$\boxed{-102\,c-}$

Case ⑤: Consider point R, where $S_2R \neq S_1R \Rightarrow$ there is a path difference of $\lambda/2$ or phase difference of π

$$S_2R - S_1R = \lambda/2$$

$$y_1 = A \sin \omega t$$
$$y_2 = A \sin(\omega t + \pi)$$
$$y_2 = -A \sin \omega t$$

$\left[\because \sin(x+\pi) = -\sin x\right]$

$y = y_1 + y_2 = 0$ giving zero amplitude or zero intensity.

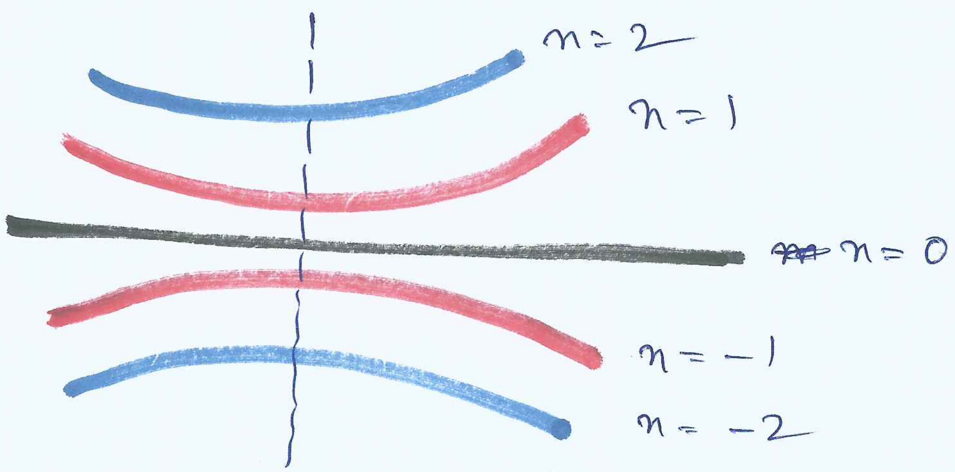
$\therefore$ ~~I = 0~~ $\therefore \boxed{I = 4A^2 = 0}$

$\therefore$ If path difference bet^n 2 waves $= \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots\ldots$
or phase — " — $= \pi, 3\pi, 5\pi \ldots\ldots$, then
the superposition is destructive (zero intensity)

In Summary:

- Condition for Constructive Superposition (max. intensity) is
  $S_1P \sim S_2P = n\lambda$; where $n = 0, 1, 2, 3, \ldots$
  path diff. bet^n two superimposing waves must be integral multiple of λ.
- Condition for Destructive Superposition (zero intensity or min. intensity)
  $S_1P \sim S_2P = \left(n + \frac{1}{2}\right)\lambda$ where $n = 0, 1, 2, 3 \ldots\ldots$
  [path difference between two superimposing waves must be odd integral multiple of $\lambda/2$]

$\therefore$ Locus of points for which $S_1P \sim S_2P = 0, \pm\lambda, \pm 2\lambda, \pm 3\lambda \ldots$ will be constructive (maximum intensity)



IMP

$\boxed{P.T.O}$

Case (iv) $\boxed{102\,d}$ Contd. from 102 C

From previous pages, path difference considered is either integral multiple of λ or odd multiple of λ.

Let us consider some arbitrary point G, where

- path difference b/w 2 two waves is neither $n\lambda$ nor $(n+\frac{1}{2})\lambda$
or – phase ——"—— ——, —— neither $0, 2\pi, 4\pi \ldots$ nor $\pi, 3\pi, 5\pi, \ldots$

but a random phase difference $= \phi$

Thus, the displacement produced by two sources S_1 and S_2 having a phase difference ϕ is given by (assuming same amplitude, $f, \lambda \ldots$)

$$y_1 = A \cos \omega t$$
$$y_2 = A \cos(\omega t + \phi)$$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos(-B) = \cos B$

