Capacitors :

A capacitor is an electronic/electrical component which has the ability to store the electric charge when the current passing through it

The ability to store electric charge is called "Capacitance".

→ "Capacitance" of a capacitor is defined as its ability to store charge and is measured by the ratio of the charge added to the capacitor to the rise in its potential.

$\Longrightarrow$ $\boxed{C = \dfrac{Q}{V}}$ IMP

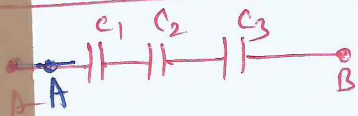
unit of capaciter = farad (SI unit)

1 Farad is the capacitance of a capacitor if its potential rises by 1 volt when a charge of 1 Coulomb is added to it.

→ $1\,\mu F = 10^{-6}$ farads

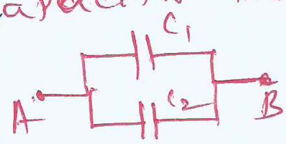
$1\,pF = 10^{-12}$ farads

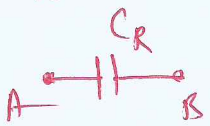
Capacitor in Series



$\Longrightarrow$ $\boxed{\dfrac{1}{C_R} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}}$

Capacitor in parallel





$\Longrightarrow$ $\boxed{C_R = C_1 + C_2}$

Info : RC = Timeconst unit is Sec , how?

we know that $Q = CV$ $\therefore C = \dfrac{Q}{V}$

and $V = IR$ $\therefore R = V/I$

$RC = \left(\dfrac{V}{I}\right)\left(\dfrac{Q}{V}\right) = \dfrac{Q}{I} = t$ (Since $I = \dfrac{Q}{t}$)

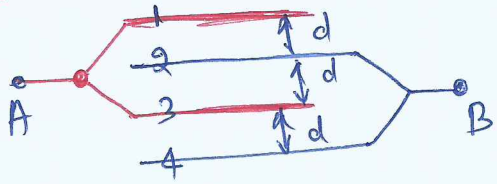
$t = \dfrac{Q}{I}$

~~41~~ ~~74~~ 75

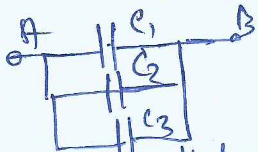
problem :-

Find the capacitance betⁿ A and B if area of each plate is A and distance betⁿ successive plates is d.

(i)



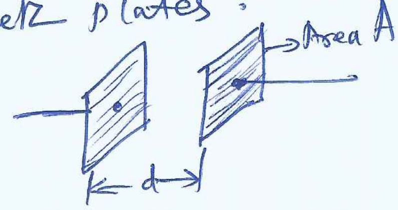
If we see carefully, there are 3 capacitors in parallel.
(capacitors → 1 & 2, 2 & 3, and 3 & 4)



$$\therefore \boxed{C_p = C_1 + C_2 + C_3}$$

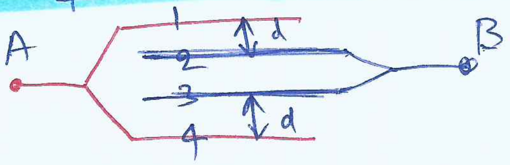
↳ C parallel.

We know that capacitance C of a parallel plate capacitor is given by $\boxed{C = \frac{\epsilon_0 A}{d}}$ where ϵ_0 = absolute electrical permittivity of free space
A = Area of a single plate
d = dist. betⁿ plates.

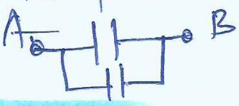


$$\therefore C_p = C_1 + C_2 + C_3$$
$$= \frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{d}$$
$$\boxed{C_p = \frac{3\epsilon_0 A}{d}}$$

(ii)



→ See ② & ③ plates are shorted, hence there are only 2 capacitors in parallel



$$C_p = C_1 + C_2$$
$$\boxed{C_p = \frac{2\epsilon_0 A}{d}}$$

problem : 27 drops, each carrying charge 'q', potential 'V' and capacitance 'C' are put together. What is the charge, potential and capacitance of bigger drop?

→ Let 'r' be the radius of each small drop
'R' ——— " ——— of big drop.

