Chapter 8

Electromagnetic Waves

Introduction :

We have learnt from

From Ch. 4 (Moving charges & Magnetism)

produces

- Electric current (moving charges) produces mag. field around it
- Two current-carrying wires exert a mag. force on each other.

From ch 6 (Electromagnetic Induction)

- "Time-varying magnetic field gives rise to an electric field"

Is the converse also true? means "Time-varying Electric field gives rise to a magnetic field".

Yes as per Maxwell.

- Maxwell concluded "magnetic field" is generated by two ways ~~Also~~
  (1) Electric current produces mag. field around it.
  (2) time-varying electric field also generates mag field.
- While applying Ampere's Circuital law [$\oint B \cdot dl = \mu_0 I$] to find $\vec{B}$ at a point outside ~~the~~ a capacitor connected to a circuit with a time-varying current, Maxwell noticed an inconsistency in Ampere's Law [$\oint B \cdot dl = \mu_0 I$]. He suggested the existence of an additional current, called displacement current to remove this inconsistency.
- Maxwells equations involving electric and mag. fields, and their sources, the charge and current densities. Following 4 eqns are known as Maxwells equations.

1. $\oint E \cdot dA = \frac{Q}{\epsilon_0}$ (Gauss's law for electricity) — $EA = Q/\epsilon_0$ ($EA = \phi_E$), $\phi_E = Q/\epsilon_0$ → LHS represents "electric flux" and so has units of electric flux density (V/m) multiplied by m2 ⇒ V-m

2. $\oint B \cdot dA = 0$ (Gauss's law for Magnetism)

3. $\oint E \cdot dl = -\frac{d\phi_B}{dt}$ (Faraday law)

4. $\oint B \cdot dl = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$ (Ampere – Maxwell law) → additional term $\epsilon_0 \frac{d\phi_E}{dt}$ displacement current suggested by Maxwell.

Together with Lorentz Force formula, all these (5) eqns mathematically express all the basic laws of electromagnetism.

$$F = q\vec{E}(r) + q\left[\vec{v} \times \vec{B}(r)\right] \equiv F_{electric} + F_{magnetic}$$

P.T.O →

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- According to Maxwell, when either of the electric field or magnetic field changes with time, the other field is produced. This leads to the production of electromagnetic disturbance or wave of time ~~time~~ varying electric and magnetic fields that propagate in space.

- From optical measurements, the speed of these e-m waves, in free space is very close to speed of light $3\times10^8\ m\,s^{-1}$.

⇒ This lead to a remarkable conclusion that LIGHT is an E-M wave.

⇒ Thus, Maxwell's work unified 3 domains
① Electricity
② Magnetism
③ LIGHT (optics)

- Hertz, in 1885, experimentally demonstrated the existence of e-m waves.

- Marconi was the first to use e-m waves for "communication" purpose. Marconi used these e-m waves betⁿ two locations of about 50 km apart, without using the wires betⁿ these locations and successfully communicated the ~~sig~~ signal ~~b~~ betⁿ these locations. This lead to the discovery of "wireless communication" and Marconi was considered as a "father of wireless communication".
Now-a-days, the wireless communication has revolutionized our ~~lives~~ life.

IMPORTANT

- It is possible to produce these e-m waves practically, and know its properties and later its immense use in many applications including medical and ~~co~~ communication fields.

IMP

Maxwell's "Displacement current" : A new Source of Mag. field.

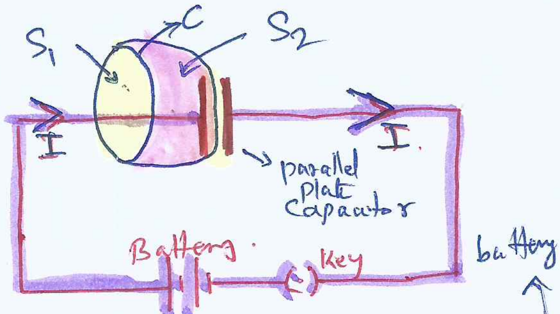
→ We know that an "electric Current" produces a "mag. field" around it

→ Maxwell's logical ~~proposal~~ suggestion is that "time varying electric field also generates a "mag field" → this effect is of great importance since it explains existance of e-m waves.

→ To see how a "time-varying electric field" gives rise to a "mag. field", let us consider a circuit of charging a capacitor and apply Ampere's Circuital law.

→ Ampere's circuital law $\oint \vec{B}\cdot\vec{dl} = \mu_0 I$ is valid if electric field in a given region does not change with time. If $\vec{E}$ changes with time, then Ampere's law needs modification. Maxwell modified Ampere's law to make it valid for constant as well as time-varying electric fields.

→ Maxwell showed that Ampere's law $\oint \vec{B}\cdot\vec{dl} = \mu_0 I$ was logically inconsistent when applied to charging or discharging of a capacitor.



→ To understand this, consider a circuit having a capacitor as shown in figure.

→ When key is closed, a current I flows through the connecting wires and the capacitor begins to charge. Current I decreases as the charge on the capacitor grows. When C is fully charged to ~~battery~~ voltage, the current I vanishes. There is no current betⁿ the plates of the capacitor during charging.

