Chapter 5

Chapter 5

Magnetism & Matter

Introduction :

A substance which attracts small pieces of iron, steel, nickel, cobalt etc.. and rests in the N-S direction when suspended freely is called a "magnet".

→ **Natural Magnets :** A naturally occurring iron ore called "Magnetite" can attract small pieces of iron & steel. A linear piece of this ore when suspended freely comes to rest along geographical N-S direction. These special properties of "Magnetite" is referred to as "Magnetism".

Magnetite is actually an iron oxide ore with the chemical formula Fe_3O_4. This natural magnet of dark blackish brown colour is also known as "Black stone" or "Lode stone" or "Load stone" or "leading stone" or "Kissing stone". The Greek discovered the "attractive property" of the natural magnet, whereas chinese discovered the "directive property" of this magnet. The earlier ships used magnetite to assess the direction of the travel. The natural magnets are not strong magnets. Earth itself behaves as a natural magnet having geographical N & S poles.

→ **Man-Made (called as "artificial") magnets :**

Shape of natural magnets are irregular and those magnets are also weak. When ordinary iron pieces are rubbed ~~against~~ with magnets, they acquire the attractive and directive properties. The magnets ~~thus~~ thus produced are known as artificial magnets. and the process is called "magnetisation".

Magnetisation can also be produced by passing ~~a~~ current thro' a coil wound over a piece of iron. These are called electromagnets.

A soft iron bar gets magnetised easily as compared to a steel bar but the steel bar does ~~not~~ lose magnetism readily.

The artificial magnets are stronger and can be made of many shapes (Bar magnets, mag. needle, horse-shoe magnets...etc..)

— Artificial magnet can be demagnetised by

(i) heating or by

(ii) hammering or by

(iii) throwing it repeatedly on the floor.

• We cannot isolate N & S poles of a magnet. If a bar magnet is broken into 2 halves, we get two similar bar magnets with weaker properties. Unlike electric charges, isolated magnetic N and S poles known as "magnetic monopoles" do not exist. It is possible to make magnets out of iron & its alloys.

Overview of this Chapter.

→ We begin with a description of a bar magnet and its behaviour in an external magnetic field.

→ Description of Gauss's law of Magnetism

→ Earth's magnetic field
  → Magnetic declination and dip & Hor. component of earth's field B_H
  → Magnetic Meridian

→ Important terms used in Magnetism → Very imp.

→ Mag. properties of Materials
  — Diamagnetism
  — paramagnetism
  — Ferromagnetism.

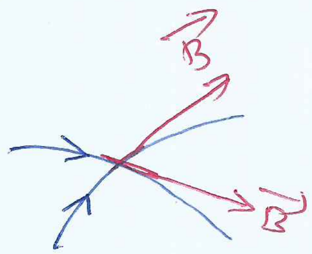
→ permanent Magnets and Electromagnets.

→ Magnetic field lines (of a bar magnet, solenoid)

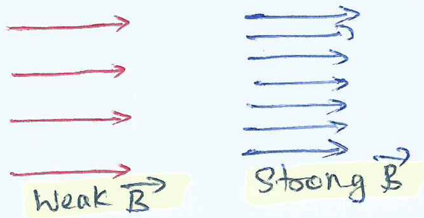
We know the arrangement of iron filings surrounding a bar magnet. The pattern mimics "magnetic field lines" (do not use word "mag. lines of force" → use only mag. field lines). The pattern suggests that ~~the~~ the bar magnet is a mag. dipole. The mag. field lines are a visual and intuitive realisation of the magnetic field.

→ properties of magnetic field lines:

- The mag. field lines of a magnet (or a solenoid) form continuous closed loops (travelling from N-pole to S-pole outside the magnet and from S-pole to N-pole inside the magnet). → This is unlike electric dipole where these field lines begin from a positive charge and end on the -ve charge or escape to infinity.
- The tangent to the field line at a given point represents the direction of the net mag. field $\vec{B}$ at that point.
- Mag. field lines do not intersect or cross each other. If they cross each other, then at the point of intersection, there will be two directions of the mag. field at a single point, which is not possible.



- The larger the number of field lines crossing per unit area, the stronger is the magnitude of the mag. field $\vec{B}$. Generally, the strength is more near the poles.



- Although mag. field lines are not real, yet they represent a magnetic field which is real.
- Mag. field lines can be plotted using compass needle whose orientation gives us the idea of the mag. field directions at various points in space.

Note :- Map. field lines inside a long Solenoid carrying current is uniform mag. field.
- Mag. field of a bar magnet is a non-uniform magnetic field.

"Magnetic Dipole" in a uniform Magnetic field.

→ A mag. dipole consists of a pair of mag. poles of equal and opposite strengths separated by a small distance.

