Effect of width of slit on diffraction pattern

We know that width of Central Max. $= \beta_0 = 2\lambda \frac{D}{a}$

——do—— ——"—— Sec Max $= \beta = \lambda \frac{D}{a}$

$\therefore \beta \text{ or } \beta_0 \propto \frac{1}{a}$

If slit width ~~decreases~~ increases, width of Central & Sec. Maxima decrease. When a is sufficiently large, secondary maxima of diffraction pattern ~~do~~ disappear and the Central maximum becomes a shar point, which is sharp image of the slit.

Factors on which width of Central Maximum depends:

(a) $\beta_0 = 2\lambda \frac{D}{a}$, Since $\beta_0 \propto \lambda$, width of Central maximum is small for violet colour and large for red colour.

(b) $\beta_0 \propto D$, β_0 will increase with increase in D

(c) $\beta_0 \propto \frac{1}{a}$. If the slit width is small, width of central maximum is large and vice-versa.

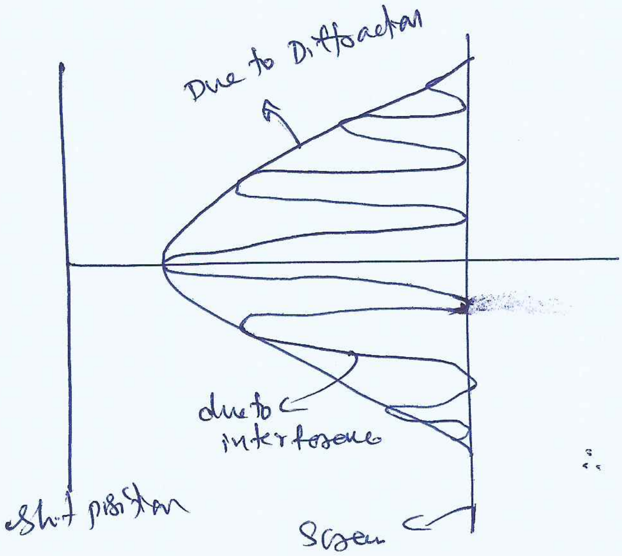
— 105 g —

Difference between Interference and Diffraction of Light is given below:

Interference	Diffraction
1. Interference is due to the superposition of two wavefronts originating from two coherent sources.	1. Diffraction is due to the superposition of secondary wavelets originating from the different points of the same wavefront.
2. In interference pattern, all the maxima *i.e.* bright fringes are of the same intensity.	2. In diffraction pattern, the bright fringes are of varying intensity. Intensity decreases away from the central maximum on each side.
3. In interference pattern, the dark fringes are usually almost perfectly dark.	3. In diffraction pattern, the dark fringes are not perfectly dark.
4. In interference pattern, the width of fringes (bright and dark) is equal.	4. In diffraction pattern, width of central fringe is double than the width of other maxima.
5. In interference, fringes (or bands) are large in number.	5. In diffraction, fringes (or bands) are a few in number.
6. In interference, fringes (or bands) are equally spaced.	6. In diffraction, fringes (or bands) are unequally spaced.

problem NCERT book (page 370) ex. 10.5

Q: what should be the width of each slit to obtain 10 maximum of the double-slit pattern within the central maximum of single-slit pattern. Given separation betw slits $d = 1\,mm$



We know that from

Diffraction: Central width $= \frac{2D\lambda}{a}$

Interference: fringe width $= \beta = \lambda \frac{D}{d}$

Given there are 10 maxima of interference fringes within the central width of diffraction envelope

$\therefore \cancel{10}^{5} \cdot \frac{\cancel{\lambda}\cancel{D}}{d} = \frac{\cancel{2}\cancel{D}\cancel{\lambda}}{a}$

$\therefore \frac{5}{d} = 1/a \quad \therefore a = \frac{d}{5} = \frac{1\,mm}{5}$

