# Total internal reflection in nature and its technological applications

## Optical fibres:

- Nowadays optical fibres are extensively used for transmitting audio and video signals through long distances. Optical fibres too make use of the phenomenon of total internal reflection. Optical fibres are fabricated with high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core is higher than that of the cladding.
- When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflections along the length of the fibre and finally comes out at the other end (Fig. 9.16). Since light undergoes total internal reflection at each stage, there is no appreciable loss in the intensity of the light signal. Optical fibres are fabricated such that light reflected at one side of inner surface strikes the other at an angle larger than the critical angle. Even if the fibre is bent, light can easily travel along its length. Thus, an optical fibre can be used to act as an optical pipe.

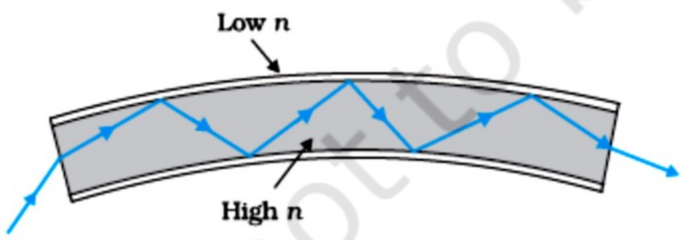



**FIGURE 9.16** Light undergoes successive total internal reflections as it moves through an optical fibre.

- A bundle of optical fibres can be put to several uses. Optical fibres are extensively used for transmitting and receiving electrical signals which are converted to light by suitable transducers. Obviously, optical fibres can also be used for transmission of optical signals. For example, these are used as a 'light pipe' to facilitate visual examination of internal organs like esophagus, stomach and intestines. You might have seen a commonly available decorative lamp with fine plastic fibres with their free ends forming a fountain like structure. The other end of the fibres is fixed over an electric lamp. When the lamp is switched on, the light travels from the bottom of each fibre and appears at the tip of its free end as a dot of light. The fibres in such decorative lamps are optical fibres.
- The main requirement in fabricating optical fibres is that there should be very little absorption of light as it travels for long distances inside them. This has been achieved by purification and special preparation of materials such as quartz. In silica glass fibres, it is possible to transmit more than 95% of the light over a fibre length of 1 km. (Compare with what you expect for a block of ordinary window glass 1 km thick.)

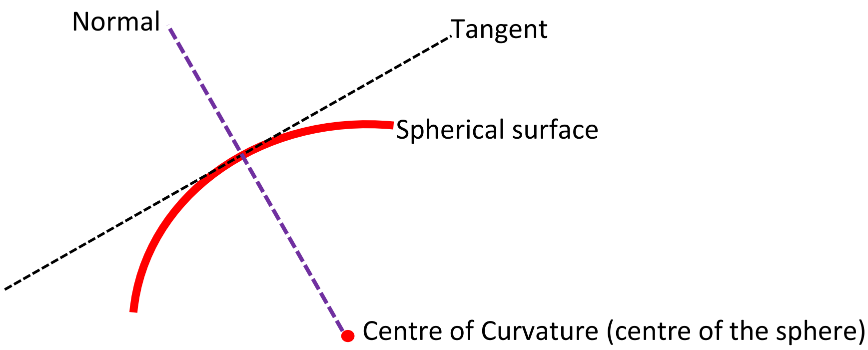
**75dg**

## Refraction of Light

1. Refraction at plane transparent surface → "Snell's Law", "Real/Apparent depth", "Total Internal Reflection"
2. **Refraction at a spherical surface**
3. Refraction by Lenses
4. Refraction through a Prism

## Refraction at a spherical surface

- We have so far considered refraction at a plane interface. We shall now consider refraction at a spherical interface between two transparent media.
- An infinitesimal part of a spherical surface can be regarded as planar and the same laws of refraction can be applied at every point on the surface. Just as for reflection by a spherical mirror, the **normal** **at the point of incidence** is **perpendicular** to the **tangent plane** to the spherical surface **at that point** and, therefore, passes through its centre of curvature.




- We first consider refraction by a single spherical surface and follow it by thin lenses. A thin lens is a transparent optical medium bounded by two surfaces; at least one of which should be spherical.
- Applying the formula for image formation by a single spherical surface successively at the two surfaces of a lens, we shall obtain the lens maker's formula and then the lens formula.

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**Further, we will derive the relation between μ, u, v and R for refraction at a spherical surface.**

**The relation is** $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$ **, where**

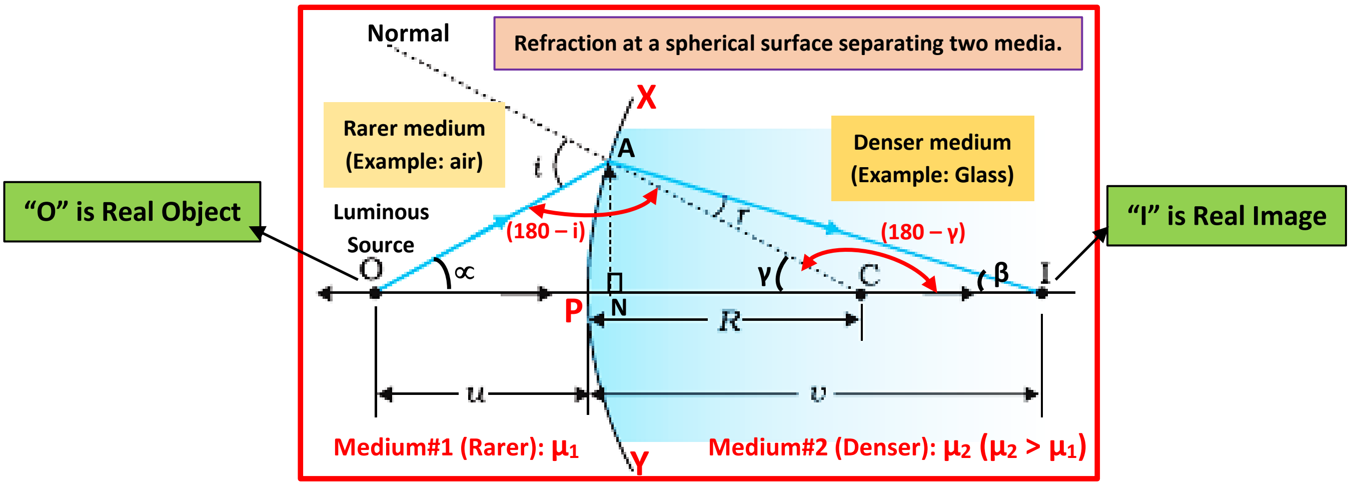
- **$\mu_1$ = refractive index of the rarer medium**
- **$\mu_2$ = refractive index of the denser medium**
- **u = object distance from the spherical surface**
- **v = image distance**
- **R = Radius of Curvature of the sphere.**

**The above equation gives us a relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface. It holds for any curved spherical surface.**

**See the next page for the derivation of the formula** $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$

# 75dh

## Deduce the relation between μ, u, v and R for refraction at a spherical surface, where the symbols have their usual meaning and when image formed is REAL



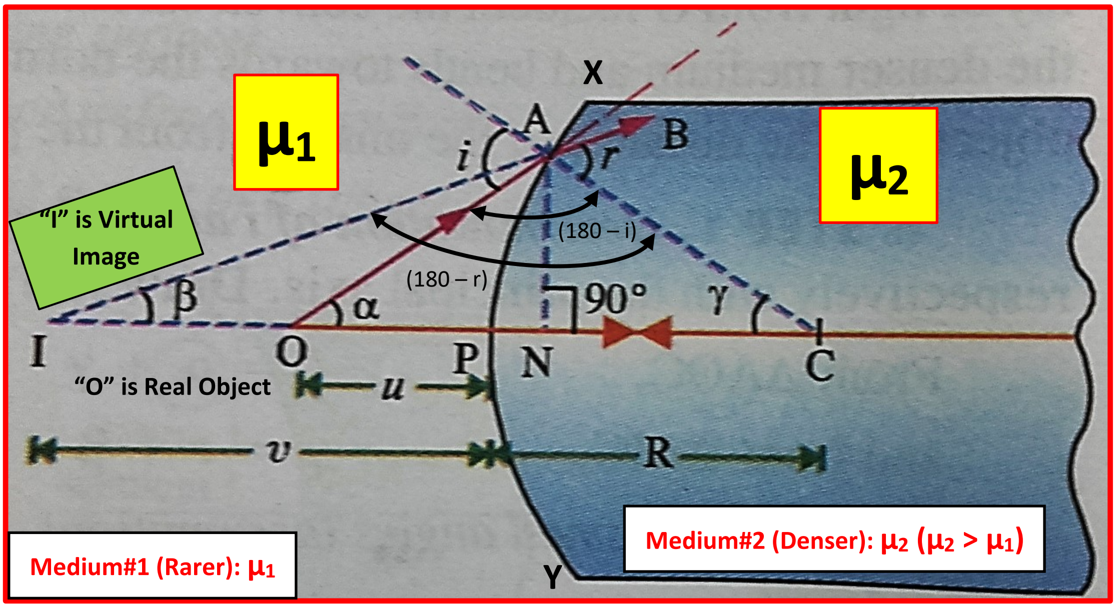

- In the diagram, XPY represents a spherical surface of radius of curvature R, separating two media of refractive indices $\mu_1$ & $\mu_2$ such that $\mu_2 > \mu_1$
- O is the luminous point object placed on the principal axis in rarer medium
- A ray OP incident along principal axis proceeds without deviation.
- Another ray OA incident at point A at an angle 'i' is refracted along AI with an angle of refraction 'r' (bending towards normal, since light is passing from rarer medium to denser medium).
- **The refracted rays converge at point I, which is the real image of the object O.**
- From $\Delta^{le}$ AOC, $\propto + \gamma + (180^0 - i) = 180^0$ $\Rightarrow$ **$i = \propto + \gamma$** ---------------------------------------------------- **(1)**
- From $\Delta^{le}$ AIC, $r + \beta + (180^0 - \gamma) = 180^0$ $\Rightarrow$ **$r = \gamma - \beta$** -------------------------------------------------------- **(2)**
- Let us take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made. So, in particular **AN = AP**.
  - **This leads to NO = PO, NI = PI, NC = PC → using these approximations, we have**
- $\tan\propto = \frac{AP}{PO}$ , $\tan\beta = \frac{AP}{PI}$ , $\tan\gamma = \frac{AP}{PC}$ ; *for small angles* $\tan\theta = \theta$, *therefore*
- $\propto = \frac{AP}{PO}$ , $\beta = \frac{AP}{PI}$ , $\gamma = \frac{AP}{PC}$
- $\boldsymbol{From\ (1),\ i = \propto + \gamma = \frac{AP}{PO} + \frac{AP}{PC}} - - - - - - - - - - - - - - - - - - - - - - - -(3)$
- $\boldsymbol{From\ (2),\ r = \gamma - \beta = \frac{AP}{PC} - \frac{AP}{PI}} - - - - - - - - - - - - - - - - - - - - - - - - - -(4)$
- $From\ Snell's\ law\ \mu_1 \sin(i) = \mu_2 \sin(r)$ ; $for\ small\ angles, \sin\theta = \theta,$ $therefore\ \mu_1 i = \mu_2 r$
- $Using\ (3)\&(4),\ \mu_1\left[\frac{\cancel{AP}}{PO} + \frac{\cancel{AP}}{PC}\right] = \mu_2\left[\frac{\cancel{AP}}{PC} - \frac{\cancel{AP}}{PI}\right]$ ; $AP\ cancels\ on\ both\ sides\ and\ after\ rearranging, we\ have$
- $\boldsymbol{\left[\frac{\mu_1}{PO} + \frac{\mu_2}{PI}\right] = \left[\frac{(\mu_2 - \mu_1)}{PC}\right]} - - - - - - - - - - - - - - - - - - - - - - - - - - - - -(5)$
- $\boldsymbol{Applying\ Cartesian\ Sign\ Convention, PO = -u,\ PC = +R,\ \ PI = +v}$
- $-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{(\mu_2 - \mu_1)}{R}$

$$\boldsymbol{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}} - - - - - - - - - - - - - - - - - - - - - - - - - - - -(6)$$

- This is the required expression which also holds good when image is Virtual (see next page for the proof)
- Equation (6) gives us a relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface.
- It holds for any curved spherical surface.
- **Special case:**
  - **if medium #1 is air and medium #2 is glass, we have $\mu_1 \approx 1$ and $\mu_2 = \mu_{glass}$**
  - **Then, we get $\frac{\mu_{glass}}{v} - \frac{1}{u} = \frac{(\mu_{glass} - 1)}{R}$**

# 75dha

**Deduce the relation between μ, u, v and R for refraction at a spherical surface, where the symbols have their usual meaning and when image formed is VIRTUAL**

