* Special cases:

① If point P lies on the axial line of the dipole, $\theta = 0^\circ$ $(\cos\theta = 1)$

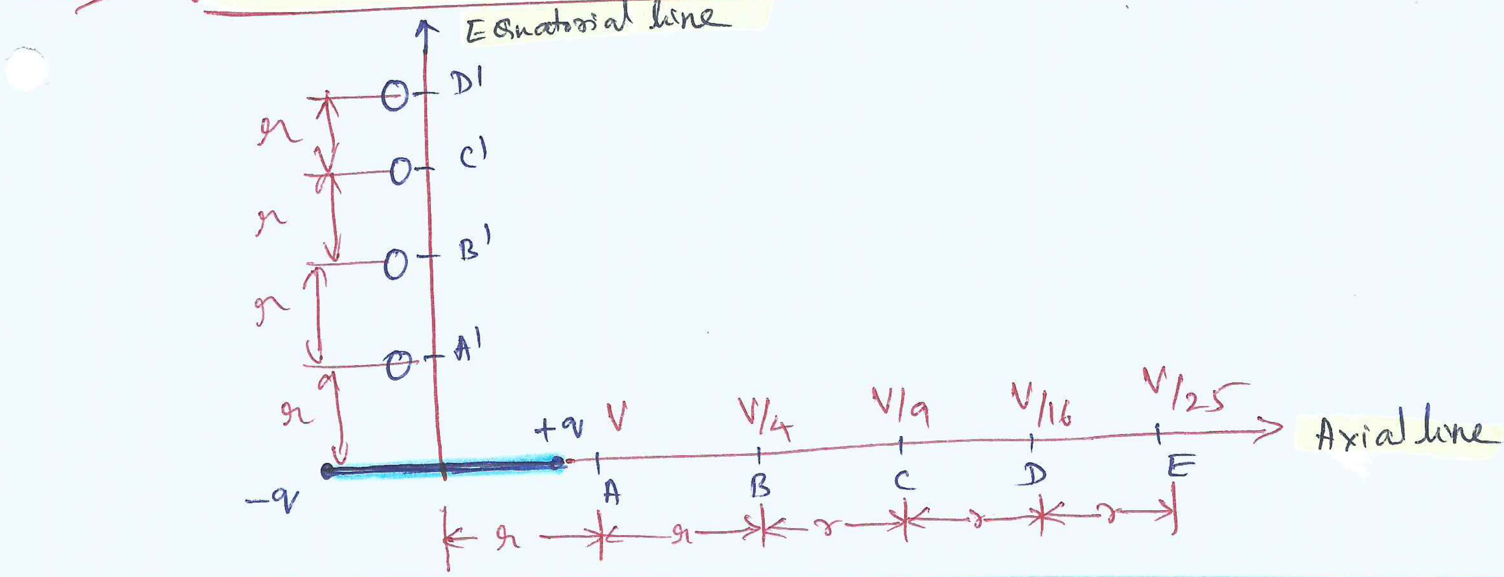
then eqn ③ becomes $V = \dfrac{p}{4\pi\epsilon_0 r^2}$ or $V \propto \dfrac{1}{r^2}$

② If point P lies on the equatorial line of the dipole, $\theta = 90^\circ$
$\cos 90^\circ = 0$

∴ eqn ③ becomes $V = 0$.

Thus, electric potential due to a dipole is zero at all points on the equatorial line of the dipole.

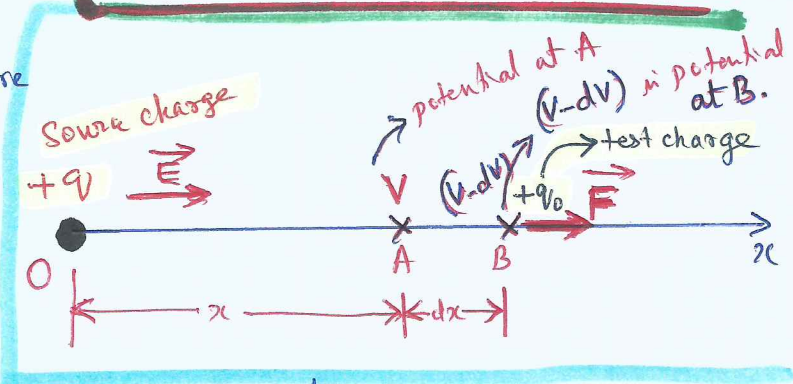
→ V due to electric dipole (for axial line / equatorial line)



V due to isolated charge	V due to electric dipole.
① V at a dist r from an isolated charge is given by, $V \propto \frac{1}{r}$	① V at a dist r from the centre of the electric dipole is given by, $V \propto \frac{1}{r^2}$
② V at a dist. r from an isolated charge q is given by, $V \propto q$	② V at a dist r from the centre of electric dipole of dipole moment (p) is given by, $V \propto p$
③ V due to an isolated point charge has same value at the points equidistant from the isolated charge	③ V due to electric dipole is same (ie zero) at all points lying on the equatorial line of the electric dipole while electric potential due to electric dipole decreases along the axial line of the electric dipole. $(\propto 1/r^2)$

Relation betn. E & V : Electric field as Gradient of electric potential.

- As shown in figure, there exists an electric field $\vec{E}$ (along x-axis) due to point charge +q at point O.
- Suppose A and B are two points, where
  OA = x
  OB = x + dx
- Let Electric potential at A = V and
  let ——"—— B = V − dV



- Let us place a test +ve charge q_0 at B. Force experienced by q_0 at B is given by

$$\vec{F} = q_0 \vec{E} \longrightarrow (1)$$

where $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

Direction of $\vec{F}$ = direction of $\vec{E}$

- If we want to move test charge q_0 from B to A (towards source charge q), an external agent will have to do work against the repulsive force $\vec{F}$ of +q. Let dw be the work done by external agent to carry charge q_0 from B to A (opposite to $\vec{F}$) through a distance −dx. (dx must be small enough to ensure that $\vec{E}$ is uniform along AB)

$\therefore$ dw = Force × distance = F(−dx)

$$\therefore \boxed{dw = -F\,dx} \longrightarrow (2)$$

pulg-in F of eqn (1) into (2)

This is from eqns (1) and (2).

$$dw = -q_0 E\,dx$$

$$\boxed{\frac{dw}{q_0} = -E\,dx} \longrightarrow (3)$$

By defn of potential energy (PE),

$$\boxed{\frac{dw}{q_0} = dV} \longrightarrow (4)$$

(potential difference betn A and B)

From (3) and (4); $dV = -E\,dx$

$$\therefore \boxed{E = -\frac{dV}{dx}} \quad (5)$$

The negative sign shows that $\vec{E}$ is in the direction of decreasing electric potential gradient.

