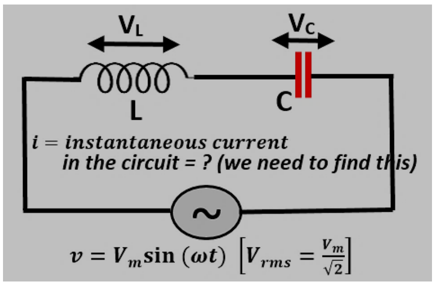
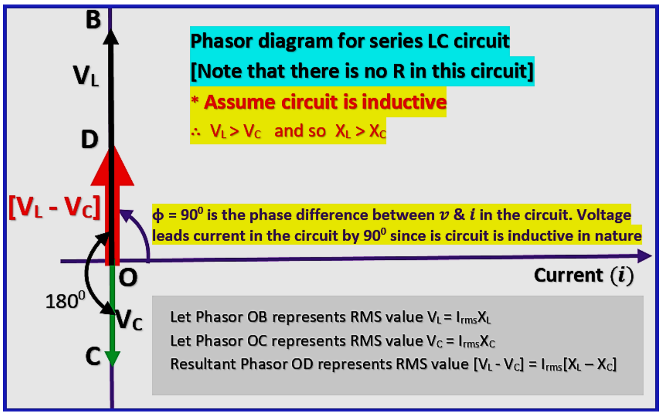
Series LC circuit → Analysis using Phasors

- An AC voltage $v = V_m \sin(\omega t)$ is applied to a __series LCR circuit__ as shown below; **hence same current i will flow through both the components**. v is the instantaneous voltage of the AC source. V_{rms} is the rms voltage of the AC source. $V_m = \sqrt{2}\, V_{rms}$





- Assume L and C are pure components meaning no resistance in those components. Assume that the resistance in the circuit = 0
- Let i be the instantaneous current in the series circuit. Since, there are active components L and C, there will be a phase difference between v and i. Let φ be the phase difference between v and i, then the instantaneous current is given by $i = I_m \sin(\omega t \pm \varphi)$ --------- (1)
- **Aim is to find I_m and φ in terms of circuit components. Let us solve this using Phasor diagram.**
- Let us refer to the phasor diagram shown.

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- Let x-axis represents current i in the circuit.
- In inductor, voltage leads current by 90^0, hence is represented by OB Phasor along +y axis. *See that OB is leading OA by 90^0.*
  - Note that current is on +x axis and the direction of rotation of phasor will be anticlockwise (standard convention)
- In capacitor, voltage lags current by 90^0, hence is represented by OC Phasor along -y axis. *See that OC is lagging OA by 90^0*
- Since V_L and V_C are 180^0 out of phase and assuming $V_L > V_C$ ($\Longrightarrow X_L > X_C$), the resultant $[V_L - V_C]$ is along +y axis & represented by OD.
- The resultant phasor = OD (***see phasor diagram***) and its value = $[V_L - V_C]$.
- **Since we measure all voltages including the source using multimeter**, __**we read the rms values**__. Therefore, **we can consider V_L and V_C as** RMS voltages measured across L and C respectively. We know the resultant phasor = OD = $[V_L - V_C]$ from the phasor diagram
- $\therefore\ [V_L - V_C] = (V_{rms}\ of\ source)$ **[assuming $V_L > V_C$]** -- **(2)**
- **Eq (2)** can be written as $V_{rms} = I_{rms}X_L - I_{rms}X_C = I_{rms}\,[X_L - X_C]\,, where\ X_L = \omega L\,, X_C = \frac{1}{\omega C}\ \&\ \omega = 2\pi f\ where\ f\ is\ the\ frequency\ of\ source\ signal$
- $\therefore I_{rms} = \frac{V_{rms}}{[X_L - X_C]}$; this can be written in peak values as {we know that $V_m = \sqrt{2}\, V_{rms}$ and $I_m = \sqrt{2}\, I_{rms}$}
- $\frac{I_m}{\sqrt{2}} = \frac{V_m/\sqrt{2}}{[X_L - X_C]} \implies I_m = \frac{V_m}{[X_L - X_C]} \implies I_m = \frac{V_m}{Z}\ , where\ Z = [X_L - X_C]\ is\ called\ impedance\ of\ the\ circuit.$
- $So, we\ have\ arrived\ at\ peak\ value\ of\ current\ using\ phasor\ technique\ \boldsymbol{I_m} = \frac{V_m}{[X_L - X_C]}$
- We calculated I_m and we have to get the expression for φ ; refer to the phasor diagram $\varphi = 90^0$; From equation (1) for current
- $i = I_m \sin(\omega t \pm \varphi) = I_m \sin(\omega t \pm 90^0)$; $\pm\varphi$ depends on values of X_L and X_C
- $\therefore$ ***For the applied voltage $v = V_m \sin(\omega t)$, the instantaneous current in the circuit is $i = I_m \sin(\omega t \pm \varphi)$, where***
  - $I_m = \frac{V_m}{[X_L - X_C]}$ **and** $\varphi = 90^0$ -- **(3)**
- Note the phasor rotates in the anticlockwise direction, the first phasor in anticlockwise direction is the leading one compared to the one coming behind it.
- **Equation (3) $\Longrightarrow I_m$ and φ specify completely the equation of instantaneous current $i = I_m \sin(\omega t \pm \varphi)$ in the given circuit. Thus we have obtained the amplitude and phase of the "current" in a series LC circuit using phasor technique**
- Since in this example we have assumed $V_L > V_C$ (see phasor diagram), voltage will be leading current in this LC circuit and the circuit is predominantly inductive and hence φ *is positive*. It is clearly depicted in the phasor diagram where $[V_L - V_C]$ phasor is leading current by 90^0. Or current i lags resultant voltage $[V_L - V_C]$ by 90^0. Therefore, current i can be written as $I_m \sin(\omega t - 90^0)$.
- If $V_L < V_C$, *then* $[V_L - V_C]$ phasor is lagging current by 90^0, then the resultant phasor OD is shown along –Y axis , then the circuit is predominantly capacitive
  - and hence φ *is negative and* $i = I_m \sin(\omega t + 90^0)$

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Special condition:

- *When $X_L = X_C$, then the impedance $Z = 0$; in this situation, the amplitude of the current in the circuit would be infinite.* This is the condition of "electrical resonance". Hence, at resonance $\omega L = \frac{1}{\omega C}$; $\omega^2 = \frac{1}{LC}$ $\implies since\ \omega = 2\pi f, we\ get\ 4\pi f^2 = \frac{1}{LC}$; $f = \frac{1}{2\pi\sqrt{LC}}$.
- This is the natural (resonant) frequency of the circuit. So, the condition for resonance is that the frequency of the applied emf v should be equal to the natural frequency of the circuit.
- Practically, there will be some resistance due to wires and we don't have pure L and C, there will be a resistance in the circuit. This case is dealt separately as LCR circuit.