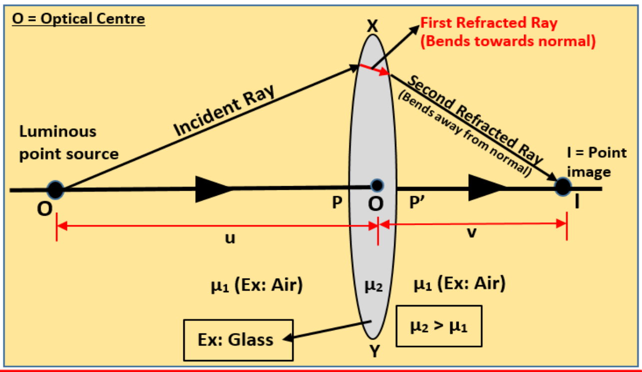



- In the diagram, XPY represents a spherical surface of radius of curvature R, separating two media of refractive indices $\mu_1$ & $\mu_2$ such that $\mu_2 > \mu_1$
- O is the luminous point object placed on the principal axis closed to spherical surface in the rarer medium such that the image formed is virtual
- A ray OP incident along principal axis suffers no deviation.
- Another ray OA incident at point A at an angle 'i' is refracted along AB with an angle of refraction 'r' (bending towards normal AC since light is passing from rarer medium to denser medium). The refracted ray AB, when produced backward, meets the principal axis at I. So, "I" is the **Virtual Image of the real object O**.
- From $\Delta^{le}$ AOC, $\propto + \gamma + (180^0 - i) = 180^0$ $\Rightarrow$ **$i = \propto + \gamma$** ------------------------------------------------------- **(1)**
- From $\Delta^{le}$ AIC, $\beta + \gamma + (180^0 - r) = 180^0$ $\Rightarrow$ **$r = \beta + \gamma$** --------------------------------------------------------- **(2)**
- Let us take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made. So, in particular **AN = AP**.
  - **This leads to NO = PO, NI = PI, NC = PC → using these approximations, we have**
- $\tan \propto = \frac{PA}{PO}$ , $\tan \beta = \frac{PA}{PI}$ , $\tan \gamma = \frac{PA}{PC}$ ; *for small angles* $\tan \theta \approx \theta$, *therefore*
- $\propto = \frac{PA}{PO}$ , $\beta = \frac{PA}{PI}$ , $\gamma = \frac{PA}{PC}$
- $\boldsymbol{From\ (1),\ i = \propto + \gamma = \frac{PA}{PO} + \frac{PA}{PC}}$ ------------------------------------------------(3)
- $\boldsymbol{From\ (2),\ r = \beta + \gamma = \frac{PA}{PI} + \frac{PA}{PC}}$ ------------------------------------------------(4)
- $From\ Snell's\ law\ \mu_1 \sin(i) = \mu_2 \sin(r)$ ; $for\ small\ angles, \sin\theta \approx \theta$, $therefore\ \mu_1 i = \mu_2 r$
- $Using\ (3)\&(4),\ \mu_1\left[\frac{\cancel{PA}}{PO} + \frac{\cancel{PA}}{PC}\right] = \mu_2\left[\frac{\cancel{PA}}{PI} + \frac{\cancel{PA}}{PC}\right]$ ; $PA\ cancels\ on\ both\ sides\ and\ after\ rearranging, we\ have$
- $\boldsymbol{\left[\frac{\mu_1}{PO} - \frac{\mu_2}{PI}\right] = \left[\frac{(\mu_2 - \mu_1)}{PC}\right]}$ ------------------------------------------------(5)
- $\boldsymbol{Applying\ Cartesian\ Sign\ Convention, PO = -u,\ PC = +R,\ \ PI = -v}$
- $-\frac{\mu_1}{u} - \frac{\mu_2}{(-v)} = \frac{(\mu_2 - \mu_1)}{R}$ $\Rightarrow$ $-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{(\mu_2 - \mu_1)}{R}$ ; $\boldsymbol{Rearranging, we\ get}$

$$\boldsymbol{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}} \text{------------------------------------------------}(6)$$

- **This is the same as previous case of real page (previous page)**
- Equation (6) gives us a relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface.

# 75di

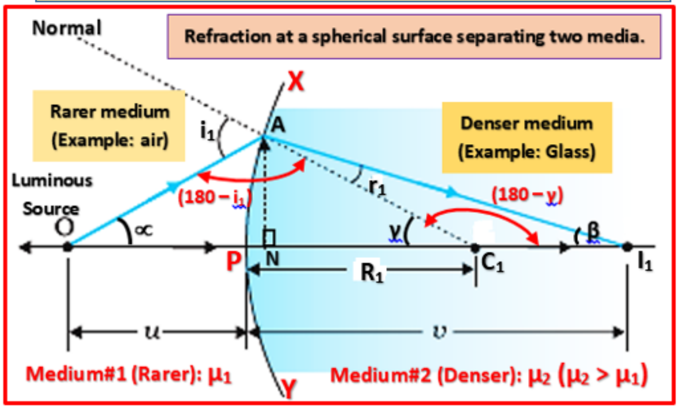



## Refraction by a thin lens: Lens maker's formula / Thin Lens formula

Figure on the right shows the geometry of image formation by a double convex lens. The image formation can be seen in terms of 2 steps:

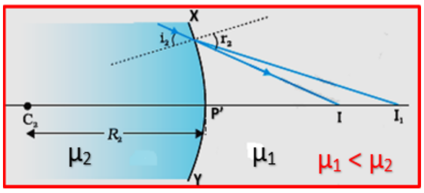
- First refraction by the lens surface **XPY** followed by the
- Second refraction by the lens surface **XP'Y**.

### Refraction by the surface XPY (we have studied this in the previous page)




- In fig, XPY represents a spherical surface of radius of curvature R, separating two media of refractive indices $\mu_1$ & $\mu_2$ such that $\mu_2 > \mu_1$
- O is the real luminous point object placed on the principal axis in rarer medium
- A ray OP incident along principal axis proceeds without deviation.
- Another ray OA incident at point A at an angle '$i_1$' is refracted along $AI_1$ with an angle of refraction '$r_1$' (bending towards normal, since light is passing from rarer medium to denser medium).
- The refracted rays converge at point $I_1$, which is the real image of the object O.
- From $\Delta^{le}$ $AOC_1$, $\propto + \gamma + (180^0 - i_1) = 180^0 \Rightarrow$ $i_1 = \propto + \gamma$ ----- **(1)**
- From $\Delta^{le}$ $AI_1C_1$, $r_1 + \beta + (180^0 - \gamma) = 180^0 \Rightarrow$ $r_1 = \gamma - \beta$ ----- **(2)**
- Let us take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made. So, in particular **AN = AP**.
  - **This leads to NO = PO, $NI_1 = PI_1$, $NC_1 = PC_1$ → using these approximations, we have**
- $\tan \propto = \frac{PA}{PO}$ , $\tan \beta = \frac{PA}{PI_1}$ , $\tan \gamma = \frac{PA}{PC_1}$ ; *for small angles* $\tan\theta \approx \theta$, $\therefore \propto = \frac{PA}{PO}$ , $\beta = \frac{PA}{PI_1}$ , $\gamma = \frac{PA}{PC_1}$
- $\boldsymbol{From}\ (1),\ \boldsymbol{i_1 = \propto + \gamma = \frac{PA}{PO} + \frac{PA}{PC_1}}$ ------------------------------------(3)
- $\boldsymbol{From}\ (2),\ \boldsymbol{r_1 = \gamma - \beta = \frac{PA}{PC_1} - \frac{PA}{PI_1}}$ ------------------------------------(4)
- *From Snell's law* $\mu_1 \sin(i_1) = \mu_2 \sin(r_1)$ ; *for small angles*, $\sin\theta = \theta$, $\boldsymbol{therefore\ \mu_1 i_1 = \mu_2 r_1}$
- *Using* (3)& (4), $\mu_1\left[\frac{PA}{PO} + \frac{PA}{PC_1}\right] = \mu_2\left[\frac{PA}{PC_1} - \frac{PA}{PI_1}\right]$ ; *Simplifying, we get* $\left[\frac{\mu_1}{PO} + \frac{\mu_2}{PI_1}\right] = \left[\frac{(\mu_2 - \mu_1)}{PC_1}\right]$
- **∴ First refraction by the lens surface XPY results in equation** $\frac{\mu_1}{PO} + \frac{\mu_2}{PI_1} = \frac{(\mu_2 - \mu_1)}{PC_1}$ ----------------(5)

The First Refracting Surface XPY would have formed real image $I_1$ of the real object O if Second Refracting Surface XP'Y were not present. Since XP'Y is present, the point $I_1$ acts as a "virtual object" for XP'Y placed in the denser medium for the surface XP'Y that forms the real image at I.




**A similar procedure applied to the XP'Y gives** $-\frac{\mu_2}{P'I_1} + \frac{\mu_1}{P'I} = \frac{(\mu_2 - \mu_1)}{P'C_2}$ ------------------ (6)

Note that the point $I_1$ is in denser medium (left side of fig) and hence $P'I_1$ is the virtual object distance for the 2nd interface and so is negative as the distance is measured against the direction of incident light. Considering thin lens $P'I_1 = PI_1$, eq (7) becomes $-\frac{\mu_2}{PI_1} + \frac{\mu_1}{P\,I} = \frac{(\mu_2 - \mu_1)}{P\,C_2}$ ---------- (7)

**(FYI: See next page how this is done)**

**Adding (5) & (7), we get**

$$\frac{\mu_1}{PO} + \frac{\mu_2}{PI_1} + \left[-\frac{\mu_2}{PI_1} + \frac{\mu_1}{P'I}\right] = \frac{(\mu_2 - \mu_1)}{P'C_1} + \frac{(\mu_2 - \mu_1)}{P'C_2} \Rightarrow \frac{\mu_1}{PO} + \cancel{\frac{\mu_2}{PI_1}} - \cancel{\frac{\mu_2}{PI_1}} + \frac{\mu_1}{P'I} = (\mu_2 - \mu_1)\left[\frac{1}{P'C_1} + \frac{1}{P'C_2}\right]$$

$\therefore \frac{\mu_1}{PO} + \frac{\mu_1}{P'I} = (\mu_2 - \mu_1)\left[\frac{1}{P'C_1} + \frac{1}{P'C_2}\right]$ ----(8) ; **If** object is at PO = ∞, then P'I = f (focal length), eq (8) becomes

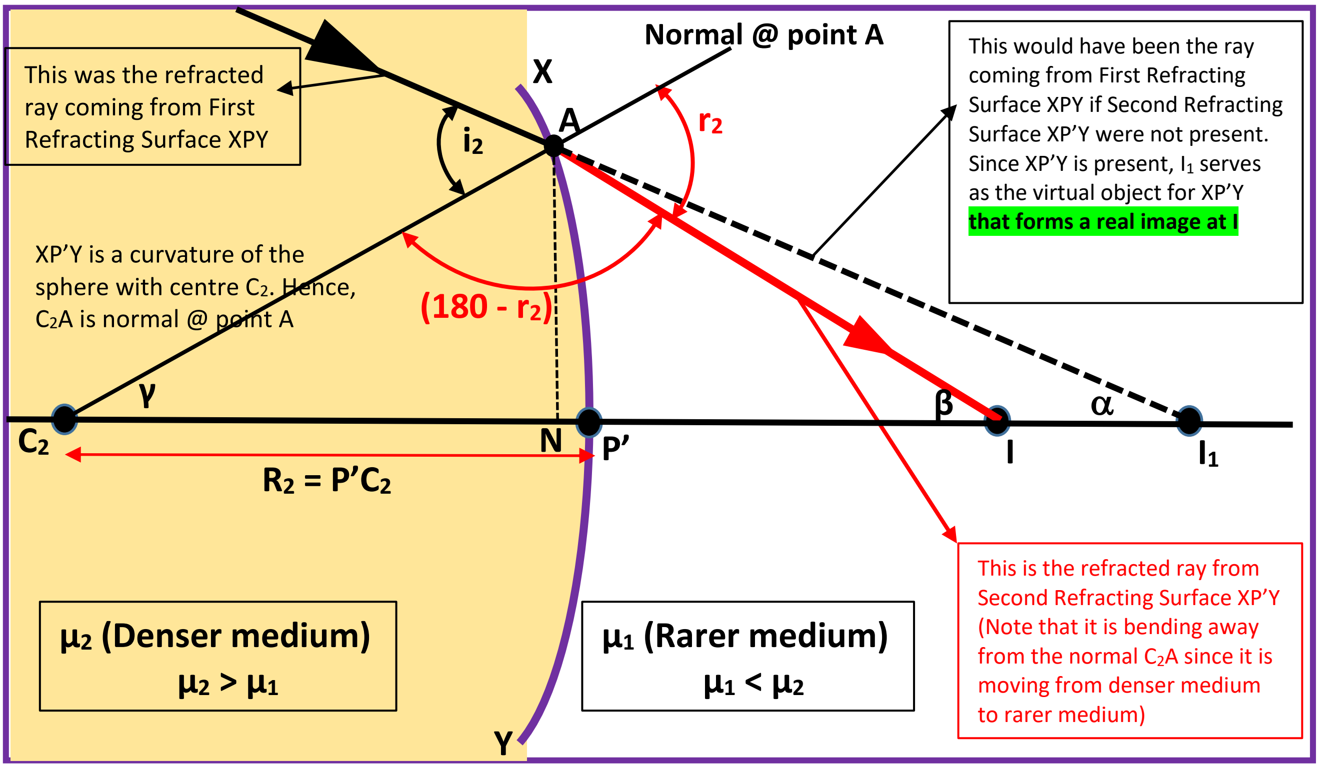
$\therefore \frac{\mu_1}{f} = (\mu_2 - \mu_1)\left[\frac{1}{P'C_1} + \frac{1}{P'C_2}\right]$ ---------(9) ; **A lens has two foci, F and F', by sign convention, $P'C_1 = +R_1$ & $P'C2 = -R_2$**

**∴Eq (9) can be written as** $\frac{1}{f} = (\mu_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ -------- (10) ; $\boldsymbol{where\ \mu_{21} = \frac{\mu_2}{\mu_1}}$

**Equation (10) is known as the lens maker's formula.** It is useful to design lenses of desired focal length using surfaces of suitable radii of curvature.