→ The quantity $\frac{dV}{dx}$ is the rate of change of potential with distance and is known as "potential gradient".

→ Thus Electric field intensity $\vec{E}$ at a point in an electric field in a given direction is equal to the negative potential Gradient in that direction.

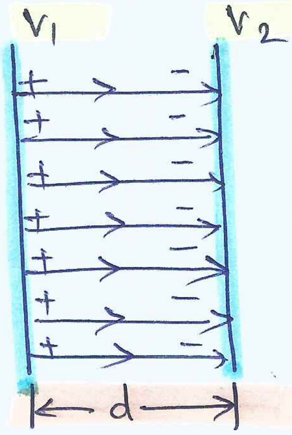
→ The −ve sign indicates that the "potential" decreases in the direction of $\vec{E}$

→ Due to $E = -dV/dx$, $\vec{E}$ can also be expressed as Vm^{-1} (compare with $\vec{E} = \vec{F}/q_0$)

$$\therefore \boxed{1\,NC^{-1} = 1\,Vm^{-1}} \longrightarrow (6)$$

P.T.O →

Fig. shows two metallic plates having +ve and −ve charges.
If plates are long in relation to distance betn plates (d), then $\vec{E}$ produced betn plates ~~are~~ is uniform and directed from +ve plate to −ve plate.



→ If the potentials of the plates be V_1 and V_2 and distance betn them d meters, then electric field $\vec{E}$ betn plates is given by

$$\boxed{E = \frac{V_1 - V_2}{d} \quad Vm^{-1}} \longrightarrow (7)$$

* Note that relation $E = -\frac{dV}{dr}$ is valid for non-uniform field also
* SI unit of $\vec{E}$ is same as that of "potential gradient" "Vm^{-1}"
  $1 Vm^{-1} = 1 Nm^{-1}$
* Electric potential is a scalar quantity ; while electric potential gradient is a vector quantity.

* Importance of the Relation betn E and V (very imp).

$E = -\frac{dV}{dx}$ → This relation enables us to calculate the electric field intensity (a vector) at a point if potential (a scalar) at that ~~point~~ point is known.

For ex:, the electric potential due to a point-charge q at a dist r is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

By symmetry, electric field $\vec{E}$ must be directed radially outwards from a positive charge. Its magnitude equals the negative potential gradient in the radial direction. Thus,

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left[\frac{1}{4\pi\epsilon_0}\frac{q}{r}\right] = -\frac{q}{4\pi\epsilon_0}\frac{d}{dr}\left(\frac{1}{r}\right)$$

$\frac{d}{dr}\left(\frac{1}{r}\right) = -\frac{1}{r^2}$

$$\boxed{E = \frac{q}{4\pi\epsilon_0}\frac{1}{r^2}}$$ So we get eqn for $\vec{E}$.

A potential difference of 50V is applied across 2 plates of 2mm apart. Calculate the magnitude of the electric field intensity betn the plates.

→ Given $d = 2\ mm = 2\times10^{-3}\ m$; $dV = 50\ V$

$$E = -\frac{dV}{dr} = \frac{50}{2\times10^{-3}} = 25\times10^{3}\ Vm^{-1}$$

How is electric field strength (E) related to potential difference (V) ?

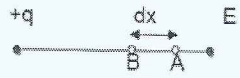
I was looking at the derivation of drift velocity of free electrons in a conductor in terms of relaxation time of electrons.

Here a metallic conductor XY of length ℓ is considered and having cross sectional area A. A potential difference V is applied across the conductor XY. Due to this potential difference an electric field E is produced. The magnitude of electric field strength is E=V/ℓ.

I don't understand this part. How is electric field strength = potential difference divided by length? Where am I lagging in my concepts?

- The electric field = - potential gradient.
- It comes from expressing energy changes in two different ways...
  - You can say work done = force x distance = qE x distance
  - However you can also say the work done = change in potential x charge moved = qV
- If you equate these two different ways of expressing the same energy change you get
- qE x distance (d) = qV → qEd = qV → q cancels → Ed = V
- E = V/d which is the potential gradient.
- The minus sign comes from the distance moved for a positive charge opposite the field counts as being negative. (You really need vectors to do it properly).

Consider two points A and B separated by a small distance dx in an electric field. Since dx is small, the electric field E is assumed to be uniform along AB. The force acting on a unit positive charge at A is equal to E.



Work done in moving a unit positive charge from A to B against the electric field is

dW = - E dx. The negative sign shows that the work is done against the direction of the field. Since the work done is equal to p.d dV between A and B then

dV = - E dx or $\boxed{E = -\frac{dV}{dx}}$

Thus, the electric field at a point is the negative potential gradient at that point.

The electric flux lines that are going **through** the conductor will be parallel to the conductor. Therefore, the electric field inside the conductor has a constant magnitude, and direction is parallel to the conductor.

We know that **E = −∇φ** , where φ is the electric potential. We then integrate along a line through the conductor

$$\boxed{V = \int_0^\ell \nabla\phi\, dx = \int_0^\ell -E\, dx = -E\ell}$$

which is the same as **E=V/ℓ** if you consider a positive E to mean a vector pointing in the –x direction instead of the +x direction.

How do you define **electric field**? It is force per unit charge (E= F/q).

