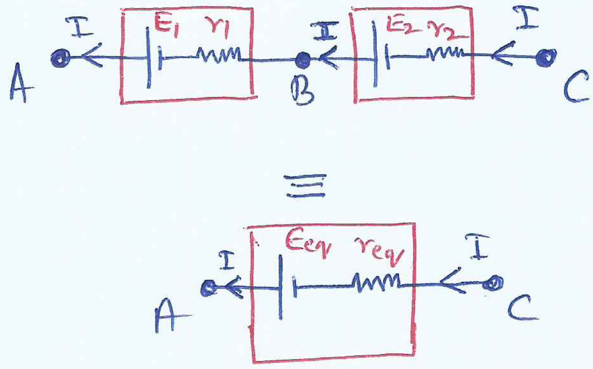
IMP

Different Cells in Series.

Consider two cells having emfs and internal resistances ϵ_1, ϵ_2 and r_1, r_2 respectively connected in Series.



V_A, V_B, V_C are potentials at A, B, C respectively.

- V_{AB} = p.d. betⁿ +ve and −ve terminals of the first cell

$\therefore V_{AB} = V_A - V_B = \epsilon_1 - I r_1$

- V_{BC} = p.d. betⁿ +ve and −ve terminals of the second cell

$\therefore V_{BC} = V_B - V_C = \epsilon_2 - I r_2$

$$\therefore V_{AC} = V_A - V_C = (V_A - V_B) + (V_B - V_C)$$
$$= V_{AB} + V_{BC}$$
$$= (\epsilon_1 - I r_1) + (\epsilon_2 - I r_2)$$
$$\boxed{V_{AC} = (\epsilon_1 + \epsilon_2) - I(r_1 + r_2)} \longrightarrow (1)$$

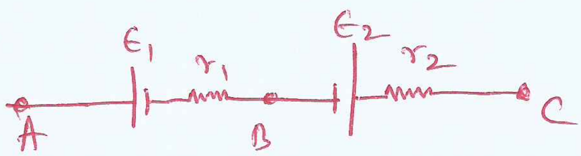
If we replace 2 cells by a single cell betⁿ A and C of emf ϵ_{eq} and internal resistance r_{eq}, we would have

$$\boxed{V_{AC} = \epsilon_{eq} - I r_{eq}} \longrightarrow (2)$$

Comparing (1) and (2), we get

$$\boxed{\epsilon_{eq} = \epsilon_1 + \epsilon_2} \quad \text{and} \quad \boxed{r_{eq} = (r_1 + r_2)}$$

- If we have cells connected as →



then $V_{BC} = -\epsilon_2 - I r_2$

$$\therefore V_{AC} = V_{AB} + V_{BC}$$
$$= (\epsilon_1 - I r_1) + (-\epsilon_2 - I r_2)$$
$$= \epsilon_1 - I r_1 - \epsilon_2 - I r_2$$
$$= (\epsilon_1 - \epsilon_2) - I(r_1 + r_2)$$

$\therefore \epsilon_{eq} = \epsilon_1 - \epsilon_2 \quad (\epsilon_1 > \epsilon_2)$

$r_{eq} = r_1 + r_2$ → which is same as previous case.

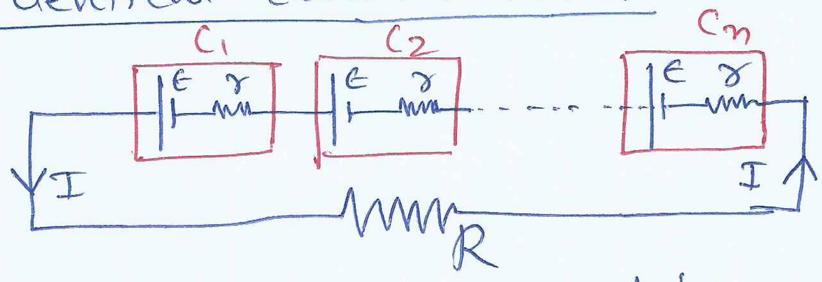
Rule:
(1) Eq. emf in series of n cells is sum of their individual emfs.
(2) Eq. internal resistance of a series combination of n cells is just the sum of their internal resistances.

P.T.O

.... Contd..

$$\boxed{\begin{aligned} \epsilon_{eq} &= \epsilon_1 + \epsilon_2 + \epsilon_3 + \cdots + \epsilon_n \\ r_{eq} &= r_1 + r_2 + r_3 + \cdots + r_n \end{aligned}}$$

Identical cells in Series.



