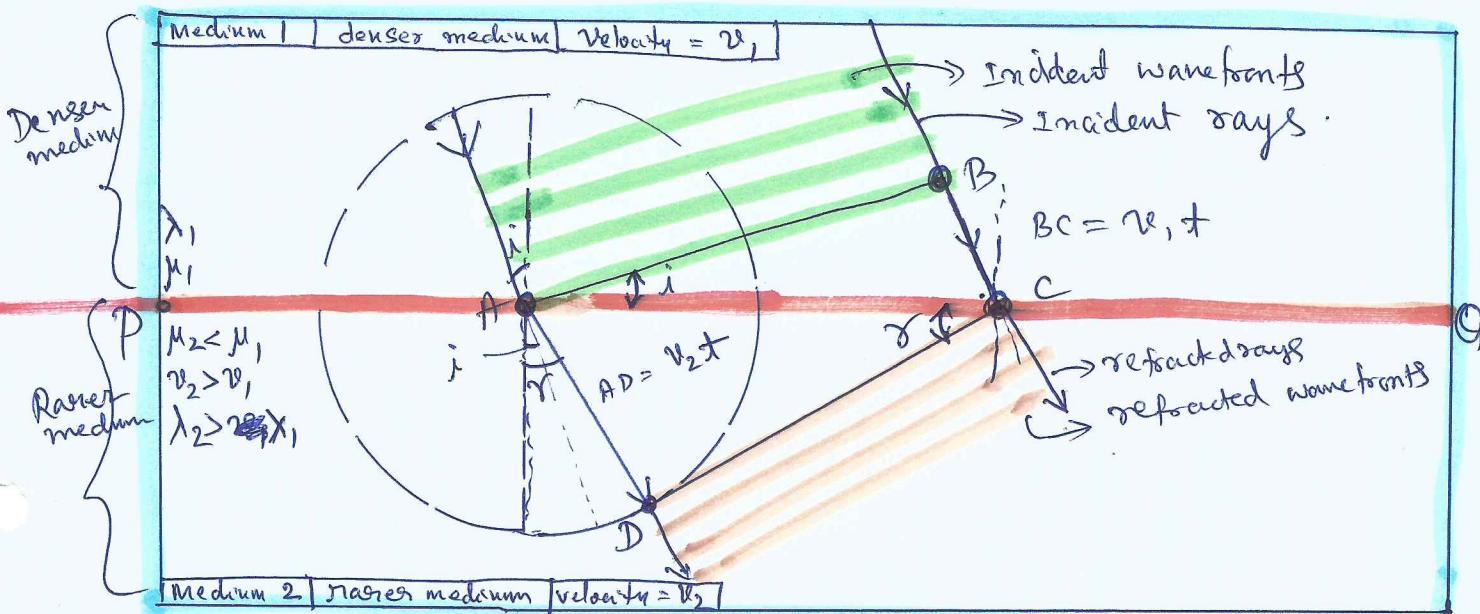


Refraction using wave model

II From Denser medium to Rarer medium



- PQ → Surface separating two media with different μ .
- AB → Incident wavefront on surface PQ making an angle i
- From figure, $\mu_2 < \mu_1$; $v_2 > v_1$ (Reflected ray bends away from normal)
- $BC = v_1 t$ and $AD = v_2 t$
- Draw a sphere of $r = v_2 t$ m/s centre taken as 'A'
- Draw a line from C to touch circle at its tip. (point D)
- DC is refracted wavefront making an angle r w.r.t. PQ

$$\begin{aligned} \Delta ABC &= \frac{\sin i}{\sin r} = \frac{BC}{AC} \\ \Delta ADC &= \frac{\sin r}{\sin i} = \frac{AD}{DC} \end{aligned}$$

$$\frac{\frac{\sin i}{\sin r}}{\frac{\sin r}{\sin i}} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1} = \mu_{21}$$

$\mu_{21} < 1$
→ Refr. index of 2nd medium wrt first
 $= \mu_{12}$ → μ of first medium wrt to second.

Also $BC = \lambda_1 = v_1 t$; $AD = \lambda_2 = v_2 t$

$$\frac{BC}{AD} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

$$\therefore \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

$$\begin{aligned} \text{Since } \mu_1 &= \frac{c/v_1}{\lambda_1} \\ \mu_2 &= \frac{c/v_2}{\lambda_2} = \frac{v_1}{v_2} \end{aligned}$$

$$\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$$

$$\rightarrow \mu_2 < \mu_1$$

∴ ④ implies, when light wave goes from Denser medium to Rarer medium, $v_2 > v_1$ and $\lambda_2 > \lambda_1$ ∴ Ratio $\frac{v_2}{v_1}$ and $\frac{\lambda_2}{\lambda_1}$ increased proportionately to keep f constant.

∴ Denser → Rarer \Rightarrow v, λ increases, Ratio const = f.
 $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$ Since $\mu_2 < \mu_1$
 $\frac{\sin i}{\sin r} < 1$ $i < r$ or $r > i$

{ Refracted Wavefront and Light Ray is deviated away from normal. }

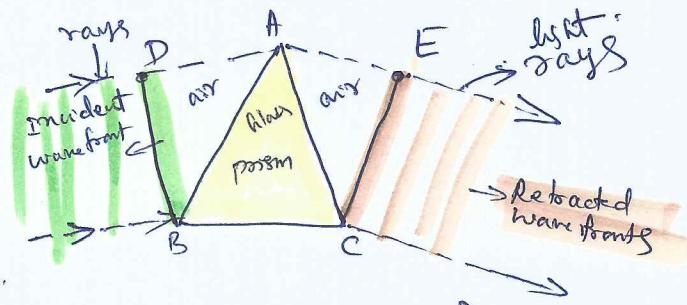
IMP

Behaviour of a prism, a lens and a spherical mirror towards plane wavefront

Once we have derived law of reflection and refraction using wave theory, let us consider how wavefronts behave when it encounters other shapes other than plane surface (Other than plane surface e.g. prism, lens, mirror etc.)