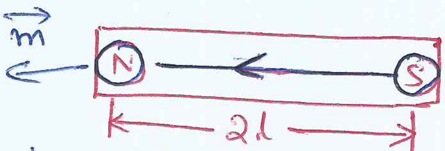
Examples of mag. dipoles are "mag. needle", "bar magnet", "current carrying solenoid" "a current loop" etc.. Atom is also considered to behave like a dipole, so the "fundamental mag. dipole in nature" is associated with the electrons.

→ Magnetic Dipole moment (Simply called as "Magnetic Moment")

Mag. dipole moment is defined as the product of ~~the~~ one of the pole strengths (q_m) and distance betn mag. poles.

* Distance betn the two poles is called mag. length and is taken as $2l$.

$$\therefore \boxed{m = q_m \times 2l}$$



* "Mag. dipole moment" is a Vector quantity.

$\vec{m} = q_m(2\vec{l})$, where $2\vec{l}$ is the mag. length from S to N pole.

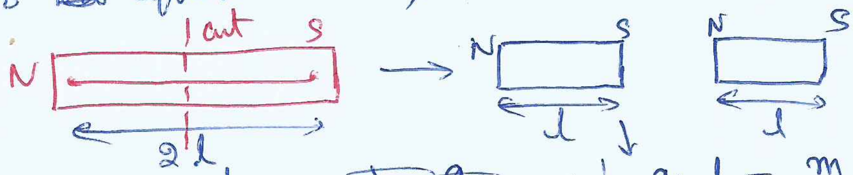
* Thus, direction of mag. dipole moment is from S to N pole

- SI unit of "mag. dipole moment" is $A\,m^2 \rightarrow [M^0 L^2 T^0 A]$
- SI unit of "pole strength" is $A\,m \rightarrow [M^0 L^1 T^0 A]$

* Geometrical length (actual length of bar magnet) is > "mag. length"

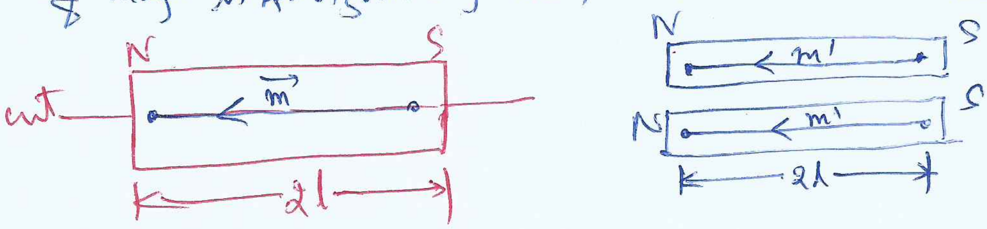
$$\frac{\text{Mag. length}}{\text{Geometrical length}} \simeq 0.84$$

* If magnet is cut into ~~two~~ equal halves, m becomes m/2



$m = q_m \times 2l$ $\quad$ ~~$m' = q_m$~~ $\quad$ $m' = q_m l = \frac{m}{2}$

* If mag. is horizontally cut, m becomes m/2



$$m' = \left(\frac{q_m}{2}\right) 2l = q_m l = \frac{m}{2}$$

pole strength reduces to half

What is magnetic moment? Several interpretations: (Unit of mag. dipole moment: $A\,m^2$)

- The magnetic moment of a magnet is a quantity that determines the **torque** it will experience in an **external magnetic field**.
  - A loop of electric current, a bar magnet, an electron (revolving around a nucleus), a molecule, and a planet all have **magnetic moments**.
- The magnetic moment may be considered to be a vector having a magnitude and direction.
- The direction of the magnetic moment points from the **South to North Pole of the magnet (inside the magnet).**
- The magnetic field produced by the **magnet** is proportional to **its** magnetic moment. More precisely, the term magnetic moment normally refers to a system's magnetic dipole moment, which produces the first term in the multipole expansion of a general magnetic field. The dipole component of an object's magnetic field is symmetric about the direction of its magnetic dipole moment, and decreases as the inverse cube of the distance from the object.
- The magnetic moment is defined as a vector relating the aligning torque on the object from an externally applied magnetic field to the field vector itself. The relationship is given by:

$\tau = m \times B$ where

τ is the torque acting on the dipole and **B is the external magnetic field**, and **m is the magnetic moment**.

m = Current I * Area A = IA ; unit is Am^2

This definition is based on how one would measure the magnetic moment, in principle, of an unknown sample.

The **preferred** classical explanation of a magnetic moment has changed over time. Before the 1930s, textbooks explained the moment using hypothetical magnetic point charges. Since then, most have defined it in terms of Ampèrian currents. In magnetic materials, the cause of the magnetic moment are the spin and orbital angular momentum states of the electrons, and varies depending on whether atoms in one region are aligned with atoms in another.

