※ Info only ※

Circular Current-loop as a magnetic dipole (Another approach by Nootan's book)

A current-carrying solenoid, or a coil, or a cir current-loop, behaves like a bar magnet. A bar magnet having N and S poles at its ends is a magnetic dipole, and so a current-loop is also a mag. dipole.

→ Let us compute the magnetic moment of a current-loop

We know that when a current-loop is suspended in a $\vec{B}$, it experiences a torque which tends to rotate the loop to ~~the~~ a position in which the axis of the loop is parallel to the field. The magnitude of the Torque acting on the current-loop in a uniform $\vec{B}$ with its axis (which is $\perp^r$ to the plane of the loop) at an angle θ with the direction of $\vec{B}$ is given by

$$\tau = I A B \sin\theta \longrightarrow (1)$$

where A is the area of the current-loop.

- We have read in electrostatics that an electric dipole placed in an $\vec{E}$ also experiences a torque which tends to align the dipole along the field. The magnitude of the torque acting on an electric dipole in a uniform electric field $\vec{E}$ with its axis at an angle θ with the direction of $\vec{E}$ is given by

$$\tau = p E \sin\theta \longrightarrow (2)$$

where p is the magnitude of the electric dipole moment.

- Compare (1) and (2), we can associate a mag. dipole moment $\vec{M}$ with the current-loop of area A and carrying a current I. The magnitude of $\vec{M}$ is $\boxed{M = IA} \longrightarrow (3)$ or $\boxed{\vec{M} = I\vec{A}} \longrightarrow (3)$

Vector form.

where $\vec{A}$ is area vector $\perp^r$ to the plane of the loop.

- SI unit of M = $A m^2$; Dimension $[AL^2]$

Thus, the direction of mag. dipole moment is $\perp^r$ to the plane of the loop. (Direction is given by RHT rule → fingers curling I, thumb → $\vec{M}$)

If instead of a current-loop, there is a current-carrying coil having N turns, then its magnetic moment is given by $\vec{M} = NI\vec{A}$

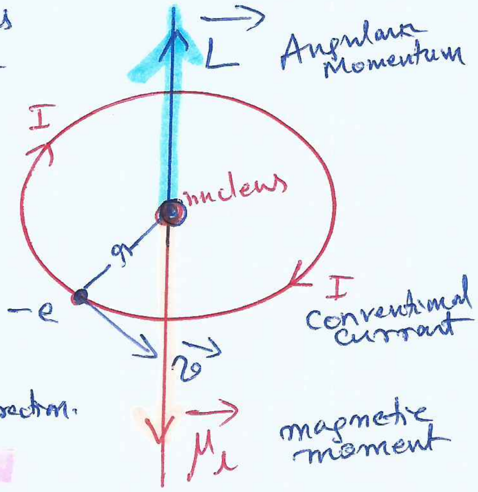
We had to use the torque expression for an electric dipole to compute the magnetic dipole moment just because of an important difference bet^n electric dipole and mag. dipole.

- The electric dipole is a system made up of 2 equal & opposite charges which have independent existence and can be separated from each other. Thus, an ~~electric~~ isolated electric charge (not the electric dipole) is the simplest electric structure.

- The mag. dipole is, however, itself the simplest mag. structure (elementary structure) because an isolated mag. pole does not exist. If we go on breaking a magnet, then even the atoms and the fundamental particles (electrons, protons etc) of the magnet are mag. dipoles (not poles). Hence, a mag. dipole cannot be assumed to be a system formed by two opposite poles. In a single current-loop (which is a magnetic dipole), the plane of the loop behaves as N-pole when observed from one side and as S-pole when observed from the other side.

over.

Magnetic Dipole moment of a revolving electron
(Show that an atom behaves as a mag. dipole ?)