Resultant Displacement is given by

$$y = y_1 + y_2$$

$$y = A\cos\omega t + A\cos(\omega t + \phi)$$

$$= A\left[\cos\omega t + \cos(\omega t + \phi)\right]$$

$$= A\left[2\cos\left(\frac{\omega t + \omega t + \phi}{2}\right)\cos\left(\frac{\cancel{\omega t} - \cancel{\omega t} - \phi}{2}\right)\right]$$

$$= A\left[2\cos\left(\frac{2\omega t + \phi}{2}\right)\cos\left(-\frac{\phi}{2}\right)\right]$$

$$= A\left[2\cos\left(\frac{2\omega t + \phi}{2}\right)\cos\frac{\phi}{2}\right]$$

$$\boxed{y = \left(2A\cos\frac{\phi}{2}\right)\cos\left(\omega t + \frac{\phi}{2}\right)}$$

$\therefore$ Amplitude of Resultant displacement $= \boxed{(2A)\cos\frac{\phi}{2}}$

$\therefore$ Intensity of —— " —— $= 4A^2\cos^2\frac{\phi}{2}$ (Since $A^2 = I_0$)

($I \propto (\text{amplitude})^2$) $= 4I_0\cos^2\frac{\phi}{2}$

$$\therefore \boxed{I = 4I_0\cos^2\frac{\phi}{2}}$$ IMP

(i) If $\phi = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi, \ldots$ ---- Constructive Interference
Maximum intensity (bright fringes) $I = 4I_0$

(ii) if $\phi = \pm\pi, \pm 3\pi, \pm 5\pi, \pm 7\pi \ldots$ ---- Destructive Interference
Min. or Zero intensity (dark fringes) $I = 0$

P.T.O

— 102 e —

Since ϕ cannot be maintained as constant value, and if ϕ changes rapidly (0 to 2π) with time and ϕ is not stable, then we will see a "time-averaged" intensity distribution.

When this happens, we will observe an average intensity that will be given by

Average value of I is

$$\langle I \rangle = 4A^2 \left\langle \cos^2 \frac{\phi}{2} \right\rangle$$

(Angular brackets represent time-averaging)

Since $\phi(t)$ varies randomly with time bet^n 0 to 2π, the time average of $\left\langle \cos^2 \frac{\phi}{2} \right\rangle = \frac{1}{2}$

Average value of $\cos^2 \phi/2$

$$2\cos^2\frac{\phi}{2} = 1 + \cos\phi$$

$$\cos^2\frac{\phi}{2} = \frac{1}{2}(1+\cos\phi)$$

$$\left\langle \cos^2\frac{\phi}{2} \right\rangle = \frac{1}{2}\cdot\frac{1}{2\pi}\int_0^{2\pi}(1+\cos\phi)\,d\phi$$

$$= \frac{1}{4\pi}\left[\phi + \sin\phi\right]_0^{2\pi}$$

(Since $\int \cos\theta \, d\theta = \sin\theta$)

$$= \frac{1}{4\pi}\left[2\pi - 0 + \sin 2\pi - \sin 0\right]$$

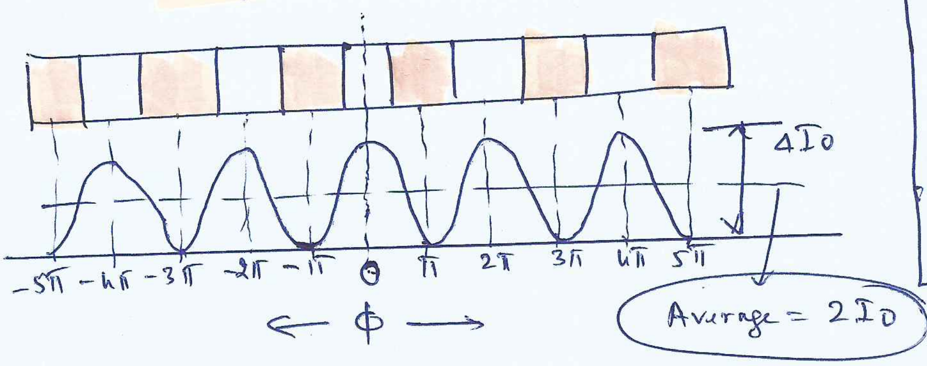
$$= 2\pi/4\pi = \frac{1}{2}$$

$\therefore$ Average value of I

$$\langle I \rangle = 4A^2 \times \frac{1}{2}$$

$$= 2A^2$$

$$\langle I \rangle = 2I_0$$



$\therefore \langle I \rangle = 2I_0 \Rightarrow$ which is equal to intensity when no interference were there $\Rightarrow$ conservation of energy.