The sources of **magnetic moments in materials** can be represented by **poles** in analogy to electrostatics. This is sometimes known as the **Gilbert model**. In this model, a small magnet is modelled by a pair of magnetic poles of equal magnitude but opposite polarity. Each pole is the source of magnetic force which weakens with distance. Since magnetic poles always come in pairs, their forces partially cancel each other because while one pole pulls, the other repels. This cancellation is greatest when the poles are close to each other i.e. when the bar magnet is short.
The magnetic force produced by a bar magnet, at a given point in space, therefore depends on two factors: **the strength q_m of its poles** (**magnetic pole strength**), and the **vector l separating them**. The magnetic dipole moment m is related to the fictitious poles as $m = q_m\, l$. (where l is the distance between two poles). Unit of "pole strength q_m" = Am

- Sometimes, the above equation is also written as $m = q_m\, 2l$; *where 2l is the distance between two poles (called as "magnetic length" → taking value as 2l will be useful to calculate torque for unequal forces forming a couple).* $q_m l$
- **"m" points in the direction from South to North pole (inside the magnet).** ∴ Unit of $\vec{m}$ = Am^2

The **magnetic moment** is a quantity that represents the magnetic strength and orientation of a magnet or other object that produces a magnetic field. Examples of objects that have magnetic moments include: loops of electric current (such as electromagnets), permanent magnets, elementary particles (such as electrons), various molecules, and many astronomical objects (such as many planets, some moons, stars, etc).

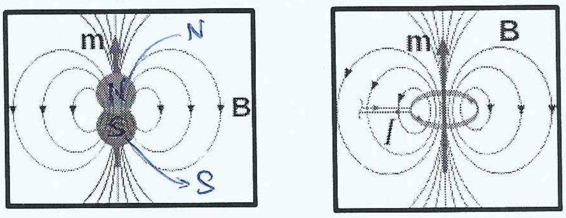
The magnetic dipole moment of an object is readily defined in terms of the **torque** that object experiences in a given magnetic field. The same applied magnetic field creates larger torques on objects with larger magnetic moments. The strength (and direction) of this torque depends not only on the magnitude of the magnetic moment but also on its orientation relative to the direction of the magnetic field. The magnetic moment may be considered, therefore, to be a vector. **The direction of the magnetic moment points from the south to north pole of the magnet (inside the magnet).**

Magnetic moment of a solenoid (m)

A generalization of the above current loop is a coil, or solenoid. Its moment is the vector sum of the moments of individual turns.

- If the solenoid has "N" identical turns (single-layer winding) and vector area "A", $\boxed{m = NIA}$ OR
- Magnetic moment in magnetism is defined as the product of the area and the current flowing through any circular coil and it is given by M= nIA. (I = current)

The **magnetic field** and **magnetic moment**, due to natural **magnetic dipoles (left)**, or an **electric current (right)**. Either generates the same field profile.



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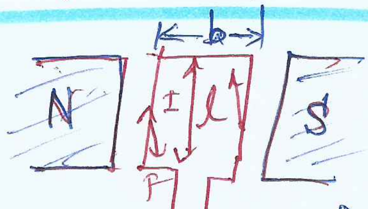
* Magnetic moment: The property of a magnet that interacts with an applied field to give a mechanical moment (torque)

* The mag. moment is a quantity that represents the magnetic strength and orientation of a magnetic material (eg. magnet) that produces a mag. field. → Examples of materials or objects that have mag. moments include:
- loops of electric current
- electromagnets, permanent magnets
- elementary particles (electrons)
- Many astronomical objects (planets, some moons, stars)

More precisely, the term "mag. moment" refers to a system's "magnetic dipole moment", the component of the mag. moment that can be represented by an equivalent "magnetic dipole"



$\vec{F} = l I B$

$\tau = \vec{F} \times$ Separation betw two forces

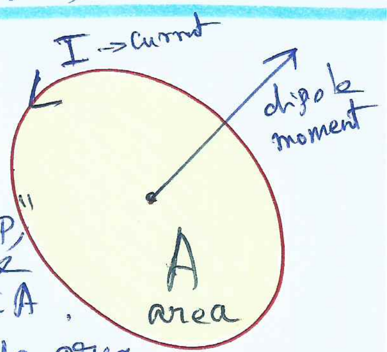
$\tau = \vec{F} \times b$

$\tau = l I B \times b$ $\quad$ ($\because lb =$ Area of rectangle)

$\tau = I A B$ $\quad$ (Mag. dipole moment $\vec{m} = I \underline{A}$; unit Am^2)

$\vec{\tau} = \vec{m} \times \vec{B}$ $\quad$ ($\tau \rightarrow$ is the torque acting on dipole)

* The strength of a magnetic dipole, called the "magnetic dipole moment" is a measure of a dipole's ability to turn itself into alignment with a given external mag. field $\vec{B}$. (IMP)



When a mag. dipole is considered as a "current loop", the magnitude of the dipole moment is α to the current multiplied by the enclosed area $|\vec{m}| = IA$.

→ The direction of dipole moment (hence '$\vec{m}$' is a vector) is ⊥r. the area.