I Prism

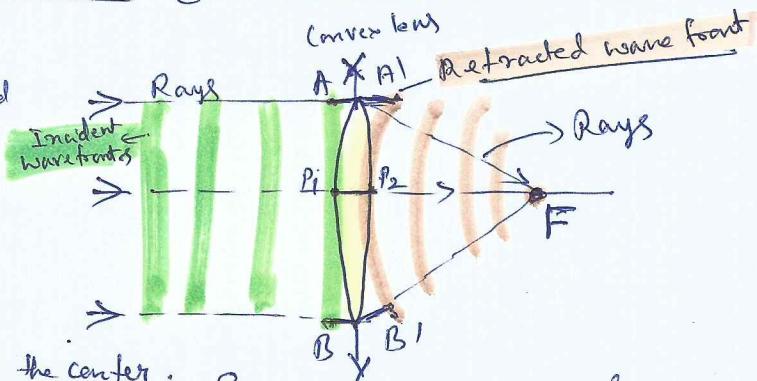
- ABC is thin prism of small angle A.
- BD is incident wavefront
- Different parts of the wavefront travel different thickness of glass prism, max. at the bottom and minimum at the top.
- We know that Speed of light in glass < C (Speed of light in air)
- Emerging wavefront CE bends towards the base of the prism.
- Time taken from D to E (D to A and then A to E) = time taken by light from B to C



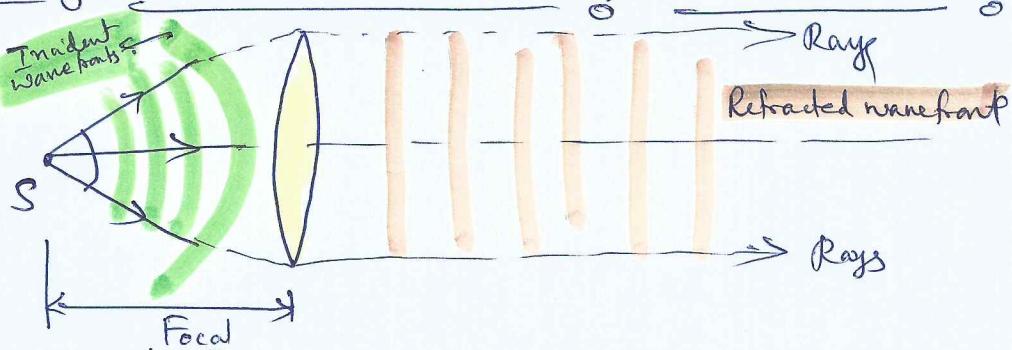
II Lens

- Figure shows incident and refracted wavefronts.
- Plane wave AB will emerge out as spherical wavefront since light passes through different thickness of glass, maximum at the center.
- Light is slowest at the center of lens, hence takes more time.

$$\frac{(Ax + xA)}{v_{\text{air}}} = \frac{By + yB'}{v_{\text{glass}}} = \frac{P_1 P_2}{v_{\text{glass}}}$$



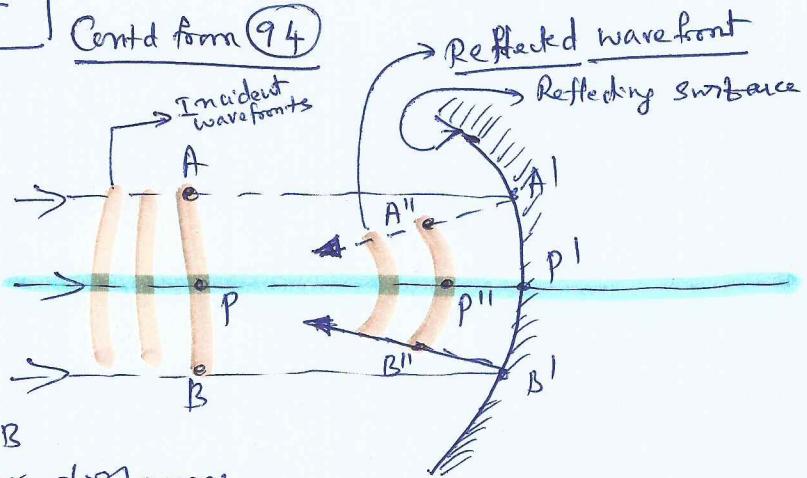
III Lens



- If point source of light is at focus of convex lens such that diverging spherical wavefronts fall on convex lens. The refracted wavefront will be a plane wavefront as shown in figure.

IV Mirrors

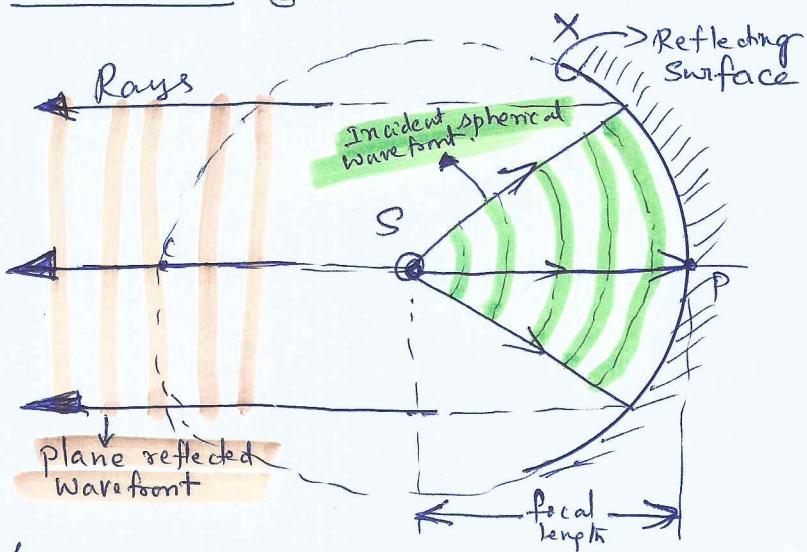
The centre of incident wavefront AB i.e. P has to travel largest distance $\rightarrow PP'$ before it gets reflected back to P'' .



