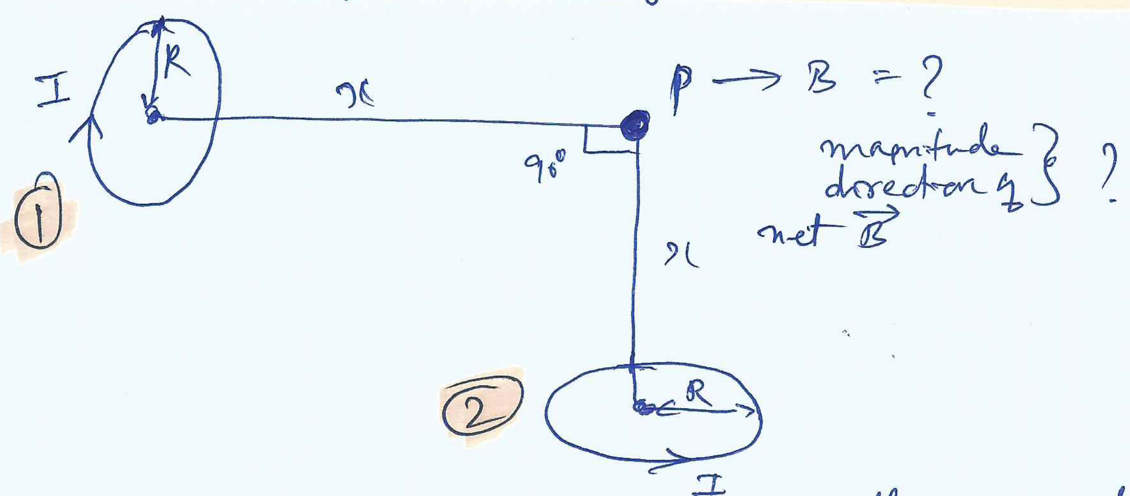
problem : Current - carrying circular loop.

problem : Two small identical circular loops carrying equal currents are placed with the geometrical axes $\perp^r$ to each other as shown in figure. Find the magnitude and direction of the net magnetic field produced at point P.



Since loops ① and ② are identical in all respects and mutually $\perp^r$ to each other, the magnitude of $\vec{B}$ will be

Take loop ① $\quad |\vec{B_1}| = \dfrac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$

and for loop ② $\quad |\vec{B_2}| = \dfrac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$

Resultant vector magnitude

$$= B = \sqrt{B_1^2 + B_2^2} = \sqrt{\frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}} + \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}}$$

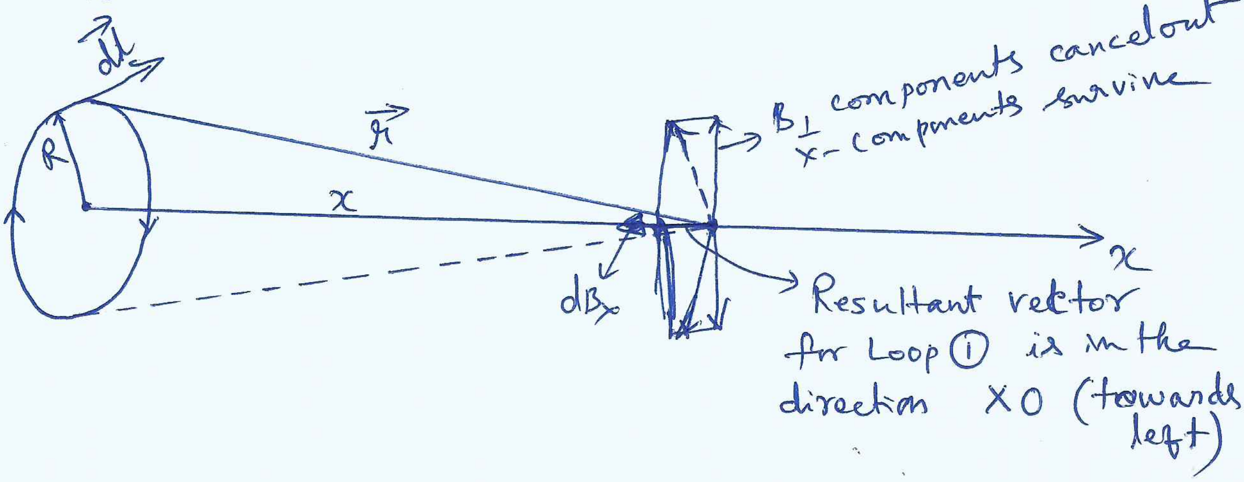
Magnitude of $\vec{B}$ as given by eqn ①.

$$\boxed{B = \sqrt{2}\left(\frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}\right)} \longrightarrow (1)$$