→ Let us consider a plane circular surface S_1 and a hemispherical surface S_2, both bounded by the same closed loop C, such that during charging, the conduction current I threads (passes) through S_1 but not through S_2 (since S_2 lies in the region betⁿ plates)

→ Now, if the closed loop C is considered to bound the surface S_1, then by Ampere's circuital law, we have

$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0 I \longrightarrow (1)$$ since "I" threads through S_1

→ If the closed loop C is considered to bound the surface S_2, then

$$\oint_C \vec{B}\cdot\vec{dl} = 0 \longrightarrow (2)$$ Since, no current threads (passes) thro' S_2.

Obviously, eqⁿ (1) and (2) both cannot be correct for the same closed loop C. Maxwell pointed out this contradiction arises because of the assumed discontinuity in current (absence of current betⁿ the plates)

→ ~~He assigned~~

— P.T.O. Contd...

5

- Maxwell argued that, during charging of capacitor, a changing electric field (with time) exists in the region betn capacitor plates.
- This electric field is equivalent to a current which exists so long as the electric field is changing and produces the same magnetic ~~field~~ effect as does the conduction current.
- The current corresponding to the changing electric field is called the "displacement current".
- Thus, during charging of capacitor, there is a conduction current I_c (say) thro' the connecting wires and a displacement current I_d in the region betn capacitor plates. Hence the current being continuous ~~in the~~ ~~circuit~~ circuit.
- Hence, according to "Ampere - Maxwell" law,

$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 (I_c + I_d)}$$

Expr for I_d = ?

- Expr for I_d : Let $+q$ and $-q$ be the charge on the left and the right plates of the capacitor respectively. Let σ be the surface charge density of the plates of the capacitor. Then, the electric field betn the plates of the capacitor is given by

$$E = \frac{\sigma}{\epsilon_0} \Rightarrow \boxed{E = \frac{q}{\epsilon_0 A}} \qquad \because \sigma = q/A$$

- Electric flux thro' the surface S_2

$$\phi_E = EA = \frac{q}{\epsilon_0} \qquad \left(\because E = \frac{q}{\epsilon_0 A}\right)$$

$$\frac{d\phi_E}{dt} = \frac{1}{\epsilon_0}\frac{dq}{dt} = \frac{1}{\epsilon_0}(I_d) = \frac{I_d}{\epsilon_0}$$

$$\therefore \boxed{I_d = \epsilon_0 \frac{d\phi_E}{dt}}$$ which is the expression for displacement current I_d.

$$\therefore \oint \vec{B}\cdot\vec{dl} = \mu_0 (I_c + I_d)$$

(I_c = Conduction current through the connecting wires)

$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 I_c + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}}$$

→ This is known as Ampere - Maxwell Law

⟹ So, the source of mag. field is not just the "Conduction current due to flowing of charges" but also the time-varying electric field.

→ More precisely, total current $I = I_c + I_d$, where $I_d = \epsilon_0 \frac{d\phi_E}{dt}$.

→ In explicit terms, this means that outside Cap. plates, we have only I_c, $I_d = 0$. However, inside the plates of capacitor, $I_c = 0$, $I_d = I$

—P.T.O

prove that $I_c = I_d$?

I_d = Displacement current betn cap. plates
I_c = Conduction current in the connecting wires.

We know that instantaneous electric field betn the plates is

$$E = \frac{q}{\epsilon_0 A}$$ where A = Surface area of each plate.

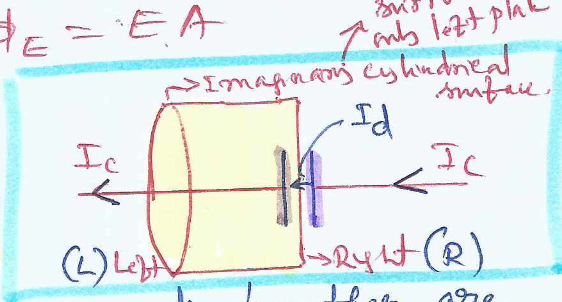
$$\frac{dE}{dt} = \frac{1}{\epsilon_0 A} \cdot \frac{dq}{dt} = \frac{1}{\epsilon_0 A} I_c$$ where $I_c = \frac{dq}{dt}$ is the instantaneous current in the connecting wires.

$$\therefore I_c = \epsilon_0 A \frac{dE}{dt} \longrightarrow ①$$

→ The displacement current I_d by definition,

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$ But $\phi_E = EA$

$$\therefore I_d = \epsilon_0 A \frac{dE}{dt} \longrightarrow ②$$



→ Comparing ① and ②, $I_c = I_d$.

→ Thus, the displacement and the conduction currents together are continuous thro' the entire circuit including the capacitor, although individually they are discontinuous.

→ From figure above, no conduction current I_c enters the imaginary cylindrical surface R, however I_c leaves thro' surface L. Consistency of Ampere's law requires $\boxed{I_d = I_c}$ to flow in betn capacitor plates.

Info : There is a difference in the magnetic effects produced by "Conduction" and "displacement" currents.

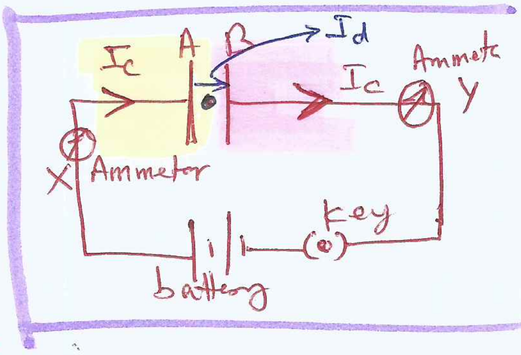
→ A conduction current in a thin wire produces mag. field at the surface of the wire much larger than the field produced by an equal displacement current at the edge of the cap. plates.