(pradeep's 1/135)

$\therefore$ Volume of big drop $= \frac{4}{3}\pi R^3$

Since volume remains unchanged,

$$27 \times \cancel{\tfrac{4}{3}} \cancel{\pi} r^3 = \cancel{\tfrac{4}{3}} \cancel{\pi} R^3 \Rightarrow 27 r^3 = R^2$$
$$\therefore \boxed{R = 3r}$$

(i) if q is the charge on small drop, then charge on big drop $= q' = 3^3 q = 27q$

(ii) Capacitance of big drop $C' = 4\pi\epsilon_0 R = 4\pi\epsilon_0 (3r)$

(iii) potential of big drop $V' = \frac{q'}{C'} = \frac{\cancel{27}\,9\,q}{4\pi\epsilon_0 \times \cancel{3} r}$

$$V' = 9\frac{q}{4\pi\epsilon_0 r} = 9V$$

Problem: (See Nootan page 147 and 173):
A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected in parallel to another uncharged capacitor. How much energy is lost in the process.

Ans:

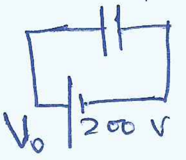


fig ①

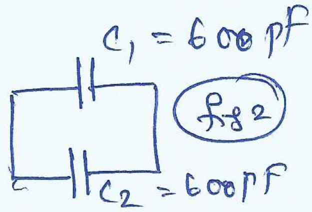


fig 2

Initially charge on C_1 is $Q_0 = C_1 V_0$. After connecting as in fig ②, Q_0 is shared betn C_1 and C_2. Let Q_1, and Q_2 be the charges on C_1 and C_2 respectively and V is common p.d. across each of them; then

$$Q_0 = Q_1 + Q_2 \quad \text{or} \quad C_1 V_0 = C_1 V + C_2 V$$

FYI
$Q_0 = C_1 V_0$
$600 \times 10^{-12} \times 200$
$= 12 \times 10^{-8}$
$= 120\,nC$

$$\therefore C_1 V_0 = V(C_1 + C_2)$$

$$\therefore V = \frac{C_1}{C_1 + C_2} \cdot V_0 = \frac{600}{600+600} \times 200$$

$$= \underline{100\,V}$$

→ Initial energy $E_1 = \frac{1}{2} C_1 V_0^2$

(Here $V_0 = 200V$)

$$= \frac{1}{2} \cdot \overset{300}{\cancel{600}} \times 10^{-12} \times (200)^2$$

$$= 300 \times 4 \times 10^{-8} = 12 \times 10^{-6}\,J$$

→ Final energy $E_2 = \frac{1}{2}(C_1 + C_2)V^2$

(here $V = 100V$)

$$= \frac{1}{2}\left(\overset{600}{\cancel{1200}}\right) \times 10^{-12} \times (100)^2$$

$$= 600 \times 10^4 \times 10^{-12} = 6 \times 10^{-6}\,J$$

$$\therefore \text{Loss of energy} = (12-6) \times 10^{-6}\,J$$

$$= \underline{6 \times 10^{-6}\,J}$$

The missing energy appears as heat and electromagnetic radiation when charge flows from the first capacitor to the second capacitor.

77 ~~40~~ ~~36~~ 77

problem: A parallel plate capacitor is charged by a battery, the battery is then removed, a dielectric slab is then inserted betn plates. Explain what will be effect on

(i) charge
(ii) capacitance
(iii) potential difference betn the plates
(iv) electric field between the plates
(v) energy stored in the capacitor

? page 163 (Nooton CBSE)

Ans $\Rightarrow$ Let Q_0, V_0, C_0, E_0 and U_0 be the charge, potential difference, capacitance, electric field and stored potential energy respectively before the dielectric slab is inserted.

Then
$$\boxed{\begin{aligned} Q_0 &= C_0 V_0 \\ E_0 &= \frac{V_0}{d} \\ U_0 &= \frac{1}{2} C_0 V_0^2 \end{aligned}}$$

(i) The charge on the plates remains Q_0 (unchanged) because the battery is disconnected,

(ii) Capacitance increases from C_0 to C $\quad (C = K C_0)$; $K \rightarrow$ dielectric constant.