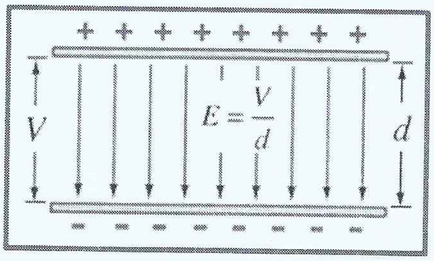
Again, what is electric force unit? Capacity to move **one unit charge** through **one unit of distance**.

Potential difference V (the voltage) is the capacity to move one unit charge through the extremes of reference points.

So if one unit charge moves from one reference point to other reference extreme, it needs some energy to move through each unit of distance.

Now combine these two statements, and you will get electric field E as voltage V divided by distance.

- Electric field describes the *force* on a charge.
- Electric potential describes how much *energy* that charge will gain or lose in moving (or being moved) from one point to another.
- What you are really asking is : " How is *force* related to how much *energy* I can give (or take) from a particle with a certain force?"
- This is exactly the concept of "work." Work is the change in *energy* moving something from point A to point B with a certain *force*.
- What's missing is the very important idea of "from A to B."
- How far is it?
- You can't define a specific potential without telling how far A is from B. That distance is the missing part of your question.
- In the same way that



work(AB) = change in potential energy(AB) = force times distance(AB)

This leads to

"potential(AB)" = change in potential energy(AB) = force times distance(AB).

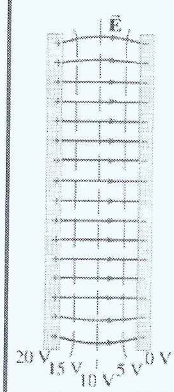
So that's the relationship.

The (AB) here is meant to suggest the ("from A to B") that most people leave out because they figure everybody *knows* that...

In the same way that they leave out the "energy" in potential energy.

Caveat: I've used distance instead of displacement. I must really use displacement because directions matter. That's why I've also left out the plus and minus signs . . . for the sake of clarity.

Equipotential Surfaces



An equipotential is a line or surface over which the potential is constant.

Electric field lines are perpendicular to equipotentials.

The surface of a conductor is an equipotential.

Electrostatic Potential Energy; the Electron Volt

One electron volt (eV) is the energy gained by an electron moving through a potential difference of one volt:

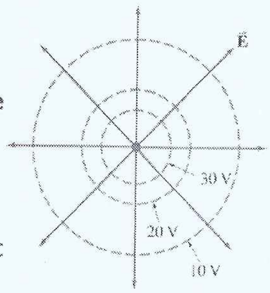
$$1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}.$$

The electron volt is often a much more convenient unit than the joule for measuring the energy of individual particles.

Equipotential Surfaces

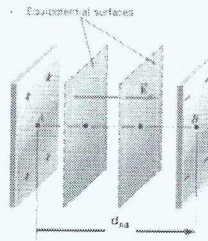
Point charge equipotential surfaces.

For a single point charge with $Q = 4.0 \times 10^{-9}$ C, sketch the equipotential surfaces (or lines in a plane containing the charge) corresponding to $V_1 = 10$ V, $V_2 = 20$ V, and $V_3 = 30$ V.



Example 9 **The Electric Field and Potential Are Related**

The plates of the capacitor are separated by a distance of 0.032 m, and the potential difference between them is $V_B - V_A = -64$V. Between the two equipotential surfaces shown in color, there is a potential difference of -3.0V. Find the spacing between the two colored surfaces.



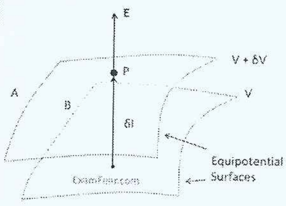
$$E = -\frac{V}{d_{AB}} = \frac{-64\ \text{V}}{0.032\ \text{m}} = 2.0 \times 10^{3}\ \text{V/m}$$

$$d = -\frac{V}{E} = -\frac{-3.0\ \text{V}}{2.0 \times 10^{3}\ \text{V/m}} = 1.5 \times 10^{-3}\ \text{m}$$

Relation between electric field and potential

According to the relation between electric field and potential,

- Electric field is in the direction in which the potential decreases steepest.
- Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.



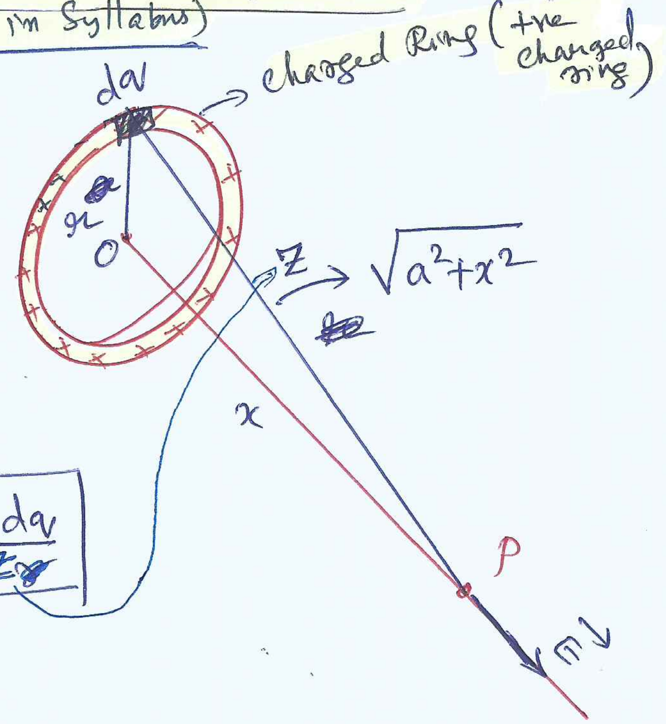
For moving unit charge along perpendicular from surface B to A,
Work done = $|E|\delta l$

Since, Potential difference = Work done
$|E|\delta l = -\delta V$
$|E| = -\delta V/\delta l = +|\delta V|/\delta l$

※ Electric potential at an axial point of a charged Ring.

