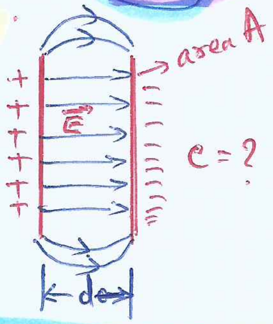
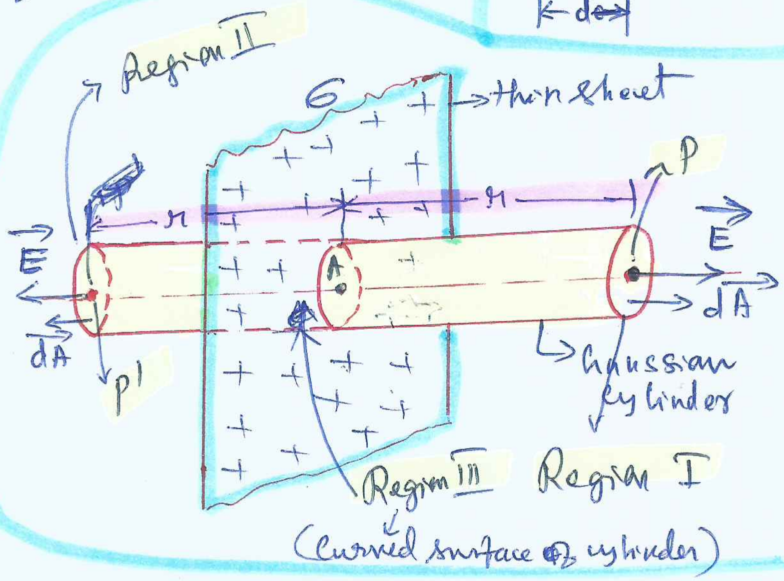
Capacitance of a parallel-plate capacitor

Let us ~~derivative~~ derive Capacitance of a parallel-plate capacitor in 4 steps.



Step 1
- Consider a thin infinite plane of non-conducting sheet having uniform charge density (i.e. charge per unit area) σ.
- To calculate electric field intensity $\vec{E}$ at a point dist r from sheet, draw a Gaussian surface in the form of a closed cylinder (pill box) of length 'r' on each side of thin sheet with end caps of area A.
- $\vec{E}$ is ⊥r to the sheet and hence ⊥r to two clar ends of gaussian cylinder.
- Charge enclosed by Gaussian Surface
  $$q = \sigma A \longrightarrow (1)$$
- According to Gauss' theorem
  $$\oint_S \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0} = \frac{\sigma A}{\epsilon_0} \longrightarrow (2)$$



- Gaussian Surface is divided into 3 parts I, II, III ⟹ 2 clar end caps and curved surface of cylinder.
- Eqn (2) becomes
  $$\oint_{I} \vec{E}\cdot d\vec{A} + \oint_{II} \vec{E}\cdot d\vec{A} + \oint_{III} \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0} \longrightarrow (3)$$
- For curved ~~Curved~~ surface of gaussian cylinder (Region III), $\vec{E}$ is ⊥r to $d\vec{A}$; ∴ $\vec{E}\cdot d\vec{s} = E\,ds\cos 90^\circ = 0$

  $$\therefore \oint_{III} \vec{E}\cdot d\vec{A} = 0$$
- Remaining regions are 2 end clar regions I and II, eqn (3) becomes
  $$\oint_{I} \vec{E}\cdot d\vec{A} + \oint_{II} \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$$
  ; Since angle betn $\vec{E}$ and $d\vec{A}$ is zero, $\cos 0 = 1$ ∴ $\vec{E}\cdot d\vec{A} = E\,dA\cos 0 = E\,dA$

  $$\therefore \oint_{I} E\,dA + \oint_{II} E\,dA = q/\epsilon_0$$
  ; Since $\vec{E}$ is constant at every point of the Gaussian Surface, so

  $$\therefore E\oint_{I} dA + E\oint_{II} dA = q/\epsilon_0$$

  $$\Rightarrow EA + EA = q/\epsilon_0 \Longrightarrow 2EA = \frac{q}{\epsilon_0} \Rightarrow \boxed{E = \frac{q}{2\epsilon_0 A}} \rightarrow (4)$$

  $$\therefore \boxed{E = \frac{q}{2\epsilon_0 A}} \text{ or } \boxed{E = \frac{\sigma}{2\epsilon_0}} \longrightarrow (4)$$

$\vec{E}$ at any point (as P or P') is directed away from the thin sheet having positive charge. If the thin sheet is having −ve charge, then $\vec{E}$ would be directed towards the sheet.