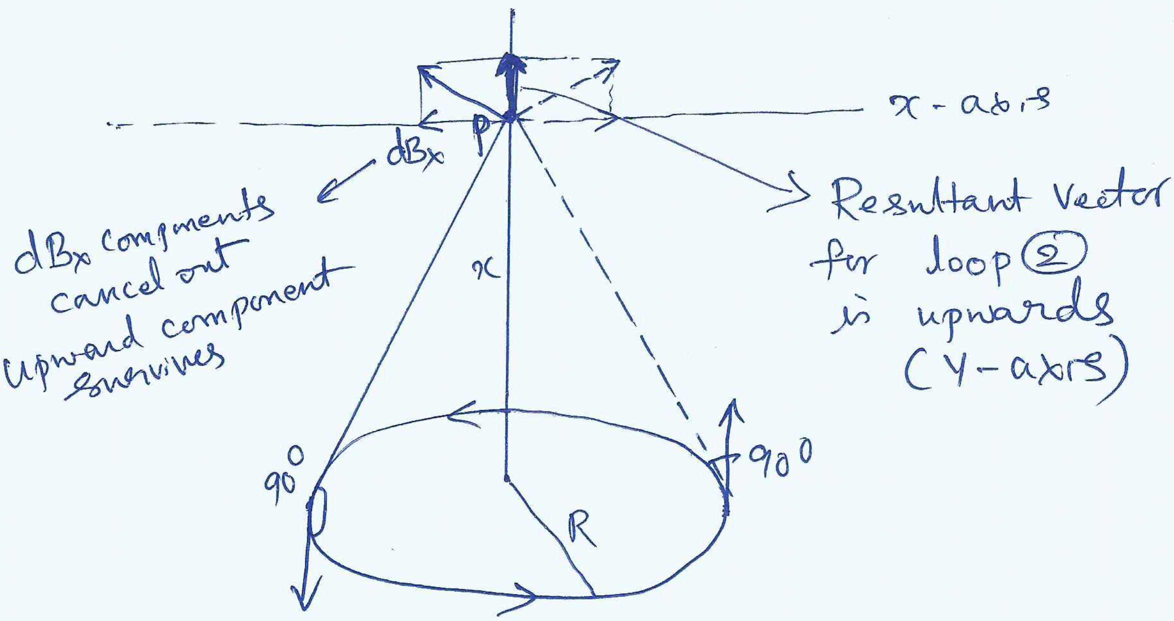
To find direction :-

P.T.O → See page ~~8b~~ 32

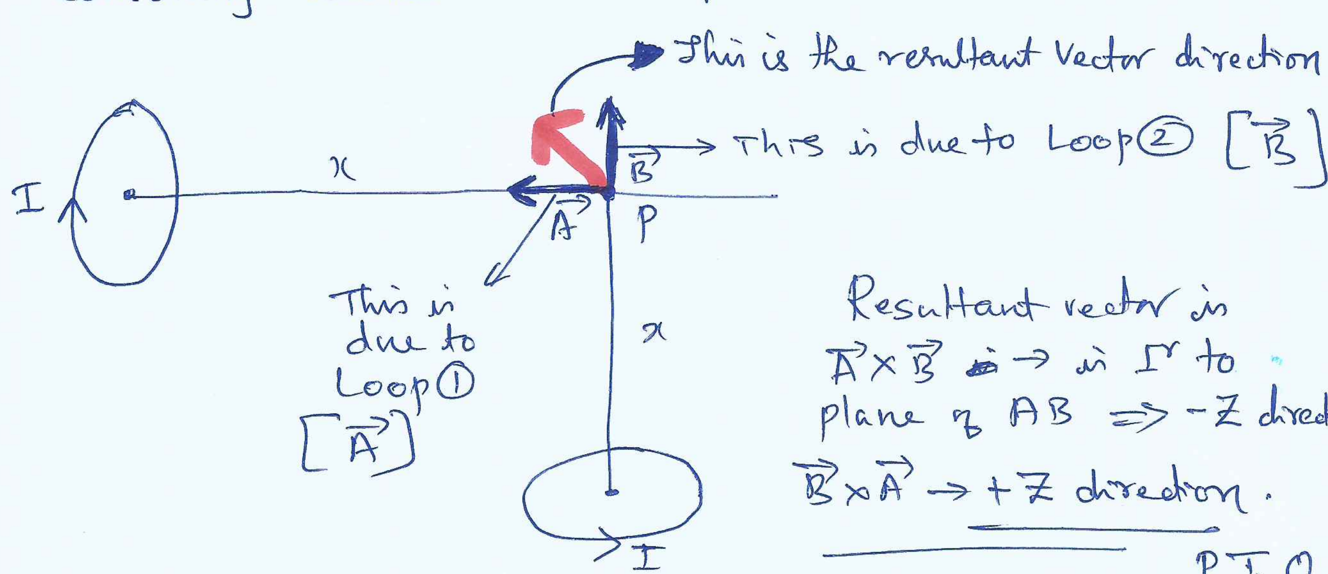
- Assume Loop ① in YZ plane and P is a point on x-axis.



- Assume Loop ② in XZ plane & P is a point on x-axis.



- Combining above two loops

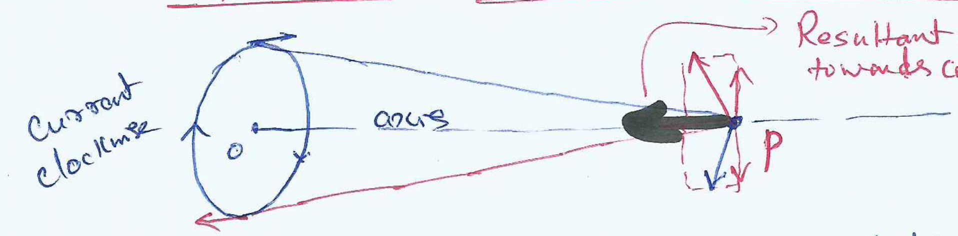


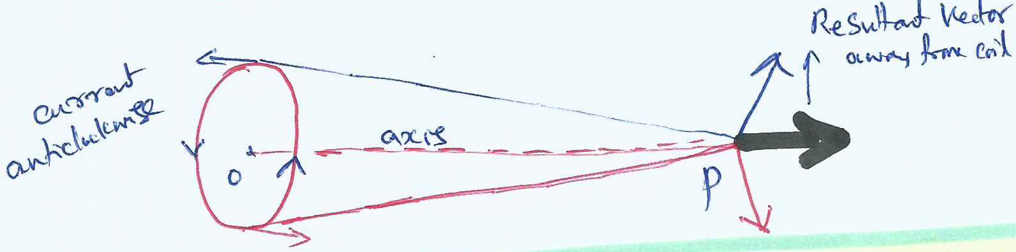
Resultant vector is $\vec{A} \times \vec{B}$ → is ⊥r to plane of AB ⇒ −Z direction.
$\vec{B} \times \vec{A}$ → +Z direction.

