Chapter 7

Alternating Current

Note :→ In this chapter page numbers ~~start from~~ 33 ~~(this page)~~ e ~~to 74~~ are

33 to 74e 33 to ~~74~~ 74e
75a

Alternating current Chapter (AC chap.)

page 33 to 74

- We know that AC is generated by rotating a coil in a magnetic field (fig 1) OR by rotating a magnetic field within a stationary coil (fig 2)

fig 1

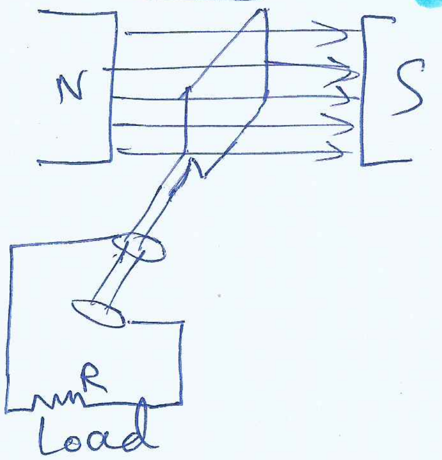
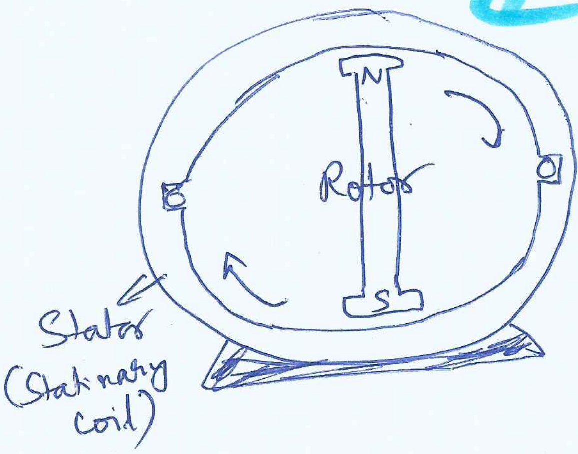


fig 2



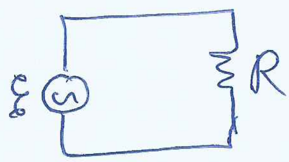
It is seen that the induced emf varies as sine function of the time angle ωt.
Consider a source that produces sinusoidally varying potential difference across its terminals. Let this pd also called as ac voltage be given by

$$\boxed{v = v_m \sin \omega t} \longrightarrow (1)$$

Where v_m is the peak value or the amplitude of the AC voltage and ω is its angular frequency.

