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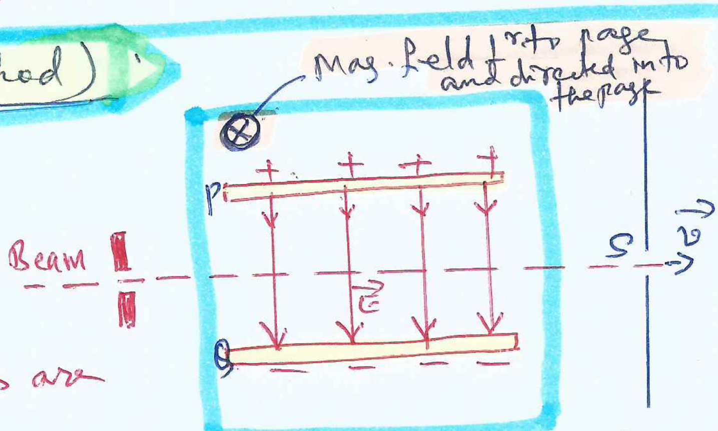
The crossed E and B fields, therefore, serve as a "velocity selector". Only particles with speed E/B pass undeflected through the region of crossed fields. This method was employed by J.J. Thomson in 1897 to measure the charge to mass ratio (e/m) of an electron.

The principle is also employed in Mass Spectrometer – a device that separates charged particles, usually ions, according to their charge to mass ratio.

Velocity Selecter (II Method)

Velocity selector is a set-up to select charged particle of a particular velocity from a beam passed through a space having crossed electric and magnetic fields.



→ Mutually $\perp^r$ electric & magnetic fields are called "crossed fields".

→ Consider two equally and oppositely charged plates such that the electric field is set-up from +ve to the −ve plate (see fig).

→ Let a mag. field $\vec{B}$ be $\perp^r$ to $\vec{E}$ and directed into the page (shown by ⊗).

→ Let a charged particle having charge q enter a region with a velocity $\vec{v}$, where mutually $\perp^r$ $\vec{E}$ and $\vec{B}$ are present

→ Force acting on the charged particle due to $\vec{E}$ is $\vec{F_e} = q\vec{E}$

This force $\vec{F_e}$ acting on the particle is in the downward direction (i.e. along the direction of $\vec{E}$)

→ Force acting on the charged particle due to $\vec{B}$ is given by $\vec{F_m} = q(\vec{v} \times \vec{B})$ or $|\vec{F_m}| = qvB$ ($\sin 90° = 1$)

This force deflects the particle in the upward direction (FLHR).

→ Values of $\vec{E}$ and $\vec{B}$ are so selected that electric force and mag. force become equal and opposite.

→ So, if the charged particle has to move undeflected (by $\vec{E}$ and $\vec{B}$), then upward force = downward force

$|\vec{F_e}| = |\vec{F_m}|$ or $\cancel{q}E = \cancel{q}vB$ $\therefore$ $\boxed{v = \frac{E}{B}}$ → (1)

→ Only those particles of the beam which move with a velocity given by eqn (1) will pass thro' slit 'S'. The particles moving with velocity greater than or less than the velocity given by eqn (1) will not pass thro' the slit.

→ This method can be used to measure specific charge (q/m) of electron

→ This principle is employed in Mass spectroscope — which is a device used to separate ions according to their specific charge.

Difference betn Electric field $\vec{E}$ and Mag. field $\vec{B}$

	Electric field ($\vec{E}$)	Magnetic field ($\vec{B}$)
①	$\vec{E}$ is produced by a stationary as well as moving electric charge.	$\vec{B}$ is produced by moving charges only.
②	Electric field $\vec{E}$ exerts a force on a stationary as well as on moving charges	Mag. field $\vec{B}$ exerts a force on moving charges only, provided the charges are not moving parallel or anti-parallel to the mag. field.
③	A charged particle is moving $\perp^r$ to the direction of $\vec{E}$. The "force" acting on this particle is in the plane of the electric field & also $\perp^r$ to the direction of motion of the charged particle. The path followed by the charged particle is parabolic.	A charged particle is moving $\perp^r$ to the direction of $\vec{B}$. The force acting on this particle is $\perp^r$ to the direction of motion of the charged particle as well as $\perp^r$ to the plane of $\vec{B}$. The path followed by the charged particle is circular.

Problem: What is the radius of the path of an electron (mass $= 9\times10^{-31}$ kg and charge $= 1.6\times10^{-19}C$) moving at a speed of 3×10^{7} m/s in a mag. field of $6\times10^{-4}T$ ⊥r to it. What is its frequency? Calculate ~~the~~ its energy in keV ($1eV = 1.6\times10^{-19}J$)

→ ① We know that $r = \frac{mv}{qB} = \frac{9\times10^{-31}kg \times \cancel{3}\times10^{7}ms^{-1}}{1.6\times10^{-19}C \times \cancel{6}\times10^{-4}T}$

$r = 26\times10^{-2}m = 26\,cm.$

② $T = \frac{2\pi r}{v}$ or $f = \frac{v}{2\pi r} = \frac{3\times10^{7}\cancel{m}s^{-1}}{2\pi(26\times10^{-2})\cancel{m}} = 2\,MHz.$

③ $E = \frac{1}{2}mv^2 = \left(\frac{1}{2}\right)(9\times10^{-31}kg)(9\times10^{14})m^2s^2$

$= 40.5\times10^{-17}J = 4\times10^{-16}J$

$= \frac{4\times10^{-16}}{\cancel{1.6}\times10^{-19}}eV = \frac{10}{4}keV = 2.5\,keV$

Cyclotron :→

Cyclotron is a device used to accelerate positively-charged particles (such as protons, deuterons, α-particles, ions etc.), to very high energies so that they could be used in nuclear disintegration expts.