P.T.O.

Tips

to find direction of $\vec{B}$ due to current-carrying circular coil (plane of coil is $\perp^r$ to its axis)

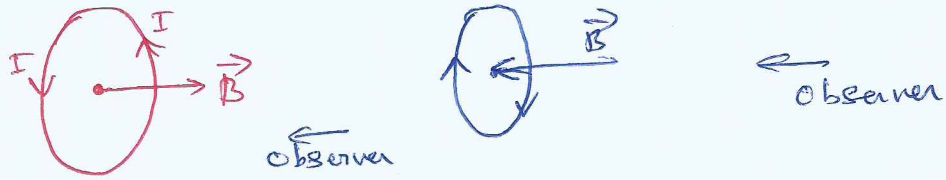


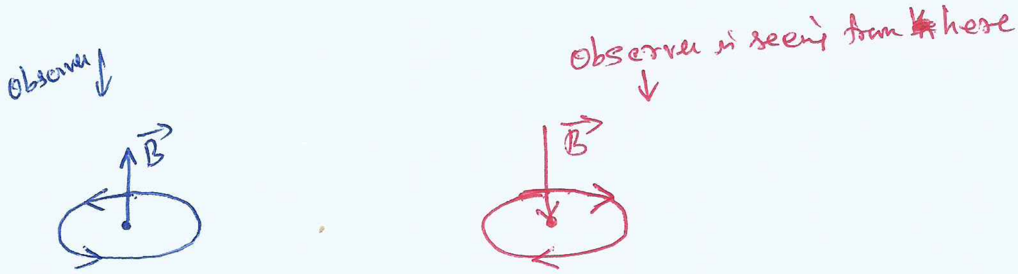


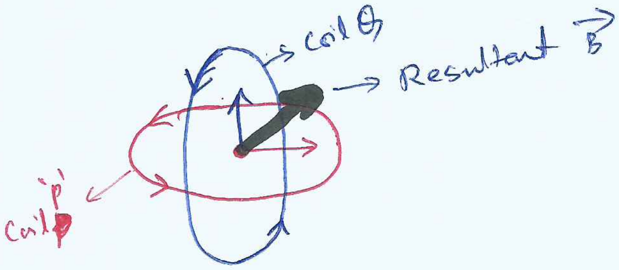
If observer facing circular coil carrying current (I)

- If I is clockwise → $\vec{B}$ towards coil (or away from observer)
- If I is anticlockwise → $\vec{B}$ away from coil (or towards observer).

Ex:





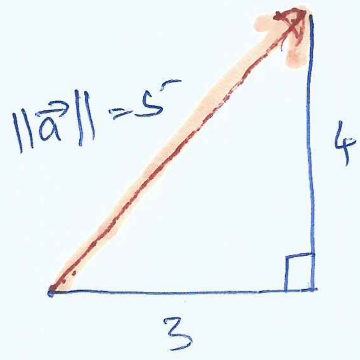


Unit Vector

For Ref

Unit vector is having magnitude of 1

eg: $\vec{a} = (3, 4)$



Magnitude of $\vec{a}$

$$= \|\vec{a}\| = \sqrt{3^2 + 4^2} = 5$$

$\therefore$ Unit vector for $\vec{a}$ is given by

$$\hat{a} = \frac{\vec{a}}{\|\vec{a}\|} = \left(\frac{3}{\|\vec{a}\|}, \frac{4}{\|\vec{a}\|}\right)$$

$$\boxed{\hat{a} = \left(\frac{3}{5}, \frac{4}{5}\right)}$$

$$\boxed{\hat{a} = (0.6, 0.8)}$$

IMP $\rightarrow$ 1

~~To verify eqn 1 whether its~~

Unit vector magnitude = 1

To verify above eq 1

$\vec{a} = \left(\frac{3}{5}, \frac{4}{5}\right)$ or $(0.6, 0.8)$

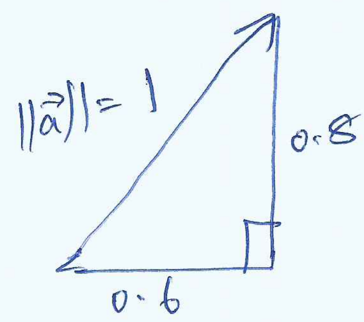
say $\vec{a}$ is a vector

$$\|\vec{a}\| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25} + \frac{16}{25}}$$

$$= \sqrt{25/25}$$

$$= 1$$



So, $\vec{a}$ is a unit vector.

35

For Ref

Two vectors

$\vec{a} = (2\hat{i} + 2\hat{j} - 5\hat{k})$

$\vec{b} = (2\hat{i} + \hat{j} + 3\hat{k})$

$\vec{c} = \vec{a} + \vec{b} = ?$

$\hat{c} = ?$

find

$\vec{c} = \vec{a} + \vec{b}$

$= (2\hat{i} + 2\hat{j} - 5\hat{k} + 2\hat{i} + \hat{j} + 3\hat{k})$

$$\boxed{\vec{C} = 4\hat{i} + 3\hat{j} - 2\hat{k}}$$

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{4\hat{i} + 3\hat{j} - 2\hat{k}}{\sqrt{4^2 + 3^2 + (-2)^2}}$$

$$= \frac{4\hat{i} + 3\hat{j} - 2\hat{k}}{\sqrt{16 + 9 + 4}} = \frac{4\hat{i} + 3\hat{j} - 2\hat{k}}{\sqrt{29}}$$

$$\boxed{\hat{c} = \left(\frac{4}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} - \frac{2}{\sqrt{29}}\hat{k}\right)}$$