$$\boxed{\text{Imp:} \quad \omega = 2\pi f = \frac{2\pi}{T}}$$

To find the ~~ac~~ AC current, consider

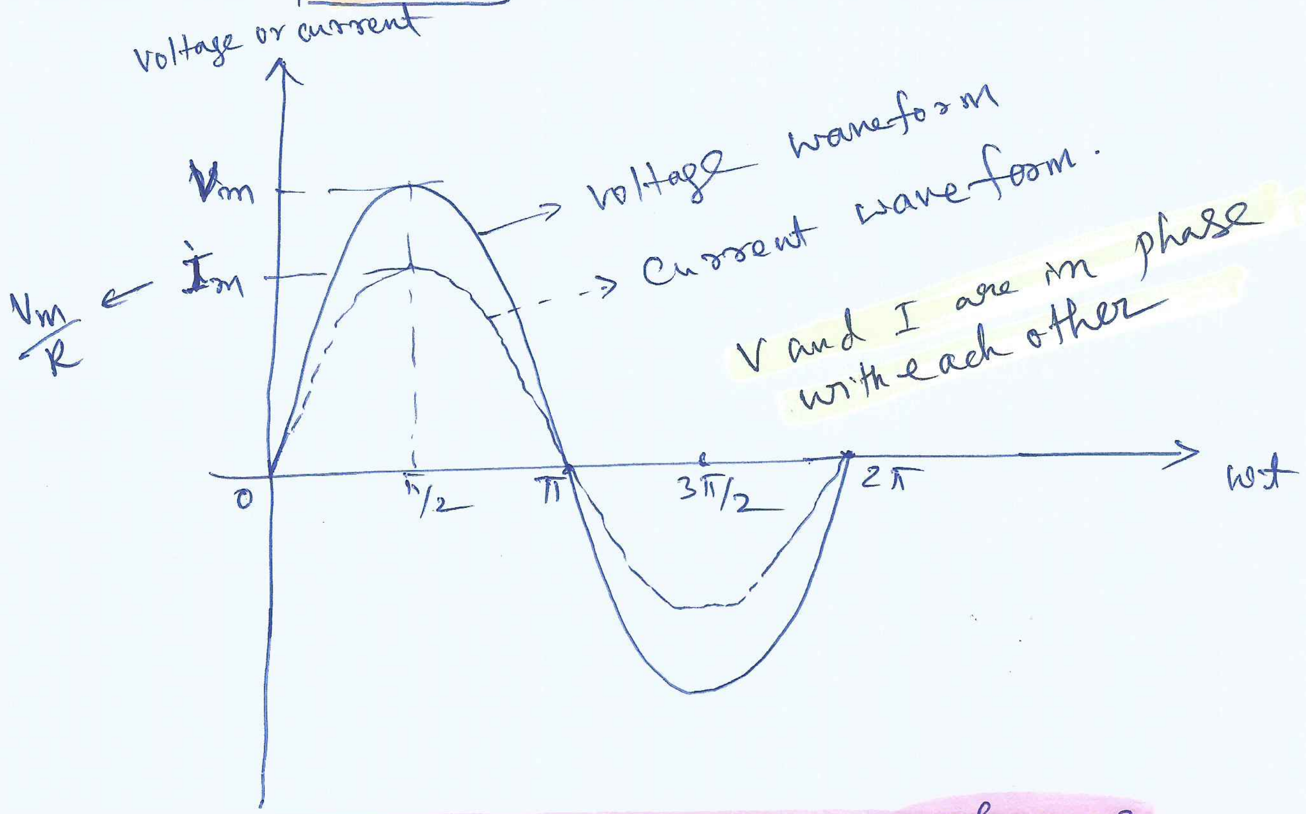


Looprule $\quad v_m \sin \omega t = iR$

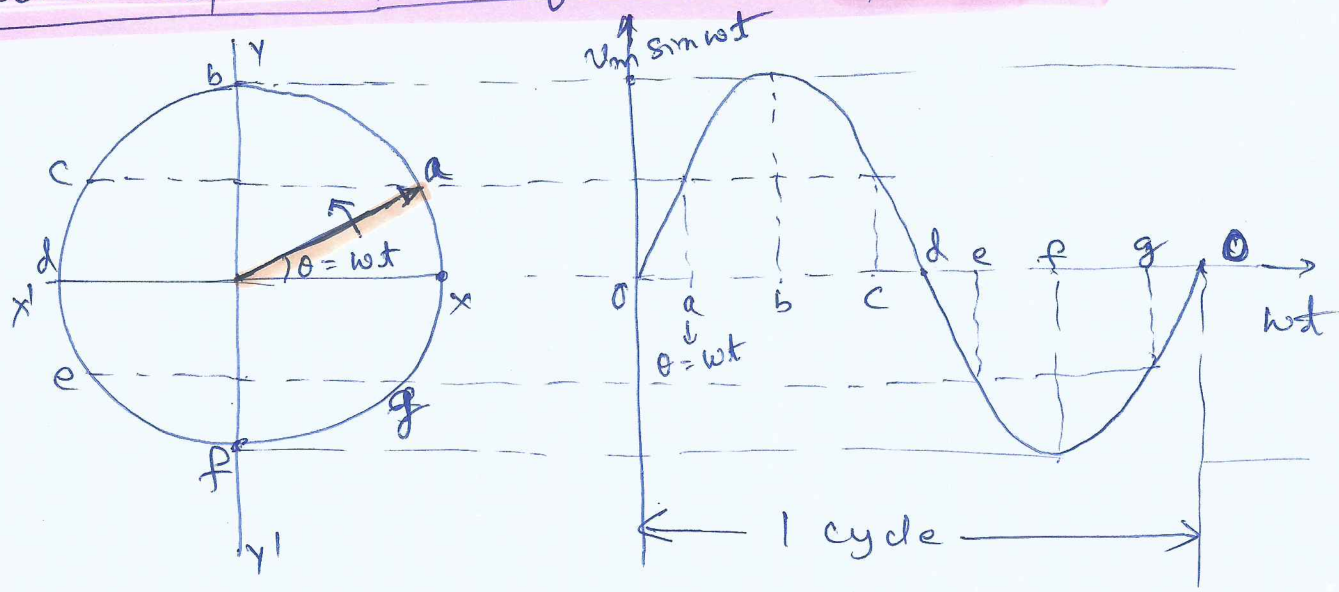
$$\boxed{i = \left(\frac{v_m}{R}\right) \sin \omega t} \longrightarrow (2)$$

$$\boxed{i = i_m \sin \omega t} \quad \text{where} \quad \boxed{i_m = \frac{v_m}{R}} \longrightarrow (3)$$

P.T.O.



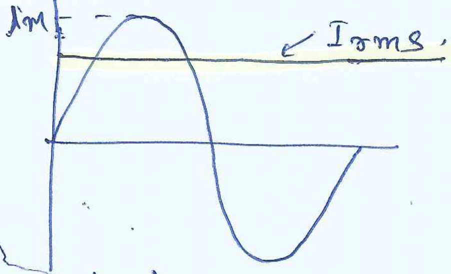
Vector Representation of AC → Phasors.



- A phasor is a vector which rotates about the origin with angular speed ω as shown above.
- To represent phase difference between AC voltage and AC current in a convenient way, we use notion of phasors. The analysis of AC circuit will be easier by the use of phasor diagram.
- From figure, the projection of voltage (or current) phasor on Y-axis represents the value of voltage (or current) at that instant.

RMS value of an AC source

- **Why RMS value:** ~~for~~ DC power $P = I^2R$, to express AC also in the same form as DC power, a special value of current (or voltage) is defined, which is called "root mean square" current (rms current or rms voltage)
- **Relation betn I (or V) with rms value is given by**



$V_{rms} = \cancel{0.707}\ v_m/\sqrt{2} = 0.707\ i_m$

$I_{rms} = i_m/\sqrt{2} = 0.707\ v_m$

- **To prove above relation.** IMP

Consider AC current (AC voltage is similar)

$i = I_m \sin \omega t = I_m \sin \theta$ → is the instantaneous value of current

The mean of the squares of the instantaneous values of current over one complete cycle is

$$= \frac{1}{2\pi}\int_0^{2\pi} i^2\, d\theta$$

We know that

$$\left[\sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

Root of the above value is I_{rms}

$$I_{rms} = \frac{1}{\sqrt{2\pi}}\sqrt{\int_0^{2\pi} I_m^2 \sin^2\theta\, d\theta}$$

$$= \frac{I_m}{\sqrt{2}\cdot\sqrt{\pi}}\sqrt{\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \frac{I_m}{2\sqrt{\pi}}\sqrt{\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \frac{I_m}{2\sqrt{\pi}}\sqrt{2\pi - 0 - 0 + 0} = \frac{I_m}{2\sqrt{\pi}} \times \sqrt{2\pi} = \frac{I_m \cancel{\sqrt{2}}\cdot\cancel{\sqrt{\pi}}}{\sqrt{2}\cancel{\sqrt{2}}\ \cancel{\sqrt{\pi}}}$$

$$\boxed{I_{rms} = I_m/\sqrt{2} = 0.707\ I_m}$$

$$|||^{ly}\quad \boxed{V_{rms} = V_m/\sqrt{2} = 0.707\ V_m}$$

∴ peak value $= I_m = \sqrt{2}\ I_{rms}$

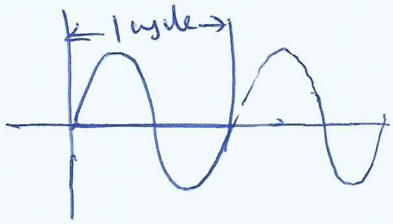
P–P value $= 2I_m = 2\sqrt{2}\ I_{rms}$

Defn: The rms value of AC is given by that steady current (dc) which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

Average value of AC :

- For one complete cycle, the average value of AC = 0

$i = I_m \sin\theta$



$I_{average}$ over one complete cycle

$$I_{av} = \int_0^{2\pi} \frac{i\,d\theta}{2\pi}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} I_m \sin\theta\, d\theta = \frac{I_m}{2\pi}\Big[-\cos\theta\Big]_0^{2\pi}$$

$$= \frac{I_m}{2\pi}\Big[\cos\theta\Big]_{2\pi}^{0} = \frac{I_m}{2\pi}\Big[1-1\Big]$$