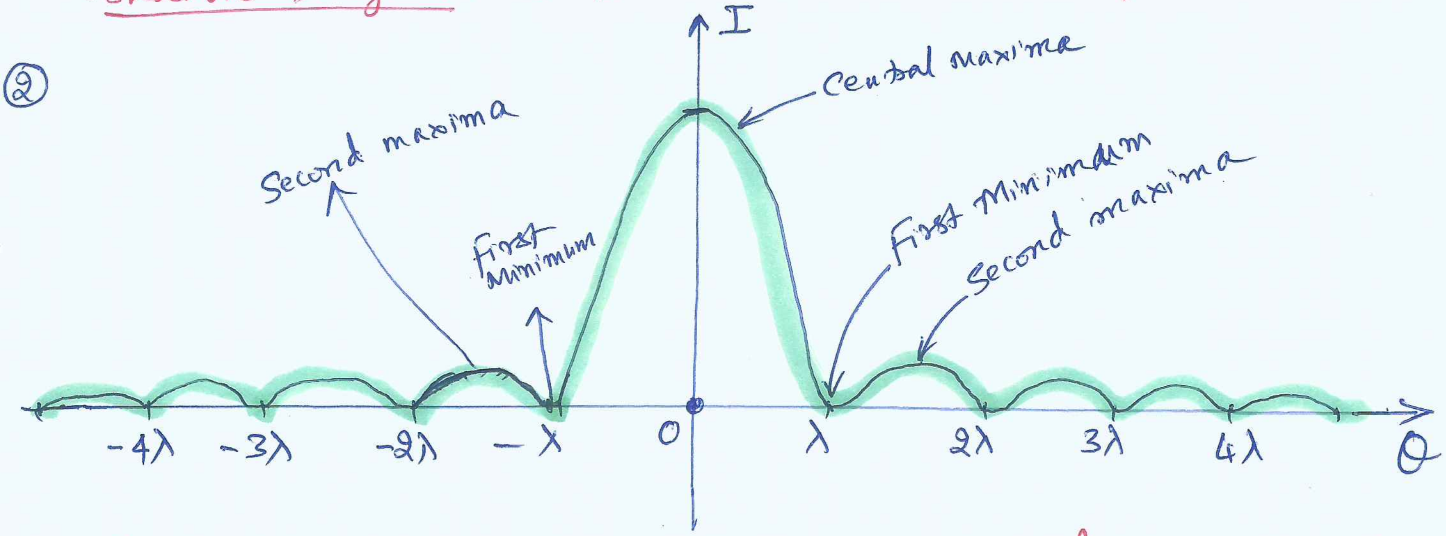
$\boxed{a = 0.2\ mm}$

Summary of Diffraction ~~134~~ — 105 gg —

① Diffraction is the bending of light around an obstacle when the obstacle size is comparable to the wavelength of the incident light.

→ The essential condition for diffraction is size of the obstacle or aperture should be comparable to the wavelength of the incident light; then there is a departure from straight line propagation and the light bends around the corners of the obstacle or aperture and enters the geometrical shadow region. Diffraction is a characteristic of wave-motion.

②



③ Since red is having longer λ, the central maximum is broadened and hence its ~~decreases~~ intensity decreases.

④ ⓐ The angular position θ of the first minimum fall is

$$\sin\theta = \frac{n\lambda}{d}$$

| $\sin\theta = \sin 30^\circ = 0.5$

$(n=1)$; $d\sin\theta = n\lambda$

$$d = \frac{\lambda}{\sin\theta} \quad \left(\begin{matrix}\text{Since } n=1 \\ \theta = 30^\circ\end{matrix}\right)$$

$$d = \frac{\lambda}{0.5} = 2\lambda = 2\times 6500\ \mathring{A} = 13{,}000\ \mathring{A} = 1.3\times10^{-6}\ m$$

ⓑ The angular position θ of the first maximum w.r.t. the central maximum is given by

$$\sin\theta = \frac{(n+\frac{1}{2})\lambda}{d} \qquad (\theta = 30^\circ) \rightarrow (\sin 30^\circ = 0.5)$$

Since $n=1$, $\sin\theta = \dfrac{3\lambda}{2d}$ $\quad\therefore d = \dfrac{3\lambda}{2\sin 30^\circ} = \dfrac{3\lambda}{2\times 0.5}$

$$\therefore d = 3\lambda = 3\times 6500\ \mathring{A} = 19{,}500\ \mathring{A} = 1.95\times10^{-6}\ m$$

— 105 h —

Resolving power of optical Instruments (briefly).

Defn: Limit of Resolution:
The minimum distance between two objects which can just be seen as separate (or just resolved) by the optical instrument.

Defn: Resolving power: (RP)
The ability of the instrument to produce distinctly separate images of two close objects.

$$RP \propto \frac{1}{\text{Limit of Resolution}}$$

① Resolving power of Telescope.

$$\boxed{RP_{telescope} = \frac{D}{1.22\lambda}}$$

$D \rightarrow$ is the diameter of the objective lens
$\lambda \rightarrow$ wavelength of light used

$$\boxed{\text{Limit of Resolution } \alpha = \frac{1.22\lambda}{D}}$$

② Resolving power of Eye.

Since eye lens is a converging lens, so that the case is same as telescope.