→ The reason is that I_c is confined to the ~~the~~ thin wire but the I_d is spread out over the entire surface of the capacitor plates.

Continuity of Current :

→ We know that the current flows through a closed and unbroken circuit. If a parallel-plate capacitor is added to the circuit, the circuit is broken since region betw capacitor could be vacuum or air or some ~~non~~ insulating ~~and~~ material (dielectric). How could we understand the continuous flow of current in such circuits where cap. is present.

→ From fig, on left side of capacitor plate A, there exists an ~~increasing~~ decreasing current (during charging of capacitor), that could be observed using ammeter X. We can also experimentally observe the same current in ammeter Y, although ~~there~~ the circuit is broken due to parallel-plate capacitor.



→ As shown in figure, a time varying current (during charging of capacitor) produces time-varying ~~mag~~ electric field → which in turn produces a mag. field. There exists a "displacement current" betw cap. plates.

→ [Therefore, betw plates of capacitor, there is I_d

→ outside of capacitor, in the connecting wires, there is I_c

→ this ensures "continuity of current" in electronic circuits involving capacitor.

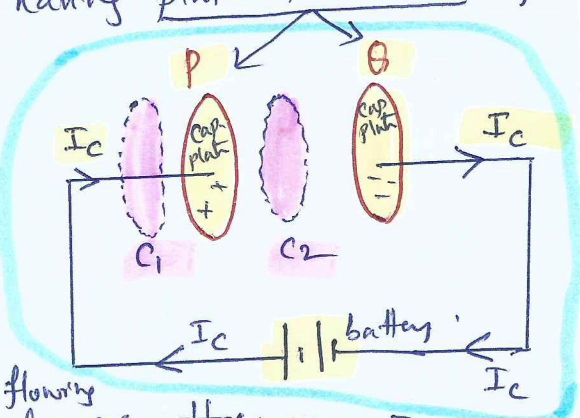
⇒ I_c is same on left side of the capacitor and the right side of the capacitor ⇒ I_d must be equal to I_c

→ how to prove analytically the "Continuity of current"? → See next page.

Continuity of current : proof

The sum of I_c and I_d is continuous along any closed path, although the individual I_c and I_d may not be continuous.

→ To prove it, consider a parallel-plate capacitor having plates P and Q being charged by a battery. Let C_1 and C_2 be the two loops, which have the same boundary as that of cap. plates. C_1 is little towards left and C_2 is little towards right of the plate P of the capacitor.



→ During charging of capacitor, at any instant of time, I_c be the conduction current flowing thro' the connecting wires as shown in figure. However, I_c cannot flow across the capacitor gap, as no charge is able to flow across the gap. ∴ Flow of I_c is ~~discontinued~~ broken at plate P of ~~cap.~~ capacitor.

→ For loop C_1, there is no electric flux (as there is no accumulation of charge on connecting wires) $\Rightarrow \phi_E = 0 \Rightarrow \frac{d\phi_E}{dt} = 0$ $\left(\because I_d = \epsilon_0 \frac{d\phi_E}{dt}\right)$

$$\therefore I_c + I_d = I_c + \epsilon_0 \frac{d\phi_E}{dt}$$

$$\boxed{\therefore I_c + I_d = I_c} \longrightarrow (1)$$

→ For loop C_2 there is no conduction current $I_c \Rightarrow I_c = 0$

we know that betw cap plates $E = \frac{q}{\epsilon_0 A}$

∴ Electric flux $\phi_E = EA$

$\phi_E = \frac{q}{\epsilon_0}$

$\frac{d\phi_E}{dt} = \frac{1}{\epsilon_0} \cdot \frac{dq}{dt}$

$$\therefore I_c + I_d = 0 + \epsilon_0 \frac{d\phi_E}{dt}$$

$$= 0 + \cancel{\epsilon_0}\left(\frac{1}{\cancel{\epsilon_0}} \cdot \frac{dq}{dt}\right)$$

$$= \frac{dq}{dt} = I_c$$

$$\boxed{I_d = I_c}$$

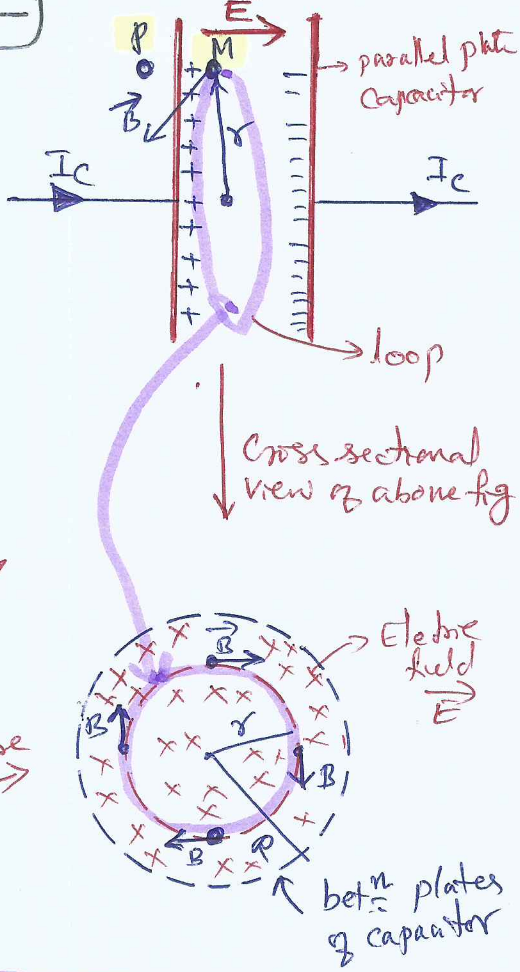
$$\therefore \boxed{I_c + I_d = I_c} \longrightarrow (2)$$