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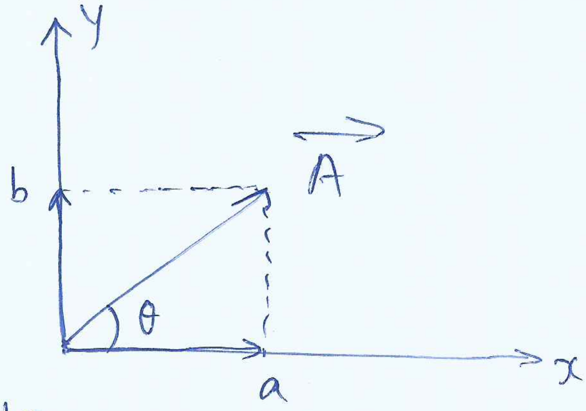
For Info only.

Phasor Algebra:



The complex quantities normally employed in ac circuit analysis can be added and subtracted like coplanar vectors. Such coplanar vectors which represent sinusoidally time varying quantities are known as "phasors"

In Cartesian system, a phasor $\vec{A}$ ~~cabe~~ can be written as

$$\vec{A} = a + jb$$

where a is x-component and b is y-component of phasor $\vec{A}$

- Magnitude of $\vec{A}$ is $|\vec{A}| = \sqrt{a^2+b^2}$
- Angle betn phasor $\vec{A}$ and +ve x-axis is $\Big\}$ $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

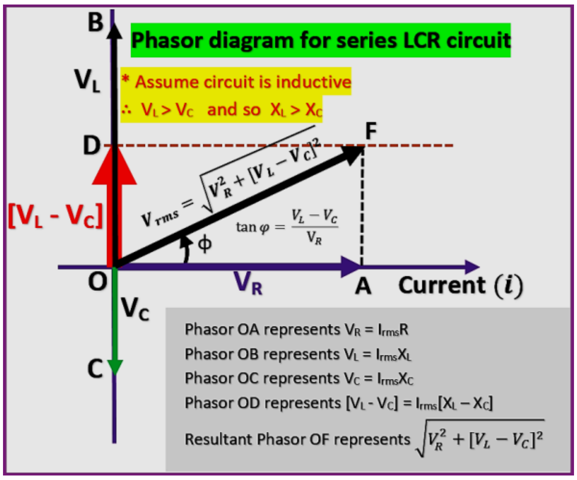
- When a given phasor $\vec{A}$, the direction of which is along the x-axis is multiplied by the operator j, a new phasor $j\vec{A}$ is obtained ~~will~~ which will be 90° anticlockwise from $\vec{A}$, i.e. along +ve y-axis.

- If the operator j is multiplied now to the phasor $j\vec{A}$, a new phasor $j^2\vec{A}$ is obtained which is along −ve x-axis and having same magnitude as of $\vec{A}$.

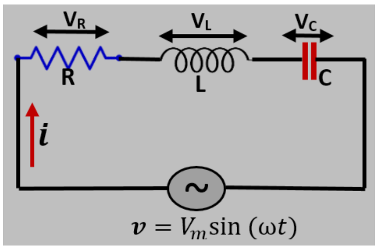
Thus

$$\boxed{\begin{array}{l} j\vec{A} = -\vec{A} \\ j^2 = -1 \quad \text{or} \quad j = \sqrt{-1} \end{array}}$$

Series LCR circuit → Analysis using Phasors



- AC voltage $v = V_m \sin(\omega t)$ is applied to a **series LCR circuit** as shown below; **hence same current i will flow through all 3 components**. v is the instantaneous voltage of the AC source. V_{rms} is the rms voltage of the AC source. $V_m = \sqrt{2}\, V_{rms}$



- Assume all components L,C & R are pure components.
- Let i be the instantaneous current in the series circuit. Since, there are active components L and C, there will be a phase difference between v and i. Let φ be the phase difference between v and i, then the instantaneous current is given by $i = I_m \sin(\omega t \pm \varphi)$.
- **Aim is to find I_m and φ in terms of circuit components. Let us solve this using Phasor diagram.**
- Let us refer to the phasor diagram shown.

