LIGHT

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→ Ray optics → up to page 81

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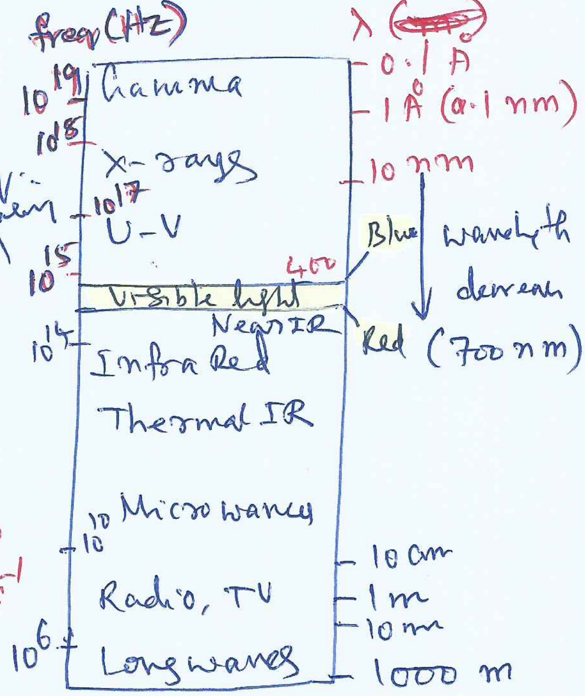
— A —

LIGHT

Light :

Nature has endowed the human eye with the sensitivity to detect e-m waves within a small range of the e-m spectrum. Electromagnetic radiation of wavelength bet^n 400nm to 750 nm is called LIGHT. It is mainly through vision that we know and interpret light and the sense of vision the world around us.



From common experience, we had concluded two things about "Light"

1. Light speed is $3\times10^{8}\ ms^{-1}$ (in vacuum)
2. Light travels in straight line

- IMP: Speed of light ≅ $3\times10^{8}\ ms^{-1}$ in vaccum. Generally for air also it is considered equal to $3\times10^{10}\ ms^{-1}$

Notation of Speed of light = c

$c = 3\times10^{8}\ ms^{-1}$

- In other materials ex. glass, water etc.. speed of light will be reduced. $c = f\lambda$ λ = wave length → length of a given wave in its own time period.

f = freq. of the given wave.

- Visible light (approx) 400 nm to 800 nm or 4000 Å to 8000 Å | $1\ nm = 10^{-9}\ m$ | $1\ Å = 10^{-10}\ m$

Knowing λ, we can calculate freq. using $c = f\lambda$

- 400 nm → 800 nm (λ)
  8×10^{5} GHz → 4×10^{5} GHz (f)

- We have indicated that light is a wave (with wavelength) and also told that light goes in st. line (rectilinear propagation of light. How to reconcile the two facts → Answer is λ is too small in the visible range compared to the size of the objects around us → then assumed that light travels from ~~po~~ one point to another in st. line. The line joining two points → the path is called a "ray of light" and bundle of such "rays" are called "beam of light".

— B —

Ray optics (Geometrical optics)

→ Sign convention (applicable for both mirrors and lens)

→ Reflection → plane surfaces

→ spherical surfaces (The "normal" in spherical case is to be taken as normal to the tangent to surface at the point of incidence).

→ Derivation → Relation betn f and R < Concave mirror / Convex mirror

→ Derivation → Mirror formula or Mirror equation

- → case (i) using concave mirror (Real image)
- → case (ii) — " — (Virtual image)
- → case (iii) using convex mirror.

— Derivation → magnification < Concave Mirror Case / Convex Mirror Case

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→ Refraction → Snell's law and Refractive index ($\mu = c/v$)

→ Lateral shift

→ Apparent depth vs real depth

- Advanced Sunrise and delayed sunset due to atmospheric refraction.

→ Total Internal reflection, critical angle

→ Total internal reflection in nature

→ Applications (~~fibre~~ optical fibre)

→ ~~Refraction~~ → Derivation — Refraction at a spherical surface

→ Derivation → Refraction by a lens

- → Lens formula derivation
- → Len's maker's formula

→ Image formation by Lens (convex and concave)

→ power of Lens $P = \frac{1}{f}$ in dioptre (D)

→ combination of thin lenses in contact (f, D and m)

→ Refraction through prism → μ of prism derivation $\boxed{n_{21} = \frac{A + D_m}{A}}$

→ Dispersion by a prism → Some natural phenomena due to sunlight

- → Rainbow
- → Scattering of light.

→ The eye → myopia, hypermetropia, astigmatism, least distance of distinct vision (or near point)

→ Microscope < Simple microscope / compound microscope } Derivation & construction

→ Telescope — Derivation and construction.

Ray Optics

Thin Lens formula: $\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$

Magnification $= m = \frac{h_i}{h_o} = \frac{v}{u}$

- For Convex lens, take f as +ve
- For Concave lens, take f as −ve

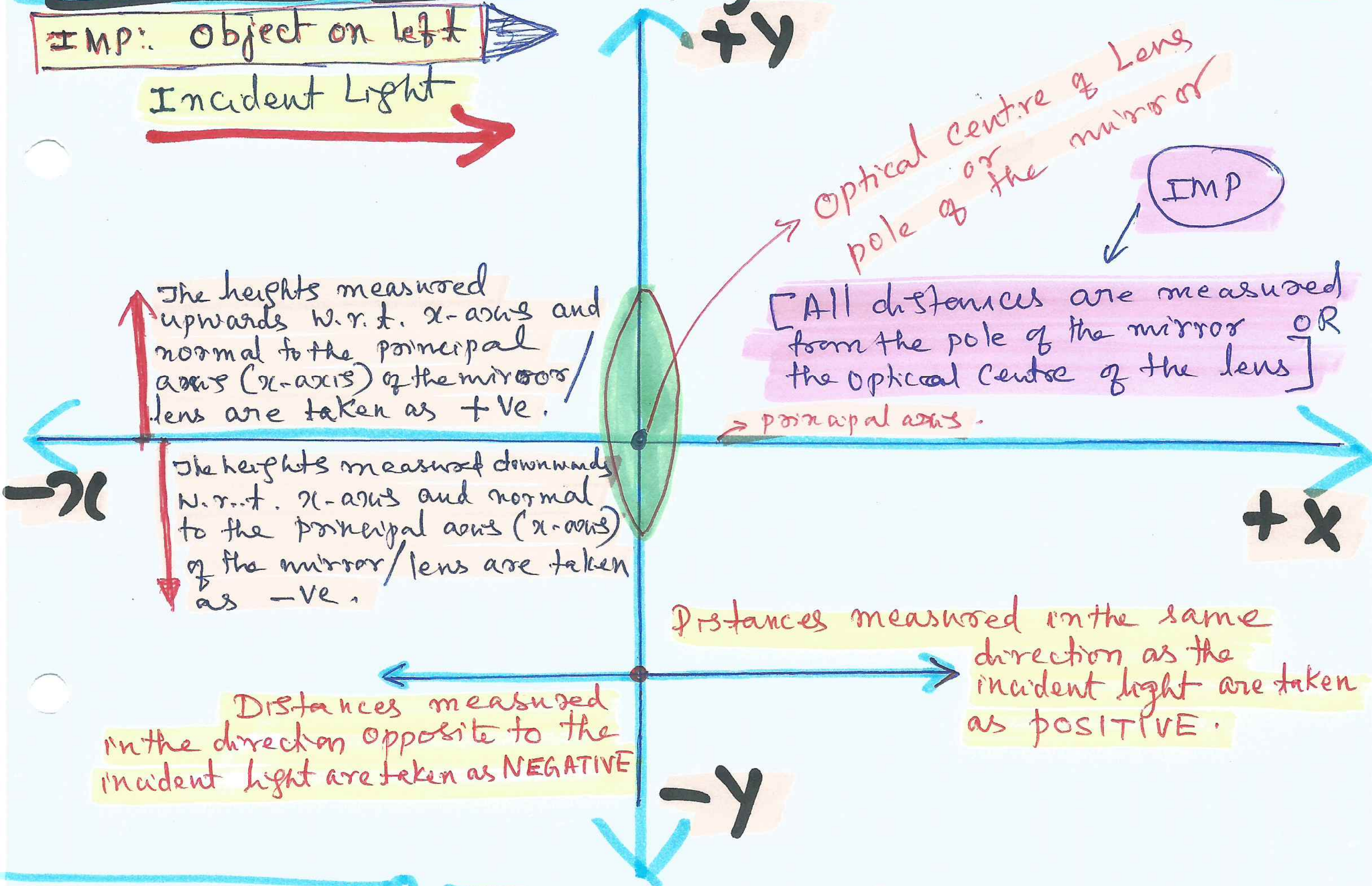
Mirror formula: $\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$

Magnification $= m = \frac{h_i}{h_o} = -\frac{v}{u}$

For Convex Mirror, take f as +ve

For Concave Mirror, take f as −ve

Use Cartesian Sign convention



IMP: ① All distances are measured from the pole of the mirror or the optical centre of the lens.

② Above Convention should be used to solve problems.

③ Note that if the problem gives "Convex lens" with a value of focal length f, then f should be taken as POSITIVE in the formula $\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$ Ex: if $f = 1\,m$, then in formula, use $f = +1\,m$

④ if concave lens is given, then f should be taken as −ve.
ex: If $f = 1\,m$, then in formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, use $f = -1\,m$.

~~Specioal~~ Sperical Mirrors (Concave or Convex).

Mirror Formula

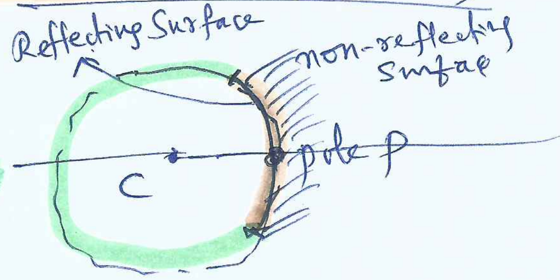
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$m = \frac{h_i}{h_o} = -\frac{v}{u}$$

f is taken as -ve for concave mirror problems

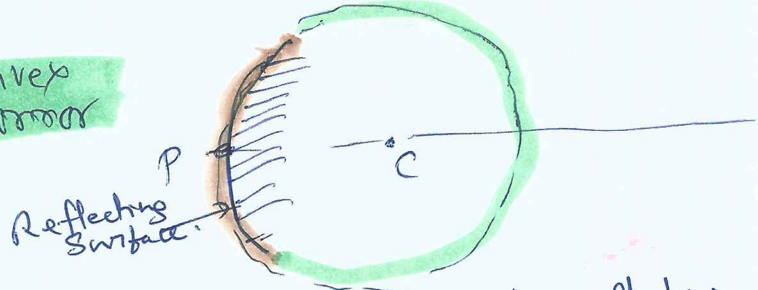
f is taken as +ve for convex mirror problems

Concave mirror



In Concave mirror, the reflecting surface is towards the centre of the sphere of which mirror is a part.

Convex mirror



In convex mirror, the reflecting surface is away from centre of sphere of which mirror is a part.