From (1) and (2)

From (1) and (2), we conclude that sum of $(I_c + I_d)$ has same value on either side of the capacitor plates. $\Rightarrow (I_c + I_d)$ has the property of continuity although individually they may not be continuous.

→ Note that I_c and I_d ~~exists~~ may exist in different regions of space. (In this case → I_d exists betw cap. plates and $I_c = 0$ betw cap. plates; and I_c exists outside the capacitor plates and $I_d = 0$)

- It is to be noted that in any general medium, both I_c and I_d will be present, giving rise to the total current.
- In a conducting medium, I_c dominates over I_d;
- In an insulating medium, I_d dominates over I_c.
- In all respects, I_d has the same physical effects as the conduction current I_c.
  - For example, during the charging of capacitor, if we measure the mag. field at a point M betn capacitor plates, it is found to be same as that just outside the capacitor at point P.
- $\vec{E}$ and $\vec{B}$ betn cap. plates is shown on figure.
- $\vec{E}$ is from +ve plate to −ve plate
- $\vec{B}$ at M is $\perp^r$ to plane of paper.
- The cross-sectional view of $\vec{E}$ and $\vec{B}$ are shown here
  - → $\vec{E}$ is shown by crosses
  - → $\vec{B}$ is tangent to a circle in the same plane.



Consequence of Displacement Current (I_d) / prediction of e-m waves.

The discovery of I_d has literally far reaching consequences as it has established more symmetry betn laws of electricity and magnetism.

→ Faraday's law of induction states that there is an induced emf equal to the rate of change of mag. flux. Now, since the emf betn two points 1 and 2 is the work done per unit charge in taking it from 1 to 2, the existence of emf implies the existence of an "electric field". So, we can rephrase Faraday's law of e-m induction as → time-varying magnetic field gives rise to an electric field.

→ Then, ~~the~~ fact that ~~or~~ time-varying electric field gives rise to a mag. field is a symmetrical counterpart, and is a consequence of the "displacement current" being a source of a mag. field.

→ Thus, time-dependent electric and magnetic fields give rise to each other. Faraday's law of e-m induction and Ampere-Maxwell law give a quantitative expression of this statement, with the current being the total current

$$\longrightarrow I = I_c + I_d = I_c + \epsilon_0 \frac{d\phi_E}{dt}$$

→ From these concepts and symmetry, Maxwell concluded the existence of e-m waves in a region where electric and magnetic fields were changing with time.

For Info only.

We Know that $I_d = \epsilon_0 \frac{d\phi_E}{dt}$ Since $\phi_E = EA$

$I_d = \epsilon_0 A \frac{dE}{dt}$ Since $E = \frac{V}{d}$

(for parallel plate capacitor)
d = dist. betn plates.

$I_d = \frac{\epsilon_0 A}{d} \frac{dV}{dt}$

Since $\boxed{C = \frac{\epsilon_0 A}{d}}$

$$\boxed{I_d = C\frac{dV}{dt}}$$

[Note $Q = CV$
$i = \frac{dQ}{dt} = C\frac{dV}{dt}$]

unit of RC

$RC \rightarrow \frac{\cancel{V}}{I} \cdot \frac{Q}{\cancel{V}} = \frac{\cancel{Q}}{\cancel{Q}\, t^{-1}}$

$= t$

unit of RC → time.

problem: How would you establish an instantaneous displacement current of 1 A in the space between parallel plate 1 μF capacitor?

→ Given $I_d = 1\,A$, $C = 1\,\mu F$

Formula $I_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt}(EA) = \epsilon_0 A \frac{dE}{dt} = \left(\frac{\epsilon_0 A}{d}\right)\frac{dV}{dt}$ $E = \frac{V}{d}$

$I_d = C\frac{dV}{dt} \Rightarrow \frac{dV}{dt} = \frac{I_d}{C} = \frac{1\,A}{1\,\mu F} = 10^6\, V s^{-1}$

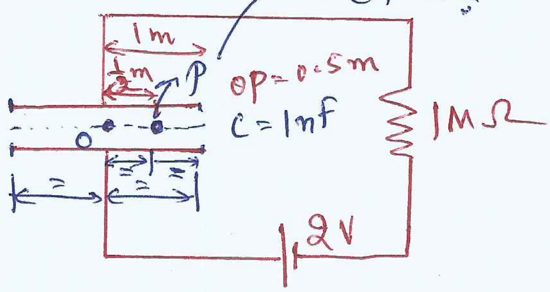
Thus an instantaneous $I_d = 1A$ can be setup by changing the p.d. across the parallel plates of capacitor at the rate of $10^6\, V s^{-1}$.

problem: In an electric circuit, there is a capacitor of reactance 100 Ω connected across the source of 220V. Find I_d.