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- Let x-axis represents current and since in resistor, there is no phase difference between v and i, V_R phasor is **also** represented along x-axis.
- In inductor, voltage leads current by 90^0, hence is represented by OB Phasor along +y axis. *See that OB is leading OA by 90^0.*
  - Note that current is on +x axis and the direction of rotation of phasor will be anticlockwise (standard convention)
- In capacitor, voltage lags current by 90^0, hence is represented by OC Phasor along -y axis. *See that OC is lagging OA by 90^0*
- Since V_L and V_C are 180^0 out of phase and assuming $V_L > V_C$ ($\Longrightarrow X_L > X_C$), the resultant $[V_L - V_C]$ is along +y axis & represented by OD
- The resultant of OA and OD is given by OF (***see phasor diagram***) and its value = $\sqrt{V_R^2 + [V_L - V_C]^2}$.
- Since we measure all voltages including the source using multimeter, **we read the rms values**. Since the voltages in the circuit are not in the same phase, we **cannot write** $[V_R + V_L + V_C] = V_{rms}$ ***of source***. They cannot be added like ordinary numbers. Therefore, the resultant voltage must be obtained using the Pythagorean theorem $\therefore \sqrt{V_R^2 + [V_L - V_C]^2} = V_{rms}$ ---------------------------------- **(1)**
  - ***In the phasor diagram shown, note that V_R, V_L and V_c are all rms voltage values***
- Equation (1) can be written as $V_{rms} = \sqrt{(I_{rms}R)^2 + (I_{rms}X_L - I_{rms}X_C)^2} = I_{rms}\sqrt{R^2 + (X_L - X_C)^2}$
- $\therefore I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$; this can be written in peak values as {we know that $V_m = \sqrt{2}\, V_{rms}$ and $I_m = \sqrt{2}\, I_{rms}$}
- $\frac{I_m}{\sqrt{2}} = \frac{V_m/\sqrt{2}}{\sqrt{R^2 + (X_L - X_C)^2}} \Longrightarrow I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}$
- *So, we have arrived at peak value of current using phasor technique* $I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}$; *this can be written as*
  - $I_m = \frac{V_m}{Z}$; *where Z is impedence of the circuit* and $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$
- **We calculated I_m and we have to get the expression for φ** ; refer to the phasor diagram
- $\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{I_{rms}(X_L - X_C)}{I_{rms}R} = \frac{(X_L - X_C)}{R} = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$
- $\therefore \varphi = \tan^{-1}\left[\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}\right]$
- *Therefore, the instantaneous current in the circuit is* $i = I_m \sin(\omega t \pm \varphi)$ *, where*
  - $I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}$ and $\varphi = \tan^{-1}\left[\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}\right]$ --- **(2)**
- **Equation (2) $\Longrightarrow I_m$ and φ specify completely the equation of instantaneous current $i = I_m \sin(\omega t \pm \varphi)$. Thus we have obtained the amplitude and phase of the "current" in a series LCR circuit using phasor technique**
- Note the phasor rotates in the anticlockwise direction, the first phasor in anticlockwise direction is the leading one compared to the one coming behind it.
- $\pm\varphi$ *depends on values of* X_L *and* X_C
- *When* $X_L > X_C$ *, then* $V_L > V_C$ *and* $\tan(\varphi)$ *is* $+ve$ *and hence* φ *is positive*. Then the current i lags the voltage v by phase angle φ ; then the circuit is predominantly inductive. (this is shown in the phasor diagram in our example). *the instantaneous **current** in the circuit is* $i = I_m \sin(\omega t - \varphi)$
- *When* $X_L < X_C$ *, then* $V_L < V_C$ *and* $\tan(\varphi)$ *is* $-ve$ *and hence* φ *is negative*. Then the current i leads the voltage v by phase angle φ ; then the circuit is predominantly capacitive. *the instantaneous **current** in the circuit is* $i = I_m \sin(\omega t + \varphi)$
- *When* $X_L = X_C$ *, then* $Z = R$*, this is condition for **minimum Z and hence maximum i***. This case is called "electrical resonance". Hence, at resonance
- $\omega L = \frac{1}{\omega C}$; $\omega^2 = \frac{1}{LC}$ $\Longrightarrow$ *since* $\omega = 2\pi f$*, we get* $4\pi f^2 = \frac{1}{LC}$; $f = \frac{1}{2\pi\sqrt{LC}}$. This is the natural frequency of the circuit. So, the condition for resonance is that the frequency of the applied emf v should be equal to the natural frequency of the circuit.
- $\varphi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right] = \tan^{-1}(0) = 0,\ \therefore\ \varphi = 0\ \&\ i = I_m \sin(\omega t \pm \varphi)$ *becomes* $i = I_m \sin(\omega t)$*, which is in phase with the applied AC voltage*

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IMP :

- Phasor analysis of series LCR AC circuit gives steady-state solution. It does not say anything about transient solution (which exists even for $v = 0$).
- The general solution is the sum of the transient solution and the steady-state solution. After a sufficiently long time, the effects of the transient solution will die out and the behaviour of the circuit is described by only the steady-state solution.

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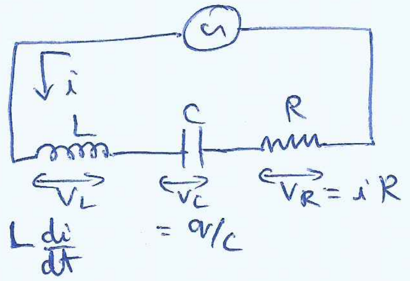
Series LCR circuit (Analytical Solution)

$v = V_m \sin \omega t$



The voltage eqn of LCR circuit is

$$L\frac{di}{dt} + iR + \frac{q}{C} = V_m \sin \omega t \longrightarrow (1)$$

we know that $i = \frac{dq}{dt}$, (1) becomes

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_m \sin \omega t \longrightarrow (2)$$

(2) is like forced, damped oscillator, assume a solution

$$\left.\begin{aligned} q &= q_m \sin(\omega t + \theta) \\ \frac{dq}{dt} &= q_m \omega \cos(\omega t + \theta) \\ \frac{d^2q}{dt^2} &= -q_m \omega^2 \sin(\omega t + \theta) \end{aligned}\right\} \longrightarrow (3)$$

Substitute (3) in (2)

$$-L q_m \omega^2 \sin(\omega t+\theta) + R\omega q_m \cos(\omega t+\theta) + \frac{q_m}{C}\sin(\omega t+\theta) = V_m \sin(\omega t)$$

$$q_m \omega\left[-\underbrace{(\omega L)}_{X_L}\sin(\omega t+\theta) + R\cos(\omega t+\theta) + \underbrace{\frac{1}{\omega C}}_{X_C}\sin(\omega t+\theta)\right] = V_m \sin(\omega t)$$

$$q_m \omega\left[R\cos(\omega t+\theta) + (X_C - X_L)\sin(\omega t+\theta)\right] = V_m \sin(\omega t)$$

Multiplying and dividing Z on LHS above

$$q_m \omega Z\left[\frac{R}{Z}\cos(\omega t+\theta) + \frac{(X_C - X_L)}{Z}\sin(\omega t+\theta)\right] = V_m \sin(\omega t)$$