The peripheral points A or B has to travel relatively smaller distances before they get reflected. This leads a plane wave to take a shape of spherical wavefront $A''P''B''$. This explains the converging action of the concave mirror.

V Mirror

When a point source is at the focus of the concave mirror, then diverging spherical wavefronts fall on concave mirror.



~~Centre of the Incident Wavefront~~ which is ~~stays~~ a diverging spherical wavefront,

the centre of the wavefront reaches faster to mirror surface compared to other ~~peripheral~~ peripheral points of incident wavefront.

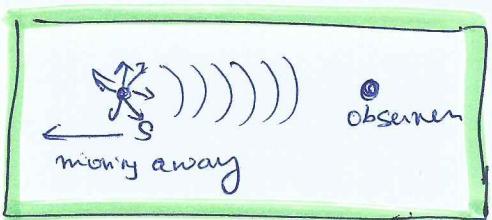
This results in conversion of spherical wavefront to plane wavefront.

Doppler's effect in light

Till now, we have assumed that source \leftrightarrow observer are stationary and hence we concluded that freq 'f' will not change when light ~~is~~ wavefront is reflected or refracted. But one should be careful in constructing the wavefronts if the source (or observer) is moving.

For example, if S is moving away from observer (O), then the

later wavefronts (wavefronts after some time) have to travel greater distance to reach the Observer and hence take longer time.



- Note that source will keep producing ^{same} wavefronts periodically whether it is stationary or moving. But for the Observer, the effect will be different.
- Time taken by two successive wavefronts is hence longer at the observer than when it is emitted at source and hence could result in change in freq. ~~as observed~~ by the observer.
- • Whenever there is a relative motion b/w the source of light and the observer, the apparent frequency of light observed is different from the actual (or true) frequency of the light emitted by the source of light. This effect is known as Doppler's effect.

Explanation:

~~S is stationary~~

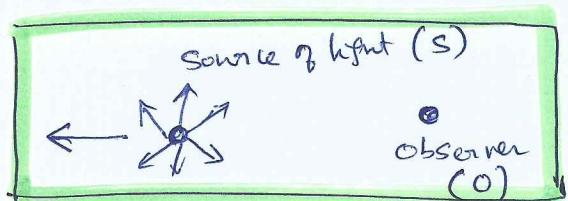
- Suppose both S and O are stationary.

$$\text{Freq. detected by } O \text{ is } f = \frac{c}{\lambda}$$

$$\Rightarrow c = 3 \times 10^8 \text{ ms}^{-1} \text{ (Speed of light in air)}$$

$$\Rightarrow \lambda = \text{Wavelength of the light emitted by Source of light (S)}$$

- Suppose S moves away from the stationary Observer. The time taken by the wavefronts to reach the Observer from the source will increase. Hence freq. of the light received by O is $<$ actual freq. of S.
- If S moves towards O, then f observed by observer $>$ actual freq. of S.
- The frequency of light received, when S and O are in relative motion is known as apparent frequency.



Doppler Effect in Light

Definition:

Whenever there is a relative motion between the source of light and the observer, the apparent frequency of light observed (by the observer) is different from the actual (or true) frequency of light emitted by the source of light. This effect is known as "Doppler effect".

Doppler Effect is a wave phenomenon, it is applicable not only for light waves but also for other electromagnetic waves and sound waves.

We need to derive 3 main cases: Let us indicate Source of light as S and Observer as O

- S is moving and O is stationary (S is moving towards and away) → 2 cases
- O is moving and S is stationary (O is moving towards and away) → 2 cases
- Both S and O are moving. (Both S & O towards and away) → 4 cases

Let us define the following notations

$V = \text{velocity of Light in air} (=3 \times 10^8 \text{ ms}^{-1})$ (**Instead of notation C, we refer C = V**)

$T = \text{Time period of one light wave}$

$f = \text{True frequency of light wave}$

$\lambda = \text{distance travelled by light wave in its own time period } T$

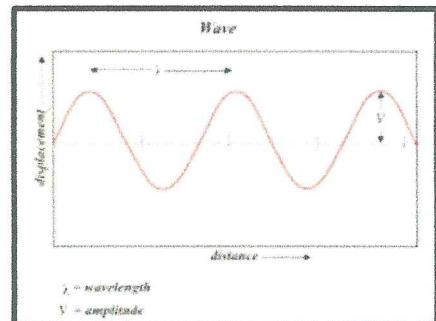
We know that $V = \text{distance/time} = \lambda/T = \lambda f$

$$\triangle \lambda = V/f = VT \quad \dots \quad (1)$$

$v_o = \text{velocity of Observer O}$

$v_s = \text{velocity of Source S}$

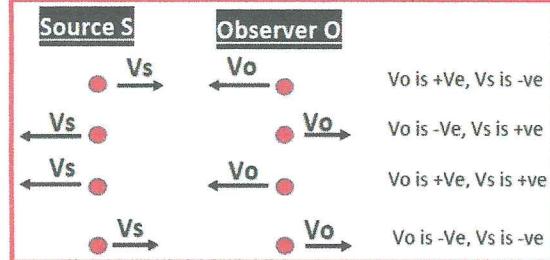
$f' = \text{Apparent frequency perceived by the Observer O}$



Sign Convention S and O velocities: (Very important)

Take Observer to Source as positive direction of velocity (O → S is +ve velocity).

