Chapter 4

Moving charges and Magnetism

- Introduction
- Motion in a Magnetic field
- Motion in combined Electric and Magnetic fields
- Magnetic Field due to a current element, B-S Law
  - Mag. field on the Axis of a Circular current loop
- Ampere's Circuital Law
- Solenoid and Toroid
- Force between two parallel currents Def^n of Ampere.
- Torque on current loop, Magnetic Dipole.
- Moving coil Galvanometer (Application).

In this chapter, we will see

1. how mag. field exerts forces on moving charged particles like
   - electrons
   - protons
   - current-carrying wires (conductors)
2. we shall learn how currents produce mag. field.
3. We shall see how particles can be accelerated to very high energies in a Cyclotron.
4. we shall see how currents and voltages are detected by a Galvanometer.

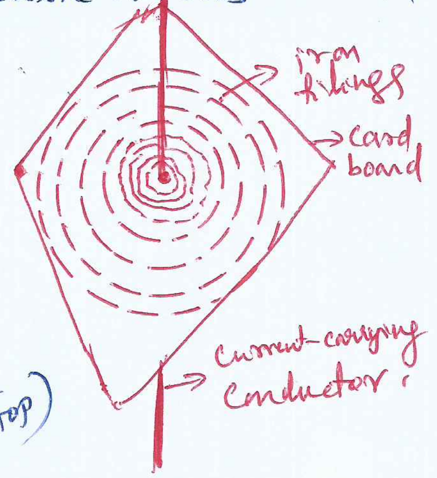
Introduction.

→ Just as static charges produce an electric field, the moving charges (or currents) produce (in addition) a magnetic field.

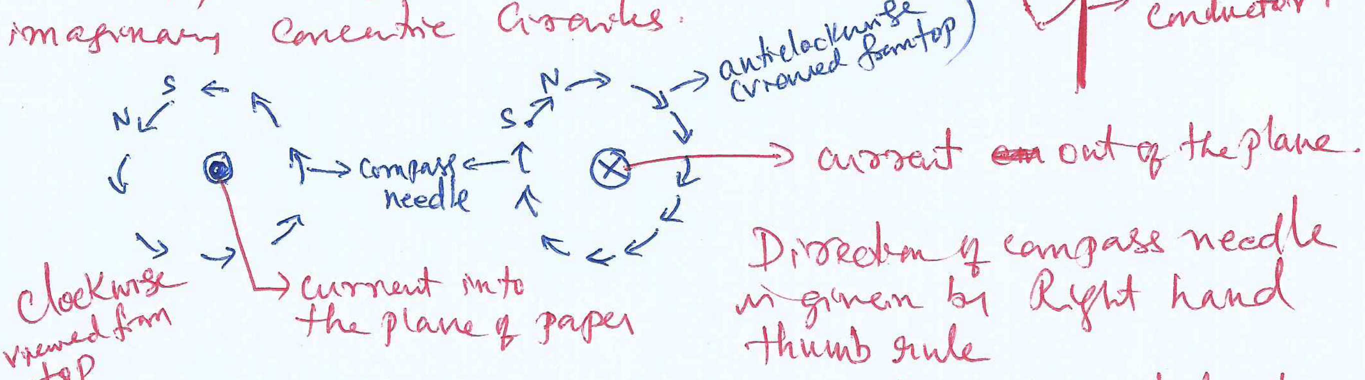
→ In 1820, Oersted proposed and concluded that "moving charges or currents produces magnetic field in the surrounding space.

How Oersted concluded this ?
it is through following experiments ---

① If a current-carrying straight wire is passed through a cardboard having iron filings spread over it, then the iron filings arrange themselves in concentric circles around the wire.



② If a compass needle is allowed to trace its path around the current-carrying conductor, once again it is found to be imaginary concentric circles.



Direction of compass needle is given by Right hand thumb rule

→ Thumb → Current direction

→ Curl line of fingers } → $\vec{B}$ direction.

IMP { [current should be large enough and compass needle sufficiently close to the wire, so that earth's mag. field may be ignored]

→ The deflection of compass needle increases when current is increased or bringing the needle close to the current-carrying conductor (wire).

Convention:

① A current or a field (electric or magnetic) emerging out of the plane of paper is depicted by a dot ⊙

② A current or a field (electric or magnetic) going into the plane of paper is depicted by a cross ⊗

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Note that there are two distinct ~~rules~~ "Right hand thumb rule" : (⇒ Thumb and curling of fingers)

I rule : For circular current loop, the curling of fingers will give direction of current and the thumb direction gives the direction of $\vec{B}$.

II rule : For long straight current-carrying conductor (wire), ~~the the thumb gives the~~ ~~the cu~~ the thumb gives the direction of current and the fingers will curl around in the direction of $\vec{B}$.

∴ Fingers and thumb play different roles in the above two situations.

Magnetic Force (See 4.2 in NCERT book):

→ We have seen that interaction bet^n two charges can be considered in 2 stages.