- In Bohr model, the electron revolves around nucleus much as a planet revolves around the Sun. The force in the ~~fo~~ former case is electrostatic (Coulomb force) while it is gravitational force for the planet-Sun case.
- In an atom, electrons revolve around the nucleus in circular orbits. The movement of an electron in circular orbit around the nucleus in anti-clockwise is equivalent to the flow of (conventional) current in the orbit in clockwise direction.
- Thus, the orbit of electrons is considered as tiny current-loop.
- If an electron revolves in anti-clockwise direction (See fig), the angular momentum acts normal to the plane of orbit in the upward direction and its magnitude is given by

$$\boxed{L = m_e v r} \quad \text{or} \quad \boxed{v r = \frac{L}{m_e}} \quad \text{(1)}$$

; m = mass of electron
v = is the velocity
r = radius of orbit

- Orbital motion of electron in anti-clockwise direction, is equivalent to the flow of conventional current in clockwise direction.

$$I = \frac{e}{T}$$ where e = charge of an electron
T = Time period of orbital motion.

$$\text{But } T = \frac{2\pi r}{v} \qquad \therefore \boxed{I = \frac{ev}{2\pi r}}$$

Orbital Magnetic moment of a current-loop,

$$\mu_l = I \times A = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2} \longrightarrow (2)$$

using (1) in (2), $$\boxed{\mu_l = \frac{eL}{2m_e}} \longrightarrow (3) \text{ orbital motion.}$$

In Vector notation $$\boxed{\vec{\mu_l} = -\left(\frac{e}{2m_e}\right)\vec{L}} \longrightarrow (4)$$

- $\vec{\mu_l}$ is ⊥r to plane of the current-loop and directed downwards.
- The negative sign in eqn (4) shows that the directions of $\vec{\mu_l}$ and $\vec{L}$ are opposite to each other in case of electron. If a +ve charged particle is considered, then minus sign will disappear and directions of $\vec{\mu_l}$ and $\vec{L}$ will be same.

P.T.O →

From eqn ③ in page 57

$$\frac{\mu_l}{L} = \frac{e}{2m_e} = \frac{1.6\times10^{-19}}{2\times 9.1\times10^{-31}} = 8.8\times10^{10}\ C\,Kg^{-1}$$

This constant is experimentally verified and is known as "gyromagnetic constant".

ⓐ According to Bohr's Quantization law, angular momentum of electrons is given by $L = n\frac{h}{2\pi}$, where $n = 1, 2, 3 \ldots$; h = planck's constant.

Then eqn $\mu_l = \frac{eL}{2m_e}$ becomes

$$\mu_l = \frac{e}{2m_e}\cdot\frac{nh}{2\pi} \quad \therefore \boxed{\mu_l = n\left[\frac{eh}{4\pi m_e}\right]} \rightarrow ⑤$$

If $n=1$, then $\mu_{l\,(\text{minimum})} = \frac{eh}{4\pi m_e}$, which is Bohr magneton. It is also denoted by μ_B. It serves as natural unit of magnetic moment.

ⓘ Bohr magneton can be defined as the orbital magnetic moment of an electron circulating in the innermost orbit of the atom.

$$\mu_B = \frac{eh}{4\pi m} = \frac{(1.6\times10^{-19})(6.6\times10^{-34})}{4\pi(9.1\times10^{-31})} = 9.27\times10^{-24}\ Am^2$$

↳ Bohr magneton (μ_B) is defined in terms of 3 fundamental constants of nature. It is a non-S.I unit. It is equal to the orbital magnetic moment of an electron circulating in an orbit with the smallest allowed value of orbital angular momentum.

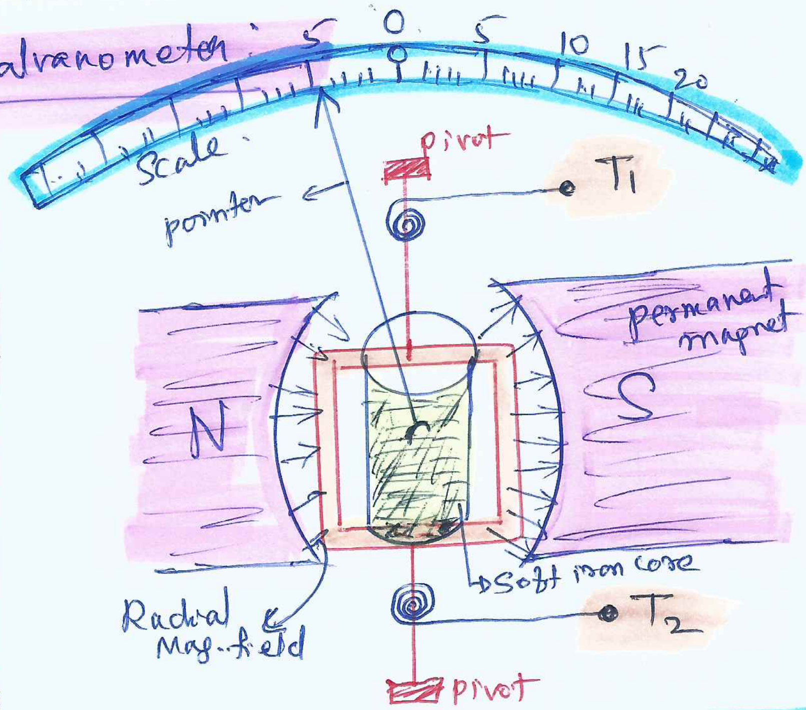
✻ In addition to orbital magnetic moment, an electron has intrinsic magnetic moment called spin magnetic moment ✻

