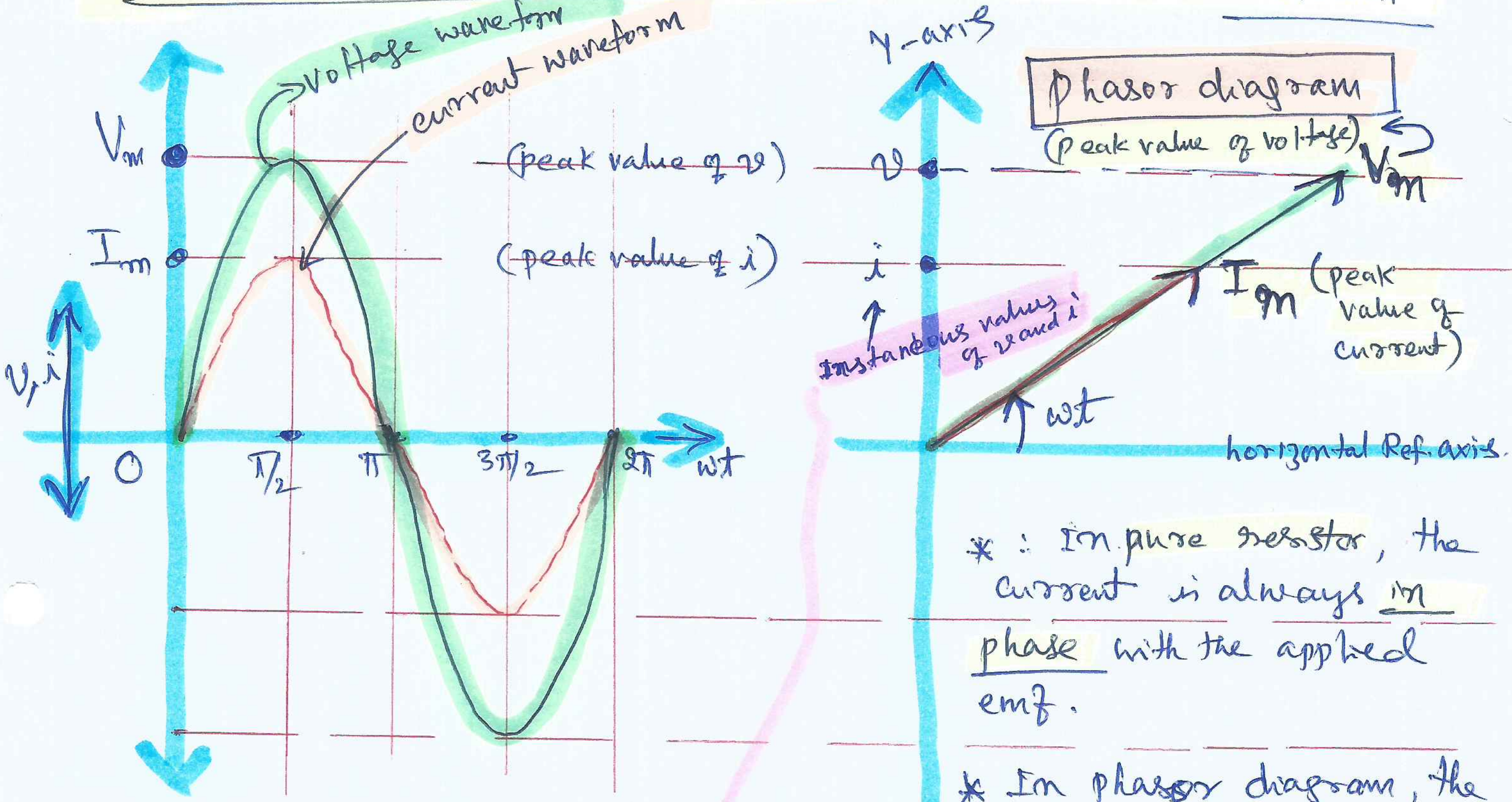
Simplified phasor diagram to represent Resistor in an ac circuit.



* : In pure resistor, the current is always in phase with the applied emf.

* In phasor diagram, the peak values V_m and I_m are represented by vectors (phasors) rotating anticlockwise with angular velocity ω w.r.t horizontal ~~ref~~ reference axis.

IMP { (IMP) → Their projections on the vertical axis give "instantaneous" values v and i of emf and current respectively.

(Use this diagram instead of phasor diagram in page 46)

.... Contd. from pre. page

Resistor in an ac - circuit :

Applied emf $v = V_m \sin(\omega t)$

$i = \frac{V_m}{R} \sin(\omega t) = I_m \sin(\omega t)$ where $I_m = \frac{V_m}{R}$

I Instantaneous power dissipated in the resistor is

$p = vi = V_m I_m \sin^2(\omega t)$; Since $V_m = I_m R$

Instantaneous power : $\boxed{p = I_m^2 R \sin^2(\omega t)}$ → ① Very imp

II Average value of power p over a cycle is

Average power = Average of Instantaneous power over a complete cycle.