$\therefore \alpha = \text{Limit of Resolution} = \frac{1.22\lambda}{D}$ $\quad D \Rightarrow$ Diameter of the pupil of eye.

$$RP = \frac{D}{1.22\lambda} = \frac{1}{\alpha}$$

③ Resolving power of Microscope

$$\boxed{RP_{microscope} = \frac{2\mu \sin\beta}{1.22\lambda}}$$

$\mu \rightarrow$ refractive index of medium (if air, $\mu = 1$)

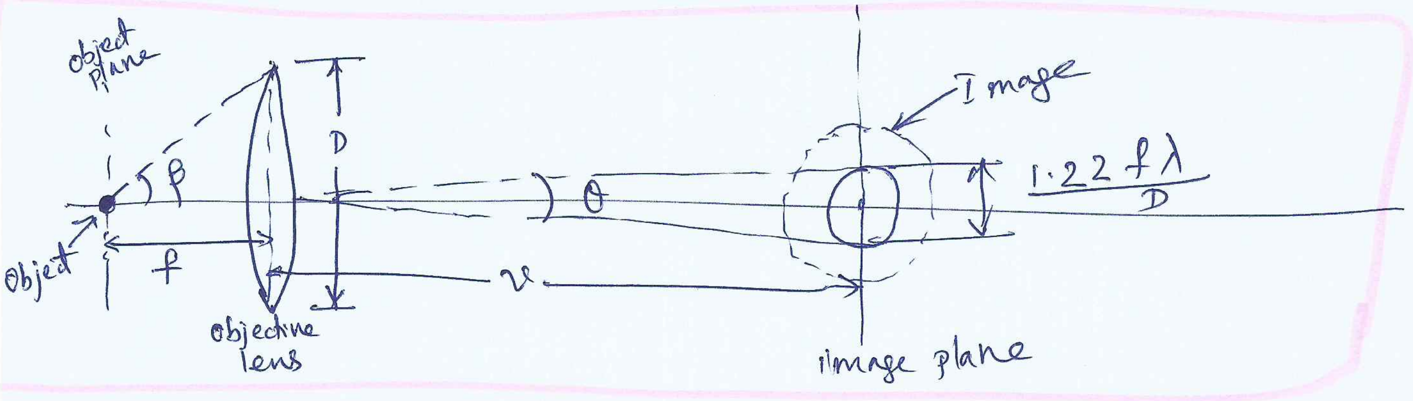
$\mu \sin\beta \rightarrow$ is called "Numerical Aperture" and is marked on the objective

$\lambda \rightarrow$ is the wavelength of light used.

2β = angle subtended by the diameter of the objective lens at the focus of the microscope.

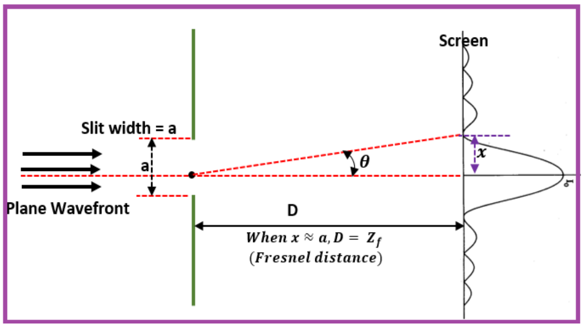
Resolving Limit of Microscope $= d_{min} = \frac{1}{RP_{microscope}}$

$$\boxed{d_{min} = \frac{1.22\lambda}{2\mu \sin\beta}}$$



Validity of Ray Optics (Fresnel distance)

Fresnel distance is defined as the distance of the screen from the slit when the spreading of light due to diffraction from the centre of the screen is equal to the size (i.e. width) of the slit. It is represented as Z_f.



- The diffraction pattern of a single slit consists of a central maximum surrounded by alternate secondary maxima and minima
- We know that in diffraction, the criteria for minima occurs when $a\ sin\theta\ = n\lambda$; If θ is very small, then $a\theta\ = n\lambda$. (θ is known as the half angular width of the central maximum). For the first minimum, n = 1, then $a\theta\ = \lambda\ \therefore\ \theta = \frac{\lambda}{a}$ ------ (1)
- From figure, $\theta = \frac{x}{D}$ --------(2) ; plugging (2) in (1), we get
- $\frac{x}{D} = \frac{\lambda}{a}$ or $x = \lambda\frac{D}{a}$; If spreading due to diffraction is comparable to "a" (implies $x\ = a$), then $D = Z_f$
- $\therefore\ \boldsymbol{Z_f = \frac{a^2}{\lambda}}$, Z_f *is called the Fresnel distance*----- (3)
- Fresnel distance is the minimum distance a beam of light has to travel before its deviation from straight line path becomes significant. This is the boundary of ray optics and wave optics.
- Validity of Ray optics can be explained on the basis of The Fresnel distance usually denoted by Z_f. It is defined as the minimum distance a light ray has to travel before it bends from its original path.
- If λ is very small (i.e. λ → 0), Z_f = ∞. This means, if wavelength of the incoming light is very small, the light will spread after travelling a very large distance. In other words, the "ray optics" is valid if λ → 0