- A cyclotron is a type of particle accelerator invented by Lawrence in 1932.

Principle :→ A cyclotron accelerates charged particles outwards from the center along a spiral path. The particles are held to a spiral trajectory by a static magnetic field and accelerated by a rapidly varying (Radio frequency) electric field.

Or

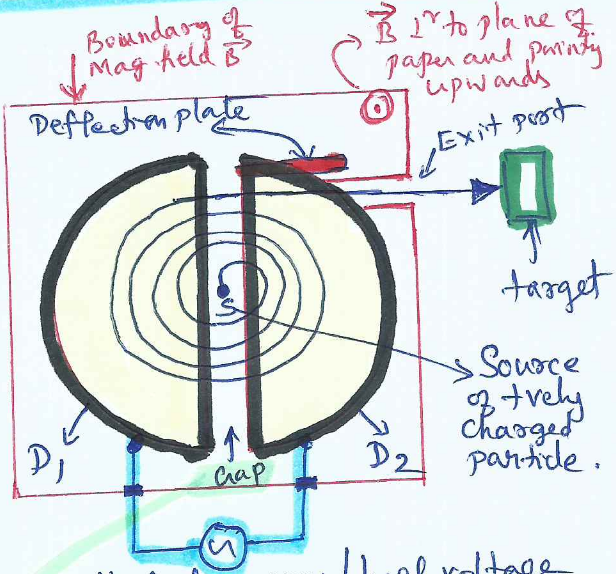
When a positively charged particle is made to move time and again in a high frequency (10-15 MHz) / high voltage ($\approx 10^5$ V) alternating voltage [electric field] and also using strong magnetic field, the particle gets accelerated and acquires sufficiently large amount of energy.

Construction ⇒ It consists of two semicircular disc like hollow metal containers D_1 and D_2 (called as Dees) with a small gap between them (See figure).

The walls of the dees provide electrical shielding so that the space within each dee is free from electric field.

However, $\vec{B}$ is not screened by the dees

- Therefore, the particle is acted upon by ~~electric~~ magnetic field inside dees. $\vec{E}$ exists only within the gap betn dees.
- However $\vec{B}$ acts on particle and makes it go around circular path inside a dee.



High frequency / high voltage oscillator of the order of 10-15 MHz and 10^5 V

→ due this high freq. electric field sets up across the gap between Dees.

→ As shown in fig, HF/HV AC is applied across dees.

→ An intense mag. field $\vec{B}$ of about 1.6 T is set up ⊥r to plane of the dees by a large electromagnet.

→ The whole space inside the dees is evacuated to a pressure of about 10^{-6} mm of mercury to minimize collisions betn ions and air molecules.

→ A negative charged "deflector plate" enables accelerated +ve particles to the exit port ~~two~~ towards a target to be ~~and~~ analysed.

→ An ion source is located at the centre S in the gap betn the dees. The ions come out thro' a small hole in the ion source and are available to be accelerated.

Contd ....

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Theory and working :

- Suppose that an ion of mass m and charge $+q$ emerges from of the ion source at an instant when D_2 is at a $-$ve potential. It will be accelerated towards D_2 by the electric field in the gap betⁿ the dees and enter D_2 with a velocity v (say). Once inside D_2, it is screened from the electric field by the metal walls of the dees. Now, under the action of mag. field $\vec{B}$, which is $\perp^r$ to the plane of dees, the ion adopts a circular path with a constant speed v and of radius r given by

$$\boxed{r = \frac{mv}{qB}} \rightarrow (1)$$

where B is the mag. field

The time 't' required by the ion to complete a semi-circle is

$$\boxed{t = \frac{\pi r}{v} = \frac{\pi m}{qB}} \rightarrow (2)$$

This shows that the time of passage of the ion thro' the dee is independent of the speed of the ion and of radius of the circle. It depends only on $\vec{B}$ and (q/m) of the ion (Greater the speed of the ion, larger will be the circle in which it travels, the period of motion remaining the same → from eqⁿ (1))

- Let freq. of applied voltage to dees has been adjusted that during the one-half cycle of AC, the ion completes a semi-circle. Then the ion will emerge from D_2 into the gap at the instant when D_1 becomes $-$ve potential. The ion is therefore further accelerated while crossing the gap and enters D_1. On account of its increased velocity, its semi-circular path in D_1 is now of greater radius. The time of passage t through D_1, however, is still the same.
- This process is repeated after every half-cycle of AC and the ion gains in speed each time it passes from one dee to the other. Finally, the ion becoming enough energetic reaches the outer edge of one dee where it is pulled out of the system by a negatively-charged deflector plate.
- So, cyclotron operates under the condition that the freq f_c of the applied voltage must be equal to the freq. f of the circular revolution of the ion. $\Rightarrow f_c = f \rightarrow (3)$

but $f = \frac{1}{2t} = \frac{qB}{2\pi m}$ $\therefore$ $\boxed{f_c = \frac{qB}{2\pi m}}$ f_c is called cyclotron frequency.

In ~~pratice~~ practice, the freq f_c is kept fixed, and $\vec{B}$ is varied until eqⁿ (3) ~~is~~ condition is satisfied.

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Limitations :

① Cyclotron cannot accelerate uncharged particles like neutrons.

② Cyclotron cannot accelerate electrons since they have very small mass. Electrons start moving at a very high speed when they gain small energy in the cyclotron. Oscillating electric field makes them to go quickly out of phase because of their high speed.