→ In fig. I is anti-clockwise, as per RH thumb rule direction of $\vec{B}$ and $\vec{m}$ are same. (South to North). When free to ~~rota~~ rotate (eg. magnet), dipoles align themselves in the direction of external $\vec{B}$.

→ The "magnetic dipole moment" (Simply called Magnetic moment) is defined as the max. amount of torque caused by magnetic force on a dipole that arises per unit value of surrounding $\vec{B}$ in vacuum.

Units: SI unit is $A.m^2 = \frac{N.m}{T} = \frac{J}{T}$

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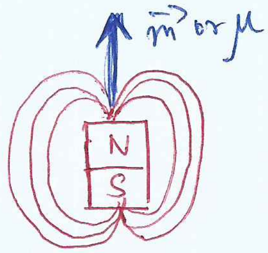
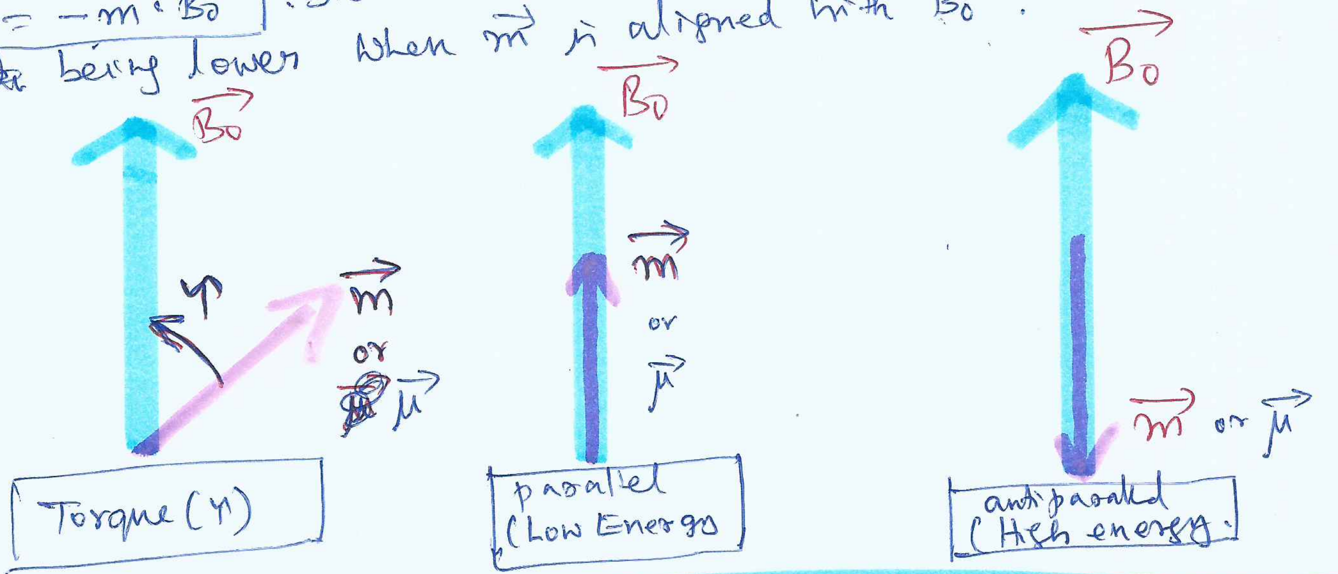


Dipole modelled as a current Loop

* mag. moment ($\vec{m}$) will seek to align with an externally applied mag. field (B_0). It ~~e~~ experiences a torque $\gamma = \vec{m} \times B_0$. / When perfectly aligned parallel to $\vec{B_0}$, $\vec{m}$ will be in its lowest energy state and experience no torque. / ~~While~~ / when pointing opposite to $\vec{B_0}$, $\vec{m}$ will be in its highest energy state because extra energy would be required to move and maintain it in this position / For any other direction, the energy (E) of the mag. moment $\vec{m}$ would be given by dot product $\boxed{E = -\vec{m} \cdot \vec{B_0}}$. The -ve sign is required to account for the energy ~~bet~~ being lower when $\vec{m}$ is aligned with $\vec{B_0}$.



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* Magnet has got two poles of equal strength & opposite direction (N, S). When it is kept in magnetic field it (magnet) experiences a torque which has got the value = ($q_m \times 2l$), where q_m = pole strength, $2l$ → length of magnet (dist betw poles). This is the magnitude of ~~dipol~~ mag. dipole moment.

IMP

* In "bar magnet", $\vec{m}$ is product of poles strength & distance between poles: $\vec{m} = q_m \times (2l)$; $2l$ → dist betw two poles.

→ In "Current carrying loop", it is product of current & area of loop: $\vec{m} = IA$

Coulomb's law of Mag. force (FY I)

The Force betn two magnetic poles of strengths q_{m_1} and q_{m_2} lying at a distance r is directly ∝ to the product of pole strengths and inversely ∝ to the square of distance betn their pole centres.

$$F \propto \frac{q_{m_1} q_{m_2}}{r^2} \quad \therefore \quad F = K \frac{q_{m_1} q_{m_2}}{r^2}$$

K → mag. force constant.