- **Note that the equation (10) is true for a concave lens also. In that case $R_1$ is negative, $R_2$ positive and therefore, f is negative.**
- **Comparing equations (8) and (9), we get** $\frac{\mu_1}{PO} + \frac{\mu_1}{P'I} = \frac{\mu_1}{f}$; **Again, in the thin lens approximation, P and P' are both close to the optical centre O of the lens. Applying the sign convention, PO = -u and P'I = v, we get** $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$
- $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ ------------------------ (11) ; *Equation* (11) *is the familiar thin lens formula*
- Though we derived it for a <u>real image formed by a convex lens</u>, the formula is equally applicable to a <u>convex lens forming a virtual image</u> and <u>also to a concave lens which forms only virtual images</u>. Note that the two foci, F & F', of a double convex or concave lens are equidistant from the optical centre. The focus on the side of the (original) source of light is called the ***first focal point***, whereas the other is called the ***second focal point***.

# 75dia




- The diagram explains in detail the refracted ray from the 2[nd] refracting surface XP'Y (Read the text boxes in fig)
- ∴ $I_1$ is the virtual object for the second refracting surface and I is the final real image.
- From $\Delta^{le}$ $AI_1C_2$, $\propto + \gamma + (180^0 - i_2) = 180^0 \Rightarrow$ **$i_2 = \propto + \gamma$** ------------------------------------------------------------------------------------- **(1)**
- From $\Delta^{le}$ $AIC_2$, $\gamma + \beta + (180^0 - r_2) = 180^0 \Rightarrow$ **$r_2 = \beta + \gamma$** ------------------------------------------------------------------------------------- **(2)**
- Let us take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made. So, in particular **NA = P'A**.
    - **This leads to NI = P'O, NI₁ = P'I₁, NC₂ = P'C₂ → using these approximations, we have**
- $\tan\propto\ = \frac{P'A}{P'I_1}$ , $\tan\beta\ = \frac{P'A}{P'I}$ , $\tan\gamma\ = \frac{P'A}{P'C_2}$ ; *for small angles* $\tan\theta \approx \theta$, $\therefore \propto\ = \frac{P'A}{P'I_1}$ , $\beta = \frac{P'A}{P'I}$ , $\gamma = \frac{P'A}{P'C_2}$
- $\boldsymbol{From\ (1),\ i_2 = \propto + \gamma = \frac{P'A}{P'I_1} + \frac{P'A}{P'C_2}}$ ------------------------------------ (3)
- $\boldsymbol{From\ (2),\ r_2 = \beta + \gamma = \frac{P'A}{P'I} + \frac{P'A}{P'C_2}}$ ------------------------------------ (4)
- $From\ Snell's\ law\ \ \mu_2 \sin(\boldsymbol{i_2}) = \mu_1 \sin(r_2)$ ; $for\ small\ angles, \sin\theta \approx \theta$, $\boldsymbol{therefore\ \mu_2 i_2 = \mu_1 r_2}$
- $Using\ (3)\&(4),\ \mu_2\left[\frac{P'A}{P'I_1} + \frac{P'A}{P'C_2}\right] = \mu_1\left[\frac{P'A}{P'I} + \frac{P'A}{P'C_2}\right]$
- $Simplifying, we\ get\ \left[\frac{\mu_2}{P'I_1} - \frac{\mu_1}{P'I}\right] = \left[\frac{(\mu_1 - \mu_2)}{P'C_2}\right]$ ; $\boldsymbol{Rearranging, we\ get}\ -\frac{\mu_2}{P'I_1} + \frac{\mu_1}{P'I} = \frac{(\mu_2 - \mu_1)}{P'C_2}$
- $\boldsymbol{\therefore\ Second\ refraction\ by\ the\ lens\ surface\ XP'Y\ results\ in\ equation}\ -\frac{\mu_2}{P'I_1} + \frac{\mu_1}{P'I} = \frac{(\mu_2 - \mu_1)}{P'C_2}$ ---- (5)
- $\boldsymbol{Equation\ \ (5)\ is\ the\ one\ that\ needs\ to\ be\ proved\ from\ the\ previous\ page}$
- **Applying the sign convention to eq (5), P'I₁ = -u , P'I = v and P'C₂ = -R₂, we get** $\left[\frac{\mu_2}{u} + \frac{\mu_1}{v}\right] = \left[\frac{(\mu_2 - \mu_1)}{-R_2}\right]$ -- (6)

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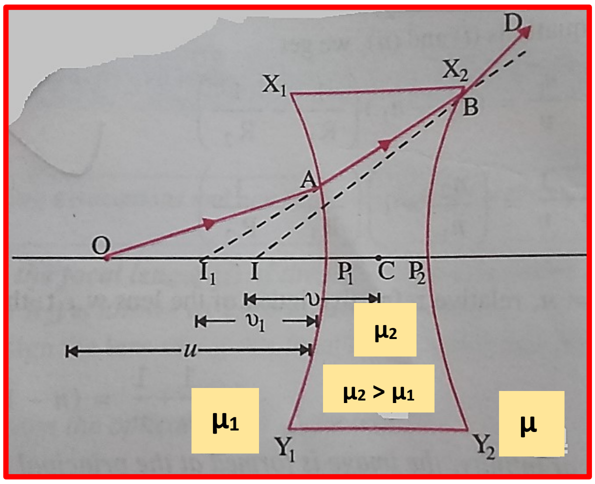
- From previous page, we had derived : First refraction by the lens surface XPY is $\frac{\mu_1}{PO} + \frac{\mu_2}{PI_1} = \frac{(\mu_2 - \mu_1)}{PC_1}$ -------- (7)
- Applying the sign convention to eq (7), PO = -u , $PI_1$ = v and $PC_1 = R_1$, we get $\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{(\mu_2 - \mu_1)}{R_1}$ ------ (8)
- Adding (6) & (8), $\frac{\mu_2}{u} + \frac{\mu_1}{v} + \frac{\mu_1}{-u} + \frac{\mu_2}{v} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ------------------------ (9)
- Note that the image distance from first refracting surface (v) = object distance for the second refracting surface (u)
    - Since, image from XPY surface serves as virtual object for XP'Y surface, the object for XP'Y is to the left of the co-ordinate system (meaning in the denser medium, hence 'u' associated with $\mu_2$ to be considered).
    - Therefore, $u = -v$ ; $\therefore\ equation\ (9)\ becomes$ $\cancel{-\frac{\mu_2}{v}} + \frac{\mu_1}{v} + \frac{\mu_1}{-u} + \cancel{\frac{\mu_2}{v}} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$
- Simplifying, $\frac{\mu_1}{-u} + \frac{\mu_1}{v} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \Rightarrow \frac{1}{-u} + \frac{1}{v} = (\mu_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$, $where\ \mu_{21} = \frac{\mu_2}{\mu_1}$ --- (10)
- If object is at ∞, then u = -∞ , v = f ; hence (10) becomes $\frac{1}{f} = (\mu_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ -- (11) $\boldsymbol{Lens\ Maker's\ Formula}$
- Comparing equations (10) & (11), we get $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ -- (12) ; $\boldsymbol{Eq(12)\ is\ the\ familiar\ thin\ lens\ formula}$

# 75dib

## Derivation of Lens Maker's Formula for a Concave Lens

- Let the concave lens have 2 spherical surfaces $X_1P_1Y_1$ and $X_2P_2Y_2$ having radius of curvature as $R_1$ and $R_2$ respectively.

### Step 1: Refraction at $X_1P_1Y_1$




- Let O be the object placed in the rarer medium on the principal axis as shown in the figure.
- The incident ray OA after refraction at A bends towards the normal $NC_1$ (*$NC_1$ is the centre of curvature of surface $X_1P_1Y_1$ → not shown in fig*) and travel along AB. It appears to come from the point $I_1$ if the second surface $X_2P_2Y_2$ were not present. So $I_1$ is the virtual image of the object O.
- Since the object lies in the rarer medium, so we have
- $-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{(\mu_2 - \mu_1)}{R}$ ; Here, $v = -v_1$ $and\ R = R_1$
- $\therefore\ -\frac{\mu_1}{u} - \frac{\mu_2}{v_1} = \frac{(\mu_2 - \mu_1)}{R_1} - - - - - - - - - - - - - - (1)$

### Step 2: Refraction at $X_2P_2Y_2$

- Ray AB inside the lens will suffer another refraction at surface $X_2P_2Y_2$. After refraction from the surface $X_2P_2Y_2$, the ray travels along BD and appears to come from the point I. So, I is the final virtual image of the object. Point $I_1$ acts as the virtual object placed in the denser medium for the surface $X_2P_2Y_2$
- $\therefore\ -\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{(\mu_1 - \mu_2)}{R}$ ; $here\ u = -v_1,\ v = -v\ and\ R = R_2$
- $\therefore\ \frac{\mu_2}{v_1} - \frac{\mu_1}{v} = \frac{(\mu_1 - \mu_2)}{R_2} \Rightarrow \frac{\mu_2}{v_1} - \frac{\mu_1}{v} = -\frac{(\mu_2 - \mu_1)}{R_2} - - - - - - - - - - - - (2)$

### Step 3: *Adding (1) and (2)*

- *Adding* (1) *and* (2), *we get* $-\frac{\mu_1}{u} - \cancel{\frac{\mu_2}{v_1}} + \cancel{\frac{\mu_2}{v_1}} - \frac{\mu_1}{v} = \frac{(\mu_2 - \mu_1)}{R_1} - \frac{(\mu_2 - \mu_1)}{R_2} \Rightarrow -\frac{\mu_1}{u} - \frac{\mu_1}{v} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$
- $\therefore -\frac{\mu_1}{u} - \frac{\mu_1}{v} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$
- $\therefore\ \ -\frac{1}{u} - \frac{1}{v} = (\mu_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ; $where\ \mu_{21} = \frac{\mu_2}{\mu_1} - - - - - - - - - - - - - - - - - - - - - - (3)$
- For u = -∞, the image is formed at principal focus, so v = -f, therefore eq (3) becomes
- $\frac{1}{f} = (\mu_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ***which is the expression for lens maker's formula – – – –(4)***

***Note that the nature of a lens depends on the medium surrounding the lens made of glass***

- The focal length of a lens made of glass is given by $\frac{1}{f} = (\mu_{ga} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ where $\mu_{ga} = \frac{\mu_g}{\mu_a}$ is the refractive index of glass w.r.t. air.
- Consider a liquid with refractive index $\mu_l$ ; refractive index of liquid w.r.t. air = $\mu_{la} = \frac{\mu_l}{\mu_a}$
- If the glass lens is immersed in the liquid, the focal length of the lens (f') is given by $\frac{1}{f'} = (\mu_{gl} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$, where $\mu_{gl} = \frac{\mu_g}{\mu_l}$ is the refractive index of glass w.r.t. the liquid.
- $\frac{1}{f'} = (\mu_{gl} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ can be written as $\frac{1}{f'} = \left(\frac{\mu_g}{\mu_l} \times \frac{\mu_a}{\mu_a} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \Rightarrow \frac{1}{f'} = \left(\frac{\mu_{ga}}{\mu_{la}} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$
- $\therefore \frac{1}{f'} = \left(\frac{\mu_{ga}}{\mu_{la}} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ , *where f' is the modified focal length of the glass lens* – – – – – – – (1)
- **Case 1: $\mu_{la} < \mu_{ga}$**
    - If the refractive index of liquid < the refractive index of glass (means $\mu_{la} < \mu_{ga}$), f' is positive. This means the nature of the lens remains the same.
- **Case 2: $\mu_{la} > \mu_{ga}$**
    - If the refractive index of liquid > the refractive index of glass (means $\mu_{la} > \mu_{ga}$), f' is negative. This means the nature of the lens changes. For example….
        - In case 2, if a convex lens is immersed in the liquid, the convex lens behaves as a concave lens.
        - In case 2, if a concave lens is immersed in the liquid, the concave lens behaves as a convex lens.
- **Case 2: $\mu_{la} = \mu_{ga}$**
    - If the refractive index of liquid = the refractive index of glass (means $\mu_{la} = \mu_{ga}$), then the glass lens cannot be distinguished from liquid. In other words, glass is not visible.