(a) The Charge Q (which is source of electric field) produces an Electric field E,

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q\hat{r}}{r^2} \longrightarrow (1)$$

where $\hat{r}$ is unit vector along $\vec{r}$ and $\vec{E}$ is vector field.

$\frac{1}{4\pi\epsilon_0} = 9\times10^9\ C^{-2}Nm^2$

ϵ_0 = permittivity of free space $= 8.854\times10^{-12}\ C^2N^{-1}m^{-2}$

(b) In the first stage, electric field is created by charge Q. Now, in the second stage, a test charge q interacts with this field and it (q) experiences a force F given by

$$\vec{F} = q\vec{E} = \frac{Qq}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} \longrightarrow (2)$$

→ $\vec{E}$ can vary with time also, however, we assume that the electric field E do not change with time (in this chapter)

※ If there are ~~el~~ more charges like Q creating electric field, the electric fields from each charge add vectorically (principle of superposition). Once the net field is known (calculated), the force on test charge q is given by eq^n (2).

→ Just as static charges produce an electric field, the currents or moving charges produce (in addition) a magnetic field, denoted by $\vec{B}(r)$, again a vector field. It has several basic properties identical to $\vec{E}$.

→ It is defined at each point in space (and can in addition depend on time).

→ Expt^lly, it is found to obey the principle of superposition → the mag. field of several sources is the vector addition of mag. field of each individual sources.

In this chapter, we will see

① how mag-field exerts "forces" on moving charged particles, like electrons, protons and current-carrying conductor (wire).

② Also learn how the current produces magnetic field.

③ how particles can be accelerated to very high energies in a cyclotron.

④ We shall see how currents and voltages are detected by a galvanometer.

Lorentz Force:

Suppose there is a point charge q moving (with a velocity 'v' and located at 'r' at a given time 't') in presence of both the electric field $\vec{E}(r)$ and the magnetic field $\vec{B}(r)$. The "Force" experienced by this point charge 'q' due to both $\vec{E}$ and $\vec{B}$ is given by Lorentz equation as

$$\vec{F} = q\vec{E}(r) + q[\vec{v} \times \vec{B}(r)] \equiv F_{electric} + F_{magnetic}$$

$$\Rightarrow \boxed{\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})} \longrightarrow ①$$

(vector product of $\vec{v}$ and $\vec{B}$)

Force is $\perp^r$ to plane containing $\vec{v}$ and $\vec{B}$

→ If $\vec{v}$ and $\vec{B}$ are $\perp^r$ to each other, then we can use "Fleming's Left Hand rule" to find the direction of "Force".

→ FLHR
- ~~Thum~~ Fore finger → Direction of $\vec{B}$
- Middle finger → Direction of current 'I' or velocity of movement of charged particle.
- Thumb → Direction of Force.

→ Force on a +ve charge (eg. proton) is given by FLHR

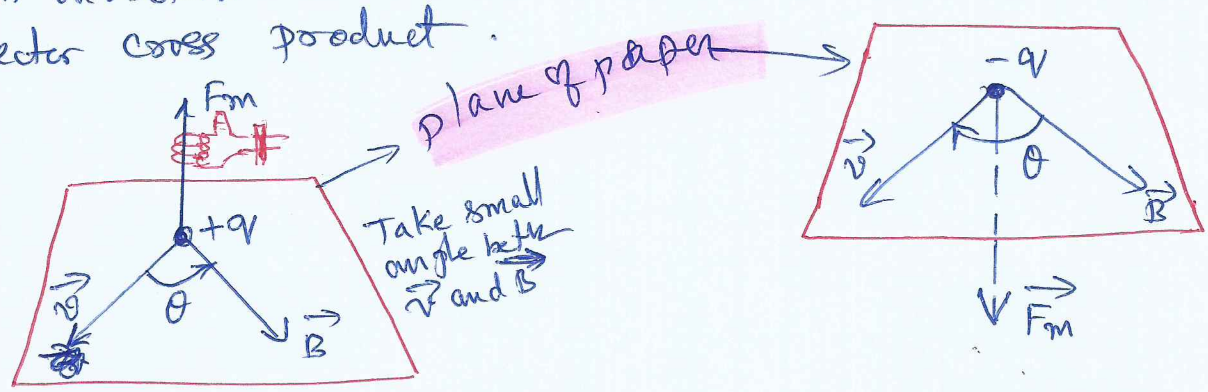
→ Force on a −ve charge (eg. electron) is opposite to that on a +ve charge.

$F_{magnetic}$ in Lorentz force is

$F_{magnetic} = q(\vec{v} \times \vec{B})$ ∴ $F_m = qvB\sin\theta$; where θ is the angle betw $\vec{v}$ and $\vec{B}$.

→ Direction of F_m is ⊥ to plane containing $\vec{v}$ and $\vec{B}$

→ This direction can be found by right hand thumb rule for vector cross product.



① If charge q moves parallel (or antiparallel) to $\vec{B}$, $\left.\begin{matrix}\theta = 0 \\ \theta = 180^\circ\end{matrix}\right\}$ $\sin\theta = 0$

Thus no mag. force is experienced by the charged particle in this case.