(iii) the P.D. betn plates falls from V_0 to V

where $V = \frac{Q_0}{C} = \frac{\cancel{C_0} V_0}{K \cancel{C_0}} = \frac{V_0}{K}$ $\quad \therefore \boxed{V = \frac{V_0}{K}}$

(iv) The electric field reduces from E_0 to E, where

$E = \frac{V}{d} = \frac{V_0}{K} \cdot \frac{1}{d} = \left(\frac{V_0}{d}\right) K \Rightarrow \boxed{E = \frac{E_0}{K}}$

(v) The stored PE decreases from U_0 to U, where

$U = \frac{1}{2} C V^2 = \frac{1}{2} (K C_0) \left(\frac{V_0}{K}\right)^2 = \frac{1}{2} \cancel{K} C_0 \cdot \frac{V_0^2}{K^{\cancel{2}}}$

$U = \frac{1}{K} \left(\frac{1}{2} C_0 V_0^2\right) \Rightarrow \boxed{U = \frac{U_0}{K}}$

Thus potential energy is decreased by a factor 1/K.

This energy is released while inserting the slab. It gives rise to forces which tend to pull the slab into the capacitor and the person inserting the slab has to restrain. If he releases the slab, the slab will be pulled into the capacitor with acceleration.

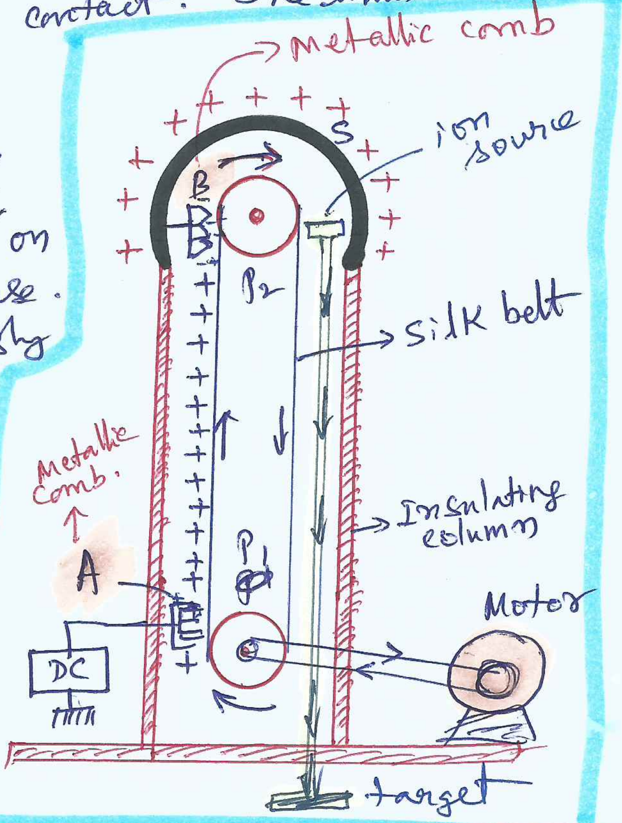
Van de Graaff's Generator :

In 1931, Van de Graaff deviced an electrostatic generator in order to produce very high potentials of the order of 10^6 volts. It is used for accelerating electrons and other charged particles required to study nuclear reactions. It is used to accelerate charged particles like deutrons and protons. In medicine, beams of truly charged accelerated particles are used to treat cancers.

Principle :→ The generator makes use of the following 2 electrostatic phenomena :→

(i) If a charged conducting object has sharp points on the surface, then the charge density (and hence electric intensity) is so high at these points that the surrounding air becomes highly conducting and produce corona discharge.

(ii) If a charged conductor is brought into "internal" contact with a second hollow conductor, all of its charge is transferred to the hollow cylinder, no matter how high the potential of the latter may be. Thus the charge and hence the potential of the hollow conductor can be raised to a quite high ~~level~~ value by successively adding charges to it by "internal" contact. The limit is set only by the insulation difficulties.



~~Contruction~~ **Construction**. As shown in figure, this generator consists of a large, hollow, smooth spherical conductor S supported on an insulating hollow column above the base. An endless silk belt moves continuously over two pulleys P_1 and P_2 driven by a motor, the upper pulley P_2 being inside the conductor S.

Two metallic combs A and B are held very close to the belt and facing it as shown in figure.

The lower comb A ~~B~~ is maintained at a very high positive (say) potential of the order of 10^4 volt by means of DC ~~supply~~ source, while the upper comb B is connected to S.

P.T.O →

Working: The comb A, which is kept +vely charged has a very high charge density at its sharp points. Therefore, due to action at points, the air around A becomes ionised.