# 75dic

- Refractive index is the relative property of **two** media. If the first medium carrying the incident ray is a vacuum (*or air → approximately for solving problems in optics*), then the ratio of $\frac{\sin i}{\sin r}$ is called the "**absolute refractive index of the second medium**".
- The relative refractive index of any two media = ratio of their absolute refractive indices.
- Therefore, if the absolute refractive indices of media 1 and 2 be $\mu_1$ and $\mu_2$ respectively, then the refractive index of medium 2 wrt medium 1 is represented as $\mu_{21} = \frac{\mu_2}{\mu_1}$ ; $\therefore \frac{\sin i}{\sin r} = \mu_{21} = \frac{\mu_2}{\mu_1}$ ***OR*** $\mu_1 \sin i = \mu_2 \sin r$ ------ (1)
- Note that there are other representations of eq (1) like $\frac{\sin i}{\sin r} = {}_1\mu_2 = \frac{\mu_2}{\mu_1}$ in various books, they will be very confusing.
- **Follow** $\boldsymbol{\mu_{21} = \frac{\mu_2}{\mu_1}}$ ***as given in NCERT book; where*** $\boldsymbol{\mu_{21}}$ ***is the refrative index of medium 2 wrt medium 1.***

**Number of images formed by a lens:**

- If a lens is made of a single material and free from chromatic and spherical aberrations, then a single image of an object is formed.
- If a lens is made of more than one material, then number of images formed by the lens = number of materials. For example, if a lens is made of two materials, then two images of an object are formed.

**Focal length of a lens depends on the wavelength used**

- Since $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \Rightarrow f \propto \frac{1}{\mu}$ {$R_1$ and $R_2$ are constants & $\mu$ is the refractive index of the medium wrt air}
- Since $\mu \propto \frac{1}{\lambda^2}$ **(Cauchy's formula), therefore** $f \propto \lambda^2$
- Since wavelength of red light is greater than the wavelength of blue light, the focal length of a lens is greater when red light is used as compared to the focal length of the lens when the blue light is used.

**Example 9.7** A magician during a show makes a glass lens with $n = 1.47$ disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

**Solution**

The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $n_1 = n_2$. This gives $1/f = 0$ or $f \to \infty$. The lens in the liquid will act like a plane sheet of glass. No, the liquid is not water. It could be glycerine.

**Problem: A double concave lens of glass of μ = 1.6 has radii of curvature of 40cm & 60cm. Calculate the 'f' of the lens in air.**

- Since $\frac{1}{f} = (\mu_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$; *where* $\mu_{21} = \frac{\mu_2}{\mu_1}$
- Given $\mu_{21} = \mu_{glass}/\mu_{air} = 1.6/1 = 1.6$ ; for concave lens $R_1$ is –ve & $R_2$ is +ve. So, $R_1$ = -40cm, $R_2$ = +60cm
- $\frac{1}{f} = (\mu_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.6 - 1)\left[\frac{1}{-40} - \frac{1}{+60}\right] = -0.6\left[\frac{60+40}{60x40}\right] = -0.6\left[\frac{100}{60x40}\right] = -\frac{1}{40}$
- $f = -40cm$

**Problem: The power of a thin convex lens of glass is 5 D. When it is immersed in a liquid, then it behaves alike a divergent lens of focal length 100 cm. Calculate the refractive index of the liquid. $\mu_{glass}$ = 1.5**

**Note:** ***Since refractive index is a relative of two media, the convention used in NCERT book is*** $\mu_{ga} = \frac{\mu_g}{\mu_a}$

- Let the focal length of the convex lens in air (or wrt air) be $f_1$
  - Given $P_1$ = 5 D, therefore $f_1 = \frac{1}{5\,D} = 0.2\,m = 20\,cm$
- Let the focal length of the lens behaving as divergent lens in a liquid be $f_2$; given $f_2$ = -100 cm (minus sign due to behaviour of convex lens as concave lens when lens is immersed in a given liquid).
- We need to find out the refractive index of the given liquid
- *Thus,* $\frac{f_1}{f_2} = \frac{20cm}{-100cm} = -\frac{1}{5}$ -------------------- **(1)**
- By Lens maker's formula, we have
- $\frac{1}{f_1} = \left(\frac{\mu_g}{\mu_a} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ---------- **(2)**
- $\frac{1}{f_2} = \left(\frac{\mu_g}{\mu_l} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ -----------**(3)** **(2) divided by (3), we get**
- $\frac{\frac{1}{f_1}}{\frac{1}{f_2}} = \frac{\left(\frac{\mu_g}{\mu_a} - 1\right)}{\left(\frac{\mu_g}{\mu_l} - 1\right)} \Rightarrow \frac{f_2}{f_1} = \frac{\left(\frac{\mu_g}{\mu_a} - 1\right)}{\left(\frac{\mu_g}{\mu_l} - 1\right)}$ *OR* $\frac{f_1}{f_2} = \frac{\left(\frac{\mu_g}{\mu_l} - 1\right)}{\left(\frac{\mu_g}{\mu_a} - 1\right)}$ ; *from* (1), *we get* $\frac{\left(\frac{\mu_g}{\mu_l} - 1\right)}{\left(\frac{\mu_g}{\mu_a} - 1\right)} = -\frac{1}{5}$ ; *given* $\mu_g$ *OR* $\mu_{ga} = \frac{\mu_g}{\mu_a} = 1.5$
- $\therefore \frac{\left(\frac{\mu_g}{\mu_l} - 1\right)}{(1.5 - 1)} = \frac{1}{5} \Rightarrow -\left(\frac{\mu_g}{\mu_l} - 1\right) = -\frac{0.5}{5} = -0.1 \therefore \mu_{gl} = \frac{\mu_g}{\mu_l} = 0.9 \Rightarrow \mu_l = \frac{\mu_g}{0.9} = \frac{1.5}{0.9} = \frac{5}{3} = 1.67$
- $\therefore$ the refractive index of liquid wrt air $= \mu_{la}$ *OR* $\mu_l = \frac{\mu_l}{\mu_a} = 1.67$

**Problem: A biconvex lens has a focal length 2/3 times the radius of curvature of either surface. Calculate the refractive index of lens material.**

- Formula: $\frac{1}{f} = (\mu_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ --------------------------- **(1)**
- Given f = (2/3) R; for biconvex lens $R_1$ is +ve & $R_2$ is -ve. So, $R_1$ = R, $R_2$ = -R ; Plugging values in equation (1)
- $\frac{3}{2R} = (\mu_{21} - 1)\left[\frac{1}{R} + \frac{1}{R}\right] = (\mu_{21} - 1)\left[\frac{2}{R}\right]$
- $\frac{3}{2} = 2(\mu_{21} - 1) \Rightarrow (\mu_{21} - 1) = \frac{3}{4} \Rightarrow \mu_{21} = 1 + \frac{3}{4} = \frac{7}{4} = 1.75$
- $\mu_{21} = 1.75$ ; ***since*** $\mu_1 = 1$, *therefore* $\mu_2$ = *refractive index of lens* = 1.75

- To find the image of an object by a lens, we can, in principle, take any two rays emanating from a point on an object; trace their paths using the laws of refraction and find the point where the refracted rays meet (or appear to meet). In practice, however, it is convenient to choose any 2 of the following rays:
    - A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the second principal focus F' (in a convex lens) or appears to diverge (in a concave lens) from the first principal focus F.
    - A ray of light, passing through the optical centre of the lens, emerges without any deviation after refraction.
    - A ray of light passing through the first principal focus (for a convex lens) or appearing to meet at it (for a concave lens) emerges parallel to the principal axis after refraction.

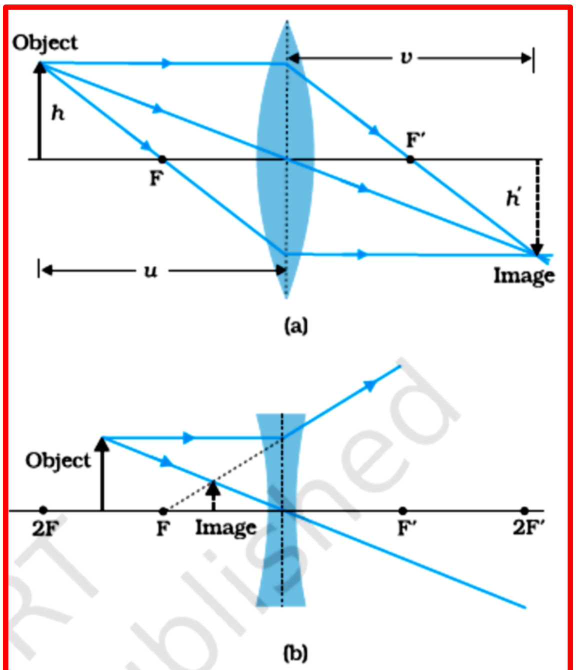



**FIGURE 9.19** Tracing rays through (a) convex lens (b) concave lens.

- Figures 9.19(a) and (b) illustrate these rules for a convex and a concave lens, respectively.
- Note that each point on an object gives out infinite number of rays. All these rays will pass through the same image point after refraction at the lens.
- Magnification (m) produced by a lens is defined, like that for a mirror, as the ratio of the size of the image to that of the object.
- $m = \frac{h'}{h} = \frac{v}{u}$
- When we apply the sign convention, we see that, for erect (and virtual) image formed by a convex or concave lens, m is positive, while for an inverted (and real) image, m is negative.

**Example 9.7** A magician during a show makes a glass lens with $n = 1.47$ disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

**Solution**

The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $n_1 = n_2$. This gives $1/f = 0$ or $f \to \infty$. The lens in the liquid will act like a plane sheet of glass. No, the liquid is not water. It could be glycerine.

**Example 9.6** Light from a point source in air falls on a spherical glass surface ($n$ = 1.5 and radius of curvature = 20 cm). The distance of the light source from the glass surface is 100 cm. At what position the image is formed?

**Solution**

We use the relation given by Eq. (9.16). Here

$u = -100$ cm, $v = ?$, $R = +20$ cm, $n_1 = 1$, and $n_2 = 1.5$.

We then have

$$\frac{1.5}{v} + \frac{1}{100} = \frac{0.5}{20}$$

or $v = +100$ cm

The image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

**75dk**

## Power of a Lens

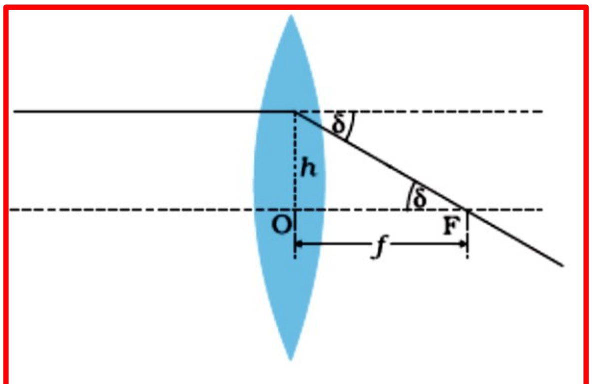



**FIGURE 9.20** Power of a lens.

- Power of a lens is a measure of the convergence or divergence, which a lens introduces in the light falling on it. Clearly, a lens of shorter focal length bends the incident light more, while converging it in case of a convex lens and diverging it in case of a concave lens.
- The power P of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distant from the optical centre (Fig. 9.20).
- $\tan\delta = \frac{h}{f}\,; if\ h = 1, then\ \tan\delta = \frac{1}{f}$
- For small values of δ, tan δ ≈ δ
- $\therefore\ \delta = \frac{1}{f}$; since tan δ = power P of the lens and for small angles of δ, tan δ ≈ δ, we therefore get
- $P = \frac{1}{f}$ ------------------ **(1)**
- The SI unit for power of a lens is dioptre (D): 1D = $1m^{-1}$.
- The power of a lens of focal length of 1 metre is one dioptre.
- Power of a lens is positive for a converging lens and negative for a diverging lens.
- Thus, when an optician prescribes a corrective lens of power +2.5D, the required lens is a convex lens of f = +40 cm.
- A lens of power of – 4.0 D means a concave lens of focal length – 25 cm.