$\epsilon_{eq} = \epsilon + \epsilon + \cdots$ upto 'n' terms $= n\epsilon$

$r_{eq} = r + r \cdots$ upto 'n' terms $= nr$

Total Resistance in the above circuit $= R + nr$

$$I = \frac{\epsilon_{eq}}{R + r_{eq}} = \frac{n\epsilon}{R + nr} \qquad \therefore \boxed{I = \frac{n\epsilon}{R + nr}}$$

Case (i): If $R >> nr$, $R + nr \approx R$

$$\therefore I = \frac{n\epsilon}{R} = \frac{n\epsilon}{R} \qquad \therefore \boxed{I = \frac{n\epsilon}{R}}$$

~~which is equal to current due to a single cell~~

$\therefore \boxed{I = n(\epsilon / R)}$ $\qquad \frac{\epsilon}{R} \rightarrow$ current due to single cell.

$\therefore$ Total current flowing in the circuit $\}$ = 'n' times the current due to a single cell.

Thus, in order to get a large amt. of current from the cells connected in Series, the ext. Resistance $R >>> nr$

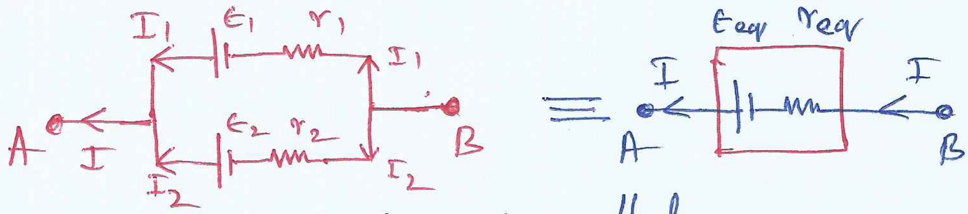
Case (ii): If $R << nr$, then $R + nr = nr$

$$\therefore I = \frac{n\epsilon}{nr} = \frac{\epsilon}{r} \qquad \therefore \boxed{I = \frac{\epsilon}{r}} \equiv \text{current due to a single cell.}$$

Thus, if large number of cells in series are connected to a very small external resistor R, then the current from combination of cells = current due to a single cell. It means, there is no use of such a combination of cells.

P.T.O.

IMP Different Cells in parallel



Consider ~~see~~ circuit where cells are connected in parallel.

If I_1 and I_2 are the currents supplied by the cells, then the resultant current $I = I_1 + I_2 \longrightarrow (1)$

Terminal voltage of the cells connected in parallel is same, Let it be V

→ for first cell $V = \epsilon_1 - I_1 r_1$ $\quad \therefore I_1 = \dfrac{\epsilon_1 - V}{r_1}$

for 2nd cell $V = \epsilon_2 - I_2 r_2$ $\quad \therefore I_2 = \dfrac{\epsilon_2 - V}{r_2}$

$$\therefore I = I_1 + I_2 = \left(\frac{\epsilon_1 - V}{r_1}\right) + \left(\frac{\epsilon_2 - V}{r_2}\right)$$

$$I = \frac{\epsilon_1}{r_1} - \frac{V}{r_1} + \frac{\epsilon_2}{r_2} - \frac{V}{r_2}$$

$$I = \left(\frac{\epsilon_1}{r_1} + \frac{\epsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$I = \left(\frac{\epsilon_1 r_2 + \epsilon_2 r_1}{r_1 r_2}\right) - V\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

$$\therefore V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \left(\frac{\epsilon_1 r_2 + \epsilon_2 r_1}{r_1 r_2}\right) - I$$

Multiplying by $\dfrac{r_1 r_2}{r_1 + r_2}$ throughout, we get

$$\boxed{V = \left(\frac{\epsilon_1 r_2 + \epsilon_2 r_1}{r_1 + r_2}\right) - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)}$$

Let parallel cells are replaced by a single cell (emf ϵ_{eq}, r_{eq})

