Chapter 13 : NUCLEI

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CHAPTER THIRTEEN

NUCLEI

Introduction:

In the chapter "Atoms", we have learnt that in every atom, the positive charge and mass are densely concentrated at the centre of the atom forming its nucleus. The overall dimensions of a nucleus are much smaller than those of an atom. Experiments on scattering of α-particles demonstrated that the radius of a nucleus was smaller than the radius of an atom by a factor of about 10^4. This means the volume of a nucleus is about 10^{-12} times the volume of the atom.

If the radius of "sphere" of Atom = 1 unit, then the radius of "sphere" of Nucleus = 10^{-4} units

Volume of sphere of atom = $\frac{4}{3}\pi r^3$ (r=1); therefore, volume of nucleus = $\frac{4}{3}\pi\,(10^{-4})^3 = \frac{4}{3}\pi 10^{-12}$

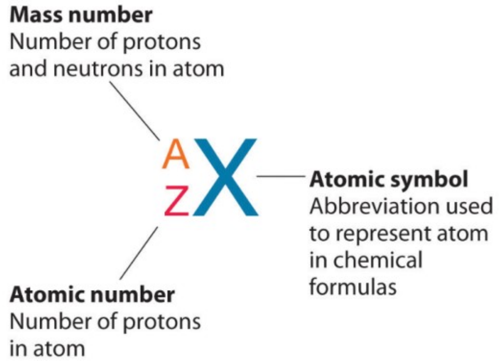
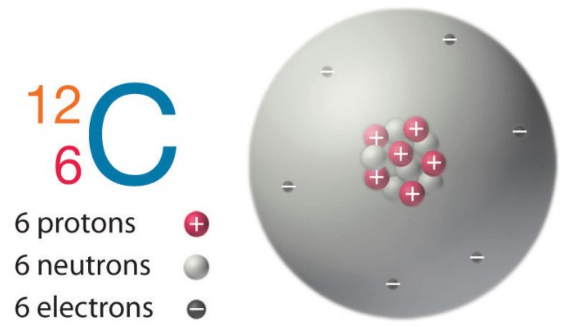
$$\boldsymbol{Volume_{nucleus} = 10^{-12}\ \text{x}\ Volume_{atom}}$$

In other words, an atom is almost empty. If an atom is enlarged to the size of a classroom, the nucleus would be of the size of pinhead. Nevertheless, the nucleus contains most (more than 99.9%) of the mass of an atom.

Does the nucleus have a structure, just as the atom does? If so, what are the constituents of the nucleus? How are these held together? In this chapter, we shall look for answers to such questions. We shall discuss various properties of nuclei such as their size, mass and stability, and also associated nuclear phenomena such as radioactivity, fission and fusion.

Detailed description of Nuclei Chapter

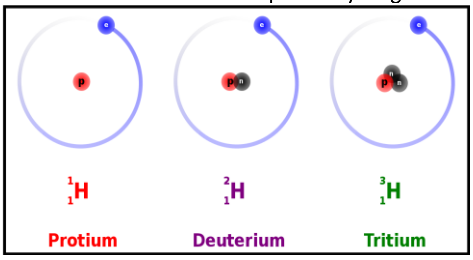
Terms	Explanation
atomic mass unit (u) 1 u = 1 a.m.u.	The atomic mass unit (u) is a very small unit of mass and it is found to be very convenient in nuclear physics. Atomic mass unit is defined as $\left(\frac{1}{12}\right) th$ of the mass of one ${}^{12}_{6}C$ atom. According to Avogadro's hypothesis, number of atoms in 12 grams of ${}^{12}_{6}C$ atom is equal to Avogadro number = $6.022140857(74) \times 10^{23}$ mol^{-1} ≈ 6.022×10^{23} mol^{-1} ∴ the mass of one carbon atom $({}^{12}_{6}C) = \frac{12}{6.022 \times 10^{23}}$ = $1.992646848267741 \times 10^{-26}$ kg ≈ 1.992647×10^{-26} kg ∴ 1 u = $\frac{\text{mass of } ({}^{12}_{6}C) \text{ atom}}{12} = \frac{1.992647 \times 10^{-26} \text{ kg}}{12}$ = $1.660539040223117 \times 10^{-27}$ kg ≈ 1.660539×10^{-27} kg **∴ 1 u = $1.660539040223117 \times 10^{-27}$ kg ≈ 1.660539×10^{-27} kg**
atomic mass unit details (Imp)	➢ The atomic masses of various elements expressed in atomic mass unit (u) are close to being integral multiples of the mass of a hydrogen atom. ➢ There are, however, many striking exceptions to this rule. For example, the atomic mass of chlorine atom is 35.46 u.
Neutron	➢ Since the nuclei of deuterium and tritium are isotopes of hydrogen, they must contain only one proton each. But the masses of the nuclei of hydrogen, deuterium and tritium are in the ratio of 1:2:3. Therefore, the nuclei of deuterium and tritium must contain, in addition to a proton, some neutral matter. The amount of neutral matter present in the nuclei of these isotopes, expressed in units of mass of a proton, is approximately equal to one and two, respectively. This fact indicates that the nuclei of atoms contain, in addition to protons, neutral matter in multiples of a basic unit. ➢ This hypothesis was verified in 1932 by James Chadwick who observed emission of **neutral radiation** when beryllium nuclei were bombarded with alpha-particles (α-particles are helium nuclei). It was found that this neutral radiation could knock out protons from light nuclei such as those of helium, carbon and nitrogen. ➢ The only neutral radiation known at that time was **photons** (electromagnetic radiation). Application of the principles of conservation of energy and momentum showed that if the neutral radiation consisted of photons, the energy of photons would have to be much higher than is available from the bombardment of beryllium nuclei with α-particles. **So, it is not photons.** ➢ The clue to this puzzle, which Chadwick satisfactorily solved, was to assume that the neutral radiation consists of a new type of neutral particles called neutrons. From conservation of energy and momentum, he was able to determine the mass of new particle 'as very nearly the same as mass of proton'. ➢ A **free** neutron, unlike a free proton, is unstable. It decays into a proton, an electron and an antineutrino (another elementary particle), and has a mean life of about 1000s. **It is, however, stable inside the nucleus.** ➢ Chadwick was awarded the 1935 Nobel Prize in Physics for his discovery of the neutron. ➢ Mass of neutron : $m_n = 1.00866$ u = 1.6749×10^{-27} kg ; kg to u conversion is given below: ➢ 1 u = $1.660539040223117 \times 10^{-27}$ kg ≈ 1.660539×10^{-27} kg ; this implies ➢ 1.660539040223117 kg x 10^{-27} kg = 1 u ➢ Mass of Neutron = 1.6749×10^{-27} kg = $\frac{1.6749 \times 10^{-27} \text{ kg}}{1.660539 \times 10^{-27} \text{ kg}} = \frac{1.6749}{1.660539}$ = 1.0087 u
Nucleons.	The nucleus consists of protons and neutrons. They are called nucleons.
Nuclides	➢ Nuclides are an atom or ion characterized by the contents of their nucleus. Eg: ${}^{12}_{6}C$ and ${}^{14}_{6}C$ are both nuclides. ➢ A nuclide is an atomic species characterized by the specific constitution of its nucleus, i.e., by its number of protons Z, its number of neutrons N, and its nuclear energy state. ○ The word nuclide was proposed by Truman P. Kohman in 1947. Kohman originally suggested nuclide as referring to a "species of atom characterized by the constitution of its nucleus" defined by containing a certain number of neutrons and protons. The word thus was originally intended to focus on the nucleus.
Nuclides Vs isotopes	➢ A nuclide is a species of an atom with a specific number of protons and neutrons in the nucleus, for example carbon-13 with 6 protons and 7 neutrons. The nuclide concept (referring to individual nuclear species) emphasizes nuclear properties over chemical properties, while the isotope concept (grouping all atoms of each element) emphasizes chemical over nuclear. ➢ The neutron number has large effects on nuclear properties, but its effect on chemical reactions is negligible for most elements. ➢ Even in the case of the very lightest elements, where the ratio of neutron number to atomic number varies the most between isotopes, it usually has only a small effect, although it does matter in some circumstances (for hydrogen, the lightest element, the isotope effect is large enough to strongly affect biological systems). ➢ Since isotope is the older term, it is better known than nuclide, and is still sometimes used in contexts where nuclide might be more appropriate, such as nuclear technology and nuclear medicine.

Notation		
	A → mass number = Z + N = total number of Protons and Neutrons = (number of Nucleons) Z → Number of Protons (atomic number) ∴ N → Number of Neutrons (neutron number) = A - Z	

Same element or different chemical element?	Number of Protons? (Z ?)	Number of Neutrons (N ?)	Mass Number A = (Z + N) ?	Called as	Examples $^{A}_{Z}X$ X = Chemical element ; A = Protons + Neutrons ; Z = Number of Protons
Same element	Same	Different	Different	Isotopes	$^{16}_{8}O,\ ^{17}_{8}O,\ ^{18}_{8}O\ \ ;\ \ ^{1}_{1}H\ ,\ ^{2}_{1}H,\ ^{3}_{1}H$
Different elements	Different	Same	Different	Isotones	$^{13}_{6}C,\ ^{14}_{7}N,\ ^{15}_{8}O$
Different elements	Different	Different	Same	Isobars	Isobar = equal in weight $^{17}_{7}N,\ ^{17}_{8}O,\ ^{17}_{9}F$ (see β decay) ; F is fluorine $^{40}_{16}S,\ ^{40}_{17}Cl,\ ^{40}_{18}Ar,\ ^{40}_{19}K,\ ^{40}_{20}Ca$ $^{3}_{1}H,\ ^{3}_{1}He$ → exception (same Z)
Different elements	Different	Different	Isodiaphers	Isodiaphers	equal neutron excess (N_1-Z_1 = N_2-Z_2) Examples are Isodiaphers with neutron excess 1. $^{13}_{6}C,\ ^{15}_{7}N,\ ^{17}_{8}O$ A nuclide and its alpha decay product are Isodiaphers
Different elements	Different	Different	Same	Mirror nuclei	neutron and proton number exchanged (Z_1=N_2 *and* Z_2=N_1) $^{3}_{1}H,\ ^{3}_{2}He$
Same	Same	Same	Same	Nuclear isomers	same proton number and mass number, but with different energy states $^{99}_{43}Tc,\ ^{99m}_{43}Tc$; where m=metastable (long-lived excited state)

Nuclides' classification

- **Isotopes – equal *Z***
- **Isobars – equal *A***
- **Isotones – equal *N***
- **Isodiaphers – equal *N – Z***
- **Isomers – equal all the above**
- **Mirror nuclei – *Z* ↔ *N***

Terms	Explanation
Isotopes	➢ A set of nuclides with equal proton number (atomic number) of the **same chemical element** but different neutron numbers, are called isotopes of the element. OR ➢ The atoms of **an element**, which have the same atomic number but different mass numbers, are called Isotopes. OR ➢ Isotopes are nuclides that have the same atomic number and **are therefore the same element**, but differ in the number of neutrons. ○ Particular nuclides are still often loosely called "isotopes", but the term "nuclide" is the correct one in general (i.e., when Z is not fixed)
Examples of Isotopes: Hydrogen	$^{1}_{1}H$ → Protium; $^{2}_{1}H$ → Deuterium; $^{3}_{1}H$ → Tritium ; The three most stable isotopes of hydrogen: Protium (A = 1), deuterium (A = 2), and tritium (A = 3).  ➢ Hydrogen (1 proton) has three naturally occurring isotopes, denoted as $^{1}_{1}H$, $^{2}_{1}H$, $^{3}_{1}H$. The first two of these are stable, while $^{3}_{1}H$ has a half-life of 12.32 years. ➢ All heavier isotopes are synthetic and have a half-life less than one zeptosecond (10^{-21} second). Of these, $^{5}_{1}H$ is the most stable, and $^{7}_{1}H$ is the least. ➢ Hydrogen is the only element whose isotopes have different names that are in common use today. The $^{2}_{1}H$ isotope is usually called deuterium, while the $^{3}_{1}H$ isotope is usually called tritium. The symbols D and T are sometimes used for deuterium and tritium. The ordinary isotope of hydrogen, with no neutrons, is sometimes called "Protium".
Examples of Isotopes: Uranium and Thorium	➢ $^{238}_{92}U$, $^{235}_{92}U$, $^{234}_{92}U$ → Abundance is $^{238}_{92}U$ ➢ Uranium is a naturally occurring radioactive element that has no stable isotopes but two primordial isotopes (uranium-238 and uranium-235) that have long half-lives and are found in appreciable quantity in the Earth's crust, along with the decay product uranium-234. ➢ The standard atomic weight of natural uranium is 238.02891(3). Other isotopes such as uranium-232 have been produced in breeder reactors. ➢ Naturally occurring uranium is composed of 3 major isotopes, uranium-238 (99.2739–99.2752% natural abundance), uranium-235 (0.7198–0.7202%), and uranium-234 (0.0050–0.0059%). All three isotopes are radioactive, creating radioisotopes, with the most abundant and stable being uranium-238 with a half-life of 4.4683×10^{9} years (close to the age of the Earth). ➢ The isotope uranium-235 is important for both nuclear reactors and nuclear weapons because it is the only isotope existing in nature to any appreciable extent that is fissile, that is, can be broken apart by thermal neutrons. The isotope uranium-238 is also important because it absorbs neutrons to produce a radioactive isotope that subsequently decays to the isotope plutonium-239, which also is fissile. ** ➢ $^{232}_{90}Th$, $^{230}_{90}Th$ → Abundance of $^{232}_{90}Th = 99.98\%$ ➢ Although thorium (90Th) has 6 naturally occurring isotopes, none of these isotopes are stable; however, one isotope, 232Th, is relatively stable, with a half-life of 1.405×10^{10} years, considerably longer than the age of the Earth, and even slightly longer than the generally accepted age of the universe.
Isotopes of plutonium	➢ Plutonium (94Pu) is an artificial element, except for trace quantities resulting from neutron capture by uranium, and thus a standard atomic weight cannot be given. Like all artificial elements, it has no stable isotopes. It was synthesized long before being found in nature, the first isotope synthesized being 238Pu in 1940. Twenty plutonium radioisotopes have been characterized. The most stable are Pu-244, with a half-life of 80.8 million years, Pu-242, with a half-life of 373,300 years, and Pu-239, with a half-life of 24,110 years. All of the remaining radioactive isotopes have half-lives that are less than 7,000 years. This element also has eight meta-states, though none are very stable; all meta states have half-lives of less than one second. ➢ The isotopes of plutonium range in atomic weight from 228.0387 u (Pu-228) to 247.074 u (Pu-247). The primary decay modes before the most stable isotope, Pu-244, are spontaneous fission and alpha emission; the initial mode after is beta emission. The primary decay products before Pu-244 are isotopes of uranium and neptunium (neglecting the wide range of daughter nuclei created by fission processes), and the primary products after are isotopes of americium.

Examples of Isotopes: Others	➢ ${}^{16}_{8}O$, ${}^{17}_{8}O$, ${}^{18}_{8}O$ → Oxygen ; Abundance is ${}^{16}_{8}O$; Half-life of 16,17, 18 are stable ➢ ${}^{35}_{17}Cl$, ${}^{36}_{17}Cl$, ${}^{37}_{17}Cl$ → Chlorine → Abundance are ${}^{35}_{17}Cl$ (76%), ${}^{37}_{17}Cl$ (24%) ➢ ${}^{204}_{82}Pb$, ${}^{206}_{82}Pb$, ${}^{207}_{82}Pb$, ${}^{208}_{82}Pb$ → Lead → all stable isotopes ➢ ${}^{3}_{2}He$, ${}^{4}_{2}He$ → Helium → Abundance is ${}^{4}_{2}He$; Half-life stable ➢ ${}^{28}_{14}Si$, ${}^{29}_{14}Si$, ${}^{30}_{14}Si$ → Silicon → Abundance is ${}^{28}_{14}Si$; Half-life stable ➢ ${}^{107}_{47}Ag$, ${}^{109}_{47}Ag$ → Silver → Abundance both ${}^{107}_{47}Ag$, ${}^{109}_{47}Ag$; Half-life stable ➢ ${}^{197}_{79}Au$ → Gold has one stable isotope, ${}^{197}_{79}Au$, and 36 radioisotopes, with 195Au being the most stable with a half-life of 186 days ; Abundance of ${}^{197}_{79}Au = 100\%$; $most\ stable$ ➢ ${}^{63}_{29}Cu$, ${}^{65}_{29}Cu$ →Copper ; half-life stable ; Abundance of ${}^{63}_{29}Cu = 69.15\%$, ${}^{65}_{29}Cu = 30.85\%$ ➢ ${}^{11}_{6}C$, ${}^{12}_{6}C$, ${}^{13}_{6}C$ → Carbon → Abundance is ${}^{12}_{6}C$; Half-life of ${}^{12}_{6}C$, ${}^{13}_{6}C$ stable. ➢ ${}^{54}_{26}Fe$, ${}^{56}_{26}Fe$, ${}^{57}_{26}Fe$, ${}^{58}_{26}Fe$ → Iron → Abundance is ${}^{56}_{26}Fe$; Half-life of all 4 are stable ➢ ${}^{14}_{7}N$, ${}^{15}_{7}N$ → Nitrogen → Abundance is ${}^{14}_{7}N$; Half-life of both are stable ➢ ${}^{22}_{11}Na$, ${}^{23}_{11}Na$, ${}^{24}_{11}Na$ → Sodium → Abundance is ${}^{23}_{11}Na$ (100%) ; ${}^{23}_{11}Na$ is only stable ➢ ${}^{39}_{19}K$, ${}^{40}_{19}K$, ${}^{41}_{19}K$ → Potassium → Abundance is ${}^{39}_{19}K$ (93.3%) ; ${}^{39}_{19}K$, ${}^{41}_{19}K$ are stable ➢ ${}^{64}_{30}Zn$, ${}^{66}_{30}Zn$, ${}^{67}_{30}Zn$, ${}^{68}_{30}Zn$, ${}^{70}_{30}Zn$ → Zinc → Abundance is ${}^{64}_{30}Zn$ (93.3%) ; all 5 stable
Mass (1 u is same as 1 a.m.u.)	Mass of Proton = m_p = 1.672621898(21) × 10^{-27} kg = 1.007276466879(91) u ➢ $m_p \approx 1.673 \times 10^{-27}$ kg = 1.0073 u Mass of Neutron = m_n = 1.674927471(21) × 10^{-27} kg = 1.00866491588(49) u ➢ $m_n \approx 1.675 \times 10^{-27}$ kg = 1.0087 u Mass of Electron = m_e = 9.10938356(11)×10^{-31} kg = 0.0005485799 u (negligible compared to proton and neutron masses) ➢ $m_e \approx 9.110 \times 10^{-31}$ kg = 0.000549 u 1 u = 1.660539040223117 x 10^{-27} kg ≈ 1.660539 x 10^{-27} kg ; this implies 1.660539040223117 kg x 10^{-27} kg = 1 u ∴ Mass of Proton = 1.673 x 10^{-27} kg = $\frac{1.673 \times 10^{-27}\ kg}{1.660539 \times 10^{-27}\ kg} = \frac{1.673}{1.660539}$ = 1.0073 u ∴ Mass of Neutron = 1.675 x 10^{-27} kg = $\frac{1.675 \times 10^{-27}\ kg}{1.660539 \times 10^{-27}\ kg} = \frac{1.675}{1.660539}$ = 1.0087 u ∴ Mass of Electron = 9.110 x 10^{-31} kg = $\frac{9.110 \times 10^{-31}\ kg}{1.660539 \times 10^{-27}\ kg} = \frac{9.110 \times 10^{-4}}{1.660539}$ = 0.000549 u
Mass of Hydrogen atom	${}^{1}_{1}H$ → Protium; ${}^{2}_{1}H$ → Deuterium; ${}^{3}_{1}H$ → Tritium ; The three most stable isotopes of hydrogen: Protium (A = 1), deuterium (A = 2), and tritium (A = 3). Abundance of ${}^{1}_{1}H = 99.98\%$, ${}^{2}_{1}H = 0.02\%$ ∴ mass of one Hydrogen atom (*taking only* ${}^{1}_{1}H$ *and ignoring* ${}^{2}_{1}H$) = {mass of one Proton atom + mass of one electron} = (1.007276 u + 0.000549 u) = 1.007825 u **∴ Mass of Hydrogen atom ≈ 1.007825 u**
Mass of Chlorine atom	Mass of Proton = m_p = 1.672621898(21) × 10^{-27} kg = 1.007276466879(91) u Mass of Neutron = m_n = 1.674927471(21) × 10^{-27} kg = 1.00866491588(49) u Mass of Electron = m_e = 9.10938356(11) × 10^{-31} kg = 0.0005485799 u Mass of Chlorine ${}^{35}_{17}Cl$ = (17 x m_p + 18 x m_n + 17 x m_e) u = (17.12369993695847 + 18.1559684859282 + 0.0093258583) = 35.28899428118667 u ≈ 35.29 u

	Mass of Chlorine ${}^{36}_{17}Cl$ = (17 x m_p + 19 x m_n + 17 x m_e) u = (17.12369993695847 + 19.1646334018131 + 0.0093258583) = 36.29765919707157 u ≈ 36.30 u Mass of Chlorine ${}^{37}_{17}Cl$ = (17 x m_p + 20 x m_n + 17 x m_e) u = (17.12369993695847 + 20.173298317698 + 0.0093258583) = 37.30632411295647 u ≈ 37.31 u Abundance of Cl are ${}^{35}_{17}Cl$ (76%), ${}^{37}_{17}Cl$ (24%) ; ∴ the weighted average of these isotopes is = $\frac{(76 \times 35.29)+(24 \times 37.31)}{100}$ = 35.7748 ≈ 35.78 u, which agrees with the atomic mass of Chlorine.
Chlorine ÷ Hydrogen mass	This is to prove that → The atomic masses of various elements expressed in atomic mass unit (u) **are close to being integral multiples of the mass of a Hydrogen atom**. **Mass of Hydrogen atom ≈ 1.007825 u ; Take for example Chlorine atom →** Mass of Chlorine ${}^{35}_{17}Cl$ = 35.28899428118667 u ≈ 35.29 u (Taking into account mass of electrons of chlorine) Mass of Chlorine ${}^{36}_{17}Cl$ = 36.29765919707157 u ≈ 36.30 u (Taking into account mass of electrons of chlorine) Mass of Chlorine ${}^{37}_{17}Cl$ = 37.30632411295647 u ≈ 37.31 u (Taking into account mass of electrons of chlorine) $\frac{\textbf{Mass of Chlorine } {}^{35}_{17}Cl = 35.28899428118667 \text{ u}}{\textbf{Mass of Hydrogen atom } {}^{1}_{1}H = 1.007825\, u}$ = **35.0150 ≈ 35** $\frac{\textbf{Mass of Chlorine } {}^{36}_{17}Cl = 36.29765919707157 \text{ u}}{\textbf{Mass of Hydrogen atom } {}^{1}_{1}H = 1.007825\, u}$ = **36.0158 ≈ 36** $\frac{\textbf{Mass of Chlorine } {}^{37}_{17}Cl = 37.30632411295647 \text{ u}}{\textbf{Mass of Hydrogen atom } {}^{1}_{1}H = 1.007825\, u}$ = **37.0167 ≈ 37** The above analysis shows that atomic mass of Chlorine is nearly equal to integral multiple of atomic mass of hydrogen atom. Similarly, it is true for all elements.

Energy equivalent of atomic mass unit (u)

We know that 1 u = 1.660539040223117 x 10^{-27} kg ≈ 1.660539 x 10^{-27} kg
Einstein's' mass-energy equivalence is E = mc^2 ; where c = speed of light = 2.998 x 10^8 ms^{-1}
Suppose mass = 1 u = 1.660539 x 10^{-27} kg
∴ the energy equivalent of 1 u = 1.660539 x 10^{-27} kg x (2.998 x 10^8 ms^{-1})^2 = 14.925 x 10^{-11} J
= 14.925 x 10^{-11} J / 1.602 x 10^{-19} eV = 9.316 x 10^8 eV = 931.6 MeV

∴ 1 u = 1 a.m.u. = 931.6 MeV

SIZE OF THE NUCLEUS

The volume of the nucleus is directly proportional to the number of nucleons (mass number) constituting the nucleus. If R is the radius of the nucleus having a mass number A, then
$\frac{4}{3}\pi R^3 \propto A$ or $R \propto A^{1/3}$ OR $R = R_0A^{1/3}$; where $R_0 = 1.2 \times 10^{-15}$ m (=1.2 fm; 1 fm = 10^{-15} m)

- This means the volume of the nucleus, which is proportional to R^3 is proportional to A. Thus the density of nucleus is a constant, independent of A, for all nuclei.
- Different nuclei are like a drop of liquid of constant density. The density of nuclear matter is approximately 2.3×10^{17} kg m^{-3}.
- This density is very large compared to ordinary matter, say water, which is 10^3 kg m^{-3}. This is understandable, as we have already seen that most of the atom is empty. Ordinary matter consisting of atoms has a large amount of empty space.
- **Example: Given the mass of iron nucleus as 55.85u and A=56, find the nuclear density?**
  - We know that $R = R_0A^{1/3}$; where $R_0 = 1.2 \times 10^{-15}$ m
  - Given mass number A = 56 ; mass of iron = m_{fe} = 55.85u
  - We know that 1 u = 1.660539040223117 x 10^{-27} kg ≈ 1.660539 x 10^{-27} kg
  - ∴ 55.85 u = 55.85 x 1.660539040223117 x 10^{-27} kg ≈ 92.74 x 10^{-27} kg ; Mass = 92.74 x 10^{-27} kg
  - $volume = \frac{4}{3}\pi R^3$; since $R = R_0A^{1/3}$, then $volume = \frac{4}{3}\pi(R_0A^{1/3})^3 = \frac{4}{3}\pi(R_0A^{1/3})^3 = \frac{4}{3}\pi R_o^3\, A$
  - $Volume = \left[\frac{4}{3} \text{x } 3.14 \text{ x } (1.2 \text{ x } 10^{-15})^3 \text{ x } 56\right] m^3$ = 405.34 x 10^{-45} m^3

- $Nuclear\ Density = \frac{Mass}{Volume} = \frac{92.74 \times 10^{-27}\ kg}{405.34 \times 10^{-45}\ m^3}$ = 0.229 x 10^{18} kg/m^3 = **2.29 x 10^{17} kgm^{-3}**

∴ The density of matter in neutron stars (an astrophysical object) is comparable to this density. This shows that matter in these objects has been compressed to such an extent that they resemble a big nucleus.

