
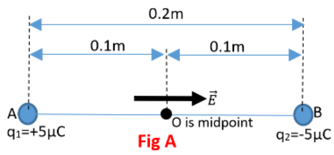


Figure 22-16 The Millikan oil-drop apparatus for measuring the elementary charge e. When a charged oil drop drifted into chamber C through the hole in plate P_1, its motion could be controlled by closing and opening switch S and thereby setting up or eliminating an electric field in chamber C. The microscope was used to view the drop, to permit timing of its motion.

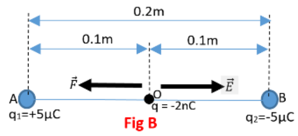
Problem 1: Two point charges q_1 = 5μC and q_2 = -5μC are located at and A and B separated by 0.2m in vacuum.

(a) What is the electric field at the midpoint "O" of the line joining the charges?

(b) If a –ve test charge of magnitude 2nC is placed at O, what is the force experienced by the test charge. [State board 2020]

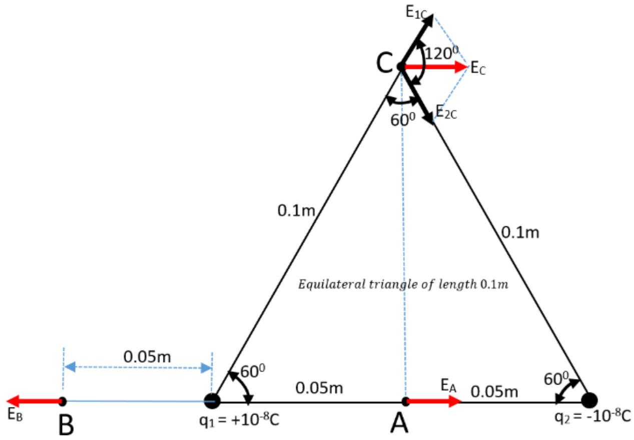


(a) Given in fig (A), q_1 = 5μC ; q_2 = -5μC and AB = 0.2m
- O is the midpoint between A and B, therefore AO = OB = r = 0.1m
- Magnitude of the electric field intensity at point O due to q_1 = $|\vec{E_1}| = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}$ $along\ AO$
- Magnitude of the electric field intensity at point O due to q_2 = $|\vec{E_2}| = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}$ $along\ OB$
- Since the direction of E_1 and E_2 are same, the magnitudes will add up. Therefore, the resultant electric field intensity at point O due to both q_1 and q_2 = $|\vec{E}| = 2x\left[\frac{1}{4\pi\epsilon_0}\right]\frac{q_1}{r^2} = 2\ x\ 9\ x\ 10^9\left[\frac{5x10^{-6}}{0.01}\right] = 9\ x\ 10^6NC^{-1}\ along\ OB$



(b) See fig (B). Since we know E at point O from above, the electric force on the test charge q = -2nC placed at O is given by F = qE = [-2 x 10^{-9} C][9 x 10^6NC^{-1}] = -1.8 x 10^{-2} N = 1.8 x 10^{-2} N along OA (opposite to E)
- The direction of force on test charge q towards the charge q_1 is obvious. Since test charge q is –ve and q_1 is +ve ⟹ attractive force along OA
- Also, if we consider test charge q and charge q_2 (which is –ve), the charge q_2 repels the test charge q (along OA)
- ∴ the test charge at O experiences attractive force from q_1 and repulsive force from q_2

Problem 2: Two point charges q_1 and q_2 of magnitude +10^{-8}C and -10^{-8}C respectively are placed 0.1 m apart. Calculate the electric field at points A, B and C as per the figure shown in the problem.



At "A" : E_{1A} at A due to q_1 points towards right and has magnitude

$E_{1A} = \frac{9\ x\ 10^9\ x10^{-8}}{(0.05)^2} = 3.6\ x\ 10^4NC^{-1}$

- E_{2A} at A due to q_2 points towards right and has same magnitude as above. The net field at A is given by
- E_A =E_{1A} + E_{1B} = $7.2\ x\ 10^4NC^{-1}$ directed towards right.

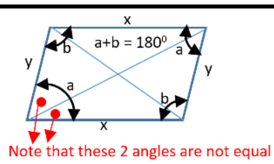
At "B" : E_{1B} at B due to q_1 points towards left and has magnitude $E_{1B} = \frac{9\ x\ 10^9\ x10^{-8}}{(0.05)^2} = 3.6\ x\ 10^4NC^{-1}$

- E_{2B} at B due to q_2 points towards right and has magnitude $E_{2B} = \frac{9\ x\ 10^9\ x10^{-8}}{(0.15)^2} = 0.4\ x\ 10^4NC^{-1}$
- Net field at "B" : E_B =E_{1B} – E_{2B} = $(3.6 - 0.4)\ x\ 10^4NC^{-1}$ = $3.2\ x10^4NC^{-1}$ directed towards left (since E_{1B} > E_{2B}).
- See page 28 also

At "C" : The magnitude of each electric field at "C" due to q1 and q2 are same and is equal to

- E_{1C} = E_{2C} = $\frac{9\ x\ 10^9\ x10^{-8}}{(0.1)^2} = 9\ x\ 10^3NC^{-1}$.
- The angle between E_{1C} and E_{2C} = 120^0. cos 120^0 = -1/2 . The net magnitude at "C" due to both charges is
- $E_C = \sqrt{E_{1C}^2 + E_{2C}^2 + 2E_{1C}E_{2C}cos120^0} = E_{1C}$ or E_{2C}
- Therefore the net magnitude of E at C = E_C = $9\ x\ 10^3NC^{-1}$
- As shown in figure, the direction of E at C is towards right.