The Moving-coil Galvanometer (MCG)

principle: It is a sensitive instrument to detect and measure electric current (of the order of ± 500 μA). Its action is based upon the principle that when electric current flows in a coil placed in a mag. field, a deflecting torque acts upon the coil whose magnitude depends upon the strength of the current. By calibrating and measuring the deflection of the coil, the strength of the current can be computed and is read on a circular scale.

There are two types:
① Suspended-coil galvanometer (not in syllabus).
② pivoted-coil (or weston) galvanometer → we will discuss only this

pivoted-coil (or weston) Galvanometer:



Construction

- It consists of a coil having a large number of turns of fine insulated copper wire wound on an aluminium frame. The ends of the axle of the frame of the coil are inserted in two pivots so that the coil may rotate about the axle.
- At both ends of the coil, near the pivots, are attached two springs which produce torsional couple on the rotation of the coil, and connect the coil to two terminals T_1 and T_2 (for external connection of the Galvanometer to the electronic circuit)
- On both sides of coil, there are pole-pieces of a permanent strong horse-shoe magnet; The coil rotates in the mag. field of these pole-pieces.
- To read the deflection of the coil, a "pointer" is attached to the coil which moves over a circular scale.
- To make $\vec{B}$ radial, the pole-pieces are cut cylindrical and a soft iron core is placed within the coil to increase the strength of $\vec{B}$.

P. T. O.

Theory: When a current flows through the coil, a torque acts on it. Torque is given by

$\tau = NIAB \sin\theta$

Since field is radial by design, $\theta = 90°$, $\sin\theta = 1$

$N \rightarrow$ No. of turns in the coil
$I \rightarrow$ current thro' the coil
$A \rightarrow$ Area of the coil.
$B \rightarrow$ Mag. field.

$\therefore \tau = NIAB$

This mag. torque NIAB tends to rotate the coil. The two springs provide a counter torque $k\phi$ that balances the magnetic torque NIAB, resulting in a steady angular deflection ϕ. In equilibrium

$$k\phi = NIAB$$

where k is the torsional constant of the spring i.e. the restoring torque per unit twist. The deflection ϕ is indicated on the scale by a pointer attached to the spring.

$$\phi = \left(\frac{NAB}{k}\right) I \longrightarrow (1)$$

; Quantity in brackets is constant for a given galvanometer.

→ **Sensitivity of a Galvanometer:**

A galvanometer is said to be sensitive if a small current flowing thro' the coil produces a large deflection.

(1) Current Sensitivity: ⟹

$$\boxed{\text{Current Sensitivity} = \frac{\phi}{I} = \frac{NAB}{k}}$$

(from eqn (1))

(2) Voltage Sensitivity $= \dfrac{\phi}{V} = \dfrac{\phi}{IR} = \left(\dfrac{\phi}{I}\right)\dfrac{1}{R}$

using eqn (1),

$$\boxed{\text{Voltage Sensitivity} = \frac{NAB}{kR}}$$

($R \rightarrow$ Resistance of the coil)

P.T.O. ⟶

Galvanometer to Ammeter : ($G \rightarrow A$)

Galvanometer is a device used to detect small currents in the range μA. If large currents (mA and A) need to be measured, another device called "ammeter" has to be used. However, one can convert $G \rightarrow A$.

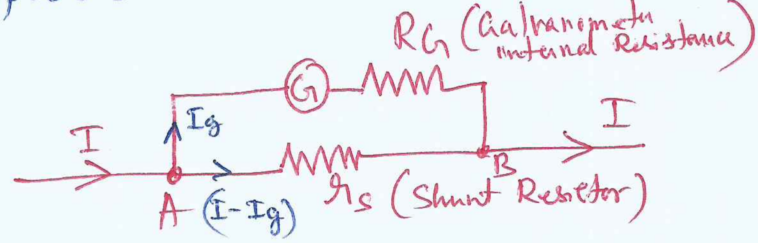
If Galvanometer is used in electric circuit with moderate current, then two things may happen.