→ Now let $\frac{R}{Z} = \cos\phi$; $\frac{(X_C - X_L)}{Z} = \sin\phi$ so that $\tan\phi = \frac{X_C - X_L}{R}$

$$\therefore q_m \omega Z\left[\cos\phi \cos(\omega t+\theta) + \sin\phi \sin(\omega t+\theta)\right] = V_m \sin(\omega t)$$

{Since $\cos(A-B) = \cos A \cos B + \sin A \sin B$}

$$q_m \omega Z \cos(\omega t + \theta - \phi) = V_m \sin(\omega t) \longrightarrow (4)$$

comparing the two sides of (4), we get

$$V_m = q_m \omega Z = i_m Z \quad ; \text{ where } i_m = q_m \omega \longrightarrow (5)$$

and $\cos(\omega t + \theta - \phi) = \sin \omega t$

$\therefore (\theta - \phi)$ must be equal to $-\pi/2$

$$\therefore \theta - \phi = -\frac{\pi}{2} \quad \text{or} \quad \boxed{\theta = -\frac{\pi}{2} + \phi} \longrightarrow (6)$$

P.T.O.

 contd. from page 59

Therefore, the current in the circuit is (from eqn 3 ...)

$$i = \frac{dq}{dt} = q_m \omega \cos(\omega t + \theta)$$ → this is from eqn 3

$$\boxed{i = i_m \cos(\omega t + \theta)}$$ (from eqn 5) $\boxed{i_m = q_m \omega}$

From 6 $\theta = \phi - \frac{\pi}{2}$

$$i = i_m \cos(\omega t + \phi - \pi/2)$$

$$= i_m \cos\left[-\left\{\frac{\pi}{2} - (\omega t + \phi)\right\}\right]$$

$$= i_m \cos\left(\frac{\pi}{2} - (\omega t + \phi)\right)$$

$$\boxed{i = i_m \sin(\omega t + \phi)} \rightarrow 7$$

We know that
- $\cos(-A) = \cos A$ — Cos is even
- $\sin(-A) = -\sin A$ — Sin is odd
- $\cos(90° - \theta) = \sin\theta$
- $\sin(90 - \theta) = \cos\theta$

where $i_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$ → 8

and $\phi = \tan^{-1}\left[\frac{(X_L - X_C)}{R}\right]$ → 8a

Or

$i_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (\frac{1}{\omega C} - \omega L)^2}}$ → 9

and $\phi = \tan^{-1}\left[\frac{X_C - X_L}{R}\right]$ → 9a

Same

We know that
$\tan(-A) = -\tan A$

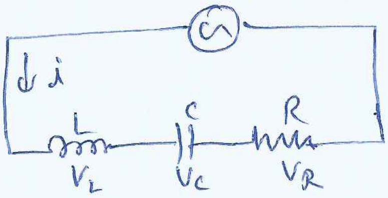
Equations 8a and 9a are valid.

(i) if $X_L > X_C$, $\tan\phi$ is +ve, ϕ is +ve, emf v leads the current i by phase angle ϕ; the circuit is predominantly inductive.

(ii) if $X_L < X_C$, $\tan\phi$ is −ve, ϕ is −ve, emf v lags behind i by phase angle ϕ; the circuit is predominantly capacitive.

Series Resonance circuit : (Also called Acceptor circuit)

$v = V_m \sin(\omega t)$



$$\left.\begin{aligned} v &= v_m \sin(\omega t) \\ i &= \frac{v_m}{Z} = \frac{v_m}{\sqrt{R^2 + (X_L - X_C)^2}} \end{aligned}\right\} \rightarrow (1)$$

If the frequency of the applied emf ~~ϕ~~ v can be varied (eg in radio or TV signal), then in series LCR circuit, there will be a particular frequency (ω_0 or f_0) where $X_L = X_C$

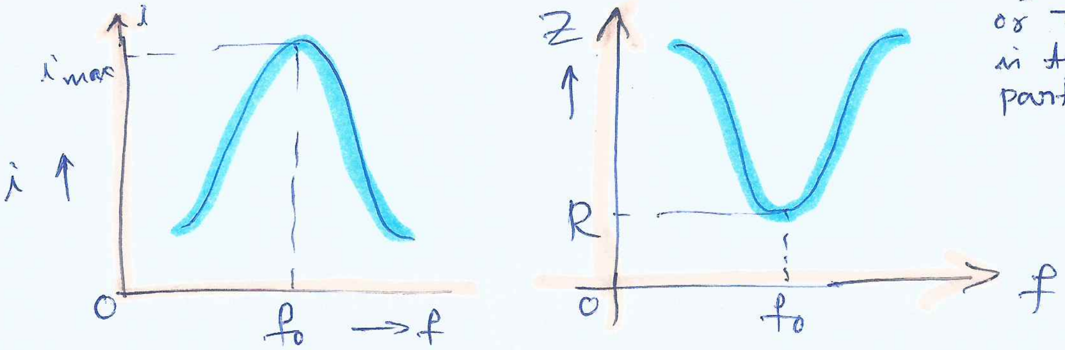
$\therefore$ then $Z = R \qquad i = i_{max} = \dfrac{v_m}{R}$

$$X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

$f_0 \rightarrow$ Resonant frequency (aka) (natural frequency of the circuit)

At resonance, $X_L = X_C$, impedance of circuit is minimum ($= R$) and the current ($i = i_{max}$) in the circuit is maximum. This is the method by which a radio or TV ~~signal~~ set is tuned at a particular frequency.



In Series resonant circuit, voltage across L & C may be > applied emf.

For ex: in above circuit $R = 5\,\Omega$, $X_L = X_C = 20\,\Omega$, $v_{rms} = 100\,V$, then

@ resonance $i_{rms} = \dfrac{E}{Z} = \dfrac{E}{R} = \dfrac{100\,V}{5\,\Omega} = 20\,A$

$V_R = iR = 100\,V$; $V_L = iX_L = 400\,V$; $V_C = iX_C = 400\,V$

Hence, a series resonant circuit gives voltage-amplification. Hence it is also called "voltage resonant" circuit. Note that resonance is possible only if both L and C are present in the circuit. Only then V_L and V_C cancel each other (both being out of phase) and total source voltage (of emf v) appears across R. $\Longrightarrow$ We cannot have resonance in a RL or RC circuit.