Law of Conservation of energy holds good during the phenomenon of Interference. When two waves superimpose out of phase at a point, destructive interference takes place. Thus, the intensity of light and hence the energy at that point is zero. It means, the light energy at that point is completely destroyed. Since energy cannot be destroyed, so the energy disappearing at the point of destructive interference appears at the point of constructive interference. So, we can say that the energy is simply re-distributed in the interference pattern.

— 102ee —

From page $\boxed{102e}$, Intensity distribution due to two waves having a phase difference ϕ at ~~some~~ some point p on the screen

$$\boxed{\langle I \rangle = 4A^2 \left\langle \cos^2\frac{\phi}{2} \right\rangle} \longrightarrow (1)$$

or $\langle I \rangle = 4 I_0 \left\langle \cos^2\frac{\phi}{2} \right\rangle$

- The general equation of (1) is given by

$$\boxed{I_R = I_a + I_b + 2\sqrt{I_a I_b}\ \cos\phi} \longrightarrow (2)$$

Where I_R = Resultant intensity of a wave due to the superposition of two ~~waves~~ coherent waves having a constant phase difference ϕ

(or) Two coherent waves with no phase difference, but at ~~a random~~ an arbitrary point with a phase difference betn two waves $= \phi$
[It is neither $n\lambda$ nor $(n+\frac{1}{2})\lambda$]
[It is neither $0, 2\pi, n\pi$ nor $\pi, 3\pi, 5\pi \ldots$)

$I_a \rightarrow$ Intensity of wave 1
$I_b \rightarrow$ ——"—— 2

- If amplitudes of 2 waves are same, their intensities are also equal. Intensity $\propto$ (amplitude)^2

If $I_a = I_b = I_0$, then [I_0 = (amplitude)^2]

$$I_R = I_0 + I_0 + 2\sqrt{I_0^2}\ \cos\phi$$
$$= 2I_0 + 2I_0\cos\phi$$
$$= 2I_0(1+\cos\phi)$$
$$= 2I_0 \times 2\cos^2\left(\frac{\phi}{2}\right)$$

$$\boxed{I_R = 4I_0\cos^2\frac{\phi}{2}}$$

Since
$1+\cos\phi = 2\cos^2\frac{\phi}{2}$

— 102 eee —

Ratio of Max and Min Intensity of light in interference pattern

$$\frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2}$$

$a \rightarrow$ Amplitude of wave 1
$b \rightarrow$ Amplitude of wave 2

$$= \frac{\left(\frac{a}{b}+1\right)^2}{\left(\frac{a}{b}-1\right)^2} = \frac{(r+1)^2}{(r-1)^2}$$

where $r = \frac{a}{b}$ is called Amplitude Ratio.

Relation between Intensity of Light source and the width of the source

Intensity of light is directly $\propto$ to the width of the source.

Let W_a and W_b be the width of slits S_1 and S_2.

$\therefore$ Intensity of light of these sources be

$I_a \propto W_a$
$I_b \propto W_b$ then $\frac{I_a}{I_b} = \frac{W_a}{W_b}$

Since $I_a \propto a^2$, $I_b \propto b^2$

$$\frac{a^2}{b^2} = \frac{W_a}{W_b}$$

$$\boxed{\frac{a}{b} = \sqrt{\frac{W_a}{W_b}}}$$

Interference of Light and Young's Experiment.