In SI unit, $K = \frac{\mu_0}{4\pi} = 10^{-7}\ \mathrm{Wb\,A^{-1}\,m^{-1}}$

μ_0 → ~~permeability~~ Absolute mag. permeability of air or free space.

$$\therefore \boxed{F = \frac{\mu_0}{4\pi} \frac{q_{m_1} q_{m_2}}{r^2}}$$

pole strength q_{m_1} or q_{m_2} unit → A m

→ If $q_{m_1} = q_{m_2} = q_m$ and $r = 1\ \mathrm{m}$

$F = 10^{-7}\ \mathrm{N}$

$q_m = \pm 1$

$\boxed{F = 10^{-7} \frac{q_m^2}{1}}$, if $q_m = \pm 1$, $F = 10^{-7}\ \mathrm{N}$

Thus, the "pole strength" is 1 Am if the pole repels equal and ~~opposite~~ similar pole with a force of 10^{-7} N, when placed 1m away from it in vacuum.

→ If q_{m_2} is a unit N-pole, then $F = \frac{\mu_0}{4\pi} \times \frac{q_{m_1} \times 1}{r^2}$

$$= \frac{\mu_0}{4\pi} \cdot \frac{q_{m_1}}{r^2}$$

$$\therefore F = \frac{\mu_0}{4\pi r^2} q_{m_1}$$

$F = q_{m_1} B$ or $F = q_m B$; $\boxed{B = \frac{F}{q_m}}$

Thus, strength of mag. field ($\vec{B}$) at any point can be defined as the ratio of force acting on an imaginary unit N-pole placed at that point to the pole strength. (q_m)

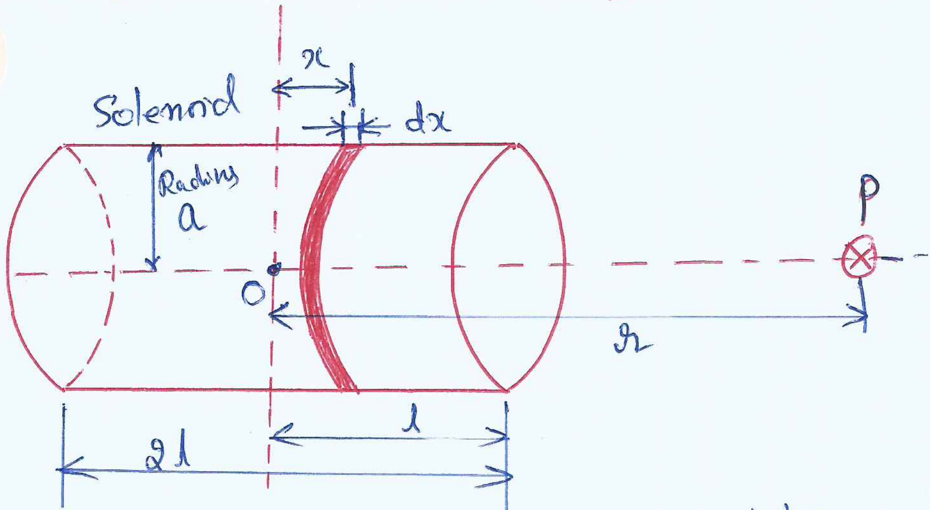
Axial field of a finite Solenoid (Sec 5.2.2 NCERT book)
(to demonstrate its similarity to that of a bar magnet)

n = No of turns/unit length

∴ No of turns in length $dx = n\,dx$

$2l = n \cdot 2l = N$

∴ $\boxed{N = 2nl}$



- As shown in figure, Consider a solenoid of length '$2l$', radius a and 'n' turns per unit length.
- The problem is to evaluate axial field at point P, at a dist r from the centre O of the solenoid. To do this, follow the procedure given below...
- Consider a circular element of thickness 'dx' of the solenoid at a distance x from centre O (see figure). This ~~cir~~ element dx consists of '$n\,dx$' turns. Let I be the current in the solenoid.
- We know that $\vec{B}$ on the axis of a circular current loop is given by

$$\boxed{B = \frac{\mu_0 N I R^2}{2(x^2+R^2)^{3/2}}} \rightarrow (1)$$

In the current problem, $N = n\,dx$, $R = a$, $x = (r - x)$ → using these eq^n (1) becomes

$$\boxed{dB = \frac{\mu_0\, n\,dx\, I a^2}{2\left[(r-x)^2 + a^2\right]^{3/2}}} \rightarrow (2)$$

The magnitude of total field $\vec{B}$ is obtained by integrating from $x = -l$ to $x = +l$

$$B = \frac{\mu_0 n I a^2}{2} \int_{-l}^{+l} \frac{dx}{[(r-x)^2+a^2]^{3/2}}$$

→ Integrating this ~~is~~ not our purpose, we will set conditions as ...

