Chapter 2
Electrostatic potential and Capacitance.

Notion of potential energy "

Introduction : Concept of "Electric potential"

→ In previous chapter, we have learnt the concept of "electric field" due to a point charge or a distribution of charges. It is represented in terms of a vector quantity $\vec{E}$, called "electric field intensity".

→ In this chapter, We shall learn that electric field can also be represented in terms of a scalar quantity V, called "electrostatic potential". This concept is of great practical value.

→ Basically, "e-s potential" of a charged body represents the degree of electrification of the body. It determines the direction of flow of charges betn two charged bodies placed in contact with each other. The charge flows from a body at higher potential to another body at lower potential. The flow of charge stops as soon as the potentials of the two bodies become equal.

IMP → When an external force does work in taking a "body" from a point to another, "against a force" (like spring force, gravitational force, coulomb force betn two stationary charges), that work gets stored as potential energy (PE) of the body (IMP).

- When the external force is removed, the "body" moves, gaining KE and losing an equal amount of PE. The sum of KE and PE is thus conserved. "Forces of this kind are called "Conservative Forces"" [Work done ≡ Energy transferred]

- Coulomb force betn two stationary charges, like the gravitational force is also a conservative force. This is not surprising since ----
  - Since both have inverse-square dependence on distance and differ mainly in the proportionality constants — the masses in Gravitation law are replaced by charges in coulomb's law.

IMP
- Thus, like the "PE" of a "mass" in a gravitational field, we can define "electrostatic PE" of a "charge" in an electrostatic field.

- IMP : The electric field produced by a charge can be represented in 2 ways
  (i) by the intensity of electric field $\vec{E}$ at a point in the field
  and (ii) by the electric potential.

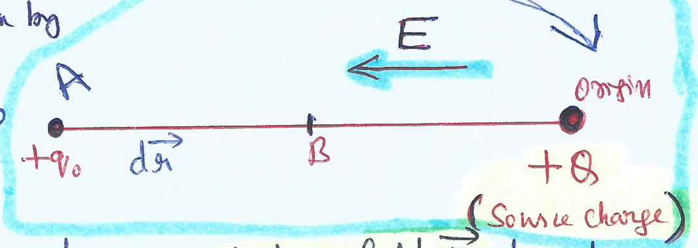
Now, $\vec{E}$ is a vector quantity while V is a scalar quantity and both of them are the characteristic properties of a point in the field.

IMP

Electric potential Energy (PE)

What is electric PE? Derive an expression for electric PE?

→ Electric PE of a charge at a point in the electric field (due to any source charge) is given by the work done by an external force in bringing a test charge from infinity to that point in the electric field.



- Let a "positive" test charge q_0 be placed in an electric field $\vec{E}$ due to another positive charge Q (called source charge). The force acting on the test charge q_0 in the electric field $\vec{E}$ is given by

$$\vec{F_e} = q_0 \vec{E} \longrightarrow ①$$

This force tends to move the test charge in the direction of the electric field $\vec{E}$.

- Suppose an external force $\vec{F_0}$ acts just to overcome this electric force $\vec{F_e}$ on the test charge to move it without any acceleration towards source charge Q.
- If the test charge (q_0) is displaced through a distance $\vec{dr}$ from point A to point B in the electric field $\vec{E}$, then the work done by the external force $\vec{F_0}$ is given by

$$W_{AB} = \int_A^B \vec{F_0} \cdot \vec{dr}$$

$\int_A^B \vec{E} \cdot \vec{dr}$ is known as line integral of electric field intensity betn points A and B.

Since $\vec{F_0} = -\vec{F_e} = -q_0\vec{E}$ (using eqn ①)

$$\boxed{W_{AB}} = -\int_A^B q_0 \vec{E} \cdot \vec{dr} = \boxed{-q_0 \int_A^B \vec{E} \cdot \vec{dr}} \longrightarrow ①$$

→ This work done (in eqn ①) gives the difference in "electrical PE" betn points A and B.

$$\therefore \boxed{W_{AB} = (U_B - U_A) = -q_0 \int_A^B \vec{E} \cdot \vec{dr}} \longrightarrow ②$$

Thus, difference in electrical PE betn two points A and B in the electric field is the work done in moving a test charge without acceleration from point A to point B in the electric field of the source charge.

- If point A is at ∞, then there is no electrostatic force betn Q and the test charge q_0. Thus the PE of the test charge q_0 at ∞ is zero. The work done to displace this test charge (without acceleration) from ∞ to point B in the electric field is given by $\boxed{U_B - U_\infty = W_{\infty B} = U_B - 0 = U_B}$

$$\therefore U_B = W_{\infty B} = -q_0 \int_\infty^B \vec{E} \cdot \vec{dr} \longrightarrow ③$$

This work done is equal to the electrical PE of the system at any point (say B) in the electric field.

P.T.O →

Imp. points : (FYI)

① $\int_A^B \vec{E} \cdot \vec{dr}$ is known as line integral of electric field intensity betn points A and B.