③ The cyclotron cannot accelerate the particles to velocities as high as comparable to velocity of light (c). The reason is that at these velocities, the mass m of the particle increases with increase in velocity of the particle (v)

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

where m_0 is the rest mass of the particle, $c \rightarrow$ velocity of light $3\times10^8\, ms^{-1}$

$\therefore$ time $t \left(= \frac{\pi m}{qB}\right)$ taken by particle to complete successive semi-circular paths goes on increasing. Thus the particle becomes more and more out of step (phase) with the applied voltage until it can no longer be accelerated further.

→ For a given energy, the velocity of an electron is much greater than that of a more massive particle like proton or deuteron and so the relativistic increase of mass is correspondingly much ~~higher~~ greater. Therefore electrons very quickly get out of phase with applied AC and hence cyclotron is not successful in accelerating electrons.

→ In some machines, the freq f_c of the applied AC is decreased as the particle accelerates in such a way that the product "$v_c m$" remains constant, and the ~~phase~~ ~~is always~~ p.d. is always in step with the rotating particle. Such machines are called as "Synchro-cyclotrons".

Kinetic energy of particles accelerated in a cyclotron :

Let R be the outside radius of the dees and v_{max} is the speed of the particle when travelling in a path of radius R, then from relation $\boxed{r = \frac{mv}{qB}}$, we have $\boxed{R = \frac{m v_{max}}{qB}}$

$$\therefore v_{max} = \frac{qBR}{m}$$

The corresponding KE of the particle (of mass 'm') is

$$\boxed{K = \frac{1}{2} m v_{max}^2 = \frac{q^2B^2R^2}{2m}} \qquad \therefore KE \quad \boxed{K = \frac{q^2B^2R^2}{2m}}$$

Contd....

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Uses of a cyclotron

1. It is used to produce radioactive material for medical purposes eg. for the purpose of diagnostics and treatment of chronic diseases.
2. It is used to synthesize ~~for~~ fresh substances.
3. It is used to improve the quality of solids by adding ions.
4. It is used to bombard the atomic nuclei with highly accelerated particles to study the nuclear reactions.

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* Info : Functions of Electric & Mag fields in a Cyclotron.

- Electric field $\vec{E}$ accelerates the charged particle in ~~between~~ the dees of cyclotron.
- Magnetic field enables the charged particle to move in a circular path inside the dees so that the particle again enters the gap betn dees for further acceleration due to electric field. Since, the charged particle moves in a circular path in the mag. field, so its motion is accelerated.
- Thus, both electric field $\vec{E}$ and magnetic field $\vec{B}$ are responsible to accelerate the charged particle in a cyclotron.

---

problem 1: In a given $\vec{B}$, 2 particles have the same cyclotron freq. what can you conclude from this?

Ans → Cyclotron freq $f_c = \dfrac{Bq}{2\pi m} = \left(\dfrac{B}{2\pi}\right)(q/m)$, Since f_c is same for both particles, so (q/m) i.e. specific charges of both the particles are same.

problem 2: Calculate freq. of $\vec{E}$ applied to dees of a cyclotron in order to accelerate α-particle if $B = 3.14\,T$. Given mass of proton $= 1.67\times10^{-27}$ Kg.

Ans → Given $B = 3.14\,T$, mass of α-particle $= m = 4m_p = 4\times1.67\times10^{-27}$ kg. $= 6.68\times10^{-27}$ kg.

Charge on α-particle, $q = 2e = 2\times1.6\times10^{-19}C = 3.2\times10^{-19}C$

Using $f = \dfrac{qB}{2\pi m} = \dfrac{3.2\times10^{-19}\times3.14}{2\times3.14\times6.68\times10^{-27}}$

$f = 2.4\times10^{7}\,Hz = \underline{24\,MHz}$

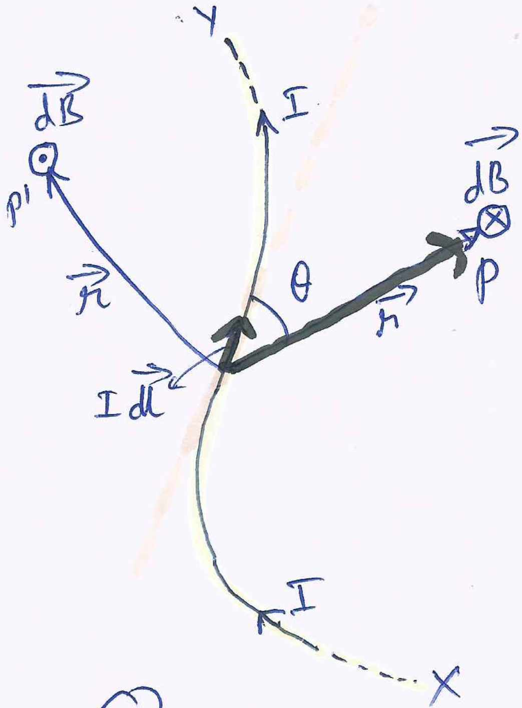
$\boxed{\text{End of cyclotron.}}$

How to Derive the Biot-Savart Law

- In the syllabus, there is no derivation of B-S law. NCERT book does not derive B-S law, but only states the law and gives the equation for calculating magnetic field due to a current element.
- If needs to be derived, one has to use Maxwell's equations, solving Poisson's equation etc.., which is not required for 12^{th} CBSE syllabus.
- However, little simplification and vector usage is given and hence we will derive it as given in book.