② If charge q moves at right angle to $\vec{B}$, $\theta = 90^\circ$, $\sin 90^\circ = 1$

then $F_m = qvB$ → charge q experiences maximum force.

if $\theta = 270^\circ$, $\sin 270^\circ = -1$ ⟹ $F_m = -qvB$

⟹ max. force in opposite direction.

③ If charged particle is at rest, $\vec{v} = 0$ ∴ $F_m = 0$, then no mag. force is experienced by the charged particle.

✱ To find direction of force

① if $\theta \neq 90^\circ$, then use right hand thumb rule for vector cross product of $\vec{v} \times \vec{B}$.

② if $\theta = 90^\circ$, we can use FLHR to find force direction.

→ Expression for mag. force helps us to define the unit of mag. field $\vec{B}$. $\vec{F} = qvB\sin\theta$

If $q = 1\,C$, $v = 1\,ms^{-1}$, $\theta = 90^\circ$, then $B = F_m$

⟹ Mag. field ($\vec{B}$) at a point may be defined as the mag. force experienced by a unit charge moving with unit velocity at right angles to the mag. field.

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Dimensional Formula of B

$$[B] = \frac{[\text{Force}]}{[\text{Charge}] \times [\text{velocity}]}$$

$$= \frac{[\text{Force}]\,[\cancel{\text{time}}]}{[\text{Ampere}]\,[\cancel{\text{velocity}}]\,[\text{time}]\,[\text{velocity}]}$$

$$= \frac{[M\cancel{L}s^{-2}]}{[A]\,[\cancel{T}]\,[\cancel{L}\cancel{T}^{-1}]}$$

$$[B] = [ML^0T^{-2}A^{-1}] \longrightarrow (2)$$

~~Charge~~ $\cancel{q = \frac{i}{t}}$

Current $i = \frac{q}{t}$

$q = it$

Force $= ma$

$= kg.ms^{-2}$

in SI unit, unit of $\vec{B}$ is tesla (T).

$\vec{B} \rightarrow$ "mag. field" or "strength of mag. field" or "mag. field strength".

$$1 \text{ tesla } (T) = \frac{1\,N}{1C \times 1ms^{-1}} = \boxed{1\,Nm^{-1}A^{-1}}$$

Thus, the magnetic field strength is said to be 1 tesla if a charge of 1C moves with a velocity of $1\,ms^{-1}$ at right angles to the mag. field experiences a force of 1N.

In CGS system, unit of $\vec{B}$ = gauss (G)

$$\boxed{1\,G = 10^{-4}\,T}$$

Problem: copper has 8×10^{28} conduction electrons per m^3. A cu wire of length 1m and cross-sectional area $8 \times 10^{-6}m^2$ carrying a current at right angles to $B = 5 \times 10^{-3}T$ experiences a force $= 8 \times 10^{-2}$ N. (Calculate Drift velocity of free electrons in the wire)

$\rightarrow$ Given $n = 8 \times 10^{28}m^{-3}$, $l = 1m$, $A = 8 \times 10^{-6}m^2$, $F = 8 \times 10^{-2}N$, $\theta = 90^0$

$B = 5 \times 10^{-3}T$

Charge on each electron, $e = 1.6 \times 10^{-19}C$

$\therefore$ No. of electrons in copper wire $= n \times$ volume of wire $= nAl$

$\therefore$ Total charge on the wire $q = nAle$

$= (8 \times 10^{28})(8 \times 10^{-6})(1)(1.6 \times 10^{-19})$

$q = 1.024 \times 10^5 C$

We know that $F = qvB\sin\theta$ (~~since θ~~ since $\theta = 90^0$, $\sin\theta = 1$)

$$\therefore v = \frac{F}{qB} = \frac{8 \times 10^{-2}N}{1.024 \times 10^5 C \times 5 \times 10^{-3}T} = 1.563 \times 10^{-4}\,ms^{-1}$$

Magnetic field Vs Electric field

Mag. field has many properties like that of electric field $\vec{E}$.

1. $\vec{B}$ is a vector quantity like $\vec{E}$
2. ~~Bt~~ $\vec{B}$ has a three dimensional (ie-space) relevance like $\vec{E}$
3. $\vec{B}$ is time dependent like $\vec{E}$.
4. $\vec{B}$ depends on permeability (μ), whereas $\vec{E}$ depends on permittivity (ϵ) of a material. Both ϵ and μ are the measure of the ~~??~~ extent to which electric field or magnetic field respectively can penetrate the material.
5. More than one mag. fields can be added vectorially to give resultant field. Thus $\vec{B}$ obeys superposition principle like E

→ Net mag. field strength $\vec{B}$ of many sources at a point is the vector sum of mag. fields due to each source say $\vec{B_1}, \vec{B_2}$ -- at that point. i.e. $\vec{B} = \vec{B_1} + \vec{B_2} + \vec{B_3} + \cdots$