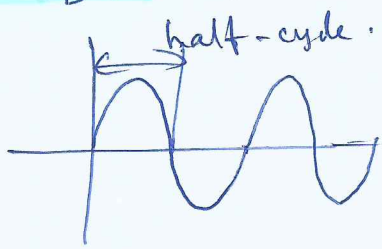
$$= 0$$

$\because \cos 2\pi = 1$
$\cos 0 = 1$
$\cos\pi = -1$

$\therefore$ $\boxed{I_{av} \text{ over one complete cycle of AC} = 0}$

- However, the Average Value over Half a cycle of AC is desired as

$i = I_m \sin\theta$



$$I_{av} \text{ over half a cycle} = \int_0^{\pi} \frac{i\,d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}\Big[-\cos\theta\Big]_0^{\pi} = \frac{I_m}{\pi}\Big[\cos\theta\Big]_{\pi}^{0}$$

$$= \frac{I_m}{\pi}\Big[1-(-1)\Big]$$

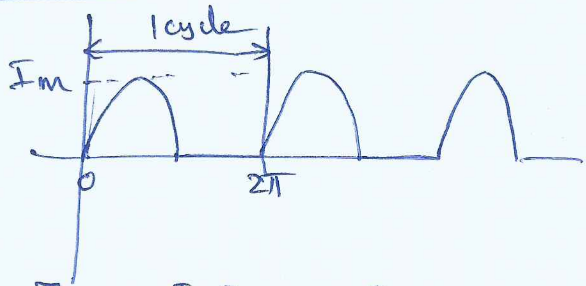
$$\boxed{I_{av} \text{ over half a cycle} = \frac{2I_m}{\pi} = 0.637\, I_m} \quad \left(\pi = \frac{22}{7}\right)$$

$\Rightarrow$ 63.7 % of peak value.

$\int \sin x\, dx = -\cos x$
$\int \cos x\, dx = \sin x$
$\int e^x dx = e^x$

Average value of Half-wave Rectified AC

$i = I_m \sin\theta$

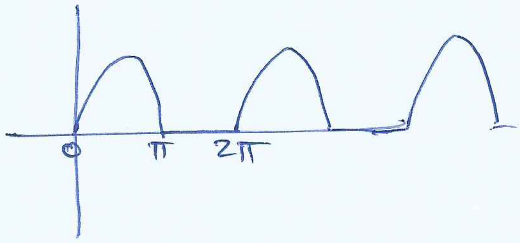


$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{2\pi}\left[\cos\theta\right]_{\pi}^{0} = \frac{I_m}{2\pi}\left[1-(-1)\right] = \frac{I_m 2}{2\pi} = \frac{I_m}{\pi}$$

$$\boxed{I_{av} = \frac{I_m}{\pi}} = 0.318\, I_m \quad (31.8\% \text{ of peak value})$$

RMS value of Half-wave Rectified AC



$$I_{RMS} = \sqrt{\int_0^{\pi} \frac{i^2 \, d\theta}{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi} \frac{(1-\cos 2\theta)}{2} \, d\theta} \qquad \left(\because \sin^2\theta = \frac{(1-\cos 2\theta)}{2}\right)$$

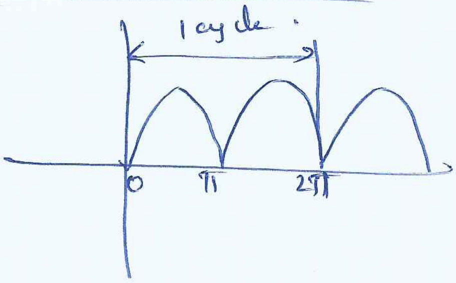
$$= \sqrt{\frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}} \qquad \begin{matrix}\sin 2\pi = 0 \\ \sin 0 = 0\end{matrix}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[(\pi - 0) - (0 - 0)\right]}$$

$$= \sqrt{\frac{I_m^2 \cancel{\pi}}{4\cancel{\pi}}} = \frac{I_m}{2}$$

$$\boxed{I_{av} = \text{RMS value of Half-wave rectified AC current} = \frac{I_m}{2}} \; \boxed{= 0.5\, I_m}$$

Average value of Full-wave rectified current



$i = I_m \sin\theta$

$$i_{av} = \frac{2 \times \int_0^{\pi} I_m \sin\theta \, d\theta}{2\pi}$$

$$= \frac{I_m}{\pi} \int_0^{\pi} \sin\theta \, d\theta = \frac{I_m}{\pi} \left(\cos\theta\right)_{\pi}^{0}$$

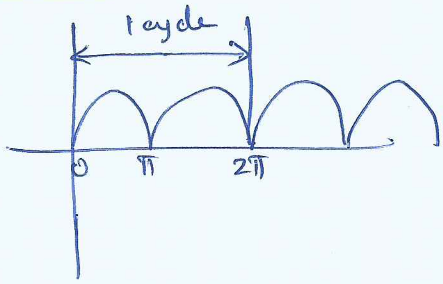
$$= \frac{I_m}{\pi}\left[1 - (-1)\right] = \frac{2 I_m}{\pi}$$

i_{av} of full-wave rectified AC current $= \frac{2}{\pi} I_m = 0.637\, I_m$

(63.7 % of peak current)

~~Avera~~ RMS value of Full-wave Rectified current

$\boxed{i = I_m \sin\theta}$



$$I_{RMS} = \sqrt{2 \times \int_0^{\pi} \frac{i^2 \, d\theta}{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{\pi} \int_0^{\pi} \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{I_m^2}{\pi} \int_0^{\pi} \frac{(1 - \cos 2\theta)}{2} \, d\theta}$$

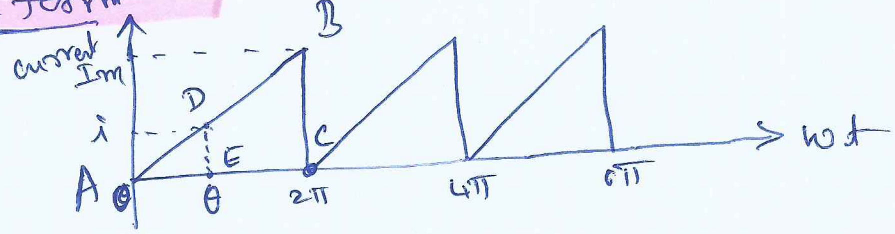
$$= \sqrt{\frac{I_m^2}{2\pi} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[(\pi - 0) - 0(0 - 0)\right]}$$

$$= \sqrt{\frac{I_m^2 \cancel{\pi}}{2\cancel{\pi}}} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{I_{RMS} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m} = \boxed{\text{Same as normal Sine wave current.}}$$

Consider wave form



① Average value of current ?