(i) Large Resistance of Galvanometer will alter the current in the circuit. (the measuring equipment should not load the circuit parameter to be measured)

(ii) Large amount of current may damage or burn galvanometer.

Ideally, if current has to be measured, the measuring device should have zero resistance and also required to measure large value of current.

→ A galvanometer can be converted into an ammeter by connecting a low resistance (called shunt resistance) parallel to the galvanometer, so that most of the current passes thro' the shunt resistor (instead of galvanometer)



∴ Since voltage across A & B are same, then we can write

$$I_g R_G = (I - I_g)\, r_s$$

$$\therefore \boxed{r_s = \left(\frac{I_g}{I - I_g}\right) R_G}$$

This is the required value of shunt resistance to convert $G \rightarrow A$ of range 0 to I ampere.

→ Effective resistance of the above circuit :

$$\frac{1}{R_{eff}} = \frac{1}{R_G} + \frac{1}{r_s} = \frac{R_G + r_s}{R_G r_s}$$

$$\therefore \boxed{R_{eff} = \frac{R_G r_s}{R_G + r_s}}$$

Since $R_G \gg r_s$ (r_s is chosen like that), then r_s can be ignored in the above eqn and hence

$$\boxed{R_{eff} = r_s}$$

Thus, an ammeter is a low resistance device. Resistance of ideal ammeter is zero.

P.T.O →

So,

* An ammeter is always connected in series in the circuit to measure current.
* If ammeter is connected in parallel to any device, due to its very low resistance, ammeter draws heavy current and may get damaged.
* To increase the range of an ammeter 'n' times its original range, the value of shunt resistance should be

$$\left(r_s = \frac{R_G}{n-1}\right) \longrightarrow \text{very IMP.}$$

* problem:- A galvanometer of $R_G = 10\,\Omega$ gives full scale deflection for a current of 4 mA. How can it be converted into ammeter of range 0 to 5 A.

Given $R_G = 10\,\Omega$, $I = 5\,A$, $I_g = 4\,mA = 4\times10^{-3}\,A$

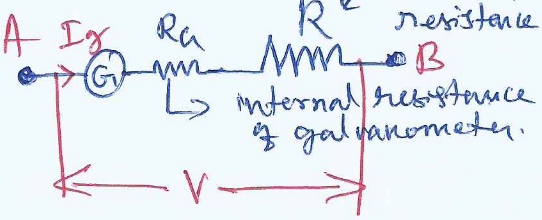
$$r_s = \left(\frac{I_g}{I-I_g}\right)R_G = \frac{(4\times10^{-3})\times 10}{5-(4\times10^{-3})} = 0.008\,\Omega.$$

Galvanometer to voltmeter ($G \to V$)

Voltmeter has to be connected across a circuit element (say R) to measure the voltage across resistance in the given circuit. If voltmeter itself draws some current from the circuit, the measurement will not be accurate. Ideally, voltmeter should draw zero current to get an accurate value of ~~Rd~~ voltage across some circuit element.

A simple galvanometer has finite resistance and if it is used as it is, then it also draws current and hence reading is not accurate in the given circuit.

If R of measuring device is very high or infinite, so that it will not draw any current and then the reading will be accurate. To ensure this, a large resistance has to be connected with galvanometer.



Let V volt be the p.d. to be measured by the voltmeter

Let I_g be the current flowing in the circuit corresponding to which the voltmeter gives the full scale deflection.

Now, p.d. betw points A and B is given by

$$\cancel{V = I_g R + I_g G} \qquad V = I_g R_a + I_g R$$

$$V = I_g (R_a + R)$$

$$\therefore R = \frac{V - I_g R_a}{I_g} \qquad \therefore \boxed{R = \frac{V}{I_g} - R_a}$$

This is the required value of Resistance which must be connected to the galvanometer to convert it into a voltmeter of range 0 − V volt

— Effective Resistance of voltmeter

$R' = R_a + R$, which is very high.

If R chosen $\gg R_a$ $\boxed{R' = R}$

Thus voltmeter is a high resistance device. Resistance of an ideal voltmeter is infinite

* A voltmeter is always connected in parallel to the circuit component across which the voltage is to be measured.