IMP

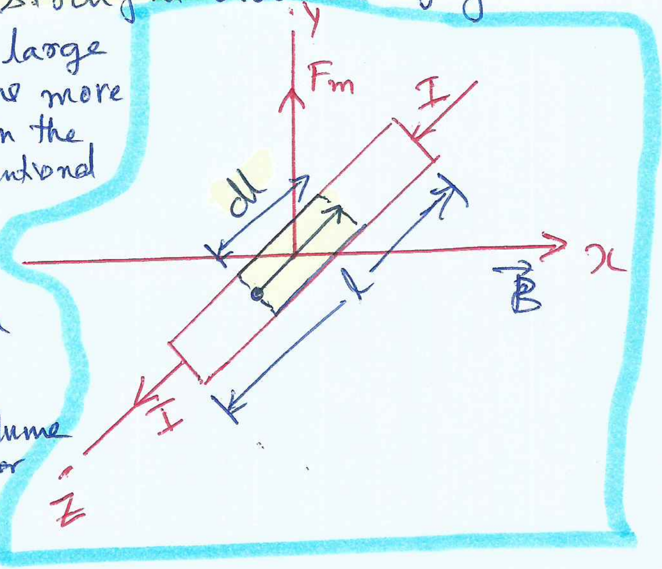
1. "Strength of mag. field" $\vec{B}$ is commonly referred to as simply "magnetic field".
2. Earth mag. field is $\simeq 3.6 \times 10^{-5}$ T
3. Force experienced by a charged particle in an electric field is independent of its speed or velocity. This means, a stationary as well as a moving charge experience force in electric field.
4. Force is experienced by only the moving charge in a mag. ~~field~~ field provided the charge is not moving parallel or antiparallel to the magnetic field.
5. A charge at rest is the source of electric field but not the source of magnetic field → however, a moving charge is the source of electric as well as magnetic field

IMP

6. Electric force on a charge accelerates or retards the charged particle.
7. Magnetic force on a charge deflects the path of the charged particle.

Magnetic Force on a Current-carrying conductor placed in a uniform magnetic field

Let us extend the analysis for force due to mag. field on a "single moving charge" to a "straight rod carrying current"



- A current-carrying conductor contains large number of free electrons. These electrons move with an average ~~dr~~ drift velocity v_d in the direction opposite to the direction of conventional current flowing in the conductor
- Consider a conductor of length l carrying a current I placed in a uniform mag. field $\vec{B}$.
- Let 'n' = no of free electrons per unit volume of the conductor
  A = Cross-sectional area of the conductor.

$\therefore$ Mag. force acting on an electron

$$\vec{f_m} = -e(\vec{v} \times \vec{B}) \longrightarrow (1)$$

- Now consider a small element of length dl of the given conductor

$\therefore$ No of electrons in the small element $= n \times$ volume of the element $= n(A\,dl)$

$\therefore$ Mag. force experienced by the element dl of the conductor

$$\vec{dF_m} = (nA\,dl)\vec{f_m} = (nA\,dl)\left[-e(\vec{v} \times \vec{B})\right]$$

$$\vec{dF_m} = -nAe\,dl(\vec{v} \times \vec{B}) \longrightarrow (2)$$

- But drift velocity $\vec{v_d} = -\frac{\vec{dl}}{dt}$ $\quad$ $\because$ ($\vec{dl}$ is in a direction opposite to the direction of $\vec{v}$)

and $(nA\,dl)e = dq \rightarrow dq$ is the charge on the small element.

$$\therefore \vec{dF_m} = dq\left[\frac{\vec{dl}}{dt} \times \vec{B}\right] = \frac{dq}{dt}(\vec{dl} \times \vec{B}) \quad \text{Since } \frac{dq}{dt} = I$$

$$\vec{dF_m} = I(\vec{dl} \times \vec{B})$$

- Since conductor is made of large number of such elements like 'dl', total force experienced by the conductor is given by

$$\vec{F_m} = \int \vec{dF_m} = \int I(\vec{dl} \times \vec{B}) = I\int \vec{dl} \times \vec{B} \quad \left(\text{Since } \int \vec{dl} = l\right)$$

$$\vec{F_m} = I(\vec{l} \times \vec{B}) \longrightarrow (3)$$

Magnitude $|\vec{F_m}| = BIl\sin\theta$, where θ is angle betw $\vec{l}$ and $\vec{B}$

Direction of $\vec{dF_m}$ is $\perp^r$ to the plane containing $\vec{l}$ and $\vec{B}$ and can be determined using FLHR. Note that $\vec{B}$ in the eqn (3) is the external field; it is not the field produced by the current-carrying straight rod. Contd...