$P_{ave} = \bar{p} = \langle I_m^2 R \sin^2 \omega t \rangle$ [$\bar{p}$ denotes average value]

$\therefore P_{ave} = I_m^2 R \langle \sin^2(\omega t) \rangle$; We know that $\left(\sin^2\theta = \frac{(1-\cos 2\theta)}{2} \right)$ (Let $\theta = \omega t$)

P_{ave} over a full cycle $= \frac{I_m^2 R}{2\pi} \int_0^{2\pi} \sin^2\theta \, d\theta$

$$P_{ave} = \frac{I_m^2 R}{2\pi} \int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2} \right) d\theta = \frac{I_m^2 R}{4\pi} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi}$$

$$P_{ave} = \frac{I_m^2 R}{4\pi} \left[(2\pi - 0) - (0 - 0) \right] = \frac{I_m^2 R}{\cancel{4\pi}\, 2} \times \cancel{2\pi}$$

$$\boxed{P_{ave} = \frac{I_m^2 R}{2}}$$

where I_m = peak value of current.

We know that $I_m = I_{rms} \times \sqrt{2}$

$$P_{ave} = \frac{(I_{rms}\sqrt{2})^2 R}{2} = \frac{I_{rms}^2 \cdot \cancel{2} \cdot R}{\cancel{2}} = I_{rms}^2 R$$

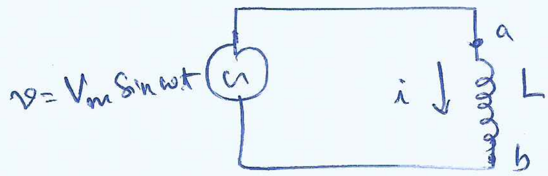
$$\therefore \boxed{P_{ave} = I_{rms}^2 R \quad \text{or} \quad P_{ave} = \frac{I_m^2 R}{2}}$$

Tips : Average value $\sin^2\theta$ (easy way)

The easiest way to Calculate average value of $\sin^2\theta$ is to note that $\sin^2\theta + \cos^2\theta = 1$. Since $\cos^2\theta$ is just $\sin^2\theta$ shifted over by 90°, the average of $\cos^2\theta$ and the average of $\sin^2\theta$ over a full period must be equal. Since the two functions must sum to 1, the average of each of them must therefore be 1/2

$$\therefore \langle \sin^2\theta \rangle = 1/2 \quad \text{and} \quad \langle \cos^2\theta \rangle = 1/2$$

Inductor in an AC-circuit :



$L \rightarrow$ Assume ~~Consider~~ pure inductor with self inductance 'L' (with resistance = 0)

As per K' Loop rule $\quad v - L\frac{di}{dt} = 0$

$\hookrightarrow$ Self induced Faraday emf in the inductor, –ve sign due to Lenz's law.

$$\therefore L\frac{di}{dt} = V_m \sin\omega t$$

$$\int L\frac{di}{dt} = V_m \int \sin(\omega t)\, dt$$

$$L i = -\frac{V_m}{\omega}\cos\omega t$$

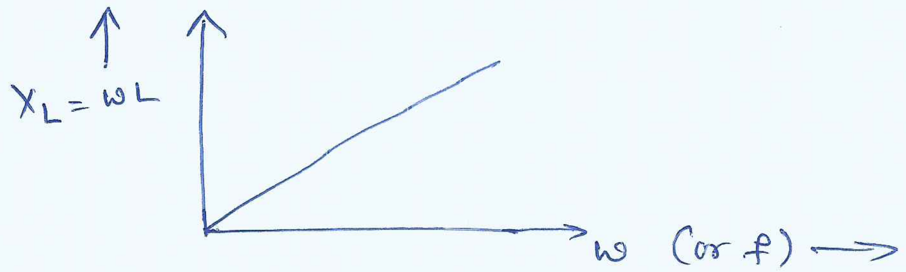
$$i = -\frac{V_m}{\omega L}\cos\omega t \quad \therefore \boxed{i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)} \longrightarrow (1)$$

$$\boxed{i = I_m \sin(\omega t - \pi/2}$$

$\therefore$ Current "Lags" voltage by phase lag of $\pi/2$

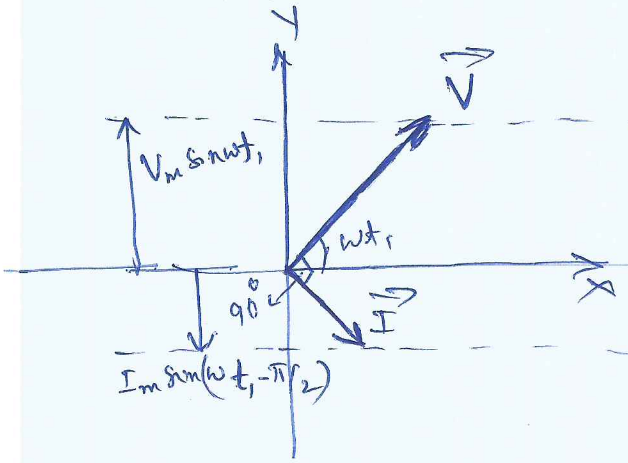
"ωL" is analogous to Resistance and is called "Inductive reactance" (X_L) $\boxed{X_L = \omega L} \longrightarrow (2)$