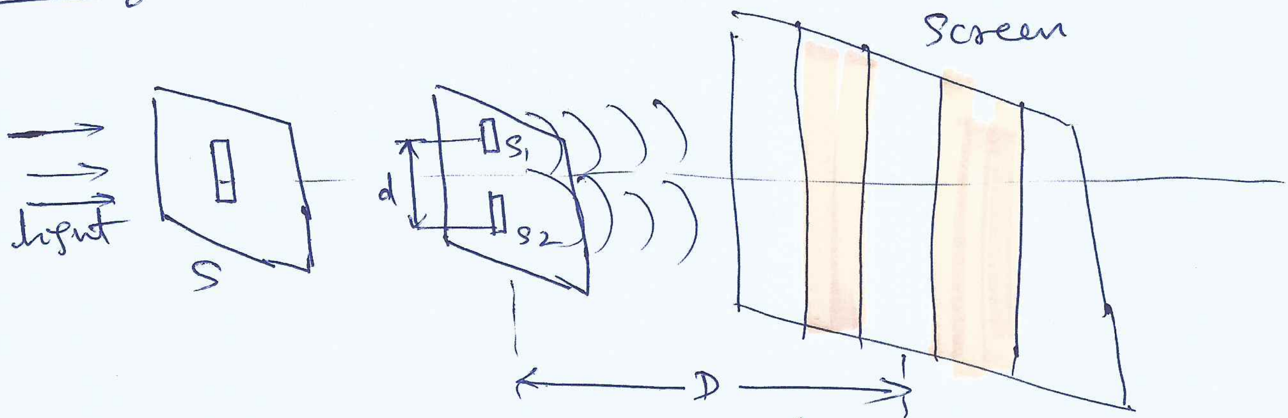
The phenomenon of redistribution of light energy due to the superposition of light waves from two coherent sources is known as interference of light.

Explanation.:

A single source of light gives light energy which spreads uniformly in all directions. But when two sources of light emitting waves of same frequency and of constant or zero phase difference superimpose, then the distribution of light energy is not uniform. At certain points, the intensity of light is maximum while at certain other points, the intensity of light is minimum (or zero) due to the superposition of these two waves. Thus, the light energy is redistributed and this phenomenon is called "Interference of Light".

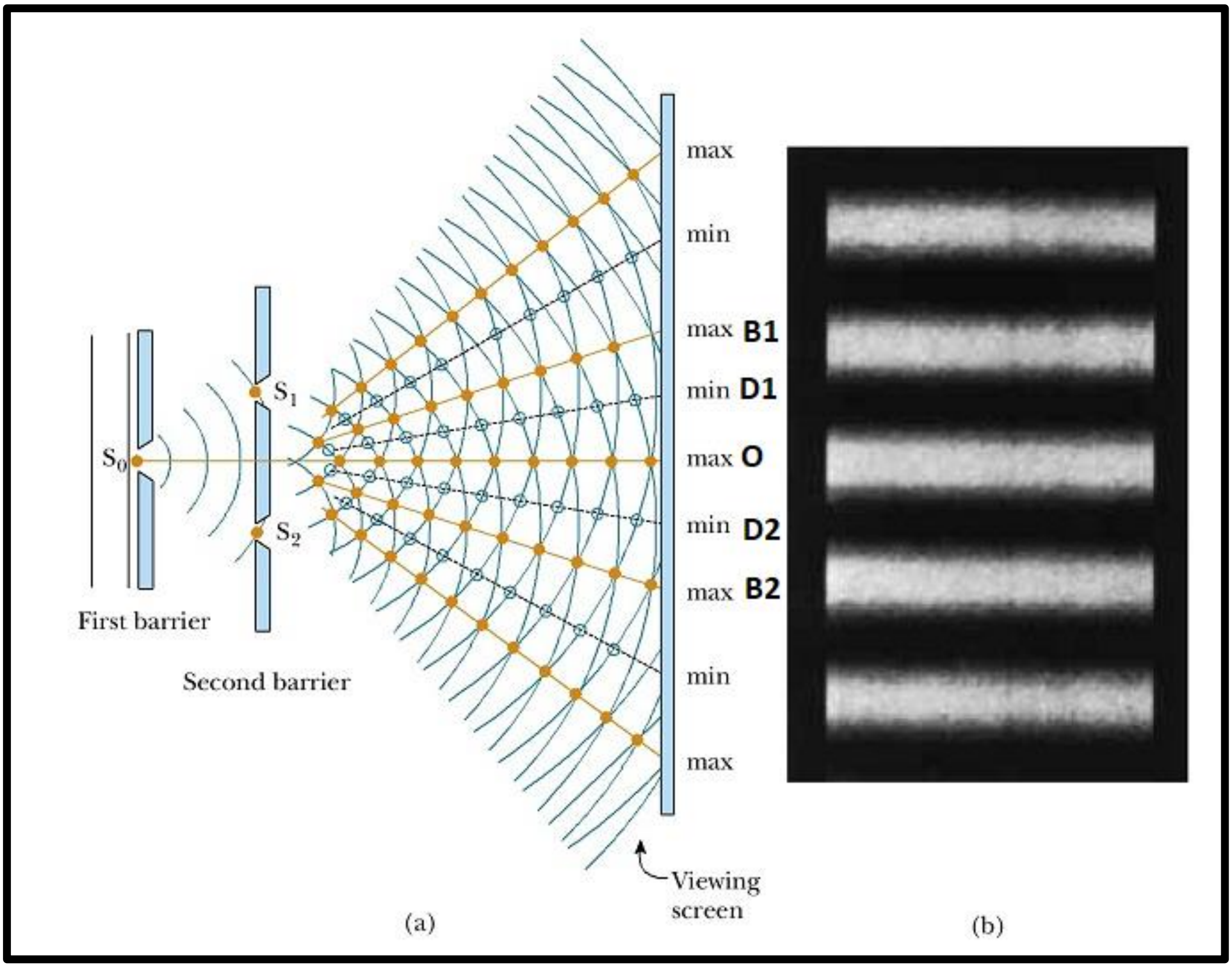
Young's Experiment (Young's double slit expt.)