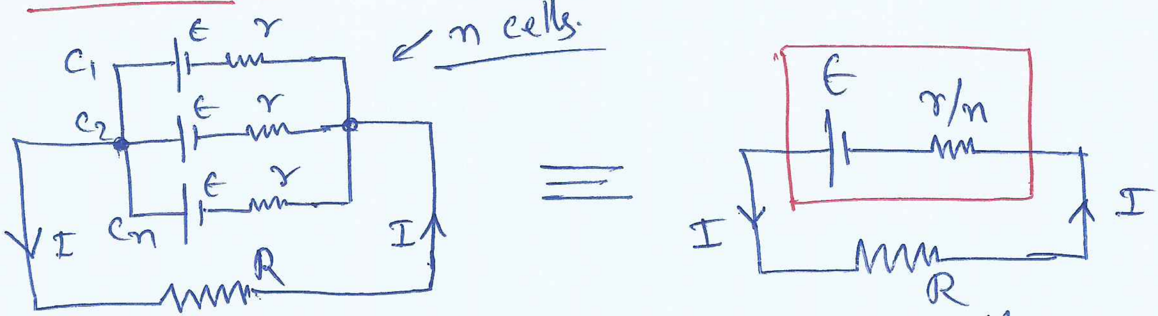
$$V = \epsilon_{eq} - I r_{eq}.$$

$\boxed{\epsilon_{eq} = \dfrac{\epsilon_1 r_2 + \epsilon_2 r_1}{r_1 + r_2}}$ and $r_{eq} = \left(\dfrac{r_1 r_2}{r_1 + r_2}\right)$ OR $\dfrac{1}{r_{eq}} = \dfrac{1}{r_1} + \dfrac{1}{r_2}$

$$\frac{\epsilon_{eq}}{r_{eq}} = \frac{(\epsilon_1 r_2 + \epsilon_2 r_1)\cancel{/(r_1 + r_2)}}{r_1 r_2 \cancel{/(r_1 + r_2)}} = \frac{\epsilon_1 r_2 + \epsilon_2 r_1}{r_1 r_2} = \frac{\epsilon_1}{r_1} + \frac{\epsilon_2}{r_2}$$

$$\longrightarrow \boxed{\epsilon_{eq} = \left[\frac{\epsilon_1}{r_1} + \frac{\epsilon_2}{r_2}\right] r_{eq}} \quad \text{IMP}$$

Identical cells in parallel



$\rightarrow$ Total emf of cells in parallel = emf of a single cell.

$$\epsilon_{eq} = \epsilon$$

$\rightarrow$ Eq. internal resistance of n cells connected in parallel is

$$\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} \text{ --- upto } n = \frac{n}{r}$$

$$\therefore \boxed{r_{eq} = \frac{r}{n}}$$

$$\therefore \text{Total Resistance} = R + \frac{r}{n}$$

$$I = \frac{\text{Total emf}}{\text{Total } R} = \frac{\epsilon}{R + \frac{r}{n}} \qquad \therefore \boxed{I = \frac{n\epsilon}{r + nR}}$$

Case (i) If $R >> r$, then $r + nR \simeq \cancel{\phi}\, nR$

$$\therefore I = \frac{\cancel{n}\epsilon}{\cancel{n}R} = \frac{\epsilon}{R} = \text{current due to a single cell.}$$

Therefore, such an arrangement of cells is of no significance.

case (ii) : If $R << r$, also $nR << r$ } then $r + nR = r$

$$\therefore I = \frac{n\epsilon}{r} = n\left(\frac{\epsilon}{r}\right) \qquad \therefore \boxed{I = n\left(\frac{\epsilon}{r}\right)}$$

Thus, I = n times the current due to a single cell. Therefore, in order to get a large current in the circuit cells may be connected in parallel to a small external resistance.

* Generally,

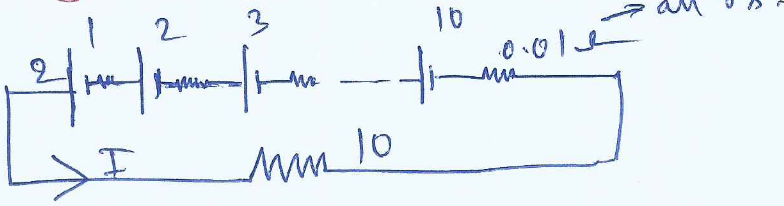
→ Series combination of cells is used when high emf is required in the circuit.

→ parallel combination of cells is used when high current is required in the circuit

→ Mixed grouping of cells is used when high power is required in the circuit.

Q! Ten cells, each of emf = 2 V and internal resistance 0·01 Ω are connected in series to provide a supply to a R = 10 Ω. What are the current and the voltage across the external resistance R.