→ Consider ~~far~~ axial point i.e. $r >> a$ and $r >> l$, then the denominator within the integral $[(r-x)^2 + a^2]^{3/2}$ becomes $\simeq r^3$

$$\therefore B = \frac{\mu_0 I n a^2}{2r^3}\int_{-l}^{l} dx = \frac{\mu_0 n I a^2}{2r^3} \times 2l$$

$$\therefore \boxed{B = \frac{\mu_0 n I a^2 (2l)}{2r^3}} \rightarrow (3)$$

Multiply & divide π in eq^n (3) → $B = \dfrac{\mu_0\, n(2l)\, I\, (\pi a^2)}{2\pi r^3}$

Magnitude of mag. dipole moment of the solenoid is $m = n(2l)\, I \cdot \pi a^2$
($m = NIA$)

$$\therefore \boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}} \rightarrow (4)$$

This is the required equation for the far axial field of a finite solenoid.

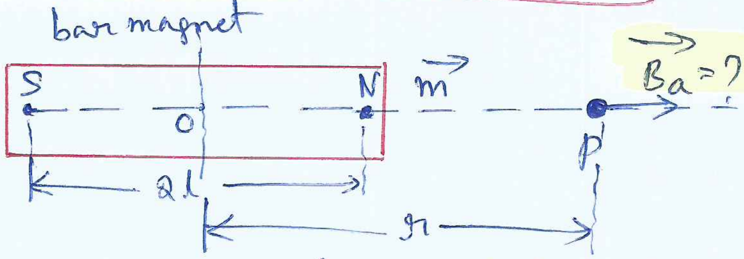
Eq (4) is same also for "far axial field" of a bar magnet. Thus bar magnet and a solenoid produce similar mag. fields. The mag. moment of a bar magnet is thus equal to the mag moment of an equivalent solenoid that produces the same magnetic field.

Info

Mag. field $\vec{B}$ at a point on the axial line of the Bar Magnet

Let O be the centre of a bar magnet of length '$2l$'. P is a point on the axial line at a dist r from O.

Let q_m be the pole strength of each pole of the magnet.



Let a unit N-pole be placed at point P. Assume that the presence of unit N-pole at P does not affect $\vec{B}$ at P due to bar magnet.

- Field at P due to N-pole of Bar Magnet, $\vec{B_1}$ = Force experienced by unit N-pole at P = $\vec{B_1} = \frac{\mu_0}{4\pi} \frac{q_m}{(r-l)^2}$ along NP.
- III'y, field at P due to S-pole of Bar Magnet $\vec{B_2} = \frac{\mu_0}{4\pi} \frac{q_m}{(r+l)^2}$ along PS

See they are opposite directions

$\therefore$ Net mag. field at P due to bar magnet is

$$B_a = B_1 - B_2 = \left(\frac{\mu_0}{4\pi}\frac{q_m}{(r-l)^2} - \frac{\mu_0}{4\pi}\frac{q_m}{(r+l)^2}\right) = \frac{\mu_0 q_m}{4\pi}\left[\frac{(r+l)^2-(r-l)^2}{(r^2-l^2)^2}\right]$$

$$B_a = \frac{\mu_0 q_m}{4\pi}\left[\frac{\cancel{r^2}+\cancel{l^2}+2rl-\cancel{r^2}-\cancel{l^2}+2rl}{(r^2-l^2)^2}\right] = \frac{\mu_0 q_m}{4\pi} \times \frac{4rl}{(r^2-l^2)^2}$$

Since $m = q_m \times 2l$

$$\boxed{B_a = \frac{\mu_0}{4\pi}\frac{2mr}{(r^2-l^2)^2} \text{ along NP}}$$

If the magnet is very small ~~leg~~ length $l^2 << r^2$

So, at far axial point $B_a = \frac{\mu_0}{4\pi}\frac{2m\cancel{r}}{r^{\cancel{4}3}}$

$$\boxed{\vec{B_a} = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{r^3}} \longrightarrow (1)$$

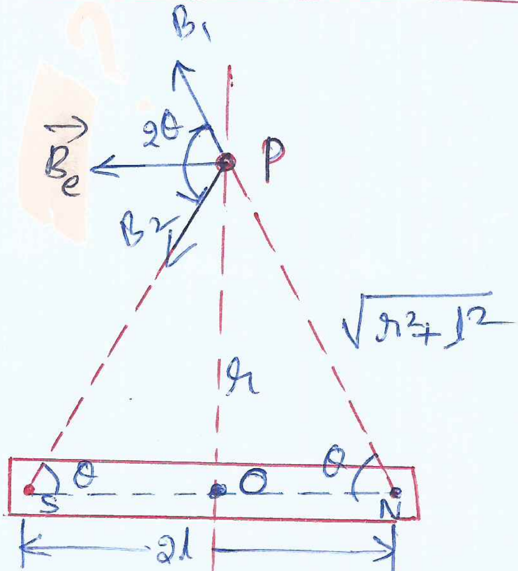
Direction of $\vec{B_a}$ is along SN extended i.e. along the direction of mag. dipole moment $\vec{m}$.