- Both S and O are moving towards each other → v_o is +ve ; v_s is -ve
- Both S and O are moving away from each other → v_o is -ve ; v_s is -ve
- S is moving away from O and O is moving towards S → v_o is +ve ; v_s is +ve
- S is moving towards O and O is moving away from S → v_o is -ve ; v_s is -ve



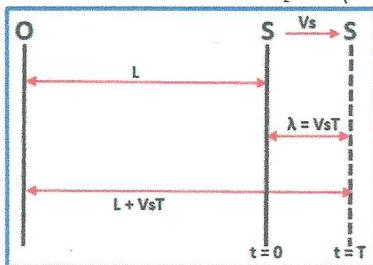
S is moving and O is stationary

If S is moving with velocity v_s away from O and O is stationary

Let distance between S and O when both are rest = L

If source is at rest at $t = 0$ and emits a crest, then the crest reaches O at time $t_1 = L/V$.

If at time t_2 , source has moved λ distance, then distance travelled by S in T secs = $v_s T$. At t_2 , S emits another crest. This reaches O at $t_2 = T + (L + v_s T)/V$.



$$\triangle t_2 - t_1 = (T + L/V + v_s T/V) - L/V$$

$$T' = T + v_s T/V = T(1 + v_s/V) \quad \dots \quad (2)$$

$$f' = f (1 + v_s/V)^{-1} \quad \dots \quad (3)$$

$$f' = f / (1 + v_s/V) \quad \dots \quad (3)$$

$$f' = f / [(V + v_s)/V] \quad \dots \quad (3)$$

$$f' = f / (V/V + v_s/V) \quad \dots \quad (3)$$

$$f' = f \frac{(V)}{(V + v_s)} \quad \dots \quad (3)$$

Approximation if $v_s \ll V$, using Binomial series on (2), we get

$$f' = f (1 - v_s/V)$$

$$f' = f \left(1 - \frac{v_s}{V} \right) \quad \dots \quad (4)$$

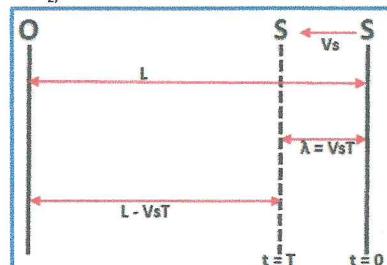
This shows $f' < f$ when light source moves away from a stationary O
(3) is an accurate equation ; (4) is an approximation of (3) if $v_s \ll V$
So, if $v_s \ll V$, use (4) ; If v_s is comparable to V, use (3)

If S is moving with velocity v_s towards O and O is stationary

Let distance between S and O when both are rest = L

If source is at rest at $t = 0$ and emits a crest, then the crest reaches O at time $t_1 = L/V$.

If at time t_2 , source has moved λ distance, then distance travelled by S in T secs = $v_s T$. At t_2 , S emits another crest. This reaches O at $t_2 = T + (L - v_s T)/V$.



$$\triangle t_2 - t_1 = (T + L/V - v_s T/V) - L/V$$

$$T' = T - v_s T/V = T(1 - v_s/V) \quad \dots \quad (5)$$

$$f' = f (1 - v_s/V)^{-1} \quad \dots \quad (5)$$

$$f' = f / (1 - v_s/V) \quad \dots \quad (5)$$

$$f' = f / [(V - v_s)/V] \quad \dots \quad (5)$$

$$f' = f / (V/V - v_s/V) \quad \dots \quad (5)$$

$$f' = f \frac{(V)}{(V - v_s)} \quad \dots \quad (5)$$

Approximation if $v_s \ll V$, using Binomial series on (5), we get

$$f' = f (1 + v_s/V)$$

$$f' = f \left(1 + \frac{v_s}{V} \right) \quad \dots \quad (7)$$

This shows $f' > f$ when light source moves towards the stationary O
(6) is an accurate equation ; (7) is an approximation of (6) if $v_s \ll V$
So, if $v_s \ll V$, use (7) ; If v_s is comparable to V, use (6)

Eg : Consider equation of apparent frequency when S is moving away from stationary Observer ($V_s \ll C$)

$$f' = f(1 - V_s/C); f' < f \quad (\text{Note notation } C \text{ for speed of light in air is used now instead of } V)$$

$$\Delta f = f' - f = (f - fV_s/C) - f = - (V_s/C)f$$

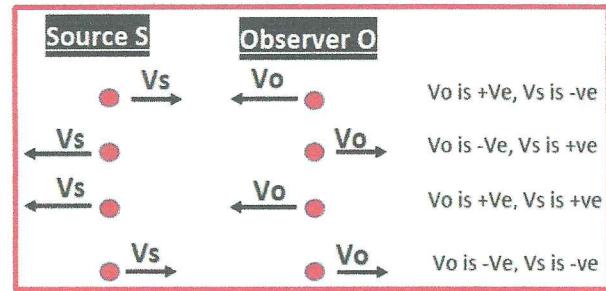
$\Delta f/f = - (V_s/C) \rightarrow$ See Sign Convention S and O velocities:

From figure, V_s (aka V_{radial}) is considered positive when S moves away from the observer; Therefore,

$$\Delta f/f = - (V_s/C) = - (V_{\text{radial}}/C)$$

Where V_{radial} is the component of the source velocity along the line joining the observer to the source relative to the observer.