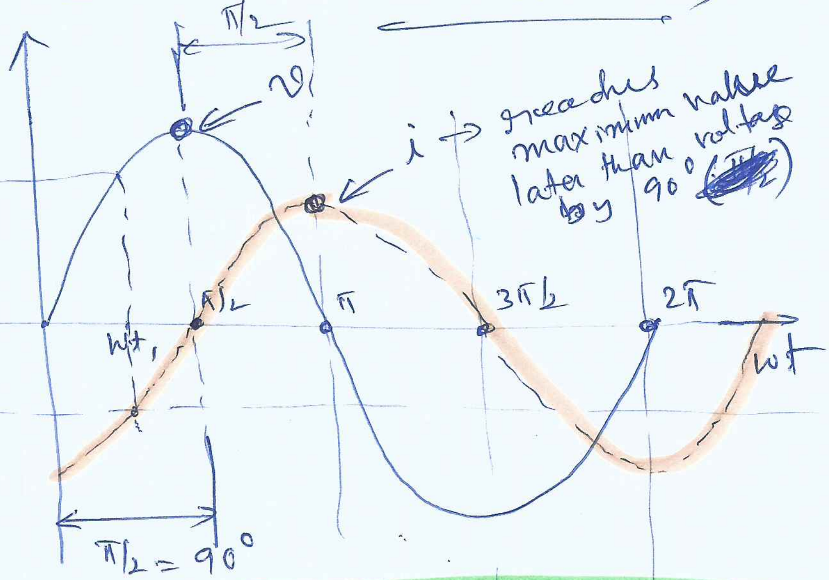
"Inductive Reactance" is the resistance offered by an inductor to the flow of AC. It is given by $X_L = \omega L$.



- Unit of inductance = Henry
- unit of Inductive Reactance $X_L = \omega L$ in Ohms.

Inductor in AC circuit : Phasor diagram (Graph of v and i versus ωt)





Note: Current through an inductor cannot raise instantaneously when AC voltage is applied. It takes time for current to build up in an inductor.

Inductor has reactance that limits current similar to resistance in a DC circuit. Does it also consume power like R ?

- Instantaneous power supplied to L.

$$P_L = iv = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \times V_m \sin(\omega t)$$

$$= -I_m V_m \cos(\omega t) \sin(\omega t)$$

$$\boxed{P_L = -\frac{I_m V_m}{2} \sin(2\omega t)} \quad (3)$$

- Average power over a complete cycle

From (3)

$$P_{avg} = -\frac{I_m V_m}{2 \times 2\pi} \int_0^{2\pi} \sin(2\omega t)\, dt$$

$$= -\frac{I_m V_m}{4\pi} \left[\frac{\cos(2\omega t)}{2\omega}\right]_0^{2\pi}$$

$$= -\frac{I_m V_m}{8\omega\pi} \left[\cos 2(2\pi) - \cos 2(0)\right]$$

$$= -\frac{I_m V_m}{8\omega\pi} \left[\cos 4\pi - \cos 0\right]$$

$$= -\frac{I_m V_m}{8\omega\pi} \left[1 - 1\right] = 0$$

OR $P_{avg} = -\frac{I_m V_m}{2T} \int_0^T \sin(2\omega t)\, dt \qquad \omega = \frac{2\pi}{T}$

$$= -\frac{I_m V_m}{2T} \left[\int_0^T \sin\left(\frac{4\pi}{T} t\right) dt\right]$$

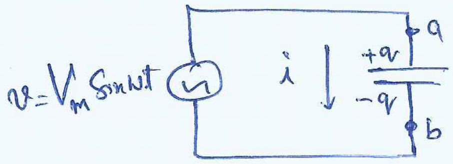
$$= -\frac{I_m V_m}{2T} \left[\frac{\cos\left(\frac{4\pi}{T}\right) t}{4\pi/T}\right]_0^T = -\frac{I_m V_m}{8\pi} \left[\cos 4\pi - \cos 0\right]$$

$$= -\frac{I_m V_m}{8\pi} \left[1 - 1\right] = 0$$

Thus, the "Average Power" supplied to an inductor over one complete cycle is Zero.

↳ See NCERT book (Vol 1) page 240 for description in terms of magnetic flux.

Capacitor in an AC Circuit



→ pure capacitor with capacitance = C Farads.