Sign convention – Mirrors :

1. Light rays are allowed to fall on the mirror from left side.
2. All distances are measured from the pole of the mirror.
3. The distances measured in the direction of the incident rays are taken as positive, while those measured in the direction opposite to the direction of the incident rays are taken as negative.
4. The lengths (heights) measured upward ⊥r to principal axis of the mirror are taken as positive, while those measured downward are taken as negative.

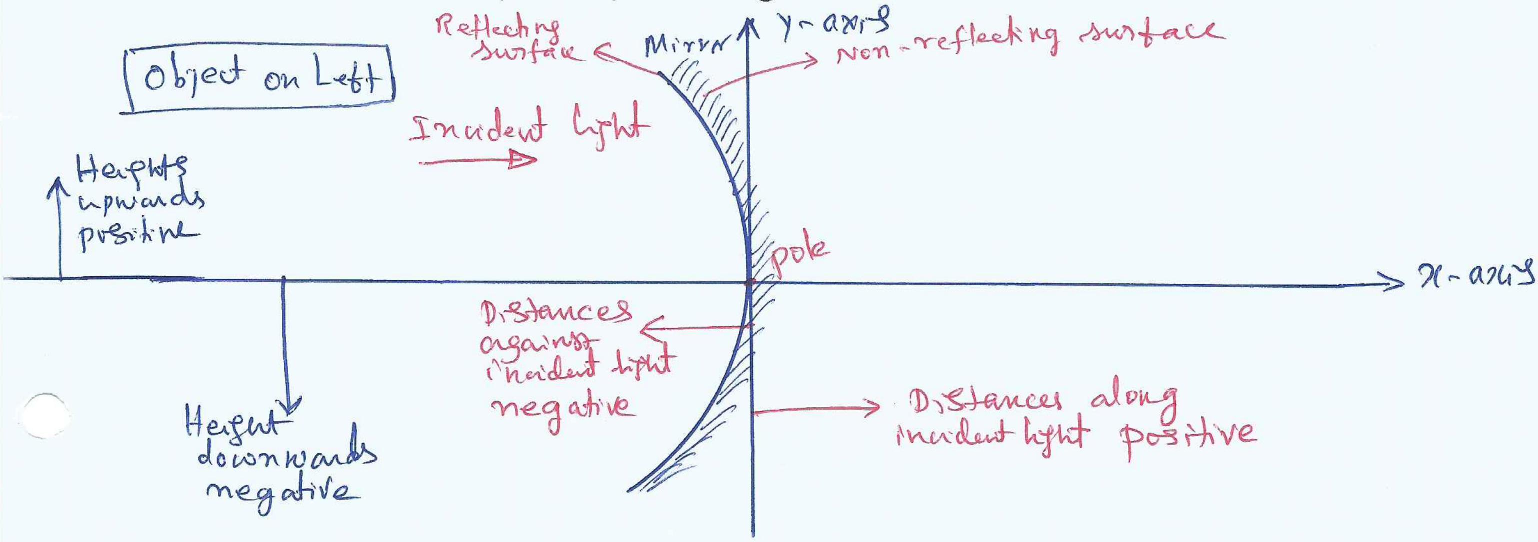
- As per ① ② and ③, f and radius of curvature of a concave mirror are negative
- As per ① ② and ③, f and Radius of curvature of a Convex mirror ~~are~~ are positive

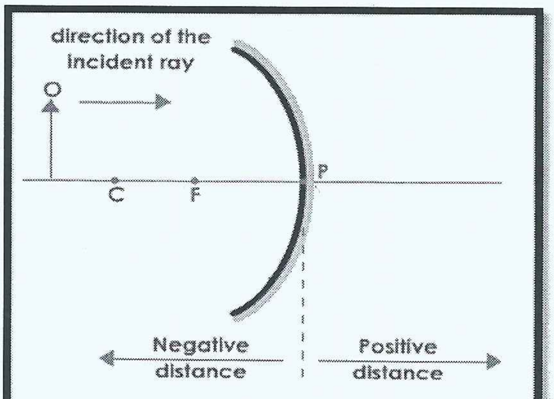
Thin Lens formula and Mirror equation

SIGN CONVENTIONS (using Cartesian co-ordinate system)

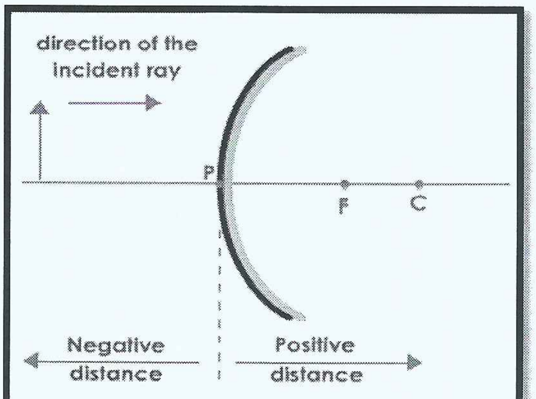
The following sign conventions are used for measuring various distances in the ray diagrams of spherical mirrors/Lens

- All distances are measured from the pole P of the mirror or from optical centre of the lens.
- Distances measured in the same direction as the incident ray are taken as POSITVE and the distances measured in the direction opposite to that of the incident rays are taken as NEGATIVE.
- The heights measured upwards with respect to x-axis and normal to principal axis (x-axis) of the mirror or lens are taken as POSITVE. The heights measured downwards are taken as NEGATIVE.
- For convex lens/mirror, focal length f should be taken as POSITVE
- For concave lens/mirror, focal length f should be taken as NEGATIVE



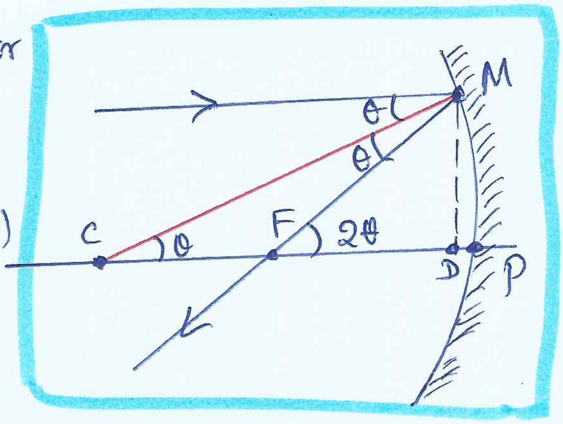


Concave Mirror



Convex Mirror

Relation between f and Radius of curvature (R) – Ex Concave mirror



- Let C be the Centre of curvature of the mirror
- Let F be the focus of the mirror
- Consider a ray parallel to principal axis striking mirror at M. Then CM is ⊥r to the mirror at M (will be normal for reflection)
- Reflected ray MF goes through focus F.

From diagram, $M\hat{C}P = \theta$ and $M\hat{F}P = 2\theta$

$\tan\theta = \frac{MD}{CD}$ and $\tan 2\theta = \frac{MD}{FD}$ for small θ, $\tan\theta = \theta$

$\therefore \theta = \frac{MD}{CD}$ and $2\theta = \frac{MD}{FD}$

$\theta = \frac{MD}{CD}$ and $2\theta = \frac{MD}{PF}$

For small aperture and for paraxial rays, M is close to P, then $FD = FP$ or $DF = PF$, $CD = PC$

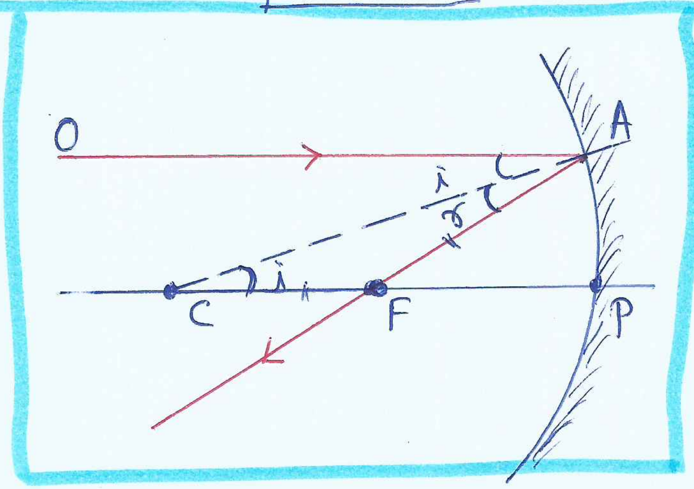
$2 \cdot \frac{\cancel{MD}}{CD} = \frac{\cancel{MD}}{PF}$

$\frac{2}{PC} = \frac{1}{PF} \Rightarrow PF = \frac{PC}{2}$

$PF = -f$
$PC = -R$

$-f = \frac{-R}{2} \quad \therefore \boxed{f = R/2}$

Another Method



- OA is incident on mirror at A and parallel to principal axis
→ CA is normal to the mirror at A.
→ $O\hat{A}C = C\hat{A}F$ (Reflection law)
$O\hat{A}C = A\hat{C}F$ (Alternate angles)
$\therefore A\hat{C}F = C\hat{A}F$
⇒ for Δ^{le} CAF, AF = FC (Isosceles Δ^{le})

For small aperture and paraxial rays, A is close to P and we can put $AF \simeq PF$

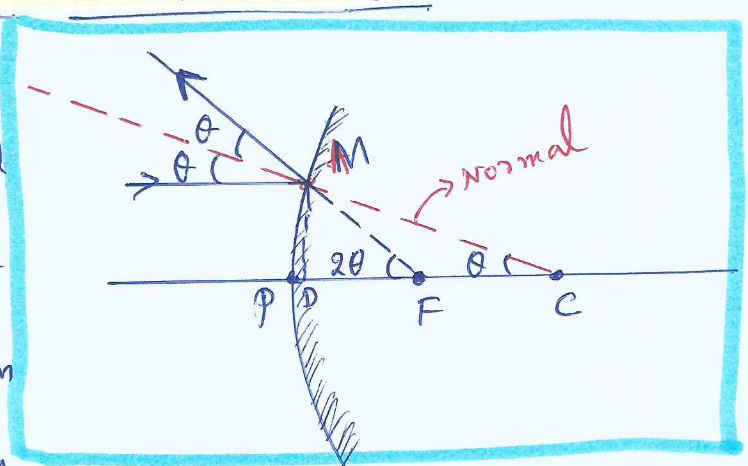
$\therefore PF = FC \qquad (PC = PF + FC)$

$\therefore PF = \frac{1}{2} PC$

Apply cartesian sign convention
$PF = -f \;;\; PC = -R$

$-f = \frac{1}{2}(-R) \qquad \therefore \boxed{f = \frac{R}{2}}$

Relation between f and R — Ex. Convex Mirror



- Let C be the centre of curvature of mirror
- Let F be the focus of mirror
- Consider a ray parallel to principal axis striking mirror at M. Then CM is $\perp^r$ to mirror at M (will be normal for reflection)
- Reflected ray appears to come from focus F
- From diagram, $M\hat{C}P = \theta$ and $M\hat{F}P = 2\theta$

$\tan\theta = \frac{MD}{DC}$ and $\tan 2\theta = \frac{MD}{DF}$ for small θ, $\tan\theta = \theta$