Examples:

If average λ of visible light = 6000 Å (6 x 10^{-7} m) and the diameter of the pupil of human eye = a = 1mm (10^{-3} m), then $\boldsymbol{Z_f = \frac{a^2}{\lambda} = \frac{10^{-3}\,x\,10^{-3}}{6\,x\,10^{-7}} = \frac{10}{6} = \frac{5}{3} = 1.67\ m}$; This implies that the width of the beam due to diffraction does not become more than 1 mm, unless distance travelled by the beam is more than 1.67 m. Hence, we can ignore broadening of beam by diffraction upto distances as large as few meters. This means that we can assume that light travels along straight line.
Hence, ray optics can be taken as a limiting case of wave optics. This is mainly because of λ of light being very small.

Estimate the distance for which ray optics is good approximation for an aperture of 4 mm and λ=400 nm.

$$\boldsymbol{Z_f = \frac{a^2}{\lambda} = \frac{4\,x\,4\,x\,10^{-3}\,x\,10^{-3}}{4\,x\,10^{-7}} = 40m}$$

Therefore, the distance for which the ray optics is a good approximation is 40 m.

Calculate the distance, a beam of wavelength 900 nm can travel without significant broadening if the slit is 3 mm wide.

$$\boldsymbol{Z_f = \frac{a^2}{\lambda} = \frac{3\,x\,3\,x\,10^{-3}\,x\,10^{-3}}{9\,x\,10^{-7}} = \frac{90}{9} = 10m}$$

Another point to be noted is that the minimum distance at which the observer should be from the obstacle to observe the diffraction of light of wavelength λ around the obstacle of size "a" "clearly" is given by $x = \frac{a^2}{4\lambda}$ (do not get confused with the Fresnel distance formula (3). Here, this formula $x = \frac{a^2}{4\lambda}$ says the diffraction is clearly observed using this formula)
If we have a hole of diameter a = 2mm which is illuminated by red light λ = 7000Å, then find the distance at which the diffraction of red light through the hole will be observed clearly by an observer?

$$x = \frac{a^2}{4\lambda} = \frac{\mathbf{2\,x\,2\,x\,10^{-3}\,x\,10^{-3}}}{4\,x\,7\,x\,\mathbf{10^{-7}}} = \frac{10}{7} = 1.43\ m$$

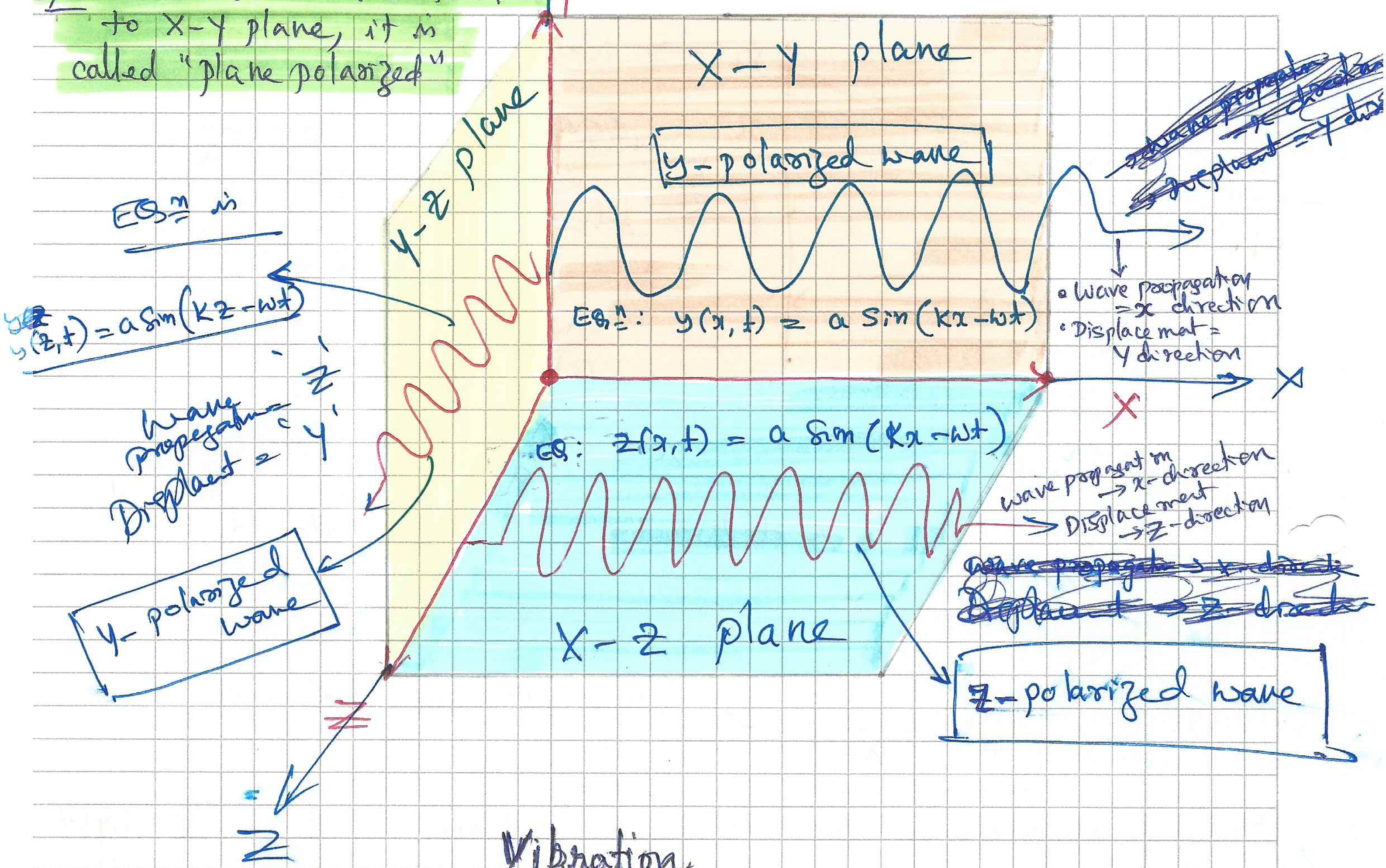
A slit 4cm wide is irradiated with microwaves of wavelength 2cm.Find the angular spread of central maximum, assuming incidence normal to the plane of the slit?