The capacitor is alternatively charged and discharged as the AC current reverses each half cycle. Let q be the charge on the capacitor at any time t. The instantaneous voltage v across the capacitor is

$$v = \frac{q}{C} \qquad \text{Since } i = \frac{dq}{dt}$$

$$V_m \sin\omega t = \frac{q}{C}$$

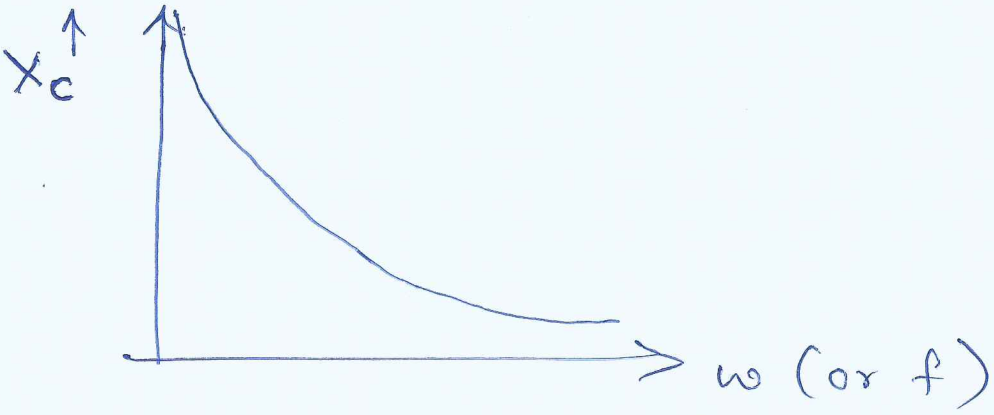
$$q = C V_m \sin \omega t \rightarrow \text{Differentiating, we get}$$

$$i = \frac{dq}{dt} = V_m C \cos\omega t \times \omega = \frac{V_m}{(1/\omega C)} \sin(\omega t + \pi/2)$$

$$\boxed{\begin{aligned} v &= V_m \sin(\omega t) \\ i &= \frac{V_m}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right) \end{aligned}} \rightarrow (1)$$

Current leads voltage by phase angle of $\pi/2$

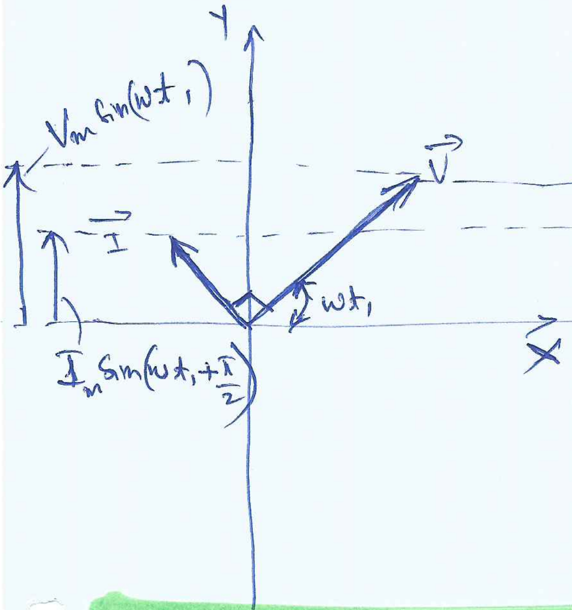
- $X_C = \frac{1}{\omega C}$ → ~~capacitance~~ "Capacitive Reactance"
- Dimension of X_C is same as resistance ⇒ X_C will be in ohms.
- "Capacitive Reactance" is the resistance offered by ~~an~~ a capacitor to the flow of AC. It is given by $X_C = \frac{1}{\omega C}$

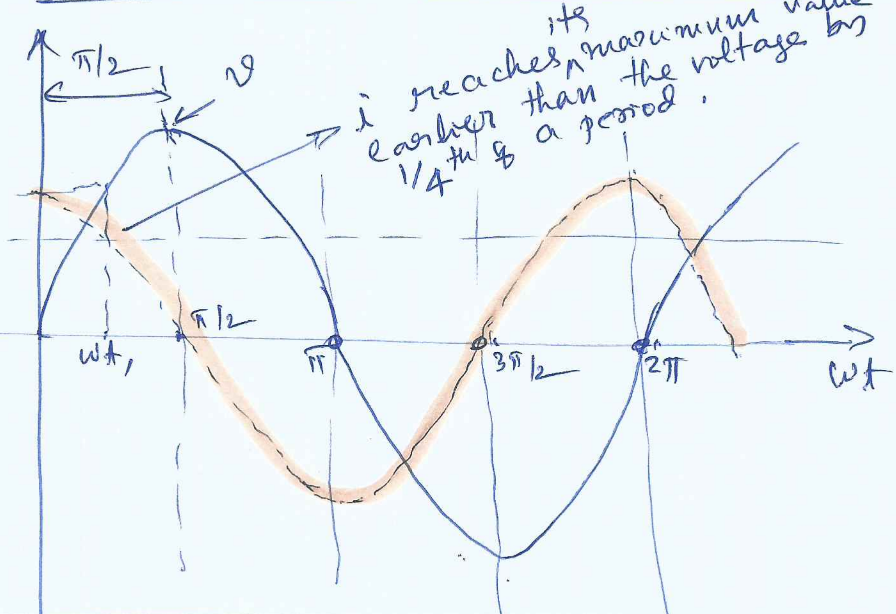


- Unit of capacitance = Farad
  $\mu F = 10^{-6}$ Farads

P.T.O.