Nuclear density:

- We know that 1 u = 1.660539040223117 x 10^{-27} kg ≈ 1.660539 x 10^{-27} kg
- We know that R = $R_0A^{1/3}$; where R_0 = 1.2 × 10^{-15} m

If an element X has mass number A, then the mass of the nucleus of the atom = A u (A a.m.u.)

∴ the mass of the nucleus in Kg = A x 1.660539040223117 x 10^{-27} kg ≈ A x 1.660539 x 10^{-27} kg

$volume\ of\ the\ nucleus = \frac{4}{3}\pi R^3$; then $volume = \frac{4}{3}\pi(R_0A^{1/3})^3 = \frac{4}{3}\pi(R_0A^{1/3})^3 = \frac{4}{3}\pi R_o^3\ A$

$$Nuclear\ Density, \rho = \frac{Mass}{Volume} = \frac{\text{Mass of the nucleus}}{\text{Volume of the nucleus}} = \frac{\text{A x } 1.660539 \times 10^{-27}\ \text{kg}}{\frac{4}{3}\pi\ [1.2 \times 10^{-15}]^3 \text{ x A } m^3}$$

= 0.229 x 10^{18} kg/m^3 = **2.29 x 10^{17} kgm^{-3}**

Note that A cancels in the above equation and hence density is independent of mass of nucleus (A)

- **The density of the nuclei of all the atoms is same as it is independent of mass number.**
- **The high density of the nucleus (≈10^{17} kgm^{-3})** suggests the compactness of the nucleus. Such examples of high densities are met in the form of neutron stars.

Energy equivalent of atomic mass unit (u) : Mass - Energy

We know that 1 u = 1.660539040223117 x 10^{-27} kg ≈ 1.660539 x 10^{-27} kg

Einstein's' mass-energy equivalence is E = mc^2 ; where c = speed of light = 2.998 x 10^8 ms^{-1}

Suppose mass = 1 u = 1.660539 x 10^{-27} kg

∴ the energy equivalent of 1 u = 1.660539 x 10^{-27} kg x (2.998 x 10^8 ms^{-1})^2 = 14.925 x 10^{-11} J

= 14.925 x 10^{-11} J / 1.602 x 10^{-19} eV = 9.316 x 10^8 eV = 931.6 MeV

∴ 1 u = 1 a.m.u. = 931.6 MeV

- Einstein showed from his theory of special relativity that it is necessary to treat mass as another form of energy. Before the advent of this theory of special relativity it was presumed that mass and energy were conserved separately in a reaction.
- However, Einstein showed that mass is another form of energy and one can convert mass-energy into other forms of energy, say kinetic energy and vice-versa. Einstein gave the famous mass-energy equivalence relation E = mc^2 ; where c = speed of light in vacuum ≈ 3 x 10^8 ms^{-1}

> **Example 13.2** Calculate the energy equivalent of 1 g of substance.
>
> **Solution**
>
> Energy, $E = 10^{-3} \times (3 \times 10^8)^2$ J
>
> $E = 10^{-3} \times 9 \times 10^{16} = 9 \times 10^{13}$ J
>
> Thus, if one gram of matter is converted to energy, there is a release of enormous amount of energy.

- Experimental verification of the Einstein's mass-energy relation has been achieved in the study of nuclear reactions amongst nucleons, nuclei, electrons and other more recently discovered particles. In a reaction the conservation law of energy states that the initial energy and the final energy are equal provided the energy associated with mass is also included.
- This concept is important in understanding nuclear masses and the interaction of nuclei with one another.

Mass Defect: Nuclear Binding Energy

- We know that nucleus is made up of protons and neutrons.
- It is expected that the "mass of nucleus" (say M) = "mass of protons" + "mass of neutrons"
- However, always **M < "mass of protons" + "mass of neutrons"**
  - Take for example Oxygen $^{16}_{8}O$
  - Mass of 8 neutrons = 8 x 1.00866 u
  - Mass of 8 protons = 8 × 1.00728 u

- Mass of 8 electrons = 8 × 0.00055 u ; Therefore, the expected mass of $^{16}_{8}O$ nucleus = proton mass + neutron mass = 8 × 2.01593 u = 16.12744 u.
- The **atomic** mass of $^{16}_{8}O$ found from **mass spectroscopy experiments** is seen to be 15.99053 u. Subtracting the mass of 8 electrons (8 × 0.00055 u) from this, we get the experimental mass of $^{16}_{8}O$ **nucleus** to be 15.98613 u. **Be careful whether mass of atom is given or mass of nucleus is given in the problem**
- Thus, we find that the mass of the $^{16}_{8}O$ nucleus is less than the total mass of its constituents by 0.14131 u.

- The difference between "sum of the masses of the nucleons constituting a nucleus" and the "rest of the nucleus" is known as "Mass defect" and is denoted by ΔM. and ΔM is given by
- **$\Delta M = [Zm_p + (A-Z)m_n] - M$; if M is the mass of nucleus given (generally mass of atom is known from experiment)**
- **$\Delta M = [Z(m_p+m_e) + (A-Z)m_n] - M$; if M is the mass of the atom given (see electron mass is also added)**
- For the above example of $^{16}_{8}O$ nucleus, the mass defect ΔM = (16.12744 - 15.98613) u = 0.14131 u

Very IMP:

- The difference between "sum of the masses of the nucleons constituting a nucleus" and the "rest of the nucleus" is known as "Mass defect" and is denoted by ΔM. ΔM is given by $\Delta M = [Zm_p + (A-Z)m_n] - M_N$; **M_N is the mass of the "nucleus".**
- This equation means that when Z free protons and (A-Z) free neutrons come in from infinity and combine together to form a nucleus, an amount of mass ΔM disappears. The disappeared mass reappears as equivalent energy $(\Delta M)c^2$ which is liberated during the formation of the nucleus. It is due to this energy that protons and neutrons remain bound in the nucleus.
  - **Conversely**, an amount $(\Delta M)c^2$ of external energy is required to break the nucleus into protons and neutrons. This is called the "binding energy" of the nucleus. **Thus, the binding energy (BE) of a nucleus is defined as the minimum energy required to separate its nucleons and place them at rest at infinite distance apart.**
  - ∴ $BE = (\Delta M)c^2 = [Zm_p + (A-Z)m_n - M_N]c^2$ ------------------------ (1)
- In the above equation (1), mass of the nucleus M_N is not known. What is known from experiment is the mass of the "atom" $^{A}_{Z}X$, that is M_A which includes the mass of Z electrons (Zm_e) and also the mass-equivalent of the energy binding the Z electrons to the nucleus.
  - If mass of **atom** M_A is given in the problem, then mass defect equation becomes $\Delta M = [Z(m_p+m_e) + (A-Z)m_n] - M_A$
  - If mass of **nucleus** M_N is given in the problem, then mass defect equation becomes $\Delta M = [Zm_p + (A-Z)m_n] - M_N$

- **Packing fraction = (mass defect) / A =** $\frac{\Delta M}{A}$

What is the meaning of the mass defect?

- It is here that Einstein's equivalence of mass and energy plays a role. Since the mass of the oxygen nucleus is less than the sum of the masses of its constituents (8 protons and 8 neutrons, **in the unbound state**), the equivalent energy of the oxygen nucleus is less than that of the sum of the equivalent energies of its constituents.
- If one wants to break the oxygen nucleus into 8 protons and 8 neutrons, this extra energy ($\Delta M \times c^2$), has to be supplied. This energy required E_b is related to the mass defect by $E_b = (\Delta M \times c^2)$
  - In a similar way, If a certain number of neutrons and protons are brought together to form a nucleus of a certain charge and mass, an energy E_b will be released in the process. The energy E_b is called the **binding energy of the nucleus**.
- The nuclear binding energy is a convenient measure of how well a nucleus is held together.
- A more useful measure of the binding between the constituents of the nucleus is the **binding energy per nucleon**, E_{bn}, which is the ratio of the binding energy E_b of a nucleus to the number of the nucleons (A) in that nucleus:
- **Binding energy per nucleon** = $E_{bn} = \frac{E_b}{A}$; we can think of binding energy per nucleon as the average energy per nucleon needed to **separate a nucleus into its individual nucleons.**

- We know that the "mass" of atomic particles (proton, neutron, electron etc.) are extremely small as compared to kilogram and hence can be conveniently expressed in a simpler unit called "atomic mass unit" (a.m.u.) and is denoted either by u or by a.m.u.
- Conversion from u to kg : 1 u = 1.660539040223117 x 10^{-27} kg ≈ 1.660539 x 10^{-27} kg
- Conversion from Kg to u : 1 kg = 0.6022140857740001 x 10^{27} u ≈ 0.6022 x 10^{27} u
- See below the masses of atomic particles and its range I kg and hence its convenient unit.

Mass of Proton = 1.673 x 10^{-27} (kg) = 1.67262189821 x 10^{-27} (0.6022140857740001 x 10^{27} u) = 1.007276467276108 u ≈ 1.0073 u
Mass of Neutron = 1.675 x 10^{-27} (kg) = 1.67492747121 x 10^{-27} (0.6022140857740001 x 10^{27} u) = 1.008664915812488 u ≈ 1.0087 u
Mass of Electron = 9.110 x 10^{-31} (kg) = 9.1093835611×10^{-31} (0.6022140857740001 x 10^{27} u) = 5.485799093212542 u ≈ 0.000549 u

- The **energy** equivalent of 1 u = 1.660539 x 10^{-27} kg x (2.998 x 10^{8} ms^{-1})^2 = 1.4925 x 10^{-10} J
- Expressing joule in eV, we get = 1.4925 x 10^{-10} J / 1.602 x 10^{-19} eV = 9.316 x 10^{8} eV = 931.6 MeV
- **Energy equivalent of 1 u = 931.6 MeV**
- **OR just 1 u = 931.6 MeV/c^2**
- For $^{16}_{8}O$ nucleus, the mass defect ΔM = 0.14131 u → here unit of ΔM is in modified "mass" unit
- ΔM = 0.14131 u = 0.14131 (u) = 0.14131 x 931.6 MeV/c^2 = 131.6 MeV/c^2
- ∴ The energy needed to separate $^{16}_{8}O$ into its constituents is thus 131.6 MeV/c^2.

Following figure is a plot of the binding energy per nucleon **E_{bn}** versus the mass number A for a large number of nuclei. We notice the following main features of the plot:

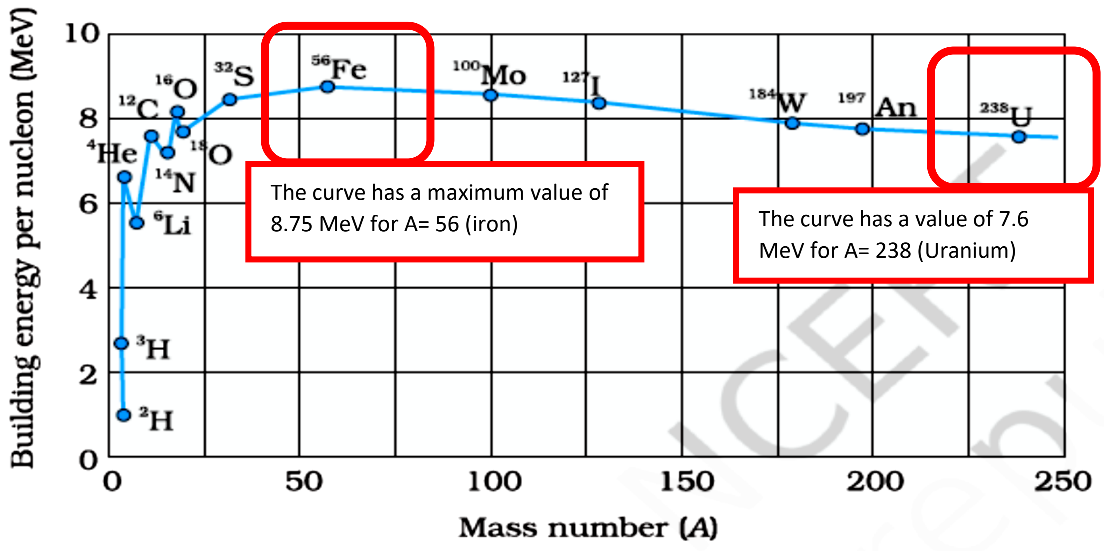


FIGURE 13.1 The binding energy per nucleon as a function of mass number.

<u>**Observations of the plot**</u>:
The binding energy per nucleon, E_{bn}, is practically constant, i.e. practically independent of the atomic number for nuclei of middle mass number (30 < A < 170). The curve has a maximum of about 8.75 MeV for A = 56 and has a value of 7.6 MeV for A = 238.
E_{bn} is lower for both light nuclei (A < 30) and heavy nuclei (A > 170).

<u>**We can draw 4 conclusions from the above two observations**</u>:
The force is attractive and sufficiently strong to produce a binding energy of a few MeV per nucleon.
The constancy of the binding energy in the range 30 < A < 170 is a consequence of the fact that the nuclear force is short-ranged.
Consider a particular nucleon inside a sufficiently large nucleus. It will be under the influence of only some of its neighbours, which come within the range of the nuclear force. If any other nucleon is at a distance more than the range of the nuclear force from the particular nucleon it will have no influence on the binding energy of the nucleon under consideration.
If a nucleon can have a maximum of ‘p’ neighbours within the range of nuclear force, its binding energy would be proportional to ‘p’. Let the binding energy of the nucleus be pk, where k is a constant having the dimensions of energy. If we increase A by adding nucleons they will not change the binding energy of a nucleon inside. Since most of the nucleons in a large nucleus reside inside it and not on the surface, the change in binding energy per nucleon would be small.
The binding energy per nucleon is a constant and is approximately equal to pk. The property that a given nucleon influences only nucleons close to it is also referred to as saturation property of the nuclear force.
Fission: A very heavy nucleus, say A = 240, has lower binding energy per nucleon compared to that of a nucleus with A = 120. Thus if a nucleus A = 240 breaks into two A = 120 nuclei, nucleons get more tightly bound. This implies energy would be released in the process. It has very important implications for energy production through fission (discussed later).
Fusion: Consider two very light nuclei ($A \leq 10$) joining to form a heavier nucleus. The binding energy per nucleon of the fused heavier nuclei is more than the binding energy per nucleon of the lighter nuclei. This means that the final system is more tightly bound than the initial system. Again energy would be released in such a process of fusion. This is the energy source of sun (discussed later).

Nuclear Force:

- The force that determines the motion of atomic electrons is the familiar Coulomb force. In the binding energy curve, we have seen that for average mass nuclei the binding energy per nucleon is approximately 8 MeV, which is much larger than the binding energy in atoms.
- Therefore, to bind a nucleus together there must be a strong attractive force of a totally different kind. It must be strong enough to overcome the repulsion between the (positively charged) protons and to bind both protons and neutrons into the tiny nuclear volume.
- We have already seen that the constancy of binding energy per nucleon can be understood in terms of its short-range. Many features of the nuclear binding force are summarised below. These are obtained from a variety of experiments carried out during 1930 to 1950.
    - The nuclear force is much stronger than the Coulomb force acting between charges or the gravitational forces between masses. The nuclear binding force has to dominate over the Coulomb repulsive force between protons inside the nucleus.
        - This happens only because the nuclear force is much stronger than the coulomb force.
        - The gravitational force is much weaker than even Coulomb force.
    - The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few femtometres (10^{-15} m). This leads to saturation of forces in a medium or a large-sized nucleus, which is the reason for the constancy of the binding energy per nucleon.
        - A rough plot of the potential energy between two nucleon as a function of distance is shown in the Fig. 13.2. The potential energy is a minimum at a distance r_0 of about 0.8 fm. This means that the force is attractive for distances larger than 0.8 fm and repulsive if they are separated by distances less than 0.8 fm.
    - The nuclear force between neutron-neutron, proton-neutron and proton-proton is approximately the same. The nuclear force does not depend on the electric charge. Unlike Coulomb's law or the Newton's law of gravitation there is no simple mathematical form of the nuclear force.

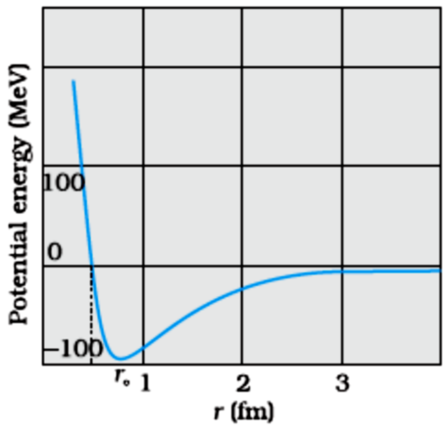
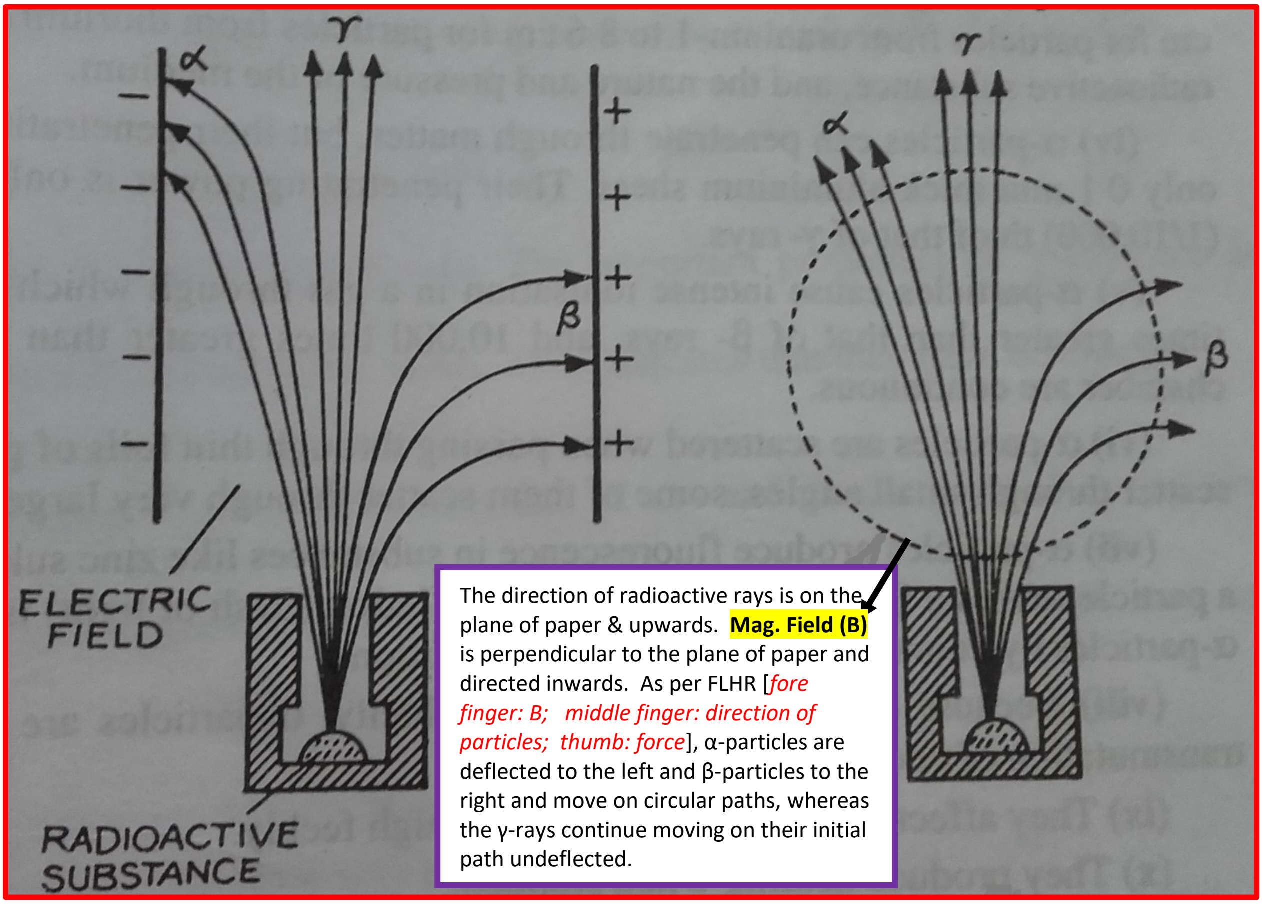


FIGURE 13.2 Potential energy of a pair of nucleons as a function of their separation. For a separation greater than r_0, the force is attractive and for separations less than r_0, the force is strongly repulsive.

Radioactivity:



The direction of radioactive rays is on the plane of paper & upwards. **Mag. Field (B)** is perpendicular to the plane of paper and directed inwards. As per FLHR [*fore finger: B; middle finger: direction of particles; thumb: force*], α-particles are deflected to the left and β-particles to the right and move on circular paths, whereas the γ-rays continue moving on their initial path undeflected.

- H. Becquerel discovered radioactivity purely by accident. In 1896, Becquerel observed that uranium and some of its salts emit spontaneously some invisible radiation, which penetrates through opaque substances and affects the photographic plate. These rays are called "radioactive rays" or "Becquerel rays". The spontaneous emission of rays from a substance is called "radioactivity" and such substances are called "radioactive" substances.
    - While studying the fluorescence & phosphorescence of compounds irradiated with visible light, he observed an interesting phenomenon. After illuminating some pieces of uranium-potassium sulphate with visible light, he wrapped them in black paper and separated the package from a photographic plate by a piece of silver. When, after several hours of exposure, the photographic plate was developed, it showed blackening due to something that must have been emitted by the compound and was able to penetrate both black paper and the silver.
- After the discovery of radioactivity in uranium, it was found that besides uranium, other elements like thorium, polonium, actinium etc. are also radioactive.
- In 1898, Piere Curie and Marie Curie discovered a new radioactive element called "radium". It is 106 times more radioactive than uranium. They worked hard to extract about 2 milligrams of radium from about 30 tons of Pitch blende (a kind a coal tar). For this Curies were honoured by Nobel Prize in 1903.

Nature and Properties of Radioactive Radiation:

- Rutherford studied the effect of electric and magnetic fields on the radioactive radiation emitted by different radioactive substances. He put a radioactive substance in a thick-walled lead box and passed the radiation emerging from a narrow opening in the box through an electrostatic field (See above left side figure).
- He observed that the radiation has 3 types of rays:
    - One that deflect towards the negative plate → α-rays
        - These are stream of particles, hence called α-particles. They are positively charged
    - One that deflect towards the positive plate → β-rays
        - These are stream of particles, hence called β-particles. They are negatively charged
        - It was observed that β-particles are deflected much more compared to α-particles. This shows that β-particles are very light compared to α-particles.
    - And the third that remain undeflected in the electric field → γ- rays
        - γ- rays are electrically neutral ; they are e-m waves like X-rays. They are also called "γ-photons".
- The same conclusion was drawn by passing the radioactive rays through a magnetic field perpendicular to their path. See figure for the explanation.

- Note that no radioactive substance emits both α-particles and β-particles simultaneously. Some substances emit α-particles and some other remit β-particles. γ- rays are emitted along with both α-particles and β-particles.

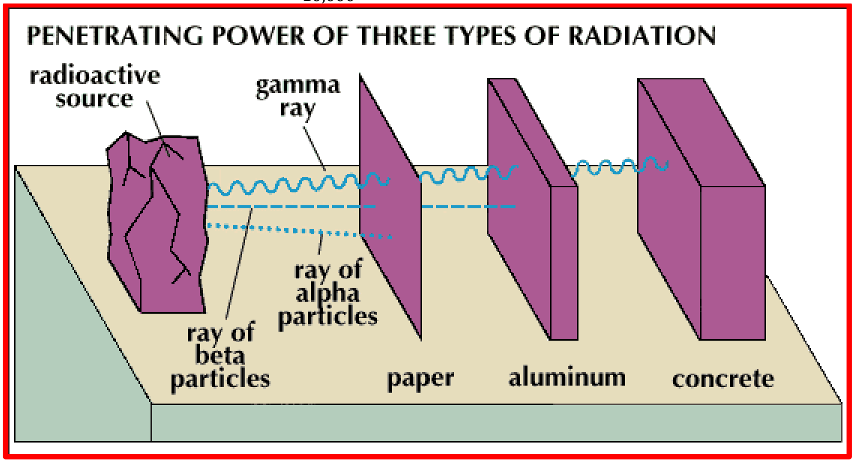
Properties of α-particles:

$\boldsymbol{Helium\ atom\ {}_{2}^{4}He - 2\ electrons = \ \alpha - particle}$ **(Info : Charge of electron = -1.602 x 10^{-19} C)**

Composition: 2 protons, 2 neutrons, 0 electrons ; Symbol : α , α^{2+}, He^{2+}, ${}_{2}^{4}He^{2+}$; Electric charge +2e = + 3.2 x 10^{-19} C

In fact, an α-particle is a **Helium nucleus** ; hence α-particle is also called "**doubly-ionized helium atom**" . Some of the important properties of α-particles are the followings:

- α-particles are deflected in electric and magnetic fields, and the direction of deflection indicates that they are positively charged. Their small deflection (compared to β-particle) shows that they are comparatively heavy particles.
- The velocity of α-particles < $\frac{1}{10}th$ of velocity of light. α-particles emitted by different elements have somewhat different velocities, however, the velocity of all α-particles emitted by the same element is same). Examples are …
  - Velocity of α-particles emitted by uranium-1 = 1.4 x 10^7 ms^{-1}
  - Velocity of α-particles emitted by Thorium-C' = 2.2 x 10^7 ms^{-1}
  - Due to this velocity-difference, there is some dispersion in α-particles in electric and magnetic fields.
- The range of α-particles in air (the distance travelled by α-particle in air at NTP) varies from 2.7cm for particles from uranium-1 to 8.6cm for particle from thorium C'. In general, the range varies with the radioactive substance, and the nature and pressure of the medium.
- α-particles can penetrate through matter, but their penetrating power is small. They are stopped by only 0.1mm thickness of aluminium sheet.
  - Penetrating power of α-particles = $\frac{1}{10}th$ of that of β-particles
  - Penetrating power of α-particles = $\frac{1}{10,000}th$ of that of γ-rays (γ-photons) (see below the diagram)



- α-particles cause intense ionisation in a gas through which they pass.
  - α-particles ionising power = 100 times greater than that of β-particles
  - α-particles ionising power = 10^4 times greater than that of γ-rays.
  - Their tracks in a cloud chamber are continuous.
- α-particles are scattered when passing through thin foils of gold or mica. While most of the particles scatter through small angles, some of them scatter through very larges angles, even greater than 90^0
- α-particles produce fluorescence in substances like zinc sulphide and barium platinocyanide. When a particle strikes a fluorescent screen, a scintillation (flash of light) is observed. We can cunt the number of α-particles by counting the number of scintillations.
- Because of their high emitting velocity, α-particles are used for bombarding the nuclei in the transmutation of one element into other.
- They affect photographic plate, though feebly.
- They produce heating when stopped.
- They cause incurable burns on human body.