Emf of each cells = 2 V



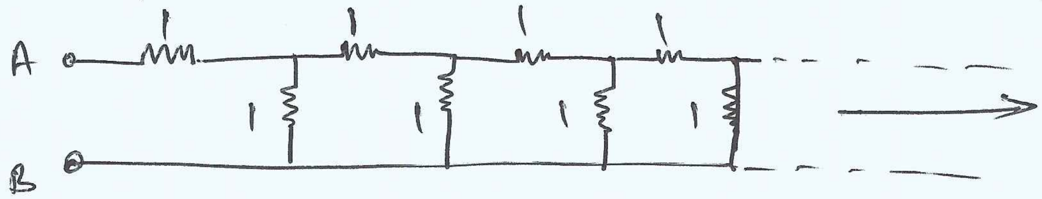
total emf = 20 V

total R = 10 + (0·01 × 10) = 10·1 Ω

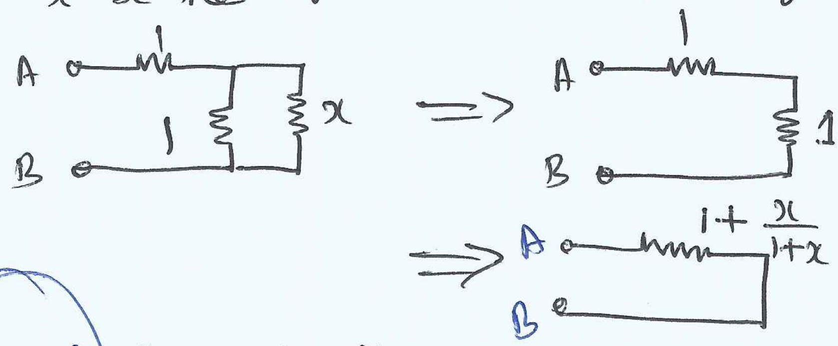
$I = \frac{20}{10 \cdot 1} = 1 \cdot 98 \text{ A}$

Voltage across R = 10 Ω = 10 × 1·98 = 19·8 V

Q: The equivalent R bet^n A and B of an infinite series (All Rs = 1 Ω)



Ans: Let x be the equivalent resistance of infinite ~~series~~ network



$1 \parallel x = \frac{x}{1+x}$

$1 \parallel x$

$\frac{1}{R'} = \frac{1}{1} + \frac{1}{x}$

$= 1 + \frac{1}{x}$

$= \frac{1+x}{x}$

$R' = (x/1+x)$

R_{EQ}: $x = 1 + \frac{x}{1+x}$

(Since it is infinite network, eq. $R = x$)

$\therefore \; x = 1 + \frac{x}{1+x} = \frac{1+2x}{1+x}$

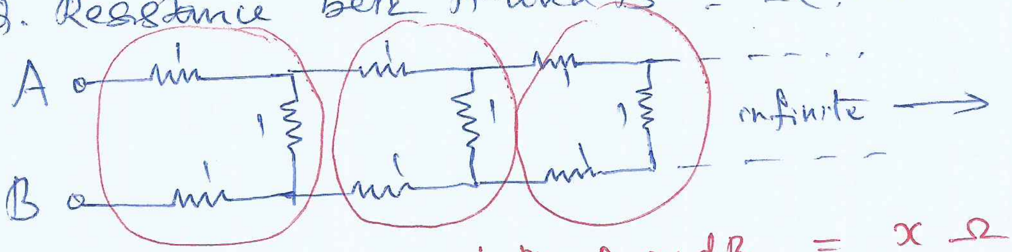
$x + x^2 = 1 + 2x$

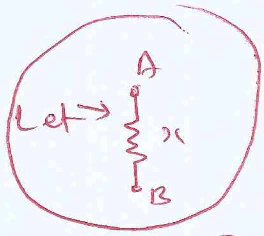
$x^2 - x - 1 = 0$

$$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2} = \frac{1+\sqrt{5}}{2} \approx 1.6\,\Omega$$

(Since negative value of R is not possible).

Q: E8. Resistance betn A and B — (All Resistances = $1\,\Omega$)



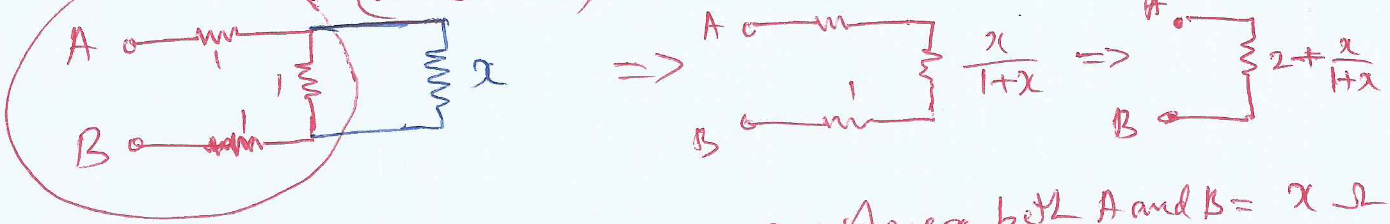


Ans: ① Let equivalent Resistance betn A and B = $x\,\Omega$

② There is symmetry in above circuit i.e. combination 3 resistors as shown in red circles.