EQ (1) is same as for finite solenoid that we have proved in the previous page.

Thus, a solenoid and a bar magnet produce similar magnetic fields and they have same magnetic moments.

Info: Mag. field $\vec{B}$ at a point on Equatorial line of bar Magnet

Let p be a point on the equatorial line as shown in fig such that its distance from centre O is r.



→ $\vec{B}$ at p due to N-pole of bar magnet

$$\vec{B_1} = \frac{\mu_0}{4\pi} \frac{q_m}{(\sqrt{r^2+l^2})^2} \text{ along NP}$$

$$\vec{B_1} = \frac{\mu_0}{4\pi} \frac{q_m}{(r^2+l^2)} \text{ along NP} \longrightarrow (1)$$

→ $\vec{B}$ at p due to S-pole of bar magnet

$$\vec{B_2} = \frac{\mu_0}{4\pi} \frac{q_m}{(r^2+l^2)} \text{ along PS} \longrightarrow (2)$$

→ $\vec{B_1}$ and $\vec{B_2}$ are inclined at angle 2θ. The resultant field is

$$B_e = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos 2\theta}$$

Since $B_1 = B_2$

$$B_e = \sqrt{2B_1^2 + 2B_1^2\cos 2\theta} = \sqrt{2B_1^2(1+\cos 2\theta)}$$

$$B_e = \sqrt{2B_1^2 \times 2\cos^2\theta}$$

$1+\cos 2\theta = 2\cos^2\theta$

$$B_e = 2B_1\cos\theta \longrightarrow (2)$$

Using eqn (1) in (2), we get

$$B_e = 2\frac{\mu_0}{4\pi}\frac{q_m}{(r^2+l^2)}\cos\theta$$

; from fig $\cos\theta = \frac{l}{\sqrt{r^2+l^2}}$

$$\therefore B_e = 2\frac{\mu_0}{4\pi}\frac{q_m}{(r^2+l^2)} \times \frac{l}{\sqrt{r^2+l^2}}$$

Since $m = q_m \times 2l$

$$\boxed{B_e = \frac{\mu_0}{4\pi}\frac{m}{(r^2+l^2)^{3/2}}} \longrightarrow (3)$$

Case: $\cancel{l^2 \gg r^2}$ $r^2 \gg l^2$, "far" equatorial point

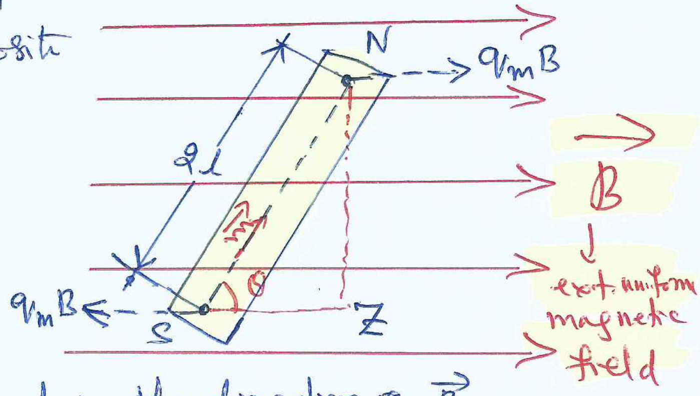
then $\boxed{B_e = \frac{\mu_0}{4\pi}\frac{m}{r^3}}$ ⟺ In contrast, for "far axial point" $B_{axial} = B_a = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$

equatorial field

$\therefore \boxed{B_a = 2B_e}$ → In a short magnet, $\vec{B}$ on axial line of magnet is twice the value on equatorial line.

Info **Derive a relation for Torque on a magnetic dipole (i.e. bar magnet) placed in a uniform mag. field.**

A bar magnet (mag. dipole) placed in a uniform $\vec{B}$ experiences equal & opposite forces.



Let mag. length of magnet = $2l$
pole strength of each pole = q_m
Strength of Mag. field = B
∠le betn $\vec{m}$ and $\vec{B}$ = θ, then

Force acting on N-pole = $q_m B$ along the direction of $\vec{B}$
Force acting on S-pole = $q_m B$ opposite to the direction of $\vec{B}$.

These two equal & opposite forces constitute a couple which tends to rotate the magnet in the direction of $\vec{B}$. Thus, bar magnet experiences a torque.

∴ Torque acting on the bar magnet or magnetic dipole is given by

τ = Magnitude of Force × ⊥r distance betn forces.