Properties of β-particles:

A β-particle has a negative charge = 1.6 x 10^{-19} C, which is the charge of electron. **Actually, β-particles are fast-moving electrons (these are not the orbital electrons of the atom, but are emitted from the nucleus)**. Some of the important properties of β-particles are the followings:

- β-particles are deflected in electric and magnetic fields, and the direction of deflection indicates that they are negatively charged. Their large deflection (compared to α-particles) shows that they are comparatively much lighter particles than the α-particles.
- The velocity of β-particles vary from 1% to 99% of the velocity of light (only in velocity, the β-particles differ from the cathode rays). There is enough variation in the velocities of β-particles emitted by the same radioactive substance. This is why enough dispersion is found in these particles in electric and magnetic fields.

- Since the velocity of β-particles is of the order of velocity of light, their mass increases with increase in their velocity. If the rest mass of a β-particle be m_0, and that in the state of velocity v be m, then according to Einstein's theory of relativity, we have
  $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$; Where c = velocity of light; as v approaches c, value of m goes on increasing. When v = c, m = ∞
- The β-particles emitted by the same radioactive substance has a continuous distribution of kinetic energy between zero and a certain maximum value, and this maximum value is different for different substances. Hence the range of β-particles is not definite (while the range of α-particles is definite).
- The penetrating power of β-particles is about 100 times larger than the penetrating power of α-particles. They can pass through 1mm thick sheet of aluminium.
- β-particles ionise gases but their ionising power is much smaller, only $(1/100)^{th}$ of the ionising power of α-particles. As β-particles cannot produce ionisation continuously, their tracks in cloud chamber do not appear to be continuous.
- β-particles produced fluorescence in calcium tungstate, barium platinocyanide and zinc sulphide.
- β-particles affect photographic plate more than do the α-particles.

Properties of γ-rays:

Like X-rays, the γ-rays are e-m waves (or photons). The energy of γ-photons is very large (of the order of 10^6 eV). The relation between wavelength and the energy of γ-rays : **E = 1.24/ λ (μm) eV, where λ in μm**

Or λ = (1.24/ E) μm (E is expressed in eV)

For 1 MeV energy γ photons, λ = (1.24/10⁶ eV) μm = (1.24 x 10⁻⁶) x 10⁻⁶ m = 1.24 x 10⁻¹² m = 0.0124 Å = 0.00124 nm

Some of the important properties of γ-rays are the followings:

- γ-rays are not deflected by electric and magnetic fields, this indicates that they have no charge.
- γ-rays travel with the speed of light (3×10^8 ms^{-1})
- The penetrating power of γ-rays is much more than that of α- and β-particles. They can pass through 30 cm thick iron sheet.
- γ-rays ionize gases, but their ionisation power is very small compared to that of α- and β-particles.
- γ-rays produced fluorescence.
- γ-rays affect photographic plate more that do β-particles.
- γ-rays are diffracted by crystals in the same way s X-rays
- Though there is much similarity between X-rays and γ-rays, yet their sources of origin are different.
  - X-rays are produced by the transition of electrons in an atom from one energy level to another energy level, that is, it is an atomic property
  - Whereas γ-rays are produced from the nucleus, that is, it is a nuclear property.
- γ-rays are absorbed by substances & give rise to the phenomenon of pair-production,. When a γ-rays photon strikes the nucleus of some atom, its energy is converted into an electron & a positron (**positively-charged electron → particles with the same mass as electrons, but with a charge exactly opposite to that of electron**) and its own existence is extinguished:

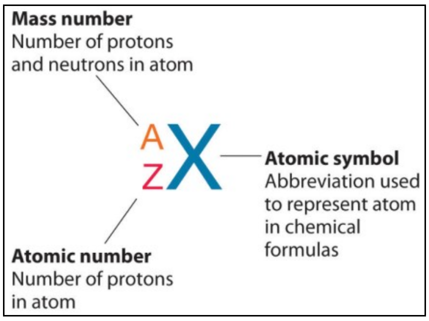
$$hf \rightarrow e^- + e^+$$

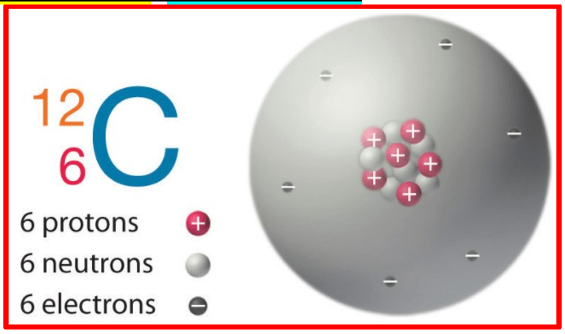
(γ-photon) (electron) (positron)

****

Comparison of the properties of α-particles, β-particles and γ-rays			
Property	**α-particle**	**β-particle**	**γ-rays**
Nature	Helium nucleus ^4_2He	Fast-moving electron	Electromagnetic waves
Charge	+ 3.2 x 10^{-19} C	- 1.6 x 10^{-19} C	zero
Rest mass	6.6 x 10^{-27} kg	9.1 x 10^{-31} kg	zero
Velocity	1.4 x 10^7 to 2.2 x 10^7 ms^{-1}	1 to 99% of the velocity of light	3 x 10^8 ms^{-1}
Ionising power	100 times that of β-particles	100 times that of γ-rays	Minimum
Penetrating power	Minimum	100 times that of α-particles	100 times that of β-particles

Law of radioactive decay and Decay Constant: Background:





We know that the atoms of a radioactive element emit α- , β- particles and γ-rays. As a result, both the atomic weight (mass number) and the atomic number (*number of protons → if this changes, it will be a different element*) of the element that has emitted those particles will get changed. Thus, the original radioactive atom is decayed and an atom of some new element is born. This phenomenon is called "radioactive decay" or "radioactive disintegration". For example, when an α-particle is emitted from a uranium atom, then it is converted into a thorium atom. This is represented as below..

$$^{238}_{92}U \text{------}\to\ ^{4}_{2}He +\ ^{234}_{90}Th$$

(*Uranium*) **(α-particle)** **(Thorium)**

If the new atom formed by α-emission is also radioactive then the new atom will also is decayed and a third new atom is obtained. Since, thorium is also radioactive, it is also decayed as shown below:

$$^{234}_{90}Th \text{------}\to\ ^{0}_{-1}\beta \ +\ ^{234}_{91}Pa + \bar{\nu}$$

(Thorium) **(β-particle)** **(Protoactinium)** **(anti-neutrino)**

Where "anti-neutrino" is having rest-mass and charge both zero. The above decay shows that the β-emission results in an atom whose mass number is same as that of the parent atom, but whose atomic number is higher by 1 than the parent atom.

- The radioactive decay continues until a stable atom is obtained (which is not radioactive).
- For example, uranium atom, through a chain of radioactive decay, is ultimately converted into a lead atom [$^{206}_{82}Pb$] which is the heaviest "stable" atom. All radioactive elements are finally converted into lead.
- **Radioactive decay is a nuclear process, that is, the radioactive rays are emitted from the nucleus of the atom.**
  - This process cannot be accelerated or slowed down by any physical or chemical process (for example, by changing temperature, pressure etc. or by mixing some other substance with the radioactive material).
  - This is because, the energy of chemical changes is of the order of 1 eV, whereas nuclear energy is of the order of MeV. So, a change of 1 or 2eV of energy cannot affect the nucleus. Similarly, ordinary temperature-changes cannot affect the rate of decay in radioactive material. **Actually, this process is a spontaneous disintegration of nucleus.**

Rutherford and Soddy Law for Radioactive Decay: (Important)

- Rutherford and Soddy made experimental study of the radioactive decay of various radioactive materials and found that the decay of all radioactive materials is governed by the same general law.
- According to this law, ***the rate of decay of radioactive atoms at any instant is proportional to the number of atoms present at that instant.***
- Let "N" be the number of atoms present in a radioactive substance at any instant "t"
- Let "ΔN" be the number of atoms that undergo decay in time "Δt", then $\frac{\Delta N}{\Delta t} \propto N$ or
- $\frac{\Delta N}{\Delta t} = \lambda N$ ---------------------------------- --- **(1)**
  - where λ is called the radioactive "**decay constant**" or "**disintegration constant**" or "**transformation constant**"
  - Do not confuse this with wavelength. In this section, λ is decay constant.

- Since, ΔN is the number of nuclei that decay, and hence is always positive. ∝α
- If dN is the change in N, which may have either sign, but here it is negative, because out of original N nuclei, ΔN have decayed, leaving (N–ΔN) nuclei.
- Therefore, the change in the number of nuclei in the sample is dN = – ΔN in time Δt. Thus the rate of change of N is (in the limit Δt → 0)
- $\frac{dN}{dt} = -\lambda N$ or $\frac{dN}{N} = -\lambda\, dt$; intergrating both sides , we get

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt \text{ -- } \mathbf{(2)}$$

$$\ln N - \ln N_0 = -\lambda\,(t - t_0) \text{ --- } \mathbf{(3)}$$

Here N_0 is the number of radioactive nuclei in the sample at some arbitrary time t_0 and N is the number of radioactive nuclei at any subsequent time t. Setting $t_0 = 0$ and rearranging Eq. (2) gives us

$$\ln \frac{N}{N_0} = -\lambda t \text{ --- } \mathbf{(4)}$$

$$\therefore\ N(t) = N_0\, e^{-\lambda t} \text{ -- } (5)$$

Equation (5) is the law of radioactive decay; where

- N_0 is the number of atoms in a radioactive substance at time t = 0
- N is the number of atoms in a radioactive substance after time t
- λ is the decay constant

As per eq (5), the decay of a radioactive substance is **exponential**, that is, the decay is rapid ion the beginning and then its rate decreases continuously. It means that a radioactive substance will take infinite time in decaying completely.

- Putting t = 1/λ, in eq (5) , we have $N(t) = N_0 e^{-1} = \frac{N_0}{e} = \frac{N_0}{2.718} = 0.368\, N_0$
- Thus, in a radioactive material, after a time-interval equal to the reciprocal of decay constant, the undecayed atoms are 36.8% of their initial number.
- Conversely, the radioactive decay constant may be defined as the reciprocal of the time during which the number of atoms in a radioactive substance reduces to 36.8% of their initial number.
  - *Note, for example, the light bulbs follow no such exponential decay law. If we test 1000 bulbs for their life (time span before they burn out or fuse), we expect that they will 'decay' (that is, burn out) at more or less the same time. The decay of radionuclides follows quite a different law, the law of radioactive decay represented by Eq. (5).*
- We are quite often interested in the decay rate R (-dN/dt) than in N itself. It gives us the number of nuclei decaying per unit time. For example, suppose we have some amount of radioactive substance. We need not know the number of nuclei present in it. But we may measure the number of emissions of α, β, or γ particles in a given time interval, say 10 seconds or 20 seconds.
- The total decay rate R of a sample is the number of nuclei disintegrating per unit time. Suppose in a time interval dt, the decay count measured is ΔN. Then dN = – ΔN. The positive quantity R is then defined as $R = -\,dN/dt$.
- Differentiating eq (5) $N(t) = N_0 e^{-\lambda t}$, we get $dN/dt = -\lambda N_0 e^{-\lambda t}$; since dN/dt = -R, we get
- $-R = -\lambda N_0 e^{-\lambda t}$
- $\mathbf{R = \lambda N_0 e^{-\lambda t}}$
- $\mathbf{R = R_0 e^{-\lambda t}}$ **; where** $\mathbf{R_0 = \lambda N_0}$ **-- (6)**
- This is equivalent to the law of radioactivity decay {eq(5)} , since you can integrate Eq. (6) to get back Eq. (5). Clearly, $R_0 = \lambda N_0$ is the decay rate at t = 0. The decay rate R at a certain time t and the number of undecayed nuclei N at the same time are related by

$$R = \lambda N \quad \text{---} \quad (7)$$

- The total decay rate R of a sample (*rather than the number of radioactive nuclei N*), is a more direct experimentally measurable quantity and is given a specific name: **activity** of that sample.
- The SI unit for activity is becquerel, named after the discoverer of radioactivity, Henry Becquerel.
- 1 becquerel is simply equal to 1 disintegration or decay per second. There is also another unit named "curie" that is widely used and is related to the SI unit as:

1 curie = 1 Ci = 3.7 × 10^{10} decays per second

1 curie = 3.7 × 10^{10} Bq

Units of Radioactivity

The activity of a radioactive substance is measured by the rate of disintegration of the substance. The traditional unit of activity is 'curie' (c). ***The curie is defined such that if*** $3{\cdot}7 \times 10^{10}$ ***disintegrations take place per second in a radioactive substance, then the activity of the substance is* 1 *curie.*** Thus

$$1 \text{ curie (c)} = 3{\cdot}7 \times 10^{10} \text{ disintegrations/second .}$$

1 curie is approximately the activity of one gram of radium. The smaller units are 'millicurie' (mc) and 'microcurie' (μc).

$$1 \text{ mc} = 10^{-3}\text{ c} = 3{\cdot}7 \times 10^{7} \text{ disintegrations/s .}$$

$$1\ \mu\text{c} = 10^{-6}\text{ c} = 3{\cdot}7 \times 10^{4} \text{ disintegrations/s.}$$

Another unit of activity is 'rutherford' (rd), which is, by definition,

$$1 \text{ rutherford} = 10^{6} \text{ disintegrations/s .}$$

The SI unit of activity is 'becquerel' (Bq); named after the discoverer of radioactivity :

$$1 \text{ becquerel (Bq)} = 1 \text{ disintegration/s}$$

$$1\text{ c} = 3{\cdot}7 \times 10^{10}\text{ Bq.}$$

so that

Half-life of a Radioactive Element:

- Different radionuclides differ greatly in their rate of decay. A common way to characterize this feature is through the notion of half-life.
- The atoms of a radioactive substance undergo continuous decay so that their number goes on decreasing.
- The time-interval in which the mass of a radioactive substance (or the number of atoms) is reduced to half its initial value is called the "half-life" of that substance.
- The half-life of radioactive substance is constant, but it is different for different substances.
- Half-life of a radionuclide (denoted by $T_{1/2}$) is the time it takes for a sample that has initially, say N_0 radio nuclei to reduce to $N_0/2$.
- We know that $N(t) = N_0 e^{-\lambda t}$,therefore $\frac{N}{N_0} = \frac{1}{2}$ when $t = T_{1/2}$
  - N_0 is the number of atoms in a radioactive substance at time t = 0
  - N is the number of atoms in a radioactive substance after time t
  - λ is the decay constant
- So, $\frac{1}{2} = e^{-\lambda T_{1/2}}$
- $-\lambda T_{1/2} = \ln(1/2) = \ln(2^{-1})$
- $-\lambda T_{1/2} = -\ln(2)$
- $\therefore \lambda T_{1/2} = \ln(2)$ (or $\log_e 2$)
- $\therefore \quad T_{1/2} = \frac{0.693}{\lambda}$ ------------------------- **(8)**
- Eq (8) is the relation between half-life and decay constant.
- We know that from eq (7) $R = \lambda N$, and $R_0 = \lambda N_0$
  - ($R_0 = \lambda N_0$ is the decay rate at t = 0)
- Clearly if N_0 reduces to half its value in time $T_{1/2}$, R_0 will also reduce to half its value in the same time according to above equation.
- Let half-life of a radioactive substance is represented as $T_{1/2}$.
- At time t = 0, if mass of the radioactive substance = m (or number of atoms = N_0), then (see also figure → → → → →)

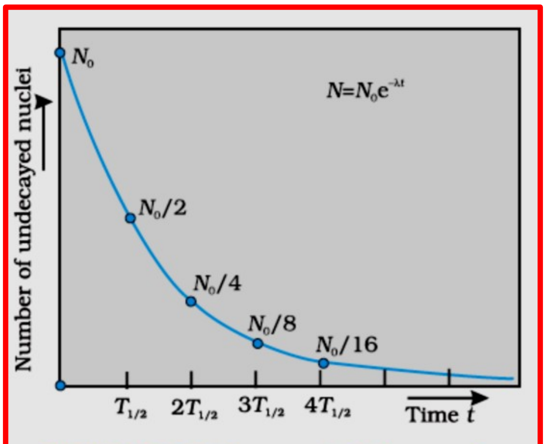


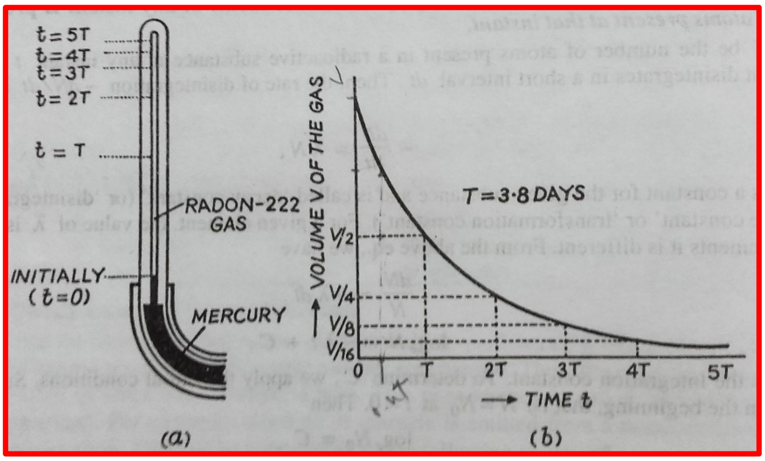
FIGURE 13.3 Exponential decay of a radioactive species. After a lapse of $T_{1/2}$, population of the given species drops by a factor of 2.

- In other words, the initial value of mass = m (or initial value of number of undecayed nuclei = N_0
  - **After time $T_{1/2}$, mass of the substance becomes m/2 and number of undecayed nuclei = $N_0/2$**
  - **After time $2T_{1/2}$, mass of the substance becomes m/4 and number of undecayed nuclei = $N_0/4$**
  - **After time $3T_{1/2}$, mass of the substance becomes m/8 and number of undecayed nuclei = $N_0/8$**
  - **After time $4T_{1/2}$, mass of the substance becomes m/16 and number of undecayed nuclei = $N_0/16$**

Info : (Note that in this paragraph, half-life is denoted by "T")

If the half-life of a radioactive substance be T, then after time T, the mass of the substance will be reduced to ½ ; after time 2T reduced to 1/4th , after time 3T reduced to 1/8th , after time 4T, reduced 1/16th **of its initial value**.

This can be seen by an experiment. In this figure (a), a gas named radon-222 is filled above mercury in a capillary tube at a constant pressure. This is a radioactive gas and its atoms are decayed into the atoms of a solid substance and are adhered to the walls of the tube. The half-life of this gas is T = 3.8 days. If this gas is left as such in the tube then, due to radioactive decay, its volume starts decreasing.



- Suppose, the initial volume (at time t = 0) of the gas is V.
- We find that after time T (= 3.8 days), it becomes V/2
- after time 2T (= 7.6 days) it becomes V/4
- after time 3T (=11.4 days), it becomes V/8
- after time 4T (= 15.2 days) it becomes V/16, and so on.
- If we plot a graph between the time and the volume of the gas, we get an exponential curve (fig (b) above).

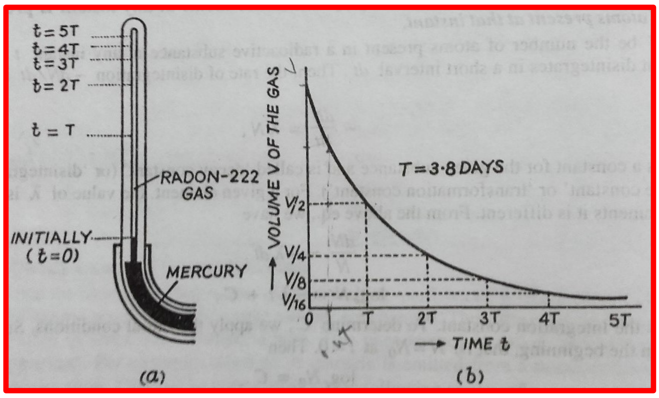
- We know that $N(t) = N_0 e^{-\lambda t}$ or simply $N = N_0 e^{-\lambda t}$; therefore $\frac{N}{N_0} = \frac{1}{2}$ when $t = T_{1/2}$
    - N_0 is the number of (**undecayed**) atoms present in a radioactive substance at time t = 0 ⇒ initial value
    - N is the number of atoms in a radioactive substance at a later time t
    - λ is the **decay constant**. For a given element, the value of λ is constant, but for different elements it is different.
- We know the relation between half-life and decay constant (by putting $t = T_{1/2}$ & $N = N_0/2$ in equation $N = N_0 e^{-\lambda t}$), we get
- $N_0/2 = N_0\, e^{-\lambda T_{1/2}}$; N_0 cancels off, we get
- $½ = e^{-\lambda T_{1/2}}$; this becomes $2 = e^{\lambda T_{1/2}}$ or $e^{\lambda T_{1/2}} = 2$
- $\lambda T_{1/2} = \log_e 2 = 0.6931 \approx 0.693$
- $T_{1/2} = \frac{0.693}{\lambda}$ **; This is the relation between half –life and decay constant ------------ (8)**

- The half-life of radioactive substance cannot be changed by any physical or chemical means. The half-life of an isotope of lead $^{214}_{82}Pb$ is 26.8 minutes. If this isotope forms some compound by chemical combination, even then its half-life will be 26.8 minutes.

- The half-life of natural radioactive substances and their isotopes vary from a fraction of a second to hundreds of millions of years. The half-life of an isotope of polonium $^{214}_{84}Po$ is 10^{-5} second, whereas the half-life of uranium $^{238}_{92}U$ is 4.5 x 10^9 years.

Estimation of Earth's age by Half-lives of Radioactive substances:

- **(Note that in this paragraph, half-life is denoted by "T", not by $T_{1/2}$)**
- Half-lives are very useful for Geologists in estimating the age of mineral deposits, rocks and the earth. For example, we know that uranium ($^{238}_{92}U$) is ultimately decay into stale lead($^{206}_{82}Pb$).
- Suppose, in some rock of the earth, the quantities of uranium and lead are found to be in the ratio of 3:1 ⇒ (out of 4 portions, remaining portion of uranium =3 and of lead = 1). We can assume that initially when the rock came into existence, there was no lead in it and that all the lead now present in it is due to the decay of uranium.
- We know that ½ of uranium decay into lead in time T (half-life) ⇒ so, remaining undecayed uranium is also ½.
- In similar lines, how much time is required for decay of 1/4th of uranium atoms? (⇒ remaining undecayed uranium is 3/4th)
- From this graph, it takes time = 0.4T for uranium to decay to 3/4th of its initial value.
- The half-life of uranium is 4.5 x 10^9 years. So, the time taken by uranium to decay to $\frac{3}{4}th$ of its original quantity is
- 0.4 X (4.5 x 10^9 years) = 1.8 x 10^9 years.
- Thus, we can say that the rock is about 1.8 x 10^9 years old.



- Similarly, it has been found by analysis that the oldest rock on earth is about 4 x 10^9 years old. This means that earth itself is about 4 x 10^9 years old.

- Info :
    - $\log_e(xy) = \log_e(x) + \log_e(y)$
    - $\log_e(x/y) = \log_e(x) - \log_e(y)$
    - $\log_e(x^y) = y \log_e(x)$
    - $\log_e(e) = 1$
    - $\log_e(1) = 0$
    - $\log_e(1/x) = -\log_e(x)$

- We know that $N = N_0 e^{-\lambda t}$

⇒ $-\lambda t = \log_e(\frac{N}{N_0})$; when t= 0, $\log_e(\frac{N}{N_0}) = 0$ → $\log_e N - \log_e N_0 = 0$ → $\log_e N = \log_e N_0$ ∴ $N = N_0$

Activity of Radioactive Substances

(Note that in this paragraph, half-life is denoted by "T", not by $T_{1/2}$)

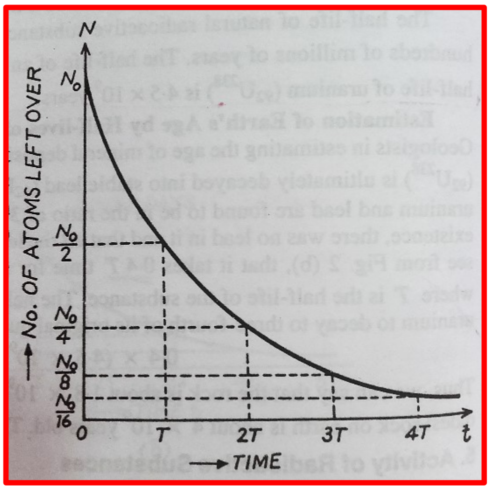
- The rate of decay of radioactive substance is called the "**activity**" R of the substance, that is,
- $R = -\frac{dN}{dt}$
- According to Rutherford and Soddy law, the rate of decay of a radioactive substance at any instant, is proportional to the number of atoms of substance left at that instant, that is,

$$-\frac{dN}{dt} \propto N$$

$-\frac{dN}{dt} = \lambda N$; where λ is decay constant for the given substance.

∴ The activity of the substance at any instant is also proportional to the number of its atoms left at that instant, thus, $R \propto N$ or $R = \lambda N$

- Suppose, initially (at time t =0) the number of atoms in a substance is N_0 and the half-life of the substance is T. Then, after one half-time (means after time T), the number of atoms left in the substance will be $N_0/2$. Therefore,
- At time t = T, $N = N_0/2$, the number of atoms left is $N = N_0\,[1/2]^1$
- At time t = 2T, $N = N_0/4$, the number of atoms left is $N = N_0\,[1/2]^2$
- At time t = 3T, $N = N_0/8$, the number of atoms left is $N = N_0\,[1/2]^3$
- At time t = 4T, $N = N_0/16$, the number of atoms left is $N = N_0\,[1/2]^4$
- Thus, the number of atoms left after n half-lives is given by
    - So, at time t = nT, $N = N_0/2^n$, the number of atoms left is $N = N_0\,[1/2]^n$



- Clearly, the number of atoms of a radioactive substance, that is, the radioactive activity of the substance decreases with time. The decrease is exponential, as shown by the graph between "the number of atoms left in a substance or activity" and the "time". The half-life T can be read from this graph.