By symmetry, we can conclude it is $\frac{I_m}{2}$

proof: from figure

$$\frac{DE}{AE} = \frac{BC}{AC} = \frac{I_m}{2\pi}$$

Consider current i at an angle θ, then

$$\frac{DE}{AE} = \frac{i}{\theta} = \frac{I_m}{2\pi} \quad \therefore \quad \boxed{i = \frac{I_m \theta}{2\pi}}$$ This is equation of current for one cycle.

$$I_{av} = \frac{I_m}{2\pi}\int_0^{2\pi} \frac{\theta}{2\pi}\, d\theta = \frac{I_m}{4\pi^2}\left[\frac{\theta^2}{2}\right]_0^{2\pi} = \cancel{\frac{4\pi^2}{4\pi^2}}\,\frac{I_m}{2}$$

$$\boxed{I_{av} = I_m/2}$$

② RMS value of above waveform :

Current equation of the waveform :

It is a St. line with slope m $\therefore$ $y = mx + c$ (m → slope, c → 0, x → wt, y → i)

$$i = \frac{I_m}{2\pi}(wt) \qquad \text{Let } wt = \theta \text{ radians}$$

$$i = \frac{I_m}{2\pi}\theta$$

$$i^2 = \frac{I_m^2}{4\pi^2}\theta^2$$

$$\text{Mean Square value} = \frac{1}{2\pi}\int_0^{2\pi} i^2\, d\theta = \frac{1}{2\pi}\int_0^{2\pi}\frac{I_m^2}{4\pi^2}\theta^2\, d\theta$$

$$= \frac{I_m^2}{8\pi^3}\left[\frac{\theta^3}{3}\right]_0^{2\pi} = \frac{I_m^2 \times \cancel{8\pi^3}}{\cancel{8\pi^3} \times 3} = \frac{I_m^2}{3}$$

$$\therefore \text{RMS} = \sqrt{\frac{I_m^2}{3}} = \boxed{\frac{I_m}{\sqrt{3}}}$$

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IMP

① The electric mains in a house in India is marked 220 V - 50 Hz. Write the eqn for instantaneous voltage.

Eq. for Instantaneous voltage $v = V_m \sin(\omega t)$

$= V_m \sin(2\pi f t)$

Given $V_{rms} = 220V$ $\therefore V_m = 220\sqrt{2} = 311\ V$

$f = 50\ Hz$ $\therefore 2\pi f = 100\pi = 314$

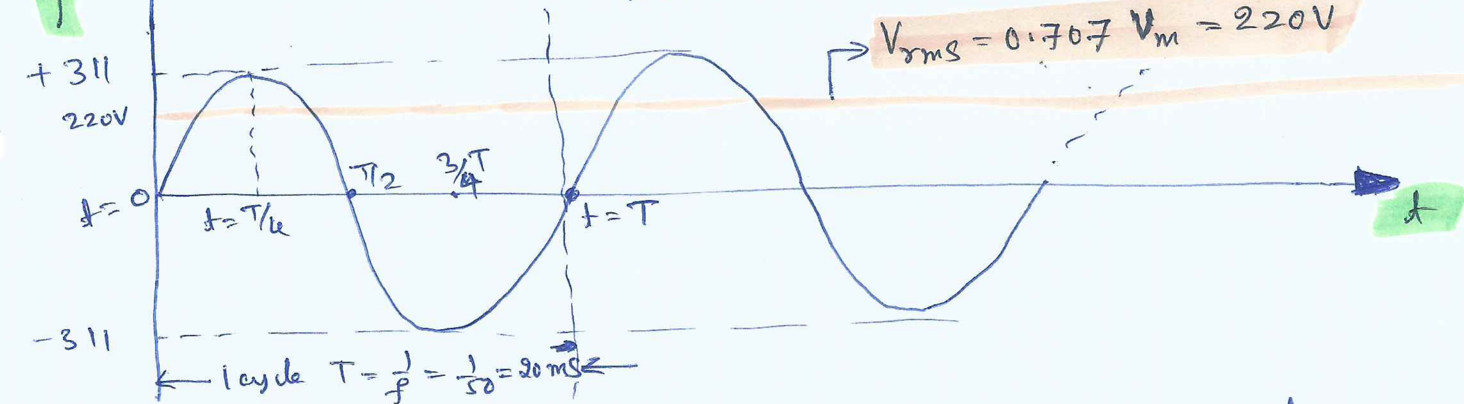
$\therefore \boxed{v = 311 \sin(314 t)}$

For US

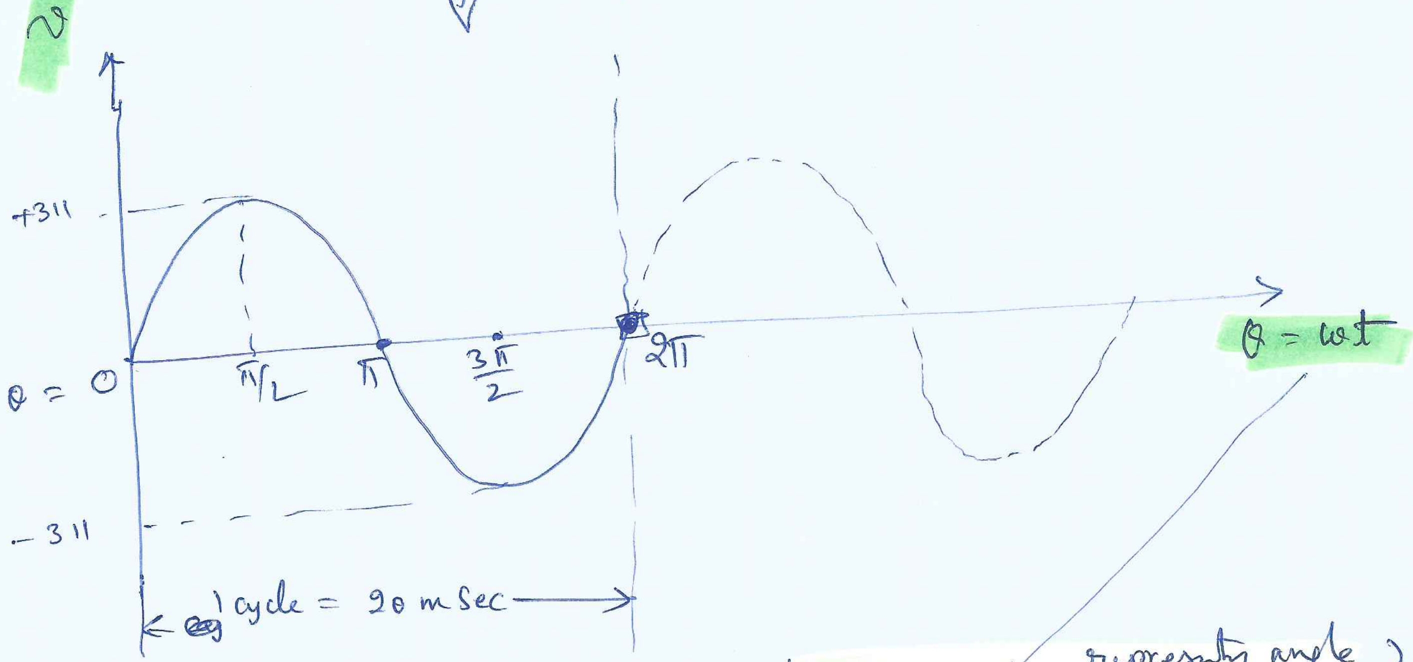
$v = 110\sqrt{2} \sin(2\pi \times 60 t)$

