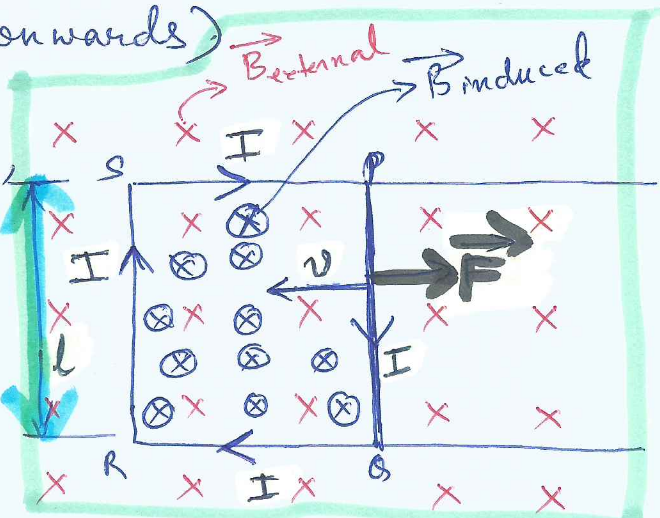
Contd from page 25

→ To find direction of induced current I in the loop PQRS

- From Lenz's law, the induced current I must be in such a direction so as to oppose the "change". Since the rod PQ is moved left, the "change" is decrease in flux ϕ_B in the loop PQRS. ~~Therefore, the current must be in~~
- Therefore, as per Lenz's law, the current must be in such a direction to produce a field that "increases or enhances" the mag. flux ϕ_B.
- ∴ "change" = decrease in flux ϕ_B → due to PQ moved left

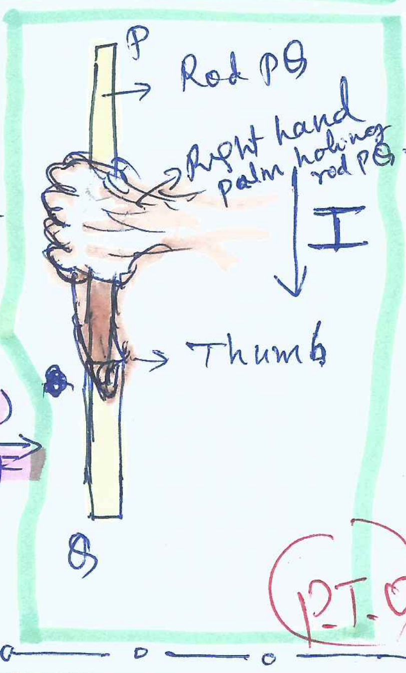
∴ Counter change as per Lenz's law → to increase flux ϕ_B by setting up a field that is parallel to the external field $\vec{B}$ within the loop "PQRS". So, $\vec{B}_{induced}$ (due to induced current) will be same as $\vec{B}$ (⟹ ⊥r to plane of paper and downwards). So, within loop PQRS, the total $\vec{B} = \vec{B}_{external} + \vec{B}_{induced}$ and $\vec{B}_{external}$ and $\vec{B}_{induced}$ are parallel and directed in the same direction (i.e. downwards)



- As per Right hand Thumb rule on rod PQ (or Right hand palm Rule #1), the direction of "current" is from P to Q.

Hold the rod PQ with right hand where 4 fingers curl in the direction of $\vec{B}$ (in the loop) and thumb will point towards the direction of current.

- **So, Direction of current is clockwise**



- The current in the loop will cause a Force $\vec{F}$ on PQ (which is moveable rod). This force must be towards right to oppose the movement of rod PQ (towards left)
- To verify the direction of $\vec{F}$, use Fleming LHR.

→ I on PQ is from P to Q (Middle finger)
→ $\vec{B}$ ⊥r to plane of paper and downwards (fore finger)
→ then thumb will point towards right, which is $\vec{F}$

$\vec{F} = I(\vec{l} \times \vec{B}) = IlB \sin 90^\circ = IlB$ (Since $I = Blv/R$)

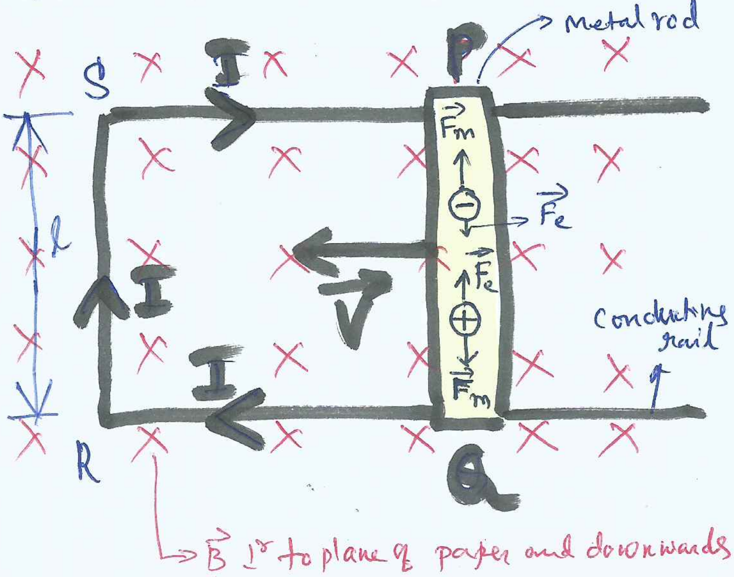
~~$F = \frac{B^2l^2v}{R}$~~ $\boxed{F = \frac{B^2 l^2 v}{R}} \rightarrow (4)$

P.T.O.

Motional EMF: Alternate Method using Lorentz Force

We know that Lorentz Force $F = q\vec{E}(r) + q[\vec{v}\times\vec{B}] = F_{electric} + F_{magnetic}$

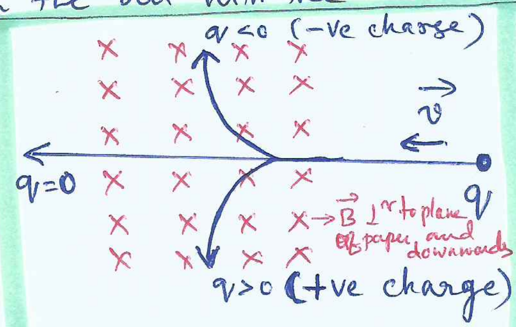
We have to find Induced current direction in the loop PQRS. Using Lorentz Force concept.



→ Figure shows metal rod PQ of length l being moved with a constant velocity $\vec{v}$ to the left in a uniform and time-independent mag. field $\vec{B}$ (which is ⊥r to plane of paper and directed downwards.

→ We know that rod PQ has positive and negative charges.

→ When the rod PQ moves left, with constant velocity $\vec{v}$, each charge q within the rod PQ moves left along with the rod with the same velocity $\vec{v}$. The charge experiences a force when entering the magnetic field



$$\Rightarrow \text{Magnetic Force } F_{mag} = |q|\,\vec{v}\times\vec{B}$$
$$= |q|\,vB\sin 90^\circ$$
$$\boxed{F_{mag} = qvB} \longrightarrow (1)$$

- Note that all charges experience the same Lorentz force irrespective of their position in the rod PQ.
- As per Fleming's LHR,
  +ve charges experience force towards Q
  −ve charges experience force towards P
- Electrons (-ve charges) drift towards P
  positive charges appears at Q end of rod.
- This process repeats till the attractive "electric force" left the accumulated charges is balanced by the magnetic force that separates the charges ($\Rightarrow$ when $F_e = F_m$)
  So, in steady state condition $F_e = F_m$

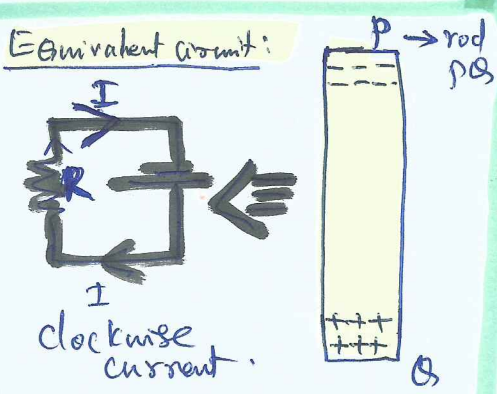
Using Lorentz force eqn } $\cancel{q}E = \cancel{q}vB$

$$\boxed{E = vB} \longrightarrow (2)$$

Fleming's LHR

Fore finger → $\vec{B}$ (⊥r to plane of paper and downwards)
Middle finger → positive Charge movement direction = Rod PQ moving Left
Thumb → Force on charge due to $\vec{B}$ and $\vec{v}$

Equivalent circuit:



- Therefore, in steady state condition, at P, there will be excess concentration of −ve charges and at Q, there will be an excess concentration of +ve charges.

P.T.O. → Contd....

So, Q is +ve wrt P. Hence an electric potential difference V is induced across the rod PQ, due to which an electric field $\vec{E}$ is created within the rod, whose magnitude is

$$\boxed{E = \frac{V}{l}} \rightarrow (3)$$ where l = length of the rod PQ.

$$\boxed{E = vB} \rightarrow (2)$$ from page 27

From (2) and (3)

$$\frac{V}{l} = vB \quad \therefore V = Blv$$

V is nothing but induced emf ϵ

$$\boxed{\epsilon = Blv} \longrightarrow (4)$$

which is same as conventional method derivation of Motional EMF.
Therefore, the separated charges across PQ give rise to an induced emf which is called Motional emf. It exists as long as rod PQ moves. When the rod PQ stops moving, magnetic force $\vec{F}_{mag}$ vanishes (since $\vec{v} = 0$, $\vec{v} \times \vec{B} = 0$). The electric field only exists and whose force reunites the separated charges across PQ and motional emf disappears. So, no emf is induced in the stationary rod.

The above eqn (4) can also be derived as follows →

- The expression for motional emf can be ~~obtained~~ obtained from Lorentz force equation $F = qvB$. This is the force experienced by any arbitrary charge $+q$ in the rod moving with constant velocity v in mag. field $\vec{B}$.

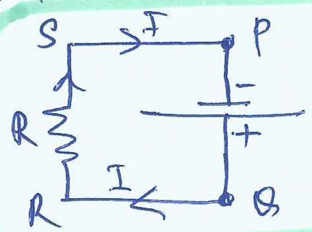
∴ Work done in moving charge $+q$ from P to Q is

$$W = F \times PQ = F \times l = qvB \times l$$

EMF induced = work done per unit charge

$$\boxed{\epsilon = \frac{W}{q} = \frac{\cancel{q} vBl}{\cancel{q}} = vBL} \rightarrow (5)$$ which is same as (4).

Since Q is positive wrt P, the rod PQ behaves like a battery as shown in this figure. Current direction will be clockwise which is in agreement with conventional method of deriving motional EMF.



see this

If R is the Resistance of the loop PQRS, then

$$\boxed{I = \frac{\epsilon}{R} = \frac{\cancel{Bl}\,Blv}{R}} \rightarrow (6)$$

Direction of current = clockwise in the given set-up

IMP

Extra (from NCERT book) page 213 vol 1

- From previous discussion, we know that when rod PQ is stationary, $v = 0$, hence contribution from mag. field towards force on charges in rod PQ is zero.
- However, if $\vec{B}$ is changing (and rod PQ is still stationary), there will be an induced emf, which should be mainly due to electric field.

$$F = q\vec{E} + q[\vec{v} \times \vec{B}] = q\vec{E}$$ (since $v = 0$, no magnetic contribution towards Force)

- Thus, any force on the charge must arise from the electric field term $\vec{E}$ alone. Therefore to explain the existence of induced emf or induced current, we must assume that a time-varying magnetic field generates an electric field. (Note that $\vec{E}$ produced by static electric charges have properties different from those produced by time-varying mag. fields)
- We have learnt
  1. Charges in motion (=> current) can exert force/torque on a stationary ~~object~~ magnet.
  2. Conversely a bar magnet (or, more generally, a changing magnetic field) can exert a "force" on the stationary charge.

This is the fundamental significance of ~~the~~ Faraday's discovery. Electricity and Magnetism are related.

Def$\underline{\underline{n}}$ of Lorentz Force: Let us suppose that there is a point charge q (moving with a velocity v and located at r at a given time t) in presence of both the electric field $\vec{E}(r)$ and the magnetic field $\vec{B}(r)$. The force on an electric charge q due to both electric and magnetic fields can be written as

$$\vec{F} = q\vec{E}(r) + q\,[\underbrace{\vec{v} \times \vec{B}}_{\text{vector product of } \vec{v} \times \vec{B}}]$$

Extra IMP

Motional emf $\epsilon = vBl$

⇒ When a straight conductor (PQ) of length l moves with velocity of magnitude v ⊥r to mag. field of magnitude B cutting its flux-lines, a potential difference $\epsilon = Bvl$ is induced betn the ends of the conductor (PQ).

⇒ If the direction of velocity of conductor (PQ) makes an angle θ with the direction of $\vec{B}$, then the component of $\vec{v}$ ⊥r to $\vec{B}$ will be $v \sin\theta$. Therefore, in this case, the p.d. induced across conductor (PQ) will be $vBl\sin\theta$. $\boxed{\epsilon = vBl\sin\theta}$

⇒ clearly, if the conductor (PQ) moves parallel to the mag. field $(\theta = 0)$, no potential difference ~~is~~ will be induced across ~~end of (PQ)~~ conductor (PQ).

Applications:

① The axle of the wheels of a train running on rails cuts the flux-lines of the vertical component of earth's mag. field and so a p.d. is induced between the ends of the axle.