→ We know that a sinθ = λ ; ∴ sinθ = $\frac{\lambda}{a} = \frac{2\,x\,10^{-2}}{4\,x\,10^{-2}} = 0.5$; $\therefore\ \theta = 30^0$

For distances << Z_f : spreading due to diffraction is smaller compared to size of the beam then the ray optics is valid.
For distances >> Z_f : spreading due to the diffraction dominates over ray optics then the wave optics is valid.

Polarization

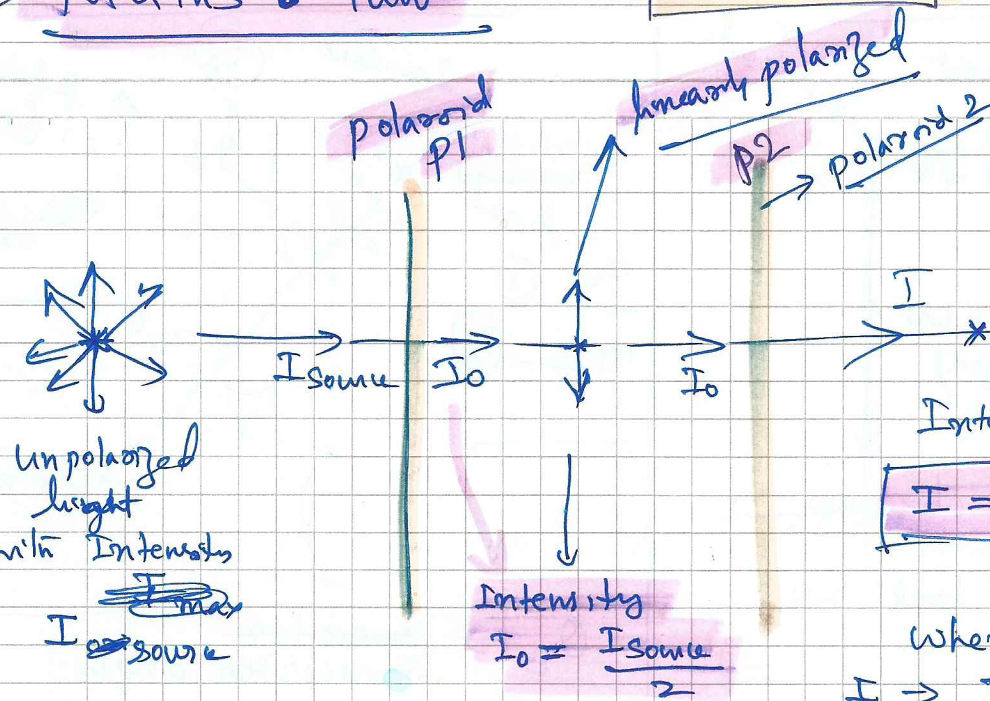
Ex 1: when a wave is confined to X–Y plane, it is called "plane polarized"