The phenomenon of interference of light was demonstrated by Thomas Young. His expt. has established the wave nature of light



- Since S_1 and S_2 are derived sources ~~of~~ from parent source S, we can conclude sources S_1 and S_2 are coherent. They pass through two ~~slits~~ slits where wavefronts are locked in phase.
- Sperical wavefronts emite from S_1 and S_2 will produce interference fringes on the screen.

P.T.O



In figure, Solid wavefront → Crests
Dotted wavefront → troughs

- Point O on screen where $S_1 0 = S_2 0$ → Bright fringe
  - Here crest of S_1 falls crest of S_2
  - Trough of S_1 falls on trough of S_2

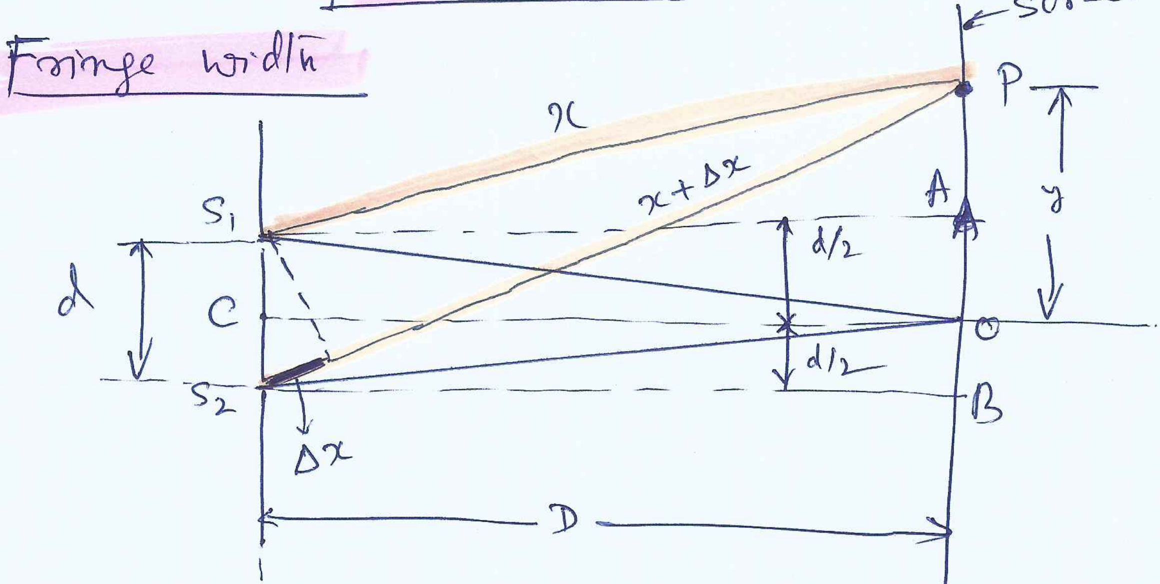
  Constructive Interference

- **Point O is Central maxima**
- At D1 and D2, crest of S_1 falls on trough of S_2 and vice versa → Destructive interference → dark fringes
- Again at B1 and B2, Crest of S_1 falls on crest of S_2 and vice versa → Constructive Interference → Bright fringes