② Since, the work done by an electrostatic force acting on a charged particle depends only on the initial and final positions of the charged particle and is independent of the path followed by the charged particle in the electric field, so the e-s force and electric field are conservative in nature.

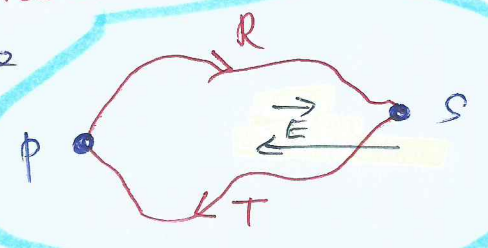
→ $\boxed{\text{Note}}$ Although line integral of electric field due to a static charge is independent of the path followed by the test charge in the electric field, yet line integral of the electric field due to a moving charge is not independent of the path followed by the test charge because the electric field due to a moving charge changes with time.

③ The work done by a conservative force in moving a charge in a "closed path" is zero.

④ **Work done = Energy transferred.**

IMP: E-S forces are Conservative :

Show that e-s forces are conservative in nature and line integral of e-s field over a closed path in an electric field is Zero.



→ No work is done in moving a unit positive charge over a closed path in an electric field, so e-s field and e-s force are conservative in nature

- Consider any closed path PRSTP in an electric field $\vec{E}$ as shown in fig.
- Work done in moving a test charge q_0 from P to S along the path PRS is

$$W_{PS} = q_0 (V_S - V_P) \longrightarrow (1)$$

- Work done in moving a test charge q_0 from S to P along the path STP is

$$W_{SP} = q_0 (V_P - V_S) \longrightarrow (2)$$

Adding (1) and (2), we get $W_{PSP} = q_0 (V_S - V_P) + q_0 (V_P - V_S)$

$$= \cancel{q_0 V_S} - \cancel{q_0 V_P} + \cancel{q_0 V_P} - \cancel{q_0 V_S}$$

$$W_{PSP} = 0$$

Since $W_{PSP} = \oint \vec{E} \cdot \vec{dr}$

where $\oint \vec{E} \cdot \vec{dr}$ is line integral of electric field $\vec{E}$ over a closed path in electric field $\vec{E}$ $\therefore \oint \vec{E} \cdot \vec{dr} = 0$

Hence, line integral of e-s field over a closed path in an electric field is zero.

END

physical meaning of Electric potential :

We know that a liquid flows always from a higher level to a lower level. Heat also flows from a body at a higher temp^r of the body to a body at a lower temp^r. Similarly, positive charge flows always from higher potential to lower potential. Just as flow of liquid does not depend upon the quantity of liquid, but depends only upon the level of liquid; and the flow of heat also does not depend upon the amount of heat; in a similar way, the flow of positive charge does not depend upon the quantity of charge.

→ Thus, the electric potential of a conductor is its electric state which determines the direction of flow of charge when the given conductor is connected to another conductor.

→ positive charge always flows from the higher potential to lower potential, when -ve charge always flows from lower to higher potential until the potentials become equal.

"Electric potential" Vs "potential difference" IMP

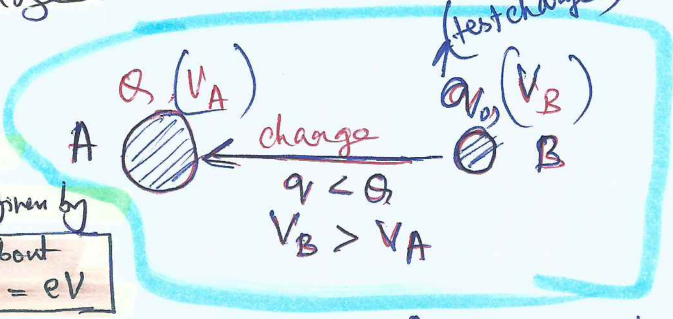
Definition Electric potential	potential difference (~~PE~~) (pd)
"Electric potential" at a point in the electric field is defined as the work done in moving a unit positive test charge from infinity to that point against the electrostatic force of the electric field of the source charge irrespective of the path followed. or Electric potential is a physical quantity which determines the flow of charge from one body to another body.	PD ~~PE~~ bet^n any two points in an electric field is defined as the work done in moving a unit positive charge from one point to the other point against the electric force of the electric field irrespective of the path followed. OR → potential difference PD ~~PE~~ bet^n any two points in an electric field is defined as the electrical ~~energy~~ potential energy difference per unit charge

P.T.O.

Electric potential : (Expression for Electric potential)

"Electric potential" at a point in an electric field is also defined as the electric potential energy per unit charge.