② The p.d. across the wings of an aeroplane flying horizontally at a definite height is also developed due to the cutting of flux-lines of the vertical component of earth's mag. field.

→ ~~If~~, however, the aeroplane is landing down and its wings are in the E-W (east-west) direction, then the wings cut the flux-lines of the horizontal component of the earth's mag. field and again a p.d. is induced.

→ If while landing, the wings of the aeroplane are in ~~north~~ north-south (N-S) direction, no flux lines due to any component of earth's mag. field is cut and no p.d. is induced.

③ If a conducting rod pointing E-W ~~then~~ is dropped down freely, it cuts the flux lines of the horizontal component of earth's magnetic field. Hence a p.d. is induced across the ends of the rod.

→ If the same rod is pointing N-S, then on dropping it down freely, it will not cut any flux-lines and no p.d. will be induced.

④ If a player is running along E-W, then his body is cutting the flux-lines of the horizontal component of earth's mag. field and so a p.d. is induced between his head and feet.

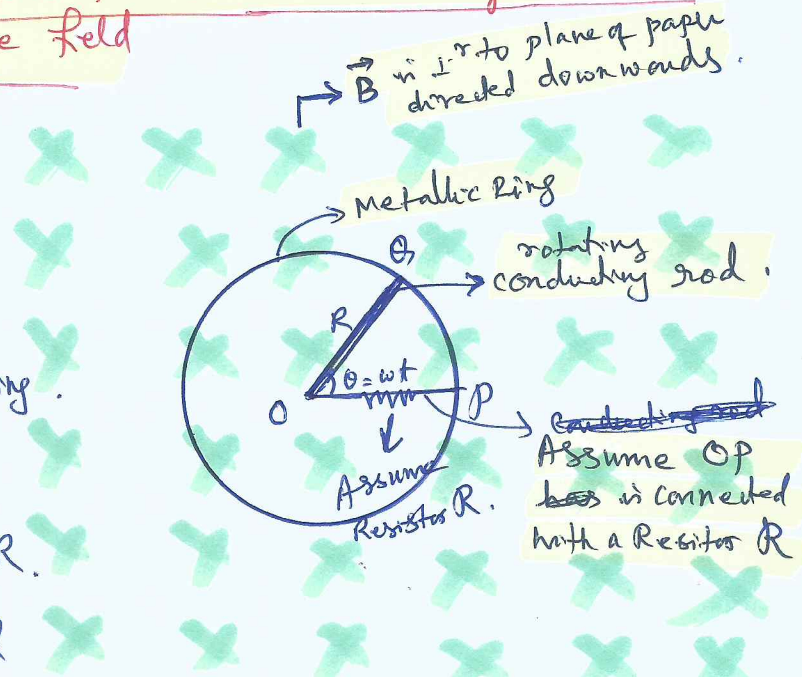
→ If, however, he runs along N-S, no p.d. is induced.

End of Motional emf

IMP

Derive an expression for induced emf in a conducting rod rotated in a uniform magnetic field

To calculate the induced emf,
let OPQ be a closed loop,
where OQ = rotating conducting rod (rotating ~~and~~ and touching the ~~ring~~ a metallic ring.
(See figure)
and imagine points O and P are connected with a Resistor R.



- The p.d. across R = induced emf = B × (rate of change of area of loop OPQ
- If θ is the angle betn the rod OQ and the radius of ~~circle~~ circle at P at time t, the area of the sector OPQ is given by

$$\pi R^2 \times \frac{\theta}{2\pi} = \frac{1}{2}R^2\theta \text{, where } R = \text{radius of the circle.}$$

$\therefore$ Induced emf $\epsilon = \frac{d\phi}{dt}$ $\quad \phi = BA$ $\quad (\cos 0^\circ = 1)$ $\Rightarrow \epsilon = \frac{d}{dt}(BA)$

$$\epsilon = B \times \frac{d}{dt}\left[\frac{1}{2}R^2\theta\right] = \frac{1}{2}R^2B\left(\frac{d\theta}{dt}\right)$$

$\frac{d\theta}{dt}$ = angular velocity = ω

$$\boxed{\epsilon = \frac{1}{2}B\omega R^2} \longrightarrow (1)$$

$\theta = \frac{PQ}{R} = \frac{S}{R}$
$S = R\theta$
$\frac{ds}{dt} = R\frac{d\theta}{dt}$
$\boxed{v = R\omega}$
$\omega = 2\pi f$

where R = Radius of the circle
ω = angular velocity = $2\pi f$ (f = freq.)
B = mag. field.

$$\epsilon = \frac{1}{\cancel{2}}BR^2 \cdot \cancel{2}\pi f \quad \therefore \boxed{\epsilon = \pi B R^2 f}$$

If R = 1m, f = 50 Hz, B = 1 Tesla, $\epsilon = \pi \cdot 1 \times 1^2 \cdot 50 = 50\pi$
$= 157$ V

(NCERT example problem 6.6 page 213, vol 1)

END

problem : (NCERT example problem 6.7, page 215, vol 1)
A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of earth's mag. field H_E at a place. If $H_E = 0.4\,G$ at the place, what is the induced emf betn the axle and the rim of the wheel. Note that $1\,G = 10^{-4}\,T$

Ans :- ~~Given~~ Formula $\mathcal{E} = \pi B R^2 f$ (from page 30)

Given $R = 0.5\,m$ Speed = 120 rev/min

$f = \cancel{60}\ 2s$ $f = \frac{\cancel{60}}{2}$ rev/sec

$B = 0.4 \times 10^{-4}\,T$

$\mathcal{E} = \pi \times (0.4 \times 10^{-4}) \times (0.5)^2 \times \cancel{60}\,2\ V$

$\mathcal{E} = \pi \times (0.4 \times 10^{-4}) \times (0.25) \times \cancel{60}\,2$

$= \pi \times (4 \times \cancel{25}\,0.25 \times \cancel{6}\,2) \times 10^{-5}$

$= \cancel{\pi (10 \times 6) \times 10^{-5}}$

$= \cancel{60\pi = 20\pi \times 10^{-5}}$

$= \pi\,(4 \times 0.25 \times 2) \times 10^{-5}\ V$

$= \pi\,(1 \times 2) \times 10^{-5}\ V$

$= 2\pi \times 10^{-5}$

$$\boxed{\mathcal{E} = 6.3 \times 10^{-5}\ V}$$

Total emf will be 6.3×10^{-5} V only since all spokes are connected in parallel. So, number of spokes is immaterial in this problem.

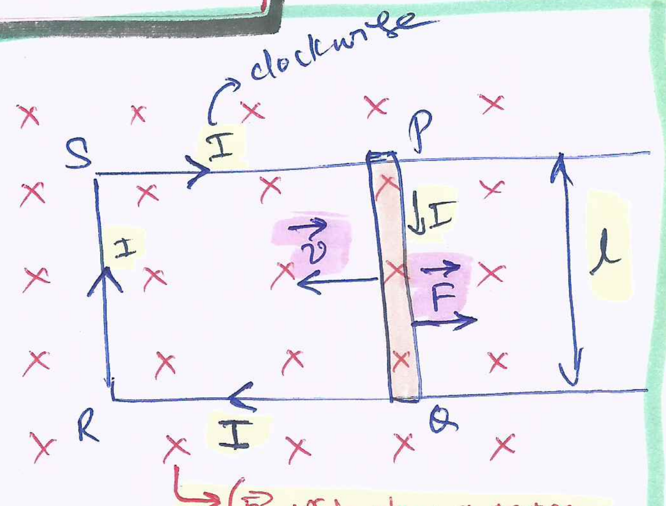
END

Energy Consideration in Motional EMF

We know that motional emf in rod PQ = $\epsilon = Bl v$

Let total Resistance of loop PQRS = R (R will be constant whether we move rod PQ or not)

$\therefore$ Induced current $\boxed{I = \frac{\epsilon}{R} = \frac{Blv}{R}}$ → (1)



→ ($\vec{B}$ $\perp^r$ to plane of paper and directed downwards)

- As per Lenz's law, the induced current opposes the movement of PQ towards left by producing suitable magnetic field.
- Therefore, PQ experiences a force towards right due to effective magnetic field produced by induced current.
- As shown in fig set up, the current must in clockwise direction to oppose movement of rod PQ towards left.
- So, I on PQ is from P to Q, B is $\perp^r$ to plane of paper and directed downwards, then ~~force~~ direction of force is given by Fleming's LHR. (Thumb → force, Fore finger → $\vec{B}$, Middle finger → current)
- So, as per Fleming's LHR, direction of Force is proved to be towards right (and hence opposing movement of rod PQ towards left)

- The magnitude of Force $\vec{F}$ on PQ is given by

$$\boxed{F = \underbrace{\cancel{BIl}}_{IlB\sin 90^\circ} = B\left(\frac{Blv}{R}\right)l = \frac{B^2 l^2 v}{R}} \rightarrow (2)$$

- The direction of $\vec{F}$ is towards right as per Fleming's LHR.

power required to push PQ with a velocity v is

$$\boxed{P = F \times v = \frac{B^2 l^2 v^2}{R}} \rightarrow (3)$$

- Since the conductor PQ is pushed mechanically (towards left with a velocity v), the mechanical energy dissipated per second is given by

$$P = I^2 R = \left(\frac{Blv}{R}\right)^2 \cdot R = \frac{B^2 l^2 v^2}{R} \rightarrow (4)$$ which is same as (3)

P.T.O. Contd.....

contd from 32

Hence, mechanical energy required to move the Conductor rod PQ is ~~convert~~ converted in to electrical energy first (i.e. the induced emf) and then to thermal energy.

$$\therefore \boxed{\text{Heat energy produced}/\text{sec} = \frac{B^2 l^2 v^2}{R}} \rightarrow (4)$$

Note that the magnetic forces do not do any work. All they do is transfer energy from mechanical to electrical

---

IMP: Amount of charge induced due to change in magnetic Flux

From Faraday's law, the magnitude of the induced emf is

$$\boxed{|\epsilon| = \frac{d\phi}{dt}} \text{ in volts} \rightarrow (5)$$

We also know that from ohm's law, if R is the resistance of the circuit, then

$$\boxed{V = IR \Rightarrow \epsilon = IR = \frac{dQ}{dt} \cdot R} \rightarrow (6)$$

From (5) and (6)

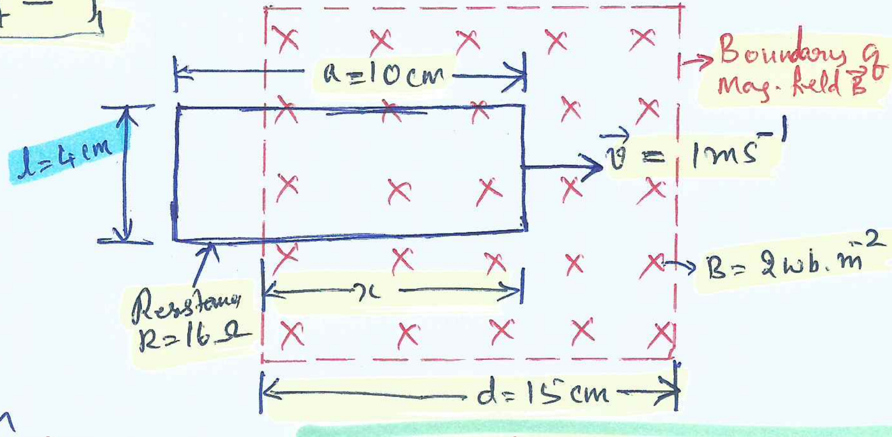
$$\left(\frac{dQ}{dt}\right) \times R = \frac{d\phi}{dt} \quad \text{or} \quad \boxed{dQ = \frac{d\phi_B}{R}} \rightarrow (7)$$

$$\text{or} \quad \boxed{\Delta Q = \frac{\Delta \phi_B}{R}}$$

$\therefore$ the amount of charge induced is equal to change in magnetic flux divided by the resistance of the circuit.

problem on Motional EMF

See figure for values:
plot ϕ_B, ϵ and power p as a function of coil position 'x'



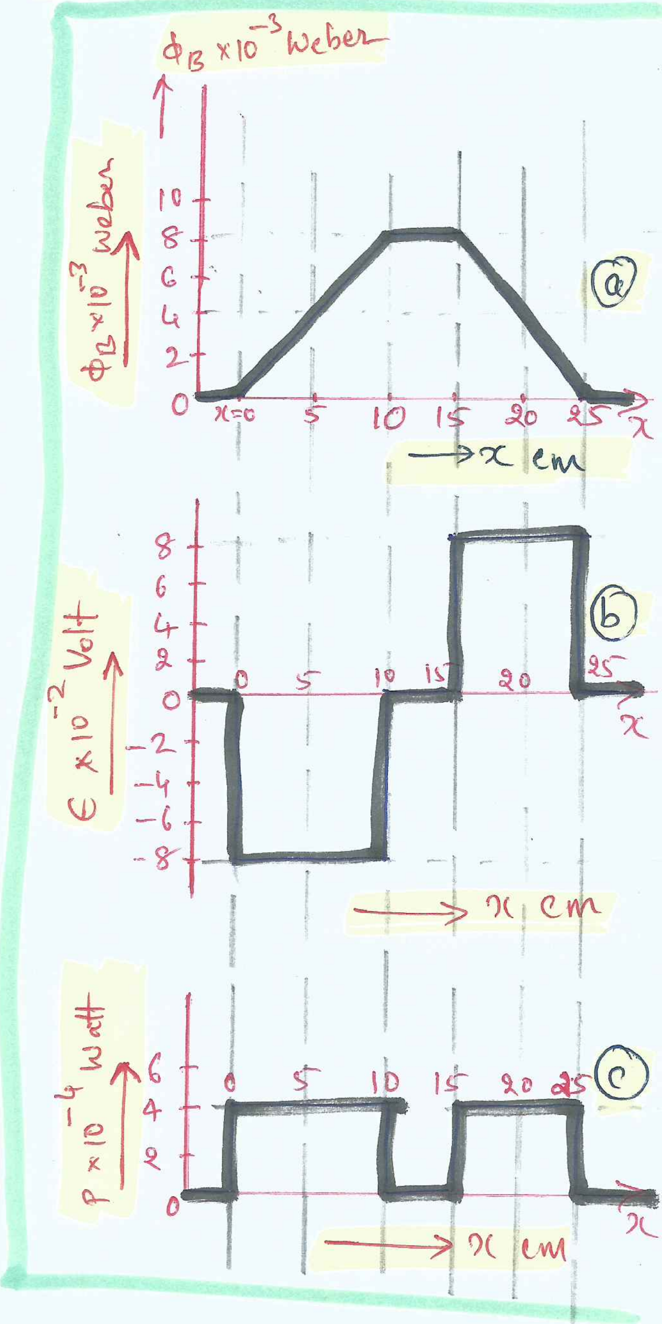
Ans: Given

$R = 16\ \Omega$
$a = 10\ cm = 0.1\ m$
$l = 4\ cm = 0.04\ m$
$B = 2\ wb.m^{-2}$ $(2T)$
$v = 1\ ms^{-1}$
$d = 15\ cm = 0.15\ m$

We know that

Flux $\boxed{\phi_B = Blx}$, where x is varying with time $\longrightarrow$ (1)

See Next page (34a)



P.T.O. contd....