Capacitor in AC circuit : phaser diagram (Graph of v and i versus ωt)





Note: voltage across capacitor can not raise instantaneously in an AC (or DC) circuit. It takes time (time constant = RC) for voltage to build up across capacitor

Capacitor has reactance that limits raise in voltage across it instantaneously. Does it also consume power like R?

- Instantaneous power supplied to the capacitor is

~~$P_c = i v = I_m \cos(\omega t) \times V_m \sin(\omega$~~

$$P_c = i v = I_m \sin(\omega t + \pi/2) \times V_m \sin(\omega t)$$

$$= I_m V_m \cos(\omega t)\sin(\omega t)$$

$$\boxed{P_c = \frac{I_m V_m}{2} \sin(2\omega t)} \longrightarrow (2)$$

- Average power supplied to a capacitor over a complete cycle

$$P_{avg} = \frac{I_m V_m}{2 \times 2\pi} \int_0^{2\pi} \sin(2\omega t)\,dt = \frac{I_m V_m}{4\pi}\left[\frac{\cos(2\omega t)}{2\omega}\right]_0^{2\pi}$$

$$= \frac{I_m V_m}{8\omega\pi}\left[\cos(2 \times 2\pi) - \cos 2 \times (0)\right] \quad \because \text{Substituting } \omega t = 2\pi \text{ and } \omega t = 0$$

$$= \frac{I_m V_m}{8\pi\omega}[1 - 1] = 0 \qquad \boxed{P_{avg} = 0} \longrightarrow (3)$$

Thus, the "Average power" supplied to a capacitor over one complete cycle is Zero (Similar to inductor circuit)

↳ see NCERT book (vol 1) page 243 for description of electric field.

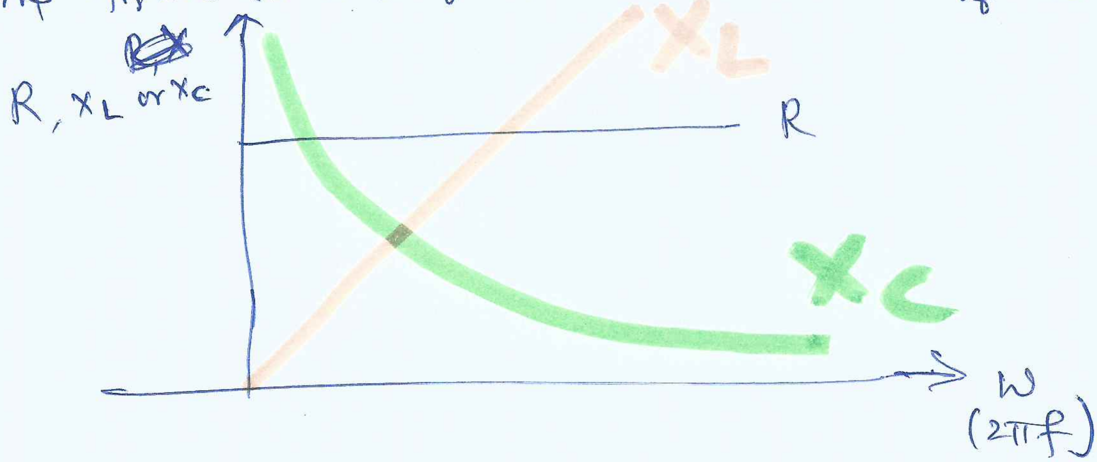
IMP

- Circuit elements with AC

Circuit elements	Amplitude Relation	Circuit quantity	phase of V
Resistor	$V_m = I_m R$	R	in phase with i
capacitor	$V_m = I_m X_C$	$X_c = \frac{1}{\omega C}$	lags i by 90°
Inductor	$V_m = I_m X_L$	$X_L = \omega L$	leads i by 90°

- In DC, $\omega = 0$, therefore $X_L = 0$ and $X_c = \infty$
- Since $X_L = \omega L = 2\pi f L$; Inductor can be used to block high frequencies (Since $X_L \propto f$); It can be used in Low pass filter circuits
- Since $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$; capacitor can be used to block DC and low frequencies ~~(Since $X_c \propto \frac{1}{\omega C}$)~~ (Since $x_c \propto \frac{1}{f}$); It can be used in high pass filter circuits.
- Remember that we can write, $V_R = iR$ (or $V_m = I_m R$), $V_{m(L)} = I_m X_L$; $V_{m(c)} = I_m X_c$ but can't write for instantaneous voltages due to 90° phase difference between voltage and current in both inductor and capacitor circuits.