<u>problem</u>: A galvanometer having a coil of $R = 12\,\Omega$ gives full scale deflection for a current of 4 mA. How can it be converted into a voltmeter of range 0 to 24 V.

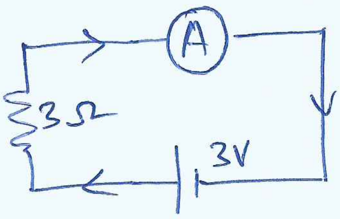
→ formula $R = \frac{V}{I_g} - R_G$

$R_G = 12\,\Omega$
$I_g = 4\,mA$
$V = 24\,V$.

$$R = \frac{24}{4 \times 10^{-3}} - 12 = 5988\,\Omega \simeq 6\,K\Omega.$$

<u>problem</u>: In the given circuit, the current is to be measured. What is the value of current if the ammeter shown
(a) is a galvanometer with $R_G = 60\,\Omega$
(b) is a galvanometer described in (a) but converted to ammeter by a shunt resistance $= r_s = 0.02\,\Omega$
(c) is an ideal ammeter with Zero Resistance.

→ (a) Given $R_G = 60\,\Omega$



Total Resistance $= 60 + 3 = 63\,\Omega$

Hence $I = \frac{3}{63} = 0.048\,A$

(b) Resistance of Galvanometer converted to an ammeter by shunt resistance $r_s = 0.02\,\Omega$

$$R_{eff} = \frac{R_G r_s}{R_G + r_s} = \frac{60 \times 0.02}{60.02} \simeq 0.02\,\Omega$$

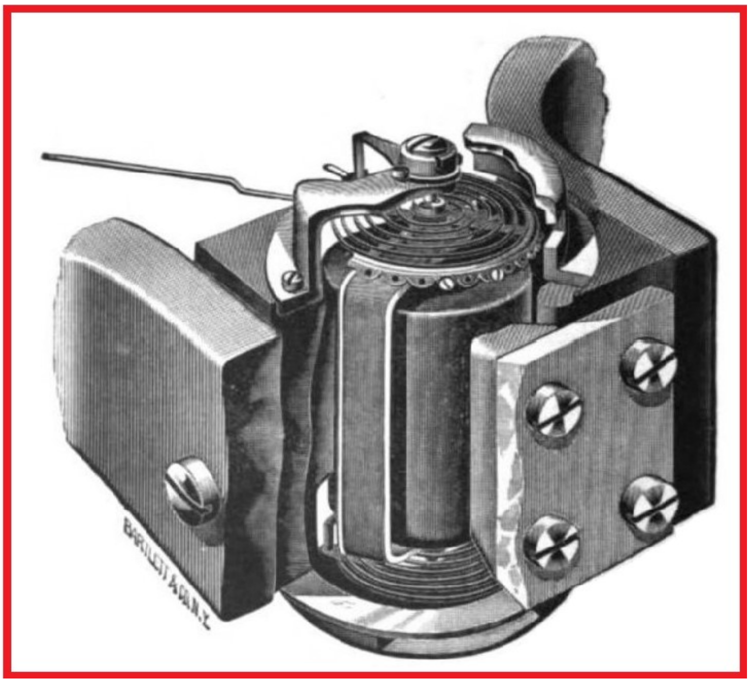
$\therefore$ Total R of circuit $3 + 0.02 = 3.02\,\Omega$

Hence $I = \frac{3}{3.02} = 0.99\,A$

(c) For an ideal ammeter with $R_G = 0\,\Omega$
$\therefore$ Total R of circuit $= 3\,\Omega$, $I = 3/3 = 1\,A$

* Note the variation of measured value with/without shunt resistance.