- As per figure V_{radial} is considered **POSITIVE** when **source moves away from the observer**. So, $(f' - f)/f = -(V_{\text{radial}}/C)$ is **NEGATIVE**; hence $f' < f$, $\lambda' > \lambda$ (Red shift)
- As per figure V_{radial} is considered **NEGATIVE** when **source moves towards the observer**. Therefore, $(f' - f)/f = -(V_{\text{radial}}/C)$ is **POSITIVE**; hence $f' > f$, $\lambda' < \lambda$ (Blue shift)
- The above bullets are valid only when $V_s \ll C$
- A more accurate formula for the Doppler Effect which is valid even when V_s and C are comparable, requires use of Einstein's special theory of relativity. The Doppler Effect for light is very important in astronomy. It is the basis for the measurements of the radial velocities of distant galaxies.



Eg : what speed should a galaxy move wrt us so that the sodium line at 589.0 nm is observed at 589.6 nm ?

Ans : Since $C = f\lambda$ and $\Delta f/f = -\Delta\lambda/\lambda$; $\Delta\lambda = \lambda' - \lambda = 589.6 - 589.0 = +0.6\text{nm}$

$$\Delta f/f = -V_{\text{radial}}/C = -\Delta\lambda/\lambda$$

$$V_{\text{radial}} = +C(\Delta\lambda/\lambda) = +C(0.6/589.0) = +306\text{ km/s}$$

Since V_{radial} is **POSITIVE** (+306 km/s), therefore the galaxy is moving away from us.

➤ Therefore, Doppler Shift in terms of **frequency** is given by

- $\Delta f = - (V_s/C)f \rightarrow \Delta f$ is negative when S is moving away from stationary Observer
- $\Delta f = + (V_s/C)f \rightarrow \Delta f$ is positive when S is moving towards the stationary Observer
- $\Delta f = \pm (V_s/C)f \rightarrow \Delta f$ is positive when S is moving towards the stationary O and Δf is negative when S is moving away from stationary observer

➤ Similarly, Doppler shift in terms of **Wavelength** is given by

Consider the equation (3) page 97 (accurate equation when S is moving away from stationary Observer)

$$f' = f [C/(C + V_s)] \quad (3) \text{ Accurate equation}$$

$$C/\lambda' = C/\lambda [C/(C + V_s)]$$

$$1/\lambda' = 1/\lambda [C/(C + V_s)]$$

$$\lambda'/\lambda = C/(C + V_s)$$

$$\lambda'/\lambda = (C + V_s)/C = 1 + V_s/C$$

$$\lambda' = \lambda [1 + V_s/C] \rightarrow \text{Compare this with approximate equation of apparent frequency } f' = f(1 - V_s/C)$$

For this case of S is moving away from stationary Observer :

$$f' < f \text{ and } \lambda' > \lambda; \text{ We conclude that :}$$

- Apparent Freq < True Freq
- Apparent wavelength > True λ

Since Red is having greater wavelength as compared to Blue, this is called "Red shift".

When waves are received from a source moving away from the observer, due to shift in wavelength towards higher wavelength, this is called Red shift. Red shift confirms that universe is expanding.

Consider the equation (6) page 97 (accurate equation when S is moving towards the stationary Observer)

$$f' = f [C/(C - V_s)] \quad (6) \text{ Accurate equation}$$

$$C/\lambda' = C/\lambda [C/(C - V_s)]$$

$$1/\lambda' = 1/\lambda [C/(C - V_s)]$$

$$\lambda'/\lambda = C/(C - V_s)$$

$$\lambda'/\lambda = (C - V_s)/C = 1 - V_s/C$$

$$\lambda' = \lambda [1 - V_s/C] \rightarrow \text{Compare this with approximate equation of apparent frequency } f' = f(1 + V_s/C)$$

For this case of S is moving towards the stationary Observer :

$$f' > f \text{ and } \lambda' < \lambda; \text{ We conclude that :}$$

- Apparent Freq > True Freq
- Apparent wavelength < True λ

Since Blue is having least wavelength as compared to Red, this is called "Blue shift".

When waves are received from a source moving towards the observer, due to shift in wavelength towards lower wavelength, this is called Blue shift.

*Comparison of Doppler's Effect in Light and Sound

The apparent frequency of light wave when source moves towards the stationary observer is same even when the observer moves towards the stationary source. Therefore, *Doppler's effect in light is symmetrical*.

In case of sound, a source moving through a medium, in which the observer is at rest, is physically different from an observer moving through that medium in which the source is at rest. That is the apparent frequency observed in both the cases is different. Hence *Doppler's effect in sound is asymmetrical*.

*Applications and Uses of Doppler's Effect in Light

1. Doppler's effect in light is used to measure the *speed of stars and galaxies* using relation

$$\Delta\lambda = \pm \frac{v}{c} \lambda \quad \text{or} \quad v = \pm \frac{\Delta\lambda}{\lambda} c$$

2. Doppler's effect is used to measure the *speed of rotation of the sun about its own axis*.

3. Doppler's effect is used to measure the temperature of plasma in *thermo nuclear reactions*.

4. Doppler's effect confirms the hypothesis that the *universe is expanding*.

5. Doppler's effect is used in *RADAR* (Radio Detection And Ranging) to estimate the speed of aircrafts, their presence and distance from the given position.

6. Doppler's effect is used in *SONAR* (Sound navigation and ranging) to detect the submarines and find the depth of the sea.



When the velocity of source of light is comparable with the velocity of light, then the expression for Doppler's effect gets modified by applying the results of *special theory of relativity*. After applying the special theory of relativity,

$$v' = \left(1 \pm \frac{v}{c}\right)v / \sqrt{1 - \frac{v^2}{c^2}}$$

... (8)

Ray optics \longleftrightarrow wave optics

-100-

23

(also called Geometrical optics)

→ Ray optics is a branch of optics where one completely neglects the finiteness of the wavelength and a "ray" is defined as the path of energy propagation in the limit of wavelength tending to zero.