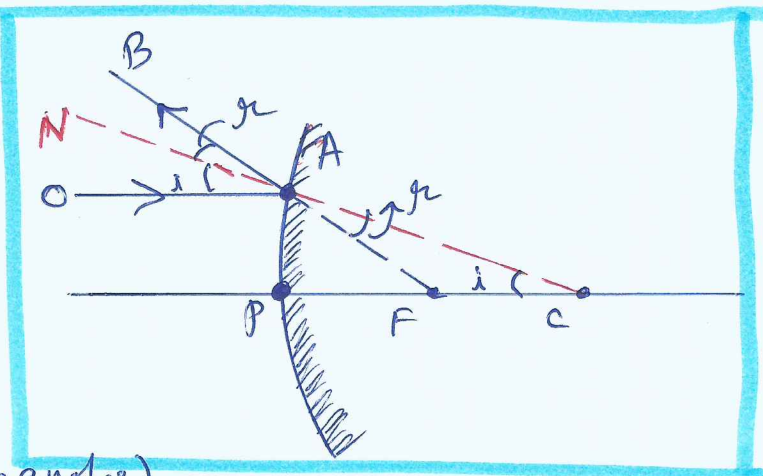
$\therefore \theta = \frac{MD}{DC}$, $2\theta = \frac{MD}{DF}$ | for small aperture and for paraxial rays, M is close to P, then $DF = PF$ and $DC = PC$

$\theta = \frac{MD}{PC}$; $2\theta = \frac{MD}{PF} \longrightarrow 2 \times \frac{\cancel{MD}}{PC} = \frac{\cancel{MD}}{PF}$

$\therefore PF = \frac{PC}{2}$ $\qquad PF = +f$; $PC = +R$

$$\boxed{f = R/2}$$

Another Method



→ OA parallel to principal axis is incident on mirror at A

→ Reflected ray AB appears to come from the focus F.

→ line NC is normal to the mirror at A

$\therefore O\hat{A}N = N\hat{A}B$ (law of reflection)

$O\hat{A}N = F\hat{C}A = i$ (corresponding angles)

$N\hat{A}B = C\hat{A}F = r$ (Vertically opposite angles)

$\therefore$ In Δ^{le} ACF $\quad \hat{A} = \hat{C}$ (Since $i = r$)

this implies for Δ^{le} ACF, $AF = FC$ (Isoceles Δ^{le})

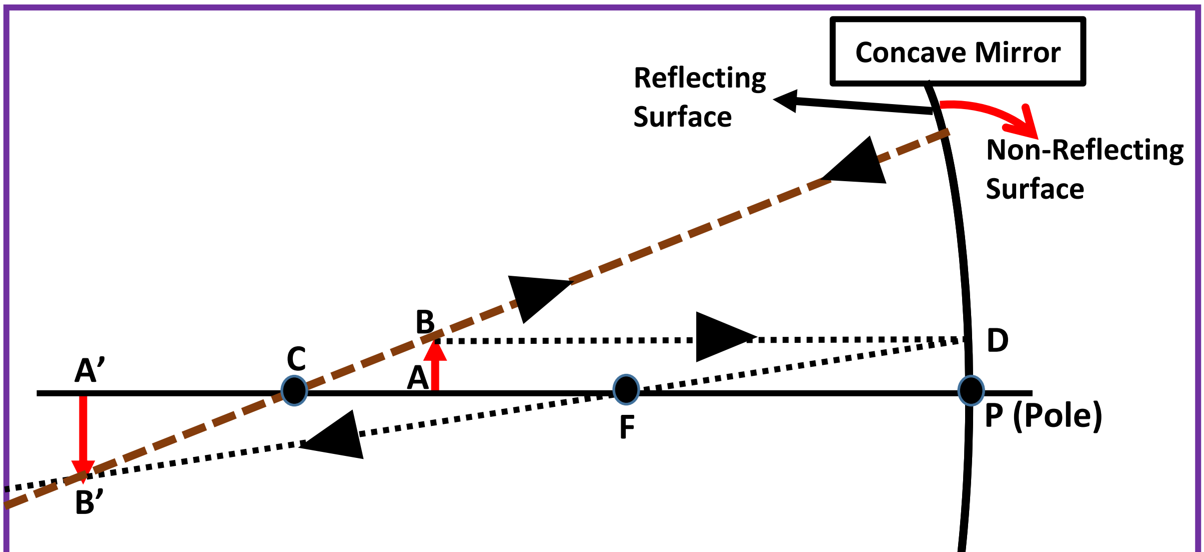
For small aperture and paraxial rays, A is close to P and we can put $AF \doteq PF$

$\therefore PF = FC$ $\qquad$ ($PC = PF + FC$)

$\therefore PF = \frac{1}{2} PC$ $\qquad$ Applying cartesian sign convention, $PF = +f$; $PC = +R$

$$\therefore \boxed{f = \frac{R}{2}}$$

Mirror Formula: Concave Mirror:
Example Case : Real Image (Object between C and F)



Let AB be the Object at a distance u from the pole P

Let A'B' be the Image at a distance v from the pole P

C is the Centre of Curvature and F is the Focus point on the principal axis

To prove Mirror Formula, we need to consider similar triangles centered at F and C that includes object and image.

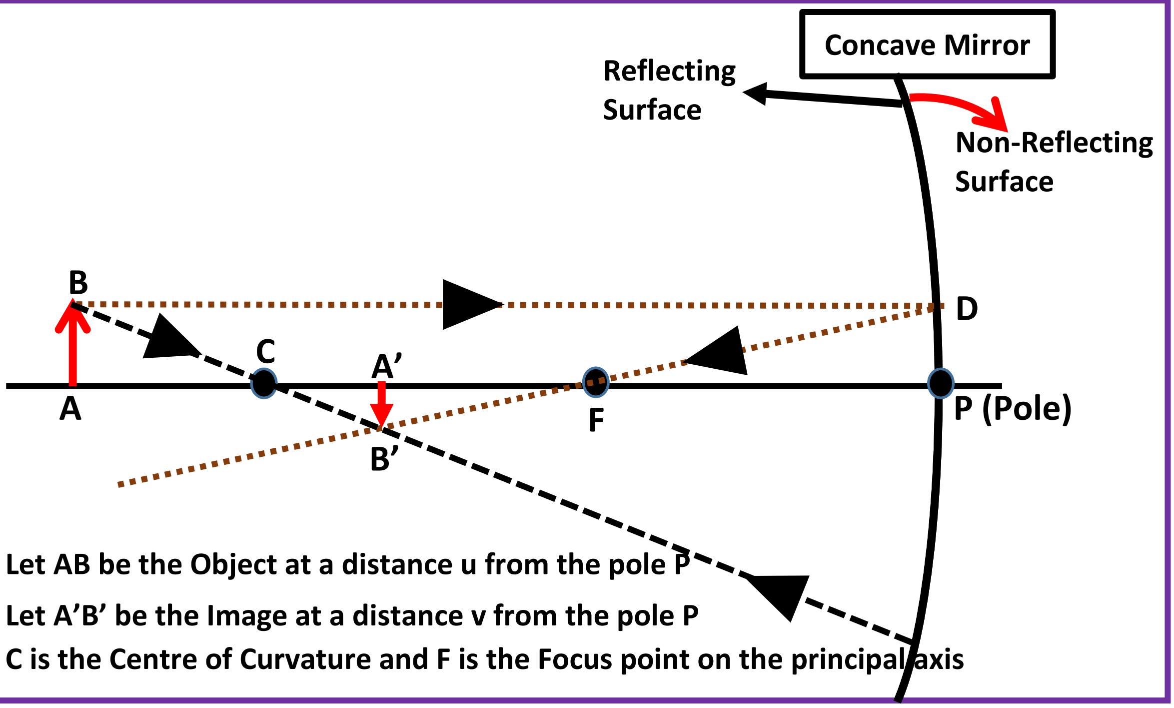
At C that includes object and image:	**At F that includes object and image:**
➢ Δ^{les} ABC & A'B'C are similar (AAA → similarity criteria)	➢ Δ^{les} PDF & A'B'F are similar (AAA → similarity criteria)
➢ $\therefore \frac{AB}{A'B'} = \frac{AC}{CA'}$ ---------- **(1)**	➢ $\therefore \frac{PD}{A'B'} = \frac{PF}{FA'}$; **since aperture is small, PD = AB**
	➢ $\therefore \frac{AB}{A'B'} = \frac{PF}{FA'}$ ----------------------- **(2)**

- From (1) and (2), we have $\frac{AC}{CA'} = \frac{PF}{FA'} \Rightarrow \quad \frac{PC-PA}{PA'-PC} = \frac{PF}{PA'-PF}$ ----------- **(3)**
- **Apply Cartesian sign convention**, PC = -2f, PA = -u, PA' = -v, PF = -f **(for concave mirror, take f as –ve)**
- Eq (3) becomes $\frac{-2f+u}{-v+2f} = \frac{-f}{-v+f} \Rightarrow \frac{u-2f}{2f-v} = \frac{f}{v-f}$; simplifying, we get
- $2f^2 - fv = uv - uf - 2fv + 2f^2 \quad \Rightarrow \quad fv = uv - uf$ or $uv = uf + fv$; Dividing LHS and RHS by uvf, we get

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$ **→ This is the required Mirror Formula ------------- (4)**

- **Mirror Formula Eq (4) is valid due to 3 assumptions:**
  - Object is placed on the principal axis
  - The aperture of the mirror is small (**Due to this we can equate PD = AB**)
  - Only "Paraxial" rays are taken
    - Rays are paraxial means → they are incident at points close the pole P of the mirror and make small angles with the principal axis.

Mirror Formula: Concave Mirror:
Example Case : Real Image (Object beyond C)



Let AB be the Object at a distance u from the pole P

Let A'B' be the Image at a distance v from the pole P

C is the Centre of Curvature and F is the Focus point on the principal axis

To prove Mirror Formula, we need to consider similar triangles centered at F and C that includes object and image.