Step 2 Consider two parallel infinite thin sheets A and B having positive charges with uniform charge densities σ_1 an σ_2 respectively. Let $\sigma_1 > \sigma_2$; Let $\vec{E}$ be taken as +ve along +x-direction and $\vec{E}$ be taken as −ve along −x direction.

P.T.O

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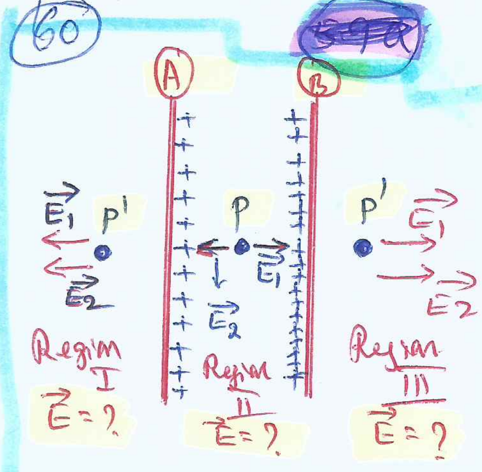


- We know that the magnitude of the electric field intensity on either side (Regions I, III) close to a plane sheet is given by

$$\boxed{E = \frac{q}{2\epsilon_0 A}} \quad \text{or} \quad \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

- ∴ Electric fields due to sheets A and B are given by

$$\vec{E}_1 = \frac{\sigma_1}{2\epsilon_0}\hat{i} \quad ; \text{ and } \quad \vec{E}_2 = \frac{\sigma_2}{2\epsilon_0}\hat{i}$$

- In region I (to the left side of sheet A), net electric field is given by $\vec{E}_I = -\vec{E}_1 - \vec{E}_2$

$$\therefore \vec{E}_I = -\frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)\hat{i} \longrightarrow (5)$$

- In region II (betn two sheets), the net electric field is given by

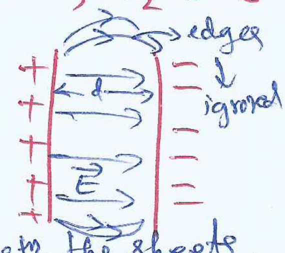
$$\vec{E}_{II} = \vec{E}_1 - \vec{E}_2 \implies \vec{E}_{II} = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)\hat{i} \longrightarrow (6)$$

- In region III (right side of sheet B), the net electric field is given by

$$E_{III} = \vec{E}_1 + \vec{E}_2 \implies E_{III} = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)\hat{i} \longrightarrow (7)$$

Step 3 Special case : Sheet A is +vely charged & Sheet B is equally −vely charged ⟹ $\sigma_1 = +\sigma$; $\sigma_2 = -\sigma$

then $\vec{E}_I = \vec{E}_{III} = 0$ ⟹ Field outside the sheets is zero everywhere



$$E_{II} = \frac{\sigma}{\epsilon_0}\hat{i} \implies \text{Magnitude } \boxed{E = \frac{\sigma}{\epsilon_0}} \longrightarrow (8)$$

※ The magnitude of E is free from the "position" of the point 'p' betn the sheets. Hence electric field betn sheets (except near edges → fringe effects of field is ignored assuming $d^2 << A$) is uniform everywhere, independent of the separation betn the sheets. It points from +ve to −ve sheet.

Thus, in this case, a uniform electric field exists only in the region betn the sheets and is zero elsewhere. In fact, this is the way of producing a uniform field in a limited region of space ; the only other requirement is that the sheets should be much larger in linear dimension than the separation betn the sheets (i.e $A >> d^2$)

Step 4 We know that $E = \frac{V}{d}$ where V = p.d. betn capacitor plates

$\therefore V = Ed \longrightarrow$ Substituting E in eqn (8) here,

$$V = \left(\frac{\sigma}{\epsilon_0}\right)d = \left(\frac{q}{\epsilon_0 A}\right)d \implies \frac{V}{q} = \frac{d}{\epsilon_0 A} \implies \frac{q}{V} = \frac{\epsilon_0 A}{d}$$

But $\frac{q}{V}$ = Capacitance C $\quad \therefore \boxed{C = \frac{\epsilon_0 A}{d}} \longrightarrow (9)$

P. T. O. →

From pre. page.

$$\boxed{C = \frac{\epsilon_0 A}{d}} \rightarrow (2)$$

Where $\epsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

So, capacitance value C depends only on the geometry of the parallel-plate capacitor.