- Following figure shows graphs of R, X_L, X_c as function of ω

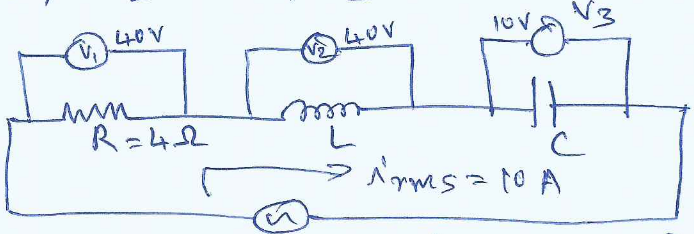


IMP Graph

problem

In the following circuit, readings of voltmeters are
$V_1 = 40V$, $V_2 = 40V$, $V_3 = 10V$

Find
(a) peak value of current
(b) — " — emf
(c) the value of L & C



$$v = V_m \sin\left(100\pi t + \frac{\pi}{6}\right) \Rightarrow f = 50Hz$$

(a) peak value of current ?

Since L and C are pure inductor and capacitors, the only resistance in circuit is $R = 4\,\Omega$
Voltmeter V_1 measures RMS voltage across R.

$$i_{rms} = \frac{V_1}{R} = \frac{40\,V}{4\,\Omega} = 10\,A \rightarrow (1)$$

$$\text{peak value of current} = i = \sqrt{2}\, i_{rms} = \boxed{10\sqrt{2}\,A} \rightarrow (a)$$

(b) peak value of emf: $\Rightarrow V_m = ?$

Since Voltmeters measure RMS voltages, but we need to calculate peak voltage $V_m = ?$

$$V_m = \sqrt{2}\,(V_{rms})_{\text{of circuit}}$$

$$= \sqrt{2}\left[\sqrt{V_1^2 + (V_2 - V_3)^2}\right]$$

$$= \sqrt{2}\left[\sqrt{40^2 + 30^2}\right] = \sqrt{2}\left(\sqrt{1600+900}\right)$$

$$\boxed{V_m = \sqrt{2} \times 10\sqrt{25} = 50\sqrt{2}\ V}$$

(c) Values of L and C ?

i_{rms} through $L = 10$ A
V_{rms} across $L = 40$ V

$$X_L = \frac{V_{rms}}{i_{rms}} = \frac{40}{10} = 4\,\Omega = X_L$$

$$X_L = \omega L = 2\pi f L = 4\,\Omega$$

$$L = \frac{4}{2\pi f} = \frac{4}{\omega} = \frac{4}{100\pi}$$

$$\boxed{L = \frac{1}{25\pi}\ H}$$

i_{rms} through $C = 10$ A
V_{rms} ~~through~~ across $C = 10$ V

$$X_c = \frac{10}{10} = 1\,\Omega$$

$$X_c = \frac{1}{\omega C} \quad \text{or} \quad X_c \omega C = 1$$

$$C = \frac{1}{\omega X_c} = \frac{1}{100\pi \times 1}$$

$$\boxed{C = \frac{1}{100\pi}\ F}$$

Problem:

A resister of resistance 130 Ω and a capacitor of 40 μF are connected to an ac source with angular frequency ω. What is the minimum value of ω for which the circuit is a pure resistive load?

This is a tricky problem.

→ We know that in an LCR (Series) circuit, impedence is given by $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

→ This LCR circuit becomes purely resistive only when $\omega L = \frac{1}{\omega C}$, then $Z = R$.

→ however, in this problem L is not given $L = 0$

$\therefore\ Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$ $\left[\text{or } Z = \sqrt{R^2 + X_C^2}\right]$

In this, $Z = R$, only when $\frac{1}{(\omega C)^2} = 0$

or $\omega C = \infty$

Since C is given a value $= 40\,\mu F$ only ω has to be infinite.

$\therefore$ the answer is ω has to be infinite, which is not possible, therefore a series RC circuit behaves as capacitative circuit only.

IMP

Time constant of an RC circuit is defined as RC

Time constant = RC seconds.

How RC will be in unit of time i.e. second

Time constant = RC

= (Ohms) (farads)

$= \dfrac{\cancel{\text{Volt}}}{\text{Ampere}} \times \dfrac{\text{coulomb}}{\cancel{\text{Volt}}}$