**Example 9.8** (i) If $f$ = 0.5 m for a glass lens, what is the power of the lens? (ii) The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. Its focal length is 12 cm. What is the refractive index of glass? (iii) A convex lens has 20 cm focal length in air. What is focal length in water? (Refractive index of air-water = 1.33, refractive index for air-glass = 1.5.)

**Solution**

(i) Power = +2 dioptre.

(ii) Here, we have $f$ = +12 cm, $R_1$ = +10 cm, $R_2$ = –15 cm.
Refractive index of air is taken as unity.
We use the lens formula of Eq. (9.22). The sign convention has to be applied for $f$, $R_1$ and $R_2$.
Substituting the values, we have

$$\frac{1}{12} = (n-1)\left(\frac{1}{10} - \frac{1}{-15}\right)$$

This gives $n$ = 1.5.

(iii) For a glass lens in air, $n_2$ = 1.5, $n_1$ = 1, $f$= +20 cm. Hence, the lens formula gives

$$\frac{1}{20} = 0.5\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

For the same glass lens in water, $n_2$ = 1.5, $n_1$ = 1.33. Therefore,

$$\frac{1.33}{f} = (1.5-1.33)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad (9.26)$$

Combining these two equations, we find $f$ = + 78.2 cm.

**75dL**

## Combination of thin lenses in contact

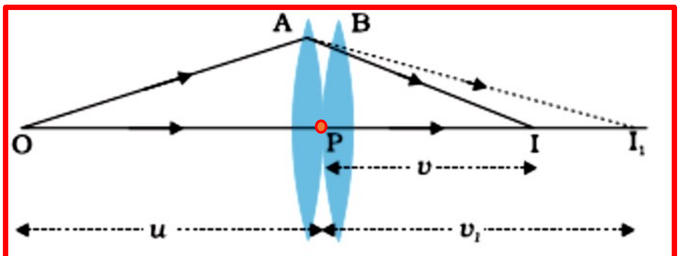



**FIGURE 9.21** Image formation by a combination of two thin lenses in contact.

- Consider two lenses A and B of focal length $f_1$ and $f_2$ placed in contact with each other. Let the object be placed at a point O beyond the focus of the first lens A (Fig. 9.21). The first lens produces an image at $I_1$. Since image $I_1$ is real, it serves as a virtual object for the second lens B, producing the final image at I.
- Note that the formation of image by the first lens is presumed **only** to facilitate determination of the position of the final image. In fact, the direction of rays emerging from the first lens gets modified in accordance with the angle at which they strike the second lens. Since the lenses are thin, we assume the optical centres of the lenses to be coincident. Let this central point be denoted by P.
- For the image formed by the first lens A, we get

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \text{ -------------------- } (1)$$

- For the image formed by the second lens B, we get

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \text{ -------------------- } (2);$$ **Adding (1) and (2), we get**

- $\frac{1}{v_1} - \frac{1}{u} + \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{f_2}$ **; simplifying, we get** $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$ ------------------ **(3)**
- If the two lens-system is regarded as equivalent to a single lens of focal length f, we have
- $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ ; *where* $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ -------------------- **(4)**
- The derivation is valid for any number of thin lenses in contact. If several thin lenses of focal length f1, f2, f3, f4... are in contact, the effective focal length of their combination is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \frac{1}{f_4} + \dots\dots \text{ -------------------- } (5)$$ **; Since power of lens** $P = \frac{1}{f}$

$$P = P_1 + P_2 + P_3 + P_4 + \text{----------} \quad \text{--------------}(6)$$

- Where P is the net power of the lens combination. Note that the sum in Eq. (6) is an **algebraic sum of individual powers**, so some of the terms on the right side may be positive (for convex lenses) and some negative (for concave lenses).
- Combination of lenses helps to obtain diverging or converging lenses of desired magnification. It also enhances sharpness of the image.
- Since the image formed by the first lens becomes the object for the second, Eq. $P = \frac{1}{f}$ implies that the total magnification m of the combination is a **product of magnification** ($m_1$, $m_2$, $m_3$,...) of individual lenses.
- Such a system of combination of lenses is commonly used in designing lenses for cameras, microscopes, telescopes and other optical instruments.

### Why combination of lenses is the product of the magnifications by the individual lenses: means why $m = m_1 \times m_2 \times m_3 \times \dots$

Ans :

- Let $m_1$ be the magnification of the "first" lens
- Let $m_2$ be the magnification of the "second" lens
- Let $h_o$ be the height of the object
- Let $h_i$ be the height of the image from the first lens,
  - which is also = height of the object for 2nd lens
- Let $h_2$ be the height of the final image

We know that,

$$m_1 = \frac{hi}{ho} \; ; \; m_2 = \frac{h2}{hi} \; ; \text{ Overall magnification } (m) = \frac{\text{height of final image}}{\text{height of object}}$$

$$m = \frac{h2}{ho} = \frac{h2}{ho} \times \frac{hi}{hi} = \frac{hi}{ho} \times \frac{h2}{hi} = m_1 \times m_2 \,;$$

Therefore **$m = m_1 \times m_2 \times m_3 \times \dots$**

# 75dm

## Refraction of Light

1. Refraction at plane transparent surface → "Snell's Law", "Real/Apparent depth", "Total Internal Reflection"
2. Refraction at a spherical surface
3. Refraction by Lenses

## 4. Refraction through a (triangular) Prism

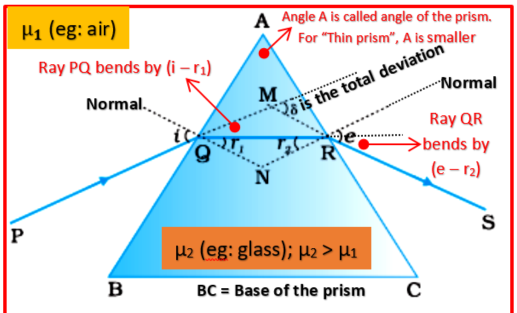



**FIGURE 9.23** A ray of light passing through a triangular glass prism.

- Figure 9.23 shows the passage of light through a **triangular [*preferably Isosceles or Equilateral*] prism** ABC. The angles of incidence and refraction at the first face AB are i and $r_1$, while the angle of incidence (from glass to air) at the second face AC is $r_2$ and the angle of refraction or emergence e. The angle between the emergent ray RS and the direction of the incident ray PQ is called the angle of deviation 'δ'.
- In the quadrilateral AQNR, two of the angles (at the vertices Q and R) are right angles. Therefore, the sum of the other angles of the quadrilateral is 180°.
- $\therefore \hat{A} + Q\hat{N}R = 180^0 - - - - - - - - - - - - (1)$
- From triangle QNR, $r_1 + r_2 + Q\hat{N}R = 180^0 - -(2)$
- Comparing (1) & (2), we get $\boldsymbol{r_1 + r_2 = A - - - (3)}$
- The total deviation δ is the sum of deviations at the two faces AB & AC, ⇒ δ = (i – $r_1$) + (e-$r_2$) ⇒ δ = i + e – ( $r_1$ + $r_2$) ⇒ **δ = i + e – A**
- **∴ δ = i + e – A ---------------------------------- (4)**
- **Since $e$ in turn depends on $i$ and μ of the medium (say glass) and since μ & A are constants for a given prism, the angle of deviation depends only on the angle of incidence.**
- **Instead of writing difficult equation for δ equating with $i$,** $A$ and μ, we will draw a simple graph (representing the equation) between the angle of deviation and angle of incidence as shown in Fig. 9.24. You can see that, in general, any given value of δ, except for $i = e$, corresponds to two values $i$ and hence of $e$. This is expected from the symmetry of $i$ and $e$ in Eq. (4), i.e., δ remains the same if $i$ and $e$ are interchanged. Physically, this is related to the fact that the path of ray in Fig. 9.23 can be traced back, resulting in the same angle of deviation.
- When the entrance and exit angles are equal ($i = e$), the deviation angle of a ray passing through a prism will be minimal. At the **minimum deviation (δ = $D_m$)**, the refracted ray inside the prism becomes parallel to its base. ∴ When $i = e,\ r_1 = r_2$. Hence eq (3) becomes $2r = A$ OR $\boldsymbol{r} = \frac{A}{2}$ ----- (5)

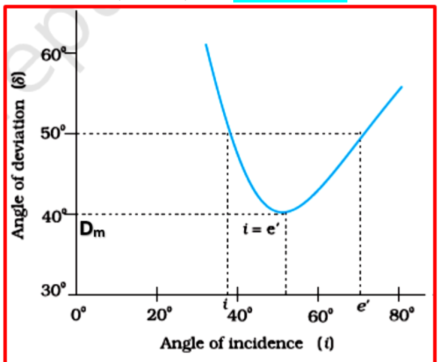



**FIGURE 9.24** Plot of angle of deviation (δ) versus angle of incidence (i) for a triangular prism.

- **Similarly, Eq. (4) gives $D_m$ = 2$i$ – A** or $\boldsymbol{i = \frac{A+D_m}{2}}$ ---------------------------------------(6)
- From Snell's law $\frac{sin(i)}{sin(r)} = \frac{\mu_2}{\mu_1} = \mu_{21}$ ∴ μ of the prism is given by
- **Using (5) & (6),** $\boldsymbol{\mu_{21} = \frac{\mu_2}{\mu_1} = \frac{sin\left[\frac{A+D_m}{2}\right]}{sin\left[\frac{A}{2}\right]}}$ -------------------- **(7)**
- The angles A and $D_m$ can be measured experimentally. Equation (7) thus provides a method of determining refractive index of the material of the prism.
- For a small angle prism (angle A being small ≈ $10^0$ to $25^0$), i.e., a thin prism, $D_m$ is **also** very small and sin(θ) ≈ θ, and we get
- $\mu_{21} = \frac{\mu_2}{\mu_1} = \frac{\left[\frac{A+D_m}{2}\right]}{\left[\frac{A}{2}\right]} \Rightarrow \mu_{21} = \frac{[A+D_m]}{[A]} \Rightarrow \mu_{21}A = \boldsymbol{A + D_m} \Rightarrow \boldsymbol{D_m} = (\mu_{21} - 1)A$
- $\boldsymbol{D_m = (\mu_{21} - 1)A}$ $\quad OR \quad D_m = \left[\frac{\mu_2}{\mu_1} - 1\right]A$ ---------------------------- (8)
- **If incident & emergent rays are in medium "air", then $\boldsymbol{\mu_1}$ = 1 & $\boldsymbol{\mu_2}$ is for glass (say μ), then $\boldsymbol{D_m = [\mu - 1]A}$ ----- (9)**
- **It implies that, thin prisms do not deviate light much.**

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- When $i$ = 90°, the incident ray grazes along the surface of the prism and the angle of refraction inside the prism becomes equal to the critical angle for glass - air. This is known as grazing incidence.
- When e = 90°, the emergent ray grazes along the prism surface. This happens when the light ray strikes the second face of the prism at the critical angle for glass - air. This is known as grazing emergence.
  - The angle of deviation is same for both the above cases (grazing incidence & grazing emergence) and it is also the maximum possible deviation if the light ray is to emerge out from the other face without any total internal reflection.

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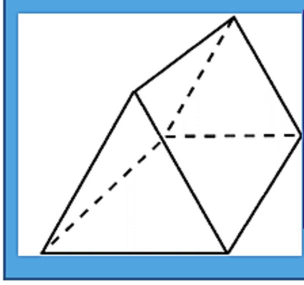
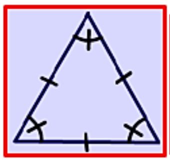


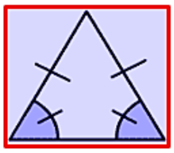
Triangular Prism shown in figure has 5 faces, 9 edges and 6 vertices

Checking with Euler's rule: V – E + F = 2, where V, E, and F are the number of vertices, edges, and faces respectively. 6–9+5=2, so our answer is correct.



**Equilateral triangle**

The Equilateral triangle has 3 congruent sides and 3 congruent angles. Each angle is 60°



**Isosceles triangle**

The Isosceles triangle has 2 equal sides and 2 equal angles.