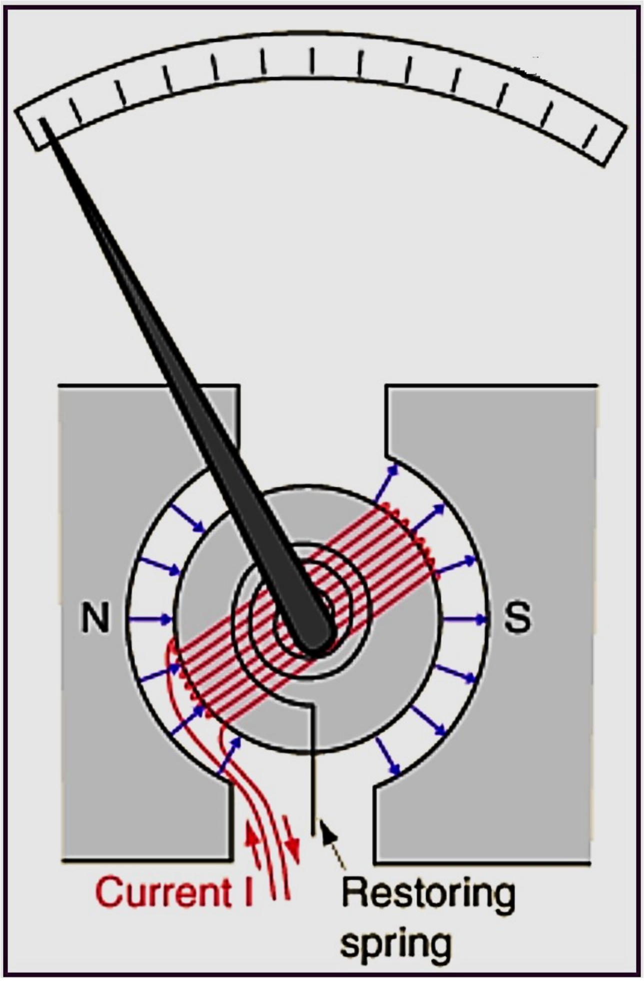
Drawing of D'Arsonval Galvanometer (ammeter) movement, of Weston type, from around 1900. This is the mechanism used in most moving-pointer (galvanometer) meters today.

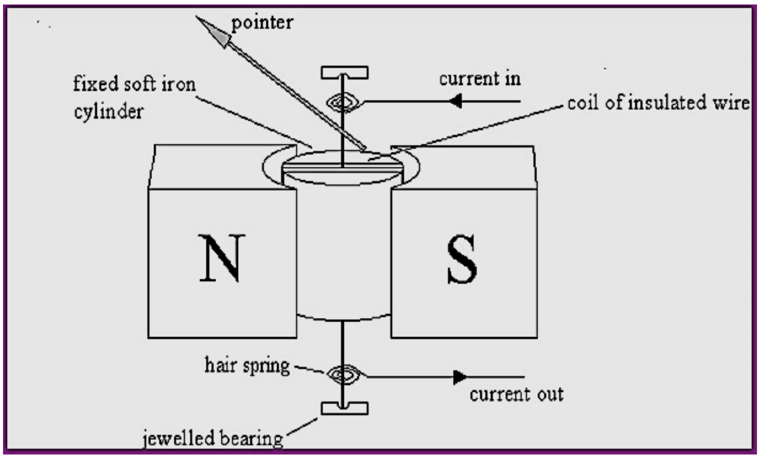


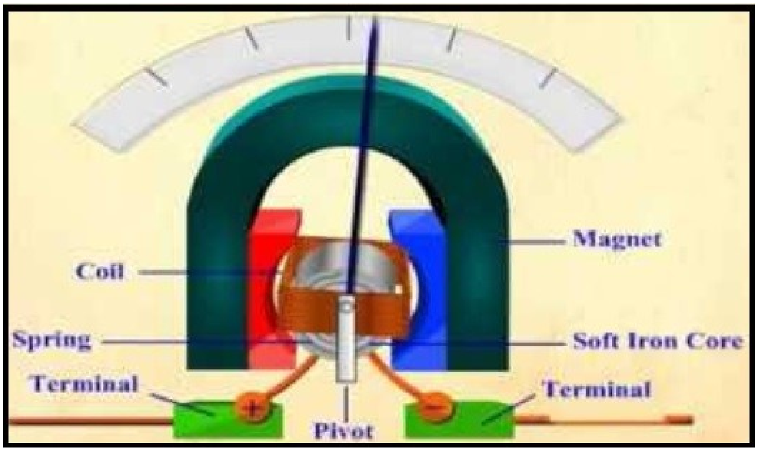
Only the pole pieces of the **horseshoe-shaped magnet** are shown, and part of the left-hand pole piece is broken out by the artist to show the coil.
The coil consists of a light coil of wire with an attached pointer, suspended on jewel bearings between the poles of a magnet, with a spiral hairspring to return it to zero.
When a current is passed through the coil, it **creates** a magnetic field that opposes the magnet's field (from external horseshoe magnet), **creating a torque** that rotates the coil against the restoring force of the spring. The coil turns through an angle until the restoring force of the spring is equal to the force of the coil. Since the angle of deflection is proportional to the force, which is proportional to the current, the deflection is proportional to the current. The screwdriver slot at the top of the axis is for adjusting the meter to zero.
****