****

Statistical Nature of Radioactive Decay:

- The radioactive decay of a substance is a nuclear process in which radioactive emission takes place from the nuclei of the atoms. Clearly, we can study the radioactive decay of a substance in bulk only, not of any particular atom. This study does not give any information about any individual atom.
- For example, by Rutherford and Soddy law, it is definite that a certain number of atoms will decay in a given time-interval, but it is not definite which atom will decay when. A particular atom may decay this very moment, after a few hours, after a few days, after a few years, or it may not decay for thousands of years to come.
- Actually, the decay of a particular atom is just a matter of chance; it does not depend upon the age or the past history of the atom. If today the probability of decay of an atom in a given time-interval is 1 in 10^6, then even after one thousand years (if this atom has decayed) the probability of decay will remain the same. Thus the radioactive decay of substances is statistical in nature.
- The nature of radioactive decay can be observed by a simple experiment. The half-life of a given radioactive substance is constant, but if we count the number of α or β particles emitted by it at intervals of 1 minute, we get different numbers like

12 9 13 11 8 14 7 ....

- Thus, in some minute 14 (or even more) particles may be emitted, while in some other minute 7 (or even less) may be emitted. If, however, we take the average of a large number of observations, it will be approximately 10 and we will say that substance is emitted 10 particles per minute.
- If we count the particles at intervals of 20 minutes, then we may get some of the following numbers:

190 205 210 208 187 213 182 ....

- The average of these numbers is nearly 200 and again we shall say that about 10 particles are emitted by the substance per minute. But now the variation in our observations is much smaller. Hence the observations should be taken for longer time-intervals

Average or Mean-life of a Radioactive Element (τ):

- Radioactive decay is a statistical process. Which means → which atom of a radioactive substance will decay when, it cannot be forecasted. An atom may take from 0 to infinite time to decay.
- The average of the lives all the atoms in a radioactive substance is called the "average life" or "mean life" of that substance. It is denoted by the symbol τ.
- We know that $N(t) = N_0 e^{-\lambda t}$ or simply $N = N_0 e^{-\lambda t}$; therefore $\frac{N}{N_0} = \frac{1}{2}$ when $t = T_{1/2}$
  - N_0 is the number of (**undecayed**) atoms present in a radioactive substance at time t = 0 ⇒ initial value
  - N is the number of atoms in a radioactive substance at a later time t
  - λ is the **decay constant**. For a given element, the value of λ is constant, but for different elements it is different.
- The **rate of decay of radioactive substance** is called the "**activity**" R of the substance, that is, $R = -\frac{dN}{dt}$
- $\therefore R(t) = \lambda N_0 e^{-\lambda t}$ → This is equivalent to the law of radioactivity decay ($\lambda N_0 = R_0$)
- The number of nuclei which decay in the time interval t to $(t + \Delta t)$ is $R(t)\Delta t$ [$= \lambda N_0 e^{-\lambda t} \Delta t$].
- Each of them has lived for time t. Thus the total life of all these nuclei = $t\,\lambda N_0 e^{-\lambda t}\Delta t = \lambda N_0 (t e^{-\lambda t}\Delta t)$
- It is clear that some nuclei may live for a short time while others may live longer. Therefore to obtain the mean life, we have to integrate this expression over all times from 0 to ∞, and divide by the total number N_0 of nuclei at t = 0. Thus,

$$\tau = \frac{\lambda N_0 \int_0^\infty t e^{-\lambda t} dt}{N_0} = \lambda \int_0^\infty t e^{-\lambda t} dt$$; integrating by parts, we get

$$\tau = \lambda \left[\left\{ \frac{t e^{-\lambda t}}{-\lambda} \right\}_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right]$$

$$\tau = \lambda \left[0 - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right]$$

$$\tau = \lambda \left[0 + \frac{1}{\lambda} \left\{ \frac{e^{-\lambda t}}{-\lambda} \right\}_0^\infty \right]$$

$$\tau = \frac{-\lambda}{\lambda^2} \left[e^{-\lambda t} \right]_0^\infty$$

$$\tau = \frac{-1}{\lambda} \left[e^{-\infty} - e^{0} \right]$$; Since $e^{-\infty} = 0$ and $e^0 = 1$, we get

$$\tau = \frac{-1}{\lambda}[0 - 1] = \frac{1}{\lambda}$$

$$\therefore \tau = \frac{1}{\lambda}$$; The average or mean life of a radioactive substance is reciprocal of the decay constant of the substance.

Relation between Half-life ($T_{1/2}$) and Mean-life (τ)

We know that $T_{1/2} = \frac{0.693}{\lambda}$ and $\tau = \frac{1}{\lambda}$

$$\therefore T_{1/2} = 0.693\,\tau \quad \text{or} \quad T_{1/2} = \tau \log_e 2$$

$$\therefore \tau = 1.443\, T_{1/2}$$

Thus, Mean-life is longer than Half-life. The reason is that the last few atoms of the radioactive substance have very long life.

We also know that the activity $R = \lambda(N_0 e^{-\lambda t})$ and $N = N_0 e^{-\lambda t}$; so $R = \lambda N$

Therefore Activity (R) in terms of half-life of a radioactive substance is given by

Since, $T_{1/2} = \frac{0.693}{\lambda}$; therefore $\lambda = \frac{0.693}{T_{1/2}}$

$$R = \lambda N = \frac{0.693\, N}{T_{1/2}}$$

$\therefore R = \frac{0.693\, N}{T_{1/2}}$ **; This shows that activity** $\propto \frac{1}{half-life\,(T_{1/2})}$

More is the half-life, less is the activity of the substance and vice-versa.

Example 13.4 The half-life of $^{238}_{92}U$ undergoing α-decay is 4.5×10^9 years. What is the activity of 1g sample of $^{238}_{92}U$?

Solution

$T_{1/2} = 4.5 \times 10^9$ y
$= 4.5 \times 10^9$ y x 3.16×10^7 s/y
$= 1.42 \times 10^{17}$ s

One k mol of any isotope contains Avogadro's number of atoms, and so 1g of $^{238}_{92}U$ contains

$$\frac{1}{238 \times 10^{-3}} \text{ kmol} \times 6.025 \times 10^{26} \text{ atoms/kmol}$$

$= 25.3 \times 10^{20}$ atoms.

The decay rate R is

$R = \lambda N$

$$= \frac{0.693}{T_{1/2}} N = \frac{0.693 \times 25.3 \times 10^{20}}{1.42 \times 10^{17}} s^{-1}$$

$= 1.23 \times 10^4\ s^{-1}$

$= 1.23 \times 10^4$ Bq

Example 13.5 Tritium has a half-life of 12.5 y undergoing beta decay. What fraction of a sample of pure tritium will remain undecayed after 25 y.

Solution

By definition of half-life, half of the initial sample will remain undecayed after 12.5 y. In the next 12.5 y, one-half of these nuclei would have decayed. Hence, one fourth of the sample of the initial pure tritium will remain undecayed.

What is meant by the 'half-life' of a radioactive element? What percentage of a given mass of a radioactive substance will be left undecayed after four half-lives?

→ The time-interval in which the mass (or number of atoms) of a radioactive element decays to on-half of its initial value, is called the 'half-life' of the element.

Thus, if the initial quantity of a radioactive element be N_0, then the quantity N of the element **left** after n half-lives is given by

$$N = N_0\,[1/2]^{\,n}$$

Therefore, the mass of the element remaining after 4 half-lives is given by

$$N = N_0\,[1/2]^{\,4} = \frac{N_0}{16} = 6.25\%$$

So, percentage of a given mass of a radioactive substance that is left undecayed after 4 half-lives = 6.25%

So, still 6.25% of undecayed radioactive substance is present after 4 half-lives.

The half-life of radon is 3.8 days. Calculate how much radon will be left out of 1024 milligram after 38 days?

→ Given initial quantity of radon N_0 = 1024mg, $T_{1/2}$ = 3.8 days ; N = ? after 38 days

By the definition of half-life, if N_0 be the initial quantity of a radioactive element, then the quantity left (undecayed) after n half-lives is given by

$$N = N_0\,[1/2]^{\,n}$$

$T_{1/2}$ = 3.8 days ; therefore, the number of half-lives in 38 days is n = 38/3.8 = 10. $\therefore \mathbf{n = 10}$

Therefore, the mass of radon remaining after 10 half-lives is given by

$$N = N_0\,[1/2]^{\,10} = \frac{1024\ mg}{2^{10}} = \frac{1024\ mg}{1024} = 1.0 \text{ mg} \quad \therefore \mathbf{N = 1\ mg}$$

So, percentage of radon that is left undecayed after 10 half-lives = 0.098%

So, still 0.098% of undecayed radon is present after 10 half-lives.

1 mg radium has 2.68 X 10^{18} atoms, its half-life is 1620 years. How many radium atoms will disintegrate from 1mg of pure radium in 3240 years?

→ Given $T_{1/2}$ = 1620 years ; the number of half-lives in 3240 years is n = 3240/1620 = 2 $\therefore \mathbf{n = 2}$

Given initial quantity of radium N_0 = 1mg

Hence the quantity of undisintegrated radium after 2 half-lives (n =2) is

$$N = N_0\,[1/2]^{\,n} = 1\text{mg x } [1/2]^2 = 1\text{mg} / 4 = 0.25\text{mg}$$

$$\therefore \textbf{mass of undisintegrated radium} = \mathbf{0.25mg}$$

$$\therefore \textbf{mass of disintegrated radium} = \mathbf{(1\ mg - 0.25\ mg) = 0.75mg}$$

Number of atoms in it = 0.75 mg x (2.68 X 10^{18} atoms) = ¾ x 2.68 x 10^{18} = 3 x 0.67 X 10^{18} = 2.01 X 10^{18}

Therefore, number of radium atoms disintegrated in 2 half-lives = 2.01 X 10^{18} atoms (out of initial value of 2.68 X 10^{18} atoms)

The half-life of radium is 1600 years. After how many years 25% of a radium block remains undecayed?

→ Suppose the initial quantity of radium is N_0. Then the quantity left after n half-lives will be

$$N = N_0\,[1/2]^{\,n}$$

Here, N = 25% of N_0 ∴ $N_0/4 = N_0[1/2]^n$ → $[1/2]^n$ = ¼ = $[1/2]^2$ ∴ n = 2

∴the time of disintegration = half –life X number of half-lives = 1660 years X 2 = 3200 years

The half-life of a radioactive substance is 1.192 X 10^7 s against α-decay. Calculate the decay rate for 3.18 X 10^{15} atoms of the substance

→ According to Rutherford and Soddy law, the rate of radioactive decay at any time t is given by $-\frac{dN}{dt} = \lambda N$, where λ is decay constant and N is the number atoms left at time t.

We know that $\boldsymbol{\lambda} = \frac{\mathbf{0.6931}}{T_{1/2}} = \frac{\mathbf{0.6931}}{\mathbf{1.192 \times 10^7}}$ = **0.5815 X 10^{-7} s^{-1}** ∴ λ = **0.5815 X 10^{-7} s^{-1}**

Given N = 3.18 X 10^{15} atoms ; substituting the values of λ and N in equation $-\frac{dN}{dt} = \lambda N$, we get

$\left|\frac{dN}{dt}\right|$ = (0.5815 X 10^{-7} s^{-1}) X 3.18 X 10^{15} atoms = 1.85 X 10^8 atoms/second

∴the magnitude of "decay rate R" = - dN/dt = $\left|\frac{dN}{dt}\right|$ = 1.85 X 10^8 atoms/second

- The level of activity of a sample of substance is directly proportional to the mass of the sample of the substance. For example : A 50 g sample of a radioactive substance has twice the activity of 25 g sample of that substance. The level of activity (disintegration/second) of a radioactive substance is measured by a device known as ***Geiger Muller Counter.***
- ***Instantaneous rate of change of the activity of a radioactive substance is inversely proportional to the square of its half-life.***

We know that, instantaneous activity of a radioactive substance is given by,

$$R = R_0 e^{-\lambda t}$$

Differentiating both sides w.r.t. 't', we get

$$\frac{dR}{dt} = R_0(-\lambda)e^{-\lambda t} = -\lambda R_0 e^{-\lambda t} = -\lambda R = -\lambda(\lambda N) = -\lambda^2 N$$

But $\lambda \frac{0.6931}{T_{1/2}}$, where $T_{1/2}$ is half life of a radioactive substance.

$$\therefore \quad \frac{dR}{dt} = -\left(\frac{0.6931}{T_{1/2}}\right)^2 N$$

or

$$\frac{dR}{dt} \propto \frac{1}{T_{1/2}^2}$$

Thus, instantaneous rate of change of activity of a radioactive substance is proportional to square of its half life.

Nuclear Reactions:

(What is a nuclear reaction? State the laws of conservation obeyed in a nuclear reaction. Distinguish between nuclear reactions and chemical reactions)

- The process by which a stable element is changed to another element when it is bombarded by an energetic particle (i.e., projectile), is called nuclear reaction. For example.

$^{27}_{13}Al$	$+$	$^{4}_{2}He$	$\rightarrow$	$^{30}_{14}Si$	$+$	$^{1}_{1}H$	$+$	**Q**
(target element)		*(α − particle or projectile)*		**(product element)**		**(proton or outgoing particle)**		**(energy)**

- First nuclear reaction was performed by Rutherford in 1919, He bombarded nitrogen with α-particle and obtained oxygen and a proton. This reaction is given below

$^{14}_{7}N + \ ^{4}_{2}He \rightarrow \ ^{1}_{1}H + \ ^{17}_{8}O$. This reaction can also be written as $^{14}_{7}N(\alpha, p) \ ^{17}_{8}O$

Following physical quantities are conserved in a nuclear reaction:

- "**Atomic Number**" (Z) is conserved ⇒ total charge before and after nuclear reaction is conserved ; ∑Z = 0
  - For example, in a nuclear reaction $^{14}_{7}N + \ ^{4}_{2}He \rightarrow \ ^{1}_{1}H + \ ^{17}_{8}O$
  - **Z = (2 + 7) = 9 before the nuclear reaction and Z = (1 + 8) = 9 after the nuclear reaction.**
- "**Mass Number**" (A) is conserved. Atomic mass (A) before and after nuclear reaction is conserved ; ∑A = 0
  - In the above example, A = (14 + 4) = 18 before the reaction and A = (1 + 17) = 18 after the reaction.
- "**Linear Momentum**" is conserved. Linear momentum of the particles before the nuclear reaction is equal to the linear momentum of the particles after the nuclear reaction ; ∑P = 0
- "**Angular Momentum**" is conserved. Angular momentum of the particles before the nuclear reaction is equal to the angular momentum of the particles after the nuclear reaction ; ∑L = 0

Chemical reaction	Nuclear reaction
In a chemical reaction, no new element is formed. For example, $2H_2 + O_2 \rightarrow 2H_2O$	In a nuclear reaction, new elements are formed $^{14}_{7}N + \ ^{4}_{2}He \rightarrow \ ^{1}_{1}H + \ ^{17}_{8}O$
A chemical reaction is said to be balanced, if number of atoms of elements before and after reaction are equal. For example, in a chemical reaction $2H_2 + O_2 \rightarrow 2H_2O$, number of hydrogen atoms + number of oxygen atoms equal before and after the chemical reaction	A nuclear reaction is said to be balanced, if atomic number (Z) and the mass number (A) are equal before and after the nuclear reaction. For example, in $^{14}_{7}N + \ ^{4}_{2}He \rightarrow \ ^{1}_{1}H + \ ^{17}_{8}O$, the atomic number (Z) and the mass number (A) before reaction are equal to atomic number and atomic mass after reaction.
In a chemical reaction, energy absorbed or released is small (in eV per atom)	In a nuclear reaction, energy absorbed or released is large (in MeV per nucleon)

Q-value or Energy of Nuclear reaction:

- The energy absorbed or released during nuclear reaction is known as Q-value of nuclear reaction. It is equal to the difference between the kinetic energy of products and reactants. That is,
- Q-value = KE of products – KE of reactants.
- Q-value of a nuclear reaction can also be defined as the difference between the mass of reactants and the mass of products. That is,
- Q-value = (mass of reactants – mass of products)c^2
- **Types of nuclear reactions on the basis of energy exchange**:
  - **Endothermic or endoergic nuclear reaction**: If mass of products of a nuclear reaction is > than the mass of the reactants of the nuclear reaction, then the value of Q is negative. (Q < 0). Such a nuclear reaction is known as endothermic reaction. For endothermic reaction to proceed, energy is supplied from outside.
  - **Exothermic or exoergic nuclear reaction**: If mass of reactants of a nuclear reaction is > than the mass of products of the nuclear reaction, then the Q-value is positive (Q > 0). Such a nuclear reaction is known an exothermic reaction. During exothermic reaction, the energy is released.
- It may be noted that exothermic reaction takes place spontaneously. Thus, radioactive decay process is always exothermic.

NUMERICAL EXAMPLE. **Find Q value of the following nuclear reaction :**

$$_7N^{14} + {}_2He^4 \longrightarrow {}_8O^{17} + {}_1H^1.$$

Given, mass of $_7N^{14}$ = 14·003 a.m.u., mass of $_2He^4$ = 4·002 a.m.u.

mass of $_8O^{17}$ = 16·999 a.m.u. mass of $_1H^1$ = 1·008 a.m.u.

Solution.

Mass of reactants = mass of $_7N^{14}$ + mass of $_2He^4$ = 14·003 + 4·002 = 18·005 a.m.u.

Mass of products = mass of $_8O^{17}$ + Mass of $_1H^1$ = 16·999 + 1·008 = 18·007 a.m.u.

$\therefore$ Q value of nuclear reaction = 18·005 – 18·007 = – 0·002 a.m.u. = – 0·002 × 931 MeV = – 1·862 MeV. ($\because$ 1a.m.u. = 931 MeV)

Since Q < 0, therefore reaction is ***endothermic***. This reaction will take place only when energy equal to 1·862 MeV is supplied from outside.

NUMERICAL EXAMPLE. **Find the Q value of the following reaction, $_5B^{10} + {}_0n^1 \longrightarrow {}_3Li^7 + {}_2He^4$. Given mass of $_5B^{10}$, $_0n^1$, $_3Li^7$ and $_2He^4$ is 10·013 a.m.u, 1·008 a.m.u. 7·016 a.m.u. and 4·002 a.m.u. respectively.**

Solution. Mass of reactants = mass of $_5B^{10}$ + mass of $_0n^1$

= (10·013 + 1·008) a.m.u = 11·021 a.m.u

Mass of products = mass of $_3Li^7$ + mass of $_2He^4$

= (7·016 + 4·002) a.m.u = 11·018 a.m.u

$\therefore$ Q-value of nuclear reaction = (11·021 – 11·018) a.m.u = 0·003 a.m.u

= 0·003 × 931 MeV ($\because$ 1 a.m.u. = 931 MeV)

= 2·793 MeV

Since, Q > 0, therefore, given reaction is **exothermic.**

Alpha decay:

A well-known example of alpha decay is the decay of uranium $^{238}_{92}U$ to thorium $^{234}_{90}Th$ with the emission of a helium nucleus ^4_2He

$$\underset{(Uranium)}{^{238}_{92}U} \text{ -------}\rightarrow \underset{(\alpha\text{-particle})}{^4_2He} + \underset{(Thorium)}{^{234}_{90}Th} \qquad \textbf{(α-decay)}$$

In α-decay, the mass number of the product nucleus (daughter nucleus) is 4 less than that of the decaying nucleus (parent nucleus), while the atomic number decreases by 2.

In general, α-decay of a parent nucleus ^A_ZX results in a daughter nucleus $^{A-4}_{Z-2}Y$

$$\underset{(Parent\ nucleus)}{^A_ZX} \rightarrow \underset{(daughter\ nucleus)}{^{A-4}_{Z-2}Y} + \underset{(\alpha\text{-particle})}{^4_2He} + \underset{\text{(Disintegration energy or Q-value)}}{Q}$$

From Einstein's mass-energy equivalence relation **E = mc²** and energy conservation, it is clear that this spontaneous decay is possible only when the total mass of the decay products is <u>less than</u> the mass of the initial nucleus. **This difference in mass appears as kinetic energy of the products.**

By referring to a table of nuclear masses, one can check that the total mass of $^{234}_{90}Th$ and $^{4}_{2}He$ is indeed **less than** that of $^{238}_{92}U$.

<u>**The disintegration energy or the Q-value of a nuclear reaction**</u> is the difference between the initial mass energy and the total mass energy of the decay products. For α-decay

$$Q = (m_X - m_Y - m_{He})\, C^2$$

Q is also the **net** kinetic energy gained in the process or, if the initial nucleus X is at rest, the kinetic energy of the products. Clearly, Q > 0 for exothermic processes such as α-decay. The disintegration energy is shared by daughter nucleus $^{A-4}_{Z-2}Y$ and alpha particle $^{4}_{2}He$ in the form of kinetic energy.

<u>**Examples of alpha decay:**</u>

- $^{238}_{92}U \text{-------}\!\!\rightarrow\ ^{234}_{90}Th\ +\ ^{4}_{2}He$
- $^{242}_{94}Pu \text{-------}\!\!\rightarrow\ ^{238}_{92}U\ +\ ^{4}_{2}He$
- $^{226}_{88}Ra \text{-------}\!\!\rightarrow\ ^{222}_{86}Rn\ +\ ^{4}_{2}He$

Example 13.6 We are given the following atomic masses:

$^{238}_{92}U$ = 238.05079 u $^{4}_{2}He$ = 4.00260 u

$^{234}_{90}Th$ = 234.04363 u $^{1}_{1}H$ = 1.00783 u

$^{237}_{91}Pa$ = 237.05121 u

Here the symbol Pa is for the element protactinium (Z = 91).

(a) Calculate the energy released during the alpha decay of $^{238}_{92}U$.

(b) Show that $^{238}_{92}U$ can not spontaneously emit a proton.

Solution

(a) The alpha decay of $^{238}_{92}U$ is given by Eq. (13.20). The energy released in this process is given by

$Q = (M_U - M_{Th} - M_{He})\, c^2$

Substituting the atomic masses as given in the data, we find

Q = (238.05079 – 234.04363 – 4.00260)u × c^2

= (0.00456 u) c^2

= (0.00456 u) (931.5 MeV/u)

= 4.25 MeV.

(b) If $^{238}_{92}U$ spontaneously emits a proton, the decay process would be

$^{238}_{92}U \rightarrow\ ^{237}_{91}Pa + ^{1}_{1}H$

The Q for this process to happen is

= $(M_U - M_{Pa} - M_H)\, c^2$

= (238.05079 – 237.05121 – 1.00783) u × c^2

= (– 0.00825 u) c^2

= – (0.00825 u)(931.5 MeV/u)

= – 7.68 MeV

Thus, the Q of the process is negative and therefore it cannot proceed spontaneously. We will have to supply an energy of 7.68 MeV to a $^{238}_{92}U$ nucleus to make it emit a proton.

Kinetic energy and speed of α-particle emitted in α-decay:

- Let m_d and m_α be the mass of daughter nucleus and α-particle respectively.
- Let v_d and v_α be the speed of daughter nucleus and α-particle respectively.
- According to law of conservation of linear momentum, we have

  $m_d v_d + m_\alpha v_\alpha = 0$ or

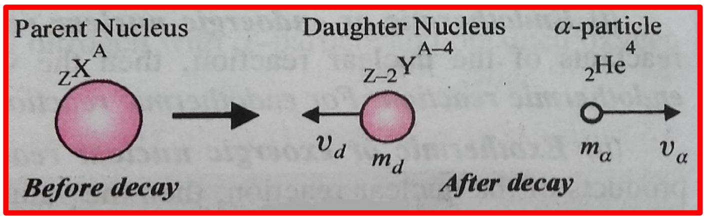
  $v_d = -m_\alpha v_\alpha / m_d$ --- (1)

  $$v_d^2 = \frac{m_\alpha^2 v_\alpha^2}{m_d^2} \quad \text{-- (2)}$$

- According to law of conservation of energy, we have

$$\frac{1}{2}\, m_d v_d^2 + \frac{1}{2}\, m_\alpha v_\alpha^2 = Q$$ --- (3) ; using eq (2), we get

$$\frac{1}{2}\, m_d \left[\frac{m_\alpha^2 v_\alpha^2}{m_d^2}\right] + \frac{1}{2}\, m_\alpha v_\alpha^2 = Q$$



$$\frac{1}{2}\,\frac{m_\alpha^2}{m_d}\, v_\alpha^2 + \frac{1}{2}\, m_\alpha v_\alpha^2 = Q \quad \text{or}$$

$$\frac{1}{2}\, m_\alpha v_\alpha^2 \left[\frac{m_\alpha}{m_d} + 1\right] = Q \quad \text{or} \quad [(KE)_\alpha]\left[\frac{m_\alpha}{m_d} + 1\right] = Q$$ where KE = Kinetic Energy

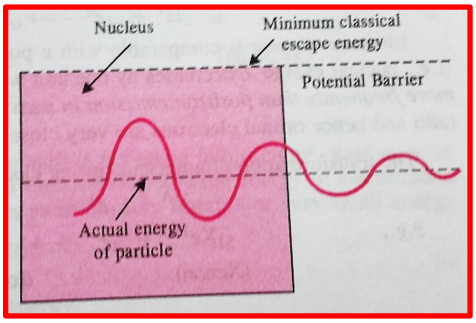
But, $m_\alpha = 4\,u \;\; and \;\; m_d = A - 4$, therefore $[(KE)_\alpha]\left[\frac{4}{A-4} + 1\right] = Q$

$$(KE)_\alpha = \left[\frac{A-4}{A}\right] Q$$

Since the daughter nucleus is very heavy as compared to the α-particle, so most of the disintegration energy (Q) appears as the kinetic energy of the α-particle. (See figure above)

$$\frac{1}{2}\, m_\alpha v_\alpha^2 = \left[\frac{A-4}{A}\right] Q \quad \text{or} \quad v_\alpha = \sqrt{\frac{2}{m_\alpha}\left[\frac{A-4}{A}\right] Q}$$ --------------------------------------- **(4)**

- α-particle can escape from a nucleus of the potential barrier of the nucleus is of the order of the KE of α-particle (about 5.4 MeV)
- It has been found that the height of the potential barrier of the nucleus is about 26 MeV.
- According to classical mechanics, α-particle having energy about 5.4 MeV is much less than the height of the potential barrier of the nucleus and hence cannot cross the height of this potential barrier.
- To cross the potential barrier, an α-particle must have energy = 26 MeV. But no α-particle emitted by a radioactive nucleus possesses this much amount of energy and hence it cannot escape the nucleus.
- The problem of escaping α-particle from the nucleus was solved by Gamow, Condon and Gurney in 1928 using quantum mechanics. According to their theory, **the motion of an α-particle in the neighbourhood of a potential barrier is considered as a wave**. It is found that there is small but definite probability that the α-particle may **tunnel through (leak through)** the potential barrier even if the KE of the α-particle is less than the height of the potential barrier of the nucleus. This effect is known as **tunnelling of the nucleus** (see fig → → → → → →).