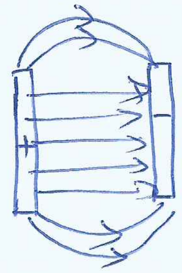
$\Rightarrow$ • C is directly $\propto$ to the area of the plates
• C is inversely $\propto$ to the distance betw plates

$\Rightarrow$ It is clear that C is determined geometrically because it depends upon the dimensions of the capacitor.

$\Rightarrow$ The C of parallel plate capacitor is independent of the charge on the capacitor or p.d. betw the plates of the capacitor.

$\Rightarrow$ value of $\epsilon_0 = 8.854 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

$\Rightarrow$ charged capacitors have attractive electrostatic forces betw the plates.



$\Rightarrow$ $\vec{E}$ betw plates is shown in figure
• $\vec{E}$ is uniform in a region betw the plates
• $\vec{E}$ is non-uniform at the edges. It is known as 'fringing' or 'edge effect'.

Non-uniformity of $\vec{E}$ at edges is negligible if we consider $d^2 << A$

$\rightarrow$ $C = \frac{\epsilon_0 A}{d}$ For $A = 1m^2$, $d = 1mm$

$$C = \frac{8.85 \times 10^{-12} \times 1}{10^{-3}} = 8.85\,nF$$

$\rightarrow$ For example if $C = 1F$ and $d = 1cm$

$$A = \frac{Cd}{\epsilon_0} = \frac{1F \times 10^{-2} m}{8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}} \simeq 1000\ km$$

$\Rightarrow$ 30 km $\times$ 30 km in length and breadth. $\Rightarrow$ This shows 1F is too big a unit in practice.

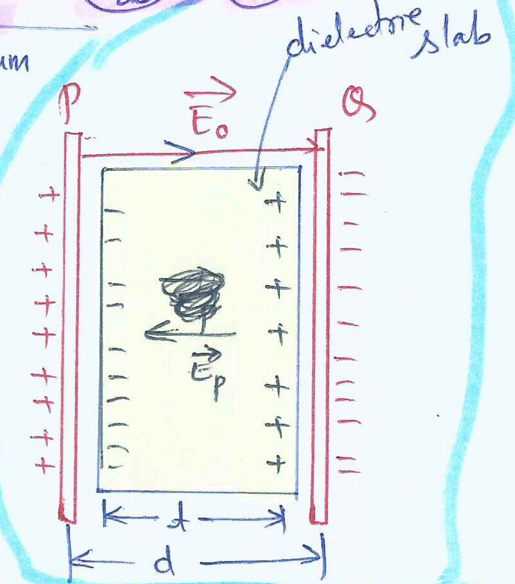
In Summary, for parallel-plate capacitor, where A is total ~~ean~~ area of each capacitor plate

$\rightarrow$ $\vec{E} = \frac{Q}{\epsilon_0 A}$

$\rightarrow$ $V = Ed = \frac{Qd}{\epsilon_0 A}$

$$\rightarrow \boxed{C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}}$$

-X- Effect of Dielectric on Capacitance (62) (62)



- Capacitance of a parallel plate capacitor having vacuum betn the plates of the capacitor is given by

$$C_0 = \frac{\epsilon_0 A}{d} \longrightarrow (1)$$

where A = ~~...~~ area of each plate, d = dist. betn plates

(Also $E_0 = \frac{Q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}$ where $\sigma = Q/A$

(Also $V_0 = \vec{E_0} d$)

- Introduce a dielectric slab of thickness 't' betn cap. plates.
- Due to polarization equal & opposite induced charges appear on the 2 faces of the dielectric slab. These induced charges give rise to induced electric field $\vec{E_p}$ which is in a direction opposite to the applied field $\vec{E_0}$.

Electric field in the dielectric is $E = E_0 - E_p$.

