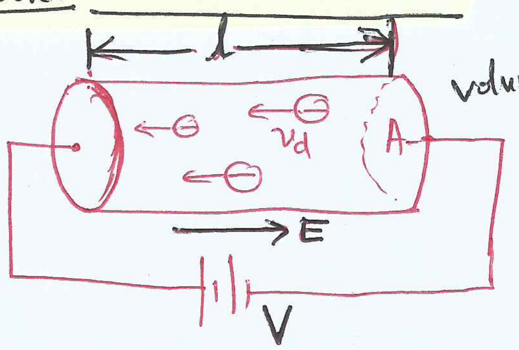
Relation between Drift velocity and Electric Current.



volume = Al

- Consider a conductor of length l and uniform cross-sectional area A.
- Voltage V is applied across conductor.
- The magnitude of electric field set up across the conductor is givean by

$$E = \frac{V}{l}$$

- Let n be the number of free electrons per unit volume
- $\therefore$ total no of free electrons in the conductor $= n \times$ Volume of Conductor $= n \times Al$
- If e is the magnitude of charge on each electron, then the total charge in the conductor

$$Q = (nAl)e \longrightarrow (1)$$

- time taken by the charge to cross the conductor length is

$$t = \frac{l}{v_d} \longrightarrow (2)$$, where v_d = drift velocity of electrons.

We know that $I = \frac{Q}{t}$ From (1) and (2)

$$I = (nA\cancel{l}e) \cdot \frac{v_d}{\cancel{l}} = nAv_d e$$

$$\therefore \boxed{I = neAv_d} \longrightarrow (3) \longrightarrow \text{IMP}$$

$\therefore I \propto v_d$ (since n, e, A are constants)

for electron $\mu_e = \frac{v_d}{E}$ or $v_d = \mu_e E$ [μ_e = mobility of electron]

$\rightarrow$ (3) becomes $\boxed{I = neA\mu_e E} \longrightarrow (4)$

Also, we know that $v_d = \frac{eE\tau}{m}$

$\rightarrow$ (3) becomes $\boxed{I = neA\left(\frac{eE\tau}{m}\right) = \frac{ne^2AE\tau}{m}} \longrightarrow (5)$

$\rightarrow$ (3) becomes $(J = \frac{I}{A})$ $\boxed{J = nev_d} \longrightarrow (6)$ $J \rightarrow$ current density

$\hookrightarrow$ (Relation betn "Current density" and "drift velocity" of free electrons.)

P.T.O.

...Contd

- Relation bet^n Current & Drift Velocity of free electrons } $I = neAv_d$

where I = current
n = no of free electrons per unit volume
$e = 1.6 \times 10^{-19}$ C (charge of one electron)
A = Cross-sectional area.
v_d = drift velocity of free electrons.

- Relation bet^n Current & mobility of electron } $I = neA(\mu_e E)$ $(v_d = \mu_e E)$

where μ_e = mobility of electron
E = magnitude of electric field $\left[E = \frac{V}{l}\right]$
$l \rightarrow$ length of conductor
$V \rightarrow$ Applied voltage.

- Relation betn Current density and drift velocity } $J = nev_d = ne(\mu_e E)$
$J = I/A$

Since J and v_d are vectors, we have
$\vec{J} = -ne\vec{v_d}$
The minus sign signifies that electron move in the direction $-\vec{J}$.

Relation bet^n J and Conductivity σ

$$J = \frac{I}{A} = \frac{1}{A} \cdot \frac{V}{R}$$

$$J = \frac{V}{AR} \cdot \frac{L}{L} = \left(\frac{L}{RA}\right)\left(\frac{V}{L}\right)$$

$$= \left(\frac{1}{\rho}\right)E = \frac{E}{\rho}$$

$$\boxed{J = \frac{E}{\rho}}$$

$$\boxed{J = \sigma E}$$

where V = voltage applied across conductor
R = Resistance of conductor
A = Cross-sectional area of conductor

$\rightarrow$ We know that $R = \rho\frac{L}{A}$
(where ρ = Resistivity of conductor
L = Length of conductor)

$\rightarrow$ We also know that $E = \frac{V}{l}$ or $\frac{V}{l} = E$

$\sigma = \frac{1}{\rho}$; σ = conductivity of the conductor.