③ ~~Therefore~~ Since, it is an infinite circuit, adding 1 more of set 3 symmetried ~~resistance~~ resistors will not alter total eq. resistance.

④ (added this)



⑤ due to ① assumption of eq. resistance betn A and B = $x\,\Omega$

$$\therefore x = 2 + \frac{x}{1+x} \implies x = \frac{2+3x}{1+x}$$

$$\implies x + x^2 = 2 + 3x \implies x^2 - 2x - 2 = 0$$

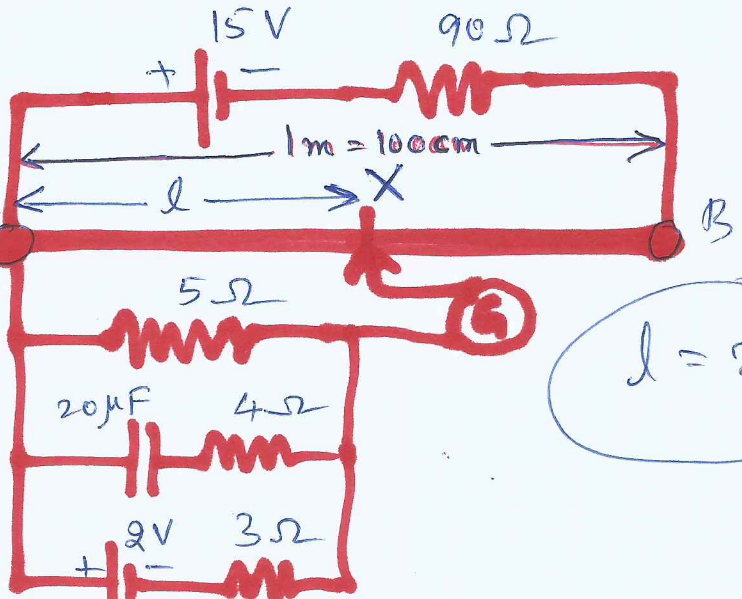
$$\therefore x = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

$(1-\sqrt{3})$ is not possible since R cannot have a negative value.

$\therefore$ R_{eff} betn A and B = $1 + \sqrt{3} \simeq 2.7\,\Omega$

Potentiometer problem.

In the circuit diagram below, the p.d. across ~~the~~ wire AB is 1 m long, and it has resistance = 10 Ω. Find the balancing length 'l' when galvanometer shows ~~null~~ null deflection?



$l = ?$

Given:

pot. wire 1 m with $R = 10\,\Omega$

$\therefore R_{AB} = 10\,\Omega$

$R_{AX} = 10\,l\,\Omega$

$\therefore l = ?$

Current in Auxillary Circuit

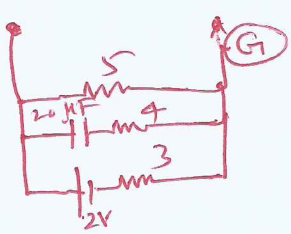
$$I = \frac{15}{R_{AB} + 90} = \frac{15}{100} = 0.15\,A$$

$$\therefore V_{AX} = R_{AX} \times I$$

$$= 10\,l \times 0.15$$

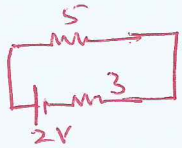
$$V_{AX} = 1.5\,l \text{ V} \longrightarrow (1)$$

To find V_{AX} : $\longrightarrow$ V_{AX} = voltage drop across 5 Ω



In this circuit, in the steady state, when capacitor is fully charged, there will be no current the branch containing C.

So, the circuit becomes (in the steady state) $\longrightarrow$



$$\therefore \text{Vol. across } 5\,\Omega = \frac{2}{8} \times 5 = 1.25 \text{ V} \longrightarrow (2)$$

From (1) and (2)

$$1.5\,l = 1.25 \text{ V}$$

$$l = \frac{1.25}{1.5} = 0.83 \text{ m} = 83 \text{ cm}.$$

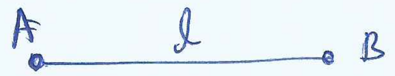
Nootan page 294

8: A potentiometer wire of cross-sectional area $8 \times 10^{-6} m^2$ carries a current ~~of 0.2 A~~ of 0.2 A. The resistivity of the material of the wire is $4 \times 10^{-7}\ \Omega\text{-}m$. The potential gradient along the wire is