At C that includes object and image:	**At F that includes object and image:**
➢ Δ^{les} ABC & A'B'C are similar (AAA → similarity criteria)	➢ Δ^{les} PDF & A'B'F are similar (AAA → similarity criteria)
➢ $\therefore \frac{AB}{A'B'} = \frac{CA}{A'C}$ ---------- **(1)**	➢ $\therefore \frac{PD}{A'B'} = \frac{PF}{FA'}$; **since aperture is small, PD = AB**
	➢ $\therefore \frac{AB}{A'B'} = \frac{PF}{FA'}$ ------------------------ **(2)**

➢ From (1) and (2), we have $\frac{CA}{A'C} = \frac{PF}{FA'} \Rightarrow$ $\boldsymbol{\frac{PA - PC}{PC - PA'} = \frac{PF}{PA' - PF}}$ ------------------- **(3)**

➢ **Apply Cartesian sign convention**, PA = -v, PC = -2f, PA' = -u, PF = -f **(for concave mirror, take f as –ve)**

➢ Eq (3) becomes $\frac{-v+2f}{-2f+u} = \frac{-f}{-u+f} \Rightarrow \frac{v-2f}{2f-u} = \frac{f}{u-f}$; simplifying, we get

➢ $2f^2 - fu = uv - vf - 2fu + 2f^2 \Rightarrow fu = uv - vf$ or $uv = uf + fv$; Dividing LHS and RHS by uvf, we get

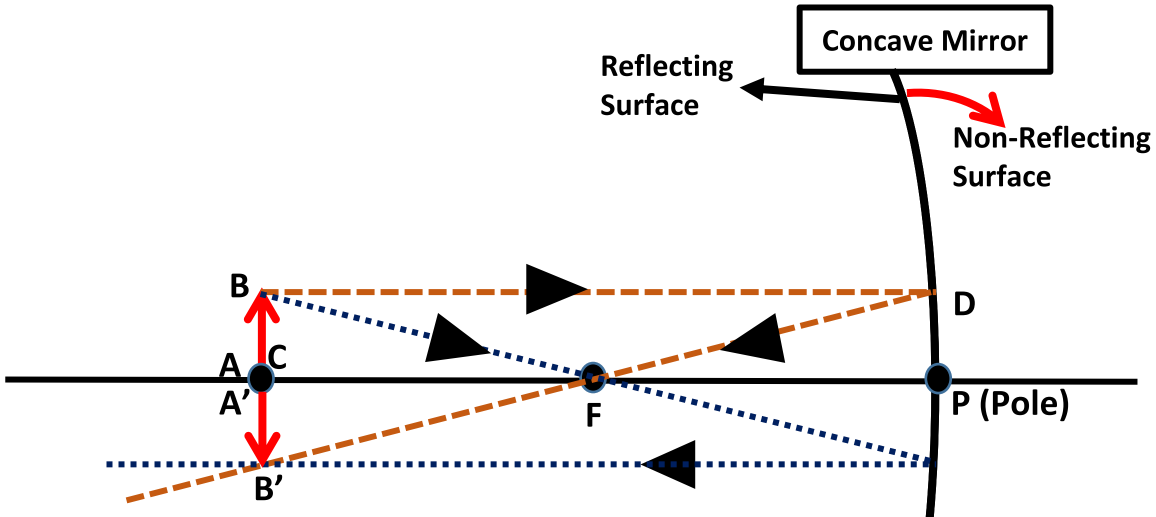
$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$ → **This is the required Mirror Formula ------------- (4)**

➢ Mirror Formula Eq (4) is valid due to 3 assumptions:

- Object is placed on the principal axis
- The aperture of the mirror is small (**Due to this we can equate PD = AB**)
- Only "Paraxial" rays are taken
  - Rays are paraxial means → they are incident at points close the pole P of the mirror and make small angles with the principal axis.

Mirror Formula: Concave Mirror:
Example Case : Real Image (Object at C)

75ccc



Let AB be the Object at a distance u from the pole P

Let A'B' be the Image at a distance v from the pole P

C is the Centre of Curvature and F is the Focus point on the principal axis

To prove Mirror Formula, we need to consider similar triangles centered at F and C that includes object and image.

At C that includes object and image:	At F that includes object and image:
This is a typical case to prove Mirror Formula. Recall when the object is beyond C, we considered triangles ABC and A'B'C. Since the object is at C itself, then triangles ABC and A'B'C merge. **So, AB = A'B'** ---------- **(1)** → **meaning same size**	➢ Δ^{les} PDF & A'B'F are similar (AAA → similarity criteria) ➢ $\therefore \frac{PD}{A'B'} = \frac{PF}{FA'}$; **since aperture is small, PD = AB** ➢ $\therefore \frac{AB}{A'B'} = \frac{PF}{FA'}$ ------------------------ **(2)** ➢ **From Eq (1), AB = A'B' ; ∴ Eq (2) becomes PF = FA'**

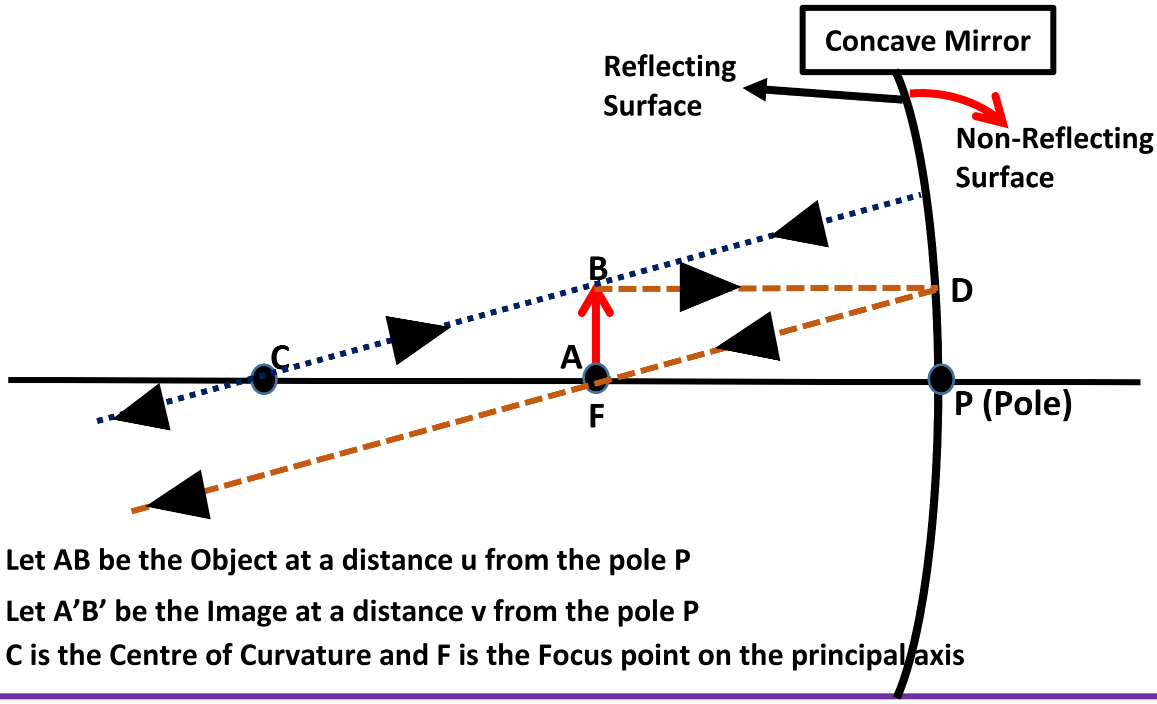
- Considering Eq (2), **PF = FA'** ⇒ **PF = PA' – PF** ---------------- **(3)**
- **Applying Cartesian sign convention**, PF = -f, PA' = -v, PF = -f **(for concave mirror, take f as –ve)**
- Eq (3) becomes -f = -v + f ⇒ 2f = v ⇒ f = $\frac{v}{2}$ **OR** $\frac{1}{f} = \frac{2}{v}$ ------- **(4)**
- **From Ray diagram shown in figure, it is proved that the object and the image are at the same distance from pole P**.
- **∴ v = u**, so we can always write $\frac{2}{v} = \frac{1}{v} + \frac{1}{u}$
- ∴ Eq (4) becomes $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ **This is the required Mirror Formula ---- (5)**
- **Mirror Formula Eq (5) is valid due to 3 assumptions:**
  - Object is placed on the principal axis
  - The aperture of the mirror is small (**Due to this we can equate PD = AB**)
  - Only "Paraxial" rays are taken
    - Rays are paraxial means → they are incident at points close the pole P of the mirror and make small angles with the principal axis.

OR simple derivation:

- We know that c = 2f
- Given case: Object is at C from pole P ⇒ PC = u ; **Applying Cartesian sign convention, -2f = -u ⇒ 2f = u**
  - **So, 2f = u ; f = u/2 OR** $\frac{1}{f} = \frac{2}{u}$ --------------------------------- **(a)**
- **From Ray diagram in figure, we have shown that the image is also formed at C from the pole P**
  - **∴ also PC = v ∴ u = v**
  - **Equation (a) can be written as** $\frac{1}{f} = \frac{1}{u} + \frac{1}{u}$ **OR since u = v,** $\frac{1}{f} = \frac{1}{u} + \frac{1}{u}$

Mirror Formula: Concave Mirror:
Example Case : Real Image (Object at Focus F)

75cccc



Let AB be the Object at a distance u from the pole P

Let A'B' be the Image at a distance v from the pole P

C is the Centre of Curvature and F is the Focus point on the principal axis

To prove Mirror Formula, we need to consider similar triangles centered at F and C that includes object and image.

At C that includes object and image:	At F that includes object and image:
➢ This is a typical case to prove Mirror Formula. At C there is no triangle. Still consider Δ^{les} ABC & A'B'C (similar Δ^{les}) ➢ From Ray diagram shown in figure, the image is formed at infinity. Therefore, **v = ∞** , <u>however let us consider v is very large and write equations</u> ➢ $\frac{AB}{A'B'} = \frac{AC}{CA'}$ ---- (1) (Considering Δ^{les} ABC & A'B'C)	➢ Δ^{les} PDF & imaginary A'B'F are similar ➢ $\therefore \frac{PD}{A'B'} = \frac{PF}{FA'}$; **since aperture is small, PD = AB** ➢ $\therefore \frac{AB}{A'B'} = \frac{PF}{FA'}$ ------------------------ (2)

➢ From (1) and (2), we have $\frac{AC}{CA'} = \frac{PF}{FA'} \Rightarrow$ $\frac{PC-PA}{PA'-PC} = \frac{PF}{PA'-PF}$ ----------- **(3)**

➢ **Apply Cartesian sign convention**, PC = -2f, PA = -u, PA' = -v, PF = -f **(for concave mirror, take f as –ve)**

➢ Eq (3) becomes $\frac{-2f+u}{-v+2f} = \frac{-f}{-v+f} \Rightarrow \frac{u-2f}{2f-v} = \frac{f}{v-f}$; simplifying, we get

➢ $2f^2$ – fv = uv –uf – 2fv + $2f^2$ ⇒ fv = uv –uf or uv = uf + fv ; Dividing LHS and RHS by uvf, we get

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ → **This is the required Mirror Formula --------- (4)**

➢ **Since in this case, the image is formed at infinity, v = ∞**

➢ $\therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ = $\frac{1}{f} = \frac{1}{u} + \frac{1}{\infty}$ $\Rightarrow$ **f = u**

<u>Mirror Formula Eq (4) is valid due to 3 assumptions:</u>

➢ Object is placed on the principal axis

➢ The aperture of the mirror is small (**Due to this we can equate PD = AB**)

➢ Only "Paraxial" rays are taken

- Rays are paraxial means → they are incident at points close the pole P of the mirror and make small angles with the principal axis.