(not in Syllabus)

ex:



Let O be the centre of a plane ring of radius a and having a charge +q uniformly distributed over it. Let p be a point on the axis of the ring at which the Electric potential is required.

Let us consider an infinitesimally-small ring-element of charge dq. Let its dist from p be $Z = \sqrt{a^2+x^2}$

$\therefore$ V due to dq at p is $\boxed{dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{Z}}$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{(r^2+x^2)^{1/2}}$$

V due to entire ring $V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{(r^2+x^2)^{1/2}}$

→ Now for the point p, x is same for all the elements and as such the term $(a^2+x^2)^{1/2}$ can be taken outside the integral, thus

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{1}{(r^2+x^2)^{1/2}}\int dq = \frac{1}{4\pi\epsilon_0}\frac{q}{(r^2+x^2)^{1/2}}} \longrightarrow (1)$$

EQ (1) is the expression for the electric potential at an axial point of a charged ring.

→ Electric Intensity: From symmetry, $\vec{E}$ at P must be along the axis of the ring (along x). We know that $\vec{E}$ at a point in any direction is the −ve gradient of V in that direction.

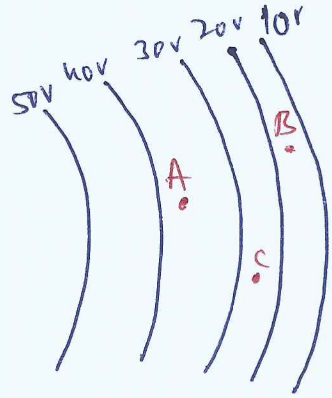
$\therefore$ the magnitude of $\vec{E}$ along the axis of the ring is

$$E = -\frac{dV}{dx} = -\frac{d}{dx}\left[\frac{1}{4\pi\epsilon_0}\frac{q}{(r^2+x^2)^{1/2}}\right] = \frac{1}{4\pi\epsilon_0}\cdot\frac{qx}{(r^2+x^2)^{3/2}}$$

$$\therefore \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{qx}{(r^2+x^2)^{3/2}}}$$

$\vec{E}$ is directed away from the +vely charged ring along the axis of the ring.

The given fig. shows lines of constant potential in a region of an electric field. The values of potentials are indicated on the curves. At which of the points A, B and C, the magnitude of the electric field is the greatest?



→ Here, V for particular curve is same throughout the curve.

∴ PD betn two consecutive curves is $dV = 10\,V$ = constant

Magnitude of electric field is given by

$$|E| = \frac{dV}{dr} \longrightarrow (1)$$

Clearly from (1), $|\vec{E}|$ is maximum when distance betn the curves is minimum.

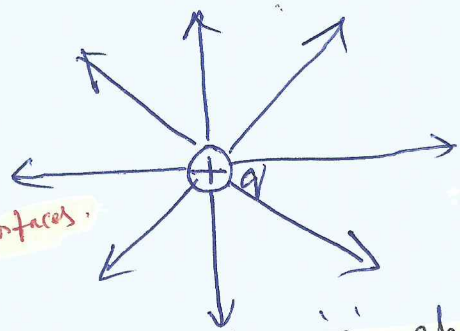
∴ magnitude of electric field $|\vec{E}|$ is the greatest at B, where the distance betn consecutive curves is the least.

Equipotential Surfaces

An equipotential surface is a surface with a constant value of potential at all points on the surface. For a single charge q, the potential is given by equation $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ $\Rightarrow$ This shows that V is a constant if r is constant. Thus, equipotential surfaces of a single point charge are concentric spherical surfaces centred at the charge.

Electric field lines for a single charge q are radial lines starting from or ending at the charge, depending on whether q is +ve or −ve. Clearly, the electric field at every point is normal to the equipotential surface passing through that point. This is true in general: for any charge configuration, equipotential surface thro' a point is normal to the electric field at that point.

(For a single charge q equipotential surfaces are spherical surfaces centered at the charge)

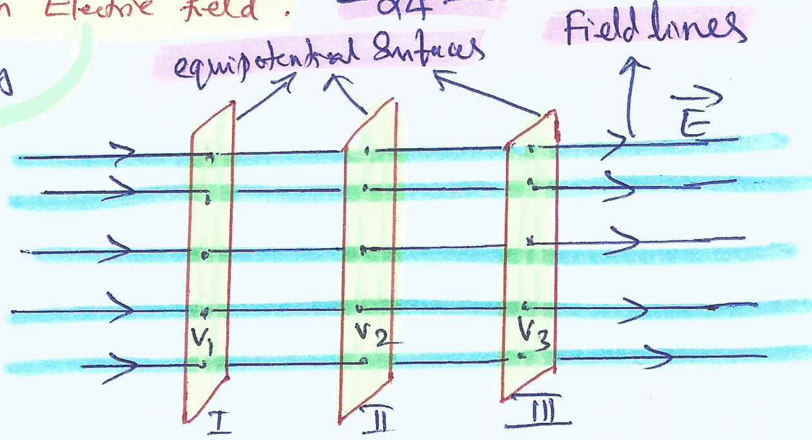


(For a single charge 'q', electric field lines are radial starting from the charge if $q > 0$.)

Defn. :→ An equipotential surface is defined as the locus of all the points in a medium at which electric potential due to a charge distribution is same.

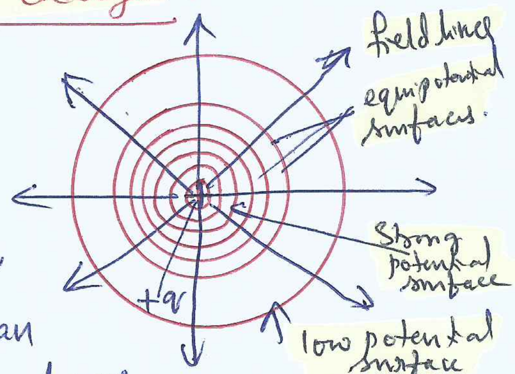
* The formation of an equipotential surface will depend upon the type of medium i.e. isotropic or non-isotropic and the amount of charge distribution.