$v = 156 \sin(120\pi t)$

↓ Graphical representation wrt time



↓ Graphical Representation wrt ~~degrees~~ angle.



how (ωt) is represents angle ?

$\omega t = 2\pi f t$ → in Sec

$= \frac{2\pi}{T} t$ → in S

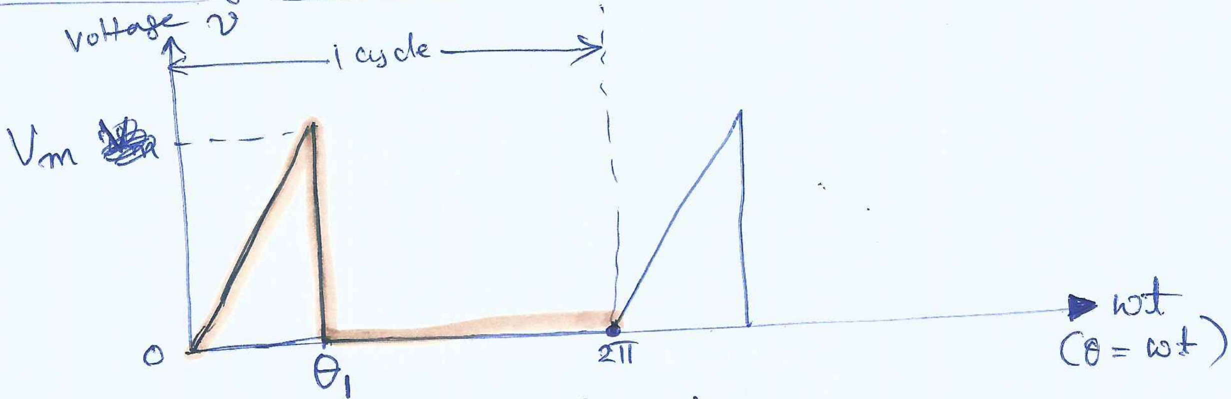
$\omega t = 2\pi$ rad.

↳ represents angle

 → See up to page 45

RMS value of Δ^{lar} waveform

- What is the RMS value of a periodic signal?
  → When a periodic signal is generated by a source connected to a load (eg. Resistor), the RMS value is the continous ~~signal~~ DC value which would deliver the same power to the load as the periodic signal.



Equation of the above waveform:

$$v = \frac{V_m}{\theta_1}(\omega t) \quad \text{or} \quad \boxed{v = \frac{V_m}{\theta_1}\theta} \rightarrow (1) \quad 0 \le \theta \le \theta_1$$

(1) is the linear function of signal and is repeated every 2π angle.

- Square of (1) $\quad v^2 = \frac{V_m^2}{\theta_1^2}\theta^2$
- Mean $\rightarrow \frac{1}{2\pi}\int_0^{\theta_1} v^2 d\theta = \frac{1}{2\pi}\int_0^{\theta_1}\frac{V_m^2}{\theta_1^2}\theta^2 d\theta$
- Square root of above $\quad v_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\theta_1}\frac{V_m^2}{\theta_1^2}\theta^2 d\theta}$

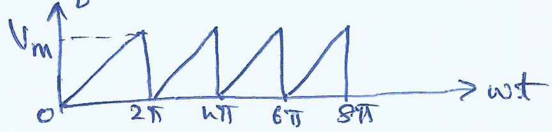
$$= \sqrt{\frac{V_m^2}{2\pi\theta_1^2}\left[\frac{\theta^3}{3}\right]_0^{\theta_1}} = \sqrt{\frac{V_m^2}{2\pi\cancel{\theta_1^2}}\frac{\theta_1^{\cancel{3}}}{3}}$$

$$= \frac{V_m}{\sqrt{2}\sqrt{\pi}} \cdot \frac{\sqrt{\theta_1}}{\sqrt{3}}$$

$$\boxed{v_{rms} = \frac{V_m}{\sqrt{3}}\sqrt{\frac{\theta_1}{2\pi}}} \rightarrow (2)$$

Here $\frac{\theta_1}{2\pi}$ is the duty-cycle

In (2) if duty-cycle is 100% $\Rightarrow$
$\theta_1 = 2\pi$

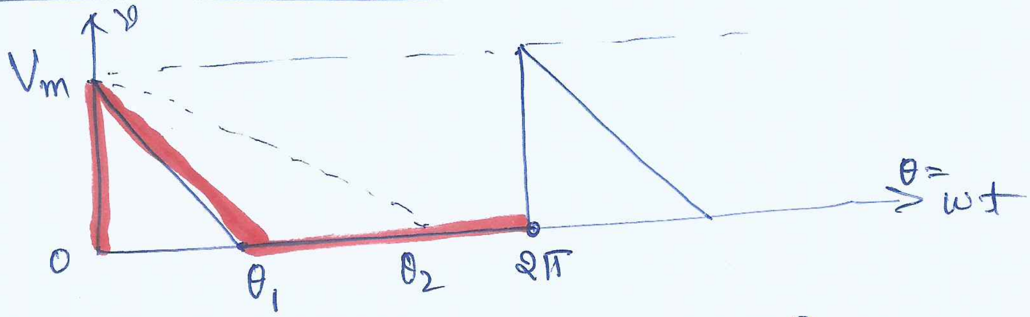


(2) becomes $\boxed{v_{rms} = \frac{V_m}{\sqrt{3}}} \rightarrow (3)$

P.T.O.