The ray optics can explain the macroscopic phenomena like straightline propagation of light, reflection and refraction of light etc. However, the microscopic phenomena like interference, diffraction and polarization could not be explained by ray optics. To explain these, the concept of waves was introduced.

The new branch of physics based on wave concept of light was called "wave optics" (or physical optics). It was proposed by Huygens and later modified by Fresnel.

According to wave theory of light, the light is a form of energy which travels through a medium in the form of transverse wave motion. The speed of light in a medium depends upon the nature of the medium.

wave optics

wave front

Huygen's principle

Wavefront of a wave is the locus of all pts ~~in~~ in the same phase during the propagation of ~~the~~ the wave.

- Reflection of plane waves
- Refraction
- Refraction in a prism
- Refraction by a convex lens
- Reflection of plane wave by a concave mirror

- Interference
- diffraction
- polarization

Interference of Light Waves

To understand the phenomenon of interference of light, it is important to understand the

→ Superposition principle

- According to superposition principle, the resultant displacement of a particle at any instant is the vector sum of the individual displacements caused to the particle by two or more waves at that instant.

→ Coherent source

- Two sources of light are said to be coherent if they emit waves of the same frequency and wavelength and are either in phase or have a constant initial phase difference.

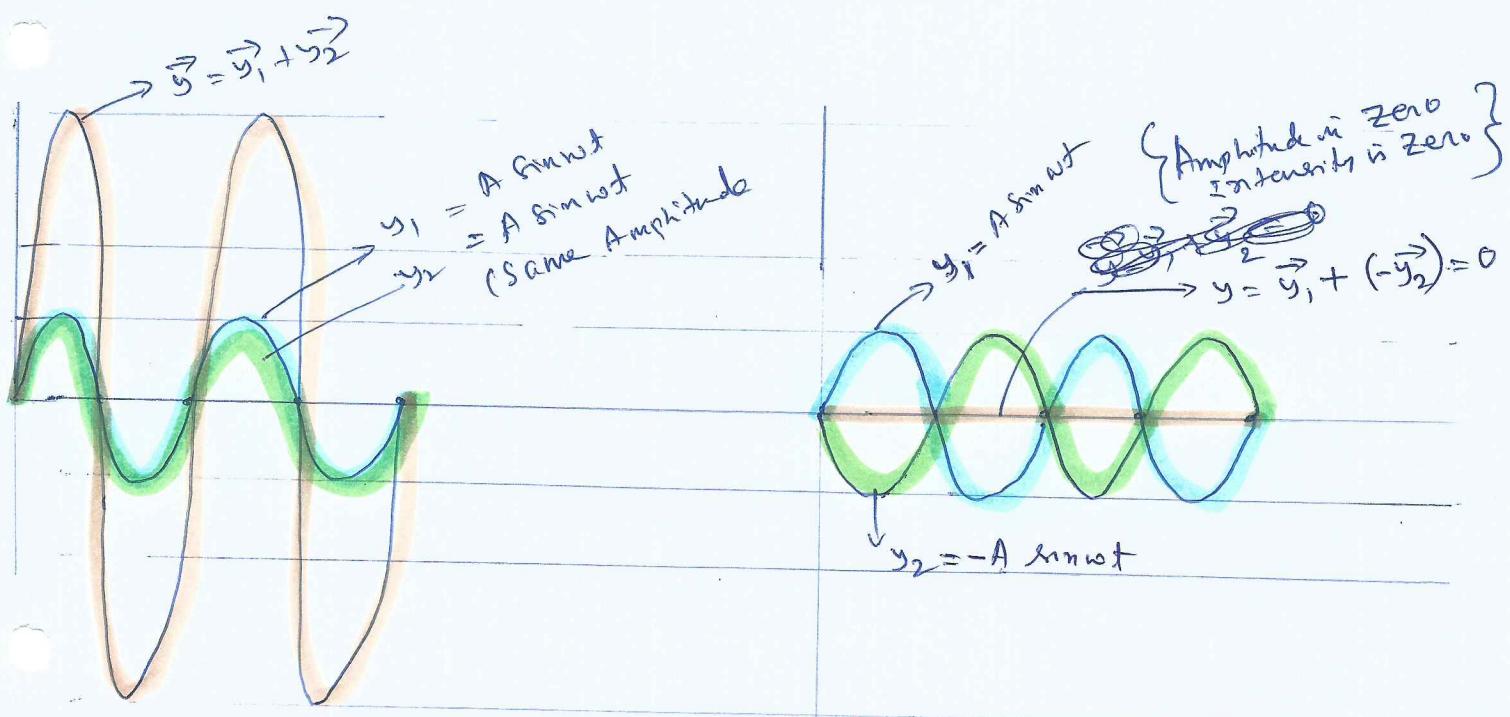
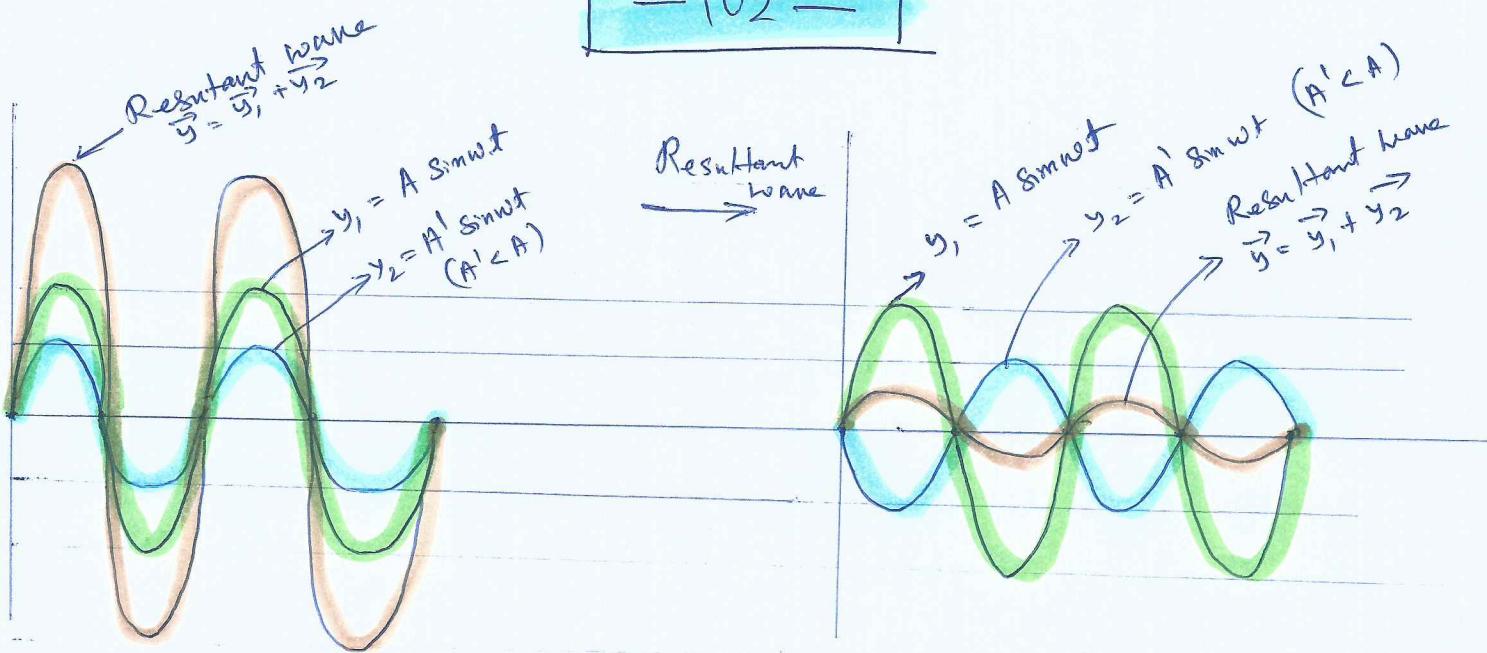
Superposition principle