# Thin Lens Formula: Convex Lens:
# Example Case : Real Image (Object between $C_1$ and $F_1$)

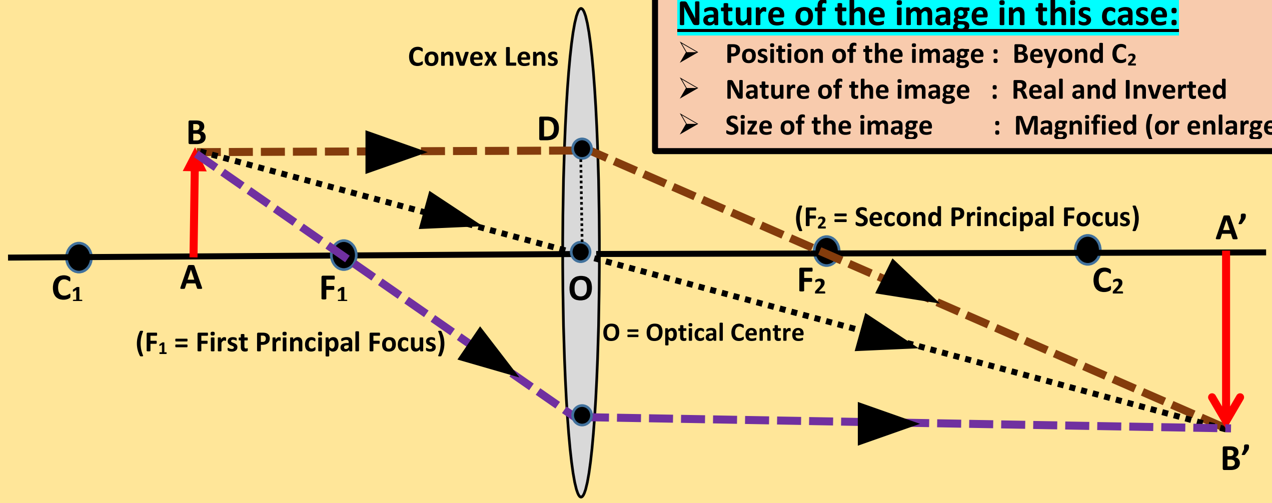
**75e**

- **A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the Second Principal Focus $F_2$.**
- **A ray of light, passing through the optical centre of the lens O, emerges without any deviation after refraction.**
- **A ray of light passing through the First Principal Focus $F_1$ emerges parallel to the principal axis after refraction.**

**Applying any 2 of the above ray principles, we can locate the position of the image formed by the lens.**

## Nature of the image in this case:

- **Position of the image : Beyond $C_2$**
- **Nature of the image : Real and Inverted**
- **Size of the image : Magnified (or enlarged)**




**Let AB be the Object at a distance u from the Optical Centre O**
**Let A'B' be the Image at a distance v from the Optical Centre O**
**$C_1$ is the First Centre of Curvature; $C_2$ is the Second Centre of Curvature**

**To prove Lens Formula, we need to consider similar triangles centered at $F_1$ and $F_2$ that includes object, image and optical centre.**

| At $F_1$ that includes object, image and O | At $F_2$ that includes object, image and O: |
|---|---|
| ➢ $\Delta^{les}$ OAB & OA'B' are similar (AAA → similarity criteria)<br>➢ $\therefore \frac{AB}{A'B'} = \frac{OA}{OA'}$ ------------------------------- **(1)** | ➢ $\Delta^{les}$ $ODF_2$ & $F_2A'B'$ are similar (AAA → similarity criteria)<br>➢ $\therefore \frac{OD}{A'B'} = \frac{OF_2}{F_2A'}$ ; **From figure, we get OD = AB**<br>➢ $\therefore \frac{AB}{A'B'} = \frac{OF_2}{F_2A'}$ ----------------------------------- **(2)** |

- **From (1) and (2),** we have $\frac{OA}{OA'} = \frac{OF_2}{F_2A'} \Rightarrow \frac{OA}{OA'} = \frac{OF_2}{OA^1 - OF_2}$ ------------------------------ **(3)**
- **Apply Cartesian sign convention**, OA = -u, OA' = +v, $OF_2$ = +f **(for Convex Lens, take f as +ve)**
- Eq (3) becomes $\frac{-u}{+v} = \frac{+f}{+v-f} \Rightarrow \frac{u}{v} = \frac{f}{f-v}$ ; simplifying, we get
- **uf – uv = vf** $\Rightarrow$ uv = uf – vf ; Dividing LHS and RHS by uvf, we get

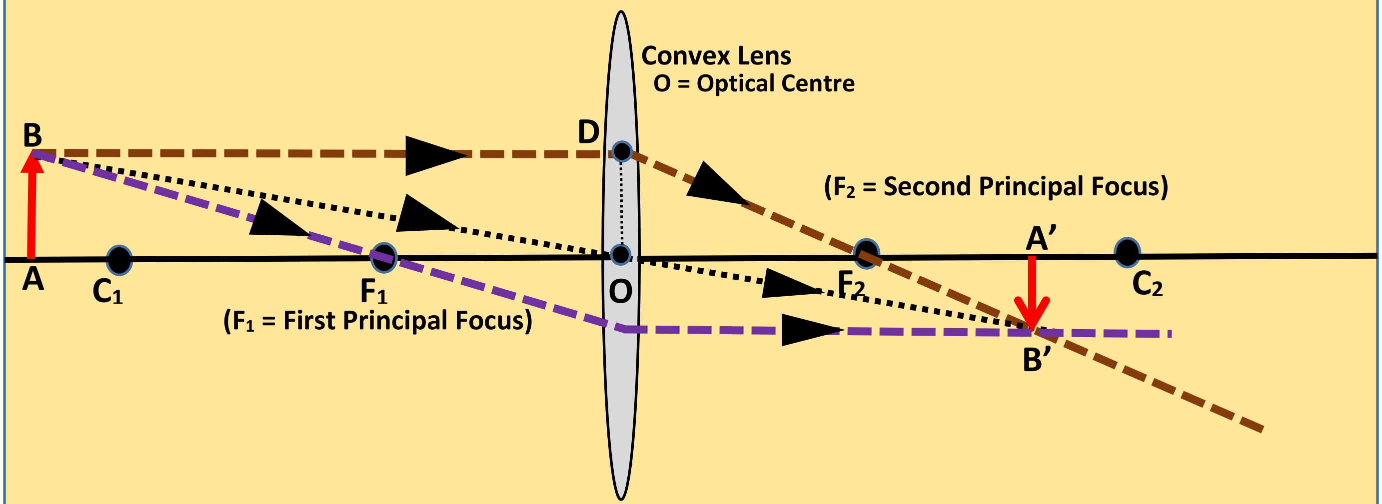
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$ **→ This is the required Thin Lens Formula --------------- (4)**

# Thin Lens Formula: Convex Lens:
# Example Case : Real Image (Object beyond $C_1$)

**75ee**

- A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the Second Principal Focus $F_2$.
- A ray of light, passing through the optical centre of the lens O, emerges without any deviation after refraction.
- A ray of light passing through the First Principal Focus $F_1$ emerges parallel to the principal axis after refraction.

Applying any 2 of the above ray principles, we can locate the position of the image formed by the lens.




Let AB be the Object at a distance u from the Optical Centre O

Let A'B' be the Image at a distance v from the Optical Centre O

$C_1$ is the First Centre of Curvature; $C_2$ is the Second Centre of Curvature

To prove Lens Formula, we need to consider similar triangles centered at $F_1$ and $F_2$ that includes object, image and optical centre.

| At $F_1$ that includes object, image and O | At $F_2$ that includes object, image and O: |
|---|---|
| ➢ $\Delta^{les}$ OAB & OA'B' are similar (AAA → similarity criteria) | ➢ $\Delta^{les}$ $ODF_2$ & $F_2A'B'$ are similar (AAA → similarity criteria) |
| ➢ $\therefore \frac{AB}{A'B'} = \frac{OA}{OA'}$ ------------------------------ (1) | ➢ $\therefore \frac{OD}{A'B'} = \frac{OF_2}{F_2A'}$ ; **From figure, we get OD = AB** |
| | ➢ $\therefore \frac{AB}{A'B'} = \frac{OF_2}{F_2A'}$ ---------------------------------- (2) |

- **From (1) and (2),** we have $\frac{OA}{OA'} = \frac{OF_2}{F_2A'}$ $\Rightarrow$ $\frac{OA}{OA'} = \frac{OF_2}{OA^1 - OF_2}$ ---------- **(3)**
- **Apply Cartesian sign convention,** OA = -u, OA' = +v, $OF_2$ = +f **(for Convex Lens, take f as +ve)**
- Eq (3) becomes $\frac{-u}{+v} = \frac{+f}{+v-f}$ $\Rightarrow$ $\frac{u}{v} = \frac{f}{f-v}$ ; simplifying, we get
- **uf – uv = vf** $\Rightarrow$ uv = uf – vf ; Dividing LHS and RHS by uvf, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$ **→ This is the required Thin Lens Formula --------------- (4)**

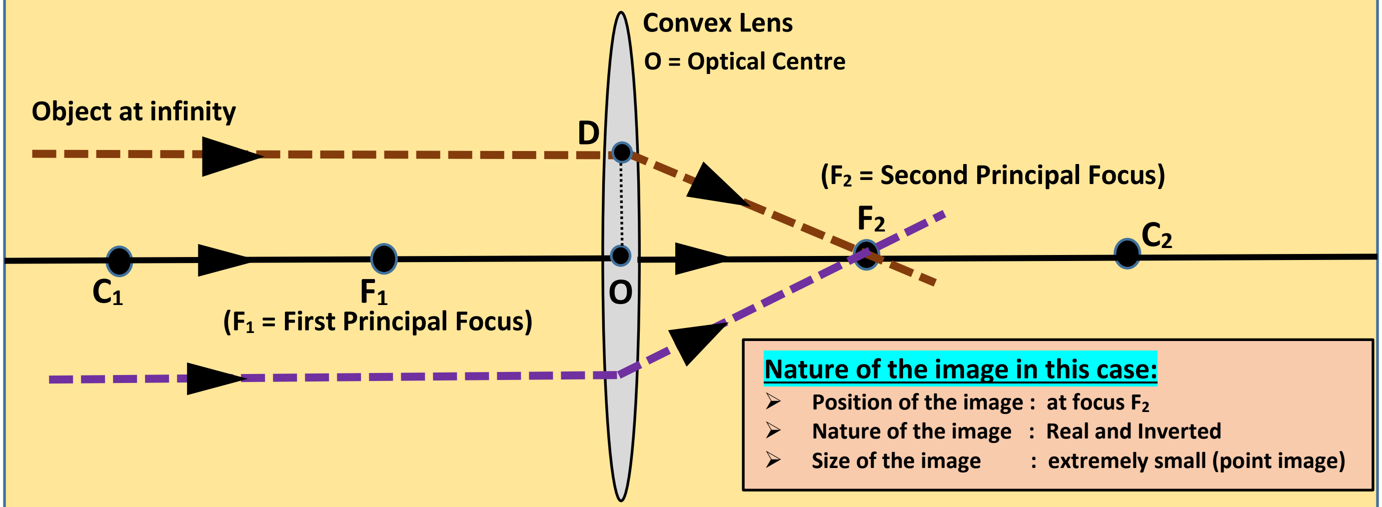
## Nature of the image in this case:

- **Position of the image : Between $F_2$ and $C_2$**
- **Nature of the image : Real and Inverted**
- **Size of the image : Diminished (or smaller compared to the object height)**

## Thin Lens Formula: Convex Lens: Example Case : Real Image (Object at infinity)

- A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the Second Principal Focus $F_2$.
- A ray of light, passing through the optical centre of the lens O, emerges without any deviation after refraction.
- A ray of light passing through the First Principal Focus $F_1$ emerges parallel to the principal axis after refraction.

Applying any 2 of the above ray principles, we can locate the position of the image formed by the lens.