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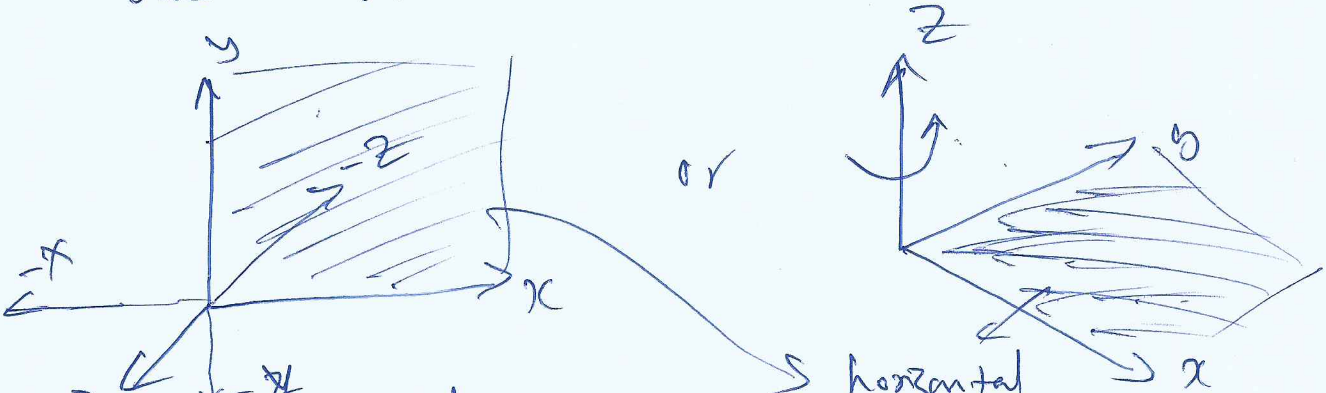
Cartesian co-ordinate systems (3-dimensions)

Two possible co-ordinate systems

① Right-handed or positive cartesian co-ordinate system

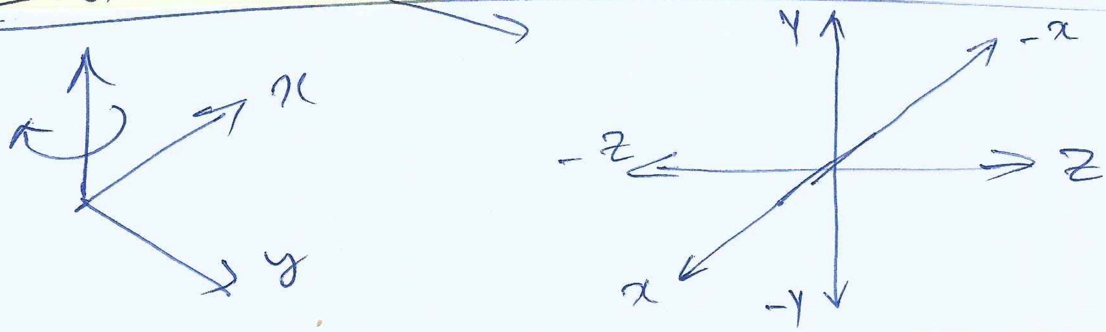
If xy plane is horizontal
and z-axis points up