CS

- Constructive Superposition: When two waves match exactly in same wavelength and in phase → meaning crest of one wave falls on the crest of the other wave or trough of one wave falls on the trough of the other wave, then the ~~is~~ resultant wave is called Constructive Superposition (CS)

→ In CS, the amplitude of the resultant wave increases.
→ In CS, hence the intensity $I \propto (\text{amplitude})^2$ also increases.

- Destuctive Superposition (DS): When two waves having the same wavelength but with 180° phase difference → means crest of one wave falls on the trough of the other wave or trough of one wave falls on the crest of the other wave, then the resultant wave is called DS.
→ In DS, the amplitude of resultant wave decreases. If amplitudes of two waves are equal, then resultant wave has ZERO amplitude.
→ In DS, the intensity $I \propto (\text{amplitude})^2$ of the resultant wave also decreases. If ~~the~~ the amplitudes of 2 waves are equal, then the resultant intensity is ZERO.

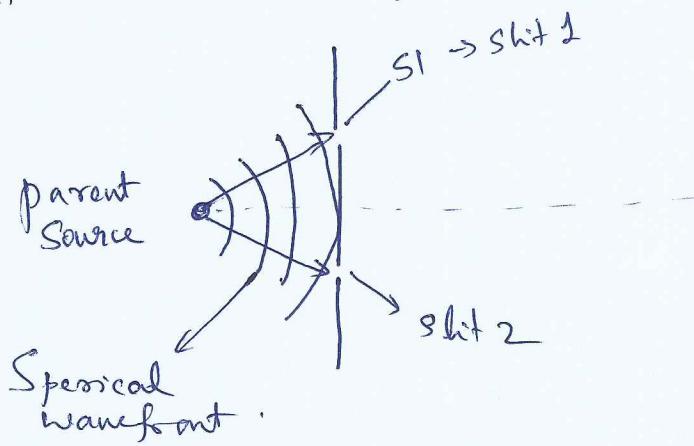


Note : ① A bone figure uses Sine wave. Instead we can also use Cosine wave.

② The above waveforms indicate Superposition principle. (Interference topic comes later)

Coherent Sources:

- Coherent sources are generally obtained by deriving them from a single ~~source~~ parent source.
- It is very difficult (almost impossible) to get coherence between 2 sources having same phase difference over a period of time.



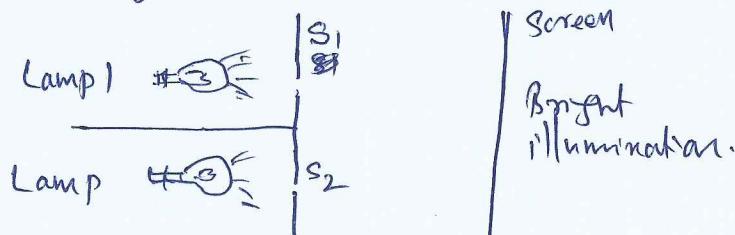
Slits 1 and 2 can be seen as two coherent sources, since it is derived from a single parent source, parent source emits wavefronts that illuminates the slits S₁ and S₂, so they have no phase difference or constant phase difference.

In Summary

- Coherent sources (S₁ and S₂) are capable of emitting spherical wavefronts with same frequency, wavelength and of ~~same~~ zero phase difference or constant phase difference.
- Coherent sources (Slits S₁ and S₂) have almost same amplitudes.