- Let U be the "electrostatic" PE of charge q_0 at B in an electric field. Then the electric potential at that point in the electric field is given by



$$\boxed{V = \frac{U}{q_0}} \longrightarrow (1)$$

think about Energy = eV

But "PE" of test charge q_0 at point B in the electric field is given by

$$U = -q_0 \int_{\infty}^{B} \vec{E} \cdot \vec{dr}$$

(from previous pages)
(Note that ~~$\vec{E}$ is due to charge~~ Q)

∴ Electric potential at B in the electric field is given by

$$\boxed{V = \frac{U}{q_0} = -\int_{\infty}^{B} \vec{E} \cdot \vec{dr}} \longrightarrow (2)$$

- Electric potential is a scalar quantity
- Dimensional formula for Electric potential :

$$= \frac{[\text{work}]}{[\text{charge}]} = \frac{[\text{work}]}{[\text{current} \times \text{Time}]} = \frac{[ML^2T^{-2}]}{[A][T]} = [ML^2T^{-3}A^{-1}]$$

- SI unit of Electric potential is volt (V)

$$V = \frac{U}{q_0} \quad \therefore \quad 1 \text{ volt} = \frac{1 \text{ Joule (J)}}{1 \text{ coulomb (C)}} = 1\,JC^{-1}$$

- Thus, electric potential at a "point" "(P)" in the electric field is 1 volt if One Joule of work is done in moving One Coulomb of charge from Infinity to that point (P) in the electric field. (Note that "Electric potential" at ∞ is taken as zero)

Find the electric potential at a point, when a work of 2.4×10^{-5} J is done to carry charge of 5×10^{-19} C to that point.

$$\longrightarrow V = \frac{W}{q} = \frac{2.4 \times 10^{-5}\,J}{5 \times 10^{-19}\,C} = 4.8 \times 10^{13}\,V$$

See next page for expression for "potential difference" (pd) →

Potential Difference

(Note previous page is on "Electric potential derivation")

- Let $\Delta U = U_B - U_A$ be the electrostatic potential Energy (PE) difference betw points A and B in the electric field.
- Let $\Delta V = V_B - V_A$ be electric potential difference (pd) betw points A & B

then as per $V = \frac{U}{q_0}$ formula,

$$\therefore \boxed{\Delta V = \frac{\Delta U}{q_0}} \rightarrow (1)$$

where $\Delta U = U_B - U_A$
$\Delta V = V_B - V_A$
q_0 = test charge.

we know that $\Delta U \left(= (U_B - U_A)\right) = -q_0 \int_A^B \vec{E} \cdot \vec{dr}$

$$\therefore \frac{\Delta U}{q_0} = -\int_A^B \vec{E} \cdot \vec{dr}$$

$$\Longrightarrow \boxed{\Delta V = -\int_A^B \vec{E} \cdot \vec{dr}} \rightarrow (2)$$

(This is to for "electric potential" equation)

→ SI unit ~~o~~ ~~volt~~ of ΔV = volt = $1\,JC^{-1}$

problem: If 100 Joule of work must be done to move electric charge of 4C from a place, where potential is $-10V$ to another place where potential is "V" volts. Find the value of V?

Ans → Given $q_0 = 4C$; $W_{AB} = 100J$
$V_A = -10V$, V_B = V volts

we know that $\Delta V = V_B - V_A = \frac{W_{AB}}{q_0}$

$$\therefore V_B - V_A = \frac{W_{AB}}{q_0} = \frac{100J}{4C} = 25\,JC^{-1}$$

$$V_B = V = 25 + V_A$$

Since $V_A = -10V$

$$\therefore V = 25 - 10 = 15 \text{ Volt}$$

$$\boxed{V = (\text{or } V_B) = 15V}$$

IMP

Electric potential due to a point charge

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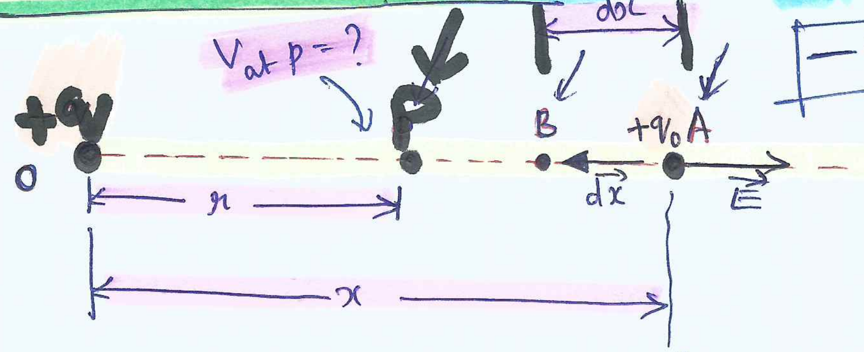


$-q$

Suppose a source charge of $+q$ Coulomb is situated at a point O (see fig) in vacuum (or air). Let 'P' be a point, distant r meter from O, at which the "electric potential" is to be determined.

→ **For this, we must calculate the work done in bringing a test ~~positive~~ charge from ∞ to P.**

→ Suppose, a test charge $+q_0$ is placed at point A, distant x from O, and away from P. By Coulomb's law, the electric force acting on q_0 is given by

$$F = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{x^2} \text{ N}$$

• This force is directed radially outwards along the line joining $+q$ and $+q_0$ (in the direction $\vec{E}$ as shown in figure)

→ Let there be another ~~charge~~ point B at an infinitely small dist dx from A to B towards O. (i.e at a distance $-dx$ from A).