 Contd from (34)

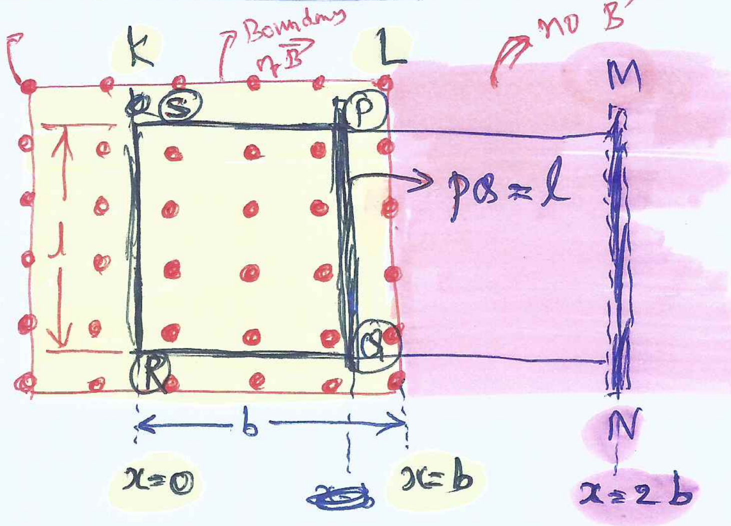
	$x \le 0$	$0 < x \le 10$	$10 < x \le 15$	$15 < x \le 25$	$x > 25$ cm.
Coil	coil out of field	coil entering field	coil is in the field	Coil is leaving the field	coil is out of mag. field
ϕ_B	$\phi_B = Blx$ $\phi_B = 0$	Flux ϕ_B linearly increases from 0 to 8×10^{-3} Wb $\phi_B = Bla$ $= 2\ Wb\,m^{-2} \times 0.04\,m \times 0.1\,m$ $\phi_B = 8\times10^{-3}$ Wb	ϕ_B remains constant at 8×10^{-3} Wb since area of coil in the field does not change. $\therefore \phi_B = 8\times10^{-3}$ Wb	ϕ_B starts decreasing linearly from 8×10^{-3} Wb to 0 Wb. eg. when $x = 20$ cm, still 5 cm of coil width is within the field, $x=$ $\phi_B = Bl \times 5\,cm$ $= 2\times0.04\times0.05$ $= 2\times4\times5\times10^{-4}$ $\boxed{\phi_B = 4\times10^{-3}\ Wb}$ See graph. eg: when $x = 25$ cm, coil is out of field. $\therefore \phi_B = 0$ Wb.	When $x > 25$ cm coil is fully out of magnetic field and hence $\phi_B = 0$ eg. $x = 25$ cm $= 0.25$ m $\phi_B = Bl \times 0\,m$ $\phi_B = 0$ weber.
Induced emf ϵ	$\epsilon = -\frac{d\phi_B}{dt}$ Since $\phi_B = 0$ $\epsilon = 0$	$\epsilon = -\frac{d\phi_B}{dt}$ $\epsilon = -\frac{d\phi_B}{dx}\times\frac{dx}{dt}$ $\epsilon = -\frac{d\phi_B}{dx}\times v$ where $v = 1\,ms^{-1}$ $\frac{d\phi_B}{dx}$ = slope of graph (a) $= \frac{8\times10^{-3}\,Wb}{0.1\,m}$ $= 8\times10^{-2}\,Wb\,m^{-1}$ $\epsilon = -v\times$ slope $= -1\,ms^{-1}\times 8\times10^{-2}\,Wb\,m^{-1}$ $\epsilon = -8\times10^{-2}$ Volt	ϕ_B is constant $\therefore \frac{d\phi_B}{dt} = 0$ $\epsilon = 0$ V	Since ϕ_B starts decreasing linearly from 8×10^{-3} Wb to 0 Wb slope $\frac{d\phi_B}{dx}$ is −ve. $\epsilon = -v\times$ slope $= -1\,ms^{-1}\times(-8\times10^{-2}\,Wb\,m^{-1})$ $\epsilon = +8\times10^{-2}$ Volt	Since $\phi_B = 0$ $\epsilon = 0$
Power $P = \frac{\epsilon^2}{R}$ (Given $R = 16\Omega$)	$P = \frac{\epsilon^2}{R}$ Since $\epsilon = 0$ $P = 0$	$P = \epsilon^2/R$ $\epsilon = -8\times10^{-2}$ V $P = \frac{4\ \cancel{64}\times10^{-4}}{\cancel{16}}$ $P = 4\times10^{-4}$ Watt	Since $\epsilon = 0$ $P = 0$	$P = \epsilon^2/R$ $\epsilon = 8\times10^{-2}$ V $P = \frac{4\ \cancel{64}\times10^{-4}}{\cancel{16}}$ $P = 4\times10^{-4}$ Watt	Since $\epsilon = 0$ $P = 0$

NCERT problem : Ex 6.8 (page 216, vol 1)

problem : Ref. figure. The arm PQ of the □lar conductor is moved from $x=0$, outwards. $\vec{B}$ is ⊥r to page and upwards. The boundary of $\vec{B}$ is only up to $x=b$; and is 0 for $x>b$. Overall Resistance = R. Consider the situation when PQ is pulled outwards from $x=0$ to $x=2b$ and is then moved back to $x=0$ with const. speed v. Obtain expression and sketch the graph for "flux", "induced emf", "the force necessary to pull the arm" and power dissipated as Joule heat.

Ans : $PQ = l$, Res. of PQ = R

- outward motion of PQ : $x=0$ to $x=2b$
- Inward —do— : $x=2b$ to $x=0$



Case (i) : PQ is at K

(a) Flux $\phi_B = 0$ since area SPQR = 0. $\therefore \phi_B = 0$

(b) Induced emf $\epsilon = -\frac{d\phi_B}{dt} = 0$ (since ϕ_B is const. and is zero)

(c) Force : Induced Current $I = \frac{Blv}{R} = 0$ since $\epsilon = 0$

$\therefore F = IlB = \frac{B^2l^2v}{R} = 0$

(d) power 'P' = $I^2R = 0$ since $I = 0$.

Case (ii) : PQ is moving from K to L with const. velocity v

(a) Flux : linearly increases from $x=0$ to $x=b$.
$\Rightarrow$ from 0 to Blb

(b) $\epsilon = -\frac{d\phi_B}{dt} = -\frac{d\phi_B}{dx}\cdot\frac{dx}{dt} = -v\frac{d\phi_B}{dx}$ ($\frac{d\phi_B}{dx}$ is slope of graph in case (i))

$$\therefore \boxed{\epsilon = -\frac{Bl\cancel{b}}{\cancel{b}} \times v = -Blv}$$

(c) Force : Induced $I = -\frac{Blv}{R}$

$F = IlB = -\frac{B^2l^2v}{R}$

(d) power = $I^2R = \frac{B^2l^2v^2}{R^2} \times \cancel{R} = +\frac{B^2l^2v^2}{R}$

P.T.O

 Contd from (35)

Case (iii) : PQ is moving from L to M

(a) Since $\vec{B} = 0$, ϕ_B = stays constant $= Blb$

(b) $\epsilon = -\frac{d\phi_B}{dt} = 0$ since $\phi_B = Blb$ = constant.

(c) $f = 0$ since ϵ and $I = 0$

(d) $P = I^2R = 0$ since $I = 0$

Case (iv) : PQ is moving from M to L

Same as case (iii)

Case (v) PQ is moving from L to K

(a) Flux : is linearly decreasing and is zero when PQ is at K.

(b) $\epsilon = -\frac{d\phi_B}{dt} = -\frac{d\phi_B}{dx} \cdot \frac{dx}{dt} = -v\frac{d\phi_B}{dx}$

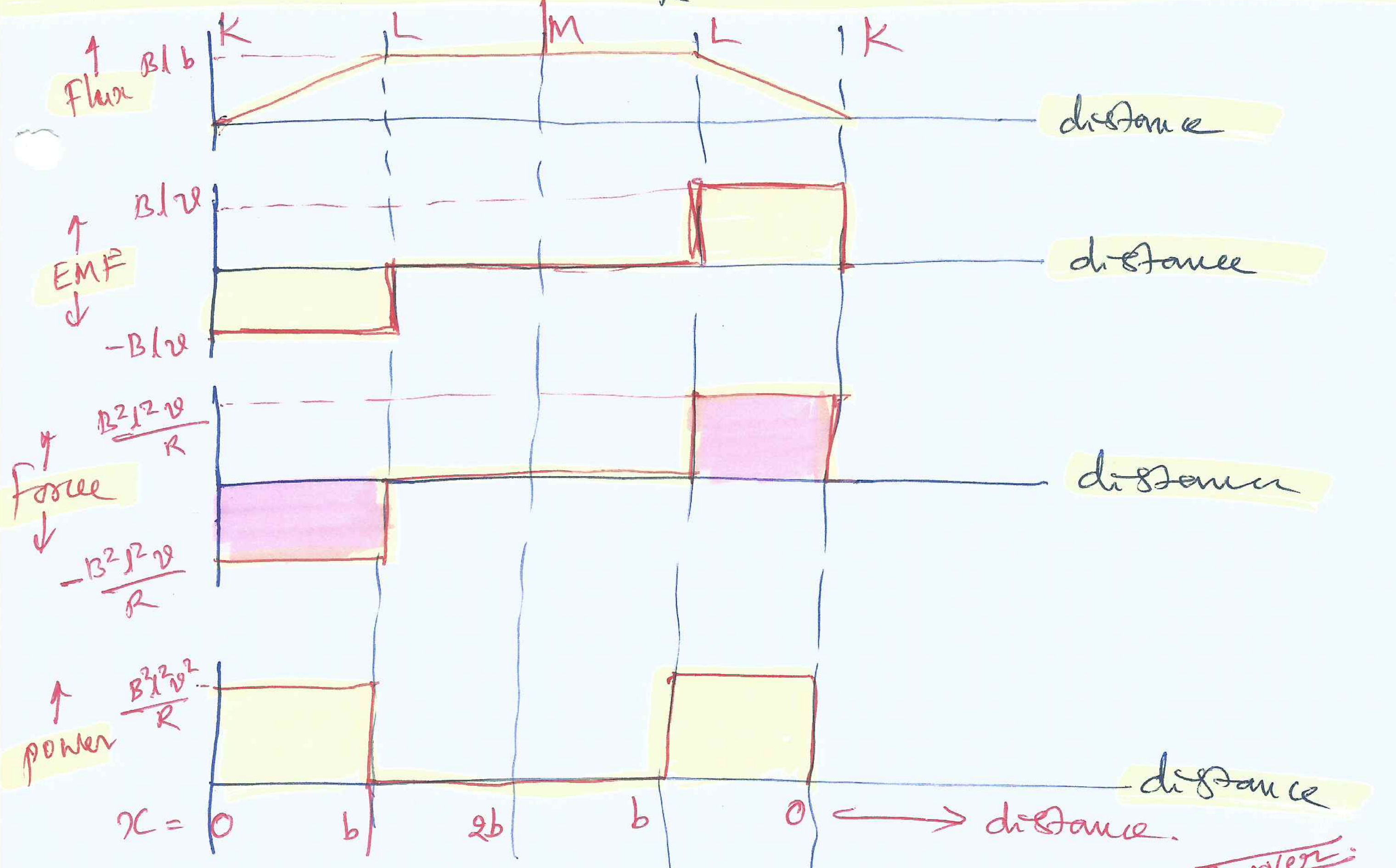
$\frac{d\phi_B}{dx}$ is the negative slope $= -\frac{Blb}{b}$