- Now, reduced value of E exists in the region of thickness 't'; E_0 exists in the region of thickness $(d-t)$
- ∴ p.d. betn cap. plates is given by

$$V = V_1 + V_2 = \underbrace{E_0}_{\text{applied electric field}}(d-t) + \underbrace{E}_{\text{Reduced Electric field}}t$$

Since $\frac{E_0}{E} = K \Rightarrow E = \frac{E_0}{K}$

$$\therefore V = E_0(d-t) + \frac{E_0}{K}t = E_0\left[(d-t) + \frac{t}{K}\right] \longrightarrow (2)$$

- We know that $E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0}$ ∴ eqn (2) becomes $\left(\because \sigma = \frac{q}{A}\right)$

$$V = \frac{q}{A\epsilon_0}\left[(d-t) + \frac{t}{K}\right] \longrightarrow (3)$$

- $C = \frac{q}{V} = \frac{\cancel{q}\, A\epsilon_0}{\cancel{q}\left[(d-t) + \frac{t}{K}\right]}$ or $C = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}}$

$$\Rightarrow C = \frac{\epsilon_0 A}{d\left[\left(1-\frac{t}{d}\right) + \frac{t}{dK}\right]}$$

Since $\frac{\epsilon_0 A}{d} = C_0$ from eqn (1)

$$\therefore \frac{C}{C_0} = \frac{1}{\left[\left(1-\frac{t}{d}\right) + \frac{t}{dK}\right]}$$

PTO →

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$$\frac{C}{C_0} = \frac{1}{\left[\left(1-\frac{t}{d}\right)+\frac{t}{dK}\right]} = \frac{1}{1-\frac{t}{d}\left(1-\frac{1}{K}\right)}$$

$$\therefore \boxed{C = \frac{C_0}{1-\frac{t}{d}\left[1-\frac{1}{K}\right]}} \longrightarrow (4)$$

It is clear that $C > C_0$

$\therefore$ C increases on the introduction of dielectric slab betn capacitor plates.

Special case:

(1) if $t = d$ (dielectric slab fits in completely betn cap. plates)

$$C = \frac{C_0}{\cancel{1}-\cancel{1}+\frac{1}{K}} = K C_0 \quad \therefore \boxed{\frac{C}{C_0} = K} \text{ or } C = C_0 K$$

Thus, Capacitance of a parallel plate capacitor increases by a factor K (dielectric constant).

(2) if $t = \frac{d}{2}$, $C = \frac{2KC_0}{K+1}$

(3) if $t = \frac{2d}{3}$, $C = \frac{3KC_0}{K+2}$

P.T.O →

when a dielectric of dielectric const k is inserted betn cap. plates and battery remains connected across the capacitor.	when a dielectric of dielectric const k is inserted betn cap. plates and the battery is disconnected.
① Charge on the plates increases ie $Q = KQ_0$	① Charge (Q) on the plates remains same $Q = Q_0$
② Capacitance of the capacitor increases $\Rightarrow C = KC_0$	② same $\rightarrow C = KC_0$
③ p.d. betn plates remains same ie $V = V_0$	③ p.d. betn plates decreases $V = \frac{Q}{C} = \frac{Q_0}{KC_0} = \frac{V_0}{K}$
④ Electric field decreases $\Rightarrow E = \frac{E_0}{K}$	④ Electric field betn plates decreases $\Rightarrow E = \frac{V}{d} = \frac{V_0}{Kd}$

problem : A parallel-plate capacitor with air betn plates has a capacitance = 10 pF. If the dist. betn plates is made double and the space is filled with ~~d~~ k = 10, find new capacitance.

$\rightarrow$ Step 1 :- C with air as medium

$$C_1 = \frac{\epsilon_0 A}{d}$$

$$10\,pF = \frac{\epsilon_0 A}{d}$$

$\rightarrow$ Step 2 C with K = 10

$$C_2 = k \cdot \frac{\epsilon_0 A}{d'}$$

Since $d' = 2d$; $C_2 = k\frac{\epsilon_0 A}{2d} \Rightarrow C_2 = \frac{k}{2}\left(\frac{\epsilon_0 A}{d}\right)$

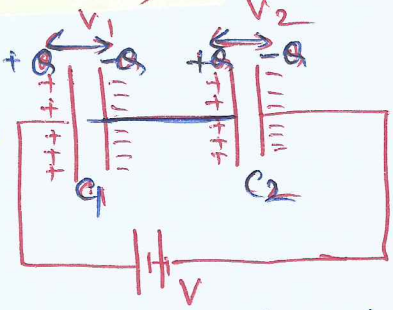
$$\therefore C_2 = \frac{\cancel{10}^{5}}{\cancel{2}}(10\,pF) = \underline{50\,pF}$$

* Combination of capacitors (Sec 2.14 NCERT)

- Series Connection
- parallel Connection

① Capacitors in Series : (Q is constant).