→ Then the work done in carrying the test charge "$+q_0$" from A to B against force $\vec{F}$ is (force × distance)

$$dw = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{x^2} \times (-dx) = -\frac{1}{4\pi\epsilon_0}\frac{q q_0}{x^2}dx \text{ joule.}$$

∴ Work done in carrying the test charge "$+q_0$" from ∞ to P is

$$w = -\int_{\infty}^{r}\frac{1}{4\pi\epsilon_0}\frac{q q_0}{x^2}dx = -\frac{q q_0}{4\pi\epsilon_0}\int_{\infty}^{r}x^{-2}dx = -\frac{q q_0}{4\pi\epsilon_0}\left[\frac{x^{-1}}{-1}\right]_{\infty}^{r}$$

$$w = \frac{q q_0}{4\pi\epsilon_0}\left[\frac{1}{x}\right]_{\infty}^{r} = \frac{q q_0}{4\pi\epsilon_0}\left[\frac{1}{r}-\frac{1}{\infty}\right] = \frac{q q_0}{4\pi\epsilon_0 r}$$

∴ $\boxed{w = \frac{q q_0}{4\pi\epsilon_0 r}}$ where $\frac{1}{4\pi\epsilon_0} = 9\times10^9 \text{ N m}^2\text{C}^{-2}$

(Since $V = W/q_0$)

→ The potential at P is the ratio of the work done in carrying a test charge from ∞ to the point P to the magnitude of the test charge

$\boxed{V = \frac{w}{q_0}}$

∴ $\boxed{V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r}}$ volt

Similarly, the potential at P due to ~~test~~ charge $-q$ is

$\boxed{V = -\frac{1}{4\pi\epsilon_0}\frac{q}{r}}$ volt.

P.T.O →

10 ∴ potential ~~due to~~ at p due to a charge $+q$ is $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

∴ potential at p due to a charge $-q$ is $V = -\frac{1}{4\pi\epsilon_0}\frac{q}{r}$

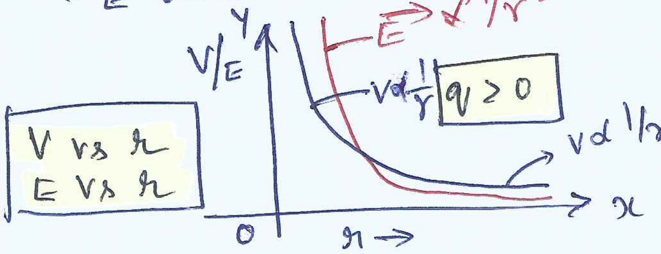
⇒ When charge is +ve, the potential is also positive ⇒ It means that work is done by an external agent in carrying the test charge from ∞ to the point P in question.

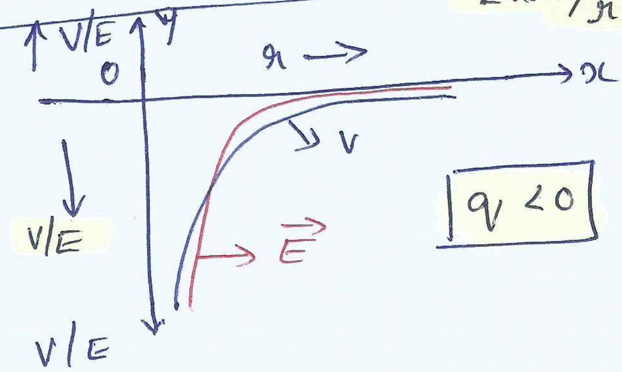
⇒ If the charge is −ve, the potential is also "negative". ⇒ It means that work is obtained (or −ve work is done) in carrying the test charge from ∞ to the point P in question.

⇒ Electric potential due to a single charge is spherically symmetric as the value of V due to this charge is same at all points distant r around the charge.

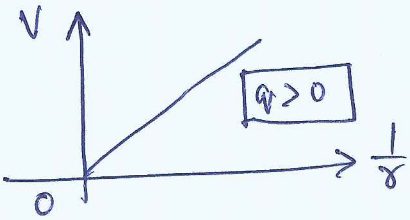
→ Although V is called "electric potential" at a point, but actually, it is equal to the "electric potential difference" between points at dist. r and ∞ from the source charge.