- **IMP**: If we want to know how many α-particles are emitted, let us take the example
- ${}^A_Z X \rightarrow {}^{A-4}_{Z-2}Y + {}^4_2He$; If ΔA is the difference in "mass number" and "n_α" is number of α-particles, then the general formula for number of α-particles emitted is $n_\alpha = \frac{\Delta A}{4}$
- ${}^{238}_{92}U \rightarrow {}^{210}_{78}Pt = {}^4_2He + {}^4_2He + {}^4_2He + {}^4_2He + {}^4_2He + {}^4_2He + {}^4_2He$
- $\therefore n_\alpha = \frac{\Delta A}{4} = \frac{(238-210)}{4} = \frac{28}{4} = 7\ (\alpha - particles)\ are\ emitted.$
- $\boldsymbol{So\ general\ rule\ is\ n_\alpha = \frac{\Delta A}{4}\ or\ n_\alpha = \frac{\Delta Z}{2}}$

Beta decay (β-decay)

Definition: **The spontaneous process of emission of β-particle from a radioactive nucleus is called β-decay.**

- Beta decay is of 3 types:
  - Beta-minus (β^-)
  - Beta-plus (β^+)
  - Electron capture (*not in 12th CBSE syllabus but mentioned here for academic interest*)

Summary :

- In beta decay, a nucleus spontaneously emits an electron (β^- decay) or a positron (β^+ decay).
- A common example of β^- decay is $^{32}_{15}P \rightarrow {}^{32}_{16}S + e^- + \bar{\nu}$ ------------------------ **(1)**
- And that of β^+ decay is $^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + e^+ + \nu$ ----------------------- **(2)**
- The decays are governed by the equations $N(t) = N_0 e^{-\lambda t}$ and $R(t) = R_0 e^{-\lambda t}$; where $\lambda N_0 = R_0$;
  - N_0 is the number of (**undecayed**) atoms present in a radioactive substance at time t = 0 ⇒ initial value
  - N is the number of atoms in a radioactive substance at a later time t
  - λ is the **decay constant**. For a given element, the value of λ is constant, but for different elements it is different.
  - When $\frac{N}{N_0} = \frac{1}{2}$, then $t = T_{1/2}$ (half-life)
  - The **rate of decay of radioactive substance** is called the "**activity**" R of the substance, that is, $R = -\frac{dN}{dt}$ ∴ $R(t) = \lambda N_0 e^{-\lambda t}$ → This is equivalent to the law of radioactivity decay ($\lambda N_0 = R_0$)
- When decays are governed by above N(t) and R(t) equations, one can never predict which nucleus will undergo decay, but one can characterize the decay by a half-life $T_{1/2}$.
- For example, $T_{1/2}$ for the decays in equations (1) and (2) is respectively 14.3 days and 2.6 years.
  - The emission of electron in β^- decay is accompanied by the emission of an antineutrino ($\bar{\nu}$).
  - In β^+ decay, instead, a neutrino (ν) is generated.
- Neutrinos are neutral particles with very small (possibly, even zero) mass compared to electrons. They have only weak interaction with other particles. They are, therefore, very difficult to detect, since they can penetrate large quantity of matter (even earth) without any interaction.
- **In both β^- and β^+ decay, the mass number A remains unchanged.**
- In β– decay, number of protons goes up by 1 and neutron goes down by 1; **Z = Z + 1 and N = N – 1**
- In β+ decay, number of protons goes down by 1 and neutron goes up by 1; **Z = Z - 1 and N = N + 1**
  - **Where Z = number of Protons and N = number of Neutrons**
- **The basic nuclear process underlying β^- decay is the conversion of neutron to proton**
  - $n \rightarrow p + e^- + \bar{\nu}$ -- (3)
- **while for β^+ decay, it is the conversion of proton into neutron**
  - $p \rightarrow n + e^+ + \nu$ --- (4)

Note that while a free neutron decays to proton, the decay of proton to neutron Eq. (4) is possible only inside the nucleus, since proton has smaller mass than neutron.

➢ β^- decay: (Note : β^- is same as ${}_{-1}^{0}e$)

- In β^- decay, the neutron inside the nucleus is converted into a proton & an electron like particle
- β^- decay is represented as $^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e + Q_\beta$
- It is surprizing that nucleus contains no electron, then how a nucleus can emit electron. This electron like particle is emitted by the nucleus during β-decay.
- As mass number (A) increases beyond 20, the number of neutrons in the nuclei increases and becomes > Z. These nuclei having more number of neutrons (N) than protons (Z) become unstable and tend to β^- decay.
- In β^- decay, neutron in the nucleus is converted into a proton and β^- particle (${}_{-1}^{0}e$) is emitted so that the ratio of neutron to proton decreases and hence the nucleus becomes stable.
- For example, $^{32}_{15}P \rightarrow {}^{32}_{16}S + {}_{-1}^{0}e + \bar{\nu}$
- In β^- decay, the process of conversion of ${}_{0}^{1}n$ into proton (${}_{1}^{1}H$) takes place according to the equation
  - ${}_{0}^{1}n \rightarrow {}_{1}^{1}H + {}_{-1}^{0}e + \bar{\nu}$

➢ β^+ decay: (Note : β^+ is same as ${}_{+1}^{0}e$)

- In β^+ decay, a proton is converted into a proton & in addition a positron (${}_{+1}^{0}e$) is emitted if a nucleus has more protons than neutrons.
- Positron is the antiparticle of an electron. The properties of a positron are same as that of an electron except that positron carries a positive-charge while electron carries a negative charge.
- For example, ${}_{11}^{22}Na \rightarrow {}_{10}^{22}Ne + {}_{+1}^{0}e + \nu$
- The process of conversion of a proton (${}_{1}^{1}H$) into a neutron (${}_{0}^{1}n$) takes place as follows:

$${}_{1}^{1}H \rightarrow {}_{0}^{1}n + {}_{+1}^{0}e + \nu$$

➢ Electron Capture:

- In electron capture, nucleus absorbs one of the inner electrons revolving around it and hence a nuclear proton becomes a neutron and a neutrino (ν) is emitted.
- This process is represented as: ${}_{1}^{1}H + {}_{-1}^{0}e \rightarrow {}_{0}^{1}n$
- Electron capture is comparable with a positron emission as both the processes lead to the same nuclear transformation (i.e., nuclear charge Z decreases by one unit while the mass number A remains unchanged).
- However, **electron capture occurs more frequently than positron emission in heavy elements**. This is because the orbits of electrons in heavy elements have small radio and hence orbital electrons are very close to the nucleus.
- The transformation of a nucleus during electron capture is given by ${}_{Z}^{A}X + {}_{-1}^{0}e \rightarrow {}_{Z-1}{}^{A}Y$
- Example: ${}_{54}^{120}Xe + {}_{-1}^{0}e \rightarrow {}_{53}^{120}I$
  - (Xenon) (electron) (Iodine)
- Electron capture process is identified by the shell or energy level from which the captured electron comes. If captured electron comes from K-shell, it is known as K-shell capture. If captured electron comes from L-shell, it is known as L-shell capture and so on.

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Energy carried by emitted β-particle (energy spectrum of β-particles)

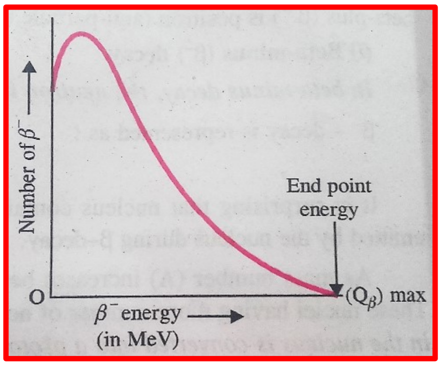
➢ β-decay is represented as follows:

$${}_{Z}^{A}X \rightarrow {}_{Z+1}{}^{A}Y + {}_{-1}^{0}e + Q_\beta$$

(Parent Nucleus) (Daughter Nucleus)

➢ The graph is plotted between the <u>energy of emitted β^- particles</u> and the <u>number of β^- particles</u>.

➢ This graph shows that:
- Most of the β particles carry small energy.
- Only a few β-particles carry maximum energy called "end point energy" $[(Q_\beta)_{max}]$
- The energy spectrum is continuous which indicates that the emitted β-particles have all possible energies from 0 to $[(Q_\beta)_{max}]$



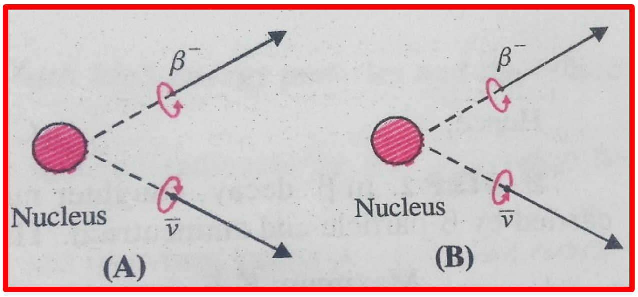
➢ The energy calculations showed that energy of a nucleus emitting β-particles decreases by an amount equal to the end-point energy of the continuous spectrum. But most of the β-particles emitted have energies smaller than the end point energy.

➢ In other words, the energy of the emitted β^- particle is not equal to the difference between the energies of the parent and the daughter nuclei. **Thus, the law of conservation of energy appears t be violated in β^- decay.**

➢ In β^- decay, another contradiction arose in respect of the law of conservation of angular momentum or spin. The β^- particle has spin ½. When it is emitted by the nucleus, the spin or angular momentum of the nucleus must change by ½. But the change in spin of the nucleus emitting β^- particle was never found to be ½. There was either no change or an integral change in nuclear spin of the nucleus emitting β^- particles. Thus, the law of conservation of angular momentum appears to be violated in β-decay.

➢ <u>Neutrino Hypothesis</u>:
- The violations of the <u>laws of conservation of energy and angular momentum</u> were resolved by Pauli in 1931. He suggested that the emission of the low energy β^- particle by any nucleus is accompanied by another neutral particle of zero rest mass. This neutral particle is called "anti-neutrino" ($\bar{\nu}$)
- The total energy of β^- particle and anti-neutrino = end point energy. **Thus, the law of conservation of energy holds good in β-decay.**

- If anti-neutrino is assumed to have spin ½, then the law of conservation of angular momentum or spin also holds good in β-decay. Thus, the nuclear spin or angular momentum of the daughter nucleus is equal to the angular momentum or spin of the parent nucleus if both β⁻ particle and the anti-neutrino spin in opposite direction (see figure A → → → →).
- If the β⁻ particle and anti-neutrino spin is in the same direction, then there will be an integral change in nuclear spin (see fig B →). This leads to the law of conservation of angular momentum in β-decay.



- **Energy in β⁻ decay (consider β-decay as β⁻ decay until or unless specified differently)**
  - **β⁻ decay** is represented as $_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e + \bar{\nu} + Q_\beta$ ------------- **(1)**
  - where $Q_\beta = \left[m\left(_{Z}^{A}X\right) - m\left(_{Z+1}^{A}Y\right)\right]c^2 \quad and \quad \bar{\nu}\ is\ anti-neutrino$ -------- **(2)**
  - *Antineutrino* is a very little particle having almost no mass. It is emitted with β⁻ particle during radioactive decay.
  - In equation (2), atomic masses are considered and mass of electron (being very small) is not taken into account.
  - Rest mass of *Antineutrino* is zero. Energy Q given by equation (2) appears as the kinetic energy of electron (K_e) and the energy of *Antineutrino* ($E_{\bar{\nu}}$). Since the product $_{Z+1}^{A}Y$ is much heavier than the electron and the *Antineutrino*, so it has only fraction or very small energy which can be neglected. Thus, $Q_\beta = K_e + E_{\bar{\nu}}$ -- (3)
  - The electron will have maximum KE if the anti-neutrino has zero energy. That is,
  - $Q_\beta = K_e = \left[m\left(_{Z}^{A}X\right) - m\left(_{Z+1}^{A}Y\right)\right]c^2$ -- (4)

- **Energy in β⁺ decay**
  - **β⁺ decay** is represented as $_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + {}_{+1}^{0}e + \nu + Q_\beta$, where energy
  - $Q_\beta = \left[m\left(_{Z}^{A}X\right) - m\left(_{Z-1}^{A}Y\right) - 2m_e\right]c^2 \quad and \quad \nu\ is\ neutrino$ -------------------- **(5)**
    - *Neutrino* is a very little particle having almost no mass. It is emitted with β⁺ particle during radioactive decay. It interacts very weakly with materials and hence not detected easily.
- **Energy in Electron capture**
  - $Q = \left[m\left(_{Z}^{A}X\right) - m\left(_{Z-1}^{A}Y\right)\right]c^2$; In electron capture, initial KE (which is small) of electron and recoil energy of nucleus are neglected. Therefore, final energy is carried by neutrino. That is $Q = E_\nu$. This energy is shared by positron and neutrino.

To sum up, the **β⁻ decay** of a nucleus is written as

- $_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e + \bar{\nu}$; eg : $_{83}^{210}Bi \rightarrow {}_{84}^{210}Po + {}_{-1}^{0}e + \bar{\nu}$

- $_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + {}_{+1}^{0}e + \nu$; eg : $_{43}^{97}Tc \rightarrow {}_{42}^{97}Y + {}_{+1}^{0}e + \nu$

- In **β⁺ decay**, a proton inside a nucleus is converted into a neutron and positron emitted along with neutrino. However, a free proton is stable and its decay is not possible.

- A weak nuclear force (one of the 4 fundamental forces of nature) is responsible for β-decay.

Complete the following radioactive decay: $_{6}^{14}C \rightarrow \ldots\ldots\ldots + {}_{-1}^{0}e + \bar{\nu}$; Also calculate the maximum KE carried by emitted β⁻ particle. Given mass of $_{6}^{14}C$ = 14.003242 u ; and mass of $_{7}^{14}N$ = 14.003074 u.

Step 1: $_{6}^{14}C \rightarrow {}_{Z}^{A}X + {}_{-1}^{0}e + \bar{\nu}$; as per law of conservation of mass number 14 = A + 0 or A = 14 ;
As per law of conservation of atomic number 6 = Z-1 or Z = 7 ; so, $_{Z}^{A}X = {}_{7}^{14}N$

Hence, $_{6}^{14}C \rightarrow {}_{7}^{14}N + {}_{-1}^{0}e + \bar{\nu}$

Step 2: In β⁻ decay, daughter nucleus $_{7}^{14}N$ is much heavier than electron and anti-neutrino. So most of the energy is carried by β⁻ particle and anti-neutrino. The maximum KE is carried by β⁻ particle because of KE of anti-neutrino is very less.

So, Maximum KE carried by β⁻ = $(mass\ of\ {}_{6}^{14}C - mass\ of\ {}_{7}^{14}N)$ x $(931\ MeV)$
= (14.003242 – 14.003974) x (931 MeV) = 0.16 MeV

Gamma decay (γ-decay):

It is the spontaneous emission of high energy photon from a radioactive nucleus

- Like an atom, a nucleus also has discrete energy levels - the ground state and excited states.
- The scale of energy is, however, very different. (*This means the energy levels of an atom and a nucleus is the spacing between the energy levels*).
- Atomic energy level spacings are of the order of eV, while the difference in nuclear energy levels is of the order of MeV. (*In an atom, the spacings between two successive energy levels is of the order of eV, on the other hand, the spacing between two successive levels of a nucleus is of the order of MeV*)
- When a nucleus in an excited state spontaneously decays to its ground state (or to a lower energy state), a photon is emitted with energy equal to the difference in the two energy levels of the nucleus. This is the so-called gamma decay. The energy (MeV) corresponds to radiation of extremely short wavelength, shorter than the hard X-ray region.
- That is, energy of a photon or γ-day is $hf = E_H - E_L$; where E_H = energy corresponding to higher energy level and E_L = energy corresponding to lower energy level or ground state.
- Typically, a gamma ray is emitted when a **α or β decay** results in a daughter nucleus in an excited state. This then returns to the ground state by a single photon transition or successive transitions involving more than one photon.
- A familiar example is the successive emission of gamma rays of energies 1.17 MeV and 1.33 MeV from the deexcitation of ${}^{60}_{28}Ni$ nuclei formed from β^- decay of ${}^{60}_{27}Co$. Another example is
- ${}^{20}_{9}F$ decays by emitting β^- particle to give a daughter nucleus ${}^{20}_{10}Ne$. This ${}^{20}_{10}Ne$ nucleus is excited state and comes to ground state by emitting γ-ray of energy 1.63 MeV.

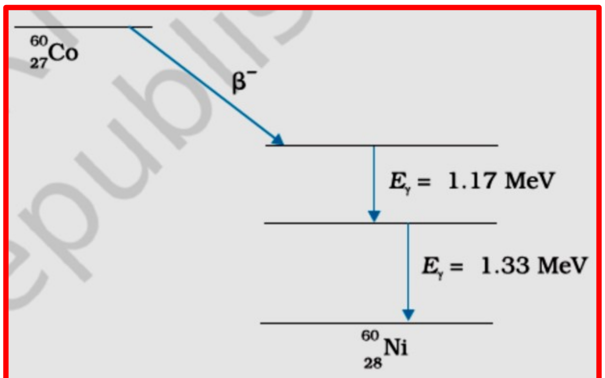
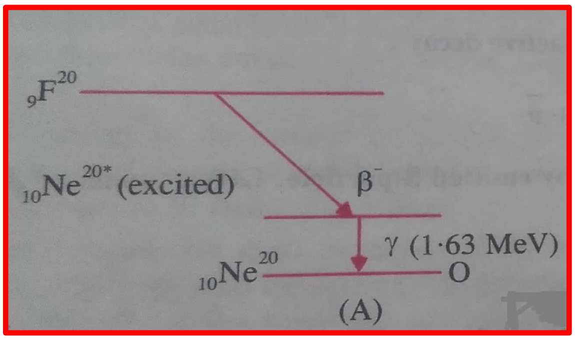


FIGURE 13.4 β-decay of ${}^{60}_{28}Ni$ nucleus followed by emission of two γ rays from deexcitation of the daughter nucleus ${}^{60}_{28}Ni$.



- γ-decay leads to nuclear de-excitation. The process of γ-decay is similar to the emission of radiation by an atom, when electron jumps from higher energy state to lower energy sate of the atom.
- During γ-decay, the atomic number (Z) and the mass number (A) of the nucleus remain unchanged.

13.6.17. Artificial Radioactivity (Radio–Isotopes)

What do you mean by artificial radioactivity ? What are radio isotopes ?

The process by which stable nuclei are made unstable by bombarding them with high energy particles and then these unstable nuclei decay to give nuclear radiation is called artificial radioactivity.

The artificially made unstable nuclei are called ***radio–isotopes.*** Artificial or induced radioactivity was discovered by ***Irene Curie Joliot*** and her husband ***Frederick Joliot.***

They bombarded aluminium with α–particles (emitted by polonium) and observed neutrons, positron ($_{+1}e^0$) and proton coming from aluminium. They observed that the *detector* is receiving some kind of penetrating radiation even after the polonium source was taken away (*i.e.* the bombardment of aluminium with α–particle was stopped). These penetrating ray were found to be *positrons* coming from aluminium.

The process is given below :

$$_{13}Al^{27} + {}_2He^4 \longrightarrow {}_{15}P^{30*} + {}_0n^1$$

(unstable)

$_{15}P^{30*}$ (unstable) is a radio isotope and it decays to become *silicon (stable)* with the emission of a positron and a neutri as shown below.

$$_{15}P^{30*} \longrightarrow {}_{14}Si^{30} + {}_{+1}e^0 + \nu.$$

The radio isotopes produced by using neutron as a *projectile* are given below :

1. $$_{29}Cu^{65} + {}_0n^1 \longrightarrow {}_{29}Cu^{66*} + \gamma\text{–ray}$$

(unstable)

$_{29}Cu^{66*}$ is a radioactive isotope and it decays by emitting an electron as under

$$_{29}Cu^{66*} \longrightarrow {}_{30}Zn^{66} + {}_{-1}e^0 + \bar{\nu}$$

(stable)

2. $$_{15}P^{31} + {}_0n^1 \longrightarrow {}_{13}Al^{28*} + {}_2He^4$$

(unstable)

$_{13}Al^{28*}$ is a radioactive isotope and it decays by emitting e^- as follows

$$_{13}Al^{28*} \longrightarrow {}_{14}Si^{28} + {}_{-1}e^0 + \bar{\nu}$$

(stable)

3. $$_{13}Al^{27} + {}_0n^1 \longrightarrow {}_{11}Na^{24*} + {}_2He^4$$

(unstable)

$_{11}Na^{24*}$ is a radioactive isotope and it decays as follows :

$$_{11}Na^{24*} \longrightarrow {}_{12}Mg^{24} + {}_{-1}e^0 + \bar{\nu}$$

(stable)

These radioactive isotopes are of great utility for mankind, so they are produced on large scale in *atomic or nuclear reactors**.

13.6.18. Applications of Radio–isotopes

Discuss the applications of radio isotopes.

1. ***Tracer Technique in the field of fluid technology :***

Tracer technique is used to investigate the blockage in a under–ground sewerage pipe or leakage of water pipe. In this technique, a small quantity of radio-isotope is introduced into the substance to be investigated and the path of radio-isotope is traced by means of a *sensitive detector (say **Geiger Muller Counter**) which detects the radiation emitted by radioactive substance.*

A rubber ball containing a radio–isotope emitting γ–rays is introduced in the pipe. This ball is stopped at the point of the *blockage* in the pipe. The detector is moved over the pipe which shows the presence of γ–rays in large quantity at the position of the stopped ball. Thus, the position of the blockage can be detected.

The *leakage* in underground water pipe can be located by injecting a small quantity of *radio–sodium.* The point where the water leaks will have more activity which is detected by the detector.

2. ***Tracer Technique in the field of Medicine :***

(i) In plastic surgery : To check whether a *skin graft* has taken place or not, a small quantity of sodium chloride solution containing radioactive sodium (Na^{24}) is injected in the skin flap. If blood is circulating satisfactorily, this solution of sodium chloride will soon drain out to other parts of the body along with blood. A detector over the point of injection will therefore indicate the flow of the blood by the rate at which the activity dies.

(ii) As a diagnostic tool : Tracing technique is used for locating brain damage and other internal injuries to the human body where X-rays photograph does not reveal the damage. This technique is also used to study the flow of atoms through metabolic processes in animals and plants.

3. *Other Uses of Radioisotopes in the medical field*

(a) *Skin Cancer* can be treated by the exposure of electron radiation emitted by radioisotopes.

(b) *Tumors and Cancer* are treated by high energy γ–rays from Co^{60}. This treatment is called *cobalt therapy*.

(c) *Radioactive iodine* (I^{131}) is used in the treatment of *hyperactive thyroid gland*. The hyperactive thyroid gland absorbs twice as much iodine as the normal one. Hence, radioactive iodine reveals the presence of hyperactive thyroid.

4. Radio–isotopes are used to investigate the flow of liquids in *chemical plants*.

5. Radio–isotopes are used to investigate the wear and tear in the machinery.

6. Radio–isotopes are used to develop new and improved varieties of plants by ***forced mutation***. Treatment of plants or seeds with intense radiations from a particular isotope changes the genes which results in a mutation.

7. Radioactive isotopes are used for the extensive study of ***photosynthesis***.

8. *Radio–active isotopes are used to determine the age of the geological structure formation in the pre–historic periods* such as rocks and old monuments. This process of knowing the age of rocks and monuments is known as ***uranium–lead dating***.

Uranium–Lead dating uses uranium (U^{238}) as parent element whose end product is lead (Pb^{206}).

9. Radioactive isotopes are used to determine the age of the ***fossils***. The process of knowing the age of fossils is called ***carbon dating***.

Information :

How to calculate the age of a rock or an old monument using Uranium Dating ?

Let N_0 be the original number of nuclei of U^{238} in an uranium bearing rock. U^{238} decays according to the relation

$$N(U^{238}) = N_0 e^{-\lambda t} \quad \text{...(i)}$$

Now the number of Pb^{206} (end product of U^{238}) after time t is given by

$$N(Pb^{206}) = N_0 - N(U^{238}) = N_0 - N_0 e^{-\lambda t} = N_0 (1 - e^{-\lambda t}) \quad \text{...(ii)}$$

Dividing eqn. (i) by (ii), we get the ratio of $N(U^{238})$ and $N(Pb^{206})$

$$\text{i.e.,} \quad R = \frac{N(U^{238})}{N(Pb^{206})} = \frac{N_0 e^{-\lambda t}}{N_0(1-e^{-\lambda t})} = \frac{e^{-\lambda t}}{1-e^{-\lambda t}} = \frac{1}{1-e^{-\lambda t}}$$

$$\text{Or} \quad (e^{\lambda t} - 1) = \frac{1}{R} \quad \text{or} \quad e^{\lambda t} = \left(\frac{1}{R}+1\right) \quad \text{Or}$$

$$\lambda t = \log_e\left(\frac{1}{R}+1\right) \quad \text{or} \quad t = \frac{1}{\lambda}\log_e\left(\frac{1}{R}+1\right)$$

Since $\lambda = \frac{0.6931}{T_{1/2}}$, where $T_{1/2}$ is half- life of U^{238}.

$$\therefore \quad t = \frac{T}{0.6931}\log_e\left(\frac{1}{R}+1\right) \quad \text{...(iii)}$$

Using this eqn. (*iii*), the age of a rock or old monument can be calculated.

How to calculate the age of a fossil using carbon Dating ?

In earth's atmosphere, high energy neutrons in cosmic rays react with the nuclei of nitrogen atom to form a radioactive isotope of carbon ($_6C^{14}$). The reaction takes place as follows :

$$_0n^1 + {}_7N^{14} \longrightarrow {}_6C^{14} + {}_1H^1.$$

All living things (*i.e.*, plants and animals) have some quantity of radioactive carbon (C-14). As soon as a plant or an animal dies, C-14 begins to decay with half life of 5730 years.