$\therefore \boxed{\epsilon = +\frac{Bl\cancel{b}}{\cancel{b}} \times v = +Blv}$

(c) force $= IlB$ $I = +\frac{Blv}{R}$

$\boxed{F = B^2l^2v/R}$

(d) $P = I^2R = \cancel{B^2l^2v} \frac{B^2l^2v^2}{R}$

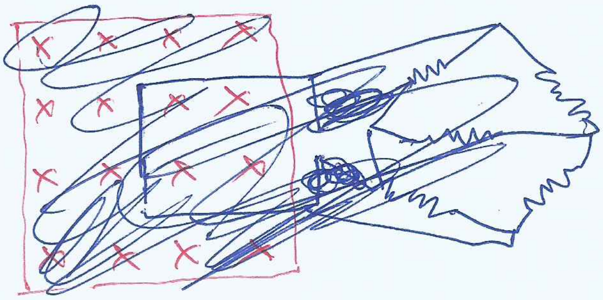


over

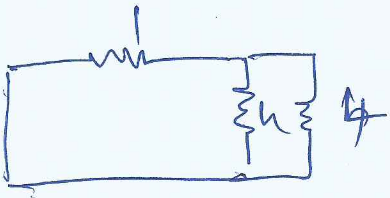
$\boxed{35\ b}$

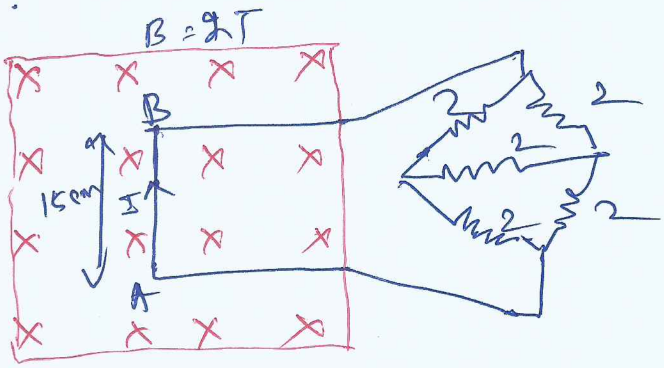
Problem on motional EMF

A metallic square ~~loop~~ wire-loop of side ~~1 cm~~ 15 cm and resistance 1 Ω is moved with a const velocity $\vec{v}$ in a uniform mag field of 2T, as shown in figure. The loop is connected to a network of resistors each of 2 Ω. Calculate ~~the speed of~~ the speed of the loop so as to have a steady current of 2 mA in the loop. Give the direction of current in the loop.



Effective resistance of the loop. Bridge circuit on the right is balanced and hence ignore that resistor.





$R = 3\ \Omega$ (loop Resistance)

$$I = \frac{B l v}{R}$$

$$v = \frac{I R}{B l} = \frac{\cancel{2} \times 10^{-3} \times 3}{\cancel{2} \times 0.15} = \frac{0.3}{15} = \frac{0.1}{5} = \frac{1}{50}\ m/s$$

$$\boxed{v = \frac{100}{50} = 2\ cm/s}$$

Direction: As per Fleming's Right Hand Rule, current I is from A to B ⟹ clockwise in the loop.

Eddy currents (Foucault currents)

- So far, we have seen that the mechanism of producing emf or induced current in a well-defined closed circuit (eg. circular loops etc..) due to change in magnetic flux linked with the closed circuit. We have ~~also~~ seen that such a mechanical energy converted to electrical energy and various usefull applications like AC, DC generators. The above explanation and applications (AC, DC generators) require a closed circuit path of conductors (or wires) to produce (induced) current.

- However, even when "bulk pieces of conductors" (where there is no closed circuit path for current to flow) are subjected to changing magnetic field (flux), induced currents are produced in them. However, the current flow patterns resemble swirling eddies in water. This effect was discovered by Foucault and these currents are called eddy currents. However, these eddy current can only produce heating effect unlike previous closed circuit circuits that could have various applications (eg. lighting etc.)

The only requirement for producing eddy currents must be that the object in question must be a conductor of electric current. **Eddy currents are only generated in conductors and not in the insulators.** The conductor could be just a piece of flat steel, aluminium plate or any other conductive object.

IMP **Defn of Eddy currents**

When a solid mass of conducting material is placed in a changing magnetic field, or when the conducting material is moved in a uniform and time-independent magnetic field, causing a change in the magnetic flux linked with the conducting material, the induced currents are set up throughout the volume of the conducting material. These currents are known as "eddy currents". The direction of circulation of these currents is such as to oppose either the motion of the conducting material Or the change in magnetic flux (Lenz's law).

R.T.O.

 Contd from (36)

※ properties of Eddy currents ※

(1) Any electrically conductive material will ~~conduct~~ produce an induced current if it is placed in a changing magnetic field (flux) OR if the conductive material is moved in a uniform magnetic field causing change in flux

(2) Eddy currents are circular induced currents.

(3) Eddy currents generate their own magnetic fields.

※ Experimental demonstration of Eddy currents :

Experiment (1) Stoppage or Dampening of oscillating Metallic plate

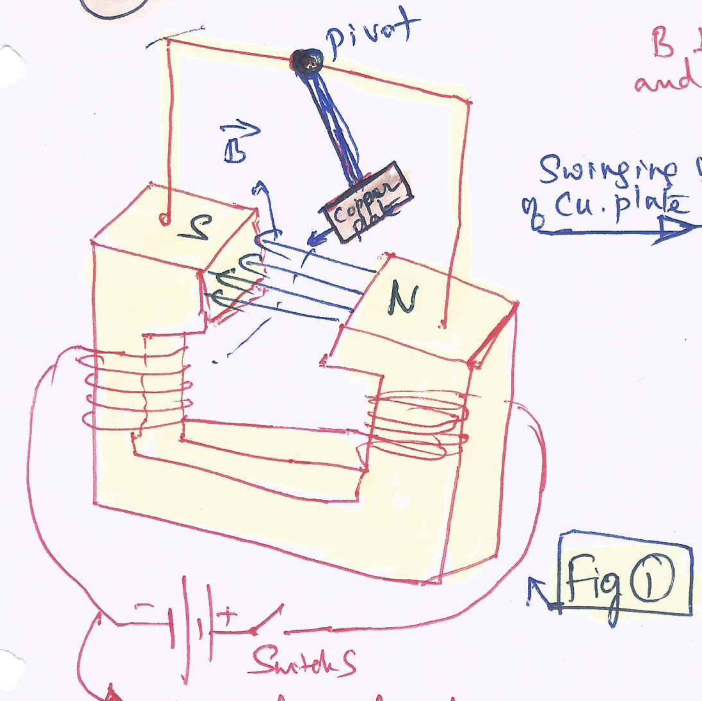


Fig (1)

Swinging of Cu. plate

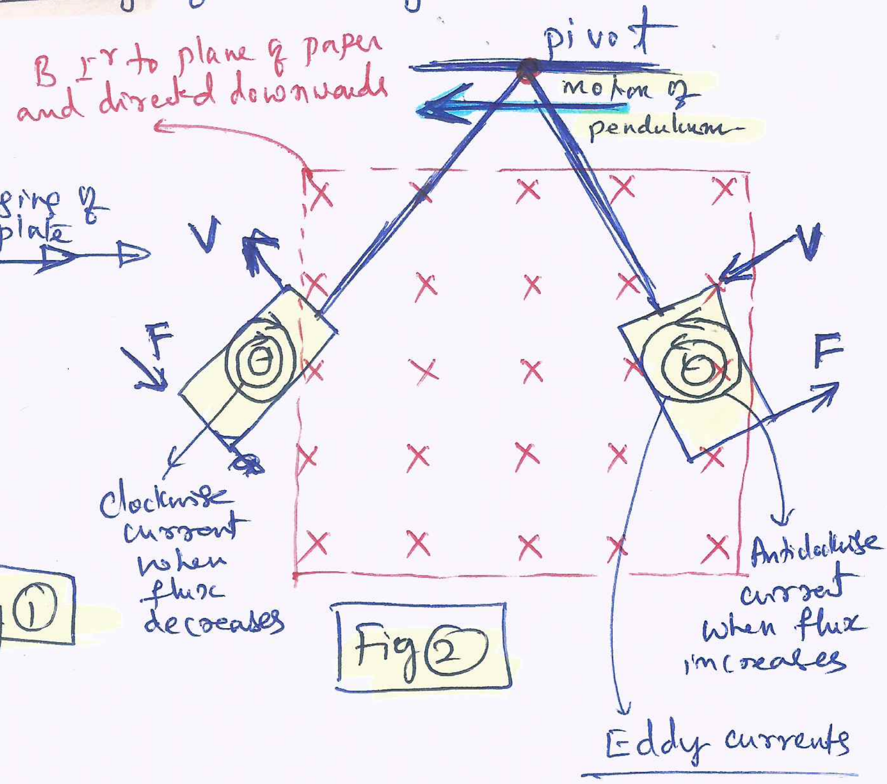


Fig (2)

As shown, a copper plate is suspended to make a pendulum so that it can swing back and forth between the poles of an electromagnet.

→ when Switch S is open (Fig①), Copper plate ~~swings~~ swings freely.

→ when S is closed, strong mag. field sets up in ferrite core and the copper motion is suddenly ~~suspended~~ arrested. As the plate enters the gap of the magnet, there is a current induced in the plate and the direction of current should be anticlockwise to oppose the increase in flux through the plate. The anticlockwise current (Lenz's law) sets up an additional magnetic field opposite to as shown in figure (means induced ~~cur~~ anti clockwise current produces an addition $\vec{B}_{induced}$ which is ⊥r to plane of paper ~~and directed upwar~~

 Contd from 37

and directed upwards to decrease the flux through the plate and the ~~pla~~ force developed pushes the plate in the opposite direction.

* if the sheet is perfect conductor (R=0), the magnitude of induced current would be so great that they would push the plate out again - it would bounce back.
* if the sheet has some resistance, the current is not so large and it brings the plate almost to a dead stop as it enters the field. Then, as the currents die down, the plate slowly settles to rest in the magnetic field.

* In a nutshell, when a metal plate is allowed to swing through a strong magnetic field, then in entering or leaving the field, the eddy currents are set-up in the plate which opposes the motion as shown in fig ② [page 37]. Since there is no closed circuit path for current, the energy is dissipated in the form of heat in the plate. plate gets heated up. The slowing down of the plate is called the "electromagnetic damping".

* The heating of ~~a~~ metal plate is undesirable in many applications. To reduce this heating, slots are made in the metal plate and these slots intercept the conducting paths and decreases the magnitude of the induced (eddy) currents, thereby reduces heating of the core.
  → By making holes or slots in the plate, it reduces the electromagnetic damping and the plate swings more freely.
  → [Note that magnetic moments of the eddy currents (which oppose motion) depend upon the area enclosed by the currents].
  → This fact is helpful in reducing eddy currents in the metallic cores of transformers, electric motors and other devices where a coil is wound over a metallic core.
  → Eddy currents are undesirable in these applications since they heat up to core and dissipate electrical energy in the form of heat. This causes laminated copper wires to lose laminations and gets shorted leading to failure of transformer function.
→ Therefore, the cores are not taken as a single solid block but are made of many thin lamination sheets tightly packed together and the laminations are separated by an insulating material like lacquer (Varnish). The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths.

This arrangement reduces the strength of the eddy currents. Since the dissipation of electrical energy into heat depends on the square of the strength of electric current, heat loss is substantially reduced.

P.T.O

→ In laminated core, the eddy currents are produced separately in each laminations instead of being produced in the entire volume of the solid core. Thus, the length of the current-path is appreciably increased, with consequent increase in resistance. Hence the currents and their heating effects are minimized.

→ IMP → The magnetization of the core, however, is not affected because the core is continuous in the direction of field.

IMP

Direction of Eddy currents

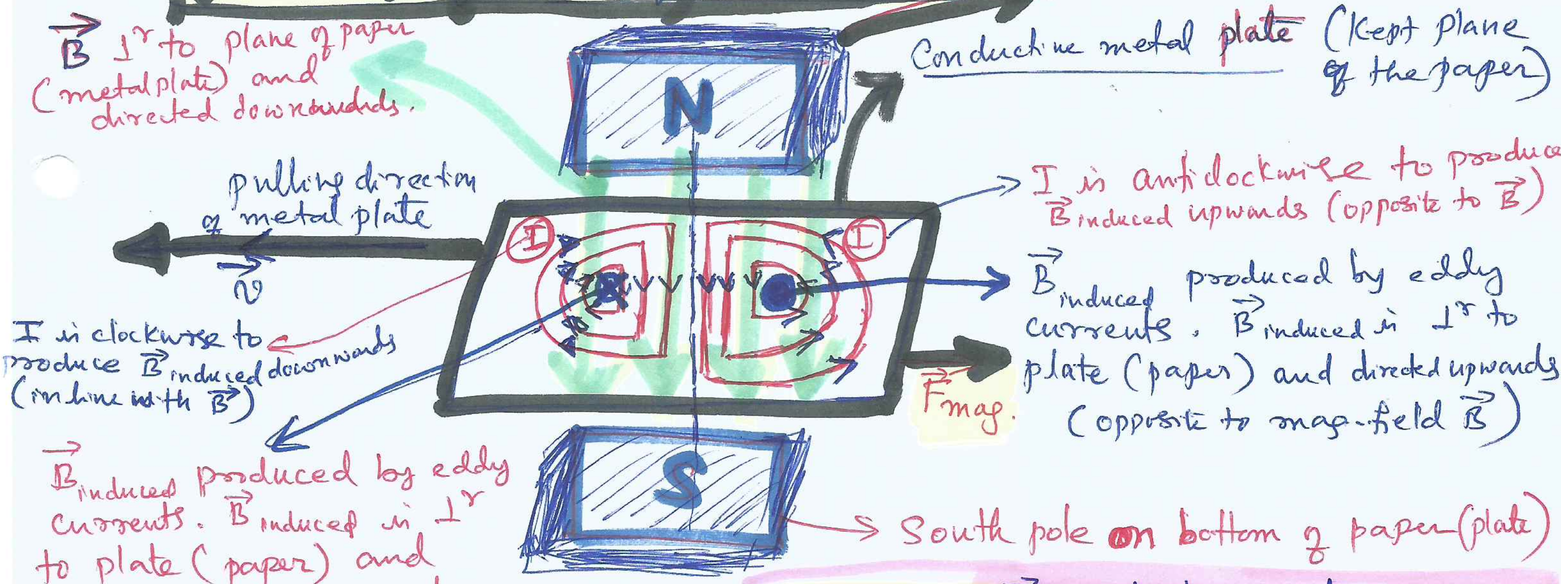


Fig (3) $\vec{B}$ → is due to magnet
$\vec{B}_{induced}$ → is due to eddy currents.