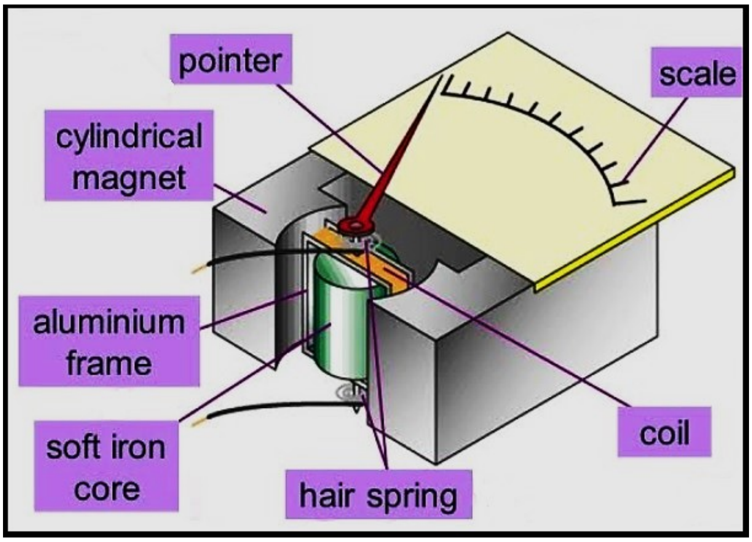
- Edward Weston replaced the fine wire suspension with a pivot, and **provided restoring torque and electrical connections through spiral springs** rather like those of a wristwatch balance wheel hairspring. He developed a method of stabilizing the magnetic field of the permanent magnet, so the instrument would have consistent accuracy over time.

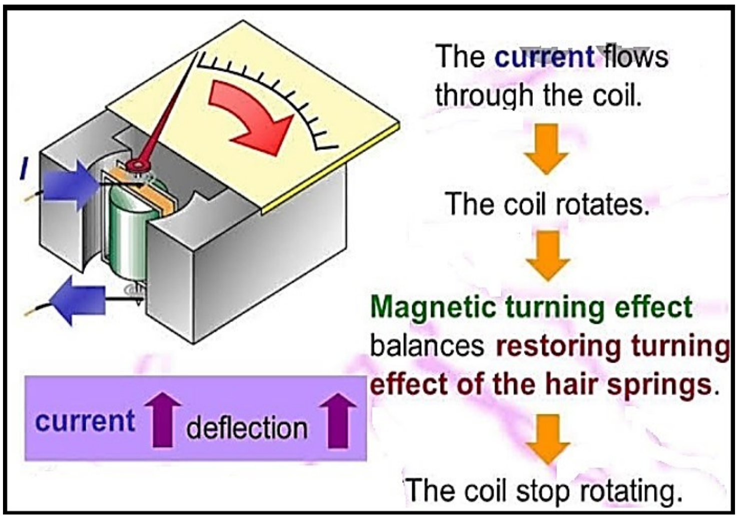
- He replaced the light beam and mirror with a knife-edge pointer that could be read directly. A mirror under the pointer, in the same plane as the scale, eliminated parallax observation error. To maintain the field strength, Weston's design used a very narrow circumferential slot through which the coil moved, with a minimal air-gap. This improved linearity of pointer deflection with respect to coil current.
- Finally, the coil was wound on a **light-weight form made of conductive metal**, which acted as a damper. By 1888, Edward Weston had patented and brought out a commercial form of this instrument, which became a standard electrical equipment component. It was known as a "portable" instrument because it was affected very little by mounting position or by transporting it from place to place. This design is almost universally used in moving-coil meters today.
- Initially laboratory instruments relying on the Earth's own magnetic field to provide restoring force for the pointer, galvanometers were developed into compact, rugged, sensitive portable instruments essential to the development of electro-technology.


N
S
Current I
Restoring spring


pointer
fixed soft iron cylinder
current in
coil of insulated wire
N
S
hair spring
current out
jewelled bearing


Magnet
Coil
Spring
Soft Iron Core
Terminal
Terminal
Pivot


pointer
scale
cylindrical magnet
aluminium frame
soft iron core
coil
hair spring


The current flows through the coil.
The coil rotates.
Magnetic turning effect balances restoring turning effect of the hair springs.
The coil stop rotating.
I
current
deflection