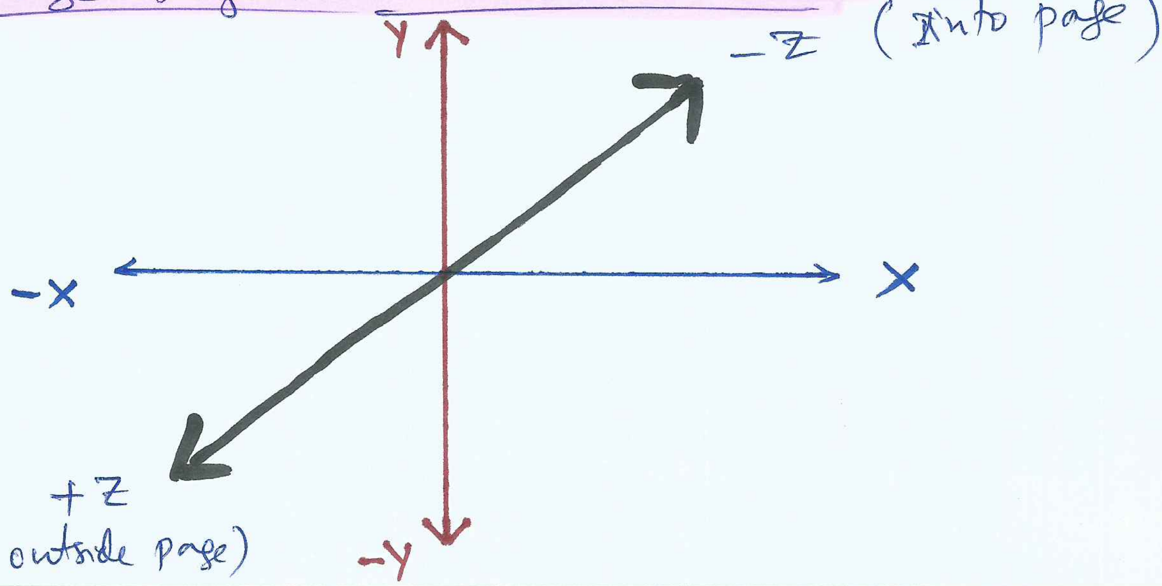
z → coming out of this page

horizontal plane

② Left-handed or negative system

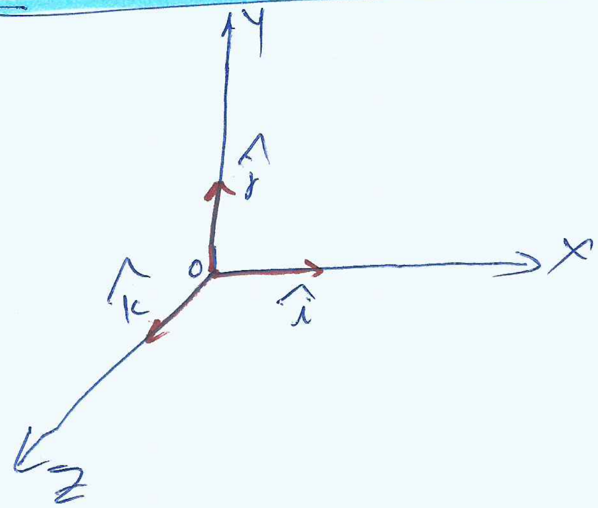
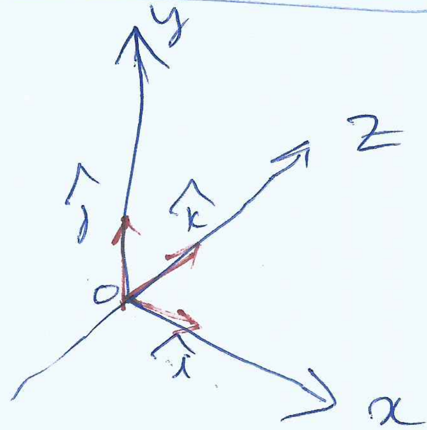
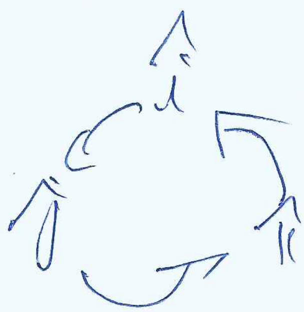
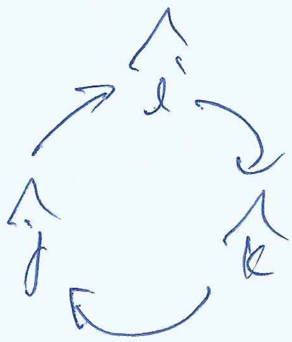


we use right-handed system



Cross product

FYI — For your Information

Right-handed co-ordinate System	Left-handed co-ordinate System.
	
$\hat{i} \times \hat{j} = \hat{k}$ $\hat{j} \times \hat{k} = \hat{i}$ $\hat{k} \times \hat{i} = \hat{j}$	$\hat{i} \times \hat{j} = -\hat{k}$ $\hat{j} \times \hat{k} = -\hat{i}$ $\hat{k} \times \hat{i} = -\hat{j}$
	

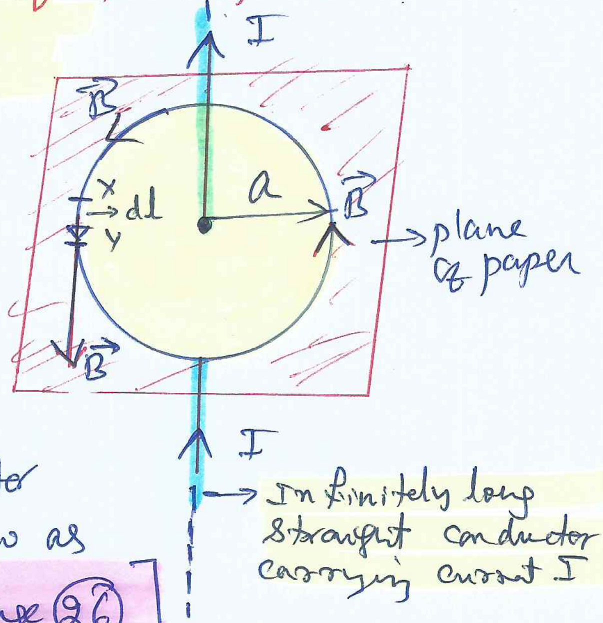
Ampere's Circuital law → This law is an alternative and appealing way in which B-S law may be expressed.

→ Ampere's Circuital Law states that the line integral of $\vec{B}$ around a closed path in free space is equal to ~~a beeolute~~ absolute permeability (μ_0) times the net current through any surface enclosed by the closed path.

Mathematically, $\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I}$ ⟶ (1)

where $\vec{B}$ is the mag. field
$\vec{dl}$ is the small element
I is the current
μ_0 = absolute permeability of free space.

→ proof : We need to prove $\oint \vec{B} \cdot \vec{dl}$ should be equal to $\mu_0 I$.



⇒ Current I in the conductor is ⊥ to paper and going upwards (See fig)

→ As per right hand thumb rule, $\vec{B}$ is concentric circles around conductor (on the plane of the paper and direction is anticlockwise as shown in figure)

→ $\vec{B}$ due to current-carrying infinite conductor at a distance 'a' is given by B-S law as

$$B = \frac{\mu_0}{2\pi}\left(\frac{I}{a}\right) \longrightarrow (2)$$ [See page 26]

→ Consider a circle of radius 'a' around the wire (called as Amperian loop). Let XY be a small element of length dl.

→ $\vec{dl}$ and $\vec{B}$ are in the same direction since direction of $\vec{B}$ is along the tangent to the ~~circle~~ circle.

$\therefore \vec{B} \cdot \vec{dl} = B\, dl \cos\theta = B\, dl \quad (\theta = 0, \cos\theta = 1)$

Taking line integral over the closed path, we get

$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl$ using eq^n (2),

$\oint B\, dl = \oint \frac{\mu_0}{2\pi} \frac{I}{a} dl = \frac{\mu_0 I}{2\pi a} \oint dl$; But $\oint dl = O^{m}$ of circle $= 2\pi a$

$= \frac{\mu_0 I}{\cancel{2\pi a}} \cdot \cancel{2\pi a} = \mu_0 I$

$\therefore \boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I}$ which is Ampere's Circuital law.