Fig. shows Series connection of 2 capacitors. The system is connected to a battery. Due to this



- The left plate of C_1 has charge $+Q$
- The right plate of C_2 has charge $-Q$

$\Rightarrow$ right plate of C_1 will have charge $-Q$ } If this is not so, the
left plate of C_2 —"— $+Q$ } net charge on each C would not be zero.

$\Rightarrow$ This arrangement results in an electric field in the conductor connecting C_1 and C_2. Charge would flow until net charge on both C_1 and C_2 is zero and there is no electric field in the conductor ~~between~~ connecting C_1 and C_2.

$\longrightarrow$ Thus, in series connection, charges on the 2 plates ($\pm Q$) are the same on each capacitor.

$\longrightarrow$ The total potential drop V across the combination $= V_1 + V_2$
where V_1 = vol. drop across C_1 and V_2 = vol. drop across C_2.

$\therefore V = V_1 + V_2 = \dfrac{Q}{C_1} + \dfrac{Q}{C_2} \longrightarrow ① \Rightarrow V = Q\left[\dfrac{1}{C_1} + \dfrac{1}{C_2}\right]$

$\Rightarrow \dfrac{V}{Q} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$; we can regard Series Combination as an effective C with charge Q and pd = V,

then effective capacitance $C = Q/V$

$$\therefore \Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Generalising $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots \dfrac{1}{C_n}$. for n capacitors.

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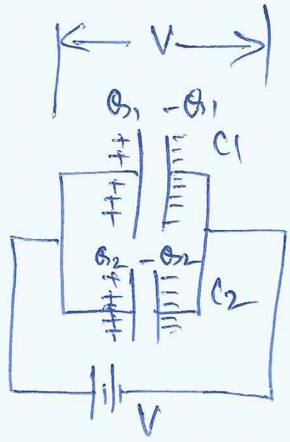
② Capacitors in parallel (V constant)



In parallel connection, the same p.d. is applied across both C_1 and C_2.

→ But the charges on ~~Q_1 and Q~~ C_1 and C_2 are not necessarily the same.

$Q_1 = C_1 V \quad , \quad Q_2 = C_2 V$

Equivalent capacitor is one with charge

$Q = Q_1 + Q_2$

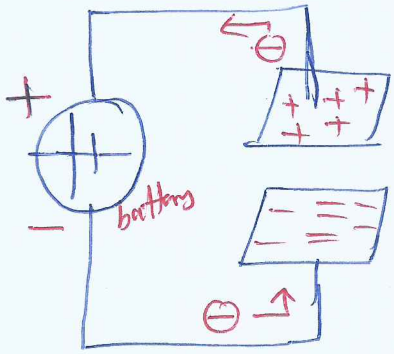
$Q = CV = C_1 V + C_2 V$

$\therefore \boxed{C = C_1 + C_2}$

Generalization $C = C_1 + C_2 + \cdots + C_n$ for n capacitors in parallel.

⁎ Energy stored in a capacitor ⁎

→ The process of charging a capacitor is equivalent to that of transferring charge from one plate to the other of the capacitor as shown in figure →



→ Some work must be done to charge a capacitor. This work done is stored as electrostatic PE in the capacitor.

→ Let at any instant a charge q be on the plate of a capacitor.
Then p.d. betw plates of the capacitor $= V = q/C$

→ If extra charge dq is transferred to the capacitor, then work done to do so is stored as electric PE in the capacitor.