→ The positive ions, being repelled from A, are sprayed on the belt which carries them upwards.

→ on arriving in front of the upper comb B, they induce a −ve charge on the ~~sphe~~ sharp points of B and positive charge on the outer ~~surface~~ surface of the sphere S (which is connected to B)

→ The negative charge on B ~~is~~ is then sprayed back on the belt by the "action at points" and neutralises the +ve charge on the belt.

→ The uncharged belt moves round to A again, where the process is repeated.

→ Thus, the sphere S receives an increasing +ve charges and its potential rises correspondingly.

→ In actual generators, the above arrangement is duplicated, positive charge being stored on one sphere and negative on the other.

→ Connected to the sphere S is an evacuated tube which serves as a particle accelerator. A particle source is located inside the tube. The high potential on the sphere S repels the particles which are accelerated towards an earthed target at the far end of the tube. The energy acquired by a charge q is qV, where V is the potential of the spherical shell S.

problem: A Van de Graaff's generator is capable of building up p.d of 15×10^6 V. The dielectric strength of the gas surrounding the electrode $= 5 \times 10^7\ Vm^{-1}$. What is the minimum radius of the spherical shell required.)

→ $$\text{Dielectric Strength} = \frac{V}{r}$$

$$\therefore r = \frac{V}{\text{dielectric Strength}} = \frac{\overset{3}{\cancel{15}} \times 10^6}{\cancel{5} \times 10^7}$$

$$r = 0.3\,m = \cancel{30\ cm}\ 30\ cm$$

Limitation of Van de Graaff's Generator

① It can accelerate only charged particles and not uncharged particles.

② Charges move along one route only and do not follow multiple paths.

③ The generator is largely obsolete because other modern accelerators produce much higher energies (voltages).

Example 34. *Eight identical spherical drops, each carrying a charge 1 nC are at a potential of 900 V each. All these drops combine together to form a single large drop. Calculate the potential of this large drop. (Assume no wastage of any kind and take the capacitance of a sphere of radius r as proportional to r).* [CBSE Sample Paper 15]

- Let r be the radius of each small drop
- Let R be the radius of bigger drop
- Let n be the number of identical spherical drops (in this problem n = 8)
- As volume of bigger drop = volume of n small drops

$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$; therefore $R = n^{1/3}\, r$

As given in the problem, the capacitance is directly proportional to the radius r, therefore
Capacitance of bigger drop = $n^{1/3}$ times the capacitance of each small drop;
We know that Q = CV or V = Q/C = total charge / capacitance

$V_{large} = \frac{nq}{4\pi\epsilon_0 R} = \frac{nq}{4\pi\epsilon_0 n^{1/3} r} = n^{2/3}\frac{q}{4\pi\epsilon_0 r} = n^{2/3}$ times the potential of each small drop (V_{small}), therefore

$\boldsymbol{V_{large} = n^{2/3} V_{small}}$

In this problem n = 8 and potential of each small drop V_{small} = 900V, therefore

$V_{large} = 8^{2/3} \times 900 = 4 \times 900 = 3600$ V

V_{large} = 3600 V ; Therefore, potential of the large drop = 3600 V

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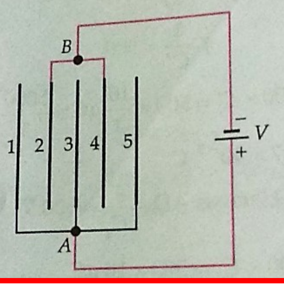
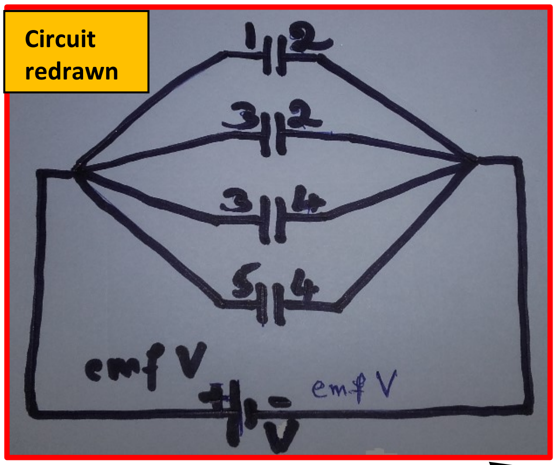
Problem