Since half life of $_6C^{14}$ is 5730 years, therefore 1 gram sample of dead wood or animal showing an activity of 8 disintegrations per minute is 5730 years old. If it shows 4 disintegrations per minute, then it is 11460 years old and so on (Figure 14).

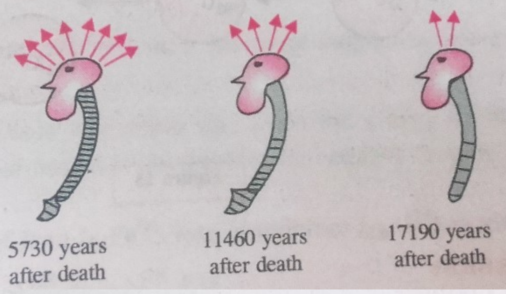


Figure 14

Let R_0 be the activity of a piece of wood obtained from a freshly cut tree and R be the activity of a sample of a burnt wood found in an archaeological site.

$$\text{We know,} \quad R = R_0 e^{-\lambda t} \quad \text{or} \quad e^{\lambda t} = \left(\frac{R_0}{R}\right)$$

$$\text{or} \quad \lambda t = \log_e\left(\frac{R_0}{R}\right) \quad \text{or} \quad t = \frac{1}{\lambda}\log_e\left(\frac{R_0}{R}\right)$$

$$\text{Since} \quad \lambda = \frac{0.6931}{T_{1/2}} \quad \therefore \quad t = \frac{T_{1/2}}{0.6931}\log_e\left(\frac{R_0}{R}\right) \quad \text{...(i)}$$

Using this eqn. (*i*), we can calculate the age of a fossil buried under earth or found on the earth.

Nuclear energy:

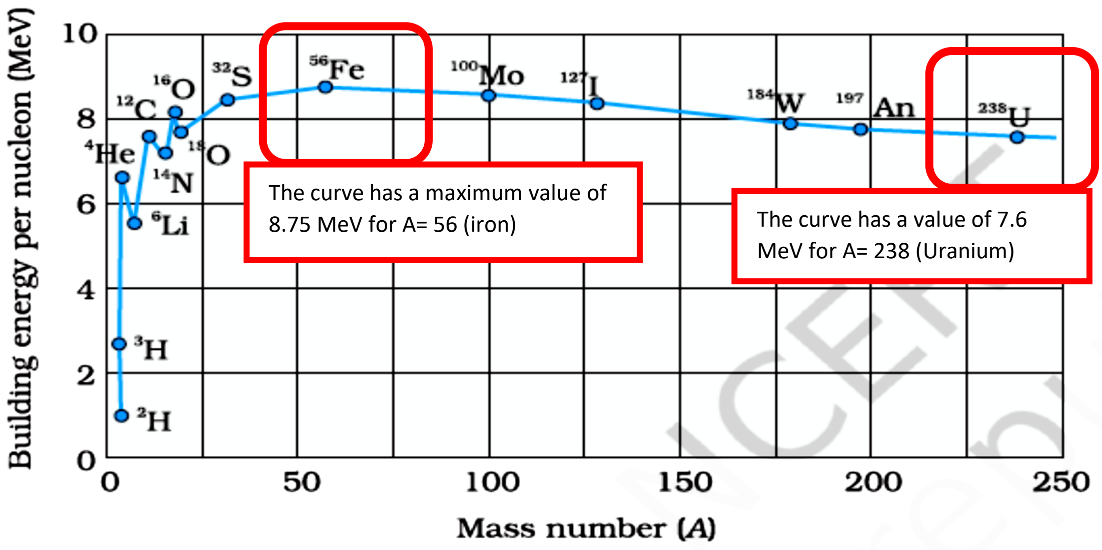


FIGURE 13.1 The binding energy per nucleon as a function of mass number.

- The curve of binding energy per nucleon E_{bn}, given above, has a long flat middle region between A = 30 and A = 170. In this region the binding energy per nucleon is nearly constant (8.0 MeV). For the lighter nuclei region, A < 30, and for the heavier nuclei region, A > 170, the binding energy per nucleon is less than 8.0 MeV, as we have noted earlier. Now, the greater the binding energy, the less is the total mass of a bound system, such as a nucleus. Consequently, if nuclei with less total binding energy transform to nuclei with greater binding energy, there will be a net energy release. This is what happens when a heavy nucleus decays into two or more intermediate mass fragments (fission) or when light nuclei fuse into a heavier nucleus (fusion).
- Exothermic chemical reactions underlie conventional energy sources such as coal or petroleum. Here the energies involved are in the range of electron volts. On the other hand, in a nuclear reaction, the energy release is of the order of MeV. Thus for the same quantity of matter, nuclear sources produce a million times more energy than a chemical source. Fission of 1 kg of uranium, for example, generates 10^{14} J of energy; compare it with burning of 1 kg of coal that gives 10^{7} J.
- The nuclear reactions involving nuclei with A > 170 constitute "nuclear fission". And the nuclear reactions involving nuclei with A < 30 constitute "nuclear fusion". Thus, two distinct ways of obtaining energy from the nucleus are (1) nuclear fission and (2) nuclear fusion.

Nuclear Fission:

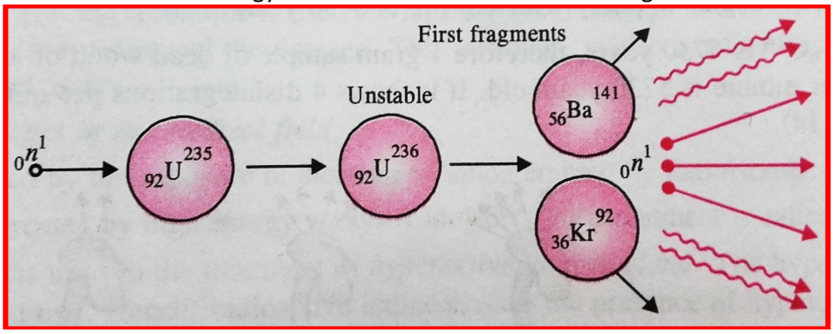
Nuclear Fission is a process of splitting a heavy nucleus (usually A > 230) into two or more lighter nuclei releasing large amount of energy. Soon after the discovery of neutron by Chadwick, Enrico Fermi found that when neutrons bombard various elements, new radioactive elements are produced.

Neutron was bombarded on a uranium target:

Nuclear fission was discovered by Otto Hahn and Strassman in 1939. They bombarded uranium ($^{235}_{92}U$) nucleus with a **thermal neutron** and found products Ba and Kr + 3 neutrons + large amount of energy. This is represented as (see also figure)

$$^{235}_{92}U + \, ^{1}_{0}n \rightarrow \underset{\text{(Unstable)}}{^{236}_{92}U} \rightarrow \, ^{141}_{56}Ba + \, ^{92}_{36}Kr + 3 \, ^{1}_{0}n + Q \quad \text{--------} \quad (1)$$

Where, Q is the amount of energy of about 200 MeV released during fission due to mass defect of about 0.215 u.



- Fission does not always produce barium and krypton. A different pair can be produced as follows:

$$^{1}_{0}n + \, ^{235}_{92}U \rightarrow \, ^{236}_{92}U \rightarrow \, ^{133}_{51}Sb + \, ^{99}_{41}Nb + 4 \, ^{1}_{0}n$$

Or, as another example,

$$^{1}_{0}n + \, ^{235}_{92}U \rightarrow \, ^{140}_{54}Xe + \, ^{94}_{38}Sr + 2 \, ^{1}_{0}n$$

- The fragment products are radioactive nuclei; they emit β particles in succession to achieve stable end products.
- The energy released (the Q value) in the fission reaction of nuclei like uranium is of the order of 200 MeV per fissioning nucleus. This is estimated as follows:
- Let us take a nucleus with A = 240 breaking into two fragments each of A = 120. Then E_{bn} for A = 240 nucleus is about 7.6 MeV, E_{bn} for the two A = 120 fragment nuclei is about 8.5 MeV. ∴ Gain in binding energy for nucleon is about 0.9 MeV.
- Hence the total gain in binding energy is 240 X 0.9 or 216 MeV.
- The disintegration energy in fission events first appears as the kinetic energy of the fragments and neutrons. Eventually it is transferred to the surrounding matter appearing as heat.
  - The source of energy in nuclear reactors, which produce electricity, is nuclear fission.
  - The enormous energy released in an atom bomb comes from uncontrolled nuclear fission.

13.7.2. Theory of Nuclear Fission

The nuclear fission was explained by **Bohr** and **Wheeler** using ***liquid drop model*** of the nucleus.

According to liquid drop model, the nucleus is treated as a drop of incompressible electrically charged nuclear liquid. There are two forces in the nucleus *(i) nuclear force just like surface tension in a liquid drop*, and *(ii)* the *coulomb's repulsive force*. Nuclear force tends to keep the spherical shape of the nucleus. On the other hand, Coulomb's repulsive force tends to destroy the spherical shape of the nucleus. The shape of the nucleus remains *spherical* as long as nuclear force balances the Coulomb's repulsive force.

When the nucleus is bombarded with a neutron, the neutron is captured by the nucleus. This provides an excitation energy to the nucleus. The excitation energy favours the Coulomb's repulsive force and tends to distort the spherical shape of the nucleus. As a result of this, *oscillations* are set up within the nucleus whose shape goes on deforming. Ultimately, the shape of the nucleus becomes like a ***dumb–bell.*** If the excitation energy is large enough, the Coulomb's repulsive force pushes the two bells apart, thereby splitting the nucleus into two nuclei of comparable masses [Figure 16].

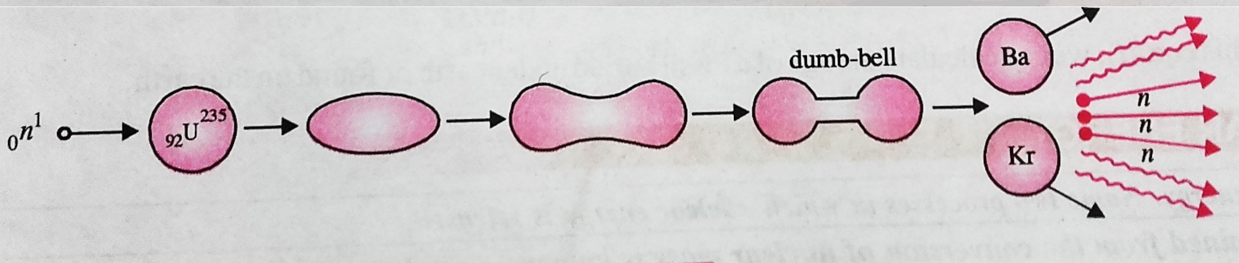


Figure 16

However, if the excitation energy is not enough, the nucleus simply emits energy in the form of γ-rays equal to the excitation energy and the 'surface tension like nuclear force' restores the spherical shape of the nucleus. This process is called a ***radiative capture*** than the fission.

The sum of the masses of product nuclei is found to be less than the mass of the reactants ($_0n^1$ and $_{92}U^{235}$). This difference in mass (called ***mass defect***) is converted into energy which is about 200 MeV per fission.

Energy release per fission of $^{235}_{92}U$

- Nuclear fission of $^{235}_{92}U$ is represented as follows:

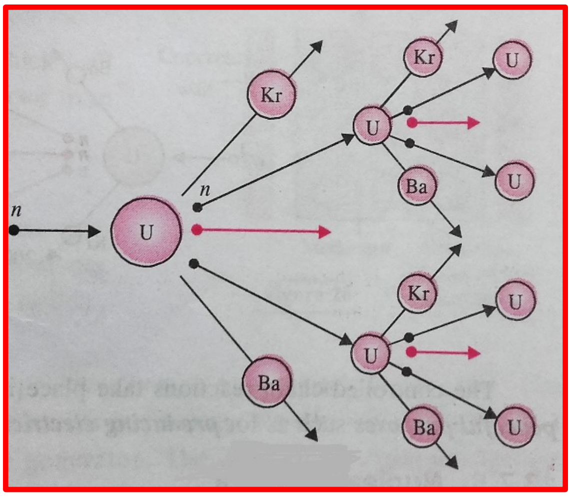
$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\ {}^{1}_{0}n + Q(energy) \quad \text{-------- (1)}$$

(unstable), removing this unstable state, we get

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\ {}^{1}_{0}n + Q(energy) \quad \text{------------------------ (2)}$$

- The energy released during the fission of $^{235}_{92}U$ is equivalent to the difference between the mass of reactants and the mass of products.
  - Mass of $^{235}_{92}U$ = 235.124 u
  - Mass of $^{1}_{0}n$ = 1.009 u
  - Total mass of reactants $^{235}_{92}U$ and $^{1}_{0}n$ = 236.133 u
  - Mass of $^{141}_{56}Ba$ = 140.958 u
  - Mass of $^{92}_{36}Kr$ = 91.926 u
  - Mass of 3 $^{1}_{0}n$ = 3.027 u
  - Total mass of products = 235.911 u
  - Difference in mass = mass of reactants – mass of products
  - = 236.133 u - 235.911 u = 0.222 u
  - We know that 1 u = 931 MeV
  - **∴ energy released per fission of $^{235}_{92}U$ = 0.222 X 931 ≈ 200 MeV**
- Energy released per fission appears as the KE if fragments, energy of released neutrons and the energy of emitted γ-radiations.
- Nuclear fission occurs only if Q value is positive, that is when the energy is released. Thus, nuclear fission is exothermic nuclear reaction. Nuclear fission is not possible for negative Q value.
- Example : in the fission of iron $^{56}_{26}Fe$ into aluminium $^{28}_{13}Al$ as given below is possible?
- $^{56}_{26}Fe \rightarrow {}^{28}_{13}Al + {}^{28}_{13}Al + Q$; Given mass of $^{56}_{26}Fe$ = 55.934940 and mass of $^{28}_{13}Al$ = 27.98191
  - Q-value = mass of $^{56}_{26}Fe - 2\ X\ mass\ of\ ^{28}_{13}Al$
    = (55.934940 – 2 X 27.98191) X 931.5 MeV/u = -0.02892 X 931.5 MeV/u
    = - 26.94 MeV ⇒ since Q is negative, so this fission process is not possible.

Chain reaction in Nuclear Fission:



- When $^{235}_{92}U$ undergoes a fission after bombarded by a neutron, it also releases extra neutron(s) and large amount of energy is released. Notice one fact of great importance in the fission reactions given in Eqs. (1) & (2). There is a release of extra neutron(s) in the fission process. Averagely, 2½ neutrons are released per fission of uranium nucleus. It is a fraction since in some fission events 2 neutrons are produced, in some 3, etc.
- The extra neutrons in turn can initiate fission processes, producing still more neutrons. These newly produced neutrons can cause further fission of more nuclei. The process continues and the number of fissions taking pace at each stage goes on increasing at fast rate. This leads to the possibility of a chain reaction, as was first suggested by Enrico Fermi. This process is called **uncontrolled chain reaction**.
- If the chain reaction is controlled suitably, we can get a steady energy output. This is what happens in a nuclear reactor.
- If the chain reaction is uncontrolled, it leads to explosive energy output, as in a nuclear bomb.

Controlled chain reaction of nuclear fission	→ Nuclear Reactors (steady energy output)
Uncontrolled chain reaction of nuclear fission	→ Nuclear Bomb (explosive energy output)

However, to maintain uncontrolled chain reaction, following difficulties are encountered.

- Ordinary uranium consists of 3 isotopes ($^{238}_{92}U$ $^{236}_{92}U$, $^{235}_{92}U$). Out of these isotopes, $^{238}_{92}U$ is in abundance (99.3%) ; $^{235}_{92}U$ is only 0.7% and $^{236}_{92}U$ is rare.
- $^{238}_{92}U$ Isotope can be fissioned only by fast neutrons having energy of the order of 1 MeV and more. Neutrons having low energy are absorbed by $^{238}_{92}U$, and hence chain reaction cannot be sustained.
- Whereas, $^{235}_{92}U$ can be fissioned by even slow neutron. To sustain chain reaction $^{235}_{92}U$ is separated from the ordinary uranium. Different methods are employed to do this. Uranium $^{235}_{92}U$ so obtained is known as "enriched uranium", which is a fissionable material. The fission of enriched uranium $^{235}_{92}U$ is possible with the fast and slow neutrons and hence chain reaction can be sustained.
- (enriched uranium → using a suitable method, the percentage of $^{235}_{92}U$ isotope in the uranium ore is increased from 0.7% to 3%. $^{235}_{92}U$ content of increased percentage (that is 3%) is known as "enriched uranium").
- Another method to sustain the chain reaction is to slow down the neutrons emitted in the fission.
- The chances of causing fission of $^{238}_{92}U$ (note this is U-238) by slow neutrons are very small (since $^{238}_{92}U$ absorbs slow moving neutrons).
- However, these slow moving neutrons are available for the fission of $^{235}_{92}U$ (note that this U-235, fissionable material or enriched uranium). Note that the fission of enriched uranium $^{235}_{92}U$ is possible with both fast and slow neutrons. It is known experimentally that slow neutrons (thermal neutrons) are much more likely to cause fission in $^{235}_{92}U$ than fast neutrons. Also fast neutrons liberated in fission would escape instead of causing another fission reaction.
- The average energy of a neutron produced in fission of $^{235}_{92}U$ is 2 MeV. These neutrons unless slowed down will escape from the reactor without interacting with the uranium nuclei, unless a very large amount of fissionable material is used for sustaining the chain reaction. What one needs to do is to slow down the fast neutrons by elastic scattering with light nuclei. In fact, Chadwick's experiments showed that in an elastic collision with hydrogen the neutron almost comes to rest and proton carries away the energy. This is the same situation as when a marble hits head-on an identical marble at rest.
  - Therefore, in reactors, light nuclei called **moderators** are provided along with the fissionable nuclei for slowing down fast neutrons. **The moderators commonly used are water, heavy water (D_2O) and graphite**. The Apsara reactor at the Bhabha Atomic Research Centre (BARC), Mumbai, uses water as moderator. The other Indian reactors, which are used for power production, use heavy water as moderator.
  - A moderator is more effective if the mass of its atoms about the same sake as that of the mass of a neutron.
  - Ordinary water (H_2O) can also act as moderator but is not used as moderator because it often absorbs neutrons and hence chain reaction ultimately stops.

- Heavy water (D_2O) is made by an isotope of hydrogen called deuterium $^{2}_{1}H$ and is more effective moderator than graphite, because deuterium in it is lighter than the carbon nucleus in graphite. So deuterium slows down the fast moving neutrons more effectively without absorbing neutrons.

- The neutrons emitted during fission are very fast and they travel a large distance before being slowed down. If the size of the fissionable material is small, the neutrons emitted will escape the fissionable material before they slowed down. Hence chain reaction cannot be sustained.
- Therefore, the size of the fissionable material should be larger than a certain “critical size”.
- The “reproduction factor” (or multiplication factor) k determines whether or not any mass of fissionable material will sustain a chain reaction. It is given by
- $\boldsymbol{k = \dfrac{rate\ of\ production\ of\ neutron}{rate\ of\ loss\ of\ neutron}}$ or $\boldsymbol{k} = \dfrac{\text{number of fission produced by a given generation of neutrons}}{\text{the number of fission of the preceding generation}}$
- k is the measure of the growth rate of the neutrons in the reactor.
- If k = 1, the rate of production of neutron = rate of loss neutron, the mass of the fissionable material is said to be “**critical**” and the chain reaction is sustained. For K = 1, the operation of the reactor is said to be **critical**, **which is what we wish it to be for steady power operation**.
- If k < 1, then chain reaction stops
- If k > 1, then reaction is accelerated. If K > 1, the reaction rate and the reactor power increases exponentially. Unless the factor K is brought down very close to unity, the reactor will become **supercritical** and can even explode. The explosion of the Chernobyl reactor in Ukraine in 1986 is a sad reminder that accidents in a nuclear reactor can be catastrophic.
- The reaction rate is controlled through control-rods made out of neutron-absorbing material such as cadmium. In addition to control rods, reactors are provided with safety rods which, when required, can be inserted into the reactor and K can be reduced rapidly to less than unity.
- Another fissionable material is **Plutonium** (see below snapshot from NCERT book)

The more abundant isotope $^{238}_{92}U$ in naturally occurring uranium is non-fissionable. When it captures a neutron, it produces the highly radioactive plutonium through these reactions

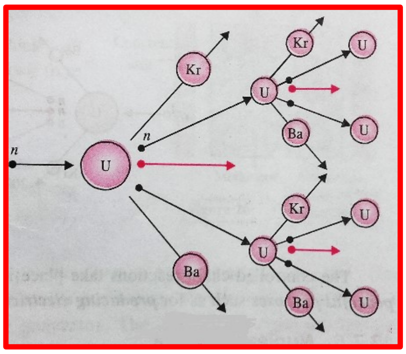
$$^{238}_{92}U + n \rightarrow {}^{239}_{92}U \rightarrow {}^{239}_{93}Np + e^- + \bar{\nu}$$

$$^{239}_{93}Np \rightarrow {}^{239}_{94}Pu + e^- + \bar{\nu} \qquad (13.29)$$

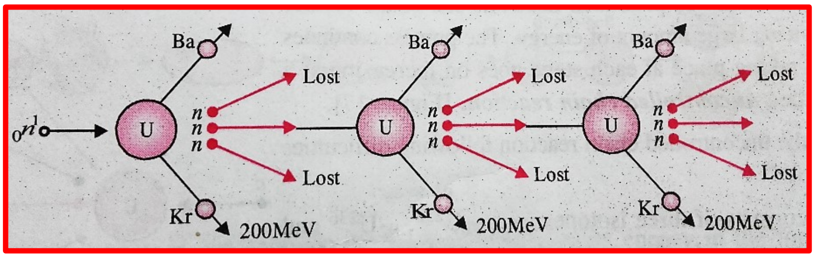
Plutonium undergoes fission with slow neutrons.

Types of Chain reactions:

- Uncontrolled chain reaction:
  - If more than one neutrons produced in a fission cause further fissions at each stage, then the number of fissions and energy released multiply rapidly. Such a chain reaction is called un-controlled chain reaction (see figure)
  - In such a chain reaction, huge amount of energy is released within a fraction of a second. This is the underlying principle of **atom bomb**.



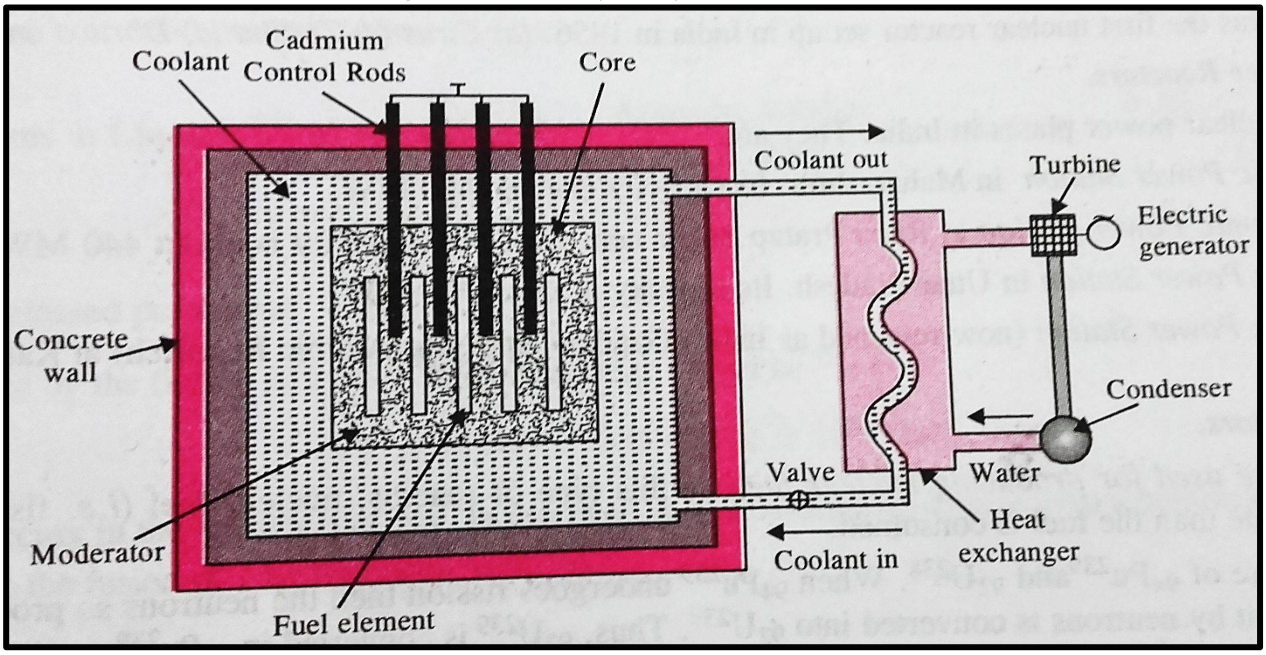
- **Controlled chain reaction:**
  - If only one neutron is available to cause further fission at each stage, then a constant amount of energy is released. Such a reaction is called controlled chain reaction (see figure).



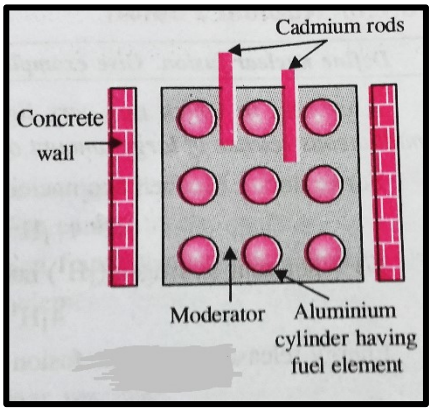
  - The controlled chain reactions take place in a nuclear-reactor and the energy obtained from such reaction is use for peaceful purposes such as for producing electricity.

Nuclear Reactor:

- Nuclear reactor is a device in which nuclear fission is maintained as a self-supporting yet controlled chain reaction to supply continuous energy (in the form of electricity). It was formerly known as atomic pile.
- Construction: The schematic diagram of a nuclear power plant is shown below.