※ Eddy currents induced in a conductive metal plate as the plate is pulled LEFT under a magnet (N). Due to this magnet, $\vec{B}$ is $\perp^r$ to plane of plate (paper) and directed downwards through the plate.

※ As the plate is pulled left, the increasing flux at the right side of the plate induces "anti-clockwise" current, which by Lenz's law creates its own magnetic field ($\vec{B}_{induced}$ directed up), which opposes field due to magnet $\vec{B}$, producing a retarding force. Similarly, at the left side of the magnet, a clockwise current, which by Lenz's law creates its own mag. field ($\vec{B}_{induced}$ directed downwards → in line with $\vec{B}$ due to magnet) also producing a retarding force.

IMP

(*) When metal plate is pulled LEFT of the magnetic field $\vec{B}$ (due to magnet)

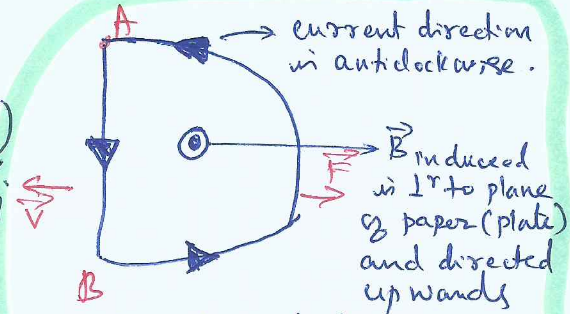
① Consider right side of eddy current loops → when plate is pulled LEFT, mag. flux (due to $\vec{B}$) increases in that part of the plate. As per Lenz's law, current should be in a direction to decrease mag. flux (opposing the ~~increase~~ increase in flux in that part of plate). So, $\vec{B}_{induced}$ should be in opposite direction to $\vec{B}$ and hence $\vec{B}_{induced}$ should be directed upwards ($\vec{B}$ is downwards). To get $\vec{B}_{induced}$ upwards, as per "Right Hand Thumb Rule",

P.T.O.

Contd from (39)

the induced (eddy) current direction should be "anti-clockwise"
(Thumb → mag. field, Curling of fingers → current direction.

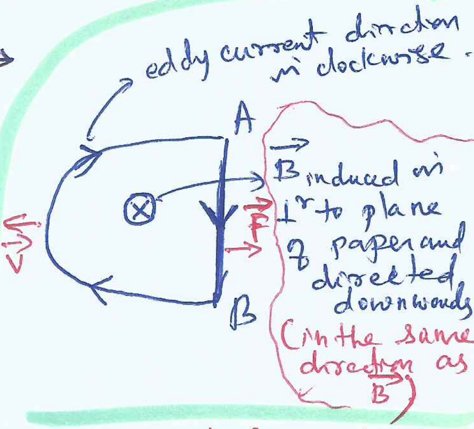
→ So, even for a solid plate moving in or out of a magnetic field produces current (called eddy currents) thus behaving like a current-carrying conductor. Hence the mechanism produces its ~~own~~ own magnetic field. In addition, it produces retarding force ~~and~~ opposite to the direction of movement of the metal plate. So, when the metal plate is pulled left, there will be a retarding force towards right opposing the metal plate being pulled left. From figure above, the retarding force towards right ~~can~~ can be verified by Fleming's LHR.



RHTR satisfies the direction of current and direction of mag. field due to eddy currents.

→ current is from A to B (middle finger)
→ there are two magnetic fields, $\vec{B}$ and $\vec{B}$ induced, the resultant is $\vec{B} - \vec{B}$ induced ; since $\vec{B}$ induced is smaller as compared to $\vec{B}$, the direction of mag. field is taken as ⊥r to plane of metal plate (paper) and directed downwards, hence fore ~~middle~~ finger is ~~in to the~~ into the page.
→ Thumb gives the retarding force direction.
⟹ So, applying this, the retarding force is towards "Right"

(2) Consider left side of eddy current loops → when the plate is pulled left, $\vec{B}$ decreases in that part of the plate. As per Lenz's law, current should be in a direction to increase mag. flux (opposing decrease in flux) in that part of the plate). So, B induced should be in the same direction as $\vec{B}$ to make up loss in flux linked with that part of the plate. So as per RHTR, direction of induced current must be clockwise in order to set-up its mag. field in the direction of $\vec{B}$.

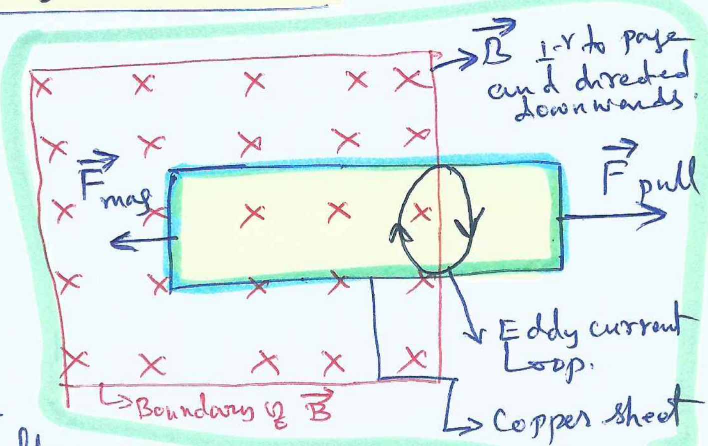


RHTR ~~satisf~~ verifies the direction of current and direction of mag. field due to eddy current
• Thumb → mag. field
• curling of fingers → current direction.

→ As far as retarding force is concerned, apply Fleming's LHR (current is from A to B, $\vec{B}$ downwards and thumb $\vec{F}$ points towards right)

Expt # 1 over

II Expt. to demonstrate Eddy Currents.



A copper plate is placed in a uniform and time-independent mag. field $\vec{B}$. If we withdraw the sheet, a repulsive force is experienced. The reason is that on withdrawing the sheet, its area A within the field decreases and so the mag. flux $\phi_B = (BA)$ linked with the sheet decreases. Because of this flux change, the current loops are induced in the sheet when the sheet is coming out of the mag. field.

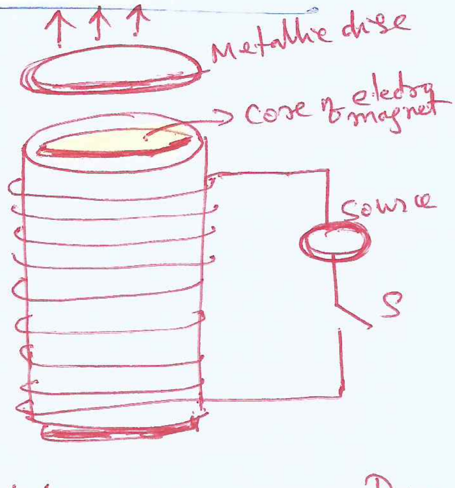
Since the sheet is pulled out, flux ~~decreases~~ linked with the ~~sheet~~ sheet decreases and as per Lenz's law, it opposes this decrease in flux $\Rightarrow$ flux has to increase ~~to give~~ give resistance to the pulling effect. So, in order to increase the flux, it has to be in the same direction as the original magnetic field $\Rightarrow \vec{B}_{induced}$ direction is same as $\vec{B}$ of magnetic field. So, $\vec{B}_{induced}$ is also ⊥r to the plane of copper sheet and directed downwards ⊗. In order to produce $\vec{B}_{induced}$ in downward direction, the eddy current must be in the clock-wise direction as per Right hand Thumb Rule.

→ As per Fleming's LHR, retarding force will be the left side opposing ~~the~~ pulling of sheet to the right side. There is some amount of ~~resistance~~ resisting force or friction in pulling out the copper sheet from the mag. field.

→ Similarly, when the copper sheets ~~is~~ is inserted into the mag. field, then eddy currents are ~~developed~~ generated in opposite direction (anticlockwise current ⊙ → that generates $\vec{B}_{induced}$ in upward direction to resist the motion of ~~the~~ the copper sheet entry into the magnetic field) Since, overall $\vec{B}$ is still in downward direction, there will be retarding force to the ~~left~~ right (Fleming's LHR)

III experimental demonstration of Eddy currents.

Jumping Disc :



An Aluminium disc is placed over the core of an electro-magnet. When the circuit is closed, the current flows in the circuit, the disc jumps up to a certain height.

When the current through the solenoid increases, the magnetic flux along the axis of the solenoid increases. Consequently, the magnetic flux linked with the disc also increases. Due to the change in the magnetic flux, induced current (eddy current) are produced in the disc and it is slightly magnetized.

If the upper face of electro-magnet acquires N polarity, then as per Lenz's law, the lower face of the disc will also acquire N-polarity. Due to the repulsion, the disc jumps up to a certain height.

3 experiments over.

I Disadvantages / Undesirable Effects of Eddy currents:

(i) The production of eddy currents in a metallic block leads to the loss of energy in the form of heat.

(ii) The heat produced due to eddy currents breaks the insulation used in the electrical machines or appliances.

(iii) Eddy currents may cause unwanted damping effect.

Eddy currents may be reduced but not completely eliminated.

→ **Minimization of losses due to eddy currents.**

In electrical devices like ~~transforming~~, dynamo, choke etc.. the metallic cores are used. Due to changing mag. flux, large eddy currents are produced in the core which heats up the core resulting in breaking the ~~insulation~~ enamel of the copper wire resulting in shorting of copper turns in a transformer thereby resulting in the failure of transformer.

To minimise losses due to eddy currents, the solid metallic core is replaced with a large number of thin sheets which are electrically insulated and are called laminations. These laminations are tightly riveted to form a core called "laminated core". The laminations in turn insulated by a material called lacquer (varnish). These sheets are arranged parallel to the magnetic flux. The insulations breaks the paths of eddy currents and keeps the eddy currents restricted to individual sheets. Hence, eddy currents produced in one sheet is not added to the current in other sheets ⇒ paths ~~are broken~~ of eddy currents are broken resulting in reducing eddy currents to a large extent. Hence, the loss ($H = I^2R$) of electrical energy due to eddy current is minimized.

II Advantages / Applications of eddy currents:

The eddy currents can be usefully employed in the following cases:

(a) **Induction Furance**: In induction furnaces, the metal to be heated is placed in a rapidly changing mag. field provided by a high-frequency AC. The eddy current set up in the metal produce so much heat that the metal melts. The process is used to extract metal from ore.

(b) **Electric power meters**: The shiny metal disc in the electric power meter (analogue type) rotates due to the eddy currents. Electric currents are induced in the disc by magnetic fields produced by sinusoidally varying currents in a coil.

(c) **Magnetic braking in trains**: Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train. As there are no mechanical linkages, the braking effect is smooth. OR

→ When a strong stationary magnetic field is suddenly applied to a rotating drum, the eddy currents set up in the drum exert a torque which stops the motion of the drum. This principle is used in stopping electric trains.

(d) **Electromagnetic damping**: Certain galvanometers have a fixed core made of non magnetic metallic material. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.

→ A similar damping device is used in some instruments such as balances, ammeters and voltmeters. A copper plate is attached to the moving system of the instrument such that it moves betn the poles of a permanent magnet when the system is oscillating.

(e) **Speedometer**: is a device used to measure the instantaneous speed of a vehicle. In a speedometer, a small magnet is attached to the ~~axle~~ axle of the wheel. This magnet is surrounded by an aluminium drum and it rotates according to the speed of the vehicle inside the aluminium drum. Due to the rotation of the magnet, the magnetic flux linked with the aluminium drum changes and hence eddy currents are produced in it. A pointer is attached to the drum is deflected in the direction of ~~the drum~~ the rotation of the drum. This speed is measured which corresponds to the deflection of the ~~meter~~ pointer.

P.T.O

 Contd from (43)

(f) Induction Cooking : Eddy currents produced in the copper pots are used for cooking food. This process of cooking is known as Induction Cooking.

(g) Induction Motor : Eddy currents are used to rotate the short circuited ~~motor~~ rotor of an inductor motor. Ceiling fans also have induction motors which run on single phase AC. Eddy currents in the rotor interact with the magnetic field of the rotor producing rotating force on the rotor.

(h) Diathermy : Eddy currents are used for the localized heating of tissues in human body. This type of treatment is called diathermy.