P.T.O.

Q: A wire of length 6 m and area of cross-section $1\,mm^2$ carries a current of 2 A. If unit cubic meter of the material of a wire contains 10^{29} free electrons, find the average time taken by an electron to cross the length of the wire.

Formula to be used (Relation betw. I & v_d) $\boxed{I = neA\,v_d}$

$I = \frac{Q}{t}$

Ampere $= \frac{C}{t}$

$C = A \cdot t$

Given
$I = 2\,A$
$n = 10^{29}$ electrons$/m^3$
$e = 1.6 \times 10^{-19}\,C$
$A = 1\,mm^2 = 10^{-6}\,m^2$

$$\therefore v_d = \frac{I}{neA} = \frac{2\,A}{10^{29}\cancel{m^{-3}}\,1.6 \times 10^{-19}\,C \times (1 \times 10^{-6}\,m^2)}$$

$$= \frac{2\,\cancel{C}\,s^{-1} \times m^{\cancel{3}}}{10^{29} \times 1.6 \times 10^{-19}\,\cancel{C} \times 10^{-6}\,\cancel{m^2}}$$

$\frac{2}{1.6} = 1.25$

$$\boxed{v_d = 1.25 \times 10^{-4}\,m\,s^{-1}}$$

$$t = \frac{l}{v_d} = \frac{6\,\cancel{m}}{1.25 \times 10^{-4}\,\cancel{m}\,s^{-1}} = 4.8 \times 10^4\,s$$

$$= \underline{\underline{13.33\,hr}}$$

Ohm's Law

Ohm's law was discovered by Georg Simon Ohm in 1828.

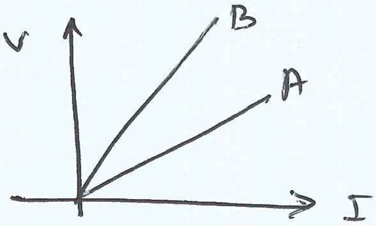
Law: According to Ohm's law, the current (I) flowing through ~~the~~ a conductor is directly proportional to the potential difference (V) across the conductor provided the physical conditions (like temp^{r}, pressure, strain etc.) of the conductor remain unchanged.

$$I \propto V \quad \text{or} \quad V \propto I$$

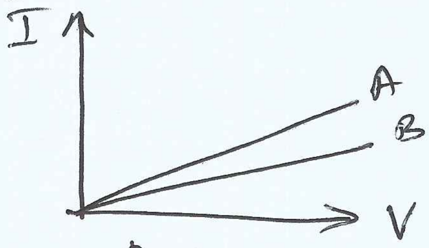
$$V = RI \quad \text{or} \quad \boxed{\frac{V}{I} = R}$$

where R is constant of proportionality and is known as "Resistance" of the conductor.

- Value of R depends on the nature of the material of the conductor, dimensions and temp^{r} of the conductor. It does not depend on the value of V and I.

- 

$R_B > R_A$



$R_B > R_A$.

- Resistance of a conductor is the opposition offered to the flow of electric charge in the conductor.

$$R = \frac{V}{I} \qquad \text{SI unit} = \text{Ohm } (\Omega)$$

$$1\,\Omega = \frac{1\,V}{1\,A} \Rightarrow 1\,\Omega = 1\,VA^{-1}$$

- Resistance of a conductor is said to be $1\,\Omega$, if current of 1 A flows through it, when the voltage 1 V is applied across it.