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We have derived an eq^n for a straight rod as

$\vec{F}_m = I(\vec{l} \times \vec{B})$ where l = length of conductor
I = Current in the conductor
B = uniform mag. field.
$\vec{F}$ = force experienced by rod.

there is a simple derivation to this as follows (Don't use it)

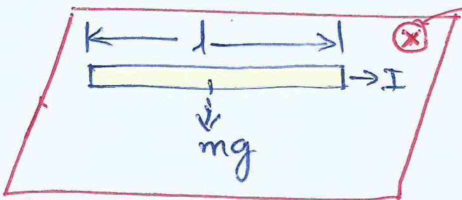
We know that ~~F_m~~ from Lorentz force,

$$F_m = q(\vec{v} \times \vec{B})$$ since $I = \frac{q}{t}$ $q = It$

$$= It(\vec{v} \times \vec{B})$$ and $v = \frac{\vec{l}}{t}$ $\therefore \vec{l} = vt$

$$\vec{F}_m = I(\vec{l} \times \vec{B})$$

problem: A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform mag. field (horizontal field) B. What is the magnitude of the mag. field.



B $\perp^r$ to plane of paper and pointing into the page

Eq: $F = I(\vec{l} \times \vec{B})$ ~~Sin~~ $= IlB\sin\theta = IlB$ (since $\theta = 90^0$)

From FLHR, ~~the~~ wire experiences an upward force $\vec{F}$ of magnitude IlB. For mid-air suspension, this force must be balanced by the force of gravity.

$$\therefore mg = IlB$$

$$B = \frac{mg}{Il} = \frac{\cancel{0.2}^{0.1}\,kg \times 9.8\,\cancel{m}s^{-2}}{\cancel{2}A \times 1.5\,\cancel{m}} = \frac{0.98\,kg\,s^{-2}}{1.5\,A}$$

$$= 0.65\,kg\,s^{-2}A^{-1}$$

$$\boxed{B = 0.65\,T}$$ (note earth's $\vec{B}$ is ignored).

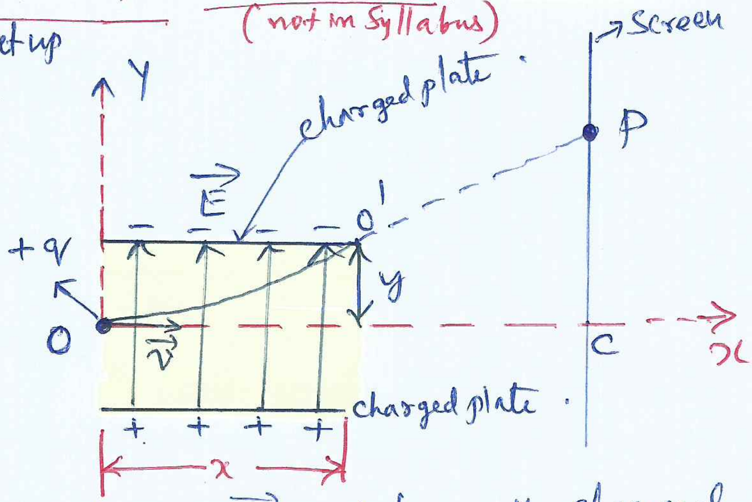
In the next page, we will consider ~~"the"~~ in greater detail

① The motion of a charged particle in a uniform $\vec{E}$

② ——— " ——— " ——— $\vec{B}$

③ ——— " ——— in a combined $\vec{E}$ and $\vec{B}$ fields.

Motion of a charged particle in a Uniform Electric field
(not in Syllabus)



Consider a uniform electric field $\vec{E}$ set up between two oppositely charged parallel plates. Let a +vely charged particle having charge $+q$ and mass m enter at O with velocity $\vec{v}$ along x-direction in the region of electric field $\vec{E}$.

- Force acting on charge $+q$ due to electric field $\vec{E}$ is $\vec{F} = q\vec{E}$

The direction of force is along the direction of $\vec{E}$ and hence the charged particle is deflected accordingly (as shown in figure). Acceleration produced in the charged particle is given by

$$\vec{a} = \frac{\vec{F}}{m} \quad \text{or} \quad \vec{a} = \frac{q\vec{E}}{m}$$

$$|\vec{a}| = \frac{qE}{m} \longrightarrow (1)$$

The charged particle will accelerate in the direction of $\vec{E}$

- As soon as the particle leaves the region of electric field, it travels in a straight line due to inertia of motion and hits the screen at point P.

Let 't' be the time taken by the charged particle to traverse the region of electric field of length 'x'. Let y be the distance travelled by the particle along y-direction (i.e. direction of $\vec{E}$)

→ Using standard eqn of motion $s = ut + \frac{1}{2}at^2 \rightarrow (2)$, find the eqns for horizontal and vertical components of motion

→ For horizontal motion, $s = x$, $u = v$ and $a = 0$ (Since no force acts on the particle along x-direction)

$\therefore$ From eqn (2), $x = vt$ or $\boxed{t = \frac{x}{v}} \rightarrow (3)$

→ For Vertical motion, $s = y$, $u = 0$ ($\because$ initially the particle was moving along x-direction)

From eqn (2), $y = \frac{1}{2}at^2$

- Using (1) and (2), $y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x^2}{v^2}\right) \Rightarrow y = \frac{qEx^2}{2mv^2} = Kx^2 \rightarrow (4)$

where $K = \frac{qE}{2mv^2}$, a constant.