Remember
$E = eV$
$E = qV$

$$\Rightarrow dU = dW = \underline{V\,dq} = \frac{q}{C}\,dq \longrightarrow (1)$$

∴ total increase in PE in charging the capacitor from $q = 0$ to $q = Q$ is the total energy stored in the capacitor.

$$\therefore U = \int dU = \int_0^Q \frac{q}{C}\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

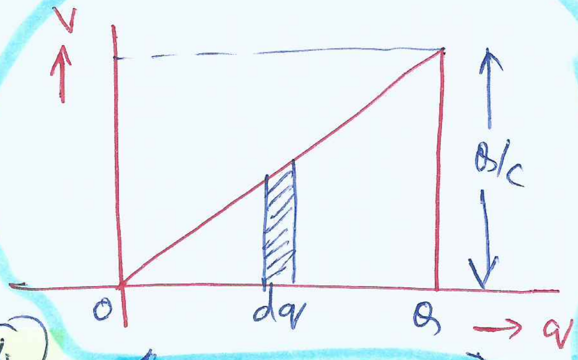
$$\boxed{U = \frac{Q^2}{2C}} \longrightarrow (2)$$

Since $Q = CV$

$$\boxed{U = \frac{1}{2}CV^2} \longrightarrow (3)$$

Since $C = \frac{Q}{V}$

$$\boxed{U = \frac{1}{2}QV} \longrightarrow (4)$$



∴ ~~Energy~~ Eqns (2), (3) & (4) are different relations for energy stored in a capacitor

→ The energy stored in C = area under V − q graph

∴ Energy stored in capacitor C = U = area under V − q graph

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2}\,Q \times \frac{Q}{C} = \boxed{\frac{1}{2}\frac{Q^2}{C}} \longrightarrow (2)$$

Electrostatic energy Density in a parallel plate capacitor

Energy stored per unit volume of the space betn the plates of the capacitor is known as energy density. Expression of energy density = ?

- Consider a parallel plate capacitor of capacitance C. Let 'A' be the area of each plate and d be the ~~tho~~ distance between cap. plates. When C is charged to vol V, energy stored in C is given by

$$U = \frac{1}{2} C V^2$$

We know that $C = \frac{\epsilon_0 A}{d}$ and $V = Ed$ (for parallel-plate capacitor)

$$\therefore U = \frac{1}{2} \cdot \left(\frac{\epsilon_0 A}{d}\right)(E d)^2 = \frac{1}{2} \epsilon_0 E^2 A d$$

Since volume of the capacitor = Ad.

$$\therefore \text{Energy stored / unit volume} = \frac{1}{2} \epsilon_0 E^2$$

But energy stored per unit volume of a capacitor is known as "Energy density" (U_d)

$$\therefore \text{Energy Density } \boxed{U_d = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 \vec{E} \cdot \vec{E}}$$

SI unit $= J m^{-3}$

Dimensional formula for energy density $= \frac{[\text{energy}]}{[\text{volume}]} = \frac{[ML^2T^{-2}]}{[L^3]}$

$$\therefore [U_d] = [ML^{-1}T^{-2}]$$

* Note :- Energy density, when a dielectric is introduced in the capacitor is given by

$$U_d = \frac{1}{2} k \epsilon_0 E^2 = \frac{1}{2} \epsilon E^2 \quad (k = \text{dielectric constant})$$

where $\epsilon = k\epsilon_0$ is the permittivity of the dielectric.

* $U_d = \frac{1}{2} \epsilon_0 E^2$ → we derived this for parallel-plate ~~capacitor~~ capacitor, the result on energy density of an electric field, in fact, very general and holds true ~~for~~ for electric field due to any configuration of charges.

Effect of ~~dielectric~~ dielectric on energy stored in capacitor

(1) When battery connected across capacitor is disconnected.

In the absence of dielectric, energy stored $= U_0 = \dfrac{Q_0^2}{2C_0}$

When dielectric is inserted & battery is disconnected, charge remains same, but $C = KC_0$

$\therefore$ Energy stored in C $= U = \dfrac{Q_0^2}{2KC_0}$

$\therefore \boxed{U = \dfrac{U_0}{K}}$ Since $K > 1$, $U < U_0$ energy stored in C decreases.

Thus, energy stored in C decreases when battery is disconnected and a dielectric (K) is inserted betn cap. plates.