→ $I_d = I_c$, So $I_d = \frac{V}{X_c} = \frac{22\cancel{0}\,V}{10\cancel{0}} = \underline{\underline{2.2\,A}}$

problem : (NCERT) : A parallel plate capacitor with circular plates of radius 1 m has a capacitance of 1 nF. At $t = 0$, it is connected for charging in series with $R = 1\,M\Omega$ across a 2 V battery. Calculate Mag. field at a point P half way betⁿ the centre and the periphery of the plates, after $t = 10^{-3}$ s (The charge on the capacitor at t is $q(t) = CV(1 - e^{-t/\tau})$ where τ is time constant is equal to RC)

$\vec{B}$ @ P = ?



→ RC time constant $\tau = RC$

$= 1\,M\Omega \times 1\,nF$

$= 10^6 \times 10^{-9} = 10^{-3}$ s

$\therefore q(t) = CV\left[1 - e^{-t/10^{-3}}\right]$

$q(t) = 2 \times 10^{-9}\left[1 - e^{-t/10^{-3}}\right]$

→ Area of cap. plate $= \pi r^2$ $(r = 1\,m)$ $\therefore \underline{A = \pi\, m^2}$

We know that $E = \dfrac{q(t)}{\epsilon_0 A} = \dfrac{q}{\epsilon_0 \pi}$ (Since $A = \pi\, m^2$)

→ Consider now a clar loop of radius ($\frac{1}{2}$ m) parallel to the plates passing through P. The mag. field B at all points on the loop is having same value. The flux thro' this loop is

$\phi_E = E\,(\text{area of loop}) = E \times \pi\left(\frac{1}{2}\right)^2 = \dfrac{\pi E}{4}$

Since $E = q/\epsilon_0\pi$

$\phi_E = \dfrac{\cancel{\pi}}{4} \cdot \dfrac{q}{\epsilon_0 \cancel{\pi}} = \dfrac{q}{4\epsilon_0}$

$\therefore \phi_E = \dfrac{\pi E}{4} = \dfrac{q}{4\epsilon_0}$

$I_d = \epsilon_0 \dfrac{d\phi_E}{dt} = \cancel{\epsilon_0} \dfrac{d}{dt}\left[q/4\cancel{\epsilon_0}\right]$

$\boxed{I_d = \dfrac{1}{4}\dfrac{dq}{dt}}$

$= \dfrac{1}{\cancel{4}_2}(\cancel{2} \times 10^{-\cancel{9}6})(\cancel{10^3}\, e^{-1})$

$\boxed{I_d = 0.5 \times 10^{-6}\, e^{-1}}$

$q = (2 \times 10^{-9})\left[1 - e^{-t/10^{-3}}\right]$

$q = (2 \times 10^{-9}) - (2 \times 10^{-9})\, e^{-10^3 t}$

$\dfrac{dq}{dt} = 0 - (2 \times 10^{-9})\,\dfrac{d}{dt}\left[e^{-10^3 t}\right]$

$= -(2 \times 10^{-9}) \times e^{-10^3 t} \cdot \dfrac{d}{dt}\left[-10^3 t\right]$

$= -(2 \times 10^{-9}) \cdot e^{-10^3 t} \times (-10^3)$

$\dfrac{dq}{dt} = (2 \times 10^{-6}) \cdot e^{-10^3 t}$

at $t = 10^{-3}$ s

$\dfrac{dq}{dt} = (2 \times 10^{-6})\left(e^{-10^3 \cdot 10^{-3}}\right)$

$dq/dt = (2 \times 10^{-6})(e^{-1})$

$\therefore \dfrac{1}{4}\dfrac{dq}{dt} = (0.5 \times 10^{-6})(e^{-1})$

$\mu_0/4\pi = 10^{-7}$; $e = 2.718$

→ Maxwell's 4th equation

$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_c + \mu_0 I_d$

Here $I_c = 0$

and LHS $\oint \vec{B} \cdot \vec{dl} = B \cdot 2\pi r$ → circular plate radius

here $r = 1/2$ m

$\therefore \oint \vec{B} \cdot \vec{dl} = B \cdot 2\pi \dfrac{1}{2} = \pi B$

$\therefore \boxed{\pi B = \mu_0 I_d}$ $\therefore B = \dfrac{\mu_0}{\pi} I_d$

$B = \dfrac{\mu_0}{\pi}(0.5 \times 10^{-6} e^{-1}) = \left(\dfrac{\mu_0}{4\pi}\right) \cdot \dfrac{2 \times 10^{-6}}{e}$

$B = \dfrac{10^{-7} \times 2 \times 10^{-6}}{2.718} = \dfrac{2}{2.718} \times 10^{-13}$ T $= \underline{0.74 \times 10^{-13}\, T}$

Maxwell's equations: (4 equations)

Maxwell formulated the basic laws of electricity and magnetism in the form of 4 fundamental eqns known as Maxwell's equations. These equations bear the same relation to electromagnetism that Newton's laws of motion do to Mechanics. In the absence of any dielectric or mag. material, the 4 Maxwell's eqns are given below.