① Equipotential Surfaces for a uniform Electric field.



uniform electric field is represented by equidistant parallel straight lines. Draw planes I, II and III ⊥r to the direction of $\vec{E}$. potential V_1 at every point on the plane I is same. So, plane I is equipotential surface. |||ly, potential V_2 is same at every point on the plane II, So the plane II is also equipotential surface. |||ly V_3. Thus, equipotential surfaces for a uniform Electric field are planes ⊥r the electric field lines representing uniform electric field.

② Equipotential surfaces for an isolated point charge.



potential due to an isolated point charge +q at a dist r is given by

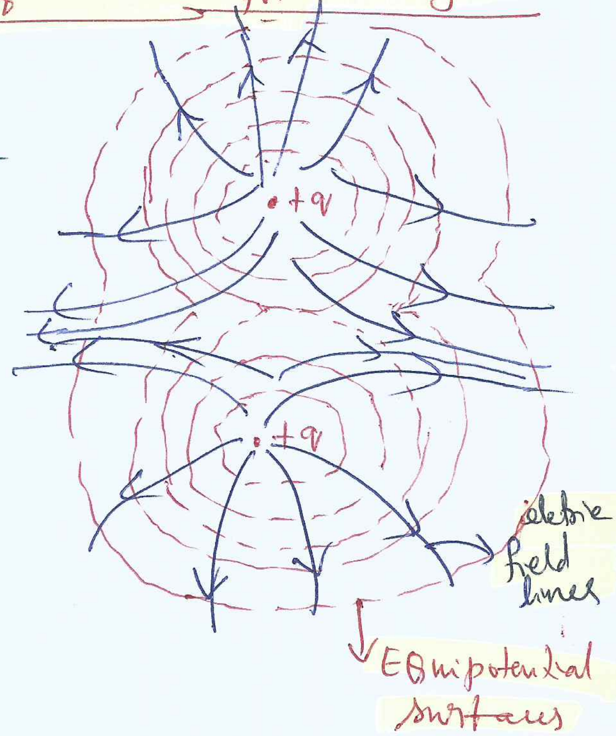
$$V = \frac{q}{4\pi\epsilon_0 r} \quad \therefore \quad V \propto \frac{1}{r}$$

If we draw a sphere of radius around +q charge, then all the point on this sphere will have the same potential (in volts). Thus, equipotential surfaces for an isolated point charge are concentric spherical surfaces around the isolated point charge.

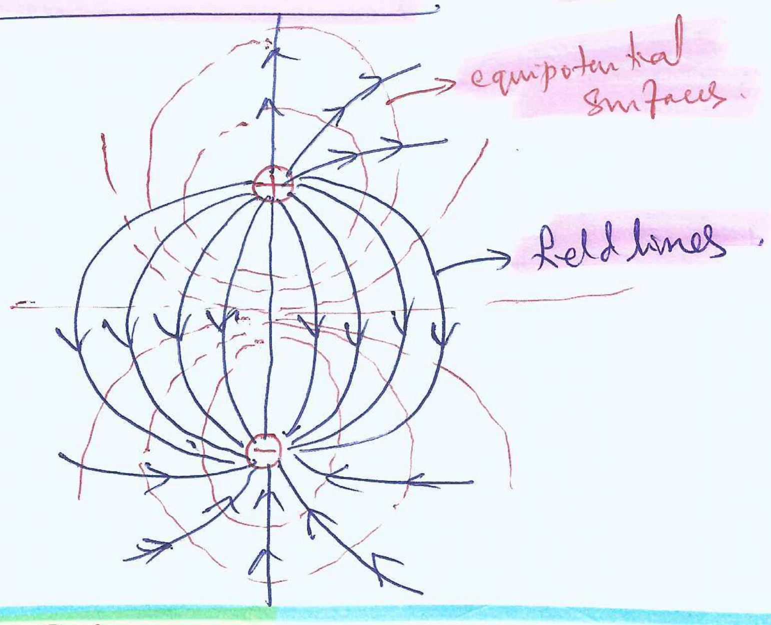
The potential decreases on surfaces as we move away from centre.

③ Equipotential surfaces for a pair of similar point charges.

The Shape of equipotential surface depends upon the algebraic sum of potentials at different point due to each charge. Figure shows the equipotential surfaces for two positive charges of same value.



(4) Equipotential Surfaces for an electric dipole.



properties of Equipotential Surfaces.

(1) No work is done in moving a test charge from one point to another point on an equipotential surface.

→ Consider 2 points A and B on an equipotential surface. potential difference betw points A and B is given by

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

Since every point on the equipotential surface has the same value of the potential i.e. $V_A = V_B$

$$\therefore \frac{W_{AB}}{q_0} = 0 \quad \text{or} \quad W_{AB} = 0$$

Thus, no work is done in moving a test charge from one point to another point on the equipotential surface.

(2) The electric field is $\perp^r$ to the equipotential surface

We know that $\frac{dW}{q_0} = \vec{E} \cdot \vec{dr}$

As no work is done in moving a test charge on the equipotential surface

i.e. $dW = 0 \quad \therefore \vec{E} \cdot \vec{dr} = 0$

$\therefore E \, dr \cos\theta = 0 \quad$ or $\cos\theta = 0 \quad$ or $\theta = 90^0$

Thus $\vec{E}$ is $\perp^r$ to $\vec{dr}$

(3) Two equipotential surfaces Cannot intersect.

If two equipotential surfaces intersect, then at the points of intersection, there will be two ~~values~~ directions ~~of the~~ of electric ~~potential~~ field due to a point charge. This is not possible. Hence, two equipotential surfaces can not intersect.

P.T.O →

④ Equipotential surfaces indicate regions of strong and weak electric fields.

Using $E = -\frac{dV}{dr}$ or $dr = -\frac{dV}{E}$

Since dV (i.e. potential difference) is constant on the equipotential surface,

so $dr \propto \frac{1}{E}$

→ If E is strong (i.e. large), dr will be small i.e. the separation of equipotential surfaces will be smaller. Thus, equipotential surfaces are closer (denser) in the region of strong electric field.