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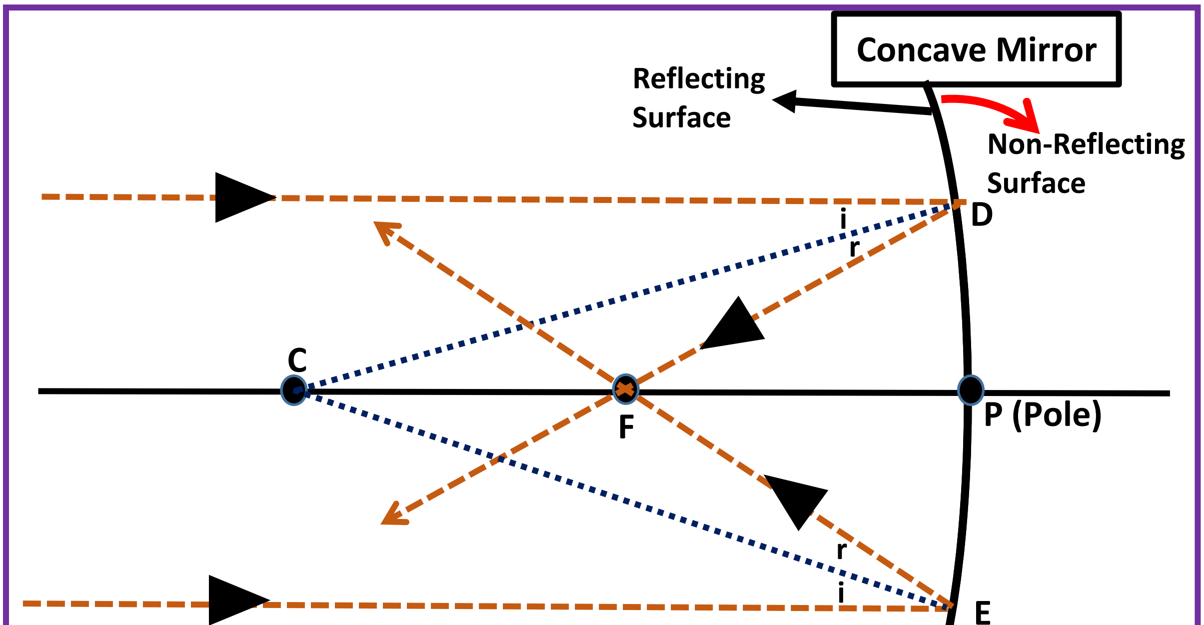
Easy way to get the Mirror Formula is:

Given Object at F, so –u = -f ; ∴u = f or $\frac{1}{u} = \frac{1}{f}$; since as per Ray diagram, image is at infinity → v = ∞, 1/v = 0 ; therefore we can write $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Mirror Formula: Concave Mirror:
Example Case : Real Image (Object at infinity)

75cccccc



From Ray diagram, when Object is at infinity, image is at focus

C is the Centre of Curvature and F is the Focus point on the principal axis

To prove Mirror Formula, we need to consider similar triangles centered at F and C that includes object and image.

At C that includes object and image:	**At F that includes object and image:**
➢ **This is a typical case to prove Mirror Formula. At C there is no triangle. Still consider Δ^{les} ABC & A'B'C (similar Δ^{les})** ➢ **From Ray diagram shown in figure, the image is formed at focus. Therefore, v = f , <u>however let us consider u is at very large distance and write equations</u>** ➢ $\frac{AB}{A'B'} = \frac{CA}{CA'}$ **---- (1) (Considering Δ^{les} ABC & A'B'C)**	➢ Δ^{les} PDF & imaginary A'B'F are similar ➢ $\therefore \frac{PD}{A'B'} = \frac{PF}{FA'}$; **since aperture is small, PD = AB** ➢ $\therefore \frac{AB}{A'B'} = \frac{PF}{FA'}$ ------------------------ (2)

➢ From (1) and (2), we have $\frac{CA}{CA'} = \frac{PF}{FA'} \Rightarrow$ $\frac{PA-PC}{PA'-PC} = \frac{PF}{PA'-PF}$ ------------ **(3)**

➢ **Apply Cartesian sign convention**, PC = -2f, PA = -u, PA' = -v, PF = -f **(for concave mirror, take f as –ve)**

➢ Eq (3) becomes $\frac{-2f+u}{-v+2f} = \frac{-f}{-v+f} \Rightarrow \frac{u-2f}{2f-v} = \frac{f}{v-f}$; simplifying, we get

➢ **2f²** – fv = uv –uf – 2fv + **2f²** $\Rightarrow$ fv = uv –uf or uv = uf + fv ; Dividing LHS and RHS by uvf, we get

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$ **→ This is the required Mirror Formula --------------------- (4)**

➢ **Since in this case, the u = ∞, from Ray diagram, image will be at focus ; So v = f**

➢ $\therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{1}{v} + \frac{1}{\infty} \Rightarrow$ **f = v**

<u>Mirror Formula Eq (4) is valid due to 3 assumptions:</u>

➢ Object is placed on the principal axis
➢ The aperture of the mirror is small (**Due to this we can equate PD = AB**)
➢ Only "Paraxial" rays are taken
➢ Rays are paraxial means → they are incident at points close the pole P of the mirror and make small angles with the principal axis.

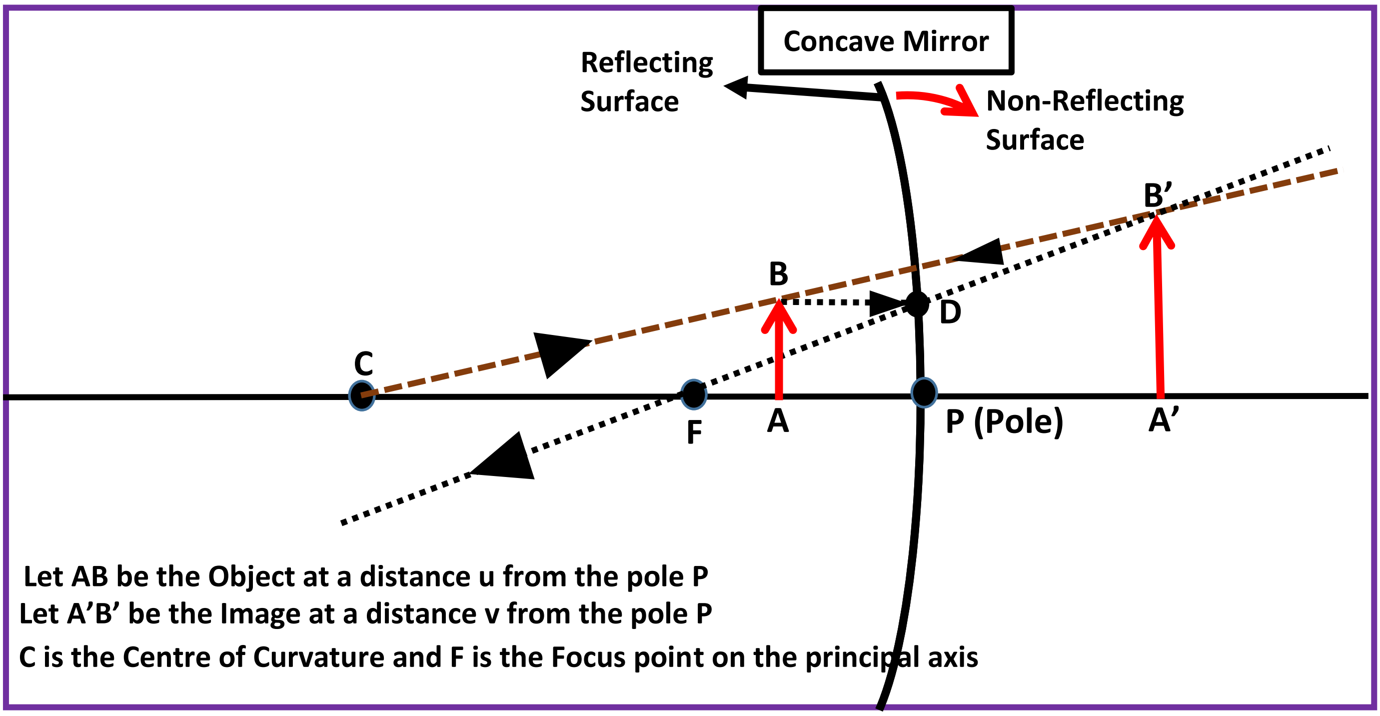
****

➢ **Easy way to get the Mirror Formula is:**

➢ **Given Object at ∞, so –u = -∞ ∴ u = ∞ OR 1/u = 0 ; since as per Ray diagram, image is at focus → v = f, therefore we can write** $\frac{1}{f} = \frac{1}{v} + 0 \Rightarrow \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

Mirror Formula: Concave Mirror:
Example Case : Virtual Image (Object between P and F)

75cccccc



Let AB be the Object at a distance u from the pole P
Let A'B' be the Image at a distance v from the pole P
C is the Centre of Curvature and F is the Focus point on the principal axis

To prove Mirror Formula, we need to consider similar triangles centered at F and C that includes object and image.

At C that includes object and image:	At F that includes object and image:
➢ Δ^{les} ABC & A'B'C are similar (AAA → similarity criteria)	➢ Δ^{les} PDF & A'B'F are similar (AAA → similarity criteria)
➢ $\therefore \frac{AB}{A'B'} = \frac{AC}{A'C}$ ---------------------------------- **(1)**	➢ $\therefore \frac{PD}{A'B'} = \frac{PF}{A'F}$; **since aperture is small, PD = AB**
	➢ $\therefore \frac{AB}{A'B'} = \frac{PF}{A'F}$ ------------------------ **(2)**

➢ From (1) and (2), we have $\frac{AC}{A'C} = \frac{PF}{A'F} \Rightarrow \frac{PC-PA}{PA'+PC} = \frac{PF}{PA'+PF}$ ------------------- **(3)**

➢ **Apply Cartesian sign convention**, PC = -2f, PA = -u, PA' = +v, PF = -f **(for concave mirror, take f as –ve)**

➢ Eq (3) becomes $\frac{-2f+u}{v-2f} = \frac{-f}{v-f} \Rightarrow \frac{u-2f}{v-2f} = \frac{f}{f-v}$; simplifying, we get

➢ $vf - 2f^2 = uf - uv - 2f^2 + 2fv \Rightarrow uf + vf = uv$ or $uv = uf + fv$; Dividing LHS and RHS by uvf, we get

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$ **→ This is the required Mirror Formula ------------ (4)**

➢ **Mirror Formula Eq (4) is valid due to 3 assumptions:**
- Object is placed on the principal axis
- The aperture of the mirror is small (**Due to this we can equate PD = AB**)
- Only "Paraxial" rays are taken
  - Rays are paraxial means → they are incident at points close the pole P of the mirror and make small angles with the principal axis.