- **Fissionable material (Fuel):**
  - The fissionable material used in the nuclear reactor is called the fuel of the reactor. Uranium isotope $^{235}_{92}U$, Thorium isotope $^{232}_{90}Th$ and Plutonium isotopes $(^{239}_{94}Pu,\ ^{240}_{94}Pu,\ ^{241}_{94}Pu)$ are the most commonly used fuels in the nuclear reactor.
- **Moderator:**
  - Moderator is used to slow down the fast moving neutrons.
  - Most commonly used moderators are graphite, water and heavy water.
  - When heavy water is used as a moderator, then ordinary or non-enriched uranium can be used as fuel because heave water has more neutrons to produce fission.
  - In case of ordinary water as moderator, enriched uranium is used as a fuel.
  - When fast moving neutrons pass through the moderator, they collide with the molecules of the moderator. As a result of this, the energy of moving neutrons decreases while that of the molecules of the moderator increases. After some time, both the neutrons and the molecules of the moderator attain the same energy. The neutrons are then in thermal equilibrium with the molecules of the moderator. Such neutrons are called **thermal neutrons**.
- **Control material:**
  - Control material is used to control the chain reaction and o maintain a stable rate of reaction. This material controls the number of neutrons available for the fission.
  - For example, cadmium rods are inserted into the core of the reactor because they can absorb the neutrons. The neutrons available for fission are controlled by moving the cadmium rod in or out of the core of the reactor.
- **Coolant:**
  - Coolant is a cooling material which removes the heat generated due to fission in the reactor,. Commonly used coolants are water, CO_2, nitrogen etc…
- **Protective shield:**
  - A protective shield in the form of a concrete think wall surrounds the core of the reactor to save the persons working around the reactor from the hazardous radiations.
  - The core of the nuclear reactor is shown in this figure →→



Working:

- A few nuclei of $^{235}_{92}U$ undergo fission liberating fast neutrons. These fast neutrons are slowed down to an energy about 0.025 eV by the surrounding **moderator through elastic collisions.** When the reactor becomes critical, self-sustained controlled chain reaction is achieved.
- The **cadmium** rods are used to control the chain reaction. The fission produces heat in the nuclear reactor core.
- The **coolant** transfers this heat from the core to the heat exchanger, where steam is formed. This steam produced at a very high pressure runs a turbine and the electricity is obtained at the generator. (In other words, the steam drives turbines and generates electricity)
- The dead steam from the turbine condenses into water and is returned to the heat exchanger.
- The process repeats and we get continuous supply of energy in the form of electricity.
- To save the persons working around the nuclear reactor from harmful radiations, a thick concrete wall known as **shield** is provided around the core of the reactor.
- **Uses** : **In addition to the production of electricity (power generation), nuclear reactor is used to produce radioactive isotopes, which are used in medical field and agriculture. They can be used to for the propulsion of ships, submarines and air crafts. They are used to produce high intensity neutron beam used in treatment** of cancel and nuclear research.
- The core is surrounded by a reflector (in addition to concrete wall) to reduce leakage.
- Like any power reactor, nuclear reactors generate considerable waste products. But nuclear wastes need special care for treatment since they are radioactive and hazardous. Elaborate safety measures, both for reactor operation as well as handling and reprocessing the spent fuel, are required. These safety measures are a distinguishing feature of the Indian Atomic Energy programme. An appropriate plan is being evolved to study the possibility of converting radioactive waste into less active and short-lived material.

Different types of Nuclear Reactors

Nuclear reactors designed to carry out atomic energy programme are of different types. These reactors are divided into four categories.

(i) Research Reactors.

These reactors have low power output and are used to produce neutrons. These neutrons are used to *(a)* study nuclear reactions *(b)* produce isotopes of elements. These reactors provide a facility for research in the field of nuclear science and technology.

Bhabha Atomic Research Centre (BARC), Trombay, near Mumbai is the core centre in India to persue research in Nuclear Science. There are five research reactors at BARC.

(a) Apsra. This was the first nuclear reactor set up in India in 1956. *(b) Cirus (c) Zerlina (d) Dhruva and (c) Purnima.*

(ii) Nuclear Power Reactors.

There are four nuclear power plants in India. They are

(a) Tarapur Atomic Power Station in Maharashtra. Its capacity is about 420 MW.

(b) Rajasthan Atomic Power station at Rana Pratap Sager near Kota. Its capacity is about 440 MW.

(c) Narora Atomic Power Station in Uttar Pradesh. Its capacity is about 470 MW.

(d) Madras Atomic Power Station (now renamed as Indira Gandhi Centre for Atomic Research) at Kalpakkam. Its capacity is 470 MW.

(iii) Breeder Reactors.

These reactors are used for producing nuclear fuel. In this type of reactor, nuclear fuel (*i.e.* fissionable material) is produced at a greater rate than the fuel is consumed.

The fuel is a mixture of $_{94}Pu^{239}$ and $_{92}U^{238}$. When $_{94}Pu^{239}$ undergoes fission then the neutrons so produced hit the $_{92}U^{238}$. The $_{92}U^{238}$ after being hit by neutrons is converted into $_{92}U^{239}$. Thus, $_{92}U^{239}$ is converted in $_{94}Pu^{239}$ as given below :

$$_{92}U^{239} \longrightarrow {_{93}Np^{239}} + {_{-1}e^0}$$

$$_{93}Np^{239} \longrightarrow {_{94}Pu^{239}} + {_{-1}e^0}.$$

$_{94}Pu^{239}$ is a nuclear fuel.

Nuclear fusion – energy generation in stars

- A process in which two very light nuclei (A ≤ 8) fuse to form a stable nucleus with a larger mass number along with simultaneous release of large amount of energy is called nuclear fusion.
- Since the larger nucleus is more tightly bound, as seen from the binding energy curve shown below.

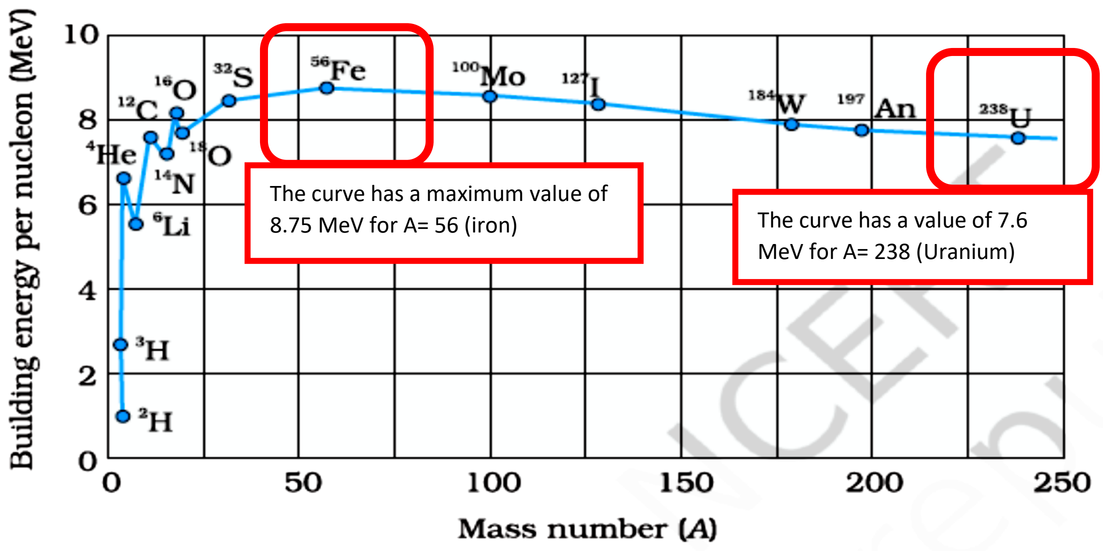


FIGURE 13.1 The binding energy per nucleon as a function of mass number.

- Some examples of such energy liberating nuclear fusion reactions are →
- In the first reaction, two protons combine to form a deuteron and a positron with a release of 0.42 MeV energy.
- In the second reaction, two deuterons combine to form the light isotope of helium.
- In third reaction, two deuterons combine to form a triton and a proton.

$${}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu + 0.42 \text{ MeV}$$
$${}^2_1H + {}^2_1H \rightarrow {}^3_2He + n + 3.27 \text{ MeV}$$
$${}^2_1H + {}^2_1H \rightarrow {}^3_1H + {}^1_1H + 4.03 \text{ MeV}$$

- **Another example of fusion is -->**
- $4\,{}^1_1H \rightarrow {}^4_2He + 2\,{}^1_0n + 2\nu + Q$ **; energy released due to fusion of 4 hydrogen nuclei is given by**

- **Q = (mass of 4 ^1_1H – mass of ^4_2He) X C^2**
- **= 4 X 1.007825 – 4.002603) u**
- **= 0.028697 X 931.5 MeV = 26.7 MeV, thus $4\,{}^1_1H \rightarrow {}^4_2He + 2\,{}^1_0n + 2\nu + 26.7\ MeV$**

- The process of nuclear fusion can be explained with the help of the concept of binding energy per nucleon of very light nuclei and the intermediate nuclei.
- Binding energy per nucleon of very light nuclei is < that of intermediate nuclei. It means light nuclei are less stable than that of intermediate nuclei. This shows the sum of masses of the individual nuclei > mass of the product nucleus formed by fusion (in other words mass of product nucleus < less than sum of masses of the lighter nuclei fusing together).
- The difference in mass (called mass defect) is released in the form of energy E = $(\Delta m)c^2$. **Thus, nuclear fusion is exothermic nuclear reaction.**
- For fusion to take place, the two nuclei must come close enough so that attractive short-range nuclear force is able to affect them. However, since they are both positively charged particles, they experience coulomb repulsion. They, therefore, must have enough energy to overcome this coulomb barrier. The height of the barrier depends on the charges and radii of the two interacting nuclei. It can be shown, for example, that the barrier height for two protons is ~ 400 keV, and is higher for nuclei with higher charges. We can estimate the temperature at which two protons in a proton gas would (averagely) have enough energy to overcome the coulomb barrier:

  (3/2)k T = K ≈ 400 keV, which gives T ≈ 3 × 10^9 K.
- When fusion is achieved by raising the temperature of the system so that particles have enough kinetic energy to overcome the coulomb repulsive behaviour, it is called thermonuclear fusion.

Stellar Energy (energy source of Stars) (Proton- Proton Cycle)

- Thermonuclear fusion is the source of energy output in the interior of stars. The interior of the sun has a temperature of 1.5 X 10^7 K, which is considerably less than the estimated temperature required for fusion of particles of average energy. Clearly, fusion in the sun involves protons whose energies are much above the average energy.
- The fusion reaction in the sun is a multi-step process in which the hydrogen is burned into helium. Thus, the fuel in the sun is the hydrogen in its core. **The proton-proton (p, p) cycle** by which this occurs is represented by the following sets of reactions:

$$ {}_1^1\mathrm{H} + {}_1^1\mathrm{H} \rightarrow {}_1^2\mathrm{H} + e^+ + \nu + 0.42 \text{ MeV} \quad \text{(i)} $$

$$ e^+ + e^- \rightarrow \gamma + \gamma + 1.02 \text{ MeV} \quad \text{(ii)} $$

$$ {}_1^2\mathrm{H} + {}_1^1\mathrm{H} \rightarrow {}_2^3\mathrm{He} + \gamma + 5.49 \text{ MeV} \quad \text{(iii)} $$

$$ {}_2^3\mathrm{He} + {}_2^3\mathrm{He} \rightarrow {}_2^4\mathrm{He} + {}_1^1\mathrm{H} + {}_1^1\mathrm{H} + 12.86 \text{ MeV} \quad \text{(iv)} $$

- **For the fourth reaction to occur, the first three reactions must occur twice, in which case two light helium nuclei unite to form ordinary helium nucleus. If we consider the combination 2(i) + 2(ii) + 2(iii) +(iv), the net effect is**

$$ 4{}_1^1\mathrm{H} + 2e^- \rightarrow {}_2^4\mathrm{He} + 2\nu + 6\gamma + 26.7 \text{ MeV} $$

$$ \text{or } (4{}_1^1\mathrm{H} + 4e^-) \rightarrow ({}_2^4\mathrm{He} + 2e^-) + 2\nu + 6\gamma + 26.7 \text{ MeV} $$

- Thus, four hydrogen atoms combine to form an ${}_2^4He$ atom with a release of **26.7 MeV of energy**.
- **2(i) + 2(ii) + 2(iii) +(iv) = 26.7 MeV of energy**
- Helium is not the only element that can be synthesized in the interior of a star. As the hydrogen in the core gets depleted and becomes helium, the core starts to cool. The star begins to collapse under its own gravity which increases the temperature of the core. If this temperature increases to about 10^8 K, fusion takes place again, this time of helium nuclei into carbon.
- This kind of process can generate through fusion higher and higher mass number elements. But elements more massive than those near the peak of the binding energy curve in the following figure cannot be so produced.

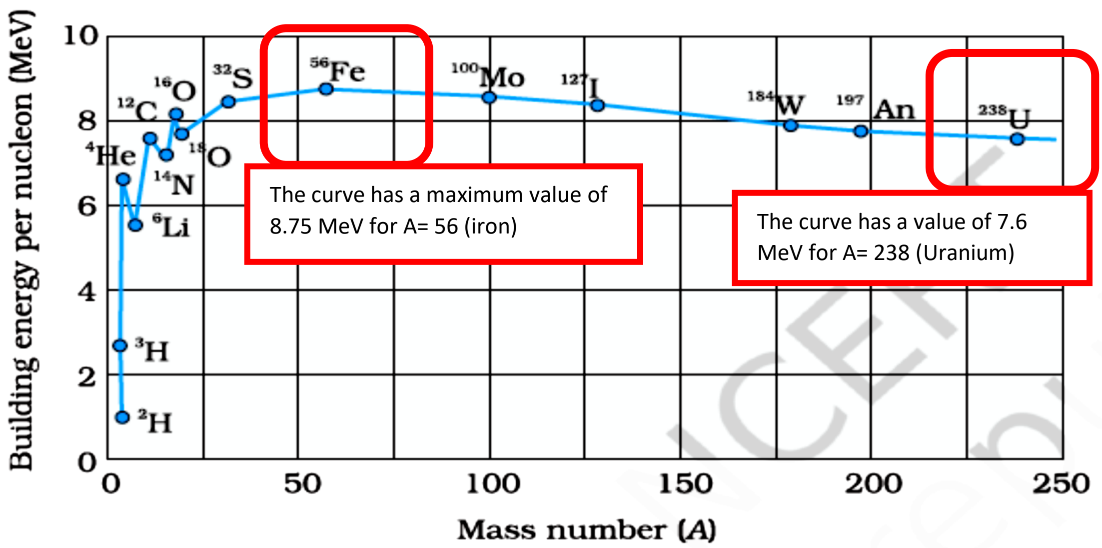


FIGURE 13.1 The binding energy per nucleon as a function of mass number.

- The age of the sun is about 5×10^9 years and it is estimated that there is enough hydrogen in the sun to keep it going for another 5 billion years. After that, the hydrogen burning will stop and the sun will begin to cool and will start to collapse under gravity, which will raise the core temperature. The outer envelope of the sun will expand, turning it into the so called **red giant**.

Stellar Energy (energy source of Stars) (Carbon - Nitrogen Cycle)

It is a thermo nuclear reaction in which carbon nuclei absorbs a succession of protons until they emit alpha particles to become carbon nuclei once more to repeat the cycle indefinitely.

Carbon–Nitrogen Cycle proceeds in the following way :

$${}_6C^{12} + {}_1H^1 \longrightarrow {}_7N^{13} + Q_1\ (MeV)$$

$${}_7N^{13} \longrightarrow {}_6C^{13} + {}_{+1}e^0 + \nu$$

$${}_6C^{13} + {}_1H^1 \longrightarrow {}_7N^{14} + Q_2\ (MeV)$$

$${}_7N^{14} + {}_1H^1 \longrightarrow {}_8O^{15} + Q_3\ (MeV)$$

$${}_8O^{15} \longrightarrow {}_7N^{15} + {}_{+1}e^0 + \nu$$

$${}_7N^{15} + {}_1H^1 \longrightarrow {}_6C^{12} + {}_2He^4$$

The total energy released in this cycle = 24·68 MeV

The above stated thermo–nuclear reactions take place in the sun and other stars and hence they are source of energy of the solar system.

Comparison of energy released in Nuclear Fission and Nuclear Fusion

Consider 1 kg of ${}^{235}_{92}U$ undergoing a nuclear fission.

Number of atoms in 1 kg of ${}^{235}_{92}U$ = No. of nuclei = $\left(\frac{Avagadro\ number}{Atomic\ weight}\ \text{x}\ mass\ in\ g\right) = \frac{6.02\ X\ 10^{23}\ X\ 10^{3}}{235}$

$\therefore$ Number of atoms in 1 kg of ${}^{235}_{92}U$ = 2.56 X 10^{24}

Since energy released per fission = 200 MeV

$\therefore$ Energy released by the fission of 2.56 X 10^{24} nuclei (that is 1 kg of ${}^{235}_{92}U$)

= 2.56 X 10^{24} X 200 = **5.12 X 10^{26} MeV**

Nuclear fusion occurs in the interior of a sun. In this case, 4 hydrogen nuclei fuse to give a helium nucleus. Let us find the energy released due to the fusion of 1 kg hydrogen in the Sun.

Number of atoms in 1 kg of 4 (${}^{1}_{1}H$) = $\frac{6.02\ X\ 10^{23}\ X\ 10^{3}}{4}$ = 1.505 X 10^{26}

We know that the energy released due to one fusion of 4 hydrogen nuclei = 26.7 MeV

$\therefore$ Energy released by the fusion of 1.505 X 10^{26} nuclei (that is 1 kg of ${}^{1}_{1}H$)

= 1.505 X 10^{26} X 26.7 = 40.2 X 10^{26} MeV

Thus, energy released by fusion is greater than the energy released by fission

Uncontrolled Fusion:

- Although the elements required for thermo-nuclear reactions are in abundance on the surface of the earth but the temperature (≈ 10^7 K) needed to fuse them cannot be attained by any known method in the laboratory. To achieve this much temperature, an atom bomb employing the process of nuclear fission is exploded. Thus, atomic explosion triggers the fusion process and simultaneous release of tremendous amount of energy. This is known as “uncontrolled fusion reactions” and is the principle of “hydrogen bomb”.

Controlled Thermonuclear Fusion:

- Fusion reactors are considered as a source of power for the future.
- The temperature of the order of 10^8 K required for thermo-nuclear reactions leads to the complete ionisation of the atoms of light elements. Thus in the sun, we have bare nuclei and electron clouds. The combination of these bare nuclei and electron clouds is called “plasma”.
- The main problem is carry out thermo-nuclear reactions in the laboratory is to contain the plasma at a temperature of 10^8 K. No solid container can remain in solid state at this much temperature and hence there is a problem of containing the plasma. If this problem of containing plasma is solved, then the large quantity of deuterium in sea water would be able to serve as an in-exhaustible source of energy.
- Nuclear fusion reactions are produced in “Tokamaks” (toroidal magnetic chamber). Plasma is contained in an evacuated doughnut shaped stainless steel vessel. A very strong magnetic field around this vessel prevents the high temperature plasma to strike the wall of the vessel.
- Many countries of the world including India are trying to build such sources of power. If fusion reactors are realised, then, the present energy crisis may be over as these reactors will supply the unlimited energy.

NUCLEAR HOLOCAUST:

- In a single uranium fission about 0.9 × 235 MeV (≈ 200 MeV) of energy is liberated. If each nucleus of about 50 kg of ${}^{235}_{92}U$ undergoes fission the amount of energy involved is about 4 × 10^{15} J. This energy is equivalent to about 20,000 tons of TNT, enough for a super explosion. Uncontrolled release of large nuclear energy is called an atomic explosion.
- On August 6, 1945 an atomic device was used in warfare for the first time. The US dropped an atom bomb on Hiroshima, Japan. The explosion was equivalent to 20,000 tons of TNT. Instantly the radioactive products devastated 10 sq. km of the city which had 3,43,000 inhabitants. Of this number 66,000 were killed and 69,000 were injured; more than 67% of the city's structures were destroyed.
- High temperature conditions for fusion reactions can be created by exploding a fission bomb. Super-explosions equivalent to 10 megatons of explosive power of TNT were tested in 1954. Such bombs which involve fusion of isotopes of hydrogen, deuterium and tritium are called hydrogen bombs. It is estimated that a nuclear arsenal sufficient to destroy every form of life on this planet several times over is in position to be triggered by the press of a button. Such a nuclear holocaust will not only destroy the life that exists now but its radioactive fallout will make this planet unfit for life for all times. Scenarios based on theoretical calculations predict a long **nuclear winter**, as the radioactive waste will hang like a cloud in the earth's atmosphere and will absorb the sun's radiation.

Distinction between nuclear fission and nuclear fusion

Similarities	Differences
➢ Both, nuclear fission and nuclear fusion are the sources of tremendous energy ➢ In both the processes, a certain mass (mass defect Δm) disappears, which appears in the form of energy as per Einstein equation E = $(\Delta m)c^2$.	➢ In fission heavy nucleus splits into two or more lighter nuclei. In fusion, two or more lighter nuclei fuse together to form a heavier nucleus. ➢ For carrying out fission, a suitable bullet or projectile like neutron is needed. For carrying out fusion, the lighter nuclei have to be brought very close to each other against electrostatic repulsion. ○ For this, suitable energy is to be made available, often by raising the temperature to the order of 10^7 K. This justifies nuclear fusion being called thermos-nuclear fusion. In actual practice, such high temperatures are generated by nuclear fission. That is why usually, **nuclear fission precedes nuclear fusion**. ➢ In a nuclear fusion reaction, energy liberated per unit mass of their nuclei is many times larger than the energy liberated per unit mass in nuclear fission reaction. **Therefore, for a given weight, hydrogen bomb (based on nuclear fusion) is far more dangerous than an atom bomb (based on nuclear fission)** ➢ The products of nuclear fission are radioactive. They produce environmental pollution and hence required very careful disposal. However, the products of nuclear fusion are not radioactive. They are harmless and can be disposed off easily. ➢ While producing nuclear energy from fission, we have learnt how to control nuclear chain reaction. But we have yet to learn controlling the thermo-nuclear fusion reactions. This would be the basis of "**fusion reactors**", which is seen as the future source of unlimited energy **without pollution**.

The mass defect in a nuclear fusion reaction is 0.3%. What amount energy will be liberated in one kg fusion reaction?

- ➔ Mass defect, Δm = 0.3% of 1 kg = (0.3/100) 1kg = (3/1000) 1kg = 3 X 10^{-3} kg
- ➔ Energy liberated = E = $(\Delta m)c^2$ = (3 X 10^{-3} kg)(3 X 10^8ms^{-1})^2 = 27 X 10^{13} kg m^{2}s^{-2} = 2.7 X 10^{14} kg m^{2}s^{-2}
  - Work (energy) = force x distance = ma x d = kg . ms^{-2} x m = kg m^{2}s^{-2} = joule
  - Energy liberated = E = 2.7 X 10^{14} joule
  - We know that 1 eV = 1.602 x 10^{-19} joule OR
  - 1 joule = 0.624 X 10^{19} eV
  - $\therefore$ 2.7 X 10^{14} joule = 2.7 X 10^{14} X 0.624 X 10^{19} eV = 1.7 x 10^{33} eV = **1.7 x 10^{27} MeV**

India's atomic energy programme:

- The atomic energy programme of our country was launched around 1950 under the leadership of Homi H. Bhabha. The major milestones achieved so far are:
    - First nuclear reactor named Apsara went critical on august 4, 1956. It used enriched uranium as fuel and water as moderator.
    - Another reactor named Canada India Reactor (CIRUS) became operative in 1960. It used natural uranium as fuel and heavy water as moderator.
    - Indigenous design and construction of plutonium plant at Trombay, it ushered in the technology of fuel reprocessing.
    - Research reactors like Zerlina, Purnima, Dhruva and Kamini were commissioned. The last one uses $^{233}_{92}U$ as fuel.
    - The fast breeder reactors which use Plutonium-239 as fuel do not need moderators. They can be used to produce fissile $^{233}_{92}U$ from $^{232}_{90}Th$ and to build power reactors based on them.
- Nuclear power is the fourth -largest source of electricity in India after thermal, hydroelectric and renewable sources of electricity. As of 2016, India has 21 nuclear reactors in operation at seven sites, having an installed capacity of 6780 MW and producing a total of 30,292.91 GWh of electricity. 11 more reactors are under construction to generate an additional 8,100 MW.
- All the 21 nuclear power reactors with an installed capacity of 6,680 MW equal to 2.0% of total installed utility capacity, are operated by the Nuclear Power Corporation of India. India ranked seventh in number of operated reactors (21) and fourteenth in total installed capacity.

Operational nuclear power plants in India

Power station	Operator	State	Type	Units	Total capacity (MW)
Kaiga	NPCIL	Karnataka	PHWR	220 x 4	880
Kakrapar	NPCIL	Gujarat	PHWR	220 x 2	440
Kudankulam[113]	NPCIL	Tamil Nadu	VVER-1000	1000 x 2	2,000
Madras (Kalpakkam)	NPCIL	Tamil Nadu	PHWR	220 x 2	440
Narora	NPCIL	Uttar Pradesh	PHWR	220 x 2	440
Rajasthan	NPCIL	Rajasthan	PHWR	100 x 1 200 x 1 220 x 4	1,180
Tarapur	NPCIL	Maharashtra	BWR PHWR	160 x 2 540 x 2	1,400
Total					6,780

Nuclear power plants and reactors under construction in India[114]

Power station	Operator	State	Type	Units	Total capacity (MW)	Expected Commercial Operation
Kakrapar Unit 3 and 4	NPCIL	Gujarat	PHWR	700 x 2	1,400	2018[115]
Kudankulam[116]	NPCIL	Tamil Nadu	VVER-1000	1000 x 2	2,000[117]	2022-2023
Madras (Kalpakkam)[118]	Bhavini	Tamil Nadu	PFBR	500 x 1	500	early 2018
Rajasthan Unit 7 and 8	NPCIL	Rajasthan	PHWR	700 x 2	1,400	Unit 7: *unknown* Unit 8: *unknown*
Gorakhpur	NPCIL	Haryana	PHWR	700 x 2	1,400	
Total					6,700	

Planned nuclear power plants in India[117][119][120]

Power station	Operator	State	Type	Units	Total capacity (MW)
Jaitapur[121]	NPCIL	Maharashtra	EPR	1650 x 6	9,900
Kovvada[122][123]	NPCIL	Andhra Pradesh	AP1000	1100 x 6	6,600
Kavali[124]	NPCIL	Andhra Pradesh	VVER	1000 x 6	6000
Gorakhpur	NPCIL	Haryana	PHWR	700 x 2	1,400[114]
Bhimpur	NPCIL	Madhya Pradesh	PHWR	700 x 4	2,800[125][121]
Mahi Banswara[121]	NPCIL	Rajasthan	PHWR	700 x 4	2,800
Kaiga	NPCIL	Karnataka	PHWR	700 x 2	1,400
Chutka	NPCIL	Madhya Pradesh	PHWR	700 x 2	1,400
Madras[121]	BHAVINI	Tamil Nadu	FBR	600 x 2	1,200
Tarapur			AHWR	300 x 1	300
Total					39,800

Radiation Hazards:

- Radiation hazards means the risk to the living tissues exposed to the natural radioactivity, X-rays and nuclear radiations (α,β, γ –rays). The damage to the human body from nuclear radiations is due to the ionisation of atoms in the living cells. The ionisation of atoms completely destroys the living cells.
- Radiation hazards lead to the following disorders or diseases:
    - Radiation damage to the chromosomes in the reproductive organs can cause genetic disorder.
    - Radiation damage to the blood producing cells in the spleen can increase the possibility of leukaemia.
    - An acute exposure to radiation weakens or even destroys the infection resistance mechanism and may lead to death.
    - Long exposure to radiation causes cancer.
    - Long exposure to radiation causes blindness.
    - Besides external exposure, radiation damage can come from inhaling air containing radio-isotopes and eating food contaminated with radio-isotopes.