P.T.O.

Contd from 61

Characteristics of Series Resonant circuit :

(1) The applied alternating emf and the resulting current are in the same phase.

(2) The impedance Z of the circuit is minimum and is equal to R in the circuit.

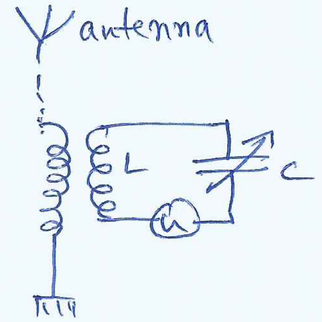
(3) The current i in the circuit is maximum and depends only upon the resistance of the circuit $i = i_{max} = \frac{V_m}{R}$

(4) The p.d. across L and C may be higher than the applied emf to the circuit.

(5) The resistance R of the circuit (if it is small) has no effect on the resonant frequency $f_0 \left(= \frac{1}{2\pi\sqrt{LC}}\right)$ of the circuit. But on increasing R, the resonant current $i_{max} = \frac{V_M}{R}$ in the circuit decreases.

Application of Series resonant (LCR) circuit :

- Series resonant circuit is used in radio (or TV) - receiver for tuning. Radio has LC series circuit which through mutual induction is connected to the antenna coil.

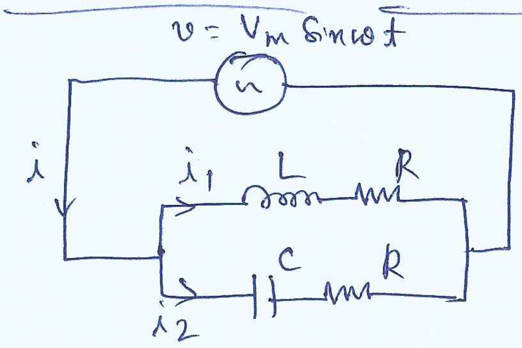


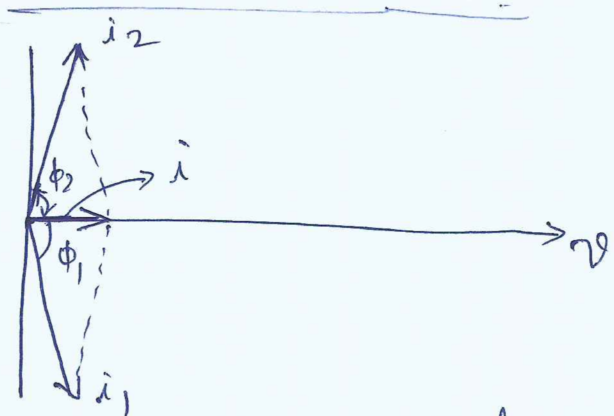
- Waves coming from different radio stations induce potentials of different frequencies in the antenna which passes them to the LC circuit through mutual induction.
- When we turn the knob of the radio and adjust the variable capacitor C so that this circuit is in resonance with the frequency of a particular radio station, then the current corresponding frequency increases appreciably (currents corresponding to other frequencies become very weak), and we listen the programme broadcasted by that radio-station only.

 (Not in ~~Sylla~~ NCERT book)

Parallel Resonant circuit (Also called Rejector circuit)

$v = V_m \sin \omega t$





A parallel resonant circuit is the circuit in which $X_L = X_C$ and main current of the circuit is in phase with the applied emf v.

$$i = i_1 + i_2$$

$$i_1 = \frac{v}{\sqrt{R^2 + X_L^2}} \quad ; \quad i_2 = \frac{v}{\sqrt{R^2 + X_C^2}}$$

If ϕ_1 is phase difference betn i_1 and v }
ϕ_2 is ——— " ——— i_2 and v } then we have

$$\tan \phi_1 = \frac{X_L}{R} \quad \text{and} \quad \tan \phi_2 = \frac{X_C}{R}$$

- Current i_1 lags behind the emf v in phase
- current i_2 leads emf v in phase

If values of L and C are such that $X_L = X_C$

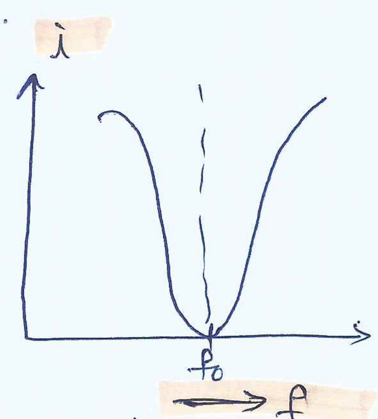
then $i_1 = i_2$

If R is very small as compared to X_L and X_C, then

$$\phi_1 = \phi_2 \cong 90^\circ$$

Note that branch currents i_1 and i_2 >> main current i.

Thus, a parallel resonant circuit gives current-amplification. Hence it is called "Current resonant" circuit.



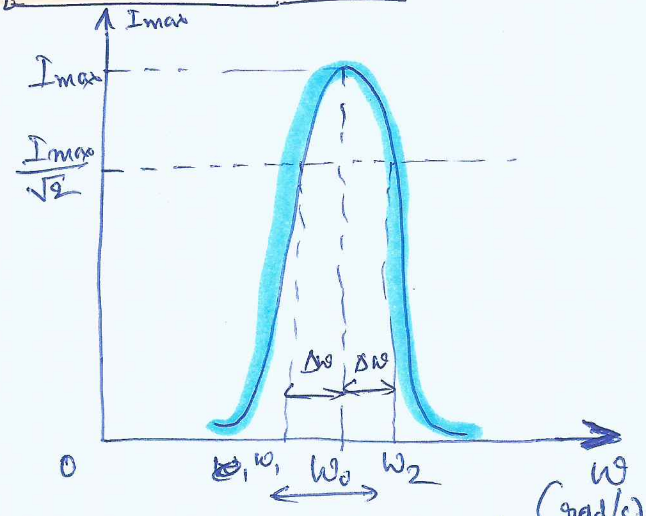
- Smaller the value of R, closer will be ϕ_1 and ϕ_2 to 90° and smaller will be the main current i.

$$f = \frac{1}{2\pi\sqrt{LC}} = f_0 \quad , \text{when } R \text{ is negligible.}$$

In this condition $i = 0$. ~~At f~~ At f_0, $i = 0$

So, this circuit does not allow alternating current to pass whose frequency is equal to its natural frequency. Hence, in parallel resonant circuit, out of many currents, the current of one particular frequency may be stopped, hence used as filter circuit. It is used in Radio transmitter.