Maxwell's First equation: (1) Gauss' Law of Electricity:

It states that the electric flux thro' any closed surface is equal to $1/\epsilon_0$ times the 'net' charge enclosed by the surface. Mathematically

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} \rightarrow (1)$$

(here q is the source of $\vec{E}$). This eqn is true for both stationary and moving charges.

Maxwell's Second equation: (2) Gauss' law of Magnetism:

It states that the magnetic flux thro' any closed surface is zero. Mathematically

$$\oint \vec{B} \cdot d\vec{A} = 0 \rightarrow (2)$$

It signifies that free magnetic poles (monopoles) donot exist. Any volume enclosed by a surface will contain equal & opposite poles leading to a net mag. pole strength zero. (On the other hand, free electric charges do exist). This eqn also signifies that mag. lines of force cannot start from a point nor end at a point, that is, they are closed curves.

Maxwell's Third equation: (3) Faraday's Law of Electromagnetic Induction

It states that an induced emf is setup in a circuit is equal to the negative rate of change of mag. flux thro' the circuit. Mathematically, $e = -\frac{d\phi_B}{dt}$

Since emf 'e' can be defined as the line integral of electric field, hence we have

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt} \rightarrow (3)$$

Thus, the law states that the line integral of electric field along a closed path is equal to the rate of change of mag. flux thro' the surface bounded by that closed path. This ~~eqn~~ eqn signifies that an electric field is produced by a changing magnetic field.

Maxwell's Fourth equation: (4) Ampere-Maxwell Circuital Law

Ampere's circuital law was extended by Maxwell on considerations of current continuity. It states that the line integral of mag. field along a closed path is equal to μ_0 times the total current (i.e. sum of conduction current I_c and displacement current I_d) linked with the surface bounded by that closed path.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 [I_c + I_d] = \mu_0 \left[I_c + \epsilon_0 \frac{d\phi_E}{dt} \right] \rightarrow (4)$$

It signifies that a "conduction current" as well as a "changing electric field" produces a magnetic field. Thus I_c as well as $\frac{d\phi_E}{dt}$ are sources of $\vec{B}$.

→ Ampere's law contains a term $\mu_0 I_c$, while Faraday's law does not contain the like term because free mag. poles (monopoles) do not exist.

→ Info: the term $\mu_0 \epsilon_0 \frac{d\phi_E}{dt}$ in 4th Maxwell's eqn is the displacement current term introduced by Maxwell. Since $\mu_0 \epsilon_0 = 1.1 \times 10^{-17}\ s^2 m^{-2}$, which is a very small quantity, this term would not contribute appreciably unless $d\phi_E/dt$ is very large, that is, the electric field must be changing very rapidly.

PTO →

→ Contd. from pre-page

(5) Lorentz force $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

These 5 eqns give a complete description of all electromagnetic interactions.

Importance of Maxwell's equations.

On the basis of 4 eqns + Lorentz force, Maxwell predicted the following

(1) An accelerated charge is the source of e-m waves.

(2) The e-m waves can propagate thro' the space with the speed of light ($= 3 \times 10^8\ m s^{-1}$)

(3) The e-m waves ~~are~~ is of transverse in nature

The e-m wave as it is transverse in nature

(4) Light itself is an e-m wave and it travels with speed ~~of~~ $= 3 \times 10^8\ m s^{-1}$.

IMP

Electromagnetic Waves (e-m waves)

What are e-m waves?

→ According to Maxwell's 3^{rd} eqⁿ (Faraday's Law), a time-varying mag. field produces an electric field.

→ According to Maxwell's 4^{th} eqⁿ (Ampere-Maxwell Law), a time-varying electric field produces a magnetic field.

⟹ It means that a change in either field produces the other field.

→ Maxwell worked out from his equations that variation in electric and magnetic fields both in space and time leads ~~the the~~ to the production of e-m disturbances in space. These disturbances have the properties of a wave, which consists of fluctuating electric and magnetic fields $\perp^r$ to each other and also $\perp^r$ to the direction of propagation of the "wave".

→ Such "waves" which can actually propagate in space even without any material medium are called "e-m waves"

Basic Source of producing e-m waves → IMP

- An electric charge at rest (stationary charge) produces an electric field in the region around it, but no magnetic field ~~is~~ produced.
- A moving charge produces both electric and magnetic fields.
  - If the charge is moving with constant velocity (i.e. the current is not changing with time), both $\vec{E}$ and $\vec{B}$ fields will not change with time and no e-m wave can be produced.
  - If the motion of the charge is accelerated, the electric and magnetic fields will change with space and time. → It then produces e-m waves.
- Hence, we conclude that "an accelerated charge emits (or radiates) e-m waves"