- Dimensional formula of R.

$$[R] = \frac{[V]}{[I]} = \frac{[\text{work}]}{[\text{charge}] \times [\text{current}]} = \frac{[\text{work}]}{[\text{current}][\text{time}] \times [\text{current}]}$$

$$[R] = \frac{[ML^2T^{-2}]}{[A^2T]} \qquad \therefore [R] = [ML^2T^{-3}A^{-2}]$$

Derivation of Ohm's law from first principles.

- Let v_d is the drift velocity of electrons thro' a section of the conductor of length l and cross-sectional area A. V is the p.d. across the section of the conductor and E is electric field.
- Relation bet^n I & v_d : $\boxed{I = neA\,v_d}$, where n is the number of electrons per unit volume in the conductor.

~~We know that v_d ... $I = neA$~~

- But magnitude of drift velocity (not ~~the~~ relation bet^n v_d and current I) is ....
  - Force experienced by a free electron in the conductor placed in the electric field is given by $\vec{F} = -e\vec{E}$
  - Accn. produced in the electron due to applied voltage (or Electric field)

$$\vec{a} = \frac{\vec{F}}{m} \quad \text{where } m = \text{mass of electron.}$$

$$\therefore \vec{a} = \frac{-e\vec{E}}{m}$$

we know that $v = u + at$ $\Rightarrow$ $\vec{v}_d = \vec{u} + \vec{a}\tau$ (τ → Relaxation time), $\vec{u} = 0$

$$\vec{v}_d = -\frac{e\vec{E}}{m}\tau$$

$$|\vec{v}_d| = \boxed{v_d = \frac{eE\tau}{m}} \qquad \text{Since } E = \frac{V}{l}$$

$$\boxed{v_d = \frac{eV\tau}{ml}}$$

$$\therefore I = neA\,v_d = neA\left(\frac{eV\tau}{ml}\right) = \left(\frac{ne^2A\tau}{ml}\right)V$$

$$\therefore \boxed{\frac{V}{I} = \frac{1}{\left(ne^2A\tau/ml\right)} = \frac{ml}{ne^2A\tau}} \qquad \left(\text{Since } \frac{V}{I} = R\right)$$

→ This is Ohm's law from first principle.

$$\therefore \boxed{R = \left(\frac{m}{ne^2\tau}\right)\frac{l}{A}} \longrightarrow (i)$$

$$\therefore \boxed{R = \rho\frac{l}{A}} \quad \text{where } \boxed{\rho = \frac{m}{ne^2\tau}}$$ is the resistivity of the material of the conductor

$$\therefore \boxed{\rho = \frac{m}{ne^2\tau}}$$ here m, e are constants

$$\therefore \rho \propto \frac{1}{n} \quad \text{and} \quad \rho \propto \frac{1}{\tau} \longrightarrow \text{IMP}$$

P.T.O.

.... Contd. $\rho \propto \frac{1}{n}$ and $\rho \propto \frac{1}{\tau}$

$\left(\rho \propto \frac{1}{n}\right)$ → Since the number of free electrons per unit volume (n) is different in different materials, therefore ρ depends on the nature of the material of the conductor.

$\boxed{\rho \propto \frac{1}{\tau}}$ → Relaxation time (τ) decreases with increase in temp^r of the conductor. Therefore, $\rho \propto \frac{1}{\tau}$ of a conductor increases with increase in temperature and vice-versa $\Rightarrow \rho \propto T$, where T is the temp^r of the conductor.

※ For ideal conductor, $\rho = 0$
※ For ideal Insulator, $\rho = \infty$
However, for practical conductors, insulators and semiconductors, ρ varies over a wide range.