- Thus alternate bright and dark fringes occur at equal spacing's on the screen if basic source of light is monochromatic.
- Using this experiment, Young calculated **λ of light** which is considered to be a property of a **wave.**
- **Slits are used to get more intensity than pin-holes (Young has used pin-holes instead of slits in the beginning)**

— 102 h —

Fringe width



- Since $S_1O = S_2O$ → we get a bright fringe at point O on screen.
- Consider a point P, where we need to find out what type of fringe (bright or dark) is formed (given conditions for constructive and destructive interference)
- The path difference between S_1P and S_2P = Δx

$$\therefore \Delta x = S_2P - S_1P \longrightarrow (1)$$

- From Right angle Δ^{le} S_2BP,

$$S_2P = \left[S_2B^2 + PB^2\right]^{1/2} = \left[D^2 + \left(y+\frac{d}{2}\right)^2\right]^{1/2} = D\left[1+\frac{\left(y+\frac{d}{2}\right)^2}{D^2}\right]^{1/2}$$

Using Binominal theorem

$$(1+x)^n = 1 + nx + \underbrace{\frac{n(n-1)}{2!}x^2 + \dots}_{\text{neglecting these higher terms}}$$

$$(1+x)^n = 1 + nx$$

$$\left[D^2+\left(y+\frac{d}{2}\right)^2\right]^{1/2}$$

$$\left[\left(D^2+\left(y+\frac{d}{2}\right)^2\right)\times\frac{D^2}{D^2}\right]^{1/2}$$

$$D\left[\frac{D^2+\left(y+\frac{d}{2}\right)^2}{D^2}\right]^{1/2}$$

$$D\left[1+\frac{\left(y+d/2\right)^2}{D^2}\right]^{1/2}$$

$$\therefore S_2P = D\left[1+\frac{\left(y+\frac{d}{2}\right)^2}{2D^2}\right] = D + \cancel{D}\cdot\frac{\left(y+\frac{d}{2}\right)^2}{2D^{\cancel{2}}}$$

$n=\frac{1}{2}$

$$\therefore \boxed{S_2P = D + \frac{\left(y+\frac{d}{2}\right)^2}{2D}} \longrightarrow (2)$$

$|||^{ly}$ from Right angled Δ^{le} $S_1\hat{A}P$

$$\boxed{S_1P = D + \left(\frac{\left(y-\frac{d}{2}\right)^2}{2D}\right)} \longrightarrow (3)$$

P.T.O.

 Contd. from 102 h

$$\therefore \Delta x = S_2P - S_1P$$

$$= \cancel{D} + \frac{(y+\frac{d}{2})^2}{2D} - \cancel{D} - \frac{(y-\frac{d}{2})^2}{2D}$$

$$= \frac{1}{2D}\left[\left(y+\frac{d}{2}\right)^2 - \left(y-\frac{d}{2}\right)^2\right]$$

$$= \frac{1}{2D}\left[\left(y+\cancel{\frac{d}{2}}+y-\cancel{\frac{d}{2}}\right)\left(\cancel{y}+\frac{d}{2}-\cancel{y}+\frac{d}{2}\right)\right]$$

$$= \frac{1}{\cancel{2}D}\left[(\cancel{2}y)(d)\right] = y\frac{d}{D}$$

$$\therefore \boxed{\Delta x = y\frac{d}{D}} \longrightarrow (4)$$

Case (i) : Bright fringes [Constructive Interference]

If path difference is an integral multiple of λ, then bright fringe will be formed at P.

$$\therefore \text{From (4)} \quad \frac{yd}{D} = n\lambda \qquad \therefore \boxed{y = \frac{n\lambda D}{d}} \longrightarrow (5)$$

- If $n = 0$, $y_0 = 0$; $n = 1$, $y_1 = \frac{\lambda D}{d}$ $\Big\}$ $y_1 - y_0 = \lambda\frac{D}{d}$ same

  $n = 2$, $y_2 = \frac{2\lambda D}{d}$ $\Big\} \rightarrow y_2 - y_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \lambda\frac{D}{d}$

$\therefore$ Distance between any two successive Bright fringes is called "Fringe width" (β)

$$\beta = y_2 - y_1 = \lambda\frac{D}{d}$$

Fringe width (Dist. betw two successive bright fringes) $= \lambda\frac{D}{d}$ $\qquad \therefore \boxed{\beta = \frac{\lambda D}{d}} \rightarrow (6)$

P.T.O.