Eqn (4) is a parabola. Hence, a charged particle moving in a uniform electric field follows a parabolic path (OO') as shown in fig.

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Note :

① $\vec{E}$ accelerates +vely charged particle if it moves in the direction of $\vec{E}$

② $\vec{E}$ ~~de~~ retards +ve charged particle if it moves opposite the direction of $\vec{E}$.

③ Energy gained by a charge (q) while passing thro' a p.d. of V volts is given by $KE = \frac{1}{2} m v^2 = qV$.

④ A positively charged particle moving along the direction of the electric field accelerates along a straight line.

⑤ A charged particle moving at right angles to $\vec{E}$, follows a parabolic path.

⑥ The deflection experienced by a charged particle moving at right angle to the direction of electric field is given by

$y = \frac{1}{2}\left(\frac{q}{m}\right)\frac{Ex^2}{v^2}$; Since E, x, v are constants, $\therefore$ $\boxed{y \propto \frac{q}{m}}$

Thus a moving charged particle experiencing a large vertical deflection in the uniform electric field has more specific charge $\frac{q}{m}$

Problem : $\vec{E}$ betn two plates of a CRO is $1.2 \times 10^5\ Vm^{-1}$. If an electron of energy 2000 eV enters at right angles to $\vec{E}$, what will be its deflection if the length of plate $= 1.5$ cm

→ Given : $q = e = 1.6 \times 10^{-19} C$

$E = 1.2 \times 10^5\ Vm^{-1}$

$x = 1.5 cm = 1.5 \times 10^{-2}\ m$

$$K.E. = \frac{1}{2} m v^2 = qV = 2000\,eV = 2000 \times 1.6 \times 10^{-19} J$$

$$y = \frac{qEx^2}{2mv^2} = \frac{qEx^2}{2 \times 2(\frac{1}{2}mv^2)} = \frac{(\cancel{1.6 \times 10^{-19}})(\overset{0.3}{\cancel{1.2}} \times 10^5)(1.5 \times 10^{-2})^2}{\cancel{4} \times 2000 \times \cancel{1.6 \times 10^{-19}}}$$

$$y = \frac{0.3 \times 2.25 \times 10^{-4} \times \cancel{10^5}\ 10^2}{2\cancel{000}}$$

$$= \frac{0.3 \times 2.25 \times 10^{-2}}{2} = 0.15 \times 2.25 \times 10^{-2}\ m$$

$$= 0.15 \times 2.25\ cm$$

$$= 0.34\ cm$$

$$\boxed{y = 3.4\ mm}$$

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Motion of a charged particle in a Uniform Magnetic field

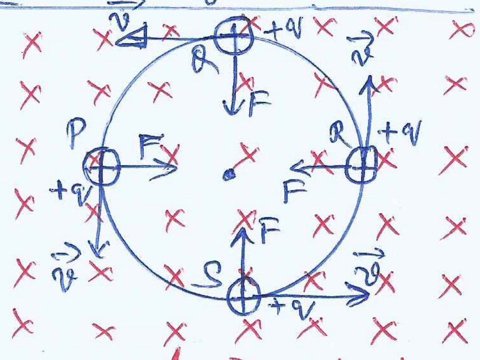
Let us consider, in greater detail, the motion of a charge moving in a mag. field.

Case ①: When motion of charged particle is parallel or antiparallel to the mag. field

$F = qvB\sin\theta \Rightarrow$ here $\theta = 0$ $\therefore F = 0 \Rightarrow$ In this case moving charged particle does not experience any force. Therefore, the charged particle in this case will continue moving along the same path with the same velocity.

(Note if charged particle is at rest in $\vec{B}$, then $v = 0$, $F = 0$, the charged particle experiences no force).

Case ② When charged particle moves $\perp^r$ to magnetic field.



$\vec{B}$ $\perp^r$ to plane of paper and pointing into the page.

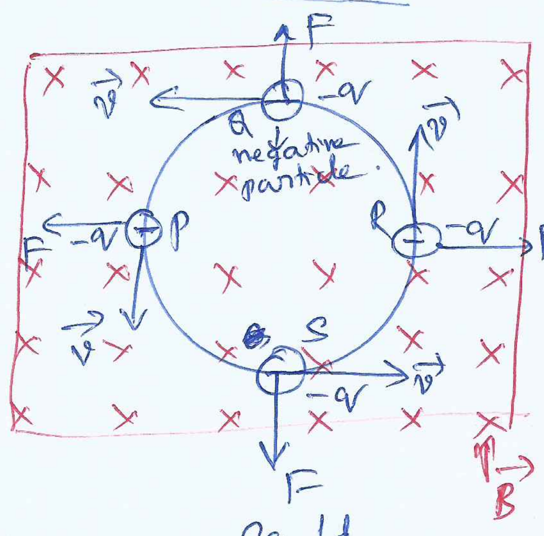
- Consider $\vec{B}$ as shown in figure.
- Suppose a particle of mass 'm' and carrying a charge +q enters the magnetic field at a point S (from ~~bottom~~ left) with velocity $\vec{v}$ directed $\perp^r$ to $\vec{B}$.
- Force acting on this charge '+q' due to $\vec{B}$ is given by $\vec{F} = qvB\sin\theta$ $(\theta = 90^\circ)$

$$F = qvB$$

Since $\vec{v}$ and $\vec{B}$ are $\perp^r$, the direction of force $\vec{F}$ is given by FLHR. As per FLHR, $\vec{F}$ is directed ~~towards~~ upwards and lying on the plane of paper.