(2) When battery remains connected ~~acro~~ across capacitor

In the absence of dielectric $U_0 = \dfrac{Q_0^2}{2C_0}$

When battery remains connected, and dielectric is inserted, then $Q = KQ_0$ and $C = KC_0$

$\therefore$ Energy stored in C $= U = \dfrac{Q^2}{2C_0} = \dfrac{K^2 Q_0^2}{2KC_0} = K\left(\dfrac{Q_0^2}{2C_0}\right)$

$\boxed{U = KU_0}$ Since $K > 1$, $U > U_0$

Thus, the energy stored in the capacitor increases, when ~~the~~ a dielectric is inserted betn cap. plates without disconnecting the battery.

Energy stored in Series/parallel connections of C

(70) (69)

1) Series Combination :

We know that $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$

We know that $U = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{2}\left[\frac{1}{C}\right]$

$$U = \frac{Q^2}{2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_n}\right]$$

$$U = \left[\frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \cdots + \frac{Q^2}{2C_n}\right]$$

$$U = U_1 + U_2 + \cdots + U_n$$

where $U_1, U_2 \cdots U_n$ are the energies stored in the individual capacitors respectively.

2) parallel Combination.

we know that $C = C_1 + C_2 \cdots + C_n$

Energy stored in a C is $U = \frac{1}{2}CV^2 = \frac{1}{2}V^2[C]$

$$U = \frac{1}{2}V^2\left[C_1 + C_2 + \cdots + C_n\right]$$

$$U = \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \cdots + \frac{1}{2}C_nV^2\right)$$

$$\boxed{U = U_1 + U_2 + \cdots + U_n}$$

Thus, the total energy stored in series or parallel combination is the sum of the energies stored in each capacitor.

※ Common potential ※

Consider 2 capacitors of different capacitance value and are charged to different potentials ⟶ are connected in parallel, then the charge flows from a capacitor at higher potential to the capacitor at lower potential. This flow of charge continues till potential of both the capacitors become equal. This equal potential of both the capacitors is known as "Common potential"

Consider 2 capacitors of capacitance values C_1 and C_2

Let V_1 and V_2 be their potentials respectively, "before sharing"

Total charge on both the capacitors

$$q = C_1 V_1 + C_2 V_2 \longrightarrow (1)$$

Let V be the common potential of both capacitors sharing the charges

∴ Total charge on both the capacitors after sharing

$$q' = C_1 V + C_2 V = (C_1 + C_2) V \longrightarrow (2)$$

Since electric charge is conserved,

∴ total charge q before sharing = total charge q' after sharing i.e. $q = q'$

$$\therefore C_1 V_1 + C_2 V_2 = (C_1 + C_2) V$$

$$\therefore \boxed{V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}}$$

Loss of energy on sharing charges

- When 2 charged capacitors are connected in parallel, no charge is lost. But some energy is lost on sharing the charge betn two capacitors. This loss of energy appears as heat energy.
- Consider 2 capacitors of value C_1 and C_2. Let these capacitors be charged to potential V_1 and V_2 respectively. When these capacitors are connected in parallel, charge flows from the capacitor at higher potential to the capacitor at lower potential till the potential of both the capacitors become equal, called the "common potential."
- Electrostatic PE of 2 capacitors before sharing

$$U = U_1 + U_2 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 \longrightarrow (1)$$

After sharing, both capacitors attain same p.d $= V$

∴ Electrostatic PE of 2 capacitors after sharing

$$U' = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 = \frac{1}{2}(C_1 + C_2) V^2$$

but $V = \dfrac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$$U' = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1 V_1 + C_2 V_2}{(C_1 + C_2)}\right)^2 = \frac{1}{2} \frac{(C_1 V_1 + C_2 V_2)^2}{C_1 + C_2} \longrightarrow (2)$$

$$U - U' = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 - \frac{(C_1 V_1 + C_2 V_2)^2}{2(C_1 + C_2)}$$

$$U - U' = \frac{1}{2(C_1 + C_2)}\left[(C_1 V_1^2 + C_2 V_2^2)(C_1 + C_2) - (C_1 V_1 + C_2 V_2)^2\right]$$

or $$U - U' = \frac{1}{2(C_1 + C_2)}\left[\cancel{C_1^2 V_1^2} + C_1 C_2 V_1^2 + C_1 C_2 V_2^2 + \cancel{C_2^2 V_2^2} - \cancel{C_1^2 V_1^2} - \cancel{C_2^2 V_2^2} - 2 C_1 C_2 V_1 V_2\right]$$