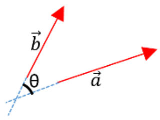
Info: Parallelogram



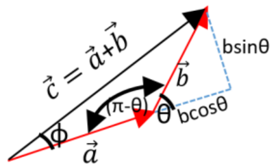
- Opposite sides are parallel
- Opposite sides are equal in length
- Diagonal angles are equal (angles "a" are same and angles "b" are same)
- Angles "a" and "b" add up to 180^0, so they are called supplementary angles.

Vectors (addition)

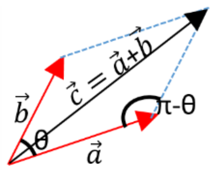
Eg: Vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle θ to each other in space. Find the sum (or resultant) of these 2 vectors



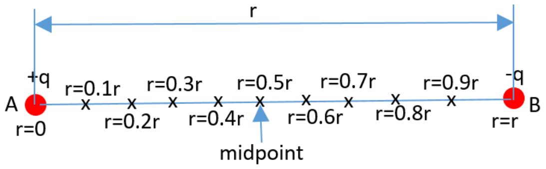
- Make the initial point (tail) of $\vec{b}$ coincide with the tip (head) of $\vec{a}$
  - Using cosine law, the 3rd side has a magnitude of
  - $c = a + b = \sqrt{a^2 + b^2 - 2ab\cos(\pi - \theta)}$; since cos(π-θ) = -cosθ
  - $c = \sqrt{a^2 + b^2 + 2ab\cos(\theta)}$
  - $and\ tan\varphi = \frac{b\ sin\theta}{a+b\ cos\theta}$



- We can also use parallelogram method
  - Since parallelogram has equal and opposite sides, the parallelogram method is equivalent to the triangle law (cosine law of triangle). Rest text is same as above.



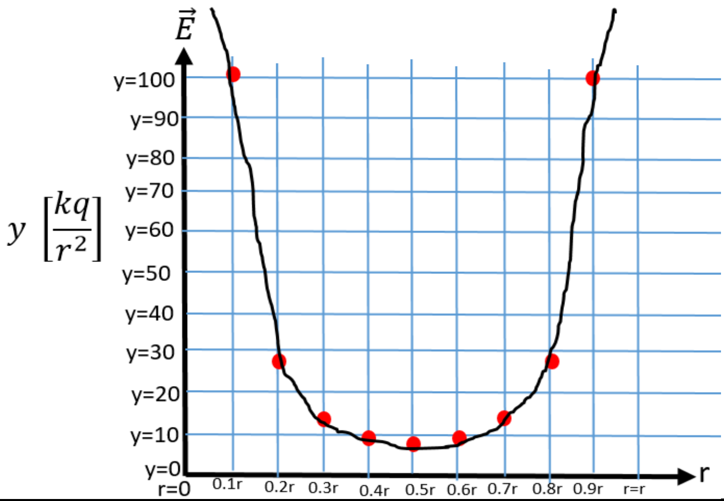
Problem : Two point charges +q and –q are placed at points A and B respectively "r"m apart in vacuum. Plot E Vs r



- Magnitude of $E = \frac{kq}{r^2}$ where $k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 Nm^2C^{-2}$
- Direction of E at all points between the straight line joining A and B is along AB. Hence the two vectors add up to give resultant value at any point between A and B (see figure above)
- @ r = 0 r and r = r, $|\vec{E}| = \infty$
- @ r = 0.1 r and r = 0.9r, $|\vec{E}| = 101\left[\frac{kq}{r^2}\right]$ .... How? See second column
- @ r = 0.2 r and r = 0.8r, $|\vec{E}| = 27\left[\frac{kq}{r^2}\right]$
- @ r = 0.3 r and r = 0.7r, $|\vec{E}| = 13\left[\frac{kq}{r^2}\right]$
- @ r = 0.4 r and r = 0.6r, $|\vec{E}| = 9\left[\frac{kq}{r^2}\right]$
- @ r = 0.5 r midpoint, $|\vec{E}| = 8\left[\frac{kq}{r^2}\right]$...... plot is given below [curve is smooth]

Take E @ r = 0.1 r

- E @ r = 0.1 r due to +q = $kq/(0.1r)^2$
- E @ r = 0.9 r due to -q = $kq/(0.9r)^2$
- Direction is along AB and vectors add up
- Resultant E @ r = 0.1 r is given by
- $E = kq\left[\frac{1}{(0.1r)^2} + \frac{1}{(0.9r)^2}\right]$
- $E = \frac{kq}{r^2}\left[100 + \frac{100}{81}\right]$
- $E = \frac{100kq}{r^2}\left[1 + \frac{1}{81}\right] = \frac{100kq}{r^2}\left[\frac{82}{81}\right]$
- $E = \frac{101kq}{r^2}$
- Similarly other points between A and B



Examples:

- If q = 3μC, r = 0.2 m ; find E at midpoint.

For midpoint $|\vec{E}| = 8\left[\frac{kq}{r^2}\right]$

Where k = 9 x 10^9; substituting all values, we get E = 5.4 x 10^6 NC^{-1}

- If q = 1μC, r = 1 m ; find E at midpoint.

For midpoint $|\vec{E}| = 8\left[\frac{kq}{r^2}\right]$; substituting all values, we get E = 7.2 x 10^4 NC^{-1}

- If q = 5μC, r = 0.5 m ; find E at midpoint.

For midpoint $|\vec{E}| = 8\left[\frac{kq}{r^2}\right]$; substituting all values, we get E = 1.44 x 10^6 NC^{-1}