(i) Inductothermy : The process of producing heating effect due to eddy current for localised heating of tissues is called inductothermy. In this process, a coil of many turns is placed around the affected part of the ~~body~~ human body (without touching the body part). An AC of high frequency is passed through the coil. Note that inductothermy is far better treatment than the electrode diathermy.

(j) Dead-beat Galvanometer : When current is passed through the coil of a galvanometer, the coil and the pointing needle (i.e. the pointer attached to the coil) get deflected in the magnetic field (provided by the two ~~poles~~ pole pieces of a magnet). They keep on oscillating for a long time before they come to rest. In order to stop the motion of the coil of the galvanometer in a shorter interval of time, the coil is wound on a non-ferrous metallic frame (made of copper or aluminium). When a current is passed through the coil, the coil along with the metallic frame oscillates. Due to rotational motion, the magnetic flux linked with the metallic frame changes and hence eddy currents are produced in it. According to Lenz's law, these eddy currents oppose the motion of the frame and hence dampen the motion of the coil. Thus, the coil comes to rest in a shorter interval of time. This type of stopping or damping is called ~~Electromag~~ "Electromagnetic damping". ~~Such~~ Such a galvanometer is called dead-beat galvanometer.

End of Eddy currents

Chapter 6.9 NCERT Syllabus:

Inductance

An electric current is induced in coil #1 by flux change produced by another coil #2 in coil #1 vicinity.

↓

Mutual Inductance

An electric current is induced in coil #1 by flux change produced by the same coil #1

↓

Self-Inductance.

- In both cases, the flux through a coil is proportional to the current I $\Rightarrow \phi_B \propto I$
- Further, if the geometry of the coil does not vary with time, then
$$\frac{d\phi_B}{dt} \propto \frac{dI}{dt}$$
- For a closely wound coil of N turns, the same mag. flux is linked with all the turns. When the flux ϕ_B through the coil changes, each turn contributes to the induced emf. Therefore, the ~~term~~ term "flux linkage" is used which is equal to $N\phi_B$ for a closely wound coil and in such a case
$$N\phi_B \propto I$$
The constant of proportionality, in the relation, is called inductance.
- The inductance depends only on the geometry of the coil and its ~~m~~ intrinsic material properties.
→ this aspect is same as Capacitance, which for a ~~so~~ parallel plate capacitor depends on the plate area and plate separation (geometry) and dielectric constant K of the intervening medium (intrinsic material property).
- Inductance is a scalar quantity.

Units of Inductance :

SI unit of inductance is henry (H)

Since $L = \frac{\epsilon}{dI/dt}$ $\therefore$ $1 H = \frac{1 \text{ volt}}{1 \text{ Amp} \cdot \text{Sec}^{-1}} = 1 V A^{-1} s$

Also $L = \frac{\phi}{I}$ $\therefore$ $1 H = \frac{1 \text{ Weber}}{1 \text{ ampere}} = 1 \text{ wb } A^{-1}$

$\therefore$ $\boxed{1 H = 1 V A^{-1} s = 1 \text{ wb } A^{-1}}$ $\boxed{1 mH = 10^{-3} H}$; $\boxed{1 \mu H = 10^{-6} H}$

Dimensional formula of L

$\because$ emf $\epsilon = \frac{\text{work done}}{\text{Charge}}$

$$\epsilon = L \frac{dI}{dt}$$

$$L = \frac{\epsilon}{dI/dt} = \frac{W/dq}{dI/dt} = \frac{W}{\frac{dq}{dt} \times dI} = \frac{W}{I \, dI}$$

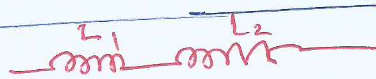
$$[L] = \frac{[ML^2T^{-2}]}{[A^2]} = [ML^2T^{-2}A^{-2}]$$

$$\boxed{[L] = [ML^2T^{-2}A^{-2}]}$$

Symbol of inductor in an electric circuit is represented as $\rightarrow$

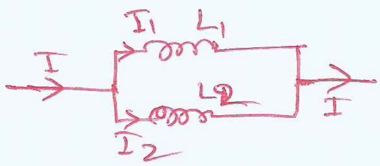
* A ideal inductor L has high value of self inductance and zero ohmic resistance.

* Coils in Series



$\Rightarrow$ $L = L_1 + L_2$

Coils in parallel

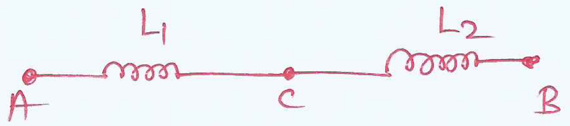


$\Rightarrow$ $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$

This is similar to resistors in series and parallel.

Grouping of coils :

(a) coils in Series :



L_1 and L_2 are connected in series in a circuit with a source.

let ϵ_1 and ϵ_2 be p.d. (or emf) across 2 coils, then the total p.d betn A and B

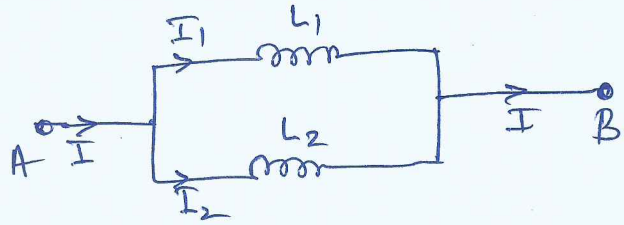
$$\epsilon = \epsilon_1 + \epsilon_2$$

Since rate of change of current is same on both coils,

$$-L\frac{dI}{dt} = -L_1\frac{dI}{dt} - L_2\frac{dI}{dt}$$

$$\therefore \boxed{L = L_1 + L_2} \quad (\text{Net inductance} = L)$$

(b) Coils in parallel :



L_1 and L_2 are connected in parallel in a circuit with a source.

$$I = I_1 + I_2 \longrightarrow (1)$$

Diff. (1) on both sides,

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} \longrightarrow (2)$$

Since, emf across both coils $= \epsilon$, then net inductance L can be calculated as follows.

$$\epsilon = -L\frac{dI}{dt} \quad \text{or} \quad \frac{dI}{dt} = -\frac{\epsilon}{L}$$

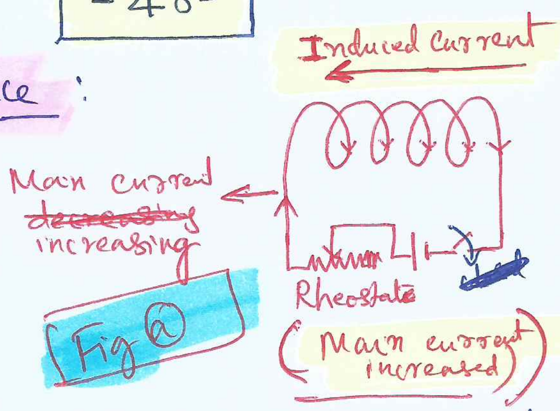
Also $\frac{dI_1}{dt} = -\frac{\epsilon}{L_1}$ and $\frac{dI_2}{dt} = -\frac{\epsilon}{L_2}$

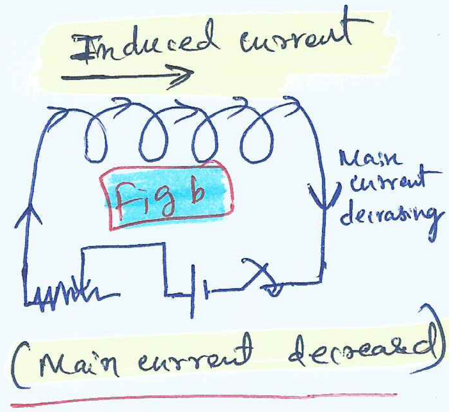
So, eqn (2) becomes

$$-\frac{\epsilon}{L} = -\frac{\epsilon}{L_1} - \frac{\epsilon}{L_2}$$

$$\therefore \boxed{\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}} \quad \text{or} \quad \boxed{L = \frac{L_1 L_2}{L_1 + L_2}}$$

Self- Inductance :





When a current flows through a coil, it produces a magnetic field around it and hence a magnetic flux is linked with the coil. If the current through the coil is changed, the flux linked with the coil also changes. Therefore, an induced emf is set up in the coil and an induced current flows through it, besides the main current. According to Lenz's law, the induced current always opposes the change in the main current. When the main current is increased, the induced current flows opposite to the main current and opposes the increase in the main current (Fig a). When the main current is decreased, then the induced current flows in the same direction as the main current and opposes the decrease in the main current (Fig b)

IMP ▷ The phenomenon of electromagnetic induction in which on changing the current in the coil, an opposing induced emf is set up in that very coil is called self-induction. The induced emf is also called "back emf".

or

IMP ▷ Self induction is the property of a coil by virtue of which it opposes the growth or decay of the current flowing through it.

Thus, when the current in a coil is ~~s~~witched-on, the self induction opposes the growth of the current, and when the current is switched off, the self induction opposes the decay of ~~the~~ the current. This is why the self-induction is also called as "inertia" of electricity. It is the electromagnetic analogue of "mass" in mechanics.

Contd....

 Contd. from (48).

"Self-inductance" or "coefficient of self-induction"

Let us consider a coil of N turns carrying a current I. Let Φ_B be the magnetic flux linked with each turn of the coil. The total number of flux-linkages $= N\Phi_B$. If no magnetic materials (iron, etc.) are present near the coil, then the number of flux-linkages with the coil is proportional to the current I,

i.e. $N\Phi_B \propto I$

$N\Phi_B = LI$ where L is a constant called the "coefficient of self-induction" or "self-inductance" of the coil.

$$\therefore \boxed{L = \frac{N\Phi_B}{I}} \longrightarrow (1)$$

If $I = 1$, then $L = N\Phi_B$ $\Rightarrow$ self-inductance of the coil is equal to the number of flux-linkages with the coil when unit current is flowing through the coil. $\Rightarrow$ L of any circuit is the total flux per unit current.

If, on changing the current through the coil, the back emf induced in the coil ϵ then, by Faraday's law

$$\epsilon = -\frac{d(N\Phi_B)}{dt}$$

but $N\Phi_B = LI$

$$\therefore \epsilon = -\frac{d}{dt}(LI) = -L\frac{dI}{dt}$$

$$\boxed{\epsilon = -L\frac{dI}{dt}} \longrightarrow (2)$$

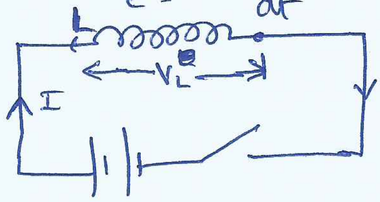
The $-$ve sign indicates that the induced emf ϵ is always in such a direction that it opposes the change of current in the coil.

$$\therefore L = \left|\frac{-\epsilon}{dI/dt}\right| \longrightarrow (3)$$

The eqn (3) states, that, the self-inductance of a circuit is the magnitude of self-induced emf per unit rate of change of current.

Derive an expression for Energy stored in an Inductor

$\epsilon = -L\frac{dI}{dt}$



Consider an inductor of inductance L connected across a battery (see figure). When current I flows through the inductor, the back emf ϵ is induced in it.

$$\boxed{\epsilon = -L\frac{dI}{dt}} \longrightarrow (1)$$

–Ve sign shows ϵ opposes the flow of main current I.

- Work needs to be done against this back emf ϵ in establishing the main current I through the inductor.
- To drive the current through the inductor against this ~~back emf~~, back emf ϵ, the external voltage is applied. Here the external voltage is emf of battery $= -\epsilon$

$\therefore$ From eqn (1), $\epsilon = L\frac{dI}{dt}$

- Let an infinitesimal charge dq be driven through the inductor, So, the work done by the external supply is given by,

$$dw = \epsilon\, dq = L\frac{dI}{dt}.dq = L\,dI\left(\frac{dq}{dt}\right) \quad \left(\text{Since } \frac{dq}{dt} = I\right)$$

$$dw = LI\,dI$$

$\therefore$ Total work done to maintain maximum value of current (say I_0) through the inductor is given by

$$\int dw = \int_0^{I_0} LI\,dI$$

$$\therefore \quad w = L\left[\frac{I^2}{2}\right]_0^{I_0} = \frac{1}{2}LI_0^2$$

$$\boxed{W = \frac{1}{2}LI_0^2} \longrightarrow (2)$$

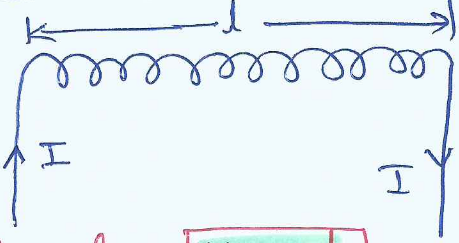
This expression reminds us of $\frac{1}{2}mv^2$ for the (mechanical) kinetic energy of a particle of mass m and shows L is analogous to m.
$\Rightarrow$ L is electrical inertia and opposes growth & decay of current in the circuit.

The work done in increasing the current flowing through the inductor is stored as energy (U) in the magnetic field of the inductor. Hence energy stored in the inductor is given by

$$\boxed{U = \frac{1}{2}LI_0^2}$$

The energy is stored in the mag. field of the inductor.