Example 64. *Five identical capacitor plates, each of area A are arranged such that the adjacent plates are at distance d apart. The plates are connected to a source of emf V, as shown in Fig. 2.78. Find the charges on the various plates.* [IIT 84]



Circuit redrawn



- Given that all the capacitor plates are identical in all respects.
- We can see that there are 5 plates and hence 4 capacitors in the problem and the circuit can be redrawn as above
- The capacitors are C_{12}, C_{23}, C_{34}, C_{45} and we need to see the arrangement of the 5 plates (of 4 capacitors) to the battery.
- Each capacitor $C = \left[\frac{\epsilon_0 A}{d}\right]$; We know that Q = CV
- Charge on each capacitor = (charge on each plate) x V = $\left[\frac{\epsilon_0 A}{d}\right] V$
- Plate 1 comes once → on +ve terminal of the battery
- Plate 2 comes twice → on -ve terminal of the battery
- Plate 3 comes twice → on +ve terminal of the battery
- Plate 4 comes twice → on -ve terminal of the battery
- Plate 5 comes once → on +ve terminal of the battery ; therefore
    - Charge on plate 1 = $+\left[\frac{\epsilon_0 AV}{d}\right]$
    - Charge on plate 2 = $-\left[\frac{2\epsilon_0 AV}{d}\right]$
    - Charge on plate 3 = $+\left[\frac{2\epsilon_0 AV}{d}\right]$
    - Charge on plate 4 = $-\left[\frac{2\epsilon_0 AV}{d}\right]$
    - Charge on plate 5 = $+\left[\frac{\epsilon_0 AV}{d}\right]$

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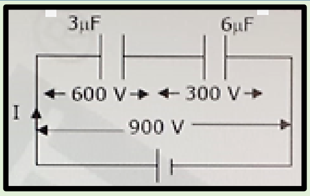
81

Two capacitors of 3 μF and 6 μF are connected in series and a potential difference of 900 V is applied across the combination. They are then disconnected and reconnected in parallel. The potential difference across the combination is

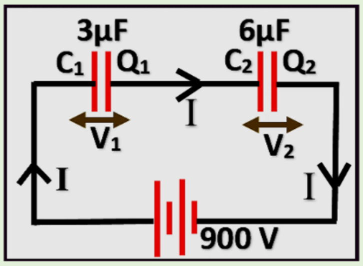
A) Zero B) 100 V C) 200 V D) 400 V



Solution:

Part 1

- When the capacitors are connected in **series** with a power source, since the current in a series circuit is same, all the capacitors will get charged to the same value irrespective of their values. This means if the capacitor C_1 is charged to Q_1 and if the capacitor C_2 is charged to Q_2, then $Q_1 = Q_2$
- We know the relation between charge on the capacitor, the value of capacitor and the voltage across the capacitor ; i.e. **Q = CV**
- $Q_1 = C_1V_1$ and $Q_2 = C_2V_2$; Since $Q_1 = Q_2$, then $C_1V_1 = C_2V_2 \Rightarrow \frac{C_1}{C_2} = \frac{V_2}{V_1}$
- Given: $C_1 = 3\mu F$, $C_2 = 6\mu F$, power source = 900 V $\Rightarrow$ $V_1 + V_2 = 900$ V & hence $V_2 = [900 - V_1]$
- $\frac{C_1}{C_2} = \frac{V_2}{V_1} \Rightarrow \frac{1}{2} = \frac{900 - V_1}{V_1} = \frac{900}{V_1} - 1 \Rightarrow \frac{900}{V_1} = \frac{3}{2} \Rightarrow V_1 = 600\ V\ \&\ V_2 = 300\ V$
- $Charge\ on\ C_1 = Q_1 = C_1V_1 = 3\mu F\ x\ 600V = 1800\ \mu C\ ;\ \ since\ Q_1 = Q_2 \Rightarrow Q_2 = 1800\ \mu F$
- Therefore, each capacitor is finally charged to 1800 μC in steady state condition.
- $\Rightarrow$ C_1 and C_2 each are charged to 1800μC, therefore total charge on C_1 and C_2 = **3600μC**