Incoherent Light Sources: Sources having different frequencies or same frequencies but no constant phase difference are known as Incoherent Sources.

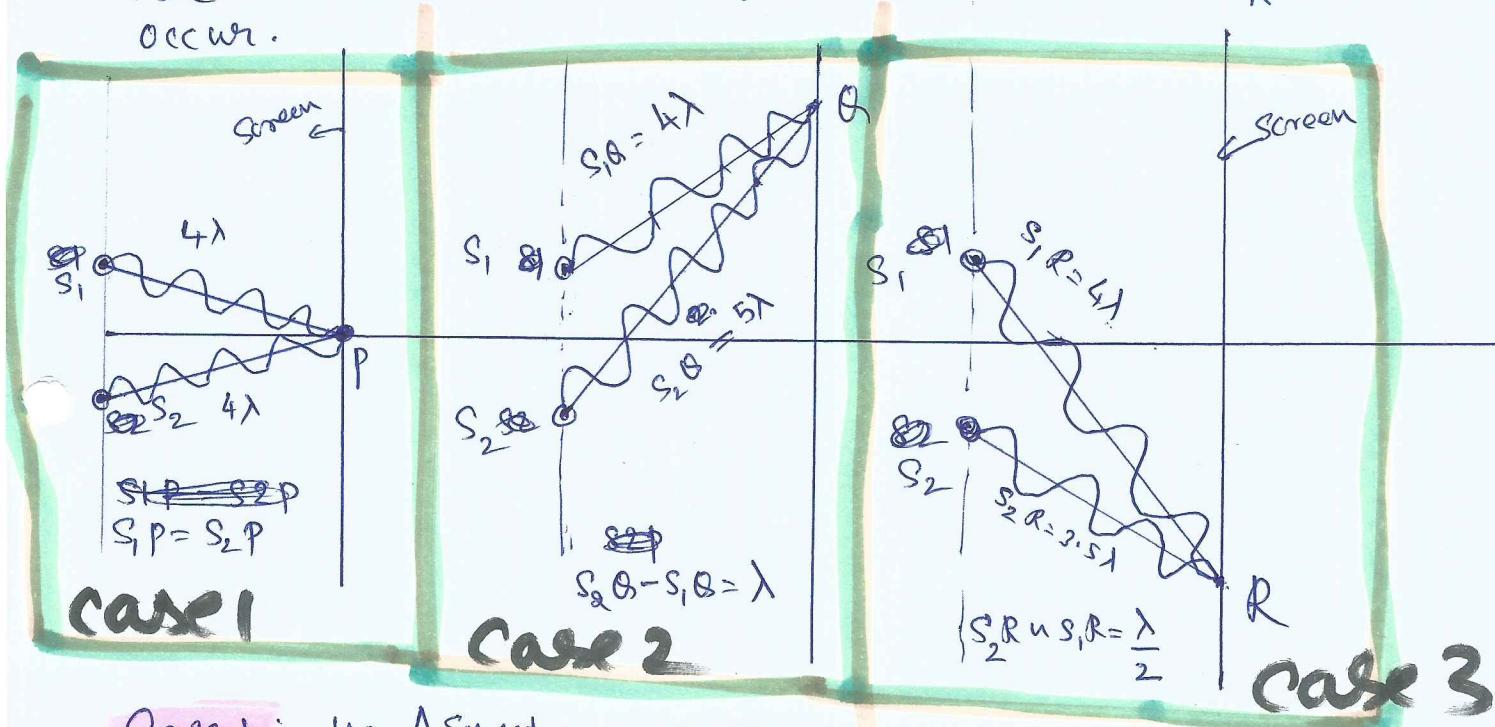
Two independent sources can emit light of same freq (or λ) but their phase difference may not be stable and it may vary from 0 to 2π .



Intensities of light from the incoherent sources just add up and the screen placed in front of these sources illuminates brightly.
 (No interference pattern)

Interference pattern

According to Superposition principle, when two waves ~~are~~ superimpose on each other ~~at certain point~~, one can see a series of bright and dark bands called as fringes. Let us see the condition when bright fringes occur and ~~when~~ dark fringes occur.



Case 1 : At point P

$$y_1 = A \sin \omega t$$

$$y_2 = A \sin (\omega t + \pi) \therefore y = (2A) \sin \omega t \quad (2A \rightarrow \text{Amplitude of Resultant wave})$$

Intensity $I \propto (\text{amplitude})^2$

$$\text{For } y_1, I = A^2 = I_0$$

$$\text{For } y_2, I = A^2 = I_0 \therefore \text{for } y, I = 4A^2 = 4I_0$$

Case 2 : Consider point Q where $S_2Q \neq S_1Q \Rightarrow$ there is a path difference.

$$S_2Q - S_1Q = \lambda \Rightarrow \text{path difference} = \lambda \Rightarrow \text{phase difference} = 2\pi$$

$$y_1 = A \sin \omega t$$

$$y_2 = A \sin (\omega t + 2\pi) = A \sin \omega t$$

$$\therefore \text{path difference } \lambda \text{ corresponds to phase difference } 2\pi$$

$$\therefore y = y_1 + y_2 = 2A \sin \omega t \quad [\text{We know that } \sin(\omega t + 2\pi) = \sin \omega t]$$

② is same as ①

$$\therefore I = 4A^2 = 4I_0$$

\therefore If path difference between 2 waves is ~~0~~, $0, \lambda, 2\lambda, 3\lambda \dots$ or the phase difference = $0, 2\pi, 4\pi, 6\pi \dots$ then case ② is same as case ①. So the superposition is constructive.

(Max. intensity)

P.T.O