~~75 ccc~~

75 ccccccc

Linear Magnification (consider concave mirror) : Linear magnification 'm' is the ratio of height of the image (h_i) to the height of the object (h_o); $m = \frac{h_i}{h_o}$

Δ^{les} A'B'P and ABP are Similar (AAA - Similarity criterion)

$\therefore \frac{A'B'}{AB} = \frac{PA'}{PA}$

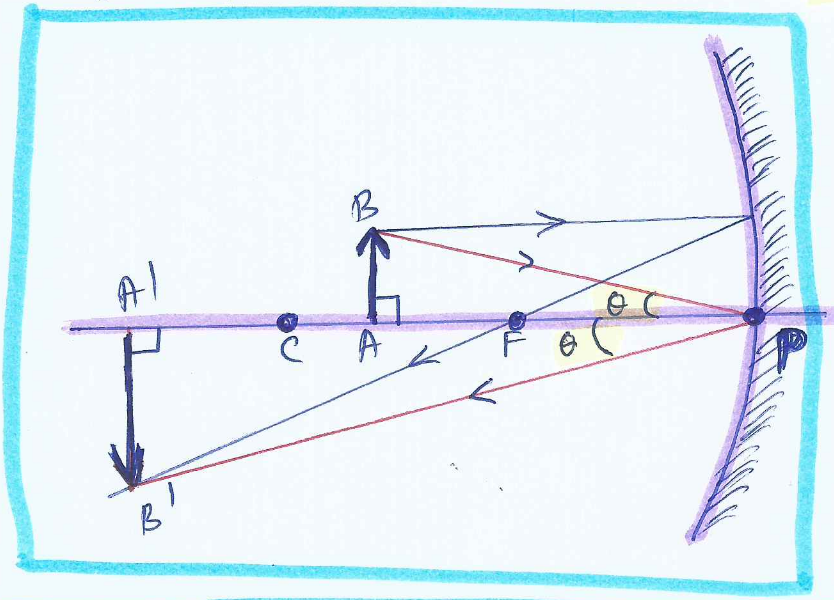
Using cartesian sign convention

$AB = +h_o$; $A'B' = -h_i$

$PA' = -V$; $PA = -u$

$\therefore \frac{-h_i}{h_o} = \frac{-V}{-u}$

$\therefore \frac{h_i}{h_o} = -\frac{V}{u}$ $\therefore$ $\boxed{m = -\frac{V}{u}}$



"m" → considering Convex Mirror

Δ^{les} ABP and A'B'P are similar (AAA → Similarity Criterion)

$\therefore \frac{A'B'}{AB} = \frac{PA'}{PA}$

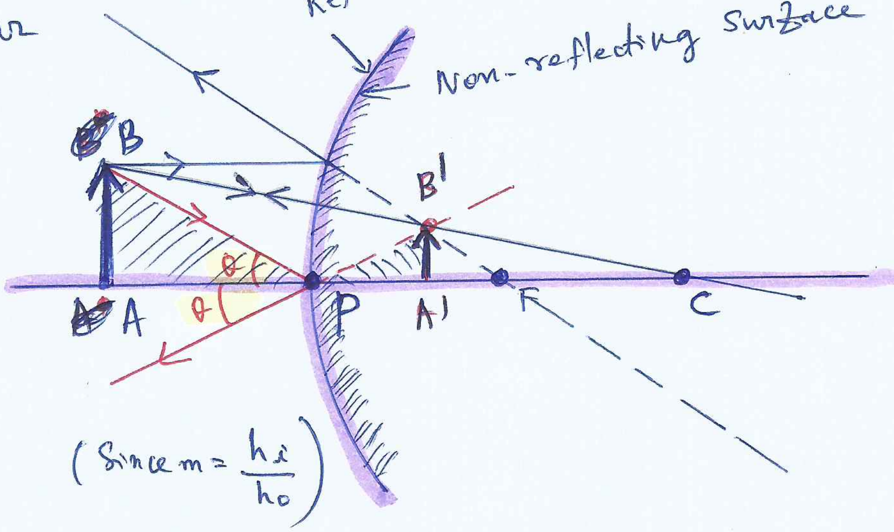
Using cartesian sign convention,

$A'B' = +h_i$, $AB = +h_o$

$PA' = +V$, $PA = -u$

$\therefore \frac{h_i}{h_o} = \frac{+V}{-u} = -\frac{V}{u}$ (Since $m = \frac{h_i}{h_o}$)

$\boxed{m = -\frac{V}{u}}$



Mirror Formula: Convex Mirror: (Only 2 possibilities)

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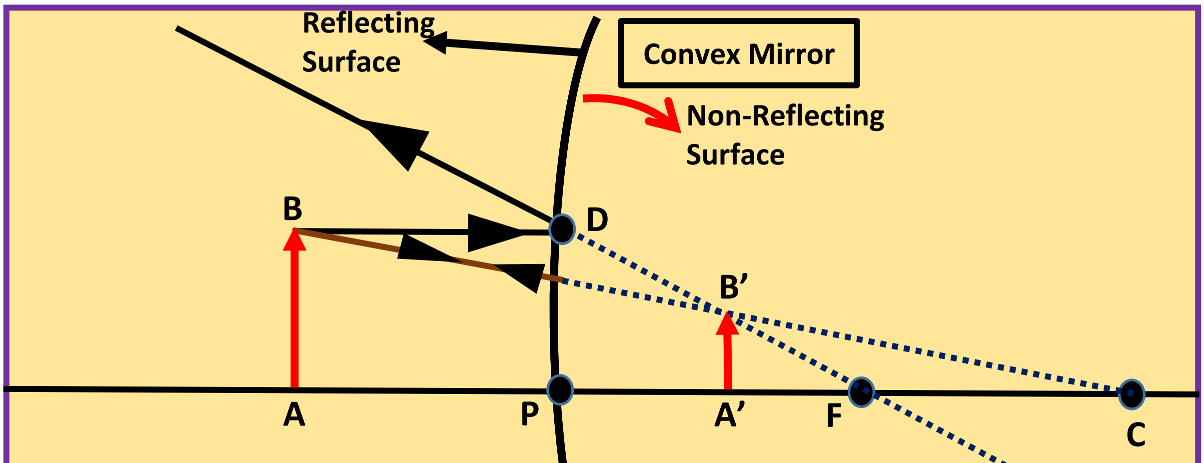
1. Position of object is between infinity and the pole of the mirror



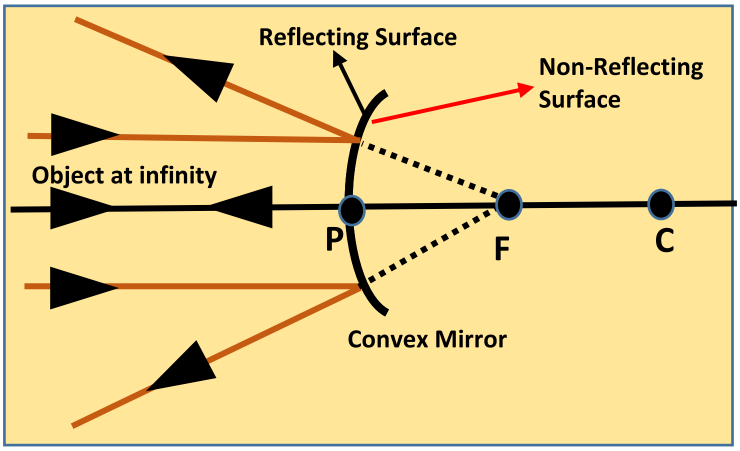
Let AB be the Object at a distance u from the pole P
Let A'B' be the Image at a distance v from the pole P
C is the Centre of Curvature and F is the Focus point on the principal axis

To prove Mirror Formula, we need to consider similar triangles centered at F and C that includes object and image.

At C that includes object and image:	**At F that includes object and image:**
➢ Δ^{les} ABC & A'B'C are similar (AAA → similarity criteria) $\therefore \frac{AB}{A'B'} = \frac{AC}{A'C}$ ------------------------------------ **(1)**	➢ Δ^{les} PDF & A'B'F are similar (AAA → similarity criteria) ➢ $\therefore \frac{PD}{A'B'} = \frac{PF}{A'F}$; **since aperture is small, PD = AB** ➢ $\therefore \frac{AB}{A'B'} = \frac{PF}{A'F}$ ------------------------ **(2)**

- From (1) and (2), we have $\frac{AC}{A'C} = \frac{PF}{A'F} \Rightarrow \frac{PC + PA}{PC - PA'} = \frac{PF}{PF - PA'}$ ---------------------------------- **(3)**
- **Apply Cartesian sign convention**, PC = +2f, PA = -u, PA' = +v, PF = +f **(for Convex mirror, take f as +ve)**
- Eq (3) becomes $\frac{2f-u}{2f-v} = \frac{f}{f-v} \Rightarrow$ $2f^2$ - 2vf - uf + uv = $2f^2$ - vf $\Rightarrow$ uv = uf + vf
- Dividing LHS and RHS by uvf, we get $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ → **This is the required Mirror Formula (4)**
- **<u>Mirror Formula Eq (4) is valid due to 3 assumptions:</u>**
  - Object is placed on the principal axis
  - The aperture of the mirror is small (**Due to this we can equate PD = AB**)
  - Only "Paraxial" rays are taken
    - Rays are paraxial means → they are incident at points close the pole P of the mirror and make small angles with the principal axis.

2. Position of Object is at infinity



- Given Object at infinity, **so u = -∞**
- From Ray diagram, we can show that image is formed at the focus F. So, PF = v
- Applying **Cartesian sign convention, we get**
- **PF = +f (for Convex mirror, take f as +ve)**
- **PF = + v ; ∴ +f = +v ⇒ f = v**
- **We can write f = v as** $\frac{1}{f} = \frac{1}{v}$
- **Since u = -∞,** $\frac{1}{u} = 0$ **, we can write**
- $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ → **This is the required Mirror Formula**

****

Note: For 'm' derivation for convex mirror, see page 75ccccccc

Till now, we have studied "Reflection of Light" by "Spherical Surfaces". Following table is just a summary.

- **First column in the following table explains reflection of light by "Plane Reflecting Surfaces"**
- **Second column gives a summary of reflection of light by "Spherical Surfaces". We have derived the mirror formula for both concave and convex mirrors in the previous pages.**

Reflection of Light

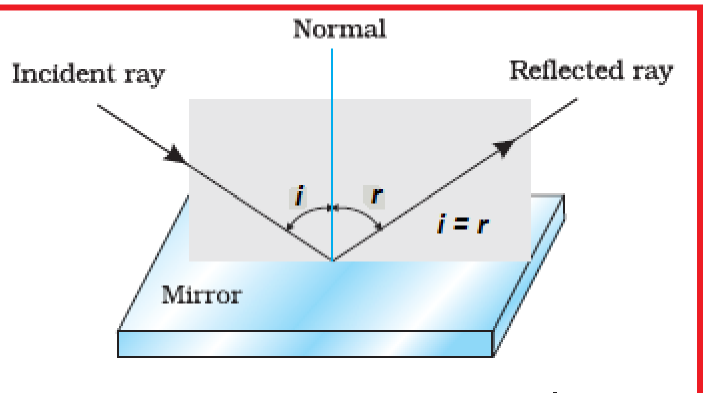
<table>
<tr><th>By "Plane Reflecting Surface" (eg : Mirror)</th><th>Reflection of light by "Curved Surface"</th></tr>
<tr><td>
➢ We have studied the reflection of light from a "Plane Reflecting Surface" (eg. Mirror) in 8th standard CBSE
➢ Laws of reflection:
➢ The angle of reflection (the angle between reflected ray and the normal to the reflecting surface) = the angle of incidence (angle between incident ray and the normal to the reflecting surface).
➢ The incident ray, reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane (Fig. 9.1).
➢ These laws are valid at each point on any reflecting surface whether plane or curved.