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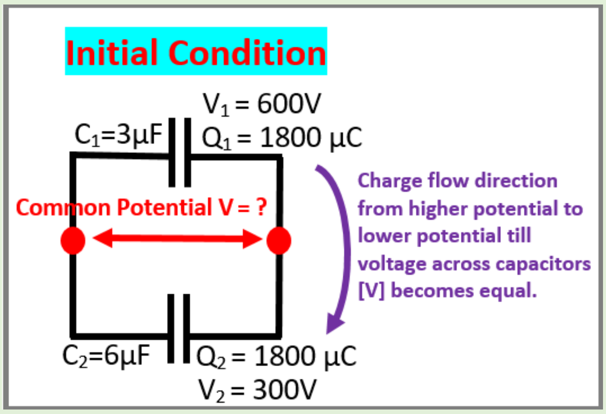
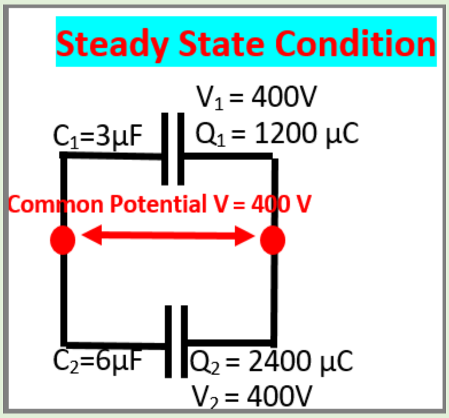
Part 2

Later the charged capacitors are disconnected from power supply and connected in parallel

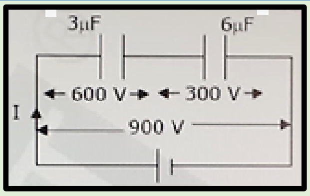
- When capacitors are connected in parallel, the voltage across them will eventually adapt to the same value [called as **Common Potential**].
- Since the value of capacitors is constant and the common potential has to be achieved, the only parameter that will get adapted is the charge. The **charge** will flow from higher potential to lower potential till the potential across capacitors becomes equal **[Called Common Potential V]**
- Due to the law of conservation of charge, the total charge will remain same [at 3600μC], however it will only be adapted across capacitors in parallel.
- ∴ Common Potential in steady state condition $V = \frac{Total\ charge\ on\ capacitors}{Effective\ value\ of\ parallel\ capacitors} = \frac{[1800+1800]\mu C}{[3+6]\mu F} = \frac{3600}{9} = 400\ V\ ; Answer = (D)$
- For information, the charge on each capacitor in the steady state condition is given in the figure. [It is calculated as follows] →
  - $Q_1 = VC_1 = [400V][3\mu F] = 1200\ \mu C$ AND $Q_2 = VC_2 = [400V][6\mu F] = 2400\ \mu C$
    - Note that the conservation of charge is preserved ; total charge [1200μC + 2400μC] = **3600μC**





Two capacitors of 3 μF and 6 μF are connected in series and a potential difference of 900 V is applied across the combination. They are then disconnected and reconnected in parallel. The potential difference across the combination is

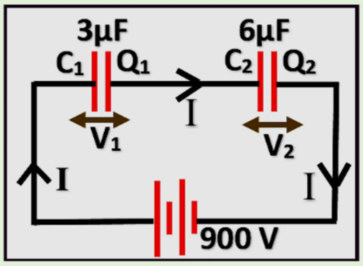
A) Zero B) 100 V C) 200 V D) 400 V



Solution:

Part 1

- When the capacitors are connected in **series** with a power source, since the current in a series circuit is same, all the capacitors **will get charged to the same value irrespective of their values**. This means if the capacitor C_1 is charged to Q_1 and if the capacitor C_2 is charged to Q_2, then $Q_1 = Q_2$. However, the potential across capacitors will be dependent on the value of the capacitor.
- We know the relation between charge on the capacitor, the value of capacitor and the voltage across the capacitor ; i.e. **Q = CV**
- $Q_1 = C_1V_1$ and $Q_2 = C_2V_2$; Since $Q_1 = Q_2$, then $C_1V_1 = C_2V_2 \implies \frac{C_1}{C_2} = \frac{V_2}{V_1}$
- Given: C_1 = 3μF, C_2 = 6μF, power source = 900 V $\implies V_1 + V_2 = 900$ V & hence $V_2 = [900 - V_1]$
- $\frac{C_1}{C_2} = \frac{V_2}{V_1} \implies \frac{1}{2} = \frac{900 - V_1}{V_1} = \frac{900}{V_1} - 1 \implies \frac{900}{V_1} = \frac{3}{2} \implies V_1 = 600\ V\ \&\ V_2 = 300\ V$
- $Charge\ on\ C_1 = Q_1 = C_1V_1 = 3\mu F\ x\ 600V = 1800\ \mu C\ ;\ \ since\ Q_1 = Q_2 \implies Q_2 = 1800\ \mu F$
- Therefore, each capacitor is finally charged to 1800 μC in steady state condition.
- $\implies$ C_1 and C_2 each are charged to 1800μC, therefore total charge on C_1 and C_2 = **3600μC**