Continuation of (41)

Consider following waveform



θ is a running value on x-axis.

Equation for waveform $\left(\text{Slope @ } \theta_1 = -\frac{V_m}{\theta_1} \right)$

$$v = \left(\frac{-V_m}{\theta_1} \right) (\theta - \theta_1)$$

$$\boxed{v = \frac{V_m}{\theta_1} (\theta_1 - \theta)} \qquad 0 \le \theta < \theta_1$$

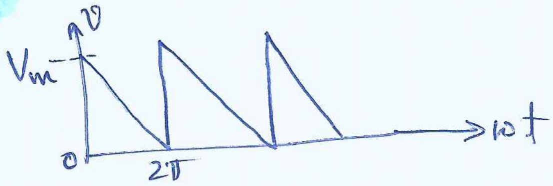
$$v^2 = \frac{1}{2\pi} \int_0^{\theta_1} \frac{V_m^2}{\theta_1^2} (\theta_1 - \theta)^2 \, d\theta \qquad \boxed{\text{Let } \theta_1 - \theta = x}$$

$$= \frac{V_m^2}{2\pi\theta_1^2} \int_0^{\theta_1} x^2 dx = \frac{V_m^2}{2\pi\theta_1^2} \left[\frac{x^3}{3} \right]_0^{\theta_1} = \frac{V_m^2}{2\pi \cancel{\theta_1^2}} \frac{\cancel{\theta_1^3}}{3}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi \times 3} \theta_1}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{3}} \cdot \sqrt{\frac{\theta_1}{2\pi}}} \rightarrow (4)$$

$\frac{\theta_1}{2\pi}$ is duty cycle



For 100% duty cycle

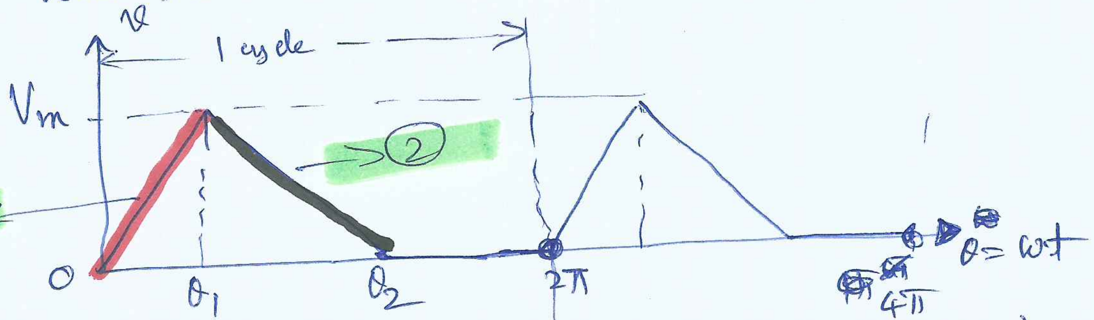
$\theta_1 = 2\pi$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{3}}}$$

PTO.

Continuation of 42

Consider waveform



Two parts in above waveform (as shown by red and black)

Eqn for ① $\quad v_1 = \frac{V_m}{\theta_1}\theta \longrightarrow 0 \le \theta < \theta_1$

eqn for ② $\quad v_2 = \cancel{V_m\left(\frac{t_2 - t}{t}\right)}$

$$= V_m\left(\frac{\theta_2 - \theta}{\theta_2 - \theta_1}\right) \qquad \theta_1 \le \theta < \theta_2$$

From eqn ② in page 41, we get

$$v_{1(rms)}^2 = \frac{V_m^2\,\theta_1}{6\pi} \longrightarrow (5)$$

From eqn ④ in page 42

$$v_{2(rms)}^2 = \frac{V_m^2(\theta_2 - \theta_1)}{6\pi} \longrightarrow (6)$$

Adding ① & ②

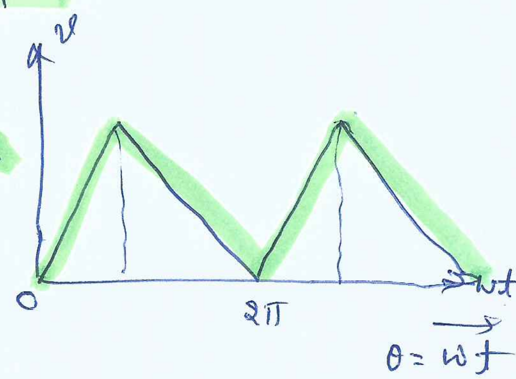
$$v_{1(rms)}^2 + v_{2(rms)}^2 = \frac{V_m^2\,\theta_1}{6\pi} + \frac{V_m^2(\theta_2 - \theta_1)}{6\pi}$$

$$v_{rms} = \sqrt{v_{1(rms)}^2 + v_{2(rms)}^2} = \sqrt{\cancel{\frac{V_m^2\theta_1}{6\pi}} + \frac{V_m^2}{6\pi}\theta_2 - \cancel{\frac{V_m^2\theta_1}{6\pi}}}$$

$$\boxed{v_{rms} = \frac{V_m\sqrt{\theta_2}}{\sqrt{6\pi}} = \frac{V_m\sqrt{\theta_2}}{\sqrt{3}\sqrt{2\pi}} = \frac{V_m}{\sqrt{3}}\sqrt{\frac{\theta_2}{2\pi}}} \rightarrow (7)$$