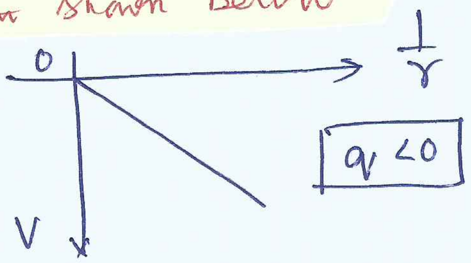
→ • V due to a point charge q varies with distance r as $V \propto \frac{1}{r}$
• $\vec{E}$ due ——— $E \propto 1/r^2$





→ Variation of V with $\frac{1}{r}$ is ~~given~~ shown below





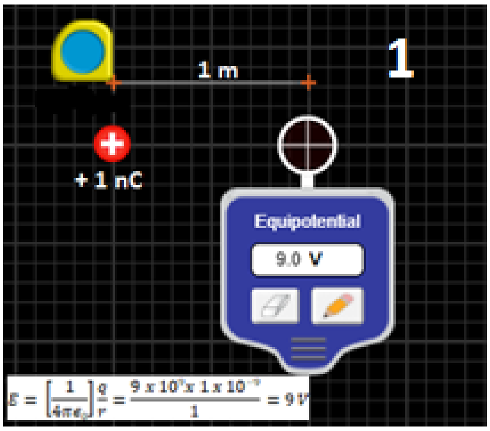
Calculate the electric potential at a distance of 9 cm from a charge 4×10^{-9} C

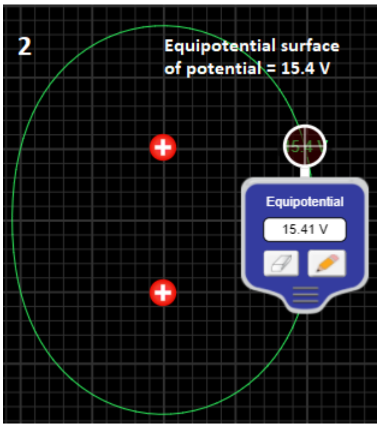
→ Given
$r = 9\text{ cm} = 9 \times 10^{-2}$ m
$q = 4 \times 10^{-9}$ C

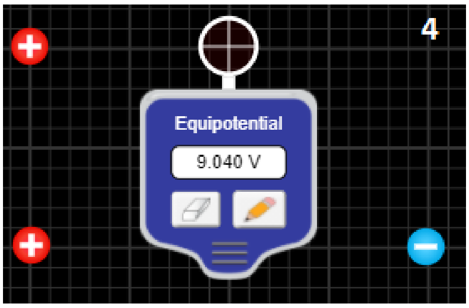
$\frac{1}{4\pi\epsilon_0} = 9 \times 10^{9}\ Nm^2C^{-2}$

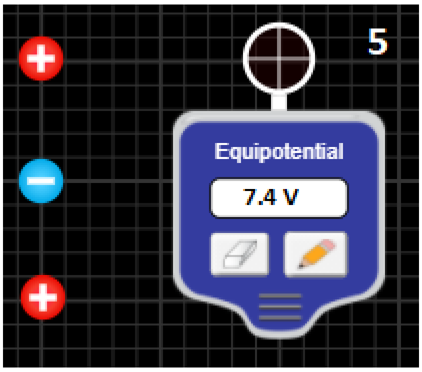
$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \cancel{9} \times \cancel{10^9}\ Nm^2C^{-\cancel{2}\,-1} \times \frac{4 \times \cancel{10^{-9}}\ \cancel{C}}{\cancel{9} \times 10^{-2}\ \cancel{m}} = \underline{\underline{400\ V}}$

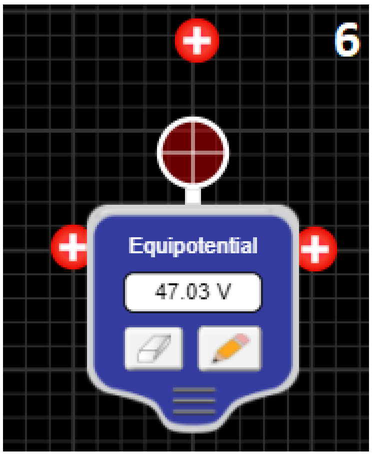
-- 10a -- Electric potential at a point due to a charge or system of charges (All charges = 1 nC)