Plane	Wave propagation direction (Example)	Vibration direction (~~Displacement direction~~)	Equation of the wave
X–Y	X	Y (y-polarized)	$y(x,t) = a\sin(kx - \omega t)$
X–Y	Y	X (x-polarized)	$x(y,t) = a\sin(ky - \omega t)$
X–Z	X	Z (z-polarized)	$z(x,t) = a\sin(kx - \omega t)$
X–Z	Z	X (x-polarized)	$x(z,t) = a\sin(kz - \omega t)$
Y–Z	Y	Z (z-polarized)	$z(y,t) = a\sin(ky - \omega t)$
Z–Y	Z	Y (y-polarized)	$y(z,t) = a\sin(kz - \omega t)$

Since wave is confined to a plane, it is called "plane polarized wave"

Malus's law



Intensity of light

$$\boxed{I = I_0 \cos^2\theta}$$

Malus' law

where

$I \rightarrow$ Final transmitted light Intensity

$I_0 \rightarrow$ light intensity after first polaroid

θ = angle between pass-axis P1 and pass-axis of P2

$$\therefore \boxed{I = I_0 \cos^2\theta \qquad \text{when } \theta = 0,\ I = I_0; \quad \theta = 90^0,\ I = 0 \ \text{(no light)}}$$

NCERT problem Ex. 10.8

Q: Discuss the intensity of transmitted light when a polaroid sheet is rotated between two crossed polaroids?

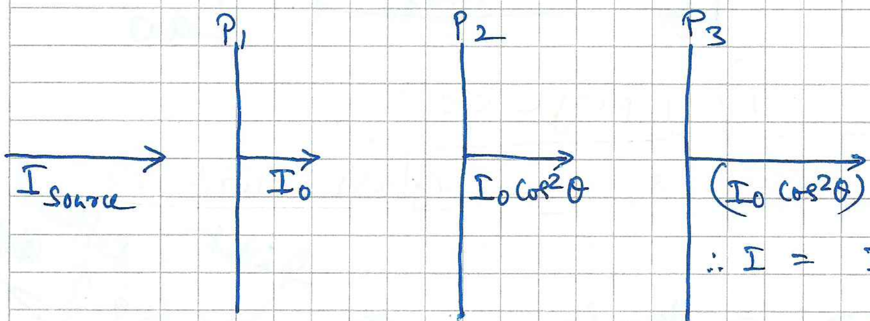
A ⇒ Given 3 polaroids P_1, P_2, P_3

Given

∠ betn. P_1 and $P_3 = 90^0$

let angle betn P_1 and $P_2 = \theta$

∴ ∠ betn P_2 and $P_3 = (90-\theta)$



$(I_0 \cos^2\theta)[\cos^2(90-\theta)]$

$\therefore I = I_0 \cos^2\theta \sin^2\theta$

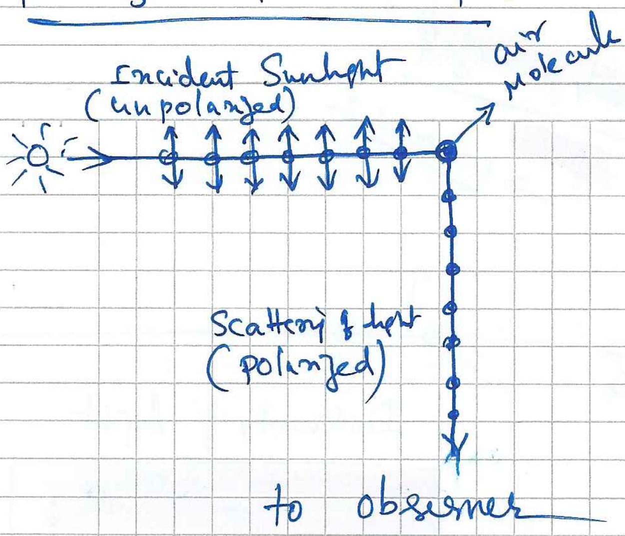
$= \frac{I_0}{4}(2\sin\theta\cos\theta)^2$

$$\boxed{I = \frac{I_0}{4}\sin^2 2\theta}$$

when $\theta = 45^0$ (angle betn P_1 and P_2)