Uses of Radiation:

- However, the controlled exposure to radiation has number of uses:
    - The dangerous disease like cancer is cured by radiation therapy. Γ-rays from Co-60 are used for this purpose.
    - Radiation are used to induce plant mutations which improves the varieties of many crops such as wheat, peas and rice.
    - Radiation are also used to eliminate agriculture pests.
    - Radiation like X-rays are used to detect the fracture in the bone and presence of foreign material in the human body.
    - Gamma rays or X-rays are used to detect the defect in metal castings and welds.
- The amount of absorbed radioactive radiation is measured in "gray" (Gy)
    - 1 gray = absorption of 1 joule of radioactive energy per kg of absorbing material.
- Radioactive exposure or dosage is expressed in "Sievert" (Sv) or "roentgen" (R)
    - A radiation dose of 100 R may cause cancer
    - 600 R may be fatal
    - 250 mR per week is safe so it is a permissible dose.

Answer the following questions:

Are the equations of nuclear reactions 'balanced' in the sense a chemical equation (e.g., $2H_2 + O_2 \rightarrow 2\,H_2O$) is? If not, in what sense are they balanced on both sides?	A chemical equation is balanced in the sense that the number of atoms of each element is the same on both sides of the equation. A chemical reaction merely alters the original combinations of atoms. In a nuclear reaction, elements may be transmuted. Thus, the number of atoms of each element is not necessarily conserved in a nuclear reaction. However, the number of protons and the number of neutrons are both separately conserved in a nuclear reaction. [Actually, even this is not strictly true in the realm of very high energies – what is strictly conserved is the total charge and total 'baryon number'. We need not pursue this matter here.] In nuclear reactions, example below, the number of protons and the number of neutrons are the same on the two sides of the equation. $^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\ {}^{1}_{0}n + Q$
If both the number of protons and the number of neutrons are conserved in each nuclear reaction, in what way is mass converted into energy (or vice-versa) in a nuclear reaction?	We know that the binding energy of a nucleus gives a negative contribution to the mass of the nucleus (mass defect). Now, since proton number and neutron number are conserved in a nuclear reaction, the total rest mass of neutrons and protons is the same on either side of a reaction. But the total binding energy of nuclei on the left side need not be the same as that on the right hand side. The difference in these binding energies appears as energy released or absorbed in a nuclear reaction. Since binding energy contributes to mass, we say that the difference in the total mass of nuclei on the two sides get converted into energy or vice-versa. It is in these sense that a nuclear reaction is an example of mass energy interconversion.
A general impression exists that mass-energy interconversion takes place only in nuclear reaction and never in chemical reaction. This is strictly speaking, incorrect. Explain.	From the point of view of mass-energy interconversion, a chemical reaction is similar to a nuclear reaction in principle. The energy released or absorbed in a chemical reaction can be traced to the difference in chemical (not nuclear) binding energies of atoms and molecules on the two sides of a reaction. Since, strictly speaking, chemical binding energy also gives a negative contribution (mass defect) to the total mass of an atom or molecule, we can equally well say that the difference in the total mass of atoms or molecules, on the two sides of the chemical reaction gets converted into energy or vice-versa. However, the mass defects involved in a chemical reaction are almost a million times smaller than those in a nuclear reaction. This is the reason for the general impression, (which is incorrect) that mass-energy interconversion does not take place in a chemical reaction.

Physical Quantity	Symbol	Dimensions	Units	Remarks
Atomic mass unit		[M]	u	Unit of mass for expressing atomic or nuclear masses. One atomic mass unit equals $1/12^{th}$ of the mass of ^{12}C atom.
Disintegration or decay constant	λ	$[T^{-1}]$	s^{-1}	
Half-life	$T_{1/2}$	[T]	s	Time taken for the decay of one-half of the initial number of nuclei present in a radioactive sample.
Mean life	τ	[T]	s	Time at which number of nuclei has been reduced to e^{-1} of its initial value
Activity of a radio-active sample	R	$[T^{-1}]$	Bq	Measure of the activity of a radioactive source.

Useful data:

$e = 1.6\times10^{-19}C$ $\quad N = 6.023\times10^{23}$ per mole

$1/(4\pi\varepsilon_0) = 9\times10^{9}\ N\ m^2/C^2$ $\quad k = 1.381\times10^{-23} J\ ^0K^{-1}$

$1\ MeV = 1.6\times10^{-13} J$ $\quad 1\ u = 931.5\ MeV/c^2$

$1\ year = 3.154\times10^{7}\ s$

$m_H = 1.007825\ u$ $\quad m_n = 1.008665\ u$

$m(^4_2He) = 4.002603\ u$ $\quad m_e = 0.000548\ u$

1. 1 atomic mass unit = 1 u = 1.67×10^{-27} kg
2. 1 mole = 6.023×10^{23} entities.
3. The radius of a nucleus of mass number A is
   $R = R_0 A^{1/3}$ (where R_0 is emperical constant)
4. For two isotopes of masses M_1 and M_2 and relative abundance x% and y% respectively, the average mass

$$M = \frac{x\times M_1 + y\times M_2}{100}$$

Calculate the radius of the nucleus of an iron atom. Given A = 56 ; R_0 = 1.2 x 10-15 m

- $R = R_0 A_{1/3}$; where A = mass number ; R = 1.2 x 10^{-15} m x $(56)^{1/3}$ = 1.2 x 10^{-15} m x 3.826 ≈ 4.6 10^{-15} m = 4.6 F

How many electrons, protons and neutrons are there in 14 gram of ${}^{14}_{6}C$. Avogadro's number = 6 x 10^{23} /mole

- The mole, symbol mol, is the SI unit of amount of substance.
- **One mole contains exactly 6 × 10^{23} elementary entities.**
- This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number. **The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities**.
- An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles. The mass of an ${}^{14}_{6}C$ atom is exactly 14 Dalton.
- In ${}^{14}_{6}C$ isotope, there are 6 protons, 6 electrons and 8 neutrons.
- The gram-atomic weight of ${}^{14}_{6}C$ is 14 g and the number of atoms in 1 gram-weight (1 mole) is 6 x 10^{23}
- Therefore, in 14 gram of ${}^{14}_{6}C$, we have
  - Number of electrons = 6 x (6 × 10^{23}) = 36 x 10^{23}
  - Number of protons = 6 x (6 × 10^{23}) = 36 x 10^{23}
  - Number of neutrons = 8 x (6 × 10^{23}) = 48 x 10^{23}

Express 1 a.m.u. in kg

- 1 a.m.u. = 1.67 x 10^{-27} kg (note that a.m.u. is also written as u)

How many electrons, protons and neutrons are there in 14 gram of ${}^{14}_{6}C$. Avogadro's number = 6 x 10^{23} /mole

- Number of protons in one atom of ${}^{14}_{6}C$ = 6
- Also by mole concept, 14g of ${}^{14}_{6}C$ will contain 6 x 10^{23} atoms
  - ∴ Number of protons in 14 g of ${}^{14}_{6}C$ = 6 x 6 x 10^{23} = 36 x 10^{23}
  - Similarly, number of electrons in 14 g of ${}^{14}_{6}C$ = 6 x 6 x 10^{23} = 36 x 10^{23}
  - Similarly, number of neutrons in 14 g of ${}^{14}_{6}C$ = 8 x 6 x 10^{23} = 48 x 10^{23}

Give the nuclear radius of ${}^{14}_{6}C$

- We know that $R = R_0 A^{1/3}$ = 1.1 x 10^{15} x $(14)^{1/3}$ = 2.65 x 10^{15} m = 2.65 fm (fermi)

The three stable isotopes of neon are ${}^{20}_{10}Ne, {}^{21}_{10}Ne, {}^{22}_{10}Ne$. The relative abundances are 90.51%, 0.27% and 9.22% respectively. The atomic masses of the three isotopes are 19.99 u, 20.99 u, 21.99 u respectively. Find the average mass of neon?

- Average mass of neon = $\frac{90.51 \, x \, 19.99 + 0.27 \, x \, 20.99 + 9.22 x 21.99}{100}$ = 20.18 u

The half-life of a radioactive sample is 30 seconds. Calculate
(a) the decay constant and (b) the time taken for sample to decay to 3/4th of its initial value?

(a) Decay constant = λ = 0.693/ $T_{1/2}$ = 0.693/30s = 0.0231 s^{-1} ; $\boxed{\lambda = 0.0231 \text{ Bq}}$ (1 Bq = 1 decay per second)

****

(b) We know that $N = N_0 e^{-\lambda t}$; where

N_0 is the number of atoms in a radioactive substance at time t = 0 (undecayed)

N is the number of undecayed atoms in a radioactive substance after time t **(Note : $\log_e X$ is same as lnX)**

λ is the decay constant

- -λt = $\log_e(N/N_o)$ OR λt = $\log_e(N_o /N)$ ∴ $t = \frac{\ln(N_0/N)}{0.0231}$ **We know that ln(x) = 2.303 $\log_{10}$(x) = 2.303 log(x) ; (10 can be omitted)**
- We need to find the time taken for sample to decay to 3/4th of its initial value ⇒ N = number of undecayed atoms left after time t and N_0 is the initial value. Since 3/4th of 'initial value of the sample has decayed' ⇒ 1/4th of the sample is still remaining = N_o/N = 4/1
- $t = \frac{2.303 \text{ x} \log(N_0/N)}{0.0231} = \frac{2.303}{0.0231} \text{ x } log\left(\frac{4}{1}\right) \approx 100 \text{ x} (\log 2^2) = 200 \; x \; 0.3010 \; \approx 200 \; x \; 0.3 =$ 60 seconds
- It is obvious that when given half-life is 30 seconds, and if there are 100 atoms initially, after $T_{1/2}$, 50 atoms will remain undecayed; so in the next half-life time $T_{1/2}$ of 30 seconds, **25 atoms will remain undecayed** (or 25 atoms gets decayed). So, without above calculation, we can easily tell t = 30 + 30 = 60 seconds. So, after 60s, out of initial 100 atoms, 75 atoms will get decayed and 25 atoms will still remain i=undecayed.

The half-life of a radioactive sample is 30 seconds. Calculate (a) the decay constant and (b) the time taken for sample to decay to 3/4th of its initial value?

(a) Decay constant = λ = 0.693/ $T_{1/2}$ = 0.693/30s = 0.0231 s^{-1} ; λ = 0.0231 Bq (1 Bq = 1 decay per second)

(b) We know that $N = N_0 e^{-\lambda t}$; -λt = $\log_e(N/N_o)$ OR λt = $\log_e(N_o /N)$ ∴ $t = \frac{\ln(No/N)}{0.0231}$; ($\log_e$ is written as ln)

- We need to find the time taken for sample to decay to 3/4th of its initial value ⇒ N = number of undecayed atoms left after time t and N_0 is the initial value. Since 3/4th of 'initial value of the sample has decayed' ⇒ 1/4th of the sample is still remaining = N_o/N = 4/1

$t = \frac{\ln\left(\frac{N_0}{N}\right)}{0.0231} = \frac{1}{0.0231} \text{ x } ln\left(\frac{4}{1}\right) = \frac{2}{0.0231} \text{ x } ln(2) = \frac{2}{0.0231} \text{ x } 0.6931 \approx 60 \; seconds$ (Note ln 2 = 0.6931 ; $\log_{10}2$ = 0.3010)

Information about logarithms and exponents:

- Constants : e = Base of natural logarithms ≈ 2.71828182846 ≈ 2.71828
- $\log_{10}(e) \approx 0.434294$; $\log_e(10) \approx 2.303$
- $\log_e(N) \approx 0.4343 \log_e(N)$; $\log_e(N) \approx 2.303 \log_{10}(N)$; $\log_{10}(\pi) \approx 0.49715$
- 1 radian ≈ $57^0\ 17'\ 45''$; π = 22/7 ≈ 3.14159265 ≈ 3.14
- Notations: $\log_e(X) \equiv \ln(X)$ (natural logarithm) ; $\log_{10}(X) \equiv \log(X)$ (logarithm to the base 10)

$\log_e(e) = x$	$\log_{10}(10) = x$	$\log_{10}(20x20) = \log_{10}(20) + \log_{10}(20)$
$\therefore e^x = e$	$\therefore 10^x = 10$	$= 1.3010 + 1.3010 \approx 2.6020$
$\Rightarrow e^x = e^1 \ \therefore\ x = 1$	$\Rightarrow 10^x = 10^1 \ \therefore\ x = 1$	OR $\log_{10}(400) = 2.6021$
$\therefore \log_e(e) = 1$ or $\ln(e) = 1$	$\therefore \log_{10}(10) = 1$ or $\log(10) = 1$	

- Since e ≈ 2.71828 ; $\log_{10}(e) = \log_{10}(2.71828) = 0.434294$; $\log_e(N) \approx (2.303) \times \log_{10}(N)$;
- Example: N = 400 ; $\log_e(400) = \log_e(4x10^2) = \log_e(4) + 2\log_e(10) = \{1.3863 + 2 \times (2.303)\} = 5.9923$ & $\log_{10}(400) = 2.6021$
- $\therefore \log_e(400) = 5.9923$ and $\log_{10}(400) = 2.6021$
- $\therefore \frac{\log_e(400)}{\log_{10}(400)} = \frac{5.9923}{2.6021} = 2.303$; $\therefore$ In general $\frac{\log_e(N)}{\log_{10}(N)} = 2.303$ $\therefore \log_e(N) \approx (2.303) \log_{10}(N)$

To prove that "ln (x) = 2.303 log (x) OR $\ln_e(x) = 2.303 \log_{10}(x)$"

Let me use notation for "natural logarithm" as ln(x) and for "logarithm to base 10" as log(x) {ignoring subscripts e & 10}

(*we know that value of e = Base of natural logarithms ≈ 2.71828182846 ≈ 2.71828*)

- **Let ln(x) = y** **----- (1)** ; Equation (1) can be written in exponential form as
- $x = e^y$; taking log on both sides, we have
- $\log(x) = \mathbf{y} \log(e)$; substitute for 'y' from equation (1), we get
- $\log(x) = \mathbf{\ln(x)} \log(e)$; rearranging keeping natural logarithm in LHS, we get
- $\ln(x) = \frac{\log(x)}{\log(e)}$; **since we know that $\log_{10}(e) = \log_{10}(2.71828) \approx 0.4343$** (*since $10^{0.4343} \approx 2.71828$*)
- $\ln(x) \approx \frac{\log(x)}{0.4343} = \frac{1}{0.4343}\log(x) = \mathbf{2.303}\log(x)$
- **ln(x) ≈ 2.303 log(x)**

Equation $N(t) = N_0 e^{-\lambda t}$ OR $N = N_0 e^{-\lambda t}$ is the **law of radioactive decay**, so $N/N_0 = e^{-\lambda t}$; where

- N_0 is the number of (**undecayed**) atoms present in a radioactive sample at time t = 0 ⇒ initial value of the sample
- N(t) or N is the number of atoms **left undecayed** in the radioactive sample at a later time t = t
- λ is the **decay constant**. For a given element, the value of λ is constant, but for different elements it is different.

$N/N_0 = e^{-\lambda t}$ $\therefore -\lambda t = \ln(N/N_0)$ or $\lambda t = \ln(N_0/N)$ or $\lambda t = 2.303 \log(N_0/N)$

$t = \frac{\ln(N_0/N)}{\lambda}$ or $t = (2.303) \times \frac{\log(N_0/N)}{\lambda}$; both these equations are same ; for example, if λ = 0.0231 s^{-1} ; then for half-life of the sample, we have $N_0 = 2$ and N = 1, then

Using natural logarithm $\rightarrow T_{1/2} = \frac{\ln(2)}{0.0231} = \frac{0.6931}{0.0231} = 30$ ---------------------------------(2)

Using logarithm to base 10 $\rightarrow T_{1/2} = 2.303 \times \frac{\log(2)}{0.0231} = 2.303 \times \frac{0.3010}{0.0231} = \frac{0.6932}{0.0231} = 30$ --------------------(3)

∴ Both the equations (2) & (3) are same

$$e^x = \frac{1}{e^{-x}} \qquad (e^x)^y = e^{x \cdot y}$$

$$e^x \cdot e^y = e^{x+y} \qquad (e^x)^{1/y} = e^{x/y}$$

$$\frac{e^x}{e^y} = e^{x-y} \qquad e^{\ln x} = x$$

$$\ln(x \cdot y) = \ln x + \ln y$$

$$\ln \frac{x}{y} = \ln x - \ln y$$

$$\ln x^y = y \cdot \ln x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ..$$

$$\ln(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ..\right)$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ..$$

For information only:

We know that $N(t) = N_0\, e^{-\lambda t}$ OR $N = N_0\, e^{-\lambda t}$ is the **law of radioactive decay**, so $N/N_0 = e^{-\lambda t}$; where

- N_0 is the number of (**undecayed**) atoms present in a radioactive sample at time t = 0 ⇒ initial value of the sample
- N(t) or N is the number of atoms **left undecayed** in the radioactive sample at a later time t = t
- λ is the **decay constant**. For a given element, the value of λ is constant, but for different elements it is different.
- e = Base of natural logarithm ≈ 2.71828182846

- **$N = N_0\, e^{-\lambda t}$ can also be written as $N = N_0\, (e)^{-\lambda t}$; $N = N_0\, (2.71828182846)^{-\lambda t}$**
- $2.71828182846 = 10^{\log(e)}$
- ∴ $N = N_0\, (10)^{-\lambda t \log(e)}$
- **$N = N_0\, (e)^{-\lambda t}$ and $N = N_0\, (10)^{-\lambda t \log(e)}$** are same. Let us verify the two equations for half-life time and say half-life = $T_{1/2}$ = 30s and calculate λ in both cases.

$N = N_0\, e^{-\lambda t}$	$N = N_0\, (10)^{-\lambda t \log(e)}$
Given N = 1, N_0 = 2, $T_{1/2}$ = 30s ; λ = ?	**Given N = 1, N_0 = 2, $T_{1/2}$ = 30s ; λ = ?**
$e^{-30\lambda} = ½$ $-30\lambda = \ln(1/2)$ $30\lambda = \ln 2$ $\lambda = (\ln 2)/30 = 0.69314718056/30$ $\lambda = 0.69314718056/30$ $\lambda = 0.02310490601\ s^{-1}$	$(10)^{-\lambda t \log(e)} = (10)^{-30\lambda \times 0.4342944819}$ $(10)^{-30\lambda \times 0.4342944819} = (10)^{-13.0288344571\,(\lambda)}$ ∴ $(10)^{-13.0288344571\,(\lambda)} = N/N_0 = ½$ $-13.0288344571\,(\lambda) = \log 2^{-1}$ $13.0288344571\,(\lambda) = \log 2$ $13.0288344571\,(\lambda) = \log 2$ $13.0288344571\,(\lambda) = 0.30102999566$ $\lambda = (0.30102999566)/(13.0288344571)$ $\lambda = (0.30102999566)/(13.0288344571)$ $\lambda = 0.02310490599\ s^{-1}$
Therefore, equations $N = N_0\, (e)^{-\lambda t}$ and $N = N_0\, (10)^{-\lambda t \log(e)}$ are same	

39 What is the approximate mass of a nucleus of uranium?

A 10^{-15} kg B 10^{-20} kg C 10^{-25} kg D 10^{-30} kg

Consider abundant uranium ${}^{238}_{92}U$

Mass of Proton = 1.673 x 10^{-27} (kg) ; Mass of Neutron = 1.675 x 10^{-27} (kg) ; we can say approximately mass of proton = mass of neutron ; Therefore, 238 x 1.7 x 10^{-27} kg ≈ 400 x 10^{-27} kg = 4 x 10^{-25} kg ;

Answer is [C]

38 The grid shows a number of nuclides arranged according to the number of protons and the number of neutrons in each.

A nucleus of the nuclide ${}^{8}_{3}$Li decays by emitting a β-particle.

What is the resulting nuclide?

number of protons							
4					A	B	
3				${}^{6}_{3}$Li	${}^{7}_{3}$Li	${}^{8}_{3}$Li	
2		${}^{3}_{2}$He	${}^{4}_{2}$He			C	D
1	${}^{1}_{1}$H	${}^{2}_{1}$H					
	0	1	2	3	4	5	6

number of neutrons

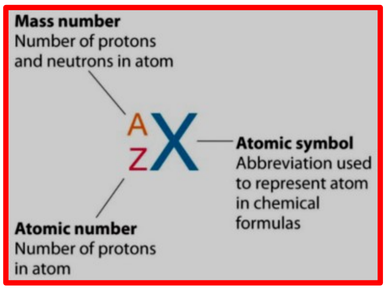
$${}^{4}_{3}\mathrm{Li} \xrightarrow{91\ \mathrm{ys}} {}^{3}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H}$$
$${}^{5}_{3}\mathrm{Li} \xrightarrow{370\ \mathrm{ys}} {}^{4}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H}$$
$$\boxed{{}^{8}_{3}\mathrm{Li} \xrightarrow{840.3\ \mathrm{ms}} {}^{8}_{4}\mathrm{Be} + \mathrm{e}^-}$$
$${}^{9}_{3}\mathrm{Li} \xrightarrow{178.3\ \mathrm{ms}} {}^{8}_{4}\mathrm{Be} + {}^{1}_{0}\mathrm{n} + \mathrm{e}^-$$
$${}^{9}_{3}\mathrm{Li} \xrightarrow{178.3\ \mathrm{ms}} {}^{9}_{4}\mathrm{Be} + \mathrm{e}^-$$
$${}^{10}_{3}\mathrm{Li} \xrightarrow{2\ \mathrm{zs}} {}^{9}_{3}\mathrm{Li} + {}^{1}_{0}\mathrm{n}$$
$${}^{11}_{3}\mathrm{Li} \xrightarrow{8.75\ \mathrm{ms}} {}^{10}_{4}\mathrm{Be} + {}^{1}_{0}\mathrm{n} + \mathrm{e}^-$$
$${}^{11}_{3}\mathrm{Li} \xrightarrow{8.75\ \mathrm{ms}} {}^{11}_{4}\mathrm{Be} + \mathrm{e}^-$$
$${}^{11}_{3}\mathrm{Li} \xrightarrow{8.75\ \mathrm{ms}} {}^{9}_{4}\mathrm{Be} + 2\,{}^{1}_{0}\mathrm{n} + \mathrm{e}^-$$
$${}^{11}_{3}\mathrm{Li} \xrightarrow{8.75\ \mathrm{ms}} {}^{8}_{4}\mathrm{Be} + 3\,{}^{1}_{0}\mathrm{n} + \mathrm{e}^-$$
$${}^{11}_{3}\mathrm{Li} \xrightarrow{8.75\ \mathrm{ms}} {}^{7}_{4}\mathrm{He} + {}^{4}_{2}\mathrm{He} + \mathrm{e}^-$$
$${}^{11}_{3}\mathrm{Li} \xrightarrow{8.75\ \mathrm{ms}} {}^{8}_{3}\mathrm{Li} + {}^{3}_{1}\mathrm{H} + \mathrm{e}^-$$
$${}^{11}_{3}\mathrm{Li} \xrightarrow{8.75\ \mathrm{ms}} {}^{9}_{3}\mathrm{Li} + {}^{2}_{1}\mathrm{H} + \mathrm{e}^-$$
$${}^{12}_{3}\mathrm{Li} \xrightarrow{<\,10\ \mathrm{ns}} {}^{11}_{3}\mathrm{Li} + {}^{1}_{0}\mathrm{n}$$

Answer :

In beta decay, a nucleus spontaneously emits an electron (β^- decay) or a positron (β^+ decay).

In both β^- and β^+ decay, the mass number A (**number of protons + number of neutrons**) remains unchanged.

- **In β^- decay, number of protons goes up by 1 and neutron goes down by 1**
- **In β^+ decay, number of protons goes down by 1 and neutron goes up by 1.**
- **In other words**
  - **In β^- decay, Z = Z + 1 and N = N – 1 ; Where N = # of neutrons**
  - **In β^+decay, Z = Z - 1 and N = N + 1**
- **In problem, When it says Li emits a β particle**
  - **→ this means β^- decay ; so, Z = Z + 1 and N = N – 1**



Answer is [A]

Info :

- **In the above problem if Li emits a positron → β^+decay**
- **→ Z = Z - 1 and N = N + 1, then answer is [D]**
- Example of β^- decay is ${}^{32}_{15}P \rightarrow {}^{32}_{16}S + e^- + \bar{\nu}$ ---------------------------------- (1)
- Example of β^+ decay is ${}^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + e^+ + \nu$ ----------------------------------(2)
- Equation (1) and (2) can also be written as (**here we can see LHS values = RHS values**)
- Example of β^- decay is ${}^{32}_{15}P \rightarrow {}^{32}_{16}S + {}^{0}_{-1}e + \bar{\nu}$
- Example of β^+ decay is ${}^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + {}^{0}_{+1}e + \nu$
- **The basic nuclear process underlying β^- decay is the conversion of neutron to proton**
  - **n → p + e^- + $\bar{\nu}$ OR ${}^{1}_{0}n \rightarrow {}^{1}_{1}H + {}^{0}_{-1}e + \bar{\nu}$ ----- (3)**
- **while for β^+ decay, it is the conversion of proton into neutron**
  - **p → n + e^+ + ν OR ${}^{1}_{1}H \rightarrow {}^{1}_{0}n + {}^{0}_{+1}e + \nu$ ----- (4)**
- Note that while a free neutron decays to proton, the decay of proton to neutron Eq. (4) is possible only inside the nucleus, since proton has smaller mass than neutron.