FIGURE 9.1 The incident ray, reflected ray and the normal to the reflecting surface lie in the same plane.
</td><td>
➢ However, we shall restrict our discussion to the special case of curved surfaces, that is, Spherical Surfaces (concave/convex mirrors) → we will study this in 12th class.
➢ The normal in this case is to be taken as normal to the tangent to surface at the point of incidence. That is, the normal is along the radius, the line joining the centre of curvature of the mirror to the point of incidence.
➢ Geometric centre of a spherical mirror is called its pole (P)
➢ Geometric centre of a spherical lens is called its optical centre (O)
➢ The line joining the pole and the centre of curvature of the spherical mirror is known as the principal axis.
➢ In the case of spherical lenses, the principal axis is the line joining the optical centre with its principal focus as you will see later.
➢ In this chapter, we will prove the spherical mirror (concave & convex) formula for objects at different distances from the pole. We have done this in previous pages.
</td></tr>
<tr><td colspan="2">
➢ When we consider light incident on "perfect plane reflecting surface" or "perfect spherical surface (concave / convex mirrors)", we talk about the phenomenon called Reflection.
➢ When we consider light incident on lenses (convex and concave lens), we have to talk about the phenomenon called Refraction
➢ When the medium is transparent, there will be both reflection and refraction.
</td></tr>
<tr><td colspan="2">We have seen the sign convention in the previous pages (useful for solving problems connected with spherical mirrors and Lenses)</td></tr>
<tr><td colspan="2">Further, we will study "Refraction of Light"</td></tr>
</table>

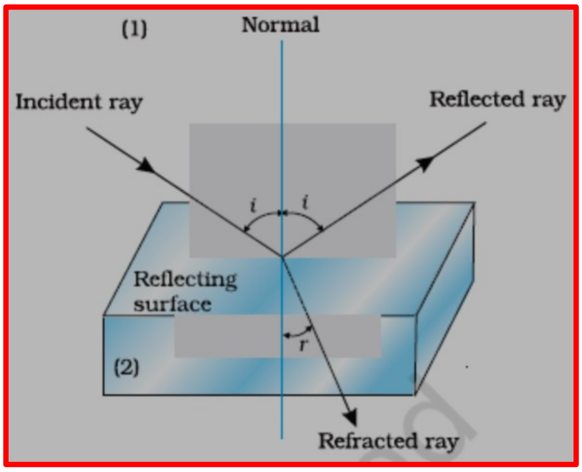
75da

Refraction of Light

1. **Refraction at plane transparent surface → "Snell's Law", "Real/Apparent depth", "Total Internal Reflection"**
2. **Refraction at a spherical surface**
3. **Refraction by Lenses**
4. **Refraction through a Prism**

1: Refraction at plane transparent surface (Snell's law of refraction)

When a beam of light encounters another transparent medium, a part of light gets reflected back into the first medium while the rest enters the other. A ray of light represents a beam. The direction of propagation of an obliquely incident (0°< i < 90°) ray of light that enters the other medium, changes at the interface of the two media. This phenomenon is called refraction of light. Snell experimentally obtained the following laws of refraction:



- The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.
- The ratio of the sine of the angle of incidence to the sine of angle of refraction is constant. Remember that the angles of incidence (i) and refraction (r) are the angles that the incident and its refracted ray make with the normal, respectively. We have

$$\frac{\sin(i)}{\sin(r)} = \frac{\mu_2}{\mu_1} = \mu_{21} \text{ ------------------------------ } \mathbf{(1)}$$

where μ_{21} is a constant, called the refractive index of the second medium with respect to the first medium.

Equation (1) is the well-known Snell's law of refraction. We note that μ_{21} is a characteristic of the pair of media (and also depends on the wavelength of light), but is independent of the angle of incidence.

- From Eq. (1), if $\mu_{21} > 1 \Rightarrow \frac{\mu_2}{\mu_1} > 1 \Rightarrow \mu_2 > \mu_1 \Rightarrow \sin(i) > \sin(r) \Rightarrow i > r$ or $r < i$, the refracted ray bends towards the normal. In such a case medium 2 is said to be optically denser (or denser, in short) than medium 1.
- If $\mu_{21} < 1 \Rightarrow \frac{\mu_2}{\mu_1} < 1 \Rightarrow \mu_2 < \mu_1 \Rightarrow \sin(i) < \sin(r) \Rightarrow i < r$ or $r > i$, the refracted ray bends away from the normal. This is the case when incident ray in a denser medium refracts into a rarer medium.

Note: Optical density should not be confused with mass density, which is mass per unit volume. It is possible that mass density of an optically denser medium may be less than that of an optically rarer medium (optical density is the ratio of the speed of light in two media). For example, turpentine and water. Mass density of turpentine is less than that of water but its optical density is higher.

- μ_{21} is the refractive index of the second medium with respect to the first medium = $\frac{\mu_2}{\mu_1}$
- μ_{12} is the refractive index of the first medium with respect to the second medium = $\frac{\mu_1}{\mu_2}$
- $\therefore\ \mu_{12} = \frac{1}{\mu_{21}}$; Also μ_{32} is the refractive index of the third medium wrt the second medium = $\frac{\mu_3}{\mu_2}$
- $\mu_{32} = \frac{\mu_3}{\mu_2} = \frac{\mu_3}{\mu_1} \text{ X } \frac{\mu_1}{\mu_2} = \mu_{31} \text{ X } \mu_{12}, \textit{where } \mu_{31} = \textit{refractive index of medium } 3 \textit{ wrt medium } 1$
- See the figure → for a rectangular slab, refraction takes place at two interfaces (air-glass and glass-air)

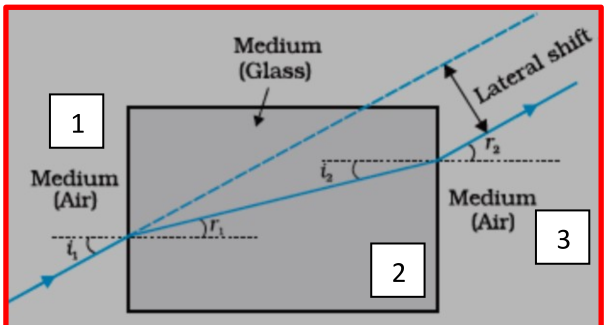


FIGURE 9.9 Lateral shift of a ray refracted through a parallel-sided slab.

- From figure, $\frac{\sin(i_1)}{\sin(r_1)} = \frac{\mu_2}{\mu_1} = \mu_{21}$; $\frac{\sin(i_2)}{\sin(r_2)} = \frac{\mu_3}{\mu_2} = \mu_{32}$
- $\frac{\sin(i_1)}{\sin(r_1)} \ x \ \frac{\sin(i_2)}{\sin(r_2)} = \frac{\mu_2}{\mu_1} \ x \ \frac{\mu_3}{\mu_2} = \frac{\mu_3}{\mu_1}$; $\boldsymbol{since\ i_2 = r_1}$ (see fig)
- $\frac{\sin(i_1)}{\cancel{\sin(r_1)}} \ x \ \frac{\cancel{\sin(r_1)}}{\sin(r_2)} = \frac{\sin(i_1)}{\sin(r_2)} = \frac{\mu_3}{\mu_1}$ **(since $\mu_3 = \mu_1$ → same medium air)**
- $\frac{\sin(i_1)}{\sin(r_2)} = 1 \ \therefore \sin(i_1) = \sin(r_2)$; $\therefore i_1 = r_2$
- **i.e., the emergent ray is parallel to the incident ray; there is no deviation, but it does suffer lateral displacement / shift wrt the incident ray.**

75db

Refraction of Light

"Real/Apparent depth"

1: Refraction at plane transparent surface ("Real/Apparent depth")

Another familiar observation due to refraction is that the bottom of a tank filled with **water** appears to be raised (See Fig 9.10). For viewing near the normal direction, it can be shown that the apparent depth (h1) is real depth (h2) divided by the refractive index of the medium (water).

Apparent depth (h1) = (Real depth h2)/μ_{water}

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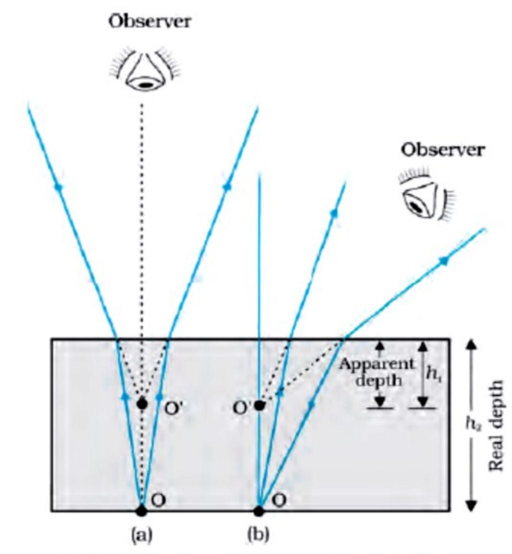

FIGURE 9.10 Apparent depth for (a) normal, and (b) oblique viewing.

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The refraction of light through the atmosphere is responsible for many interesting phenomena. For example, the Sun is visible a little before the actual sunrise and until a little after the actual sunset due to refraction of light through the atmosphere (Fig. 9.11). By actual sunrise we mean the actual crossing of the horizon by the sun. Figure 9.11 shows the actual and apparent positions of the Sun with respect to the horizon. The figure is highly exaggerated to show the effect. The refractive index of air with respect to vacuum is 1.00029. Due to this, the apparent shift in the direction of the Sun is by about half a degree and the corresponding time difference between actual sunset and apparent sunset is about 2 minutes (see Example 9.5). The apparent flattening (oval shape) of the Sun at sunset and sunrise is also due to the same phenomenon.

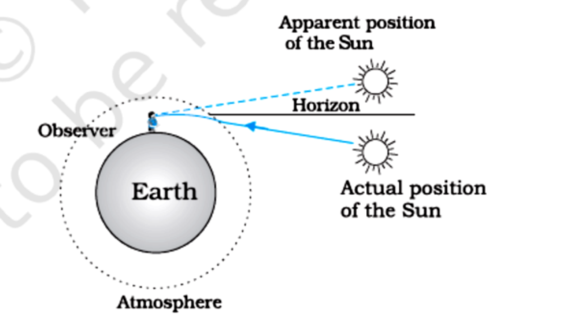


FIGURE 9.11 Advance sunrise and delayed sunset due to atmospheric refraction.

EXAMPLE 9.5

Example 9.5 The earth takes 24 h to rotate once about its axis. How much time does the sun take to shift by 1° when viewed from the earth?

Solution

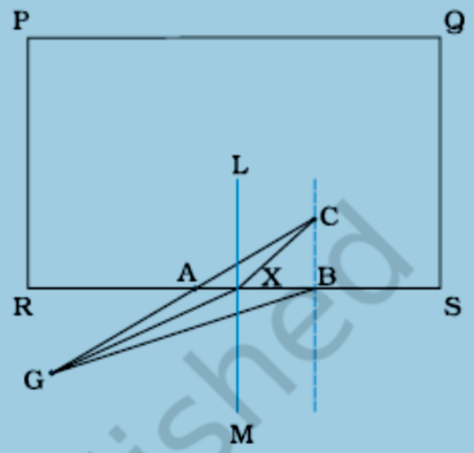
Time taken for 360° shift = 24 h

Time taken for 1° shift = 24/360 h = 4 min.