Magnetic field due to a current element, Biot-Savart law

- Oersted's experiment in 1820 showed that a current-carrying conductor produces a magnetic field around it. French scientists Biot and Savart performed series of experiments to study magnetic fields produced by various current-carrying conductors and formulated a law to determine the magnitude and direction of the fields so produced. This law is known as "B-S law".
  - Let XY be a conductor of an arbitrary shape carrying current I and P is the point in vacuum at which magnetic field dB is to be determined.
  - Let us divide XY into infinitesimal current-elements. Let $I\,\vec{dl}$ be one such vector element. Let $\vec{r}$ be displacement vector from the element to the point P.
  - According to B-S law, $|dB|$ is proportional to different parameters as follows
    - $|\vec{dB}|$ is proportional to current I
    - $|\vec{dB}|$ is proportional to current-element length $|\vec{dl}|$
    - $|\vec{dB}|$ is inversely proportional to square of the distance r
    - IMP The direction of $\vec{dB}$ is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$
  - Thus, in vector notation



$$\vec{dB} \propto \frac{I\,\vec{dl}\times\vec{r}}{r^3}$$

$$\vec{dB} = \frac{\mu_0}{4\pi}\cdot\frac{I\,\vec{dl}\times\vec{r}}{r^3} \longrightarrow (1)$$

Where $\mu_0/4\pi$ is dimensionless proportionality constant.

$\mu_0/4\pi = 10^{-7}\ Tm/A$ or $10^{-7}\ NA^{-2}$; μ_0 is the permeability in Vaccum

→ Eq (1) is valid when medium is vacuum and eq (1) is called vector form of B-S law.

→ The magnitude of $\vec{dB}$ at point P is

$$\boxed{|dB| = \frac{\mu_0}{4\pi}\times\frac{I\,dl\sin\theta\cdot r}{r^3}}$$ where θ is the angle between $I\vec{dl}$ and $\vec{r}$

$$|dB| = \frac{\mu_0}{4\pi r^2}\times\frac{I\,dl\sin\theta}{}$$

→ The direction of $|\vec{dB}|$ is given by right hand rule of cross product.

- At point P, right hand fingers curl from $\vec{dl}$ to $\vec{r}$, hence field is ~~inside the paper~~ downwards ($\perp^r$ to plane of paper and going inside)
- At point P', field is "upwards"

→ The resultant field is given by

$$\vec{B} = \int d\vec{B}$$

$$\left.\begin{aligned} \vec{B} &= \frac{\mu_0}{4\pi}\int \frac{I\,(\vec{dl} \times \vec{r})}{r^3} \\ &= \frac{\mu_0}{4\pi}\int \frac{I\,(\vec{dl} \times \hat{r})}{r^2} \end{aligned}\right\}$$

B-S law for magnetic field differs on few aspects as compared to coulomb's law for electrostatic field.

① Both are inverse square laws $\Rightarrow \propto \frac{1}{r^2}$

② Charge element dq is scalar, whereas current element $I\,dl$ is vector $(I\vec{dl})$ whose direction is in the direction of current. [means e-s field is produced by scalar source, e-m field by vector source)

③ The direction of mag. field is not along the displacement vector $\vec{r}$ as in the case of the electric field but is instead $\perp^r$ to the plane containing the velocity vector and radius vector.

④ There is an angle dependency in B-S Law, which is not present in e-s case. As in figure, the magnetic field at any point in the direction of dl (colored line) is zero. Along this line, $\theta = 0$, $\sin\theta = 0$, $|dB| = 0$

For Ref

I Note that in eq$\underline{\underline{n}}$ ①

$$\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{I\, \vec{dl} \times \vec{r}}{r^3}$$

We can write it like

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\left(\vec{dl} \times \hat{r}\right) \cancel{r}}{r^{\cancel{3}\,2}}$$

Since $\vec{r} = |\vec{r}|\,\hat{r}$ — $\hat{r}$: unit Vector; $|\vec{r}|$: Magnitude of r

$$= \frac{\mu_0}{4\pi} \frac{I\left(\vec{dl} \times \hat{r}\right)}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta\, |\hat{r}|}{r^2}$$

Since $|\hat{r}| = 1$

$$\boxed{\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}}$$

II In a cross product between two Vectors A and B, the direction of resultant vector is in the direction $\perp^r$ to plane ~~and~~ of A and B.

If A and B are on the plane of this paper, the direction of cross product $\vec{A} \times \vec{B}$ is $\perp^r$ to the plane of paper, could be inside or outside.

From ~~Recent~~ figure
- ⟹ that is decided by right hand rule
- ⟹ for $\vec{A} \times \vec{B}$ ~~lying~~ on the plane of paper, ~~fingers~~ fold fingers from $\vec{A}$ to $\vec{B}$ (right hand), then (take small angle between A and B) direction is $\perp^r$ to paper and downwards

⟹ $\vec{dl} \times \vec{r} \longrightarrow \perp^r$ to paper and downwards (point P)

⟹ At point P', $\vec{dl} \times \vec{r}$ is $\perp^r$ to paper and upwards.