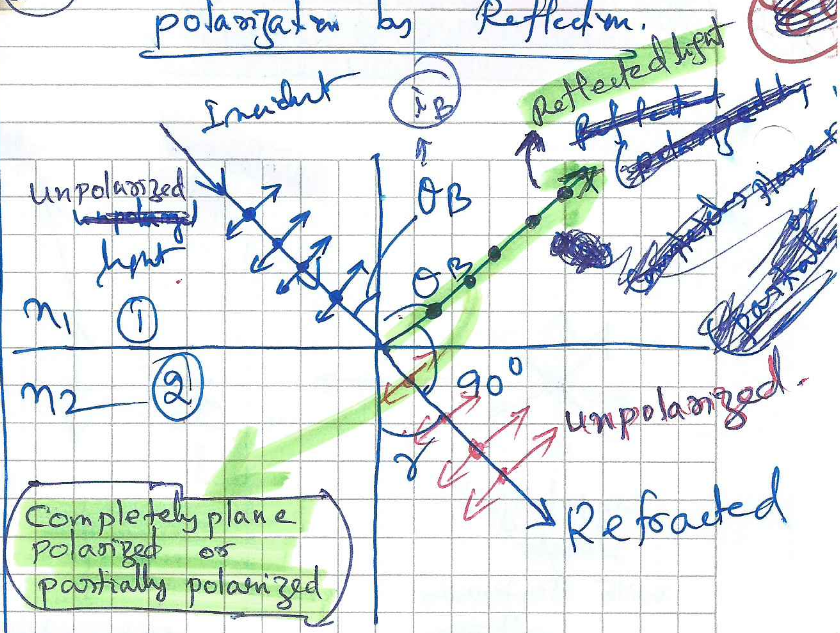
then intensity is maximum $I = \frac{I_0}{4}$

polarization by Scattering



See in NCERT book why it happens

polarization by Reflection.



$\theta_B \rightarrow$ ~~Brewster~~ Brewster angle

is the angle of incidence when Reflected ray is $\perp^r$ to refracted ray

$$\frac{n_2}{n_1} = n_{21} = \frac{\sin i_B}{\sin r}$$

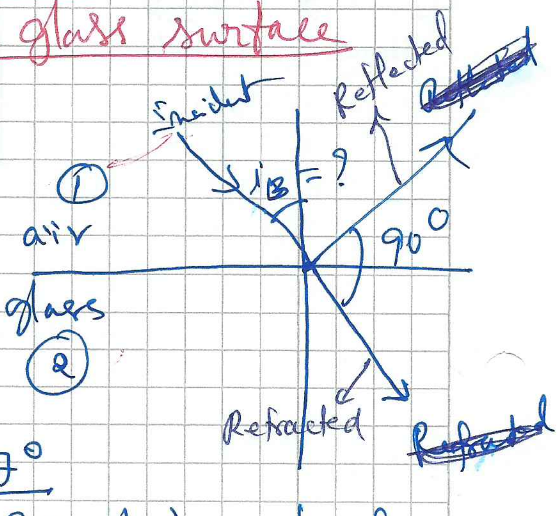
$$= \frac{\sin i_B}{\sin(180 - 90 - i_B)}$$

$$= \frac{\sin i_B}{\sin(90 - i_B)} = \frac{\sin i_B}{\cos i_B}$$

$$\boxed{n_{21} = \tan i_B}$$ Brewster's Law

problem: NCERT book (10.9) (page 381)

Unpolarized light is incident on a plane glass surface What should be the i_B?



We know that $\frac{n_2}{n_1} = n_{21} = \tan i_B$

$n_1 = 1$, $n_2 = 1.5$ (glass)

$$\frac{1.5}{1} = \tan i_B$$

$\therefore \tan i_B = 1.5$ $\quad \therefore i_B = \tan^{-1} 1.5 = 57^\circ$

This is the Brewster's angle for air to glass interface.

IMP

Brewster's Law is

$$\boxed{\frac{n_2}{n_1} = n_{21} = \tan i_B}$$

where $i_B \rightarrow$ angle of incidence (Brewster's angle) when reflected light is $\perp^r$ to refracted light.

$n_1 \rightarrow$ Refractive index of incident light medium

$n_2 =$ —— do —— of refracted light medium.

Info (IMP)

Time average value of $\cos^2\theta$

$\rightarrow$ is represented by $\langle \cos^2\theta \rangle$ is given by

$$\frac{1}{2\pi}\int_0^{2\pi} \cos^2\theta \, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{(1+\cos 2\theta)}{2} d\theta$$

$$= \frac{1}{4\pi}\left[\int_0^{2\pi} d\theta + \int_0^{2\pi} \cos 2\theta \, d\theta\right]$$

$$= \frac{1}{4\pi}\left[2\pi + \left.\frac{\sin 2\theta}{2}\right]_0^{2\pi}\right]$$

$$= \frac{1}{4\pi}\left[2\pi + 0\right] = \frac{1}{2}$$

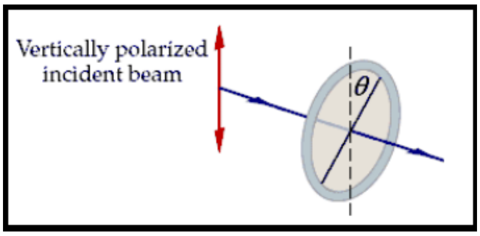
$\therefore$ $\boxed{\langle \cos^2\theta \rangle = \frac{1}{2}}$ Also $\boxed{\langle \sin^2\theta \rangle = \frac{1}{2}}$

Since $\sin^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)$

This is also intuitively obvious that the function $\cos^2\theta$ or $\sin^2\theta$ will randomly vary between 0 and 1 and therefore the average value will be $\frac{1}{2}$.