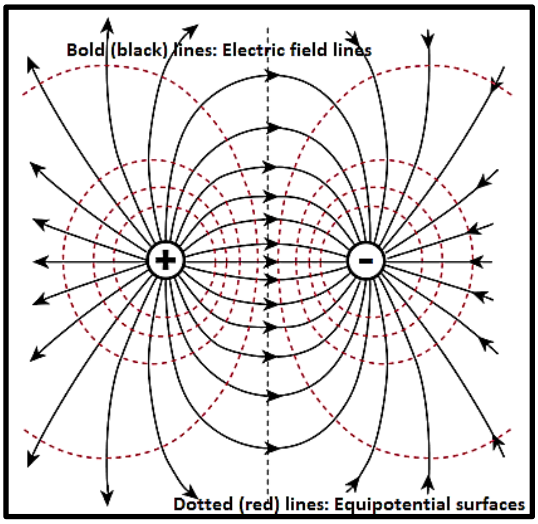
→ On the other hand, equipotential surfaces are farther apart in the region of weak electric field.

→ Thus, the spacing between the equipotential surfaces will be less where E is strong and vice-versa. Thus, equipotential surfaces can be used to give a general description of electric field in a certain region of space.

⇒ So, both field lines (electric field lines) and the equipotential surfaces can be used to depict electric field in ~~space~~ space. The advantage of using equipotential surfaces over the field lines is that they give a visual picture of both the magnitude and the direction of the electric field.

-- 26a --

Problem: Draw the equipotential surface for an "electric dipole"

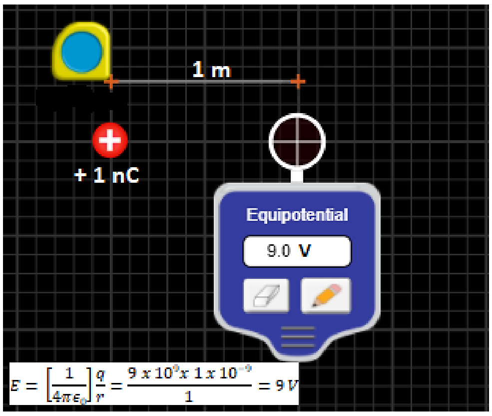


- In figure, the dotted (red) lines represent equipotential surfaces for an electric dipole.
- Note that when drawing equipotential surfaces, make sure that the lines are closer (denser) in the region of strong electric field and are farther apart in the region of weak electric field. In figure, the equipotential lines (surfaces) are closer inside the dipole (where E is stronger inside the dipole) and are farther apart outside the dipole (where E is weaker).
- Note that a dotted line surface will represent the same potential (or voltage).
  - Note that for the same potential surface (or for a given dotted loop), the potential line is closer to the charge q inside the dipole and away outside the dipole. See the explanation below.
- We know that $E = -\frac{dV}{dr}$ $or\ dr = -\frac{dV}{E}$; since dV is constant on the equipotential surface, so $dr \propto \frac{1}{E}$; If E is stronger (or large), dr will be small, that is separation between equipotential surfaces will be smaller.
  - Or $E = -\frac{V}{r}$ $or\ r = -\frac{V}{E};$ $so\ r \propto \frac{1}{E}$; this means for given potential surface, the distance between the charge and the potential surface is less inside the dipole (E is stronger) and more outside the dipole (E is weaker)
- Thus, the spacing between the equipotential surfaces will be less where E is strong and vice-versa. Thus, equipotential surfaces can be used to give a general description of electric field in a certain region of space.
- Therefore, both electric field lines and the equipotential surfaces can be used to depict electric field in space. The advantage of using equipotential surfaces over the electric field lines is that they give a visual picture of both magnitude and direction of the electric field.
- **Electric field lines are normal to the equipotential surfaces.**

****

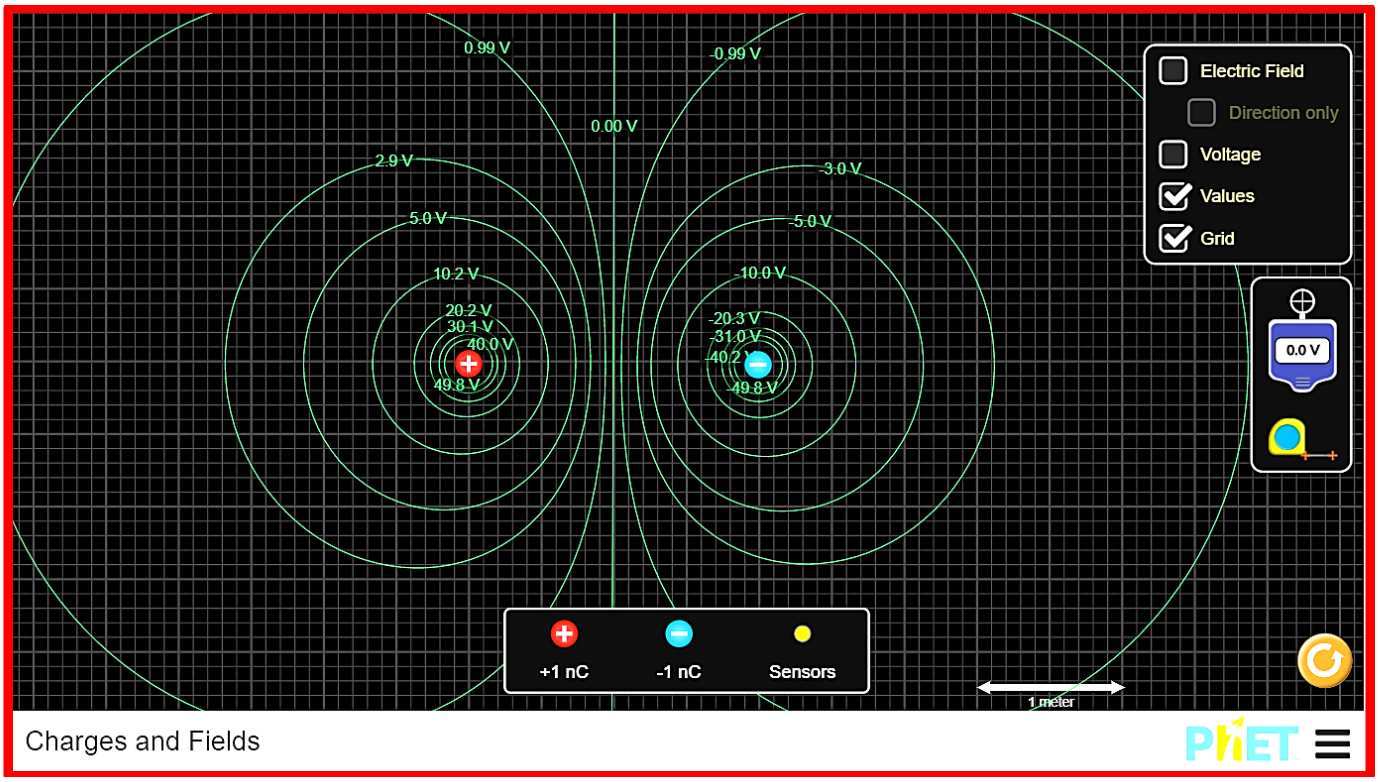
Right side is a simulated figure. It gives the potential at a distance of 1 m from charge of + 1nC. As per equation of electric potential, the simulated answer is verified.