Since F is $\perp^r$ to v, it does not change the magnitude of the velocity; it changes only the direction of velocity. Thus, the particle moves under a force whose magnitude remains constant but direction changes continuously and is always $\perp^r$ to the velocity. It therefore describes an anticlockwise circular path with constant speed v, the force $\vec{F}$ is working as the centripetal force. So, the path of the particle is circular.

- If the particle is negatively charged (eg. electron), the force at S is downwards and the particle would have described a "clockwise" circle.



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If m is the mass of the particle and r be the radius of the circular path, then the necessary centripetal force required by the particle to move in a $\boxed{\text{circular path} = \frac{mv^2}{r}}$

Cen. force $= mv(v/r) = mv/t = ma$

$F = ma$

This centripetal force is provided by the magnetic Lorentz force $F = qvB$

$F = ma = mv/t = mv(v/r) = \frac{mv^2}{r}$

$$\therefore \frac{mv^2}{r} = q\cancel{v}B$$

$$\therefore \boxed{r = \frac{mv}{qB}} \rightarrow (1)$$

mv → momentum

r is $\propto$' to momentum of the particle

If K is the KE of the particle,

$$\boxed{r = \frac{\sqrt{2mK}}{qB}} \rightarrow (2)$$

$\left(K = \frac{1}{2}mv^2\right)$

$v^2 = \frac{2K}{m}$

$v = \sqrt{\frac{2K}{m}}$

- The particle traverses a distance $2\pi r$ in one revolution

$\therefore$ Time period $= \frac{2\pi r}{v}$

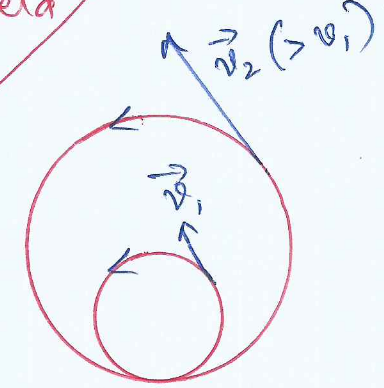
$$T = \frac{2\pi}{\cancel{v}} \times \frac{m\cancel{v}}{qB}$$

$$\therefore \boxed{T = \frac{2\pi m}{qB}} \rightarrow (3)$$

$$\text{Freq. } \boxed{f = \frac{1}{T} = \frac{qB}{2\pi m}} \rightarrow (4)$$

Eqns. (3) and (4) show that T and f of the particle is independent of the speed v of the particle. If the speed of the particle increases, its radius also increases, so that the time taken to complete one revolution remains same.

- If two identical charged particles enter the field with different speeds v_1 and $v_2 (> v_1)$, then they move along circles of smaller and larger radii respectively.



- Angular freq $\omega = 2\pi f = \cancel{2\pi}\left(\frac{qB}{\cancel{2\pi} m}\right)$

$$\boxed{\omega = \frac{qB}{m}} \rightarrow (5) \rightarrow$$

This angular freq. is called gyro frequency.

From (5), it is clear that angular freq is independent of speed of the particle (provided $v << c$). Thus, all the charged particles take the same time to complete the circular orbits of small and large radius, provided their specific charge (q/m) is same.

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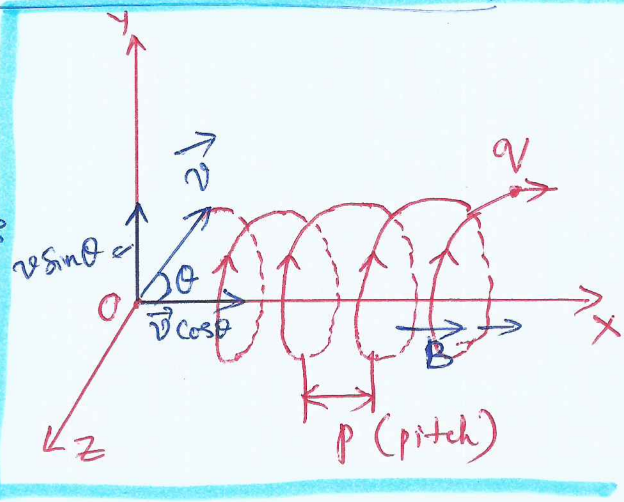
eq^n ① from pre. page $\boxed{r = \frac{mv}{qB}}$

if both electron and proton enter the same magnetic field with same velocity such that the velocity of both is $\perp^r$ to field $\vec{B}$, then the electron describes a circular path of smaller radius than described by a proton. ~~(Since q is same~~

[(Since magnitude of q is same for proton and electron, but mass of electron (9.1×10^{-31} kg) < mass of proton (1.6×10^{-27} kg)) and since $r \propto m$, electron traverses small radius path as compared to the proton].