or $BL = \mu_0 I$

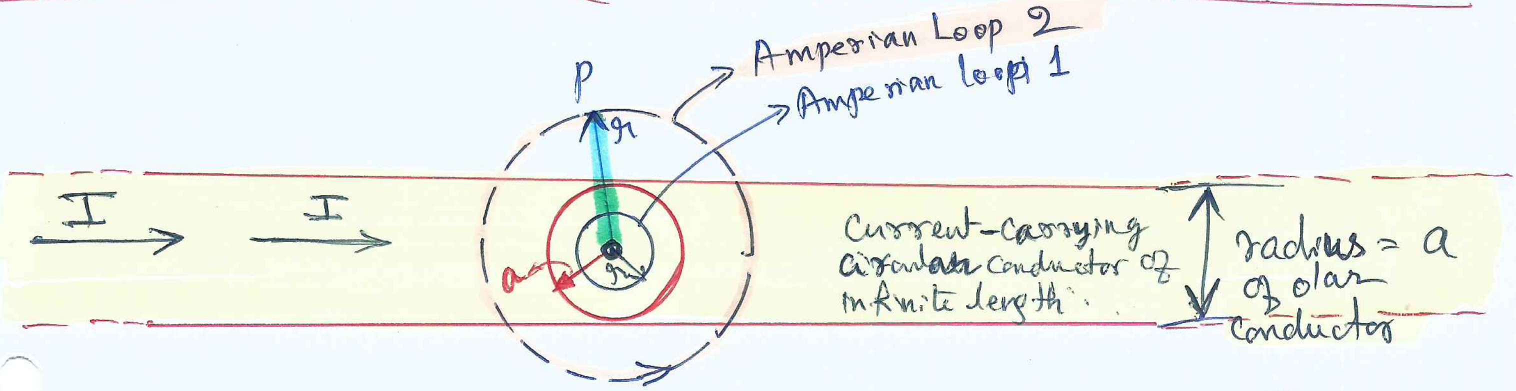
Contd....

* Limitation of Ampere's Circuital law

1. Ampere's law is not a universal law
2. Ampere's law directly deals with steady currents only.

IMP

Find $\vec{B}$ inside and outside of current-carrying circular wire of infinite length (using Ampere's law)



Case ①: ~~r<a~~ | r > a {Amperian loop #2}

For this loop #2, $\oint dl = 2\pi r$

I_e = current enclosed by the amperian loop $= I$

Ampere's eqn $B(2\pi r) = \mu_0 I_e = \mu_0 I$

$$\therefore \boxed{B = \frac{\mu_0 I}{2\pi r}} \longrightarrow ① \qquad \therefore \boxed{B \propto \frac{1}{r}} \text{ for } r > a$$

Case ② r < a {Amperian Loop # 1}

for this loop #1, let us take radius of loop #1 as "r"

Now current ~~…~~ enclosed by loop #1 I_e is not I ; $I_e < I$, by loop 1

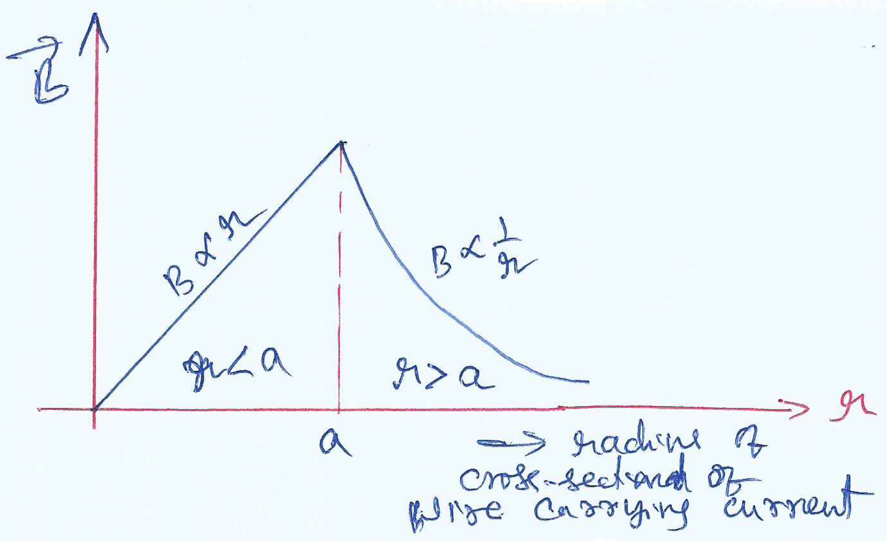
Since current distribution is uniform, the current enclosed is

$$I_e = I\,\frac{\pi r^2}{\pi a^2} = \frac{I r^2}{a^2}$$

using Ampere's law $B(2\pi r) = \mu_0\left(\frac{I r^2}{a^2}\right)$

$$\therefore \boxed{B = \left(\frac{\mu_0 I}{2\pi a^2}\right) r}$$

$$\therefore \boxed{B \propto r} \text{ for } r < a$$



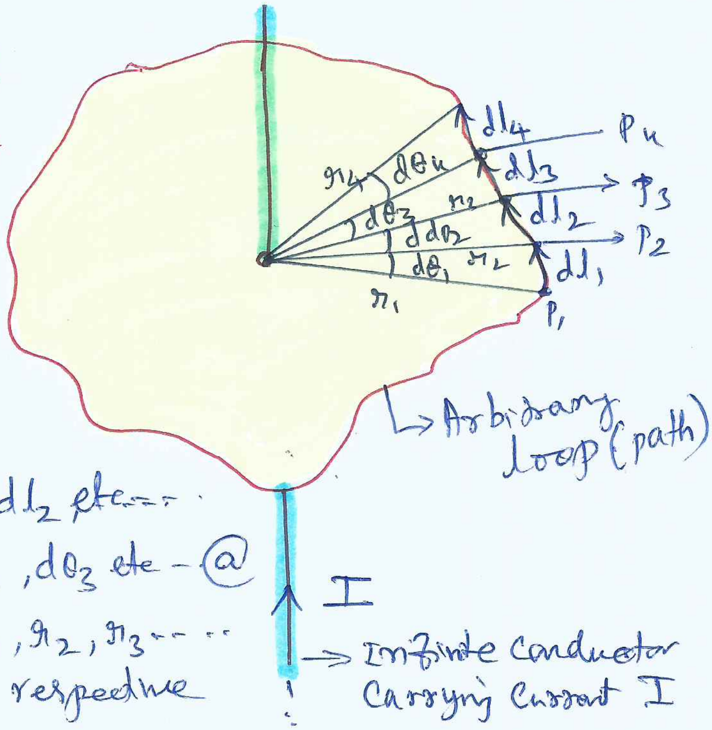
Contd...