Material	ρ (Ω-m) @ 20°C
Conductors	
Silver →	1.6×10^{-8}
copper →	1.8×10^{-8}
Al →	2.8×10^{-8}
Tungsten →	5.6×10^{-8}
Brass →	7.0×10^{-8}
iron →	11×10^{-8}
Mercury →	96×10^{-8}
Nichrome →	100×10^{-8}
Used to make electric heaters	Very small

Material	ρ (Ω-m) at 20°C	
Insulators		
Glass →	10^{12}	Very large
Wood →	10^{13}	
Amber →	5×10^{14}	
Mica →	10^{11} to 10^{15}	
Quartz →	7.5×10^{16}	
Semiconductor		
Carbon →	3.5×10^{-5}	in between Conductor and Insulator
Ge →	0.6	
Si →	2300	

→ Conductance $G = \frac{1}{R}$

Conductivity (σ) $= \frac{1}{\rho}$

These terms are used in many places for convenience.

Resistivity ρ Vs T (temp.) [Temp.^{re} dependence of Resistivity] ↳ Sec 3.8 NCERT

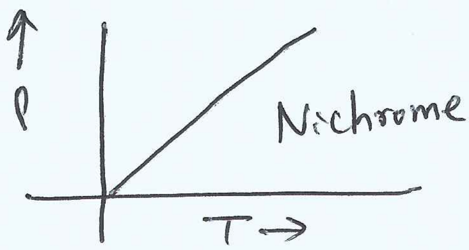
$$\rho = \rho_0 [1 + \bar{\alpha}(T - T_0)]$$, where

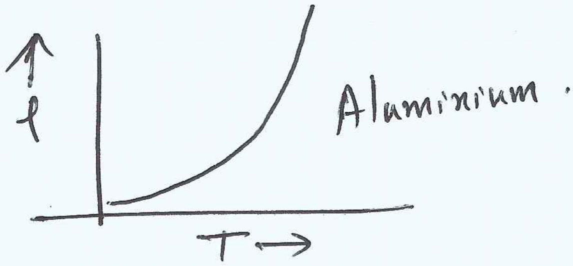
ρ_0 = resistivity at ~~absolute zero temp.~~ $T = 0°C$ or $273 K$.
ρ = " temp. T.
$\bar{\alpha}$ = temp. coefficient of resistivity.

$$\boxed{\bar{\alpha} = \frac{(\rho - \rho_0)}{\rho_0 (T - T_0)} = \frac{\Delta\rho}{\rho_0 \Delta T}}$$

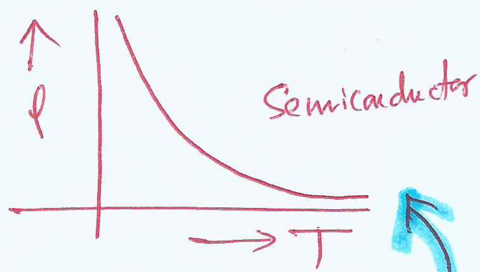
$\bar{\alpha}$ is defined as change in resistivity per unit original resistivity per unit change in temp.^{re}. $\bar{\alpha}$ is diffrent for different materials.

(1) **Metals/Conductors**: $\bar{\alpha}$ is POSITIVE $\therefore \rho$ increases with increase in temperature.
Some materials have linear relationship bet. ρ and T
Some other materials have non-linear relationships.





(2) **Semiconductors**: $\bar{\alpha}$ is Negative. (ex: Ge, Si)
At 0K, SCs behave as insulators, but at room temperature, they behave as conductors.



ρ of SC depends on 2 factors
(a) temp.^{re} variation
(b) Suitable impurities added to SC.
→ a small quantity of a specific impurity added to SC decrease their ρ to a large extent.

→ Also, with the increase in temp., ρ of SCs decreases more rapidly as shown

(3) **Insulators**: (wood, glass etc) → ρ decreases exponentially with increase in temp.^{re}. [ρ of insulators at Ab. zero is infinitely large]

The relation $\rho = \rho_0 e^{E/2KT}$ is valid for both insulators and SCs.
where E → energy gap bet. conduction band and valence band.
K → Boltzmann's Constant = (1.381×10^{-23}) J mole^{-1} K^{-1}

$E_{insulators} >> E_{SCs}$.