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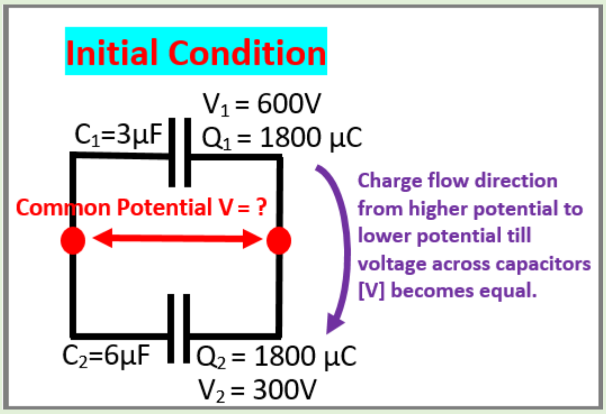
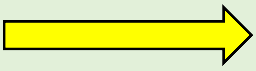
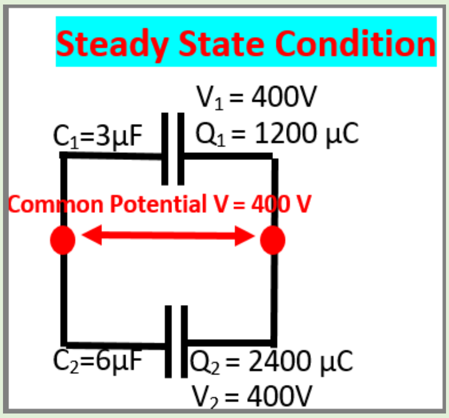
Part 2

Later the charged capacitors are disconnected from power supply and connected in parallel

- When capacitors are connected in parallel, the voltage across them will eventually adapt to the same value [called as **Common Potential**].
- Since the value of capacitors is constant and the common potential has to be achieved, the only parameter that will get adapted is the charge. The **charge** will flow from higher potential to lower potential till the potential across capacitors becomes equal **[Called Common Potential V]**
- Due to the law of conservation of charge, the total charge will remain same [at 3600μC], however it will only be adapted across capacitors in parallel.
- ∴ Common Potential in steady state condition $V = \frac{Total\ charge\ on\ capacitors}{Effective\ value\ of\ parallel\ capacitors} = \frac{[1800+1800]\mu C}{[3+6]\mu F} = \frac{3600}{9} = 400\ V\ ;Answer = (D)$
- For information, the charge on each capacitor in the steady state condition is given in the figure. [It is calculated as follows] →
  - $Q_1 = VC_1$ = [400V][3μF] = 1200 μC AND $Q_2 = VC_2$ = [400V][6μF] = 2400 μC
    - Note that the conservation of charge is preserved ; total charge [1200μC + 2400μC] = **3600μC**

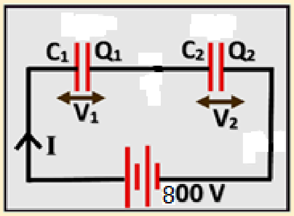






Question : Two capacitors of the same value are connected in series with a power source of 800 V. They are then disconnected and reconnected in parallel. The potential across the combination is

(A) 800 V (B) 400 V (C) 800 V (D) 200 V



Note :

- Don't start deriving it. Understand the question & use logic to select the answer. In fact, circuit in the question is redundant and the text in the question is sufficient to select the answer quickly.
- Since in series circuit, the charge Q on each capacitor is same and value of the capacitors is same, they are charged to the same potential. Given power source = 800 V, the potential across each capacitor = 400 V. The quantity of the charge on each capacitor is also same since $C_1 = C_2$ and $V_1 = V_2$
- After they are disconnected and reconnected in parallel, since $C_1 = C_2$ and $Q_1 = Q_2$, there is no redistribution of the charge since Q is same on capacitors, so the potential across the combination = 400 V