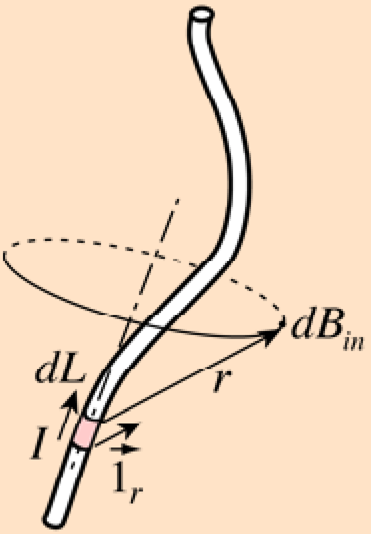
Biot-Savart Law (B-S law) → not in NCERT syllabus (for info only)

- The B-S Law relates **magnetic fields** to the **currents,** which are **their sources**.
- Similarly, Coulomb's law relates **electric fields** to the **point charges,** which are **their sources**.
- Finding the magnetic field resulting from a current distribution involves the vector product, and is inherently a calculus problem when the distance from the current to the field point is continuously changing.
- In physics, specifically electromagnetism, the B-S law is an equation describing the magnetic field generated by a **constant** electric current. It relates the magnetic field to the magnitude, direction, length, and proximity of the electric current. The Biot–Savart law is fundamental to magnetostatics, playing a role similar to that of Coulomb's law in electrostatics.
- The Biot–Savart law is used for computing the resultant magnetic field B at position r in 3D-space generated by a **steady current (I)** (for example due to a wire). A steady (or stationary) current is a continual flow of charges which does not change with time and the charge neither accumulates nor depletes at any point. The law is a physical example of a line integral, being evaluated over the path C in which the electric currents flow (e.g. the wire). The equation in SI units is
- $\vec{B}(r) = \frac{\mu_0}{4\pi} \oint \frac{I\,\vec{dl} \times \vec{r}}{|r^3|}$; Where $\vec{dl}$ is a vector along the path C whose magnitude is the length of the differential element of the wire in the direction of conventional (steady) current. $\vec{r}$ is the displacement vector from the wire element dl to the point at which the field is being computed and μ_0 is the magnetic constant. Alternatively,
- $\vec{B}(r) = \frac{\mu_0}{4\pi} \oint \frac{I\,\vec{dl} \times \vec{\hat{r}}}{|r^3|}$;where $\vec{\hat{r}}$ *is the unit vector, whose magnitute* $|\vec{\hat{r}}| = 1$
- The integral is usually around a closed curve, since **stationary** electric currents can only flow around closed paths when they are bounded. However, the law also applies to infinitely long wires (as used in the definition of the SI unit of electric current - the Ampere).
- In the special case of a steady constant current I, the magnetic field **B** is (current I can be taken out of the integral)

$$\vec{B}(r) = \frac{\mu_0 I}{4\pi} \oint \frac{\vec{dl} \times \vec{r}}{|r^3|}$$

Derive the B-S law →not in NCERT syllabus (for info only)

- **The B-S law is an expression of the magnetic field generated by a steady electric current.**
- Deriving this law involves starting from Maxwell's equations, obtaining and solving Poisson's equation for all three components of the **Vector Potential A** and taking the curl of the result.



Magnetic field of a current element

$$d\vec{B} = \frac{\mu_0 I d\vec{L} \, x \, \vec{1}_r}{4\pi r^2}$$

where

$d\vec{L}$ = infinitesmal length of conductor carrying electric current I

$\vec{1}_r$ = unit vector to specify the direction of the the vector distance r from the current to the field point.

Totally there are 5 Steps

1 **Begin with the definition of the vector potential.** Gauss' law of magnetism tells us that magnetic fields are always divergence-free, via $\nabla \cdot \mathbf{B} = 0.$ From vector calculus, we also recognize the identity $\nabla \cdot (\nabla \times \mathbf{F}) = 0,$ for any vector field $\mathbf{F}.$ In other words, the divergence of a curl is always zero. Therefore, we can write a divergence-free field in terms of the curl of another vector field, called the vector potential.

- $\mathbf{B} = \nabla \times \mathbf{A}$

2 **Rewrite Ampere's law in terms of potentials to obtain Poisson's equation.** In doing so, we have a certain degree of freedom in how we write this. Potentials are not unique, and in the case of the vector potential, we can arbitrarily add a gradient of a scalar field without affecting the magnetic field, since the curl of a gradient is always zero. This is called a gauge transformation. This means that we can choose a potential that is convenient for us. Here, we will choose the Coulomb gauge, where $\nabla \cdot \mathbf{A} = 0.$

- $$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$
  $$\nabla \times (\nabla \times \mathbf{A}) = \mu_0 \mathbf{J}$$
- An identity for simplifying the vector triple product is BAC-CAB. For curls, this is $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}.$ Now, we remember that we chose $\nabla \cdot \mathbf{A} = 0.$
  - $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$
- This is Poisson's equation for the vector potential. Though the expression above is actually three equations, we can solve for all three components simultaneously because the equations are uncoupled.

3 **Solve Poisson's equation.** One way to do this is via Fourier transforms. See the article linked for more details. Assuming you transformed correctly, you should obtain the general solution below.

- $\mathbf{A} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \mathrm{d}^3\mathbf{x}'$
- Notice that this is very similar in form to the solution for the scalar potential, of which a charge density ρ associated with a stationary point charge allows us to reduce the solution to Coulomb's law, the law that describes electrostatics. Now, we wish to obtain the Biot-Savart law, the law that describes magnetostatics.

4 **Take the curl of $\mathbf{A}$.** Doing so recovers the magnetic field. Note that all the variables with primes in them are dummy variables, so the curl is taken with respect to $\mathbf{x}$, allowing us to put the del operator under the integral.

- $\nabla \times \mathbf{A} = \frac{\mu_0}{4\pi} \int \nabla \times \left(\frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \right) \mathrm{d}^3\mathbf{x}'$

5 **Use the product rule $\nabla \times (f\mathbf{v}) = \nabla f \times \mathbf{v} + (\nabla \times \mathbf{v})f$ to simplify the above expression.** Since $\mathbf{J}(\mathbf{x}')$ does not depend on $\mathbf{x}$, that term vanishes. Note the chain rule when taking the gradient.