P.T.O.

Ohm's law (Relation ~~bet~~ among J, σ, E)

We know that $I = neA v_d \longrightarrow$ (1)

Also we know that $v_d = \dfrac{eE\tau}{m} \longrightarrow$ (2)

$$\therefore I = neA\left(\frac{eE\tau}{m}\right) = \frac{ne^2AE\tau}{m}$$

$$\therefore \boxed{\frac{I}{A} = J = \frac{ne^2E\tau}{m}} = \left(\frac{ne^2\tau}{m}\right)E$$

We know that $\rho = \dfrac{m}{ne^2\tau}$

$$\therefore \frac{I}{A} = J = \frac{E}{\rho}$$

but $\dfrac{1}{\rho} = \sigma$ (conductivity of the material)

$$\therefore \boxed{J = \sigma E} \longrightarrow (3)$$

which is microscopic form of Ohm's law.

From eqn (1), $v_d = \left(\dfrac{I}{A}\right) \times \dfrac{1}{ne} = J/ne$

Using (3), $\boxed{v_d = \dfrac{\sigma E}{ne}} \longrightarrow$ (4)

✗ v_d in terms of applied voltage across the conductor

We know that $v_d = \dfrac{I}{neA}$ (from eqn (1) above)

Since $I = \dfrac{V}{R}$

$$v_d = \frac{V}{neAR}$$

; but $R = \rho\dfrac{l}{A}$ $\therefore RA = \rho l$

$$\therefore \boxed{v_d = \frac{V}{ne\rho l}} \longrightarrow (5)$$

$\therefore v_d \propto$ applied voltage V

$v_d \propto \dfrac{1}{\text{length}}$ of the conductor.

* Classification of materials on the basis of Conductivity σ *

(1) Conductors (Materials have large σ): when a conductor is connected across a cell, large current flows thro' it.
eg: All metals → Silver, copper, Al, tungsten, human body)

(2) Insulators (Materials having very low or zero σ): → No current passes thro' insulator when connected to a cell.
eg: glass, wood, paper, rubber, pvc etc.

(3) Semiconductors (Materials having σ in betn conductors & insulators)
→ No current passes thro' a SC at 0 K
→ At room temp^{re}, current passes when cell is connected across SC.
eg: Si, Ge.

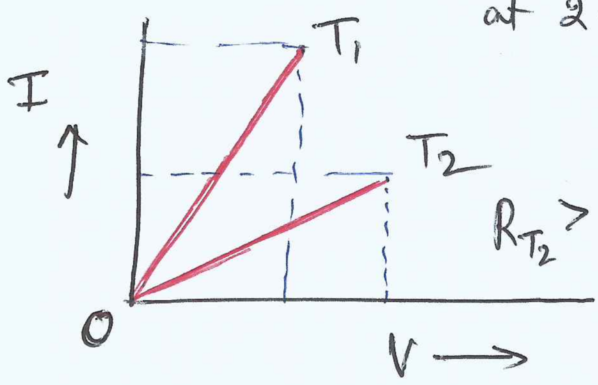
Effect of temp^{re} on the R of the conductor.

$R = R_0 (1 + \alpha \Delta\theta)$ where

R = Resistance of conductor at $\theta°C$ (theta)
R_0 = " " at $0°C$
α = temp^{re} coefficient of resistance
$\Delta\theta$ = Diff. in temp^{re}

$T_2 > T_1$?
$T_1 > T_2$?

Metallic wire at 2 temp^{res}



$R_{T_2} > R_{T_1}$

Since $R = \frac{V}{I}$

Slope of OT_2 < Slope of OT_1

{ Resistance of wire at two temp^{res} T_1 and T_2

→ $R_{T_2} > R_{T_1}$

⟹ Resistance of wire at temp^{re} T_2 > ~~te~~ R of wire at temp^{re} T_1.