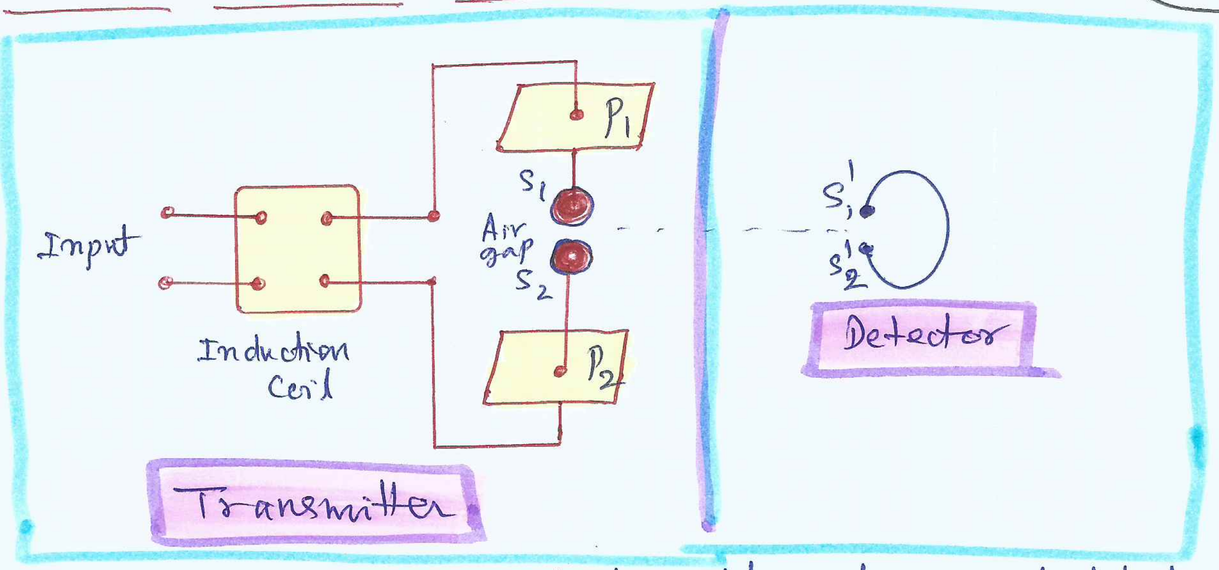
Example: ① In an oscillatory LC circuit, charge oscillates across the capacitor plates. An oscillating charge has a non-zero acceleration; hence it emits e-m waves of frequency same as that of the oscillating charge.

② In an atom, an electron circulating around the nucleus in a stable orbit, although accelerating, does not emit e-m waves. E-M waves are emitted only when it falls from higher energy ~~level~~ orbit to a lower energy orbit.

③ E-M waves (e.g. X-rays) are also produced when fast-moving electrons are suddenly stopped by a metal target of high atomic number.

P.T.O →

Hertz demonstration of e-m waves (in 1887)



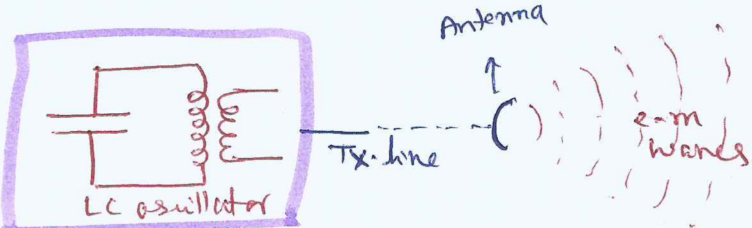
- Hertz performed expts that could produce and detect e-m waves of frequencies much smaller than those of light. His set-up is given above.
- S_1 and S_2 are two large metallic spheres which are attached to two large plates P_1 and P_2 respectively (P_1 and P_2 acts as a capacitor)
- The above assembly is connected to an induction coil as shown in fig.
- When currents in the induction coils are changed, a high voltage is induced across the gap S_1 and S_2. As a result, the air in the gap is ~~ioni~~ ionized and a spark is produced across the gap. The spark consists of electrons and ions (obtained from ionised air) which oscillate back and forth. These oscillating, and hence accelerated, charges emit e-m waves. The freq. of these waves depends on the value of inductance & capacitance of the rods that form the gap. ($f = \frac{1}{2\pi\sqrt{LC}}$)
- Hertz detected these e-m waves by means of a detector consisting of a single loop of wire connected to two spheres S_1' and S_2'.
- The e-m waves reaching the gap, by virtue of their strong electric field, produce a high potential difference across the gap $S_1'S_2'$ and cause a spark across the gap $S_1'S_2'$. The observation of small sparks establishes the detection of e-m waves.
- Hertz also found that sparks across the gap $S_1'S_2'$ are observed only when the detector gap $S_1'S_2'$ is parallel to the source gap S_1S_2. No sparks are observed at $S_1'S_2'$ when the two gaps (S_1S_2 and $S_1'S_2'$) are at right ∠es to each other. This established the "transverse nature of e-m waves.

Follow-up of Hertz work :

- The e-m waves produced by Hertz were of small frequency, which could not be detected over a large distance.
- After 7 years, Bose succeeded in producing and observing e-m waves of very short wavelength (~~2cm~~ 25 mm to 5 mm). But his expt. remained confined to the laboratory.
- In 1895, Marconi made an important discovery which helped in producing e-m waves of commercial ~~use~~ value. He was successful in transmitting e-m waves upto a few Kms.
- Marconi's expt. marks the beginning of the field of wireless communication using e-m waves.

A simple LC oscillator circuit (backed up by external energy source to produce undamped oscillations) can produce waves of different frequencies by changing L and C.

The undamped oscillations from LC circuit is taken via Tx. line and fed to an antenna.



Most efficient antennas are those which have a size comparable to the wavelength of e-m wave, they emit or receive. Antenna is a device that radiates the electrical signal as e-m waves.

Another Source producing EM waves.