If Duty cycle = 100%, $\theta_2 = 2\pi$

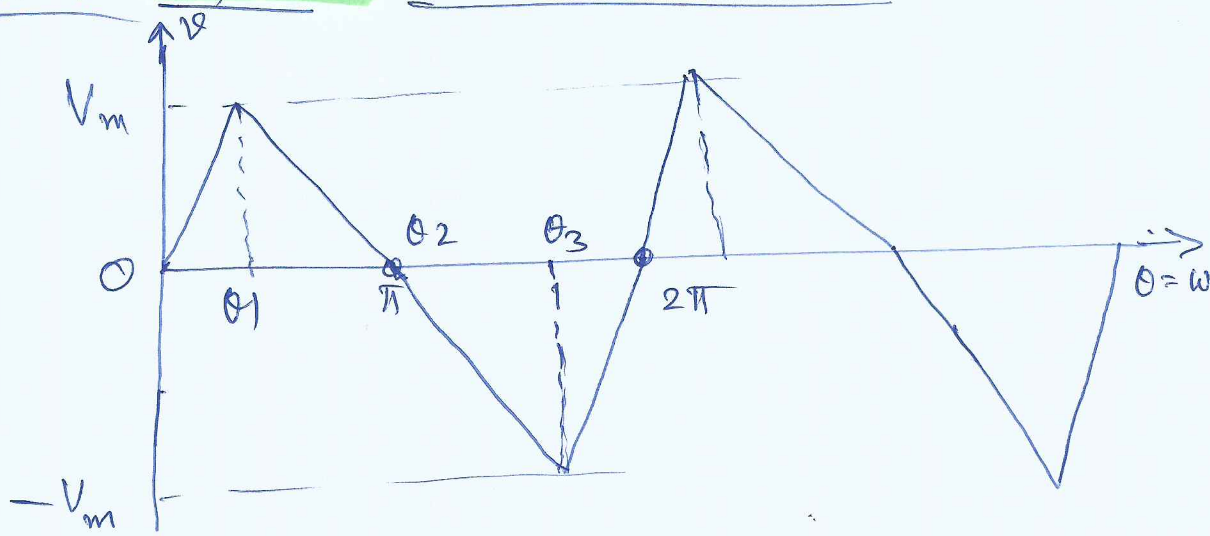
$$\boxed{v_{rms} = \frac{V_m}{\sqrt{3}}}$$



P.T.O

(continuation of 43)

Consider bipolar Δ^le as shown below



- From (7) page (43), RMS value from 0 to θ_2 is known.

$$V_{rms\,(0\ to\ \theta_2)} = \frac{V_m}{\sqrt{3}}\sqrt{\frac{\theta_2}{2\pi}} \rightarrow (7)$$

- RMS value from θ_2 to 2π is same as 0 to θ_2, with the difference that we need to replace θ_2 with $(2\pi - \theta_2)$

$$V_{rms\,(\theta_2\ to\ 2\pi)} = \frac{V_m}{\sqrt{3}}\sqrt{\frac{2\pi - \theta_2}{2\pi}} \rightarrow (8)$$

- The reason is it does not matter whether the signal is positive or negative, the power delivered to the load is the same.

$$\therefore\ V_{rms\_triangle\_bipolar} = \sqrt{V^2_{rms(0\ to\ \theta_2)} + V^2_{rms\,(\theta_2\ to\ 2\pi)}}$$

From (7) and (8)

$$V_{rms\_triangle\_bipolar} = \sqrt{\frac{V_m^2}{3}\frac{\theta_2}{2\pi} + \frac{V_m^2}{3}\frac{(2\pi - \theta_2)}{2\pi}}$$

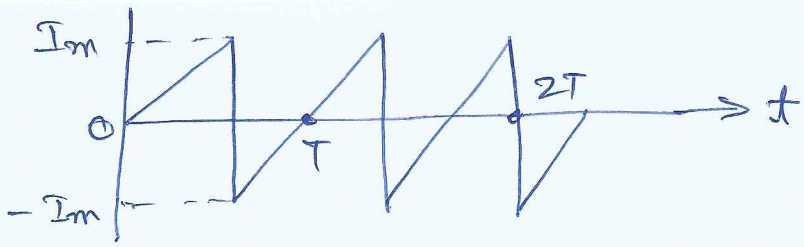
$$= \sqrt{\cancel{\frac{V_m^2}{3}\cdot\frac{\theta_2}{2\pi}} + \frac{V_m^2 \times \cancel{2\pi}}{3 \times \cancel{2\pi}} - \cancel{\frac{V_m^2}{3}\cdot\frac{\theta_2}{2\pi}}}$$

$$\boxed{V_{rms\_t\_b} = \frac{V_m}{\sqrt{3}}} \rightarrow (9)$$

P.T.O.

Continuation of (44)

Consider waveform



(1) Average value = Average current = 0 since half cycle is +ve and next half cycle current is negative

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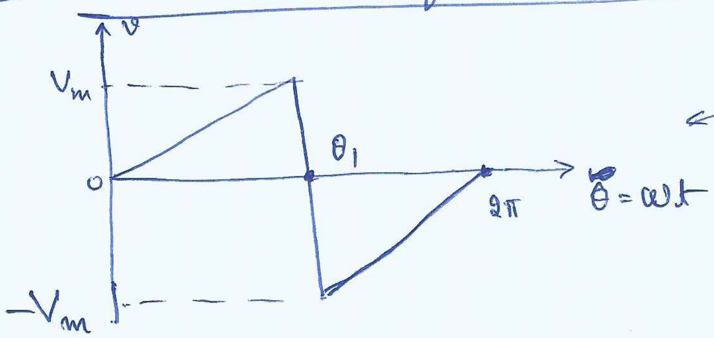
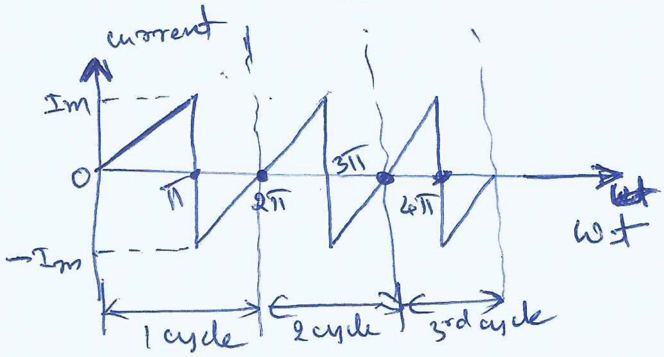
(2) RMS value of current





This is ~~an~~ same as in page (44)

$$v_{rms\,(0\ to\ \theta_1)} = \frac{V_m}{\sqrt{3}}\sqrt{\frac{\theta_1}{2\pi}} \longrightarrow (10)$$