$= \dfrac{\text{Coulomb}}{\text{Ampere}}$

$= \dfrac{\text{Current} \times \text{time}}{\text{Ampere}}$

$= \dfrac{\cancel{\text{Ampere}} \times \text{time}}{\cancel{\text{Ampere}}}$

= time

we know that

$Q = VC$

$C = \dfrac{Q}{V}$

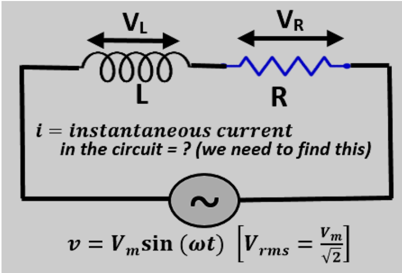
$i = \dfrac{dq}{dt}$

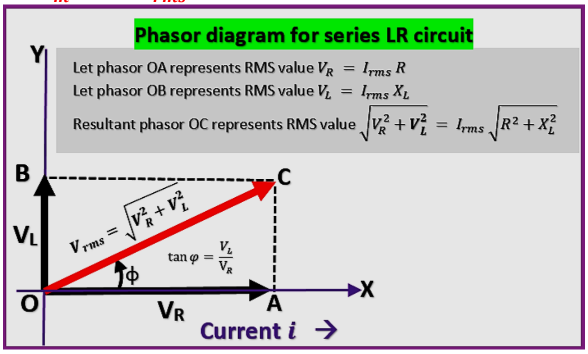
$i = \dfrac{q}{t}$

RC is having unit of time

Series LR circuit → Analysis using Phasors

- AC voltage $v = V_m \sin(\omega t)$ is applied to a **series LR circuit** as shown below; **hence same current i will flow through both the components**. v is the instantaneous voltage of the AC source. V_{rms} is the rms voltage of the AC source. $V_m = \sqrt{2}\, V_{rms}$





- Assume both the components R and L are pure components.
- Let i be the instantaneous current in the series circuit. Since, there is an active components L, there will be a phase difference between v and i. Let φ be the phase difference between v and i, then the instantaneous current is given by $i = I_m \sin(\omega t \pm \varphi)$ -------------- (1)
- **Aim is to find I_m and φ in terms of circuit components. Let us solve this using Phasor diagram.**
- Let us refer to the phasor diagram shown.

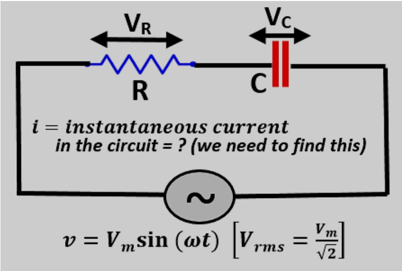
****

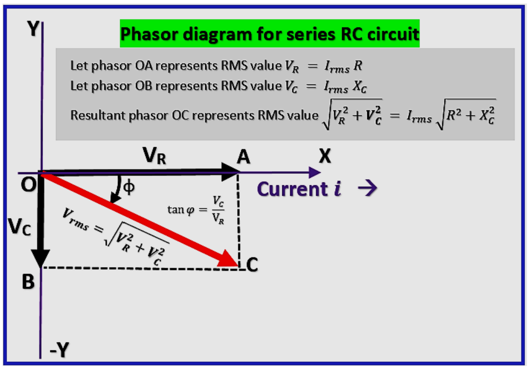
- Let x-axis represents current i in the circuit.
- In resistor, there is no phase difference between v and i, V_R phasor is **also** represented along x-axis.
- In inductor, voltage leads current by 90⁰, hence is represented by OB Phasor along +y axis. *See that OB is leading OA by 90⁰.*
  - Note that current is on +x axis and the direction of rotation of phasor will be anticlockwise (standard convention)
- The resultant of OA and OB is given by OC (***see phasor diagram***) and its value = $\sqrt{V_R^2 + V_L^2}$.
- Since we measure all voltages including the source using multimeter (voltmeter), **we read the rms values**. Since the voltages in the circuit are not in the same phase, we **cannot write** $[V_R + V_L] = V_{rms}$ ***of source***. They cannot be added like ordinary numbers. Therefore, the resultant voltage must be obtained using the Pythagorean theorem $\therefore \sqrt{V_R^2 + V_L^2} = V_{rms}$ ----------------------------------- **(2)**
  - ***In the phasor diagram shown, note that V_R and V_L are rms voltages.***
- Equation (2) can be written as $V_{rms} = \sqrt{(I_{rms}R)^2 + (I_{rms}X_L)^2} = I_{rms}\sqrt{R^2 + X_L^2}$
- $\therefore I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_L^2}}$; this can be written in peak values as {we know that $V_m = \sqrt{2}\, V_{rms}$ and $I_m = \sqrt{2}\, I_{rms}$}
- $\frac{I_m}{\sqrt{2}} = \frac{V_m/\sqrt{2}}{\sqrt{R^2 + X_L^2}} \implies I_m = \frac{V_m}{\sqrt{R^2 + X_L^2}}$
- *So, we have arrived at peak value of current using phasor technique* $I_m = \frac{V_m}{\sqrt{R^2 + X_L^2}}$; *this can be written as*
  - $I_m = \frac{V_m}{Z}$; *where Z is impedence of the circuit* and $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$
- **We calculated I_m and we have to get the expression for φ** ; refer to the phasor diagram
- $\tan\varphi = \frac{V_L}{V_R} = \frac{I_{rms}(X_L)}{I_{rms}R} = \frac{X_L}{R} = \frac{\omega L}{R} \therefore \varphi = \tan^{-1}\left[\frac{\omega L}{R}\right]$; since the circuit behaves predominantly as an inductive circuit, the **current lags resultant voltage by an angle ϕ**. This is shown in the phasor diagram. **If resistance R is removed [⟹ R = 0], $\tan^{-1}(\infty) = 90^0$; *It is a pure inductive circuit*.**
- Note the phasor rotates in the anticlockwise direction, the first phasor in anticlockwise direction is the leading one compared to the one coming behind it.
- $\therefore$ *the instantaneous current in the circuit is* $i = I_m \sin(\omega t - \varphi)$ *as compared to resultant phasor OC where*
  - $I_m = \frac{V_m}{\sqrt{R^2 + X_L^2}}$ and $\varphi = \tan^{-1}\left[\frac{\omega L}{R}\right]$ -- (3)
- **Equation (3) ⟹ I_m and φ specify completely the equation of instantaneous current $i = I_m \sin(\omega t - \varphi)$. Thus we have obtained the amplitude and phase of the "current" in a series LR circuit using phasor technique**

