Two capacitors of unknown capacitances C1 and C2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of C1 and C2. Also calculate the charge on each capacitor in parallel combination.

For series combination, energy stored

$$u_1 = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}V^2$$

$$0.045 = \frac{C_1C_2}{2(C_1+C_2)}(100)^2 \text{ ....(i)}$$

For parallel combination, energy stored

$$u_2 = \frac{1}{2}(C_1+C_2)V^2$$

$$0.25 = \frac{1}{2}(C_1+C_2)(100)^2$$

$$\therefore C_1 + C_2 = 0.5 \times 10^{-4} \text{ ....(ii)}$$

From (i), $0.045 = \dfrac{C_1C_2}{2(0.5 \times 10^{-4})} \times 10^{-4}$

$$C_1C_2 = 0.045 \times 10^{-8}$$

Now, $C_1 - C_2 = \left[(C_1+C_2)^2 - 4C_1C_2\right]^{1/2}$

$$= \left[(0.5 \times 10^{-4})^2 - 4(0.045 \times 10^{-8})\right]^{1/2}$$

$$= [(0.25 \times 0.180)10^{-8}]^{1/2}$$

$$C_1 - C_2 = 0.26 \times 10^{-4} \text{ ...(iii)}$$

From (ii) and (iii), we get

$$C_1 = 0.38 \times 10^{-4}F \text{ and } C_2 = 0.12 \times 10^{-4}F$$

Charges on C_1 and C_2 in parallel combination

$$q_1 = C_1V = 0.38 \times 10^{-4} \times 100 = 0.38 \times 10^{-2}C$$

$$q_2 = C_2V = 0.12 \times 10^{-4} \times 100 = 0.12 \times 10^{-2}C.$$

... an expression for its

(ii) Two capacitors of different capacitances are connected first (1) in series and then (2) in parallel across a dc source of 100 V. If the total energy stored in the combination in the two cases are 40 mJ and 250 mJ respectively, find the capacitance of the capacitors. 5

OR

Question: Two capacitors of different capacitances are connected first (1) in series and then (2) in parallel across a dc source of 100 V. If the total energy stored in the combination in the two cases are 40 mJ and 250 mJ respectively, find the capacitance of the capacitors.

- Given, for series combination, energy stored is $u_s = 0.04$ J
- Given, for parallel combination, energy stored is $u_p = 0.25$ J
- Given the dc source voltage = V = 100 V
- For series combination, energy stored is given by the formula
  - $u_s = \frac{1}{2}\left[\frac{C_1C_2}{C_1+C_2}\right] V^2 \implies 0.04 = \frac{1}{2}\left[\frac{C_1C_2}{C_1+C_2}\right](100)^2 \implies 0.08 = \left[\frac{C_1C_2}{C_1+C_2}\right]10^4 \implies 8 = \left[\frac{C_1C_2}{C_1+C_2}\right]10^6$
  - $\therefore C_1C_2 = 8[C_1+C_2]10^{-6} ------(1)$
- For parallel combination, energy stored is given by
  - $u_p = \frac{1}{2}[C_1+C_2]V^2 \implies 0.25 = \frac{1}{2}[C_1+C_2](100)^2 \implies 0.5 = [C_1+C_2](10)^4$
  - $\therefore [C_1+C_2] = 50\ x\ 10^{-6} ----- (2)$
- Plug (2) in (1), we get
- $C_1C_2 = 8[50\ x\ 10^{-6}]10^{-6} = 400\ x 10^{-12}\ ----- (3)$
- Now, $C_1 - C_2 = \sqrt{(C_1+C_2)^2 - 4C_1C_2} ----------(4)$
- Plug (2) and (3) in (4), we get
- $C_1 - C_2 = \sqrt{(50\ x\ 10^{-6})^2 - 4x400\ x 10^{-12}}$
- $C_1 - C_2 = \sqrt{2500-1600}\ x\ 10^{-6} = 30\ x\ 10^{-6}\ ----(5)$
- Solving equations (2) and (5), we get
- Adding (2) and (5), $2C_1 = 80\ x\ 10^{-6}$; $\therefore\ C_1 = 40\ \mu F$
- Subtracting (2) and (5), $2C_2 = 20\ x\ 10^{-6}$; $\therefore\ C_2 = 10\ \mu F$
- Extra 1 : Parallel combination
  - Charge on each capacitor in parallel combination = $Q_1 = C_1V = 40 \times 10^{-6} \times 100 = 4 \times 10^{-3}$ C = 4 mC
  - $Q_2 = C_2V = 10 \times 10^{-6} \times 100 = 1 \times 10^{-3}$ C = 1 mC
  - Total charge $Q_P = Q_1 + Q_2$ = 5 mC (we can do this simple addition in parallel combination, not in series combination)
  - Also total charge in both the capacitors in parallel combination = $Q_P = (C_1+C_2)V$ = 50μF x 100 = 5 mC
  - Verifying energy in this case:
    - u_p = ½ QV = ½ x 5mC x 100 = 250 mJ (verified)
    - u_p = ½ Q^2/C = 0.5 x (25 x10^{-6})/(50 x 10^{-6})= 0.5 x 0.5 = 0.25 J = 250 mJ (verified)
    - u_p = ½ CV^2 = ½ x 50 x 10^{-6} x 10^4 = 25 x 10^{-2} = 250 mJ (verified)
- Extra 2: Series combination
  - In series combination, charge on both the capacitors are same ⟹ $Q_1 = Q_2$
  - $C_1V_1 = C_2V_2$ and V_1+V_2 = 100 ; $C_1 = 40\ \mu F$ and $C_2 = 10\ \mu F$
  - $C_1/C_2 = 4 = V_2/V_1$; $V_2 = 4V_1$ ∴ V_1 = 20 V and V_2 = 80 V
  - $Q_1 = C_1V_1 = 40\ \mu F\ x\ 20 = 800\ \mu C$
  - $Q_2 = C_2V_2 = 10\ \mu F\ x\ 80 = 800\ \mu C$
  - In series combination, charge on both the capacitors are same ⟹ $Q_1 = Q_2$
  - In series combination, the charge on each capacitor = 800 μC = 0.8 mC
  - Effective capacitance in series combination : $C = \frac{C_1C_2}{C_1+C_2} = \frac{400\ x\ 10^{-12}}{50\ x\ 10^{-6}} = 8\mu F$
  - Total charge in the series circuit = Q_S = $\left[\frac{C_1C_2}{C_1+C_2}\right]V = 8\mu F\ x\ 100V$ = 800 μC = 0.8 mC
  - Note that $Q_S \neq Q_1 + Q_2$ in series combination. Since in series combination, all capacitors are charged to the same value (Q is same), Q_S = charge on any one capacitor
  - Verifying energy in series case:
    - u_p = ½ QV = ½ x 800 μC x 100 = 400 x 10^{-4} = 40 mJ (verified)
    - u_p = ½ Q^2/C = [0.5 x (0.64 x10^{-6})]/(8 x 10^{-6})= 0.5 x 0.64/8 = 0.04 J = 40 mJ (verified)
    - u_p = ½ CV^2 = ½ x 8 x 10^{-6} x 10^4 = 4 x 10^{-2} = 40 mJ (verified)