Sharpness of Resonance : Q-factor of an LCR circuit



$(\omega_2 - \omega_1) \rightarrow$ bandwidth of the LCR circuit.

When Ac emf $V_m \sin\omega t$ is applied to LCR circuit, we know that

$$I_{max} = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{V_m}{Z}$$

$Z \rightarrow$ impedance of the circuit.

- At resonance, $\omega L = \frac{1}{\omega C}$

  i is max $= I_{max} = \frac{V_m}{R}$

- At other values of ω, I decreases and Z is $> R$.

→ The rapidity with which the current falls from its resonant value $\frac{V_m}{R}$ with change in applied frequency is known as "Sharpness of Resonance"

→ Bandwidth of the circuit is defined as the frequency range with which power of the circuit becomes half or I and V becomes $I_{max}/\sqrt{2}$ or $V_{max}/\sqrt{2}$

→ From figure $BW = \omega_2 - \omega_1$

$$\boxed{\text{Sharpness of Resonance}} = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\text{Resonant frequency}}{\text{Bandwidth}} = \frac{\omega_0}{2\Delta\omega} \quad (1)$$

(Smaller the BW, sharper is the resonance).

Expression for BW (to get expression in terms of L or C and R) and arrive at Quality (Q-factor) of LCR circuit.

- Q-factor (Quality factor Q) is quantitative representation of "sharpness of resonance".

To find Q : At resonant ~~freq~~ freq ω_0, impedance $= R$. Therefore, at ω_1 and ω_2, it must be $\sqrt{2}\,R$

$$Z = \sqrt{R^2 + (\omega L - \tfrac{1}{\omega C})^2} = \sqrt{2}\,R$$

$$R^2 + (\omega L - \tfrac{1}{\omega C})^2 = 2R^2$$

$$(\omega L - \tfrac{1}{\omega C})^2 = R^2$$

@ ω_1 or ω_2

P.T.O.

Contd from 64

$$\therefore \quad \omega L - \frac{1}{\omega C} = \pm R$$

Thus, if $\omega_2 > \omega_1$, we can write

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \longrightarrow 2$$

$$\omega_2 L - \frac{1}{\omega_2 C} = +R \longrightarrow 3$$

→ Adding 2 and 3

$$\omega_1 L - \frac{1}{\omega_1 C} + \omega_2 L - \frac{1}{\omega_2 C} = 0$$

$$(\omega_1 + \omega_2) L - \frac{1}{C}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = 0$$

$$(\omega_1 + \omega_2) L - \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right) = 0$$

$$\cancel{(\omega_1 + \omega_2)} L = \frac{1}{C}\left(\frac{\cancel{\omega_1 + \omega_2}}{\omega_1 \omega_2}\right)$$

$$\therefore \quad \omega_1 \omega_2 = \frac{1}{LC} \longrightarrow 4$$

→ Subtracting 2 from 3, we get

$$\omega_2 L - \frac{1}{\omega_2 C} - \left(\omega_1 L - \frac{1}{\omega_1 C}\right) = R - (-R) = 2R$$

$$(\omega_2 - \omega_1) L + \frac{1}{C}\left[\frac{(\omega_2 - \omega_1)}{\omega_1 \omega_2}\right] = 2R$$

$$\omega_2 - \omega_1 \left[L + \frac{1}{C\omega_1\omega_2}\right] = 2R$$

(Since $\omega_1 \omega_2 = \frac{1}{LC}$ from 4)

$$\omega_2 - \omega_1 \left[L + \frac{1}{\cancel{C}\left(\frac{1}{L\cancel{C}}\right)}\right] = 2R$$

$$\omega_2 - \omega_1 \left[2L\right] = 2R$$

$$\boxed{BW = (\omega_2 - \omega_1) = \frac{R}{L} \longrightarrow 5}$$

We can also write ω_1 and ω_2 in terms of R and L, From figure in page 64

$$\left.\begin{aligned} \omega_2 &= \omega_0 + \frac{(\omega_2 - \omega_1)}{2} = \omega_0 + \frac{R}{2L} \\ \omega_1 &= \omega_0 - \frac{(\omega_2 - \omega_1)}{2} = \omega_0 - \frac{R}{2L} \end{aligned}\right\}$$

P.T.O.

contd from (65)

In page (64), eqn (1)

$$\boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\text{Resonant freq.}}{\text{Bandwidth}}}$$

$$Q = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R}$$

(From eqn (5) in ~~page~~ page 65)

$$\therefore \boxed{Q = \frac{\omega_0 L}{R}} \qquad \text{|||}^{ly} \quad \boxed{Q = \frac{1}{\omega_0 RC}} \longrightarrow (6)$$

$\therefore Q = \frac{\omega_0 L}{R}$ since $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Q = \frac{L}{R} \cdot \frac{1}{\sqrt{L}\sqrt{C}} = \frac{\sqrt{L}\cancel{\sqrt{L}}}{R\cancel{\sqrt{L}}\sqrt{C}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\therefore \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}} \longrightarrow (7)$$

When we consider $Q = \frac{1}{\omega_0 RC} = \frac{1}{\frac{1}{\sqrt{LC}} \cdot RC} = \frac{\sqrt{L}\cancel{\sqrt{C}}}{R\sqrt{C}\cancel{\sqrt{C}}}$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$ Same result as (7).

From (6)

$\therefore$ If BW is smaller (either R is low, L is large), Q is ~~higher~~ large and we say selectivity of the circuit is more (or circuit is more selective).