****

Series RC circuit → Analysis using Phasors

- AC voltage $v = V_m \sin(\omega t)$ is applied to a **series RC circuit** as shown below; **hence same current i will flow through all 2 components**. v is the instantaneous voltage of the AC source. V_{rms} is the rms voltage of the AC source. $V_m = \sqrt{2}\, V_{rms}$





- Assume both the components R and C are pure components.
- Let i be the instantaneous current in the series circuit. Since, there are active components L and C, there will be a phase difference between v and i. Let φ be the phase difference between v and i, then the instantaneous current is given by $i = I_m \sin(\omega t \pm \varphi)$ -- **(1)**
- **Aim is to find I_m and φ in terms of circuit components. Let us solve this using Phasor diagram.**
- Let us refer to the phasor diagram shown.

****

- Let x-axis represents current i in the circuit.
- In resistor, there is no phase difference between v and i, V_R phasor is **also** represented along x-axis.
- In capacitor, voltage lags current by 90^0, hence is represented by OB Phasor along -y axis. *See that OB is lagging OA by 90^0*
  - Note that current is on +x axis and the direction of rotation of phasor will be anticlockwise (standard convention)
- The resultant of OA and OB is given by OC (***see phasor diagram***) and its value = $\sqrt{V_R^2 + V_C^2}$.
- Since we measure all voltages including the source using multimeter (voltmeter), **we read the rms values**. Since the voltages in the circuit are not in the same phase, we **cannot write** $[V_R + V_C] = V_{rms}\ of\ source$. They cannot be added like ordinary numbers. Therefore, the resultant voltage must be obtained using the Pythagorean theorem $\therefore \sqrt{V_R^2 + V_C^2} = V_{rms}$ ------------------------------------ **(2)**
  - ***In the phasor diagram shown, note that V_R and V_C are rms voltages.***
- Equation (2) can be written as $V_{rms} = \sqrt{(I_{rms}R)^2 + (I_{rms}X_C)^2} = I_{rms}\sqrt{R^2 + X_C^2}$
- $\therefore I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}}$; this can be written in peak values as {we know that $V_m = \sqrt{2}\, V_{rms}$ and $I_m = \sqrt{2}\, I_{rms}$}
- $\frac{I_m}{\sqrt{2}} = \frac{V_m/\sqrt{2}}{\sqrt{R^2 + X_C^2}} \implies I_m = \frac{V_m}{\sqrt{R^2 + X_C^2}}$
- *So, we have arrived at peak value of current using phasor technique* $I_m = \frac{V_m}{\sqrt{R^2 + X_C^2}}$; *this can be written as*
  - $I_m = \frac{V_m}{Z}$; *where Z is impedence of the circuit* and $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$
- **We calculated I_m and we have to get the expression for φ** ; refer to the phasor diagram
- $\tan\varphi = \frac{V_C}{V_R} = \frac{I_{rms}(X_C)}{I_{rms}R} = \frac{X_C}{R} = \frac{\left(\frac{1}{\omega C}\right)}{R}$
- $\therefore \varphi = \tan^{-1}\left[\frac{\left(\frac{1}{\omega C}\right)}{R}\right]$ or $\varphi = \tan^{-1}\left[\frac{1}{\omega RC}\right]$; **since the circuit behaves predominantly as an capacitive circuit, the current leads resultant voltage by an angle ϕ. This is shown in the phasor diagram. If resistance R is removed [⟹ R = 0],** $\tan^{-1}(\infty) = 90^0$; ***It is a pure capacitive circuit***
- Note the phasor rotates in the anticlockwise direction, the first phasor in anticlockwise direction is the leading one compared to the one coming behind it.
- $\therefore$ *the instantaneous current in the circuit is* $i = I_m \sin(\omega t + \varphi)$ *as compared to resultant phasor OC, where*
  - $I_m = \frac{V_m}{\sqrt{R^2 + X_C^2}}$ and $\varphi = \tan^{-1}\left[\frac{1}{\omega RC}\right]$ -- **(3)**
- **Equation (3) ⟹ I_m and φ specify completely the equation of instantaneous current $i = I_m \sin(\omega t + \varphi)$. Thus we have obtained the amplitude and phase of the "current" in a series RC circuit using phasor technique**

****