- $$\begin{aligned}
\mathbf{B}(\mathbf{x}) &= \nabla \times \mathbf{A} \\
&= \frac{\mu_0}{4\pi} \int \nabla \left(\frac{1}{|\mathbf{x} - \mathbf{x}'|} \right) \times \mathbf{J}(\mathbf{x}') \mathrm{d}^3\mathbf{x}' \\
&= \frac{\mu_0}{4\pi} \int \frac{-(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \times \mathbf{J}(\mathbf{x}') \mathrm{d}^3\mathbf{x}' \\
&= \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \mathrm{d}^3\mathbf{x}'
\end{aligned}$$
- The above expression is the Biot-Savart law. It takes into account the thickness of the conductor through which the current is going through. Although it appears as an inverse cube law, the presence of the displacement vector $\mathbf{x} - \mathbf{x}'$ in the numerator ensures that the magnetic field falls off as the square of the distance and points in the proper direction.
- For an infinitely narrow wire, we can neglect the thickness and replace $\mathbf{J}(\mathbf{x}')\mathrm{d}^3\mathbf{x}'$ with $I\mathrm{d}\mathbf{l}$ and convert the integral to a line integral. Letting $\mathbf{r} = \mathbf{x} - \mathbf{x}'$ be the displacement vector, we recover the familiar form of the Biot-Savart law.
  - $\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{I\mathrm{d}\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^3}$

Magnetic Field due to a Current-carrying Straight Conductor of Finite Length.

Not in NCERT syllabus

XY is Straight Conductor carrying current I from $X \to Y$

$\vec{B}$ at P = ?

Let $PQ = R$

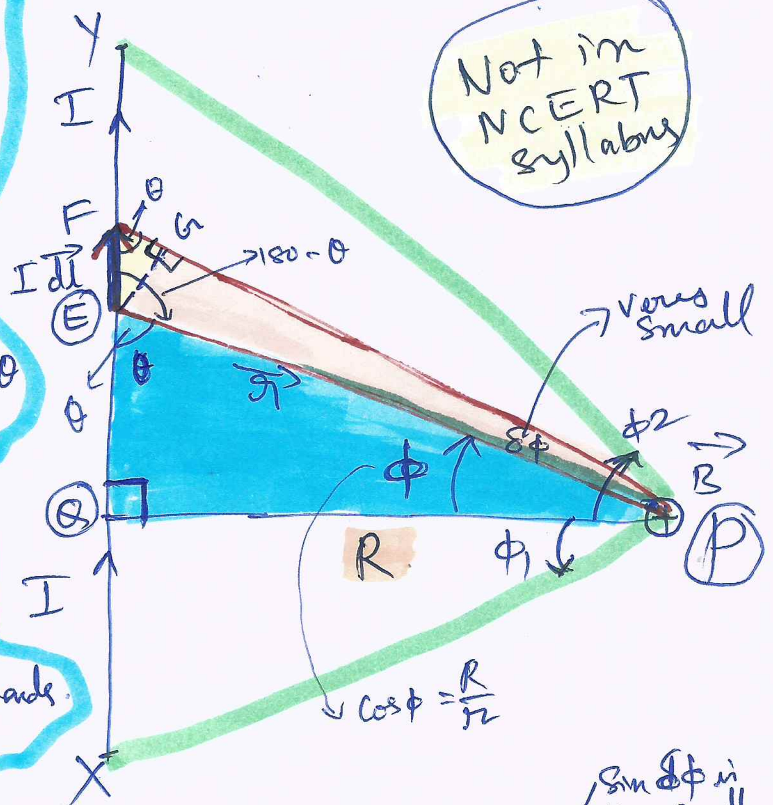
$\vec{r} \to$ displacement Vector

$\to$ Angle betn $I\vec{dl}$ & $\vec{r} = 180° - \theta$

$$\delta B = \frac{\mu_0}{4\pi} \frac{i\,dl \sin(180-\theta)}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{r^2} \longrightarrow (1)$$

By Right-hand rule, $\vec{\delta B}$ at P is $\perp^r$ to the page directed downwards.



sin δφ is very small

Now in Δ^s EFG and PEG,

$\underline{EG} = \vec{dl}\sin\theta$ and $\underline{EG} = \vec{r}\sin\delta\phi = \vec{r}\,d\phi$

$\therefore \vec{dl}\sin\theta = r\,\delta\phi \longrightarrow$ Substitute this in (1), we get

$$\boxed{\delta B = \frac{\mu_0}{4\pi} \cdot \frac{I r\,\delta\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,\delta\phi}{r}}$$

(Since $\cos\phi = \frac{R}{r}$)

From Δ^{le} EQP, $r = R/\cos\phi$

$$\therefore \delta B = \frac{\mu_0}{4\pi} \frac{I\cos\phi\,\delta\phi}{R} \longrightarrow (2)$$

Let $\angle QPX = \phi_1$ (anticlockwise) and $\angle QPY = \phi_2$ (clockwise), then B due to whole conductor XY is

$$B = \int_{-\phi_1}^{\phi_2} \frac{\mu_0}{4\pi}\frac{I}{R}\cos\phi\,\delta\phi = \frac{\mu_0}{4\pi}\frac{I}{R}\Big[\sin\phi\Big]_{-\phi_1}^{\phi_2} = \frac{\mu_0}{4\pi}\frac{I}{R}\Big[\sin\phi_2 - \sin(-\phi_1)\Big]$$

$$\therefore \boxed{B = \frac{\mu_0}{4\pi}\frac{I}{R}(\sin\phi_1 + \sin\phi_2)} \longrightarrow (3)$$