End of case ②

Case ③ When the charged particle moves at an angle θ to the mag. field (other than 0°, 90° and 180°)



Suppose the velocity $\vec{v}$ of the particle entering the field $\vec{B}$, instead of being $\perp^r$ to $\vec{B}$, makes an angle θ with it.

- Then $\vec{v}$ may be resolved into 2 components parallel to $\vec{B}$ (See fig.)

→ $v_{\parallel} = v\cos\theta$ parallel to $\vec{B}$

→ $v_{\perp} = v\sin\theta$ $\perp^r$ to $\vec{B}$

- The component $v_{\parallel}$ gives a linear path and the component $v_{\perp}$ gives a circular path to the particle.
- The resultant of these two is a helical path whose axis is parallel to the magnetic field
- The radius of the circular path of the helix is

$$r = \frac{mv_{\perp}}{qB} = \boxed{\frac{mv\sin\theta}{qB}}$$

- Time period $T = \frac{2\pi r}{v\sin\theta} = \frac{2\pi}{\cancel{v\sin\theta}} \times \frac{m\cancel{v\sin\theta}}{qB} = \boxed{\frac{2\pi m}{qB}}$

- $\boxed{f = \frac{qB}{2\pi m}}$

- The linear distance travelled by the particle in the direction of the mag. field $\vec{B}$ in one complete cycle is $P = v_{\parallel} \times T$

$$= v\cos\theta \times \frac{2\pi m}{qB}$$

$$\therefore \boxed{P = \frac{2\pi m v\cos\theta}{qB}}$$

pitch.

Contd.....

..... Contd from pre. page

Kinetic energy of charged particle moving in uniform $\vec{B}$

$$K = \frac{1}{2} m v^2$$

since $r = \frac{mv}{qB}$

$v = \frac{rqB}{m}$

$$\therefore K = \frac{1}{2} \cancel{m} \frac{r^2 q^2 B^2}{m^{\cancel{2}}}$$

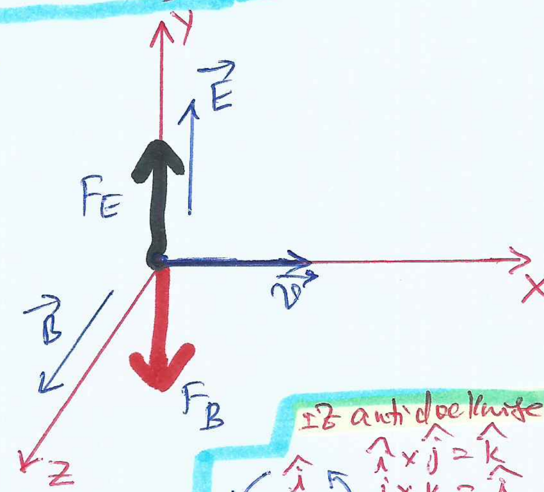
$$\boxed{K = \frac{r^2 q^2 B^2}{2m}}$$

End of case (3)

Motion in Combined Electric & Magnetic fields.

I Velocity Selector :

First method. (as per NCERT book)



We know that a charge q moving with velocity $\vec{v}$ in presence of both $\vec{E}$ and $\vec{B}$ fields experiences a force which is given by Lorentz force as

$$F = q\vec{E} + q(\vec{v}\times\vec{B}) = F_E + F_B$$

→ Consider $\vec{E}$ and $\vec{B}$ are $\perp^r$ to each other and also $\perp^r$ to velocity of the particle (See figure)

$$\vec{E} = E\hat{j}, \quad \vec{B} = B\hat{k}, \quad \vec{v} = v\hat{i}$$

$$F_E = q\vec{E} = qE\hat{j}$$

$$F_B = q(\vec{v}\times\vec{B}) = q(v\hat{i}\times B\hat{k}) = -qvB\hat{j}$$

$$\therefore F = q(E - vB)\hat{j}$$

If anticlockwise
$\hat{i}\times\hat{j}=\hat{k}$
$\hat{j}\times\hat{k}=\hat{i}$
$\hat{k}\times\hat{i}=\hat{j}$

$\hat{i}\times\hat{k} = -\hat{j}$

If clockwise
$\hat{j}\times\hat{i}=-\hat{k}$
$\hat{i}\times\hat{k}=-\hat{j}$
$\hat{k}\times\hat{j}=-\hat{i}$

Thus electric and magnetic forces are in opposite directions as shown in figure. Suppose, we adjust the value of E and B such that the magnitude of the two are equal. Then, total force on the charge is zero and the charge will move in the fields undeflected. This ~~happens~~ when

$$\cancel{q}E = \cancel{q}vB \quad \text{or} \quad \boxed{v = \frac{E}{B}}$$

This condition can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds (irrespective of their charge and mass).

Contd —