———— o ———— o ———— o ——

Power in an AC circuit – (power factor)

The rate of dissipation of energy in an electrical circuit is called the "power." The power of an ac circuit depends upon the phase difference bet^n the emf and the current.

power = vi ; if i is ampere, V in volt, then p will be in watts.

- Let in a series LCR circuit, the phase difference bet^n v and i be ϕ. $(v = V_m \sin(\omega t))$ $[i = I_m \sin(\omega t - \phi)]$ ($\pm$ depending upon values of X_L and X_C)
- "Instantaneous power" in the circuit is

$$P_{Ins} = vi = V_m I_m \sin(\omega t)\sin(\omega t - \phi)$$
$$= V_m I_m \sin(\omega t)[\sin \omega t \cos\phi - \cos\omega t \sin\phi]$$
$$= V_m I_m [\sin^2(\omega t)\cos\phi - \sin(\omega t)\cos\omega t \sin\phi]$$
$$P_{Ins} = V_m I_m [\sin^2(\omega t)\cos\phi - \tfrac{1}{2}\sin(2\omega t)\sin\phi]$$

- The Average power over a complete cycle of a.c. thro' LCR circuit is given by

~~Since~~ average value ~~over~~ for one complete cycle → $\langle \sin^2\theta \rangle = \frac{1}{2}$ and $\langle \sin(2\omega t) \rangle = 0$

$$\boxed{\text{Average power } p = \frac{1}{2} V_m I_m \cos\phi}$$

$$\therefore p = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos\phi$$

$$\therefore \boxed{p = V_{rms} I_{rms} \cos\phi} \rightarrow (1) \quad \text{or} \quad \boxed{p = I_{rms}^2 Z \cos\phi}$$

where $\cos\phi$ is known as "power factor" of the circuit and its value depends upon the nature of the circuit.

Case(i): Circuit containing pure Resistor R ONLY, the v and i are in the same phase $\Rightarrow \phi = 0$; $\cos\phi = 1$ ∴

$$\therefore p = V_{rms} I_{rms} = V_{rms}^2/R = I_{rms}^2 R \longrightarrow$$ power loss or Maximum dissipation

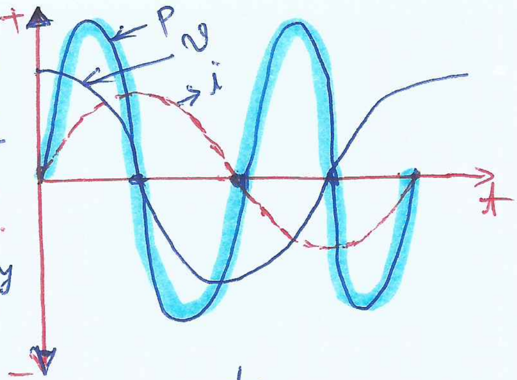
Case(ii): AC circuit having pure Inductor L or pure capacitor C only.

In pure L or pure C case, $\phi = 90°$; $\cos\phi = 0 \Rightarrow P = 0$.

Thus, no power loss takes place in a circuit having either L only or C only. although current flows in the circuit, yet the ave. power remains zero. The current in such a circuit is called "Wattless current". In practice, however, a wattless current is not a reality because no circuit can be entirely resistanceless.

→ Thus, if the resistance in an a.c. circuit is zero, ⇒ there is no energy dissipation in the circuit.

↳ Extra Info for case(ii): Fig shows curves for instantaneous current i, emf v and power p have been drawn for an ac circuit containing pure L.



The power curve is symmetrical about the time axis, which shows that, for one complete cycle, the ave. power is zero.

In $\frac{1}{4}$th cycle, the inductor draws energy from the a.c. source and stores it in the form of mag. field ; in the next $\frac{1}{4}$th cycle, the ~~mg~~ mag. field of the inductor becomes zero i.e. inductor returns the energy to the ac source.

Thus, the inductor does not draw any net energy from the source.

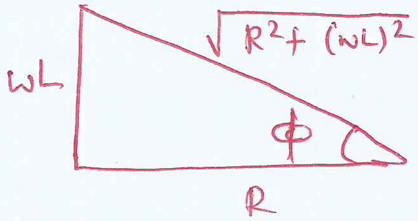
→ The same is true for an a-c circuit containing only pure Capacitor C. The energy which the capacitor draws from the a.c. source in $\frac{1}{4}$th cycle in the form of electric field, it returns the same energy to the source in the next $\frac{1}{4}$th cycle.

It is for this reason that in a.c. circuits, either an L or a C is used for controlling the current.

Contd from pre. page.

Case (iii): In an L-R Series circuit, the phase difference betn v and i is given by $\tan\phi = \frac{\omega L}{R}$

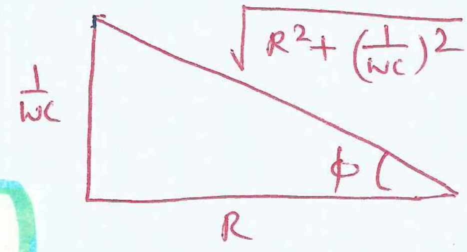
$$\therefore \cos\phi = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$



$$\boxed{\text{Average power } p = v_{rms}\, i_{rms} \times \frac{R}{\sqrt{R^2 + (\omega L)^2}}}$$

Case (iv): In RC circuit, the phase difference bet v and i is given by $\tan\phi = \frac{X_C}{R} = \frac{(1/\omega C)}{R}$

$$\therefore \cos\phi = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

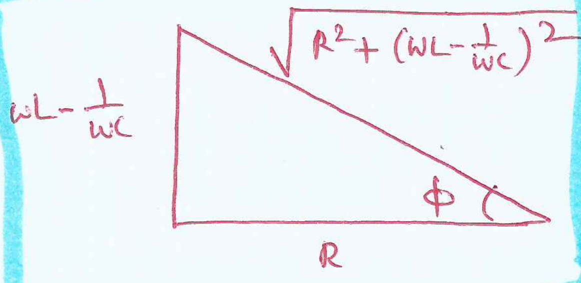


$$\boxed{\text{Average power } p = v_{rms}\, i_{rms} \times \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}}$$

Case (V): In LCR circuit, the Phase difference betn v and i is

$$\tan\phi = \frac{\omega L - 1/\omega C}{R}$$

$$\therefore \cos\phi = \frac{R}{\sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}} = \frac{R}{Z}$$



$$\therefore \boxed{p = v_{rms}\, i_{rms} \times \frac{R}{Z}}$$

↳ ϕ may be non-zero in LR or RC or LCR circuit. Even in such cases, power is dissipated only in the resistor.