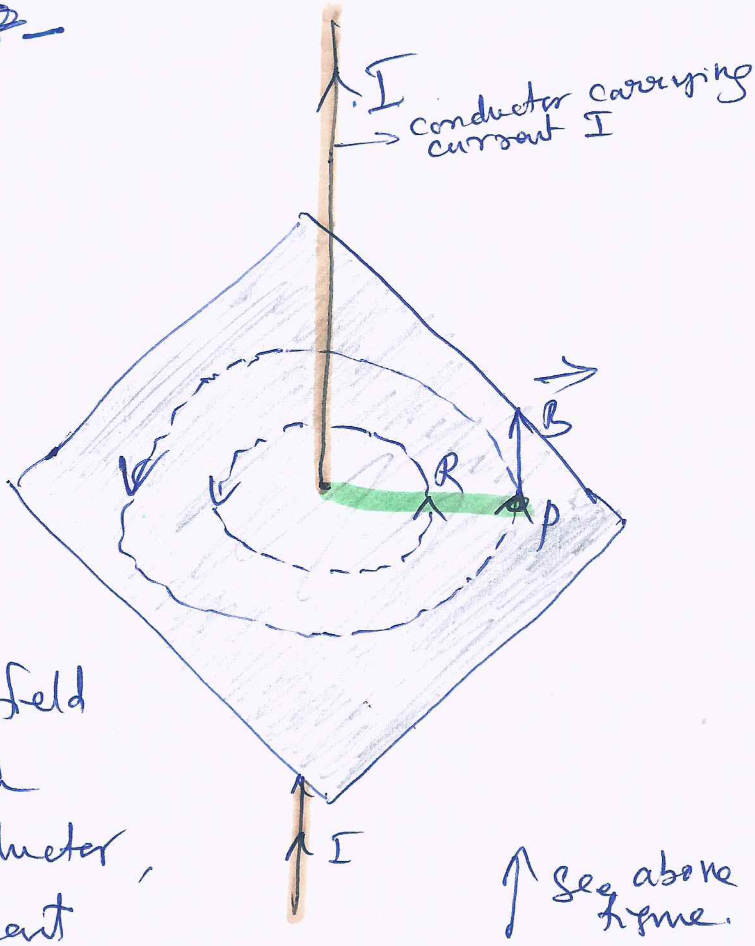
IMP

Special case, if XY is infinite, $\phi_1 = \phi_2 = 90°$ ($\sin 90° = 1$)

$$\boxed{B = \frac{\mu_0}{2\pi}\frac{I}{R}} \longrightarrow (4)$$

From (3) or (4), magnitude of field $\vec{B}$ at P is $\propto^l$ to current I & inversely $\propto^l$ to ~~the~~ the distance R of point from the conductor.