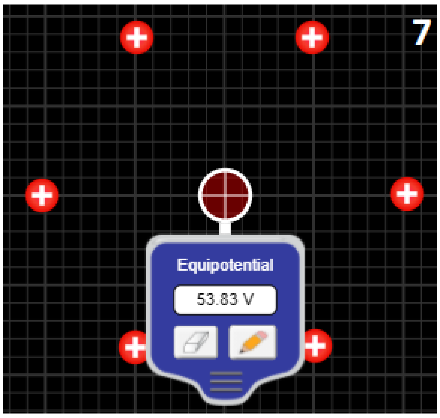


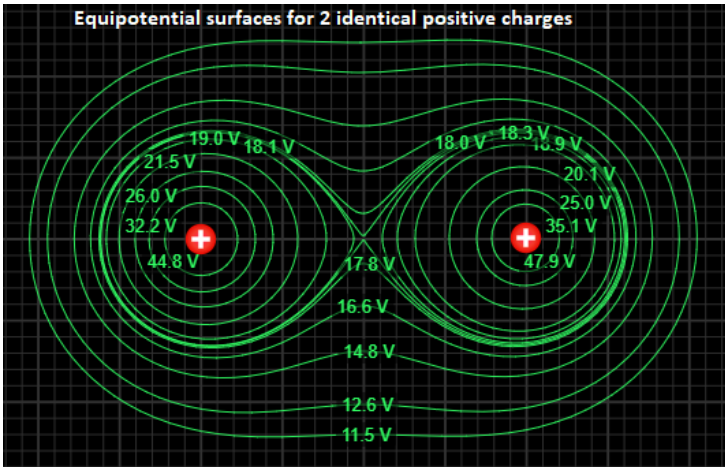


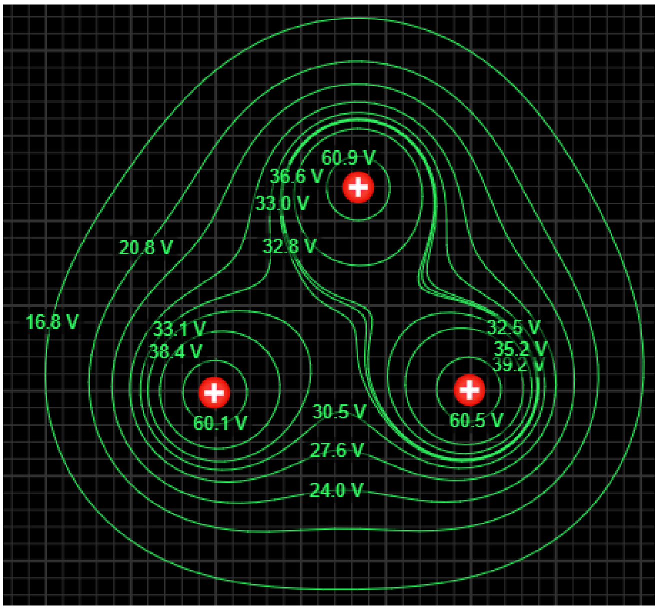




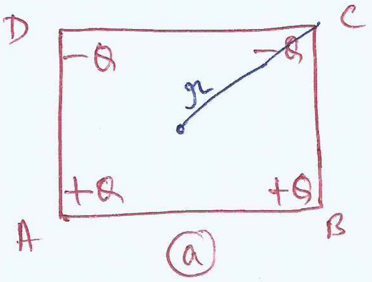


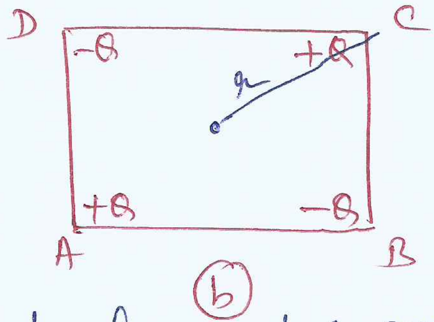






State whether the electric potentials at the centres of the squares shown in figures (a) and (b) are same or different. Justify.





The centre of a square is equidistant from each corner of the square. So, the magnitude of electric potential at the centre of the square due to all charges placed at 4 corners will be same, although sign depends on sign of charge.

(a) V at centre is given by

$$V = V_A + V_B + V_C + V_D$$

$$V = \cancel{\frac{1}{4\pi\epsilon_0}} \frac{Q}{r} + \cancel{\frac{1}{4\pi\epsilon_0}} \frac{Q}{\cancel{r}} + \cancel{\frac{1}{4\pi\epsilon_0}} \frac{(-Q)}{r} + \cancel{\frac{1}{4\pi\epsilon_0}} \frac{(-Q)}{r} = 0$$

(b) |||ly V at centre is given by

$$V = \cancel{\frac{1}{4\pi\epsilon_0}} \frac{Q}{r} + \cancel{\frac{1}{4\pi\epsilon_0}} \frac{(-Q)}{r} + \cancel{\frac{1}{4\pi\epsilon_0}} \frac{Q}{r} + \cancel{\frac{1}{4\pi\epsilon_0}} \frac{(-Q)}{r} = 0$$

∴ Electric potentials at the centres of both figures is same.

Electric potential of a charged liquid drop. Find V of a liquid drop formed when n ~~sam~~ small droplets of a liquid each of radius r and each having charge q combine.

→ V of each droplet $V_s = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ (V_s → small drop)

when n droplets combine to form a big drop of radius R, then

Volume of big drop = Volume of n droplets

$$\Rightarrow \cancel{\frac{4}{3}}\cancel{\pi} R^3 = n \times \cancel{\frac{4}{3}}\cancel{\pi} r^3$$

$$\therefore \boxed{R = n^{1/3} r}$$

$$\Rightarrow \text{Charge on big drop} = Q = nq$$

$$\Rightarrow V \text{ of big drop}, V_{big} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} = \frac{1}{4\pi\epsilon_0} \cdot \frac{nq}{n^{1/3} r} = n^{2/3} \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r} \right)$$

$$\therefore \boxed{V_{big} = n^{2/3} V_{small}}$$

Thus, electric potential of big drop formed by n small droplets = $n^{2/3}$ times electric potential of a small droplet.

problem

(a) calculate the potential at a point P due to a charge of $4 \times 10^{-7} C$ located 9cm away.