Derive a relation for self inductance of a Solenoid



Consider a long solenoid of length l, area of cross section A, and number of turns per unit length n.
Let I be the current through the solenoid and N is the total number of turns in the solenoid $\Rightarrow \boxed{N = nl}$

- The magnetic field inside this solenoid is uniform and given by

~~$B = \mu_0 n I$~~ $B = \mu_0 n I \longrightarrow (1)$

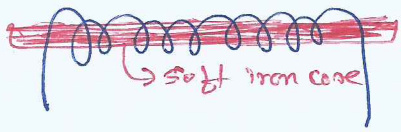
- Mag. flux linked with each turn of the solenoid $= B \times A$

$= \mu_0 n I A \longrightarrow (2)$

- Total mag. flux linked with the whole solenoid,
ϕ = mag. flux linked with each turn $\times$ number of turns in the solenoid.

$\phi = \mu_0 n I A \times nl = \mu_0 n^2 I A l \longrightarrow (3)$

Also $\phi = LI \longrightarrow (4)$



$\therefore L\cancel{I} = \mu_0 n^2 \cancel{I} A l$

$\boxed{L = \mu_0 n^2 A l} \longrightarrow (5)$

If we fill the inside of the solenoid with a material of relative permeability μ_r (for example soft iron, which has value of relative permeability)

$\therefore \boxed{L = \mu_0 \mu_r n^2 A l} \longrightarrow (6)$

So, self-inductance of the coil depends on its geometry and on the permeability of the medium.

Since $n = \frac{N}{l}$, eq^{n} (6) can be written as

$L = \mu_0 \mu_r \frac{N^2}{l^{\cancel{2}}} A \cancel{l}$ $\quad \therefore \boxed{L = \mu_0 \mu_r \frac{N^2}{l} A} \longrightarrow (7)$

- L increases with the increase in the number of turns of the coil and vice-versa.
- L increases if air core of the coil is replaced by an iron core.

P.T.O.

 Contd. from (51)

Magnetic energy stored in a solenoid is given by

$$U_m = \frac{1}{2} L I_0^2 \longrightarrow (8)$$

In case of solenoid,

$$B = \mu_0 n I_0 \quad \text{or} \quad \boxed{I_0 = \frac{B}{\mu_0 n}} \rightarrow (9)$$

$$\text{and} \quad L = \mu_0 \mu_r n^2 A l \longrightarrow (10)$$

$\therefore$ using (9) and (10) in (8), we get

$$U_m = \frac{1}{2} \cancel{\mu_0} \mu_r \cancel{n^2} A l \times \frac{B^2}{\mu_0^{\cancel{2}} \cancel{n^2}} = \frac{1}{2} \frac{B^2 A l \mu_r}{\mu_0}$$

$$\boxed{U_m = \frac{1}{2}\left(\frac{\mu_r}{\mu_0}\right) B^2 A l} \longrightarrow (11)$$

For air core, $\mu_r = 1$

Magnetic energy density is defined as the magnetic energy per ~~unit~~ unit volume. It is denoted by

$$\overline{U_m} = \frac{U_m}{V} = \frac{U_m}{Al}$$

$$\therefore \boxed{\overline{U_m} = \frac{B^2}{2\mu_0 \mu_r}} \quad \text{For air core} \rightarrow \boxed{\overline{U_m} = \frac{B^2}{2\mu_0}} \quad (12)$$

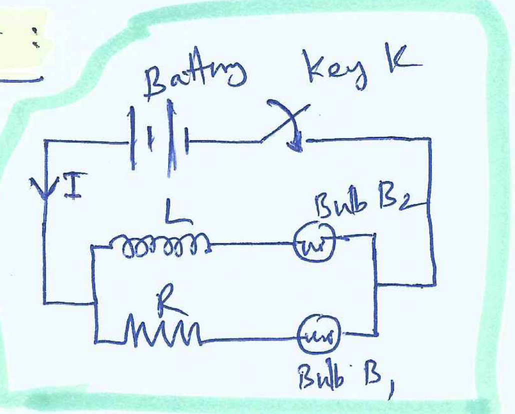
Comparison of mag. energy per unit volume and electrostatic energy per unit volume in a parallel plate capacitor

Electrostatic energy per unit volume in a parallel plate capacitor is

$$\boxed{U_E = \frac{1}{2} \epsilon_0 E^2} \longrightarrow (13)$$

- Comparing (12) and (13), in both cases the energy is directly proportional to square of the field strength.
- Magnetic energy per unit volume and electrostatic energy per unit volume are valid in any region of space in which magnetic field and electric field exist.

I Demonstration of Self Induction effect :

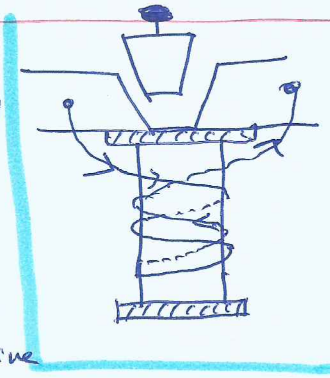


- When K is closed, B_1 glows immediately with brilliance and B_2 glows slowly.
- When current flows through the coil, an emf is induced ~~in~~ in it, which opposes the growth of main current I in the circuit.
- Hence glow of B_2 is slow. On the other hand, no induced emf is produced in resistor and hence B_1 receives maximum current as soon as K is closed and B_1 glows at once.
- When K is opened, lamp B_1 stops glowing at once but lamp B_2 takes time to stop glowing → In this case, the main current I begins to decrease through the coil, so again back emf is induced in it. This induced emf opposes the decay of current in the circuit of lamp B_2.

II Spark is produced in the electric switch when the light is switched off :

When light is switched off, the current begins to decrease rapidly. As a result of this, a large induced emf is set-up across the switch contacts which tries to maintain the current in the circuit. This emf is sufficient to break down the insulation of the air between switch contacts and hence spark is produced.

III In resistance-boxes the coils are doubly-wound :



There are number of coils of wire in a resistance-box. They have different resistances. To prepare these coils, a 'doubled' wire is wound on wooden cylinders. Thus the current at every place in the coil flows in two opposite directions. Hence the mag. flux linked with the coil remains almost zero. This is called 'non-inductive winding' of coils.

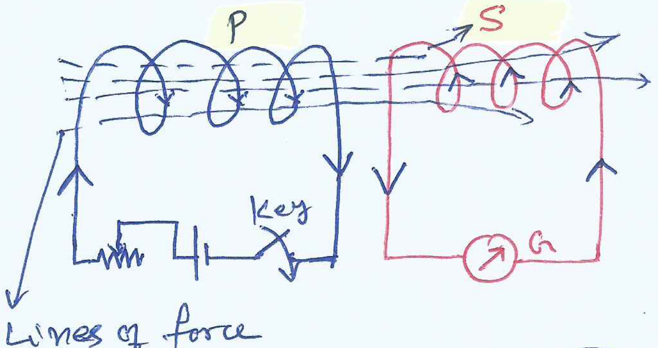
Its advantage is that the effect of self-inductance in the coil becomes negligible. So, when the resistance-box is connected in a circuit and the current in the circuit is increased or decreased, then no induced current is produced in the circuit.

Mutual Inductance

Mutual Inductance

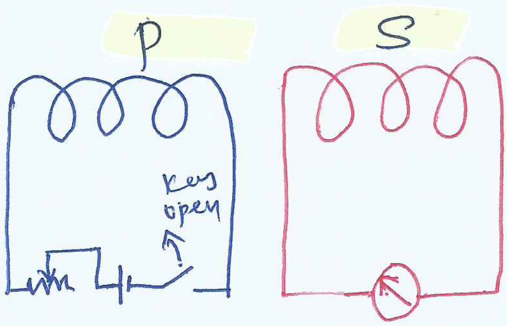
First, we will talk about what is meant by "Mutual Induction"

Mutual Induction: If we place two coils near each other and pass electric current in one of them, or change the current already passing through it, or stop the current, then an emf is induced in the second coil. This phenomenon of electromagnetic induction is called "mutual induction". The first coil is called the 'primary coil' and the second is called the 'secondary coil'.



[On pressing the key]

When key is pressed, a momentary deflection is produced in the galvanometer. The reason is that when key is pressed, a current passing ~~through~~ in the coil P and a mag. field is produced around it. Some lines of mag. flux pass through coil S also. Thus, on pressing the key, the number of flux-lines passing through the coil S increases from zero to a definite value. Due to this change in flux lines in S, an emf is induced in the coil S and a current flows through it. Hence a momentary deflection is produced in the galvanometer. According to Lenz's law, the induced current opposes the creation of flux-lines. Hence its direction is opposite to the direction of current flowing in the coil P.



[On Releasing the key]

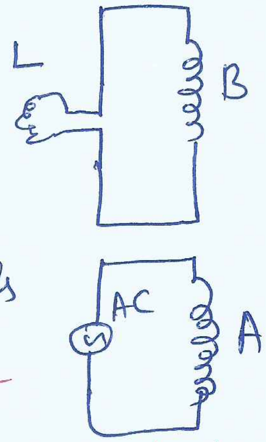
When key is released, again a momentary deflection is produced in the galvanometer, but now in the opposite direction. The reason is that on releasing the key, the current flowing in P stops and the flux-lines passing through S disappear, that is, the number of flux lines becomes zero. Hence, again an emf is induced in S and a current flows. According to Lenz's law, now this current opposes the disappearance of flux-lines that is, it tends to maintain the flux-lines. So, now the direction of current in S is same as was in P.

Similarly, when current in P is varied using Rheostat, there is change in flux-lines passing through S. Hence as long as the current in P is changing, an induced current in S flows.

Defn ↓

Mutual Induction is the property of two coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an opposing e.m.f.

Imp:: If some ferromagnetic material (iron) is placed in the primary and secondary coils, or wind both the coils on the same core, then the emf induced in the secondary coil is increased. The induction coil and the transformers are based on this principle.



Question: A coil B is connected to low voltage bulb L and placed near another coil A as shown in figure. Explain the following observations.

(a) Bulb lights
(b) Bulb gets dimmer if the coil B is moved upwards

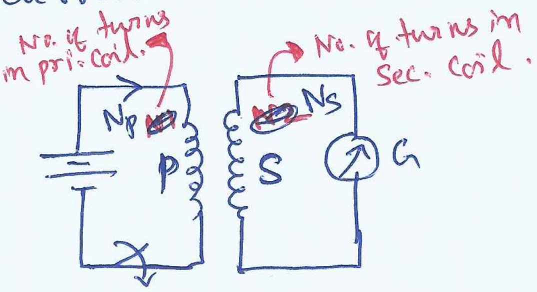
Ans

(a) The bulb lights due to mutual induction phenomenon.

(b) When B is moved upwards, flux linked with it decreases, induced emf decreases and also the current and hence the ~~bulb~~ bulb L connected to it gets dimmer.

"Mutual Inductance" or "Coefficient of Mutual Induction"

- We know that mutual induction is the phenomenon of inducing emf in a coil due to the change of current with time in a nearby coil.
- The coil which is connected to the source (AC or DC) is called primary coil.
- The coil in the nearby vicinity that gets an induced emf (and hence current) due to current change in primary coil is called secondary coil.



- Mag. flux linked with the secondary coil is directly proportional to the current flowing through the primary coil.

i.e. $N_S \phi_S \propto I_P$ or $\boxed{N_S \phi_S = M I_P} \rightarrow (1)$

where M is a constant of proportionality called "mutual inductance" or "coefficient of mutual induction".

- If $I_P = 1$, then $\boxed{M = N_S \phi_S}$, thus

"magnetic inductance" of two coils is equal to the number of magnetic flux-linkages in one coil when a unit current flows in the other coil.

- According to Faraday's law of electromagnetic induction

$$\epsilon_S = -\frac{d\phi_S}{dt}$$ if N_S is number of turns in secondary coil,

then as per Faraday's law $\boxed{\epsilon_S = -N_S \frac{d\phi_S}{dt}} \rightarrow (2)$

$$\epsilon_S = -\frac{d}{dt}[M I_P] = -M\frac{dI_P}{dt}$$

$$\boxed{\epsilon_S = -M\frac{dI_P}{dt}} \rightarrow (3)$$

where $\frac{dI_P}{dt}$ is rate of change of primary coil current.

M is the mutual inductance in henries.

−ve sign indicates that the direction of emf induced in secondary coil is always such that it opposes any change in current in the primary coil. From (3), we get $\boxed{M = -\frac{\epsilon_S}{dI_P/dt}} \rightarrow (4)$

If $\frac{dI_P}{dt} = 1$, then $M = \epsilon_S$ (numerically). Hence, the ~~mutual~~ mutual inductance of two coils is equal to the numerical value of induced emf in one coil when the rate of change of current in the other coil is unity.

problem: Calculate the ~~mutual induc~~ mutual inductance between two coils when a current of 2 A changes to 6 A in 2 s and induces an emf of 20 mV in the secondary coil.

Ans: emf induced in secondary coil is given by

$$\epsilon_s = -M\frac{dI_p}{dt} \Rightarrow |\epsilon_s| = M\frac{dI_p}{dt}$$

$$\text{or} \quad M = \frac{|\epsilon_s|}{dI_p/dt} = \frac{|\epsilon_s|}{\Delta I_p/\Delta t}$$

$$= \frac{20\,mV}{(6-2)\,A/2s} = \frac{20}{2}\,mV.A^{-1}s$$

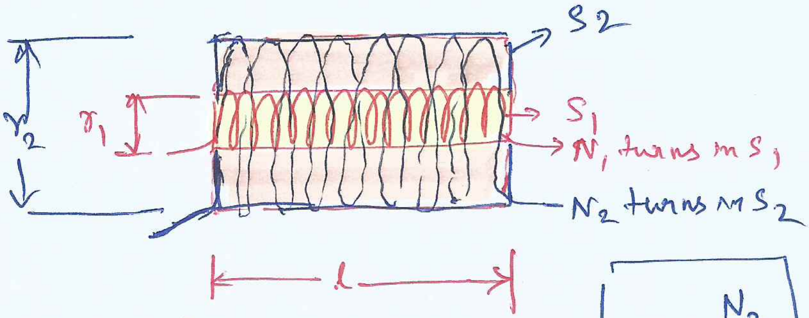
$$\boxed{M = 10\ mH}$$

IMP

Derivation:

"Mutual Inductance" of two long co-axial solenoids

Fig shows two long solenoids, each of length l.