**Nature of the image in this case:**
- Position of the image : at focus $F_2$
- Nature of the image : Real and Inverted
- Size of the image : extremely small (point image)

Let AB be the Object at a distance u from the Optical Centre O (here u = -∞)
Let A'B' be the Image at a distance v from the Optical Centre O
$C_1$ is the First Centre of Curvature; $C_2$ is the Second Centre of Curvature

- Given u = -∞ ; $\therefore \frac{1}{u} = \frac{1}{-\infty}$ or $\frac{1}{\infty} = -\frac{1}{u}$ ---------------------------- (1)
- From Ray diagram, image is formed at focus $F_2$, therefore $OF_2$ = +v
- However, focal length $OF_2$ = +f (for Convex Lens, take f as +ve) $\therefore$ f = v OR $\frac{1}{f} = \frac{1}{v}$ OR $\frac{1}{f} = \frac{1}{v} + \frac{1}{\infty}$
- From eq (1), we can write $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ → This is the required Thin Lens Formula (2)

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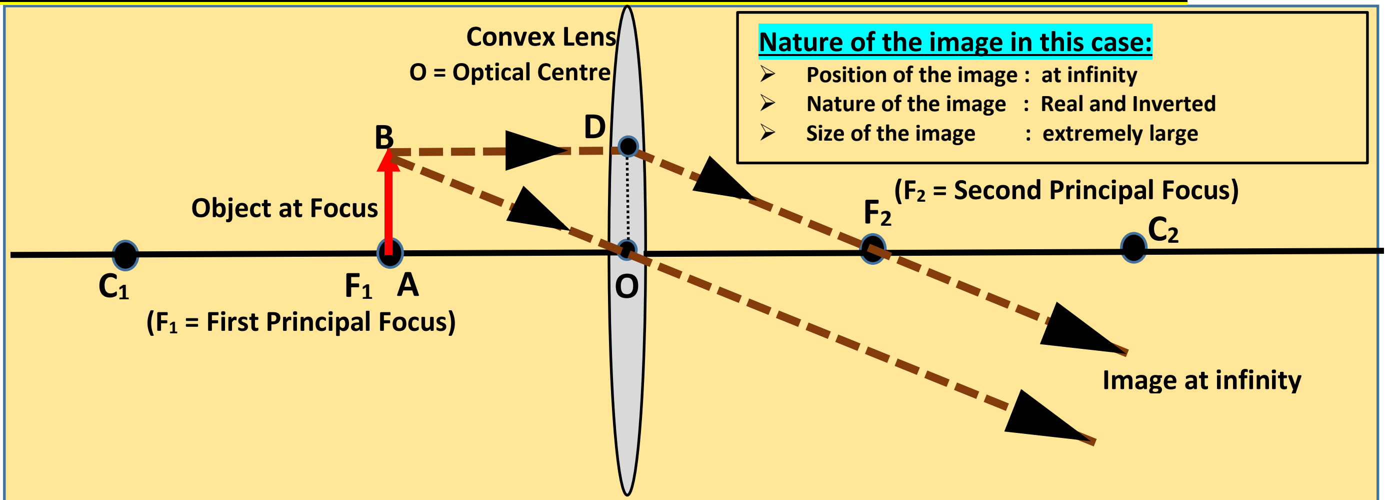
## Thin Lens Formula: Convex Lens: Example Case : Real Image (Object at Focus $F_1$)




**Nature of the image in this case:**
- Position of the image : at infinity
- Nature of the image : Real and Inverted
- Size of the image : extremely large

Let AB be the Object at a distance u from the Optical Centre O (here u = -f)
Let A'B' be the Image at a distance v from the Optical Centre O (here v = ∞)
$C_1$ is the First Centre of Curvature; $C_2$ is the Second Centre of Curvature

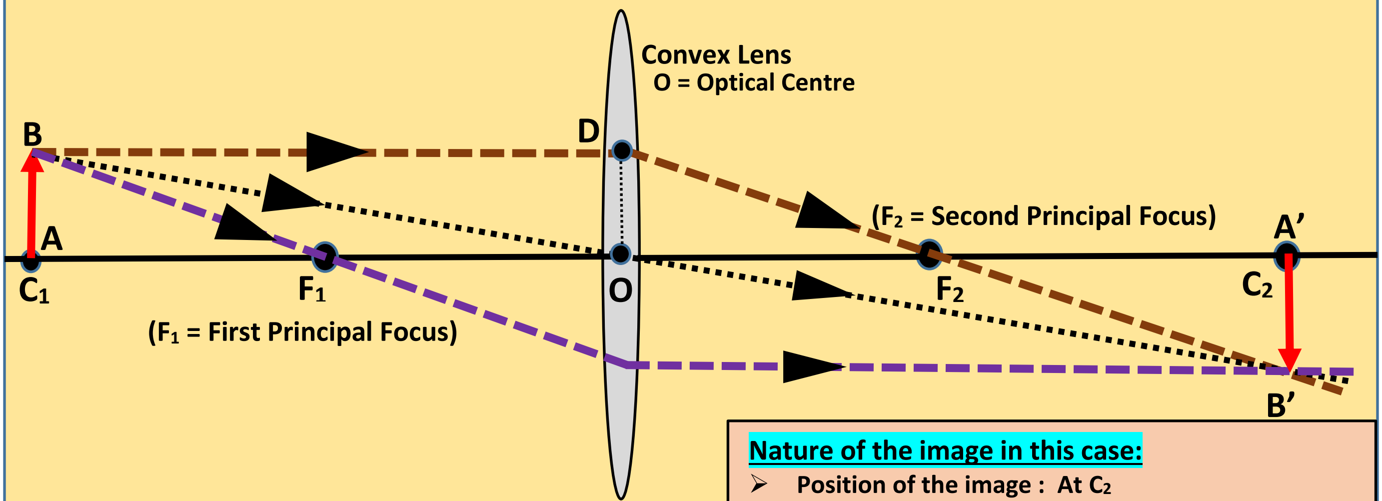
- Given u = -f ; $\therefore \frac{1}{u} = -\frac{1}{f}$ or $\frac{1}{f} = -\frac{1}{u}$ ---------------------------------------------------------------- (1)
- From Ray diagram, image is formed at infinity, therefore v = +∞ OR $\frac{1}{v} = 0 = \frac{1}{\infty}$ ---------------- (2)
- From (1) & (2), we can write $\frac{1}{f} = 0 - \frac{1}{u}$ OR $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ → This is the required Thin Lens Formula (3)

75eeee

# Thin Lens Formula: Convex Lens: Case : Object at $C_1$

- A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the Second Principal Focus $F_2$.
- A ray of light, passing through the optical centre of the lens O, emerges without any deviation after refraction.
- A ray of light passing through the First Principal Focus $F_1$ emerges parallel to the principal axis after refraction.

Applying any 2 of the above ray principles, we can locate the position of the image formed by the lens.




**Nature of the image in this case:**
- Position of the image : At $C_2$
- Nature of the image : Real and Inverted
- Size of the image : Same size as that of object

Let AB be the Object at a distance u from the Optical Centre O

Let A'B' be the Image at a distance v from the Optical Centre O

$C_1$ is the First Centre of Curvature; $C_2$ is the Second Centre of Curvature

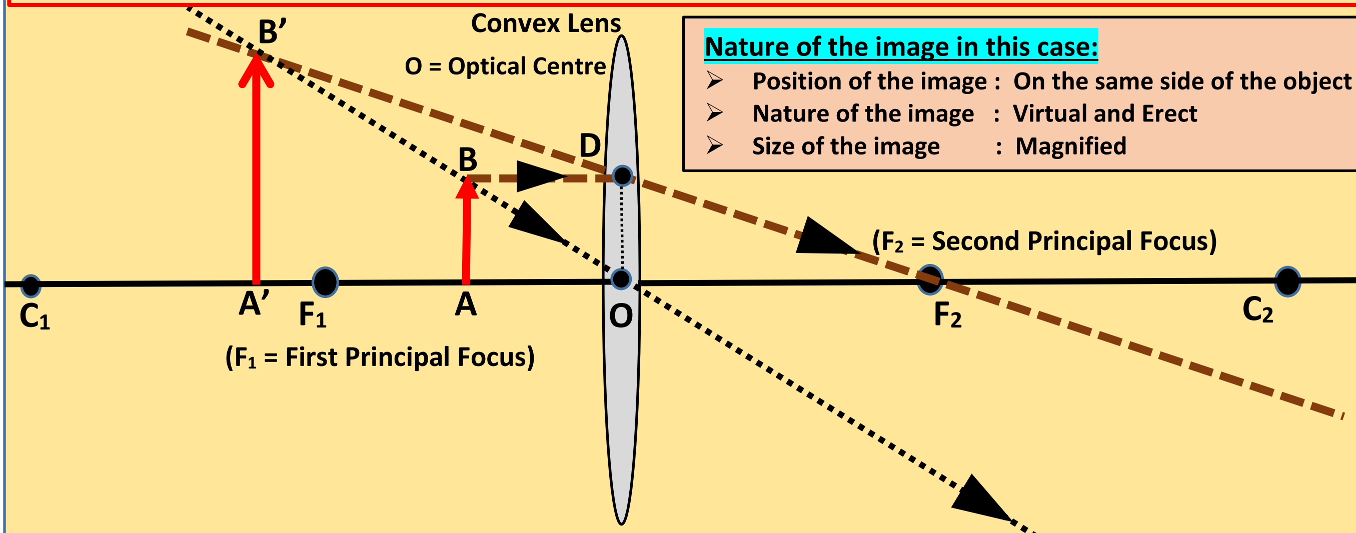
- Given $OC_1 = u$ ; Apply Cartesian sign convention $u = -2f$ OR $f = -\frac{2}{u}$ $\therefore \frac{1}{f} = -\frac{2}{u}$ -------------- (1)
- From Ray diagram, image is formed at $C_2$, $\therefore OA' = v$ ; Apply Cartesian sign convention $v = 2f$ ; $\frac{1}{f} = \frac{2}{v}$ ---------(2)
- Adding (1) & (2), we get $\frac{2}{f} = \frac{2}{v} - \frac{2}{u}$ OR $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ → This is the required Thin Lens Formula -- ------ (3)

# Thin Lens Formula: Convex Lens:
# Case : Object between $F_1$ and O

**75eeeee**

- **A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the Second Principal Focus $F_2$.**
- **A ray of light, passing through the optical centre of the lens O, emerges without any deviation after refraction.**
- **A ray of light passing through the First Principal Focus $F_1$ emerges parallel to the principal axis after refraction.**

**Applying any 2 of the above ray principles, we can locate the position of the image formed by the lens.**




**Nature of the image in this case:**
- **Position of the image : On the same side of the object**
- **Nature of the image : Virtual and Erect**
- **Size of the image : Magnified**

**Let AB be the Object at a distance u from the Optical Centre O**
**Let A'B' be the Image at a distance v from the Optical Centre O**
**$C_1$ is the First Centre of Curvature; $C_2$ is the Second Centre of Curvature**

**To prove Lens Formula, we need to consider similar triangles centered at $F_1$ and $F_2$ that includes object, image and optical centre.**

| **At $F_1$ that includes object, image and O** | **At $F_2$ that includes object, image and O:** |
|---|---|
| ➢ $\Delta^{les}$ OAB & OA'B' are similar (AAA → similarity criteria) | ➢ $\Delta^{les}$ ODF₂ & F₂A'B' are similar (AAA → similarity criteria) |
| ➢ $\therefore \frac{AB}{A'B'} = \frac{OA}{OA'}$ ------------------------------ **(1)** | ➢ $\therefore \frac{OD}{A'B'} = \frac{OF_2}{F_2A'}$ ; **From figure, we get OD = AB** |
| | ➢ $\therefore \frac{AB}{A'B'} = \frac{OF_2}{F_2A'}$ ---------------------------------- **(2)** |

- **From (1) and (2),** we have $\frac{OA}{OA'} = \frac{OF_2}{F_2A'} \Rightarrow \frac{OA}{OA'} = \frac{OF_2}{OA^1 + OF_2}$ ---------------------------- **(3)**
- **Apply Cartesian sign convention**, OA = -u, OA' = -v, $OF_2$ = +f **(for Convex Lens, take f as +ve)**
- Eq (3) becomes $\frac{-u}{-v} = \frac{+f}{-v+f} \Rightarrow \frac{u}{v} = \frac{f}{f-v}$ ; simplifying, we get
- **uf – uv = vf** $\Rightarrow$ uv = uf – vf ; Dividing LHS and RHS by uvf, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$ **→ This is the required Thin Lens Formula --------------- (4)**

# Thin Lens Formula: Concave Lens:

# Only 2 cases for Concave Lens



1. Object between Infinity and Optical Centre
2. Object at infinity (see Page 75ff)

## Object between Infinity and Optical Centre

- A ray emanating from the object parallel to the principal axis of the lens after refraction appears to diverge from the First Principal Focus $F_1$ in the case of concave lens.
- A ray of light, passing through the optical centre of the lens O, emerges without any deviation after refraction.
- A ray of light appears to meet at First Principal focus $F_1$ of a concave lens travels parallel to Principal axis after refraction.