(b) Hence obtain the work done in bring a charge of $2 \times 10^{-9} C$ from ∞ to the point P. Does the answer depend on the path along which the charge is brought?

(a) $$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{\cancel{9} \times 10^{9} \times 4 \times 10^{-7}}{\cancel{9} \times 10^{-2}} = \underline{4 \times 10^{4}\ V}$$

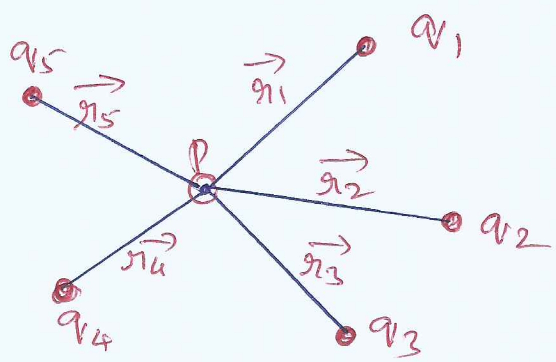
(b) $$W = qV = 2 \times 10^{-9} C \times 4 \times 10^{4} V$$
$$= 8 \times 10^{-5} J$$

No, work done will be path independent. Any arbitrary infinitesimal path can be resolved in to 2 $\perp^r$ displacements → one along $\vec{r}$ and another $\perp^r$ to $\vec{r}$. The work done corresponding to later will be zero (since $W = FS \cos 90^\circ = 0$)

✗ Potential due to a System of charges (Principle of superposition of potentials)

→ Principle of superposition of potentials states that the "net potential at any point due to 'n' discrete charges is given by the algebraic sum of their individual potentials at that point."

→ Consider n discrete charges $q_1, q_2 \dots q_n$ at distances $r_1, r_2 \dots r_n$ respectively from a point p at which net potential is to be calculated.



→ potential at p due to q_1 ; $V_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1}$

— — q_2 ; $V_2 = \frac{1}{4\pi\epsilon_0} q_2/r_2$

⋮

— — q_n ; $V_n = \frac{1}{4\pi\epsilon_0}\frac{q_n}{r_n}$

Applying principle of superposition of potentials for a group of charges, at p due to n charges, we get net electric potential V

$$V = V_1 + V_2 + \dots + V_n$$

$$V_{net} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \dots + \frac{q_n}{r_n}\right]$$

or $$\boxed{V_{net} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

potential is a scalar quantity. Therefore, the potential at any point due to a group of point charges is found by calculating the potential due to each charge (as if the other charges were not present) and then adding algebraically the quantities so obtained.

→ If charge distribution is continuous, then $\Sigma \xrightarrow{\text{replaced by}} \int$

∴ $$\boxed{V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}}$$ where dq is a differential element of the charge distribution and r is its distance from the point at which V is to be calculated.

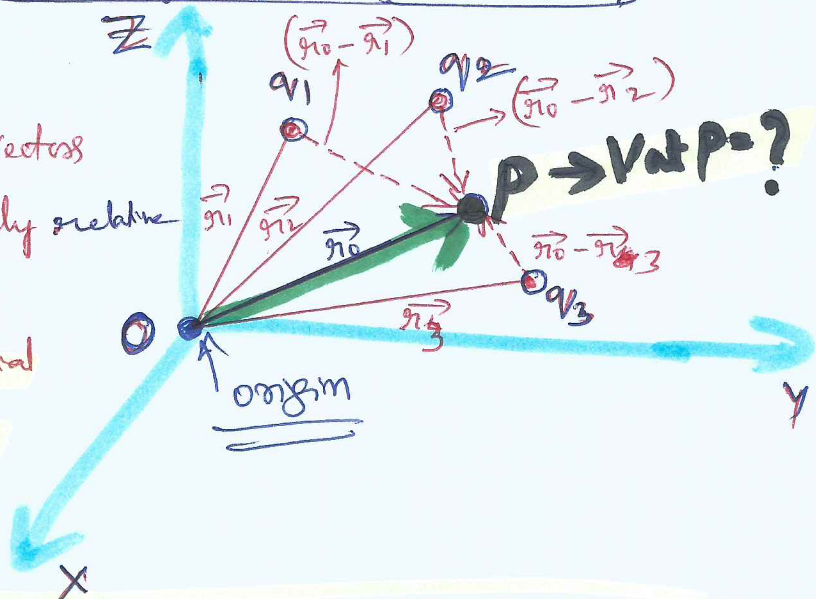
→ If charge is spread continuously over an area A, then $dq = \sigma\, dA$, where σ is surface density of charge, then

$$V = \frac{1}{4\pi\epsilon_0}\int_A \frac{\sigma\, dA}{r}$$; where $\int_A$ is the surface integral.

→ $|||^{ly}$ if charge is distributed continuously within a volume V, then

$$V = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho\, dv}{r}$$; where ρ is volume density of charge and $\int_V$ is the volume integral.