$$E = \left[\frac{1}{4\pi\epsilon_0}\right]\frac{q}{r} = \frac{9\ x\ 10^9 x\ 1\ x\ 10^{-9}}{1} = 9\ V$$



Below is a simulation of equipotential surfaces for an electric dipole.

Approximately, the equipotential surfaces represent equal differences in potentials



Kinetic Energy of a charged particle accelerated through a potential Difference in an electric field : Electron volt.

→ when a particle of mass m and carrying a charge q is placed in a 'uniform' electric field, the field exerts a force F on the particle, where

$$F = qE$$

If the particle is left free, then it performs accelerated motion under the force F. The acceleration (= force/mass) of the particle is given by

$$a = \frac{F}{m} = \frac{qE}{m}$$

Since E is constant, the acceleration a is also constant. Hence, the motion of the particle is "uniformly accelerated".

→ Let the particle start from rest. Suppose its velocity becomes v after travelling a distance x in the field. According to the formula $v^2 = 2ax$, we have

$$v^2 = 2\left(\frac{qE}{m}\right)x \longrightarrow (1)$$

→ If V is the p.d. betn the starting point and the point situated at a distance x, then

$$E = \frac{V}{x} \longrightarrow (2)$$

Substituting this value of E in eqn (1), we get

$$v^2 = \frac{2q\cancel{x}}{m}\cdot\left(\frac{V}{\cancel{x}}\right) = \frac{2qV}{m}$$

$$\therefore \boxed{v = \sqrt{\frac{2qV}{m}} \longrightarrow (3)}$$

From this formula, we can determine the velocity of the particle acquired in going from one point to another having a potential difference V.

The K.E. acquired by the particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{\cancel{2}}\cancel{m}\cdot\frac{\cancel{2}qV}{\cancel{m}} = qV$$

$$\boxed{K = qV} \longrightarrow (4)$$

If q is in coulomb, V in volt, then K will be in joule.

→ If an electron is accelerated thro' a p.d. of 1 volt, it would acquire KE given by

$$K = eV = (1.6\times10^{-19}C)(1V) = 1.6\times10^{-19}\,J$$

$$\therefore 1\,eV = 1.6\times10^{-19}\,J$$

∴ The KE of 1.6×10^{-19} Joule is known as 1 electron-volt (1 eV)

Electric potential Energy (Sec 2.7 of NCERT)

When a test charge is moved in an electric field from one point to other against the force exerted on it by the field, the work done by some external agent moving the charge is acquired by the charge as potential energy. This is known as "electric potential energy" of the charge. Because, electric field is conservative, the PE depends only on the locations of the two points in the field, and is independent of the path followed by the charge. Thus, like "electric potential", the "electric potential energy" is defined only in a conservative field.

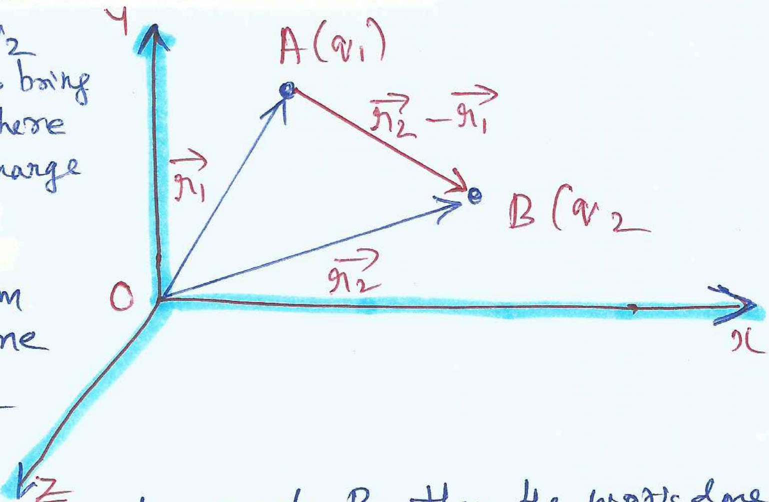
Electric PE of a system of point charges:

Here we consider 3 cases. Derive expressions for PE of a system of

1. two point charges
2. three —— " ——
3. n point charges

Electrical PE of a system of point charges is defined as the total work done in bringing these charges to their respective locations from infinity to form a system of charges.