— 102 j —

Contd from 102 i

Case ii : Dark Fringes (Destructive Interference)

If path difference is odd integral multiple of $\lambda/2$, then dark fringe will be formed at P.

From (4) page 102 i: $y\frac{d}{D} = \left(n+\frac{1}{2}\right)\lambda$ where $n = 0, 1, 2, 3 \ldots$

$$\boxed{y = \frac{D\lambda}{d}\left(n+\frac{1}{2}\right)} \longrightarrow (7)$$

- If $n=0$, $y_0 = \frac{D\lambda}{2d} = \frac{1}{2}\frac{D\lambda}{d}$
- If $n=1$, $y_1 = \frac{3}{2}\cdot\frac{D\lambda}{d}$
- If $n=2$, $y_2 = \frac{5}{2}\frac{D\lambda}{d}$

$y_1 - y_0 = \frac{3}{2}\frac{D\lambda}{d} - \frac{1}{2}\frac{D\lambda}{d} = \lambda\frac{D}{d}$

$y_2 - y_1 = \frac{5}{2}\frac{D\lambda}{d} - \frac{3}{2}\frac{D\lambda}{d} = \lambda\frac{D}{d}$

same

∴ Distance between any two successive Dark fringes is also called "Fringe width" $\beta = \lambda\frac{D}{d}$

⟹ Distance between any two successive Bright fringes = Distance between any two successive Dark Fringes.

$$\boxed{\beta = \lambda\frac{D}{d}} \longrightarrow (8)$$

⟹ $\beta \propto \lambda$, larger the λ of light, larger is fringe width & vice-versa

⟹ Fringe width β is greater for Red color than for Violet

⟹ $\beta \propto D$, if distance betw screen and slits increases, Fringe width increases

⟹ $\beta \propto \frac{1}{d}$, smaller the distance betw the coherent sources (S_1 and S_2), larger will be the Fringe width.

P.T.O

— 102K —

Shape of Interference ~~Fringes~~ Fringes :

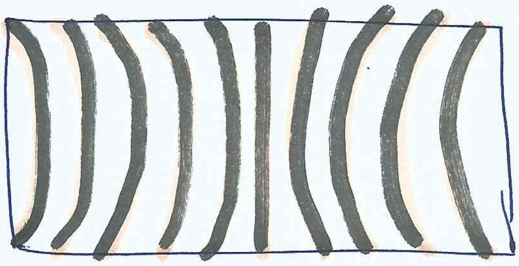
We know that $S_2P - S_1P = n\lambda$ for Bright fringes

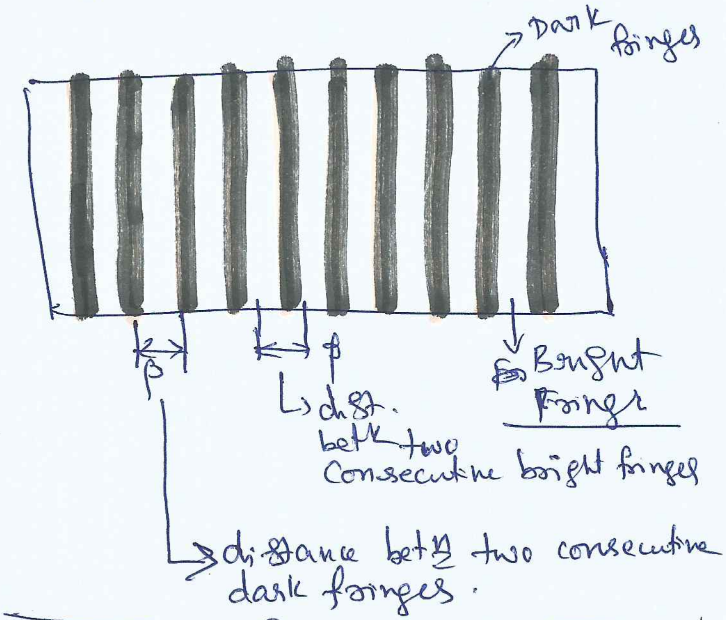
$S_2P - S_1P = (n+\frac{1}{2})\lambda$ for Dark —do—