(a) 0.01 V/m (b) 10 V/m

(c) 1.6×10^{-2} V/m (d) 5 V/m

Ans →



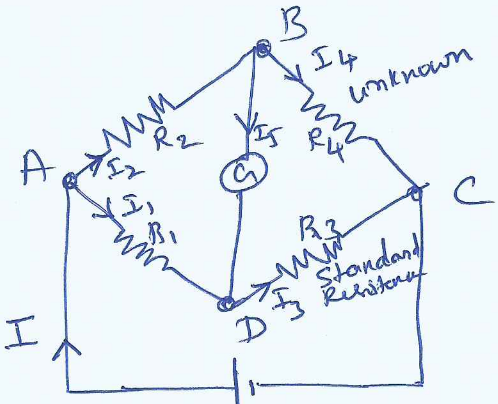
$$R_{AB} = \rho\, l/A = \frac{\cancel{4}^{\,1} \times \cancel{10^{-7}}^{\,10^{-1}}}{\cancel{8}_{\,2} \times \cancel{10^{-6}}} \times l = \frac{l}{20}\ \Omega$$

$$V_{AB} = RI = \left(\frac{l}{\cancel{20}_{\,10}}\right) \times \cancel{0.2}^{\,0.1} A = \frac{l}{100} = 0.01\, l \text{ Volts}$$

$$\therefore \text{potential gradient} = \frac{V_{AB}}{l} = \frac{0.01\, \cancel{l}}{\cancel{l}} = 0.01\ V/m$$

New Section:

Wheatstone's Bridge (current Electricity)



Assume cell has internal Resistance = 0

When the bridge is balanced after ~~after~~ ~~and resistan~~ appropriately adjusting resistors, $i_g = I_5 = 0$

(1) Using Kirchhoff's Junction rule

$$\left.\begin{array}{l} I_2 = I_4 \\ I_1 = I_3 \end{array}\right\} \text{Since } I_5 = 0 \longrightarrow (1)$$

(2) Use Kirchhoff's Loop rule: Take Loops ABDA and BCDB

ABDA: $I_2 R_2 - I_1 R_1 = 0$

BCDB: $I_4 R_4 - I_3 R_3 = 0 \longrightarrow$ using (1)

$I_2 R_4 - I_1 R_3 = 0$

$\therefore \quad I_1 R_1 = I_2 R_2$
$\quad\;\; I_1 R_3 = I_2 R_4$

$\therefore \quad \dfrac{\cancel{I_1} R_1}{\cancel{I_1} R_3} = \dfrac{\cancel{I_2} R_2}{\cancel{I_2} R_4}$

$\therefore \quad \dfrac{R_1}{R_3} = \dfrac{R_2}{R_4} \longrightarrow$ This is the balance condition to give zero or null deflection in the galvanometer G.

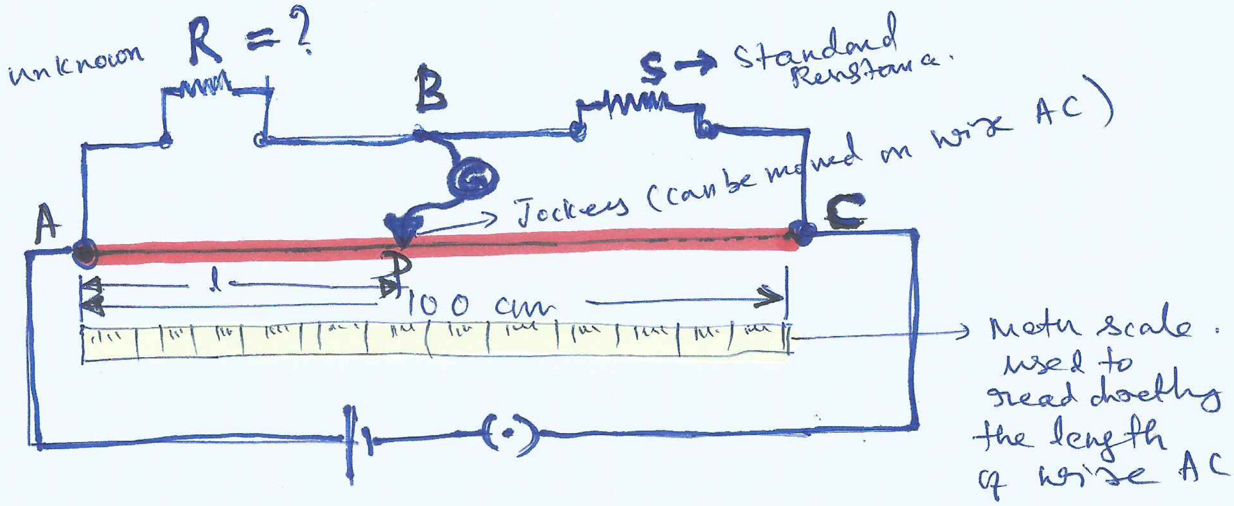
unknown Resistor $R_4 \longrightarrow \boxed{R_4 = \dfrac{R_2 R_3}{R_1}}$