75dc

THE DROWNING CHILD, LIFEGUARD AND SNELL'S LAW

Consider a rectangular swimming pool PQSR; see figure here. A lifeguard sitting at G outside the pool notices a child drowning at a point C. The guard wants to reach the child in the shortest possible time. Let SR be the side of the pool between G and C. Should he/she take a straight line path GAC between G and C or GBC in which the path BC in water would be the shortest, or some other path GXC? The guard knows that his/her running speed v_1 on ground is higher than his/her swimming speed v_2.



Suppose the guard enters water at X. Let GX $= l_1$ and XC $= l_2$. Then the time taken to reach from G to C would be

$$t = \frac{l_1}{v_1} + \frac{l_2}{v_2}$$

To make this time minimum, one has to differentiate it (with respect to the coordinate of X) and find the point X when t is a minimum. On doing all this algebra (which we skip here), we find that the guard should enter water at a point where Snell's law is satisfied. To understand this, draw a perpendicular LM to side SR at X. Let $\angle$GXM $= i$ and $\angle$CXL $= r$. Then it can be seen that t is minimum when

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

In the case of light v_1/v_2, the ratio of the velocity of light in vacuum to that in the medium, is the refractive index n of the medium.

In short, whether it is a wave or a particle or a human being, whenever two mediums and two velocities are involved, one must follow Snell's law if one wants to take the shortest time.

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Refraction of Light

"Total Internal Reflection"

1: Refraction at plane transparent surface ("Total Internal Reflection")

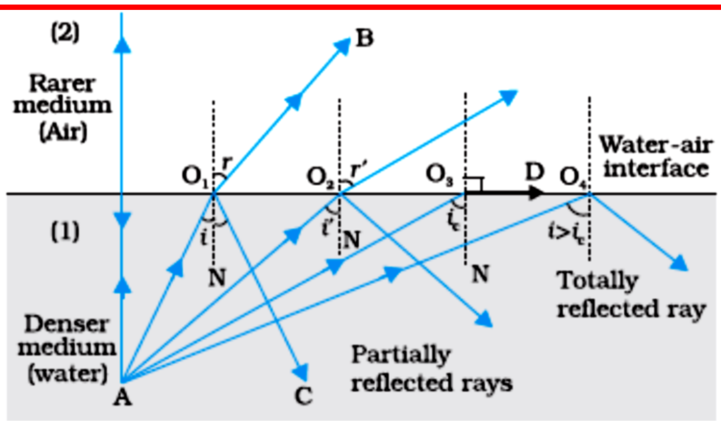

FIGURE 9.12 Refraction and internal reflection of rays from a point A in the denser medium (water) incident at different angles at the interface with a rarer medium (air).

When light travels from an optically denser medium to a rarer medium at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. This reflection is called the internal reflection.

When a ray of light enters from a denser medium to a rarer medium, it bends away from the normal, for example, the ray AO_1B in Fig. 9.12. The incident ray AO_1 is partially reflected (O_1C) and partially transmitted (O_1B) or refracted, the angle of refraction (r) being larger than the angle of incidence (i). As the angle of incidence increases, so does the angle of refraction, till for the ray AO_3, the angle of refraction is $\pi/2$. The refracted ray is bent so much away from the normal that it grazes the surface at the interface between the two media. This is shown by the ray AO_3D in Fig. 9.12. If the angle of incidence is increased still further (e.g., the ray AO_4), refraction is not possible, and the incident ray is totally reflected. **This is called total internal reflection**.

When light gets reflected by a surface, normally some fraction of it gets transmitted. The reflected ray, therefore, is always less intense than the incident ray, howsoever smooth the reflecting surface may be. **In total internal reflection, on the other hand, no transmission of light takes place.**

The angle of incidence corresponding to an angle of refraction 90°, say $A\widehat{O_3}N$, is called the critical angle (i_c) for the given pair of media. We see from Snell's law $\frac{\sin(i)}{\sin(r)} = \frac{\mu_2}{\mu_1} = \mu_{21}$ that if the relative refractive index is less than one then, since the maximum value of sin r is unity, there is an upper limit to the value of sin i for which the law can be satisfied, that is, i = i_c such that $\frac{\sin(i_c)}{\sin(90)} = \frac{\mu_2}{\mu_1} = \mu_{21} \quad \Rightarrow \quad \sin(i_c) = \frac{\mu_2}{\mu_1} = \mu_{21}$

For values of i larger than i_c, Snell's law of refraction cannot be satisfied, and hence no refraction is possible.

The refractive index of denser medium 1 with respect to rarer medium 2 will be $\mu_{12} = \frac{1}{\sin(i_c)}$

Some typical critical angles are listed in Table 9.1

TABLE 9.1 CRITICAL ANGLE OF SOME TRANSPARENT MEDIA WITH RESPECT TO AIR

Substance medium	Refractive index	Critical angle
Water	1.33	48.75
Crown glass	1.52	41.14
Dense flint glass	1.62	37.31
Diamond	2.42	24.41

A demonstration for total internal reflection

All optical phenomena can be demonstrated very easily with the use of a laser torch or pointer, which is easily available nowadays. Take a glass beaker with clear water in it. Add a few drops of milk or any other suspension to water and stir so that water becomes a little turbid. Take a laser pointer and shine its beam through the turbid water. You will find that the path of the beam inside the water shines brightly.

Shine the beam from below the beaker such that it strikes at the upper water surface at the other end. Do you find that it undergoes partial reflection (which is seen as a spot on the table below) and partial refraction [which comes out in the air and is seen as a spot on the roof; Fig. 9.13(a)]? Now direct the laser beam from one side of the beaker such that it strikes the upper surface of water more obliquely [Fig. 9.13(b)]. Adjust the direction of laser beam until you find the angle for which the refraction above the water surface is totally absent and the beam is totally reflected back to water. This is total internal reflection at its simplest.

Pour this water in a long test tube and shine the laser light from top, as shown in Fig. 9.13(c). Adjust the direction of the laser beam such that it is totally internally reflected every time it strikes the walls of the tube. This is similar to what happens in optical fibres.

Take care not to look into the laser beam directly and not to point it at anybody's face.

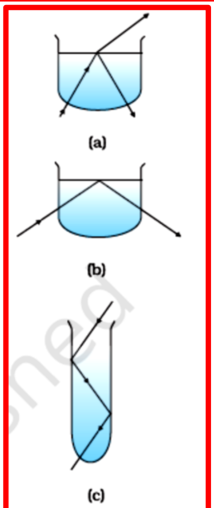

FIGURE 9.13 Observing total internal reflection in water with a laser beam (refraction due to glass of beaker neglected being very thin).

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Total internal reflection in nature and its technological applications

Mirage:

- On hot summer days, the air near the ground becomes hotter than the air at higher levels. The refractive index of air increases with its density. Hotter air is less dense, and has smaller refractive index than the cooler air. If the air currents are small, that is, the air is still, the optical density at different layers of air increases with height. As a result, light from a tall object such as a tree, passes through a medium whose refractive index decreases towards the ground. Thus, a ray of light from such an object successively bends away from the normal and undergoes total internal reflection, if the angle of incidence for the air near the ground exceeds the critical angle. This is shown in Fig. 9.14(b). To a distant observer, the light appears to be coming from somewhere below the ground. The observer naturally assumes that light is being reflected from the ground, say, by a pool of water near the tall object. Such inverted images of distant tall objects cause an optical illusion to the observer. This phenomenon is called mirage. This type of mirage is especially common in hot deserts. Some of you might have noticed that while moving in a bus or a car during a hot summer day, a distant patch of road, especially on a highway, appears to be wet. But, you do not find any evidence of wetness when you reach that spot. This is also due to mirage.

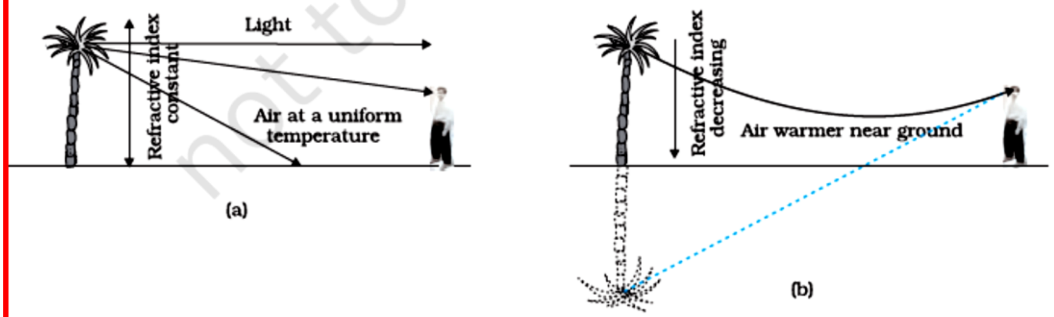

FIGURE 9.14 (a) A tree is seen by an observer at its place when the air above the ground is at uniform temperature, (b) When the layers of air close to the ground have varying temperature with hottest layers near the ground, light from a distant tree may undergo total internal reflection, and the apparent image of the tree may create an illusion to the observer that the tree is near a pool of water.

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Diamond:

- Diamonds are known for their spectacular brilliance. Their brilliance is mainly due to the total internal reflection of light inside them. The critical angle for diamond-air interface (@ 24.4°) is very small, therefore once light enters a diamond, it is very likely to undergo total internal reflection inside it. Diamonds found in nature rarely exhibit the brilliance for which they are known. It is the technical skill of a diamond cutter which makes diamonds to sparkle so brilliantly. By cutting the diamond suitably, multiple total internal reflections can be made to occur.

Prism:

- Prisms designed to bend light by 90° or by 180° make use of total internal reflection [Fig. 9.15(a) and (b)]. Such a prism is also used to invert images without changing their size [Fig. 9.15(c)]. In the first two cases, the critical angle i_c for the material of the prism must be less than 45°. We see from Table 9.1 that this is true for both crown glass and dense flint glass.

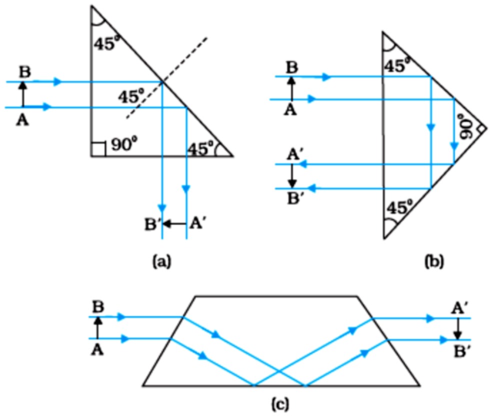

FIGURE 9.15 Prisms designed to bend rays by 90° and 180° or to invert image without changing its size make use of total internal reflection.

TABLE 9.1 CRITICAL ANGLE OF SOME TRANSPARENT MEDIA WITH RESPECT TO AIR

Substance medium	Refractive index	Critical angle
Water	1.33	48.75
Crown glass	1.52	41.14
Dense flint glass	1.62	37.31
Diamond	2.42	24.41