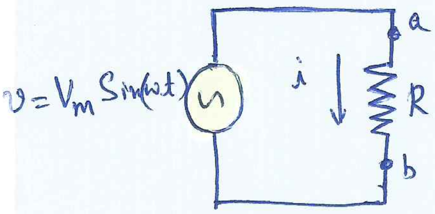
$$v_{rms\,(\theta_1\ to\ 2\pi)} = \frac{V_m}{\sqrt{3}}\sqrt{\frac{2\pi-\theta_1}{2\pi}} \longrightarrow (11)$$

$$v_{rms\_bipolar} = \sqrt{\frac{V_m^2}{3}\,\frac{\theta_1}{2\pi} + \frac{V_m^2}{3}\cdot\frac{(2\pi-\theta_1)}{2\pi}}$$

$$= \sqrt{\cancel{\frac{V_m^2}{3}\,\frac{\theta_1}{2\pi}} + \frac{V_m^2\cdot 2\cancel{\pi}}{3\cdot 2\cancel{\pi}} - \cancel{\frac{V_m^2\,\theta_1}{3\times 2\pi}}}$$

$$\boxed{v_{rms\_bipolar} = \frac{V_m}{\sqrt{3}}} \longrightarrow (12)$$

Resistor in an AC-Circuit



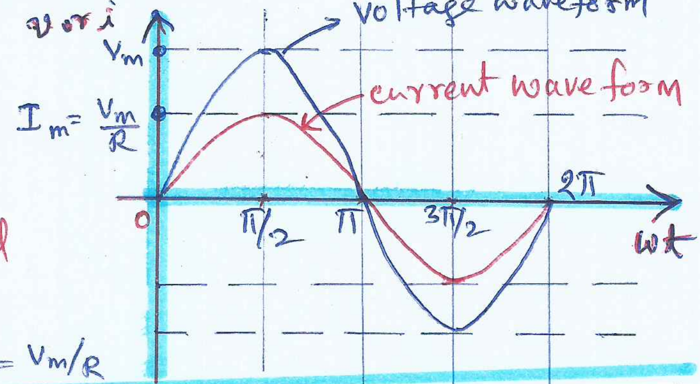
Using Kirchhoff's loop rule, $v = iR$

$\therefore V_m \sin \omega t = iR$

$\therefore i = \left[\frac{V_m}{R}\right] \sin \omega t$ $\therefore$ $\boxed{i = I_m \sin \omega t}$ where $I_m = \frac{V_m}{R}$ → (1)

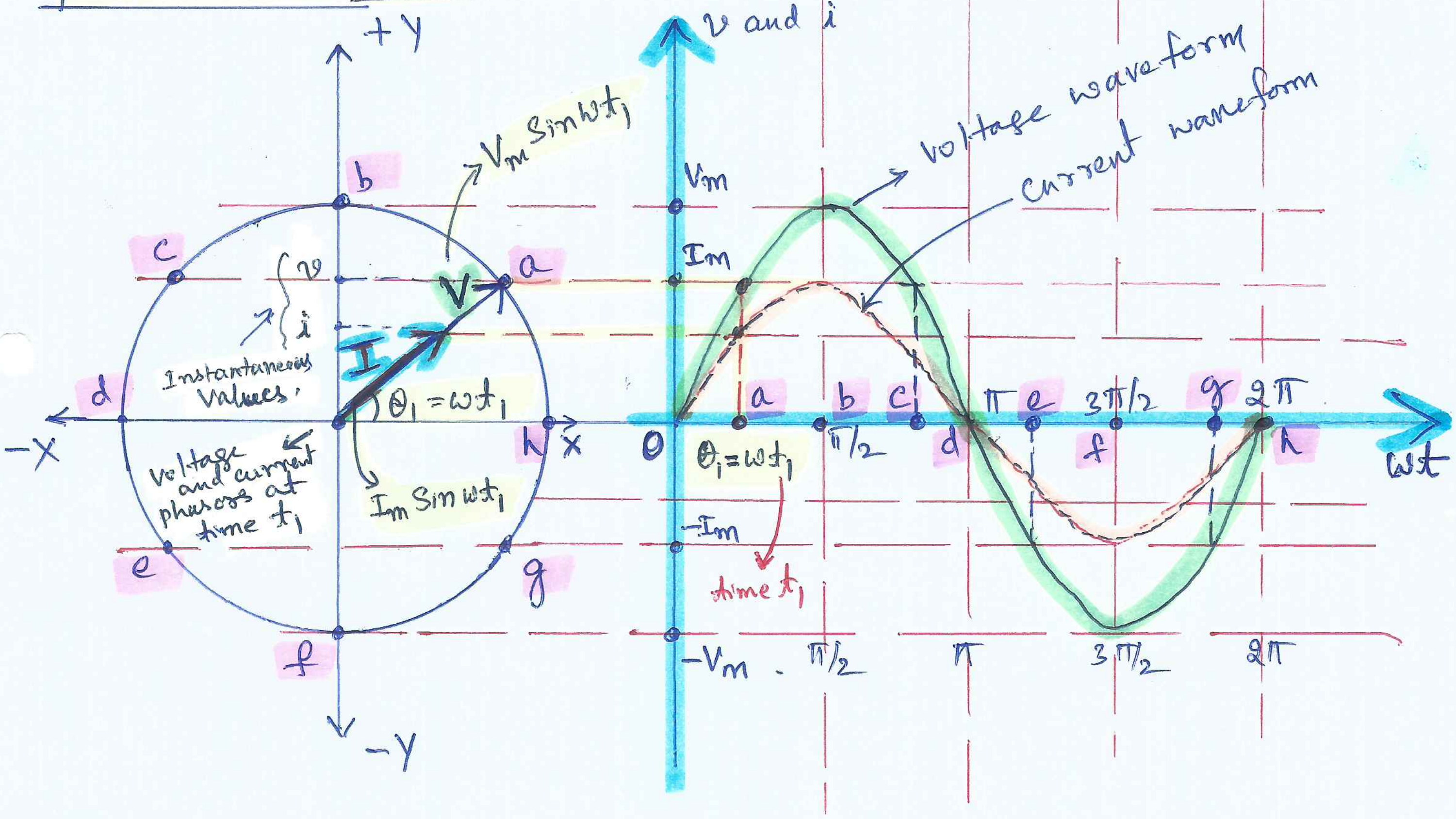
and $\boxed{v = V_m \sin \omega t}$ → (2)

Since V_m and R are constants for a given waveform, the shape of the voltage and the current will be sinusoidal.



- **v and i are in phase**
  - ⇒ When v goes to peak, i also peaks ; when $v = 0$, $i = 0$.
  - → only difference is amplitude of current < amplitude of voltage (based on selected R)
  - → V_m = peak value of voltage waveform
  - → I_m = peak value of current waveform $= V_m/R$

phasor (vector) representation of Resistor in AC circuit (NCERT book)



This diagram shows that the phase difference betn I and V is zero.

Simplified diagram in the next page.

P.T.O.