> Note: Loop rule → Clockwise direction is taken for both loops ABDA and BCDB. See page # ~~29~~ 67 for explanation. → Kirchhoff's laws

Using this wheatstone bridge principle, a practical device called "Meter Bridge" is devised to find the ~~unkn~~ resistance of a conductor (or wire).

15 ~~15~~ 50 Meter bridge (practical instrument using wheatstone bridge principle)

Meter bridge is a sensitive device based on wheatstone bridge principle for the determination of resistance of a conductor (wire)



- Set some standard resistor to S
- jockey is moved on AC and fixed at a point (say D) so that current in galvanometer = 0.
- Measure AD and DC (DC = 100 − AD) in ~~cms~~ cms.
- ~~AD~~ Let AD = l, then DC = $(100-l)$
- Resistance of $l = \rho \frac{l}{A}$
- Resistance of $(100-l) = \frac{\rho(100-l)}{A}$
- As per wheatstone bridge (when balanced)

$$\boxed{\frac{R}{S} = \frac{\rho\, l/A}{\frac{\rho(100-l)}{A}} = \frac{l}{100-l}}$$

$$\boxed{R = S\left(\frac{l}{100-l}\right)}$$ IMP

Imp: Thick copper strips are used for AC. This is to minimize the resistances of connections which are not accounted for in the bridge formula

Choose various values of S, and find corresponding 'l'. and ~~ca~~ for each calculate R.

Take average R to minimize error.

IMP :→ It is shown that percentage error in R can be minimised by adjusting null point near the middle of the meter-scale. i.e when l is close to 50 cm. This requires a suitable choice of S.

Meter bridge | -51- | ~~-15a-~~ **IMP**

Question (in pre-board exam) on Meter bridge.

(a) State with the help of circuit diagram, the working principle of a meter bridge. obtain the expression for determining the unknown resistance.

(b) In the meter bridge, the balance point is found to be at 39.5 cm from the end A. The resistance of the ~~right arm~~ is 12.5 Ω. Find unknown Resistor R. why are resistors in a meter bridge connected by thick copper strips?

(c) why is it considered important to obtain the balance point near the mid-point of the wire?

(d) Determine the balance point if resistance R and 12.5 Ω are interchanged.

(e) What happens if the galvanometer and cell are interchanged at the balance point of the bridge?

Ans: (a) See page 15

(b) $\dfrac{R}{12.5\,\Omega} = \dfrac{39.5\text{ cm}}{(100-39.5)\text{ cm}}$ ∴ $R = \dfrac{12.5 \times 39.5}{60.5} = 8.16\,\Omega$

Connections are made by thick copper strips to minimize the resistances of connections which are not accounted in the meter bridge formula $\dfrac{R}{S} = \dfrac{l}{(100-l)}$.

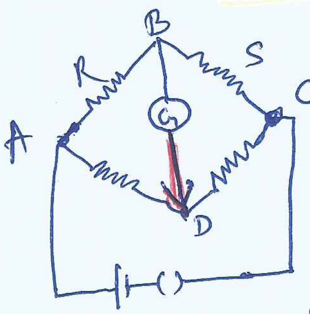
(c) Meter bridge formula $R = S\dfrac{l}{(100-l)}$. The "percentage of error in finding value R" is minimised by adjusting the null point near the middle of the meter scale (i.e. near 50 cm which requires a suitable choice to be made for standard Resistor S.

Extra info ▷ Note that it is not the sensitivity of the bridge which is increasing, it is rather reducing "percentage of error in calculating R". 'Sensitivity of bridge is maximum when resistances of all 4 arms of the bridge are almost same.

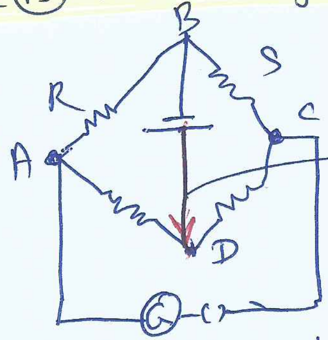
(d) On interchanging R and 12.5 Ω, the balance point will lie at 100 − 39.5 = 60.5 cm from end A.