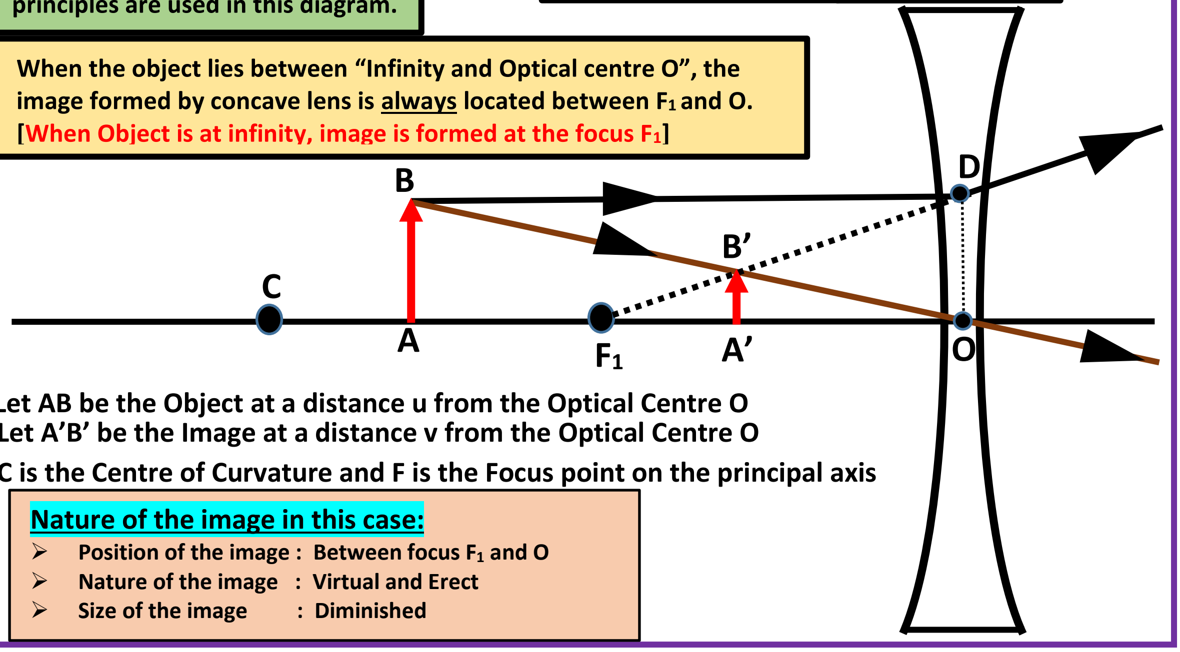
Applying any 2 of the above ray principles, we can locate the position of the image formed by the lens.

The above first two ray diagram principles are used in this diagram.

O is the optical centre

Concave Lens

When the object lies between "Infinity and Optical centre O", the image formed by concave lens is always located between $F_1$ and O.
[When Object is at infinity, image is formed at the focus $F_1$]




Let AB be the Object at a distance u from the Optical Centre O
Let A'B' be the Image at a distance v from the Optical Centre O

C is the Centre of Curvature and F is the Focus point on the principal axis

**Nature of the image in this case:**

- Position of the image : Between focus $F_1$ and O
- Nature of the image : Virtual and Erect
- Size of the image : Diminished

**To prove Lens Formula, we need to consider similar triangles centered at $F_1$ and $F_2$ that includes object, image and optical centre.**

| At $F_1$ that includes object, image and O | At $F_1$ that includes object, image and O: |
|---|---|
| ➢ $\Delta^{les}$ OAB & OA'B' are similar (AAA → similarity criteria) | ➢ $\Delta^{les}$ $ODF_1$ & $F_1A'B'$ are similar (AAA → similarity criteria) |
| ➢ $\therefore \frac{AB}{A'B'} = \frac{OA}{OA'}$ ----------------------------- (1) | ➢ $\therefore \frac{OD}{A'B'} = \frac{OF_1}{F_1A'}$ ; From figure, we get OD = AB |
| | ➢ $\therefore \frac{AB}{A'B'} = \frac{OF_1}{F_1A'}$ ---------------------------------- (2) |

- From (1) and (2), we have $\frac{OA}{OA'} = \frac{OF_2}{F_2A'} \Rightarrow \frac{OA}{OA'} = \frac{OF_1}{OF^1 - OA'}$ -------------------------- **(3)**
- **Apply Cartesian sign convention**, OA = -u, OA' = -v, $OF_1$ = -f **(for Concave Lens, take f as -ve)**
- Eq (3) becomes $\frac{-u}{-v} = \frac{-f}{-f+v} \Rightarrow \frac{u}{v} = \frac{f}{f-v}$ ; simplifying, we get
- **uf – uv = vf** ⇒ uv = uf – vf ; Dividing LHS and RHS by uvf, we get

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ → **This is the required Thin Lens Formula;** ---------------------------------- **(4)**

**The Formula is same for both Convex and Concave lens.**

# Thin Lens Formula: Concave Lens:
# Only 2 cases for Concave Lens

1. Object between Infinity and Optical Centre
2. Object at infinity (this page)

75ff

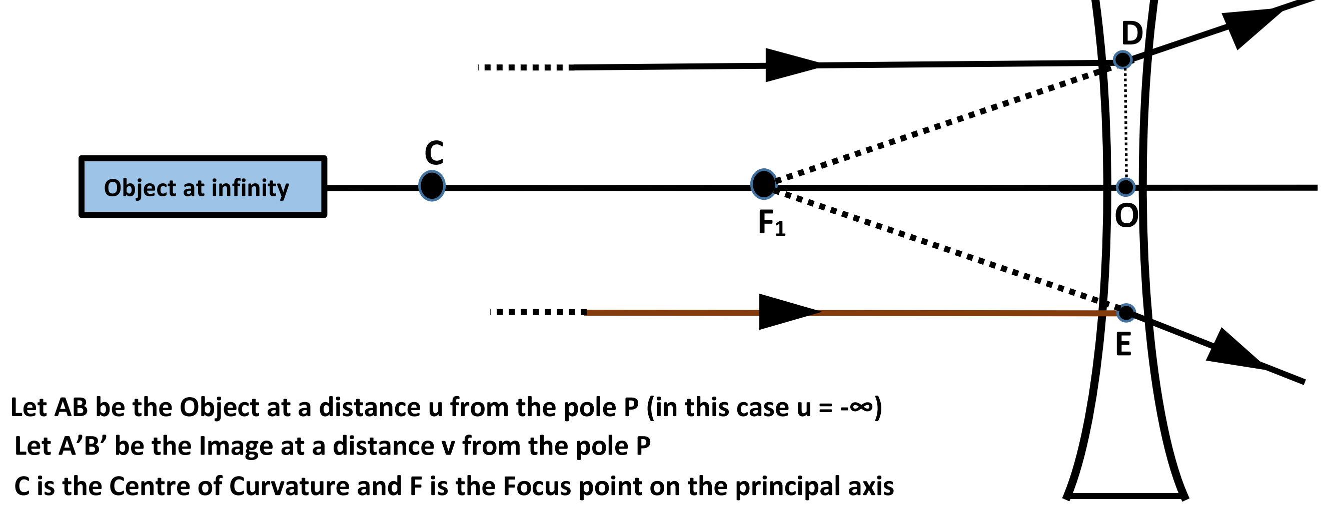
## Object at Infinity

- A ray emanating from the object parallel to the principal axis of the lens after refraction appears to diverge from the First Principal Focus $F_1$ in the case of concave lens.
- A ray of light, passing through the optical centre of the lens O, emerges without any deviation after refraction.
- A ray of light appears to meet at First Principal focus $F_1$ of a concave lens travels parallel to Principal axis after refraction.

Applying any 2 of the above ray principles, we can locate the position of the image formed by the lens.

When Object is at infinity, image is formed at the focus $F_1$
(When the object lies between "Infinity and Optical centre O", the image formed by concave lens is always located between $F_1$ and O)

Nature of the image in this case:
- Position of the image : at focus $F_1$
- Nature of the image : Virtual and Erect
- Size of the image : extremely small (point image)

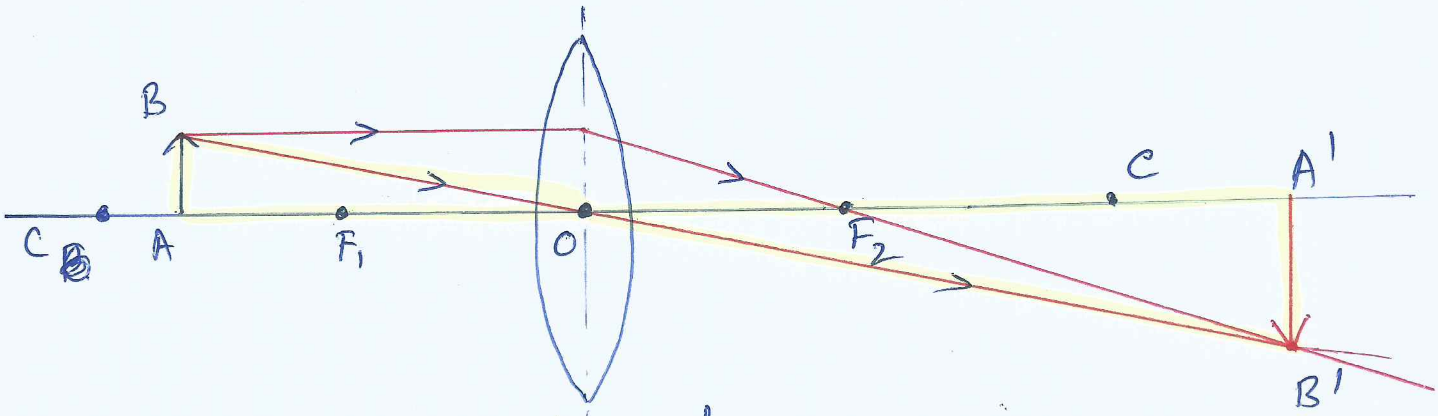



Let AB be the Object at a distance u from the pole P (in this case u = -∞)
Let A'B' be the Image at a distance v from the pole P
C is the Centre of Curvature and F is the Focus point on the principal axis

- Given u = -∞ ; $\therefore \frac{1}{u} = \frac{1}{-\infty}$ or $\frac{1}{\infty} = -\frac{1}{u}$ ------------------------------------------------------------ (1)
- From Ray diagram, image is formed at focus $F_1$, therefore $OF_1$ = -v
- However, focal length $OF_1$ = -f (for Concave Lens, take f as -ve) $\therefore$ f = v OR $\frac{1}{f} = \frac{1}{v}$ OR $\frac{1}{f} = \frac{1}{v} + \frac{1}{\infty}$
- From eq (1), we can write $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ → This is the required Thin Lens Formula (2)

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ → This is the required Thin Lens Formula; ---------------------------------- (4)
The Formula is same for both Convex and Concave lens.

**Linear Magnification in Lens** $= \dfrac{\text{height of image}}{\text{height of object}}$




- Let height of object $= h_o$
- Let height of image $= h_i$

In figure, $\Delta^{le}\, AOB$ and $\Delta^{le}\, A'OB'$ are similar

(Since $B\hat{A}O = B'\hat{A}O = 90^\circ$)
($A\hat{O}B = A'\hat{O}B' \rightarrow$ Vertically opposite angle)
(AA - Similarity criterion)

$$\therefore \frac{A'B'}{AB} = \frac{OA'}{OA} \quad \text{(Sides are proportional)}$$

Applying Cartesian sign convention, $OA' = +V$, $OA = -u$

$A'B' = -h_i$, $AB = +h_o$

$$\therefore \frac{A'B'}{AB} = \frac{OA'}{OA} \longrightarrow \frac{-h_i}{+h_o} = \frac{+V}{-u} \rightarrow -\frac{h_i}{h_o} = -\frac{V}{u}$$

$$\therefore \frac{h_i}{h_o} = \frac{V}{u}$$

Since $\boxed{m = \dfrac{h_i}{h_o}}$ $\therefore$ $\boxed{m = \dfrac{V}{u}}$ for both Convex and concave lens.

**IMP** Combination of lenses $\rightarrow$ Net magnification.

- Let $m_1$ be the magnification of the "First" lens and
- let $m_2$ ——— " ——— Second lens.
- Let $h_o \rightarrow$ height of object
- let $h_i \rightarrow$ height of the first image $=$ ~~obje~~ height of object for 2nd Lens
- Let $h_2 \rightarrow$ height of final image.

$\therefore m_1 = \dfrac{h_i}{h_o}$ ; $m_2 = \dfrac{h_2}{h_i}$ $\therefore$ Net magnification

$$m = \frac{h_2}{h_o} = \frac{h_2}{h_o} \times \frac{h_i}{h_i} = \frac{h_2}{h_i} \times \frac{h_i}{h_o} = \frac{h_i}{h_o} \times \frac{h_2}{h_i}$$

$$\therefore m = m_1 \times m_2$$

**Why combination of lenses is the product of the magnifications by the individual lenses: means why $m = m_1 \times m_2 \times m_3 \times \ldots$..**

Ans :

- Let $m_1$ be the magnification of the "first" lens
- Let $m_2$ be the magnification of the "second" lens
- Let $h_o$ be the height of the object
- Let $h_i$ be the height of the image from the first lens,
  - which is also = height of the object for 2nd lens
- Let $h_2$ be the height of the final image

We know that,

$m_1 = \frac{hi}{ho}$ ; $m_2 = \frac{h2}{hi}$ ; Overall magnification (m) = $\frac{\text{height of final image}}{\text{height of object}}$

$$m = \frac{h2}{ho} = \frac{h2}{ho} \times \frac{hi}{hi} = \frac{hi}{ho} \times \frac{h2}{hi} = m_1 \times m_2 \;;$$

Therefore **$m = m_1 \times m_2 \times m_3 \times \ldots$..**