- 110a –

Problem#1 : Vertically polarized light with intensity 0.55 W/m^2 passes through a polarizer oriented $\theta = 65^0$ from the vertical. Calculate the intensity after the polarizer?



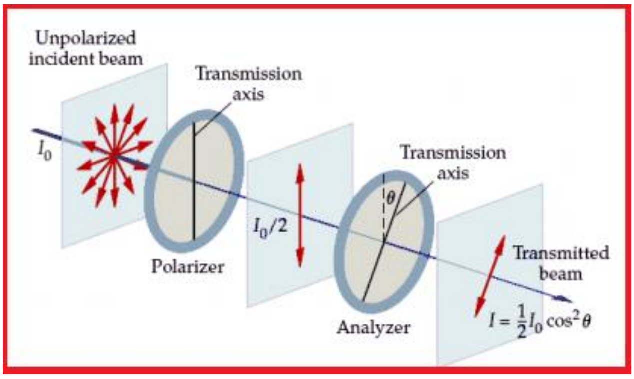
Answer: Using Malus's law $I = I_0 \cos^2\theta$

$I = (0.55)\cos^2 65^0 = 0.55 \times \cos 65^0 \times \cos 65^0$

$I = (0.55) \times 0.423 \times 0.423 = 0.098\ W/m^2$

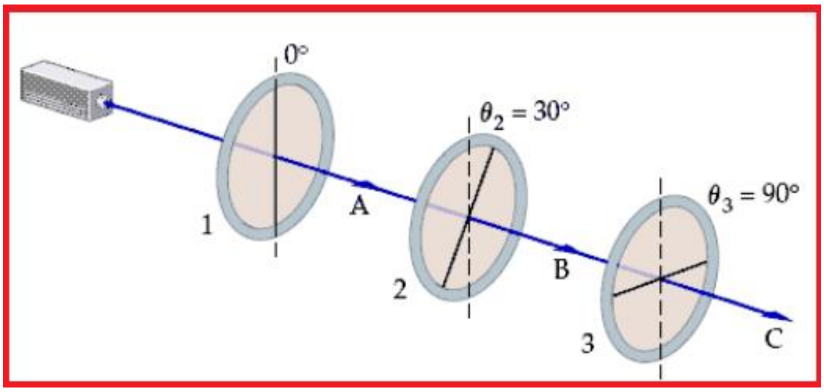
$\boxed{I = 0.098\ W/m^2}$

Problem#2 : The image shows unpolarized light incident upon two polarizers, the transmission axes of which are oriented at some angle with respect to each other ($\theta = 30^0$). Calculate the intensity after the second polarizer. Divide the result by the initial intensity to determine the relative intensity



- **Answer** : Using Malus's law $I = I_0 \cos^2\theta$
- Intensity after the first polarizer: $I_1 = ½\ I_0$
- Intensity after the second polarizer: $I_2 = I_1 \cos^2 30^0 = ½\ I_0 (0.87)\times(0.87) = 0.375\ I_0$
- Intensity after the second polarizer: $I_2 = 0.375\ I_0$
- **Relative intensity = I_2/I_0 = 0.375**

Problem#3 : The image shows unpolarized laser light from a source passing through three polarizers. Calculate the intensity after each of the polarizers.



→ **Answer**: Using Malus's law $I = I_0 \cos^2\theta$ (I_0 = incident intensity from the source)

Intensity at A : $I_A = ½\ I_0$	**$I_A = ½\ I_0$**
Intensity at B : $I_B = I_A \cos^2 30^0 = (½\ I_0)\times(0.87)\times(0.87) = 0.375\ I_0$	**$I_B = 0.375\ I_0$**
Intensity at C : $I_c = I_B \cos^2(90^0\text{-}30^0) = (0.375\ I_0)\times(0.5)\times(0.5) = 0.0938\ I_0$	**$I_c = 0.0938\ I_0$**
Intensity at C with the second polarizer removed: $I_c = I_A \cos^2 90^0 = 0$	**$I_c = 0$**