✻ Electric potential due to a group of charges in terms of position vectors ✻



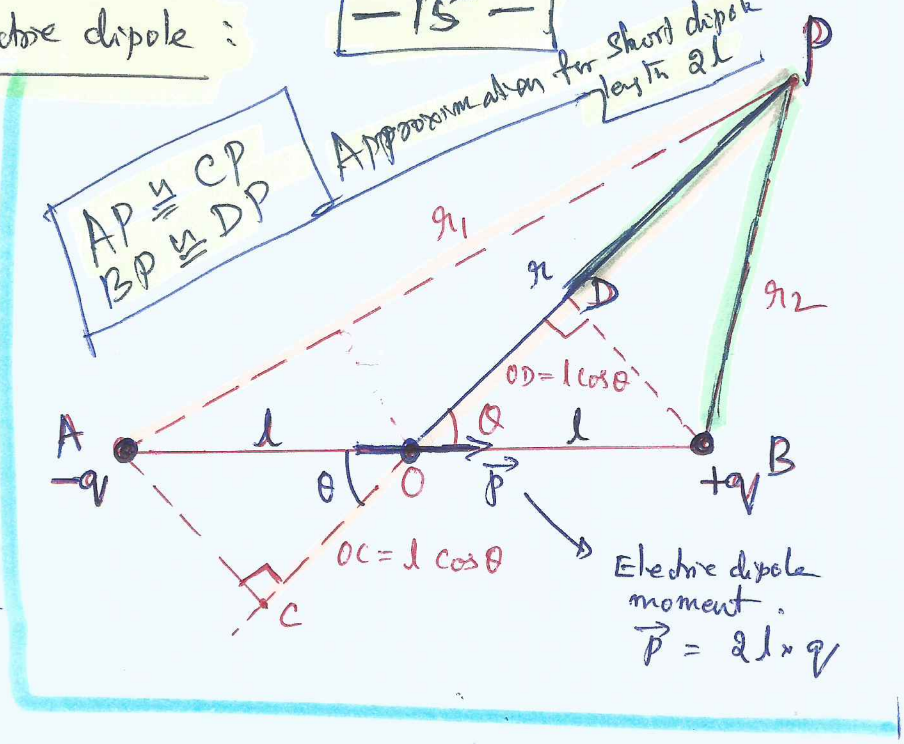
- Let $\vec{r_1}, \vec{r_2} \ldots \vec{r_n}$ be the position vectors of charges $q_1, q_2 \ldots q_n$ respectively relative to some origin O.
- Let p be the point having position vector $\vec{r_0}$, where electric potential due to the group of charges is to be calculated.

Electric potential at p due to all these charges:

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{|\vec{r_0}-\vec{r_1}|} + \frac{q_2}{|\vec{r_0}-\vec{r_2}|} + \cdots\cdots + \frac{q_n}{|\vec{r_0}-\vec{r_n}|}\right]$$

$$\therefore \boxed{V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{|\vec{r_0}-\vec{r_i}|}}$$

Potential due to an Electric dipole :



Consider any point 'P' at a dist 'r' from the centre (O) of the electric dipole formed by AB.

Let OP make an angle θ with the dipole moment $\vec{p}$. Let r_1 & r_2 be the distances of point P from $-q$ and $+q$ of the dipole respectively (see fig).

- Potential at P due to $-q$ charge

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{(-q)}{r_1}$$

potential at P due to $+q$

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{r_2}$$; using superposition principle,

potential at P due to the dipole $V = V_1 + V_2$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \longrightarrow (1)$$

We need to find r_1 and r_2?

- Draw ⊥r from A which meets the line OP at C. Also draw BD ⊥r to OP

From figure $AP \cong CP$ and $BP \cong DP$

$\therefore r_1 = OP + OC = r + l\cos\theta$

$r_2 = OP - OD = r - l\cos\theta$

Substituting these in eqn (1),

- $$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r - l\cos\theta)} - \frac{1}{(r + l\cos\theta)}\right] = \frac{q}{4\pi\epsilon_0}\left[\frac{\cancel{r} + l\cos\theta - \cancel{r} + l\cos\theta}{r^2 - l^2\cos^2\theta}\right]$$

$$V = \frac{2ql\cos\theta}{4\pi\epsilon_0(r^2 - l^2\cos^2\theta)}$$

Since p = Electric dipole moment = $2l \times q$

$$\boxed{V = \frac{p\cos\theta}{4\pi\epsilon_0(r^2 - l^2\cos^2\theta)}} \longrightarrow (2)$$

- If $r >> l$, eqn (2) becomes

$$\boxed{V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}} \longrightarrow (3)$$

Since $p\cos\theta = \vec{p}\cdot\hat{r}$, where $\hat{r}$ is the unit vector along OP

$$\therefore \boxed{V = \frac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2}} \quad \text{or} \quad \boxed{V = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3}} \longrightarrow (4)$$

P.T.O