PE of a system of two point charges:



- Consider 2 point charges q_1 and q_2 initially lying at ∞. Work done to bring a charge q from ∞ to a point where electric potential due to any source charge is $V(r)$ is given by $\boxed{W = qV(r)}$
- When charge q_1 is brought from ∞ to a position A, no work is done as $V(r) = 0$ at A in the absence of any source charge.
- When charge q_2 is brought from ∞ to point B, then the work done is given by $W = q_2 \times V_1 \longrightarrow (1)$

where $V_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1}{|\vec{r_2}-\vec{r_1}|}$ is the electric potential at position B due to charge q_1

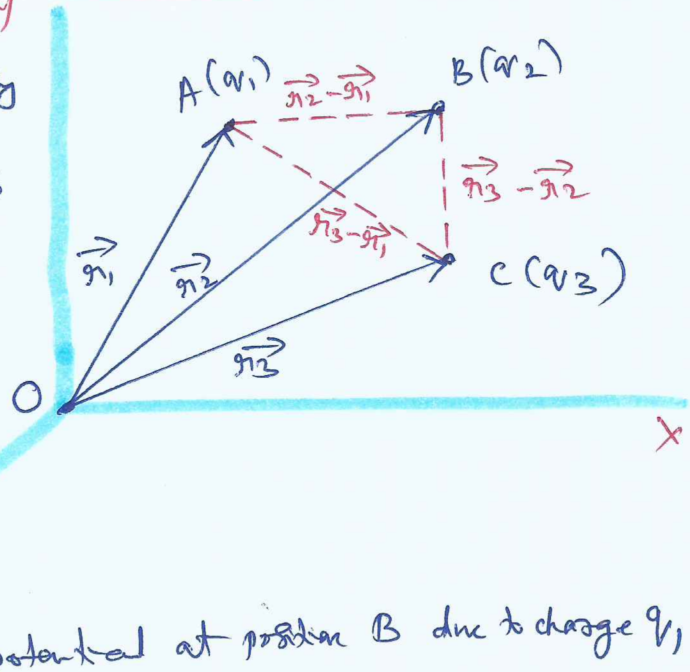
$$\therefore W = q_2 \times \frac{1}{4\pi\epsilon_0}\frac{q_1}{|\vec{r_2}-\vec{r_1}|} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r_2}-\vec{r_1}|}$$

Since $|\vec{r_2}-\vec{r_1}| = r_{12}$, distance betn charges q_1 and q_2 at their respective positions A and B

$$\therefore W = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}$$

This work done (W) is stored as PE (U) of the system of two charges $\boxed{U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r_{12}}}$ → which is the expression for the PE of a system of 2 point charges.

※ PE of a system of 3 point charges:



- Consider a system of 3 point charges q_1, q_2 and q_3, such that they are initially at infinity.
- Let $\vec{r_1}$, $\vec{r_2}$ & $\vec{r_3}$ are position vectors of charges q_1, q_2 & q_3 respectively.

Step 1: No work is done to bring q_1 from ∞ to A, when q_2 & q_3 are at ∞.

Step 2: Work done to bring q_2 from ∞ to B, when q_1 is at A is given by

$$W_{12} = q_2 \times V_1$$

where $V_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{|\vec{r_2}-\vec{r_1}|}$ is the electric potential at position B due to charge q_1

$$\therefore W_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r_2}-\vec{r_1}|} \longrightarrow (1)$$

Step 3: Work done in bringing q_3 (in the presence of charges q_1 and q_2) from ∞ to point C

$$W_{123} = V_1 \times q_3 + V_2 \times q_3 \longrightarrow (2)$$

where V_1 = Electric potential at C due to charge $q_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{|\vec{r_3}-\vec{r_1}|}$

V_2 = —— " —— $V_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{|\vec{r_3}-\vec{r_2}|}$

Hence eqn (2) becomes

$$W_{123} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_3}{|\vec{r_3}-\vec{r_1}|} + \frac{q_2 q_3}{|\vec{r_3}-\vec{r_2}|}\right] \longrightarrow (3)$$

Step 4: The total work done to bring all 3 charges from ∞ to their respective position is given by

$$W = W_{12} + W_{123}$$

$$W = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{|\vec{r_2}-\vec{r_1}|} + \frac{q_1 q_3}{|\vec{r_3}-\vec{r_1}|} + \frac{q_2 q_3}{|\vec{r_3}-\vec{r_2}|}\right]$$

But work done, W = PE (U) of the system of 3 charges

$$\Rightarrow U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{|\vec{r_2}-\vec{r_1}|} + \frac{q_1 q_3}{|\vec{r_3}-\vec{r_1}|} + \frac{q_2 q_3}{|\vec{r_3}-\vec{r_2}|}\right] \longrightarrow (4)$$

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

P.T.O →

$$\therefore \quad U = \frac{1}{2}\left[\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{q_i q_j}{|\vec{r}_j - \vec{r}_i|}\right] \quad i \neq j \longrightarrow (5)$$

The factor $\frac{1}{2}$ is introduced ~~for~~ in eqn (5) because each term gets counted twice in the above manner of writing the expression. For ex: when $i=1$, $j=2$ and ~~f~~ $i=2$, $j=1$, Contribution is got from same pair of charges q_1 and q_2. As such factor $1/2$ is required to include only one term in each pair.

Rewriting (5), we get $U = \frac{1}{2}\sum_{i=1}^{3} q_i \left[\sum_{j=1}^{3}\frac{1}{4\pi\epsilon_0}\frac{q_j}{|\vec{r}_j - \vec{r}_i|}\right] \quad i \neq j$

$$\Rightarrow \boxed{U = \frac{1}{2}\sum_{i=1}^{3} q_i V_i} \longrightarrow (6)$$

where, V_i is the potential at $\vec{r}_i$ due to all other charges.

(3) PE of a system of n charges:

Consider a system of n charges $q_1, q_2 \ldots q_n$ placed at position $\vec{r}_1, \vec{r}_2 \ldots \vec{r}_n$ respectively. by as per eqn (5) PE of the system of n charges is given

$$U = \frac{1}{2}\left[\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q_i q_j}{|\vec{r}_j - \vec{r}_i|}\right] \quad i \neq j \longrightarrow (7)$$

$$U = \frac{1}{2}\sum_{i=1}^{n} q_i V_i$$

Note (Relation betn electric potential (V) and electrical PE (U) is given by $U = qV$)