Inner solenoid S_1:

Let r_1 be the radius and total no. of turns = N_1 $\therefore$ $\boxed{n_1 = N_1/l}$

Outer Solenoid S_2:

Let r_2 be the radius and total no of turns in $S_2 = N_2$ $\therefore$ $\boxed{n_2 = \frac{N_2}{l}}$

- When current I_2 is set-up in S_2, it in turn sets-up a magnetic flux ϕ_1 through S_1. The corresponding flux-linkages with S_1 is

$$\boxed{\phi_1 = M_{12} I_2} \longrightarrow (1)$$

where M_{12} is mutual inductance of S_1 w.r.t. S_2. We need to calculate M_{12} in this case.

- The magnetic field due to current I_2 in S_2 is

$$\boxed{B_2 = \mu_0 n_2 I_2} \longrightarrow (2)$$

$\therefore$ The mag. flux through S_1 is $\phi_1 = B_2 A_1 N_1$ using (2)

$$\therefore \phi_1 = (\mu_0 n_2 I_2)(\pi r_1^2)(n_1 l)$$

$$\boxed{\phi_1 = (\mu_0 n_1 n_2 \pi r_1^2 l) I_2} \longrightarrow (3)$$

From (2) and (3),

$$\boxed{M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l} \longrightarrow (4)$$

If a material of relative permeability μ_r fills the space inside S_2, we get

$$\boxed{M_{12} = \mu_0 \mu_r n_1 n_2 \pi r_1^2 l} \longrightarrow (5)$$

Note that we have neglected the "edge effects" and considered the magnetic field to be uniform throughout the length and width of the solenoid S_2. This approximation is valid for $l >> r_2$.

- Let us consider the reverse case. Current I_1 is passed through S_1 and the flux linkages with coil S_2 is

$$\boxed{\phi_2 = M_{21} I_1} \to (6)$$

Where M_{21} is mutual inductance of S_2 w.r.t. S_1.

IMP

- Since the solenoids are very long compared to their radii ($l >> r_2$), the flux due to the current I_1 in S_1 can be assumed to be confined solely inside S_1. Hence magnetic flux linked with S_2 is

$$\phi_2 = B_1 A_1 N_2 \quad \text{(Note } A_1 \text{ is used due to above explanation)}$$

$$\phi_2 = (\mu_0 n_1 I_1)(\pi r_1^2)(n_2 l) = (\mu_0 n_1 n_2 \pi r_1^2 l) I_1 \longrightarrow (7)$$

From (6) & (7) $\therefore$ $$\boxed{M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l} \longrightarrow (8)$$

P.T.O →

 Contd. from (57)

From eqns (4) and (8), we get $M_{12} = M_{21} = M$ (say)

$$\therefore \boxed{M = \mu_0 n_1 n_2 \pi r_1^2 l} \longrightarrow (9)$$

If a medium of relative permeability ~~μ_r~~ μ_r had been present, then

$$\boxed{M = \mu_0 \mu_r n_1 n_2 \pi r_1^2 l} \longrightarrow (10)$$

Thus, M of two coils depends on their geometry, their separation ~~and~~ and relative orientation.

Extra reading

[The equality $M_{12} = M_{21}$ is very useful in many situations ~~such as~~ where ~~de~~ calculating M_{21} would be very difficult. If inner solenoid is much ~~smaller~~ shorter (in length) than the outer solenoid, then we could still have calculated the flux linkage $N_1 \phi_1$ because the inner solenoid is effectively immersed in a uniform mag. field due to outer solenoid. In this case, the calculation of M_{12} would be easy.
→ However it would be ~~very~~ extremely difficult to calculate the flux linkage with the outer solenoid as the magnetic field due to inner solenoid would vary across the length as well as cross-section of the outer solenoid. Hence, calculation of M_{21} would be extremely difficult in this case. The equality $M_{12} = M_{21}$ is very useful in such situations.]

We can rewrite eqn (10) as

$$M = \mu_0 \mu_r \left(\frac{N_1}{l}\right)\left(\frac{N_2}{\cancel{l}}\right) \underbrace{\pi r_1^2 \cancel{\times l}}$$

$$\boxed{M = \mu_0 \mu_r N_1 N_2 A / l} \longrightarrow (11)$$

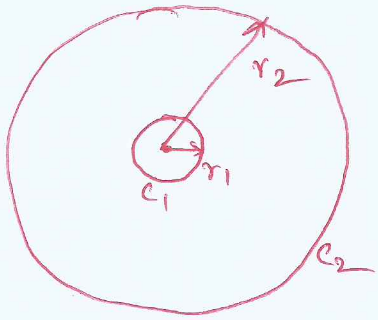
where N_1 = total number of turns in solenoid S_1
N_2 = ——— do ——— S_2
l = length of longer solenoid (when two solenoids happen to be of different lengths)

$A = \pi r_1^2$ = area of cross-section of inner solenoid (Note that we should not take area of ~~outer~~ cross section of outer solenoid because there is no magnetic field between the two solenoids)

——— o ——— o ———

Derivation (special case)

Two concentric olar coils, one of small radius r_1 and the other of large radius r_2, such that $r_1 << r_2$, are placed co-axially with centers coinciding. Obtain the mutual inductance of the arrangement.



Ans: Let a current I_2 flow through the outer olar coil.

∴ The magnetic field at the centre of the coil is

$$\boxed{B_2 = \frac{\mu_0 I_2}{2 r_2}} \longrightarrow (1)$$

Since the other co-axially placed coil has a very small radius, B_2 may be considered constant over its cross-sectional area. Hence flux associated with inner coil is

$$\phi_1 = \pi r_1^2 B_2 = \pi r_1^2 \left(\frac{\mu_0 I_2}{2 r_2}\right)$$

$$\boxed{\phi_1 = \left(\frac{\mu_0 \pi r_1^2}{2 r_2}\right) I_2 = M_{12} I_2} \longrightarrow (2)$$

$$\therefore M_{12} = M_{21} = \boxed{M = \frac{\mu_0 \pi r_1^2}{2 r_2}} \longrightarrow (3)$$

IMP (Note that we calculated M_{12} in eqn (2) from an approximate value of ϕ_1, assuming the magnetic field B_2 to be uniform over the area πr_1^2. However, we can accept this value because $r_1 << r_2$)

Sample problem: A 2 m long solenoid with dia = 4 cm and 2000 turns has a secondary of 1000 turns wound closely near its mid-point. Calculate mutual inductance b/w the two coils.

Ans: Given $l = 2m$, $r = \frac{4}{2}$ cm $= 2$cm $= 2 \times 10^{-2}$ m, $\boxed{\mu_0 = 4\pi \times 10^{-7}}$

$N_1 = 2000$, $N_2 = 1000$, $M = ?$

$$A = \pi r^2 = \pi (2 \times 10^{-2})^2 m^2 = 4\pi \times 10^{-4} m^2$$

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{\cancel{4}^{2}\pi \times 10^{-4} \times 2000 \times 1000 \times 4\pi \times 10^{-7}}{\cancel{2}}$$

$$= 16\pi^2 \times 10^{-5} = 158 \times 10^{-5} \text{ H}$$

$$\boxed{M = 1.58 \text{ mH}}$$

Information : (from NCERT book vol 1, page 223)

Consider the general case of currents flowing simultaneously in two nearby coils. The flux linked with one coil will be the sum of two fluxes which exist independently. Hence

$$N_1 \Phi_1 = M_{11} I_1 + M_{12} I_2$$

where M_{11} represents inductance due to the same coil.

Using Faraday's law,

$$\epsilon_1 = -M_{11}\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}$$

(Since M_{11} is the self-inductance and is written as L_1,

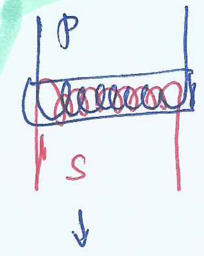
therefore
$$\boxed{\epsilon = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}}$$

Co-efficient of coupling : (K) between two coils having self inductances L_1 and L_2 and mutual inductance is given by

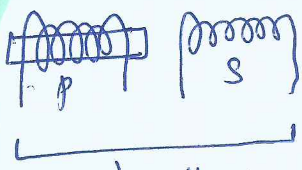
$$K = \frac{M}{\sqrt{L_1 L_2}}$$

[K value always lies b/w 0 and 1
K is always < 1]

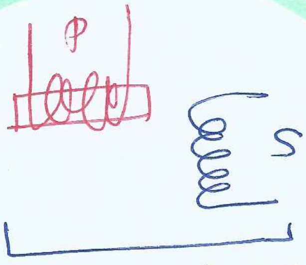
when $K = 1$, the coupling of two coils is tight and $M = \sqrt{L_1 L_2}$
when $K = 0$, the coupling of two coils is loose
when $K < 1$, $M < \sqrt{L_1 L_2}$



K is maximum



K is less than previous case

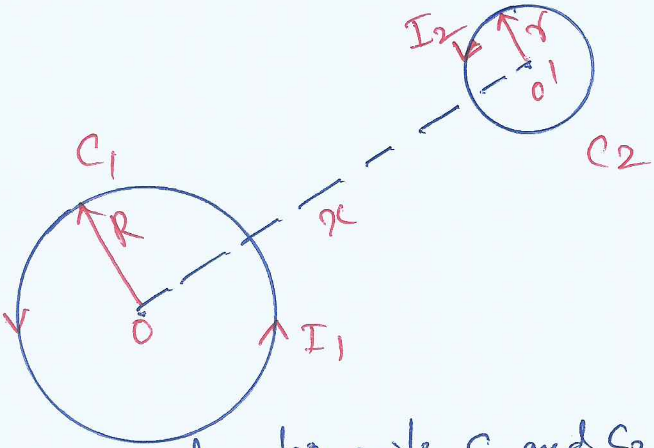


K is minimum

Imp :→ If two coils are so connected in series that their K = 1, then we can show that $L = L_1 + L_2 + 2M$ when current in two coils is in the same direction, and $L = L_1 + L_2 - 2M$, when the current in two coils is in opposite direction.

-60- (Not in NCERT Syllabus)

Derive a formula for mutual inductance of two co-axial coils



Consider two co-axial olar coils C_1 and C_2 of radii R and r and $(R >> r)$. The distance betn the centres of two coils $= x$ Let I_1 be the current in C_1 and I_2 is the current in C_2.

- Magnetic field at the centre O' of coil C_2 due to current carrying coil C_1,

$$B_1 = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi R^2 I_1}{(R^2+x^2)^{3/2}}$$

- $\therefore$ Magnetic flux linked with coil C_2 due to mag. field B_1 is

$$\phi_2 = B_1 A_2 = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi R^2 I_1}{(R^2+x^2)^{3/2}} \times \pi r^2$$

$$\phi_2 = \left\{\left(\frac{\mu_0}{4\pi}\right)\frac{2\pi^2 r^2 R^2}{(R^2+x^2)^{3/2}}\right\} I_1$$

(We know that $\phi_2 = M I_1$)

$\therefore$ Mutual inductance

$$\boxed{M = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi^2 R^2 r^2}{(R^2+x^2)^{3/2}}} \longrightarrow (1)$$

End of Mutual Inductance

problem

CET2018 problem: The dimension of the ratio of magnetic flux and permeability (μ) is

(a) $[M^0L^1T^0A^1]$ (b) $[M^0L^{-3}T^0A^{-1}]$ (c) $[M^0L^1T^1A^{-1}]$ (d) $[M^0L^2T^0A^1]$

dimension of $\frac{\phi}{\mu} = ?$

Dimension of ϕ

$\phi = BA\cos\theta$

Find dimension of B since $A = [L^2]$

$|\vec{F}| = qvB\sin\theta$

$$B = \frac{F}{qv} = \frac{[M\cancel{L}T^{-2}]}{[A\cancel{T}][\cancel{L}\cancel{T^{-1}}]}$$

$$B = [ML^0T^{-2}A^{-1}]$$

$$BA = [ML^0T^{-2}A^{-1}][L^2]$$

$$\phi = [ML^2T^{-2}A^{-1}] \rightarrow (1)$$

Dimension of μ

$B = \mu H$

$H = \frac{B_0}{\mu_0}$ $\mu_0 = 4\pi 10^{-7}\ TmA^{-1}$

$$H = \frac{\text{unit of } B_0}{\text{unit of } \mu_0} = \frac{\cancel{T}}{\cancel{T}mA^{-1}}$$

$$[H] = [L^{-1}A]$$

$$\therefore \mu = \frac{B}{H} = \frac{[ML^0T^{-2}A^{-1}]}{[L^{-1}A]}$$

$$\mu = [MLT^{-2}A^{-2}]$$

$$\therefore \frac{\phi}{\mu} = \frac{[\cancel{M}L^{\cancel{2}}\cancel{T^{-2}}A^{-1}]}{[\cancel{M}\cancel{L}\cancel{T^{-2}}A^{-2}]}$$

$$\frac{\phi}{\mu} = [M^0L^1T^0A^1]$$

Ans.. (a)