.... Contd. from pre. page

- Note that Ampere's circuital law holds for any loop. However, it may not always facilitate an evaluation of the mag. field in ~~ea~~ every case.
- For example, (see page 29), mag. field at the centre of the loop $B = \frac{\mu_0 I}{2R}$ $\longrightarrow$ Ampere's law cannot be applied to this simple case.
- However, there exists a large number of situations of high symmetry where Ampere's law can be conveniently applied.
- Using Ampere's circuital law, we can easily calculate the mag. field produced by two commonly used and very useful magnetic systems (in electronics) $\longrightarrow$ the Solenoid and the Toroid.

IMP

prove Ampere's Circuital law $\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 I}$ for the following arbitrary loop. (ACL)



Ans: $\boxed{\text{Info}}$ In page 39, we have proved ACL for a circular loop. The problem in question is ~~a given~~ arbitrary loop (path) is given and we have to prove $\oint \vec{B}\cdot\vec{dl} = \mu_0 I$

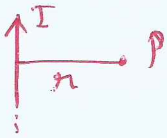
Ans → Divide the given arbitrary loop in to several infinitesimal segments dl_1, dl_2 etc.... which subtends angles $d\theta_1, d\theta_2, d\theta_3$ etc - @ the point where conductor penetrates the loop ~~to the centre of conductor~~. Let r_1, r_2, r_3 .... be the displacement vector to the respective segments.

→ As we know, there will be mag. field around the current carrying conductor. So, at P_1, P_2, P_3 ...., the field $\vec{B}$ is tangential at those points. ~~This f~~ These fields B_1, B_2, B_3 ... are parallel corresponding to segments dl_1, dl_2, dl_3 ... and hence $B_1 \cdot dl_1 = B_1 dl_1$ $(\cos 0 = 1)$

$$\rightarrow \oint \vec{B}\cdot\vec{dl} = B_1 dl_1 + B_2 dl_2 + B_3 dl_3 + B_4 dl_4 + \dots \quad \rightarrow (1)$$

→ We know from B-S law for infinite st. conductor carrying current I

$$\vec{B} = \frac{\mu_0 I}{2\pi R} \rightarrow (2)$$



Using (2) in eqn (1), we get

$$\rightarrow \oint \vec{B}\cdot\vec{dl} = \frac{\mu_0 I}{2\pi r_1} dl_1 + \frac{\mu_0}{2\pi r_2} dl_2 + \dots = \frac{\mu_0 I}{2\pi}\left[\frac{dl_1}{r_1} + \frac{dl_2}{r_2} + \dots\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[d\theta_1 + d\theta_2 + d\theta_3 + \dots\right]$$

$$= \frac{\mu_0 I}{\cancel{2\pi}} \times \cancel{[2\pi]} = \mu_0 I$$

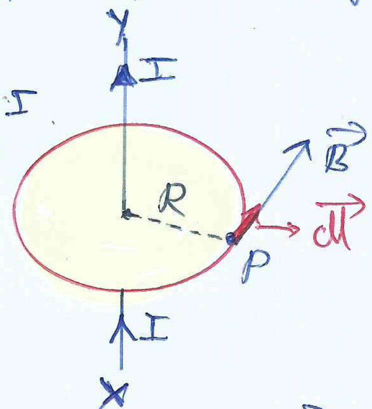
proved.

Applications of Ampere's Circuital law : Let us apply this law to compute magnetic fields due to

1. a long straight ~~wire carrying~~ current-carrying wire
2. a long straight Solenoid.
3. The Toroid.

① Field $\vec{B}$ of a Long Straight wire : (See also page 26)

Let $\vec{B}$ be the mag. field at a point P distant R from a long straight wire carrying a current.



- Let XY be a long straight Conductor carrying current I.
- We need to find $\vec{B}$ at a point P, distant R from current-carrying conductor. We can do this using B-S law also. However, Ampere's Circuital law is very simple.
- As per Ampere's Circuital Law, draw a circle of radius R around the wire. By symmetry, $\vec{B}$ is tangent to the ~~circl~~ circle everywhere. By Ampere's law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I \rightarrow (1)$$ where I is ~~the~~ the current enclosed by the circle.

- Since $\vec{B}$ and $\vec{dl}$ are along the same direction, we have

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl \quad (\text{Since } \theta = 0,\ \cos\theta = 1)$$

$$= B\oint dl \quad (\text{~~magnitude of $\vec{B}$ is un~~ Since } \vec{B} \text{ is uniform})$$

$$\oint \vec{B}\cdot\vec{dl} = B(2\pi R) \rightarrow (2)$$

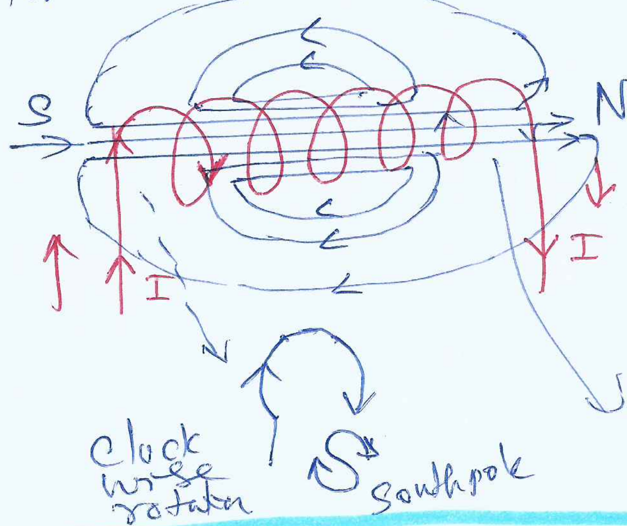
$$\therefore\ B(2\pi R) = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R}} \quad \text{or} \quad \boxed{B = \frac{\mu_0}{2\pi}\cdot\frac{I}{R}}$$