$= q_m B \times (ZN) = q_m B\,(2l \sin\theta)$

$= (q_m \times 2l)\, B \sin\theta$ ($q_m \times 2l = m$, mag. dipole moment)

∴ In Scalar form $\boxed{\tau = mB\sin\theta}$

In Vector form $\boxed{\vec{\tau} = \vec{m} \times \vec{B}}$

When B = 1 unit and $\theta = 90°$, then $\tau = m$

∴ **Defn of magnetic dipole moment** → It can be defined as the Torque acting on a magnetic dipole placed normal to a uniform magnetic field of value 1 T.

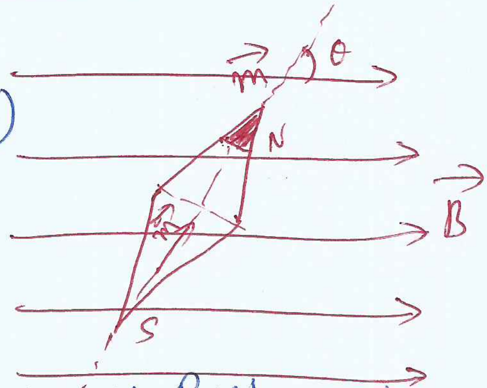
NCERT

Find magnitude of magnetic field using a compass

Consider a small compass needle of
- magnetic dipole moment $\vec{m}$
- moment of inertia I (not current)

placed in a uniform $\vec{B}$.

θ is the angle betw $\vec{m}$ and $\vec{B}$



Torque acting on needle $\tau = m \times B$

$\tau = mB \sin\theta$ → This torque is known as Restoring torque.

But $\tau = I(\alpha) = I\left(\frac{d^2\theta}{dt^2}\right)$ (where $\frac{d^2\theta}{dt^2}$ = angular acceleration α)

$\therefore mB\sin\theta = -I\frac{d^2\theta}{dt^2}$ (Note I → moment of inertia)

(-ve sign shows that restoring force acts opposite to the direction of the displacement θ of the needle)

Since θ is small, $\sin\theta = \theta$

$\therefore I\frac{d^2\theta}{dt^2} = -mB\theta \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta$ (or $\alpha \propto \theta$)

Thus, motion of needle is S.H.M.

Time period of oscillation is given by

using this

$$T = 2\pi\sqrt{\frac{\theta}{d^2\theta/dt^2}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{I}{mB}}}$$

where I → Moment of inertia.

$$\therefore \boxed{B = \frac{4\pi^2 I}{mT^2}}$$ where I = Moment of inertia. $\vec{m}$ = magnetic moment of compass needle.

Ex: A mag. needle completes 20 oscillations in 10 s when placed in a uniform B. Moment of inertia of this needle $= 10^{-5}$ kg m^2 and mag. moment $= 0.1$ A m^2. Find B?

$B = \frac{4\pi^2 I}{mT^2}$ where $I = 10^{-5}$ kg m^2, $T = \frac{10}{20} = 0.5$ s, $m = 0.1$ A m^2, we get

$$B = \frac{4 \times (3.14)^2 \times 10^{-5}}{0.1 \times 0.5 \times 0.5} = 1.58 \times 10^{-2}\ T$$

NCERT IMP

IMP

Define and derive an expression for potential energy of a magnetic dipole placed in a uniform mag. field.

※ Work done to rotate a mag. dipole in a uniform $\vec{B}$ leads to PE of the magnetic dipole.

→ If a mag. dipole of mag. dipole moment $\vec{m}$ is placed at an angle θ with respect to uniform $\vec{B}$, then torque experienced by dipole is

$$\tau = m \times B = mB \sin\theta$$

⇒ If the dipole rotates thro' an angle $d\theta$, then work done is

$$dW = \tau \, d\theta = mB \sin\theta \, d\theta$$

∴ Total work done to rotate the dipole from θ_1 to θ_2 position is given by

$$W = \int_{\theta_1}^{\theta_2} mB \sin\theta \, d\theta = mB \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = mB\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$\boxed{W = -mB\left[\cos\theta_2 - \cos\theta_1\right]}$$

By defn, PE, $U = W$

$$\therefore \quad \boxed{U = -mB\left[\cos\theta_2 - \cos\theta_1\right]} \quad 1$$

If $\theta_1 = 90^\circ$, $\theta_2 = \theta^\circ$, then $\cos\theta_1 = 0$, ~~$\cos\theta_1 = 0$~~ and $\theta_2 = \theta$.

$$U = W = -mB\cos\theta$$

$$\boxed{U = -\vec{m}\cdot\vec{B}} \quad 2$$

Sp. Cases:

① When $\vec{m}$ and $\vec{B}$ are anti-parallel $\theta = 180^\circ$, ($\cos\theta = -1$) the dipole has maximum PE and ~~it~~ it is in unstable equilibrium

$$\boxed{U = mB}$$

② When $\vec{m}$ and $\vec{B}$ are parallel, $\theta = 0$ ($\cos\theta = 1$), the dipole has minimum PE and it is in stable ~~positon~~ equilibrium

$$\Rightarrow \boxed{U = -mB}$$