(e) When galvanometer and cell are interchanged at the balance point of the bridge, the balanced position of meter bridge (i.e. null point on meter bridge) is not affected. The galvanometer will not show any deflection.

Extra Info | from page 15 meter bridge figure



when galvanometer and cell are interchanged →



null point on the meter bridge.

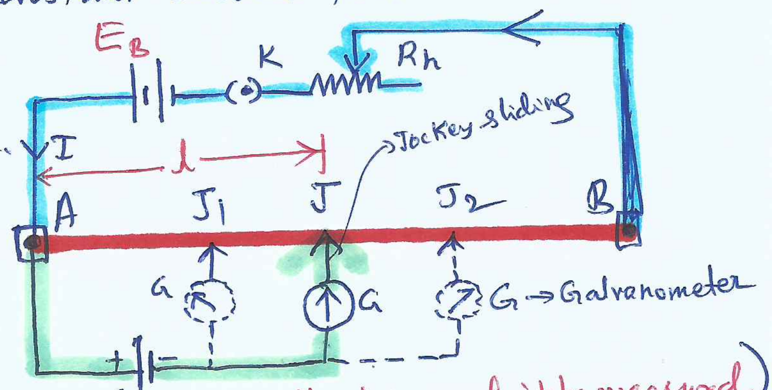
Since A and C will be at same potential, so no current will flow through galvanometer. Given bridge is comparable to balanced wheatstone bridge, galvanometer will not show any deflection.

Potentiometer (Defn)

A potentiometer is an accurate instrument for measuring the emf of a cell without drawing any current from the cell. The device can also be used to measure internal resistance of a cell; comparing emf's of two cells, comparing resistances etc.

Principle: The potentiometer works on the principle that the p.d. across any part of wire of uniform cross-sectional area is directly ∝ to the length of that portion, provided a steady constant current flows thro' the wire.

Measurement of emf of a cell



(E is a cell whose emf is to be measured)

AB is a long uniform manganin or constantan wire fixed on a wooden board.

- Set up the circuit as shown in fig.
- Adjust Rh to get a steady constant current I from battery E_B.
- Ignoring the bottom circuit below AB, the current I flows from A to B. The drop in potential per unit length of the wire is called "potential gradient" along the wire.
- We need to measure emf of an (unknown) cell E. Connect E as shown.

→ Jockey at J1 on wire AB :→ Figure shows galvanometer shows deflection indicating current in the loop AEJ_1 ⟹ current is drawn from E and hence position J1 will not give emf of E.

→ Jockey at J_2 on wire AB :→ Similarly as discussed above, galvanometer shows deflection (opposite side to first case), again indicating that at position J2, current is drawn from E and point J2 will not give emf of cell.

→ Jockey at J on wire AB : Galvanometer shows no deflection ⟹ no current is drawn from ~~E~~ E. Hence the length AJ (l) on wire AB, the p.d. betn A and J ~~gives~~ = emf of the cell E. J is called "null point".

- From ohm's law $V = IR$ $\quad (R = \rho l/A)$ $\quad \therefore V = \left(\frac{\rho l}{A}\right) I$

$$\therefore V = \left(\frac{\rho I}{A}\right) l = Kl \quad ; \text{ where } K = \frac{\rho I}{A}$$

ρ, I, A are constants.

$\therefore \boxed{V = Kl} \rightarrow (1)$: $V \propto l$ provided I, ρ, A are constants.

V in eqn (1) represents emf of the cell E

$\therefore \boxed{E = Kl}$ or $\boxed{E \propto l}$

Since at null point (J), no current is drawn from cell E (open circuit), the p.d. betn A and J gives emf E of cell E.

→ Note: To find K, use standard known emf cell in place of E (eg. Weston cadmium cell whose emf = 1.0184 V). Find the null point (length l')

$\therefore Kl' = 1.0184$ $\qquad \therefore \boxed{K = \frac{1.0184}{l'}}$ volt cm^{-1}

Sensitivity of potentiometer:

The smallest p.d. that can be accurately measured by a potentiometer is known as the "sensitivity of the potentiometer".

→ A potentiometer can be made more sensitive by decreasing the potential gradient i.e.

(a) by decreasing the potentiometer current using rheostat Rh and/or

(b) by increasing the length of wire of the potentiometer → In practice, wire of 4-12 m long uniform wire spread on a wooden board in the form of parallel pieces, each of length 1m.