(This is same eqn as proved in page # 26)

The Solenoid: → A cylindrical coil of many tightly wound turns of insulated wire with diameter of the coil smaller than its length is called solenoid.

A solenoid has enamelled wire wound in the form of a "helix". Each turn of the solenoid can be regarded as a *olar loop*. ∴ Total Mag. field for whole solenoid = vector sum of mag. field of each turn

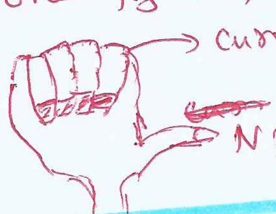


(Similar to bar magnet

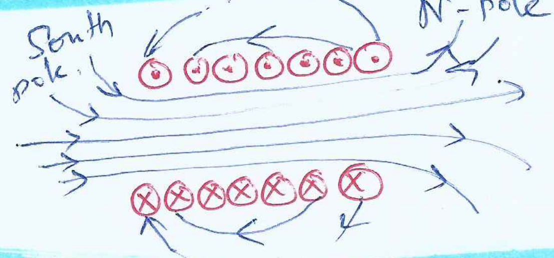
See from this direction

[Right hand grip rule → find N-pole of solenoid → In the above figure, current is going towards right

↳ current flowing like this



N pole direction (direction of $\vec{B}$ inside solenoid)



- $\vec{B}$ inside Solenoid is uniform and strong field.
- $\vec{B}$ at a point outside the long solenoid is non-uniform and weak field

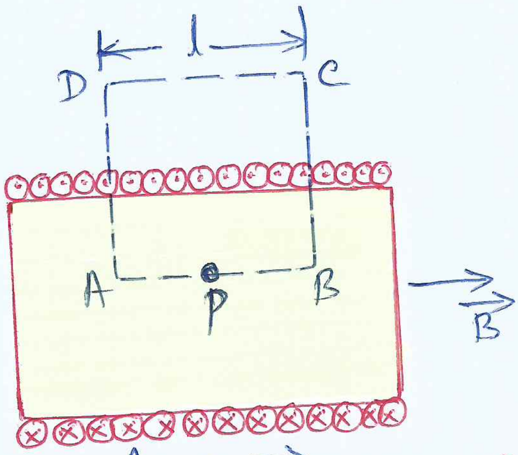
⟹ In ideal Solenoid (a solenoid in which turns are tightly packed and their number is large), the field $\vec{B}$ at a point outside the solenoid is practically zero.

→ Consider a very long solenoid having 'n' turns per unit length of solenoid. Let I be the current flowing thro' solenoid. Figure shows, inside solenoid, $\vec{B}$ is uniform & strong and directed along the axis of solenoid.

→ $\vec{B}$ outside solenoid is very weak and can be neglected ⟹ 0.

P.T.O →

<u>Step 1:</u> → Let P be a point within the solenoid, Consider $\square$'ar loop ABCD (Known as Amperian loop) passing thro' P as shown in figure →



Then $\oint \vec{B}\cdot\vec{dl}$ = Line integral of $\vec{B}$ across loop ABCD

$$= \int_A^B \vec{B}\cdot\vec{dl} + \int_B^C \vec{B}\cdot\vec{dl} + \int_C^D \vec{B}\cdot dl + \int_D^A \vec{B}\cdot dl \longrightarrow (1)$$

→ $\vec{B}$ is $\perp^r$ to paths BC and AD ∴ ∠le betn $\vec{B}$ and $\vec{dl}$ = 90° ∴ cos 90° = 0

$$\therefore \int_B^C \vec{B}\cdot\vec{dl} = \int_D^A \vec{B}\cdot\vec{dl} = 0$$

→ Since path CD is outside Solenoid, we have taken B as zero outside solenoid, so $\int_C^D \vec{B}\cdot\vec{dl} = 0$.

→ For path AB, the direction of $\vec{dl}$ and $\vec{B}$ are same, θ = 0°

∴ eqn (1) becomes

$$\oint \vec{B}\cdot\vec{dl} = \int_A^B \vec{B}\cdot\vec{dl} = \int_A^B B\, dl \cos\theta = \int_A^B B\, dl$$

or $\oint \vec{B}\cdot\vec{dl} = B\int_A^B dl$ (Since B is uniform inside Solenoid)

But $\int_A^B dl$ = total length of AB = l ⟶ (2)

$$\therefore \oint \vec{B}\cdot\vec{dl} = Bl$$

<u>Step 2:</u>— As per Ampere's Circuital law, $\oint \vec{B}\cdot\vec{dl} = \mu_0 I_e$

$\oint \vec{B}\cdot\vec{dl}$ = μ_0 × net current enclosed by loop ABCD

= μ_0 × number of turns in loop ABCD × I

= $\mu_0 (nl) I$ ⟶ (3)

(n = no of turns per unit length)

<u>Step 3</u> → Comparing (2) and (3)

$B\cancel{l} = \mu_0 n \cancel{l} I$ ∴ $B = \mu_0 n I$

Thus, magnetic field well within an infinitely long solenoid is given by $\boxed{B = \mu_0 n I}$ ⟶ (4)

P.T.O →