P.T.O →

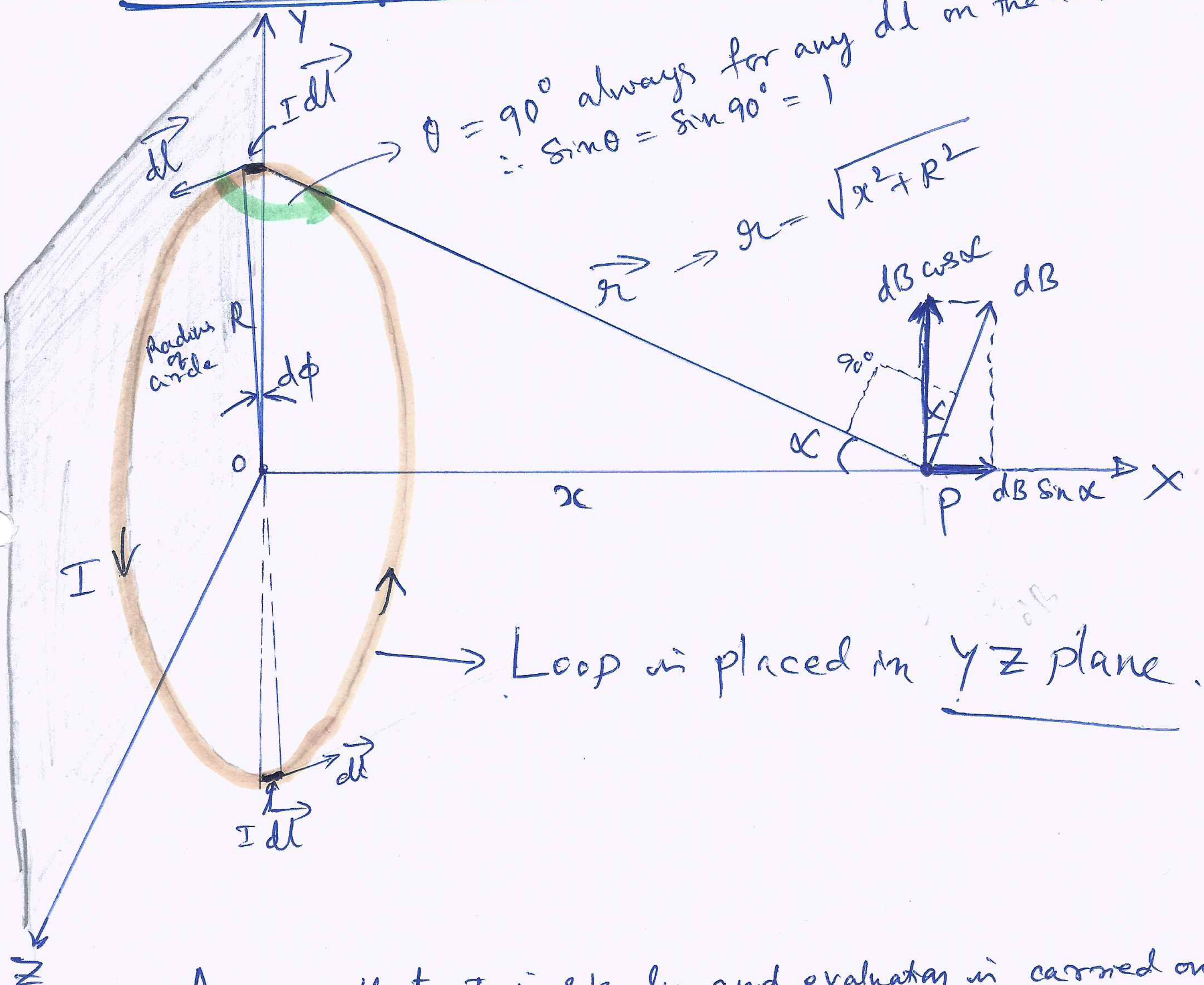


→ The line of constant magnetic field $\vec{B}$ near a linear current-carrying conductor are concentric circles around the conductor in a plane $\perp^{r}$ to conductor

→ The direction of the field $\vec{B}$ at a point P with distance R from conductor, will be along the tangent drawn on a circle of radius R around the conductor.

↑ See above figure.

Magnetic field on the axis of a Circular current loop (Derivation)



- Assume that I is steady and evaluation is carried out in vacuum.
- Assume loop is placed in YZ plane with its centre at the Origin of co-ordinate system and has a radius R. The x-axis is the axis of the current loop.
- As mentioned in figure θ between $\vec{dl}$ and $\vec{r}$ is always $\perp^r$, hence $\theta = 90^\circ$, $\sin\theta = \sin 90^\circ = 1$
- Choose an element of length dl on the loop subtending an angle $d\phi$ at the center. Then

$$dl = R\,d\phi \longrightarrow (1)$$

From B-S law, $dB = \dfrac{\mu_0}{4\pi r^2} I\,dl \sin\theta$

$$dB = \frac{\mu_0}{4\pi r^2} I\,dl \quad (\theta = 90^\circ) \longrightarrow (2)$$

Displacement Vector $\vec{r}$ makes an angle α with the axis of the loop where ~~tan~~ $\sin\alpha = R/\sqrt{x^2+R^2} \longrightarrow (2a)$

- From right hand rule ; $\vec{dB}$ is $\perp^r$ to $\vec{r}$ as shown in figure. Resolve $\vec{dB}$ into components dB_x along the axis of the loop and $dB_\perp$ in the plane $\perp^r$ to the axis of the loop.
- From symmetry, $dB_\perp$ when summed up for whole loop, they cancel out to get null result. (For example, $dB_\perp$ due to dl is cancelled ~~out~~ by the contribution due to the diametrically opposite dl element). Thus only x-component dB_x survives. The net contribution along x-direction is obtained by integrating dB_x over the loop.

$$B = \int dB \sin\alpha \longrightarrow ③$$ ~~from~~ using ②,

$$B = \int \frac{\mu_0}{4\pi r^2} \frac{I\, dl \sin\alpha}{}$$ using ① $dl = R\, d\phi$

$$= \frac{\mu_0 I R \sin\alpha}{4\pi r^2} \int_0^{2\pi} d\phi$$

$\boxed{\text{Since } \int_0^{2\pi} d\phi = 2\pi}$ ~~(circumference of the coil)~~ → total angle of the circle in radians.

$$B_x = \frac{\mu_0 I R \sin\alpha}{\cancel{2}\cancel{4\pi} r^2} \times \cancel{2\pi}$$ (Since $\int_0^{2\pi} d\phi = 2\pi$) or $\int dl = 2\pi R$

$$B_x = \frac{\mu_0}{2r^2} \cdot I R \sin\alpha$$ From ②a ($\because \sin\alpha = \frac{R}{\sqrt{x^2+R^2}}$) and $r = \sqrt{x^2+R^2}$

$$B_x = \frac{\mu_0 I R}{2(x^2+R^2)} \times \left(\frac{R}{\sqrt{x^2+R^2}}\right)$$

$$\boxed{B = B_x \hat{i} = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}} \hat{i}} \longrightarrow ④$$

ⓐ Sp. case, to obtain field at the centre of the loop, then $x=0$, $\boxed{B = \frac{\mu_0 I}{2R} \hat{i}} \rightarrow ⑤$

PTO

~~Case (b)~~

Case (b) if $x = \infty$, $\boxed{B_x = \frac{\mu_0 I R^2}{2x^3}}$

Case (c), if the coil has N turns, then each turn will contribute equally to B, then from eq^n 4

$$\boxed{B = B_x \hat{i} = \frac{\mu_0 N I R^2}{2(x^2+R^2)^{3/2}}}$$

where N = no. of turns in the circular current loop.

The mag. field lines due to circular current loop form closed loops (See fig below). The direction of $\vec{B}$ is given by RH Thumb rule (See that fingers and thumb representations are interchanged here → thumb gives $\vec{B}$.

Curl the palm of your right hand around the wire with the fingers in the direction of "Current". The RH thumb gives the direction of the magnetic field.