or $$U - U' = \frac{C_1 C_2 (V_1^2 + V_2^2 - 2 V_1 V_2)}{2(C_1 + C_2)}$$

or $$\boxed{U - U' = \frac{C_1 C_2 (V_1 - V_2)^2}{2(C_1 + C_2)}}$$

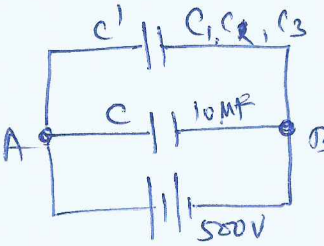
$U - U' > 0$ or $U > U'$

$U - U'$ is a '+ve' quantity because there is some loss of energy when 2 charged capacitors are connected together. This loss of energy appears as heat energy and heats up the connecting metallic wire.

Problem: A network of four 10 μF capacitors is connected to a 500 V supply. Determine (a) equivalent capacitance of the network and (b) charge on each capacitor. (73)

73

(a) Equivalent circuit is

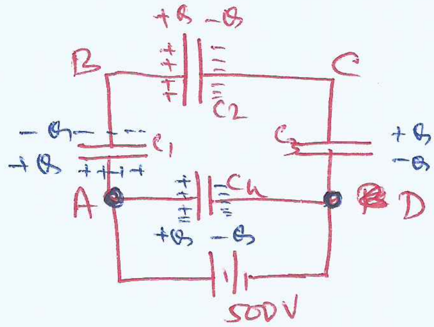


C_1, C_2, C_3 in series

$$1/C' = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C}$$

$$= \frac{3}{10\,\mu F}$$

$$\boxed{C' = \frac{10}{3}\,\mu F}$$



C' is parallel to C $\quad = \quad C_{eq} = 10\,\mu F + \frac{10}{3}\,\mu F = 13.3\,\mu F$.

(b) Since C_1, C_2, C_3 are in series, they have equal charges (say Q)

Vol. across $C_1 = Q/C_1$

" " $C_2 = Q/C_2$

" " $C_3 = Q/C_3$

But $V_1 + V_2 + V_3 = 500\,V$

$$Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] = 500\,V$$

$Q = VC$

$V = Q/C$

and also $\frac{Q'}{C_4} = 500\,V$; Q' = charge on C_4.

$$\therefore Q = 500\,V \times \frac{10}{3}\,\mu F = 1.7 \times 10^{-3}\,C \text{ and}$$

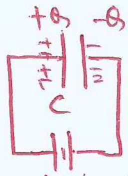
$$Q' = 500 \times 10\,\mu F = 5 \times 10^{-3}\,C$$

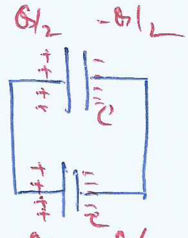
problem: (a) A 900 pF C is charged by 100 V. How much energy is stored in C.
(b) C is removed from battery and connected to another 900 pF C. What is the e-s energy stored by the system?

(a) Given $C = 900 \times 10^{-12}\,F$

$V = 100\,V$

$\therefore Q = CV = 9 \times 10^{-8}\,C$





Energy stored in capacitor is

$$= \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2} \times 9 \times 10^{-8} \times 100 = 4.5 \times 10^{-6}\,J$$

$Q = CV$

$V = Q/C$

(b) In steady state condition, V across parallel combination is same → common potential V' (say). By charge conservation, $Q' = Q/2$

$\Rightarrow V' = V/2$ (Since $V = Q/C$ and when Q becomes $Q/2$, $V = V/2$)

$$\therefore \text{Energy of system} = 2 \times \left(\frac{1}{2}Q'V'\right) = Q'V' = \frac{Q}{2} \cdot \frac{V}{2} = \frac{1}{4}QV$$

$$\text{Total energy of system} = \frac{1}{4} \times (9 \times 10^{-8}\,C)(100)^{25} = 225 \times 10^{-8} = 2.25 \times 10^{-6}\,J$$

Thus, in going from (a) to (b), though no charge is lost, final energy $= \frac{1}{2}$(initial energy)

Where has remaining energy gone.

→ There is a transient period before system comes to a steady state. During this transient period a transient current flows from the first capacitor to the second. Energy is lost during this time in the form of heat and electromagnetic radiation.