→ Since R increases with temp^{re} (we know this for metals)

$\therefore T_2 > T_1$

α for copper $= 3.9 \times 10^{-3}\ °C^{-1}$; At what temp^{re}, R of Cu will get doubled?

$R = R_0 [1 + \alpha(\theta_2 - \theta_1)]$; $\theta_1 = 0°C$

$\therefore \cancel{2R_0} = \cancel{R_0}[1 + 3.9 \times 10^{-3}\ \theta_2]$

$\therefore \theta_2 = \frac{1}{3.9 \times 10^{-3}} = \underline{256°C}$

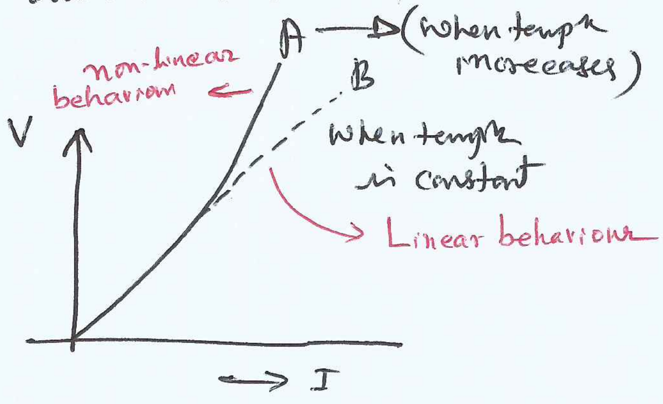
$\therefore$ R of Cu will get doubled at 256°C.

Limitations of Ohm's Law :

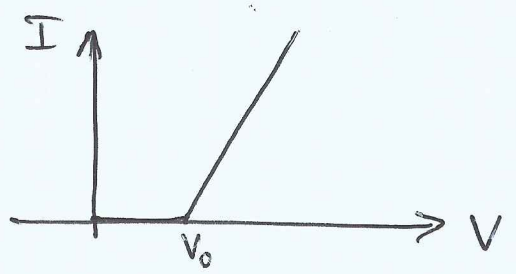
Ohm's law is not considered ~~as~~ to be a fundamental law. Although Ohm's law has been found valid over a large class of materials, there do exist materials and devices used in electric circuits where $V \propto I$ does not hold.

Following types are the deviations :

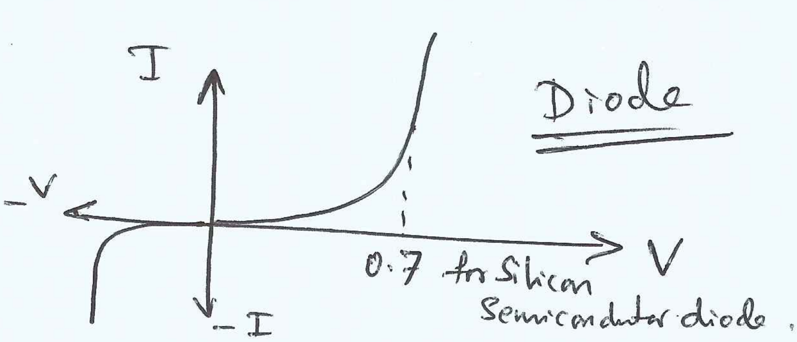
① V ceases to be $\propto$' to I



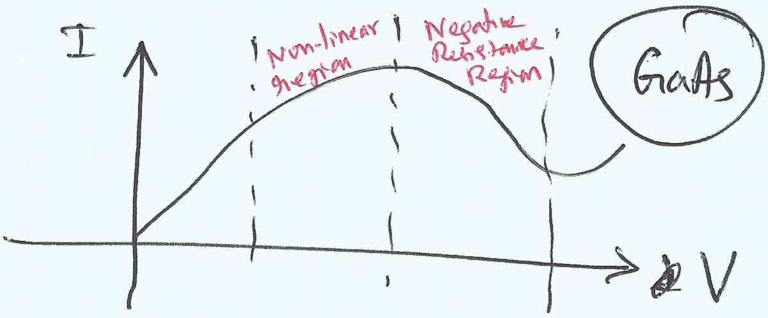
② In water voltameter, current begins to flow when $V = V_0$. when $V < V_0$, $I = 0$ → is against ohm's law