Since n and λ are constant, hence

$$\boxed{S_2P - S_1P = \text{Constant}}$$

Therefore, locus of point P lying in the x–y plane such that $S_2P - S_1P$ = constant, is a hyperbola. Thus the fringe pattern will strictly be a hyperbola. However if screen distance from slits 'D' is very large as compared to the fringe width β, then the fringes will be nearly straight lines (as shown below)



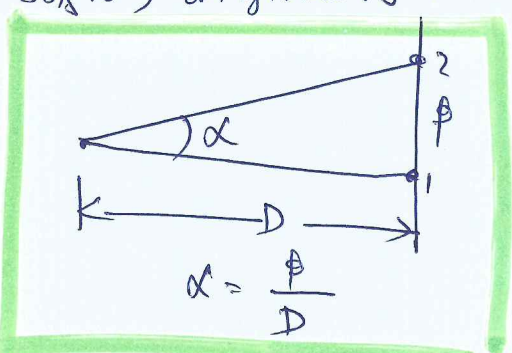


Angular width (α) of a fringe (dark or bright) is given by.

we know that

$$\beta = \lambda \frac{D}{d}$$

$$\alpha = \frac{\beta}{D} = \frac{\lambda}{d} \quad \therefore \boxed{\alpha = \frac{\lambda}{d}}$$



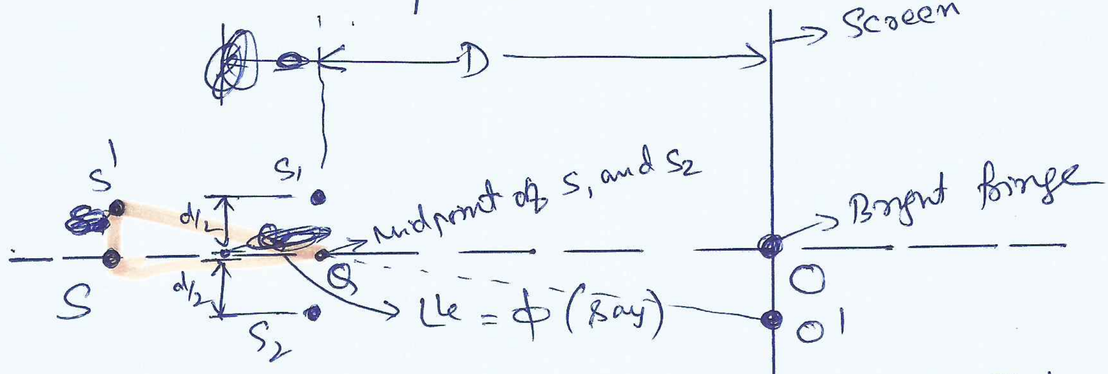
$\alpha = \frac{\lambda}{d}$ remains constant.

α is called angular separation of fringes, which is constant (Since as D increases, β also increases and hence α remains constant).

α is proportional to λ ⟶ α decreases if the whole experiment is done in a liquid of refractive index μ; since $\mu_{liquid} > \mu_{air}$ $\therefore \lambda_{liquid} < \lambda_{air}$

— 102 L —

We have seen the Young's experiment ...



→ We have considered paral source S on the ⊥r bisector of the two slits S_1 and S_2, which is shown as line SO.

→ If S is slightly moved away from ⊥r bisector to some point S', and say if Q is the midpoint of S_1 and S_2 of slits, then angle $S'\hat{Q}S = \phi$ (say). Then the central Bright fringe occurs at an angle $-\phi$, on the other side.

→ Thus, if S is on ⊥r bi-sector, then central fringe occurs at O, also on the ⊥r bi-sector.

→ If $S \rightarrow S'$ by an $\angle le\ \phi$, then the central fringe ~~occurs at~~ appears at a point O' at angle $-\phi \Rightarrow$ which means that it is shifted by the same angle ϕ on the other side of bisector.

→ Thus Source S', the midpoint Q (of S_1 and S_2) and the point O' of the central fringe are in a straight line.