Case (vi): power dissipated at Resonance in Series LCR circuit:

At, Resonance, $X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$ $\therefore \cos\phi = 1$ $\therefore \phi = 0$

$\therefore p = I^2 Z$ since $Z = R$ $\Rightarrow \boxed{p = I^2 R}$ $\Rightarrow$ Maximum power is dissipated in a circuit (through R) at resonance.

NOTE ▷ For transporting electric power without much I^2R loss is very much required. Since $p = v_{rms}\, i_{rms} \cos\phi$, where $\cos\phi$ = power factor.

To supply a given power at a given voltage, if $\cos\phi$ is small, then we have to increase the current accordingly; but this leads to large power loss (I^2R) in transmission. The solution is to improve power factor (tending to 1) by making $Z \to R$. This can be done by connecting a suitable capacitor in parallel and then $p \simeq I^2R$.

In a Series LCR circuit, let ϕ be the phase difference between v and i. The "Instantaneous power" is given by

$$P_{ins} = v\,i = V_m \sin(\omega t) \times I_m \sin(\omega t - \phi)$$

$(\omega t \pm \phi)$ depending upon values of X_L & X_C ; After Simplifying

$$P_{ins} = V_m I_m \left[\sin^2(\omega t)\cos\phi - \frac{1}{2}\sin(2\omega t)\sin\phi\right] \longrightarrow (1)$$

Circuit	$v = V_m \sin(\omega t)$, $i = I_m \sin(\omega t)$ where $I_m =$	Instantaneous Power (P_{ins}) $=$	Average power in one complete cycle $P_{ave} = ?$
R only	$\frac{V_m}{R}$	Since phase difference between v and $i = 0 \Rightarrow \phi = 0$. Therefore $P_{ins} = V_m I_m \sin^2(\omega t) = I_m^2 R \sin^2(\omega t)$	Since $\langle \sin^2(\omega t)\rangle = 1/2$ and $\langle \sin 2\omega t\rangle = 0$ $P_{ave} = \frac{V_m I_m}{2} = V_{rms} \times i_{rms}$ $P_{ave} = \frac{V_{rms}^2}{R} = i_{rms}^2 R$
L only	$\frac{V_m}{\omega L}$	Since $\phi = 90^\circ$ $P_{ins} = -\frac{V_m I_m}{2} \sin(2\omega t)$	Since $\phi = 90^\circ$, $\cos\phi = 0$ and $\langle \sin 2\omega t\rangle = 0$ $P_{ave} = 0$
C only	$\frac{V_m}{(1/\omega C)}$	Since $\phi = -90^\circ$ (say) $P_{ins} = \frac{V_m I_m}{2} \sin(2\omega t)$	Since $\phi = -90^\circ$, $\cos\phi = 0$ and $\langle \sin 2\omega t\rangle = 0$ $P_{ave} = 0$
LCR circuit	$\frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$	In Series LCR circuit, we know that $\tan\phi = (\omega L - 1/\omega C)/R$ $\therefore \cos\phi = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{R}{Z}$ $\therefore P_{ins}$ = Same as eqn (1), where $\cos\phi = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$	Since $\langle \sin^2(\omega t)\rangle = 1/2$, $\langle \sin(2\omega t)\rangle = 0$ $P_{ave} = \frac{V_m I_m}{2} \cos\phi$ $P_{ave} = v_{rms}\, i_{rms} \cos\phi$, where $\cos\phi = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$
LR circuit	$\frac{V_m}{\sqrt{R^2 + (\omega L)^2}}$	In Series LR circuit, we know that $\tan\phi = \omega L / R$ $\therefore \cos\phi = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$ P_{ins} = same as eqn (1), where $\cos\phi = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$	Since $\langle \sin^2(\omega t)\rangle = 1/2$ and $\langle \sin(2\omega t)\rangle = 0$ $P_{ave} = \frac{V_m I_m}{2} \cos\phi$ $P_{ave} = v_{rms}\, i_{rms} \cos\phi$, where $\cos\phi = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$
RC circuit	$\frac{V_m}{\sqrt{R^2 + (\frac{1}{\omega C})^2}}$	In Series RC circuit, we know that $\tan\phi = (1/\omega C)/R$ $\therefore \cos\phi = \frac{R}{\sqrt{R^2 + (\frac{1}{\omega C})^2}}$ P_{ins} = Same as eqn. (1), where $\cos\phi = \frac{R}{\sqrt{R^2 + (\frac{1}{\omega C})^2}}$	Since $\langle \sin^2(\omega t)\rangle = 1/2$ and $\langle \sin(2\omega t)\rangle = 0$ $P_{ave} = \frac{V_m I_m}{2} \cos\phi$ $P_{ave} = v_{rms}\, i_{rms} \cos\phi$, where $\cos\phi = \frac{R}{\sqrt{R^2 + (\frac{1}{\omega C})^2}}$
LC circuit	Since $R = 0$ $I_m = \frac{V_m}{\left(\omega L - \frac{1}{\omega C}\right)}$	(not @ Resonance) Since $R = 0$, $\phi = 90^\circ$; $\cos\phi = 0$ $\therefore P_{ins} = -\frac{V_m I_m}{2} \sin(2\omega t)$ or $P_{ins} = \frac{V_m I_m}{2} \sin(2\omega t)$ depending upon values of X_L and X_C	Since $\phi = 90^\circ$, $\cos\phi = 0$ and $\langle \sin 2\omega t\rangle = 0$ $P_{ave} = 0$ No energy dissipation in the circuit (wattless current)