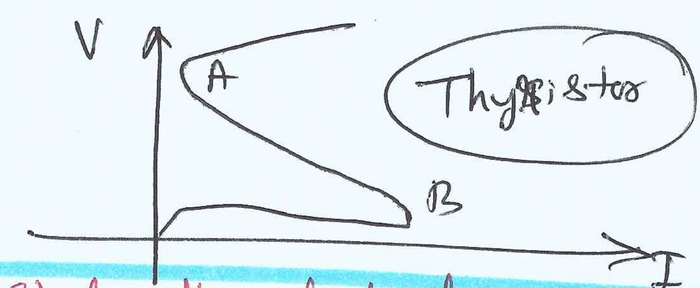
③ In diode, I starts after cut-in voltage (0.7 for Silicon). Reversing V changes the direction of I. Diode is a typical example of completely violating ohm's law.



④ In GaAs, the relation betw V and I is not unique, there is more than one value of V for same I. eg. GaAs.



⑤ Thyristor can be compared to a diode. Here, there is a decrease in current (AB) with increase in V.



In this chapter, we will study the electrical circuits that obey ohm's law → IMP

P.T.O

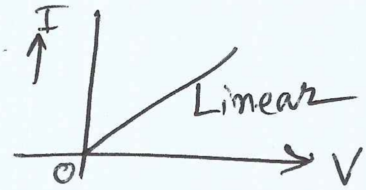
Defn :

① Ohmic materials or circuit elements.

→ The materials or circuit elements that strictly obey Ohm's law are known as Ohmic materials or Ohmic circuit elements.

eg. Metals are ohmic resistors.

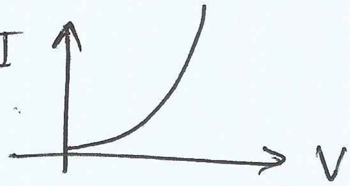
→ V vs I graph is linear passing thro' origin.



② Non-ohmic circuit elements. → which do not obey ohm's law partially or completely are known as non-ohmic circuit elements.

eg. Vaccum tubes, diodes, transistor, thermistor etc..

V vs I graph is not straight line.



In such cases, ratio of change of V to the change of I for a given voltage gives the dynamic resistance of the circuit element.

$$r = \frac{\Delta V}{\Delta I}$$

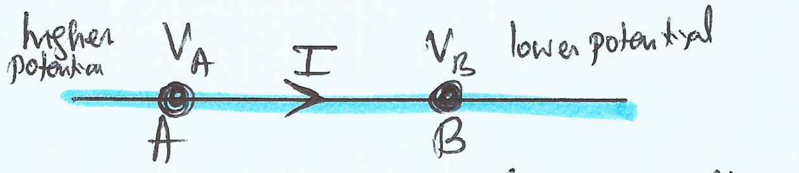
IMP

Electrical Energy and power (Sec 3.9 NCERT)

- Electric potential at A is V_A
- ———— ,, ———— B is V_B
- p.d. betn A and B = V_{AB} or V

$\therefore V = V_A - V_B > 0$

V = p.d. across AB



$V_A > V_B$ (Since I is flowing from A to B)

- In time Δt, an amt. of charge $\Delta Q = I\,\Delta t$ travels from A to B
- Potential Energy (PE) at A $= Q V_A$; PE at $B = Q V_B$

$\therefore$ Change in PE when charge moves from A to B (Decrease) =

$$\Delta U_{pot} = \text{final PE} - \text{initial PE} = \Delta Q (V_B - V_A)$$
$$= -\Delta Q \times V$$
$$\Delta U_{pot} = -I\,\Delta t \times V \longrightarrow (1)$$

- If charges moved without collisions thro' the conductor, their KE would also change so that total energy is conserved.

$\Longrightarrow \Delta K = -\Delta U_{pot} \implies \Delta K = I V \Delta t > 0$ (using (1)) (without collisions)

- However, in real situation, electrons have a steady drift velocity due to collisions with +ve ions and atoms. Due to this collision, the atoms start vibrating more vigorously $\Rightarrow$ conductor heats up. Thus, in actual conductor, an amt of energy dissipated as heat in the conductor during the time interval Δt is

$$\Delta W = I V \Delta t \qquad \text{power} = \frac{\text{work}}{\text{time}}$$

$\therefore$ Energy dissipated per unit time $\boxed{P = \frac{\Delta W}{\Delta t} = VI \text{ or } I^2R \text{ or } \frac{V^2}{R}}$

- P is the power loss due to R of conductor. It is this power that heats up the conductor (eg. incandescence filament ...)
- Where does this power come from $\rightarrow$ which ~~get~~ heats up conductor

As we know, we have applied "external source" to keep steady current thro' the conductor. It is clearly this source which must supply this power. So, some part of "source energy" is dissipated as unnecessary heat instead of some useful work done using this energy.

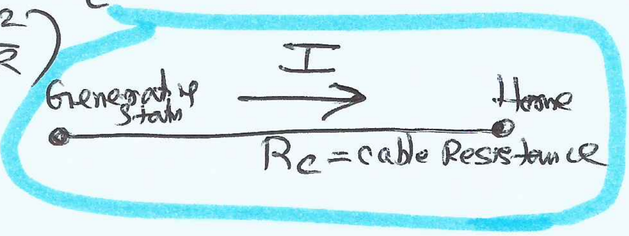
P.T.O.

.... Contd...

Application: $P = VI$ or $P = I^2R$ or $P = \frac{V^2}{R}$ → Wasted power.

↳ Imp. application in in power transmission from generating station to consumer premises. Long ~~wires~~ cables have been installed from power generating station to consumer premises.

- Requirement is to minimize this power loss (ohmic loss) which is due to resistance of the long cable..
- For home, we need 230 Vrms for operating ~~applian~~ electrical appliances. If we transmit this voltage (230 Vrms) from generating station to home over a long distance, there is nothing we will get at home since ~~the~~ power is completely lost in the R of the cable during transmission.
- What is the solution?
  → Consider a home device ~~have~~ with load resistance R_L to which a power P is to be delivered.
  → Transmission cable resistance R_C → which is responsible for dissipating power $\left(P = \frac{V^2}{R}\right)$
  → power dissipated (as heat) in R_C = $P_C = I^2 R_C$



$$\text{Since } I = \frac{P}{V}$$

$$\therefore \boxed{P_C = \frac{P^2 R_C}{V^2}} \longrightarrow (1)$$

Where P = power at generating station.

From (1), to drive a device (at home) with power P, the power wasted in the ~~cable~~ transmission cable is inversely proportional to V^2. Since, the distance betⁿ generating station and home is hundred of kms, R_C is very large. We don't have much control over R_C and hence to deliver power with less wastage, Voltage V has to be stepped up at the generating station. Since $P_C \propto \frac{1}{V^2}$, if V is very high P_C ~~is less~~ can be reduced and the cables carry current at enormous values of V ⟹ this is the reason for high voltage danger signs on Tx lines — which is a common sight from generating station to delivery premises.

- Generally, generating station generates ~~power~~ voltage is 400 kV
- This 400 kV will go thro' transmission line over long distance (suffers voltage drop due to R of cable) and finally at distribution system (eg. homes), it is stepped down in various stages to generate 230 Vrms