→ Electric Dipole as a basic source of em waves

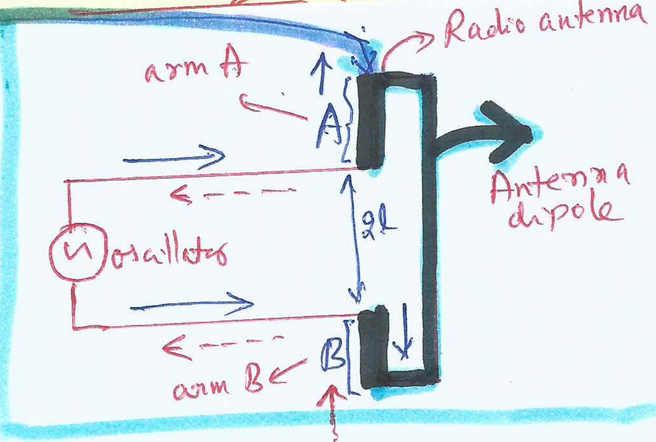
Question : Show that electric dipole is a basic source of em waves.

→ See next page.

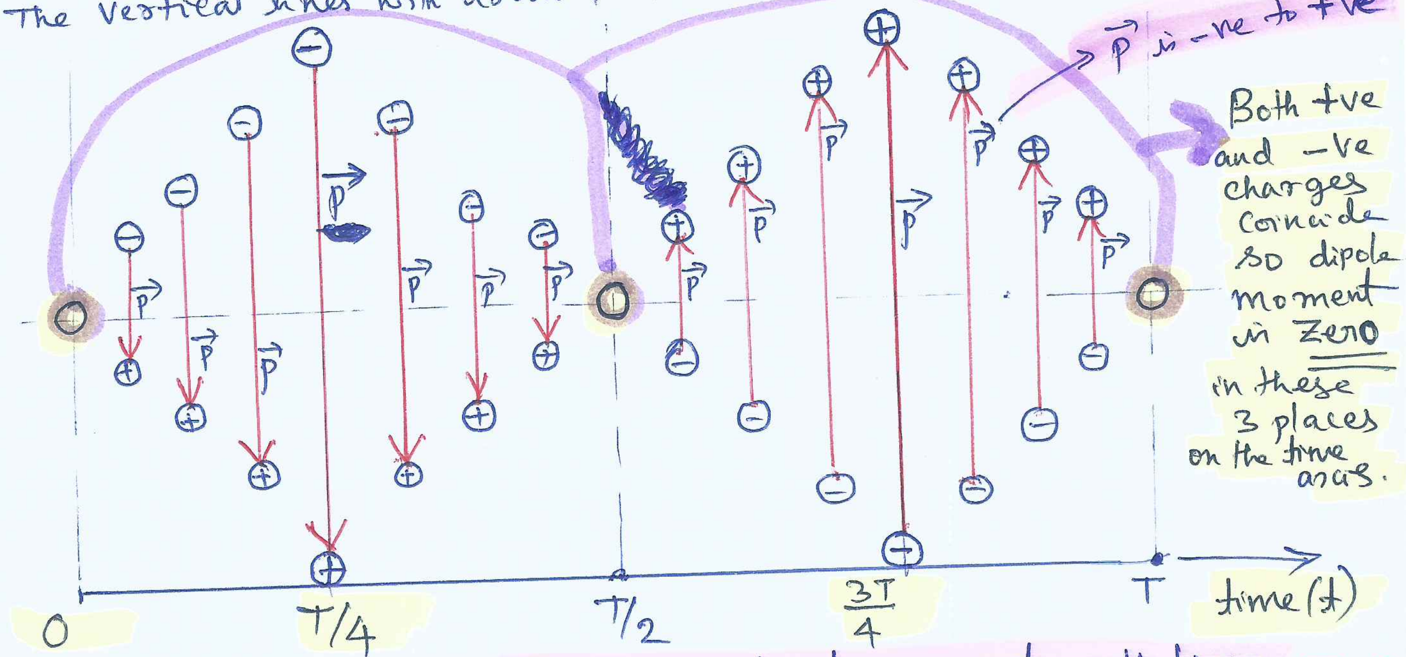
* Show that electric dipole is a basic source of EM wave

General (for info): We have seen transmitting antennas from TV and Radio services. These antennas are essentially ~~radiating~~ converting an amplified electrical signal in to EM waves and radiated into space. Consumers receive these EM signals and retrive the signal information. Basically, these antennas are electric dipoles and the property of an electric dipole will be acting as a source to produce and radiate EM waves. Let us see in detail the function of this electric dipole ~~used~~ acting as a basic source of EM waves.

IMP *: Electric Dipole as a basic source of EM wave *



- Consider a "radio antenna" connected to an oscillator. A and B are the arms of the antenna from which wire is drawn to connect to an oscillator.
- Due to oscillator, the charges accelerate in one direction (→) and then in the other direction (←) alternately in the arms A and B of the antenna.
- The charges in the arms of antenna is equivalent to an electric dipole.
- We know that, electric dipole moment $p = q \times 2l$, where $2l$ is the distance of separation bet^n -ve and +ve charges of the dipole.
- The direction of $\vec{p}$ is from -ve to +ve charge.
- Since the charges are oscillating alternately (due to oscillator), the magnitude and direction of $\vec{p}$ changes with time (See fig. below)
- The vertical lines with arrows show the magnitude & direction of $\vec{p}$.



Change in dipole moment with time

Contd.

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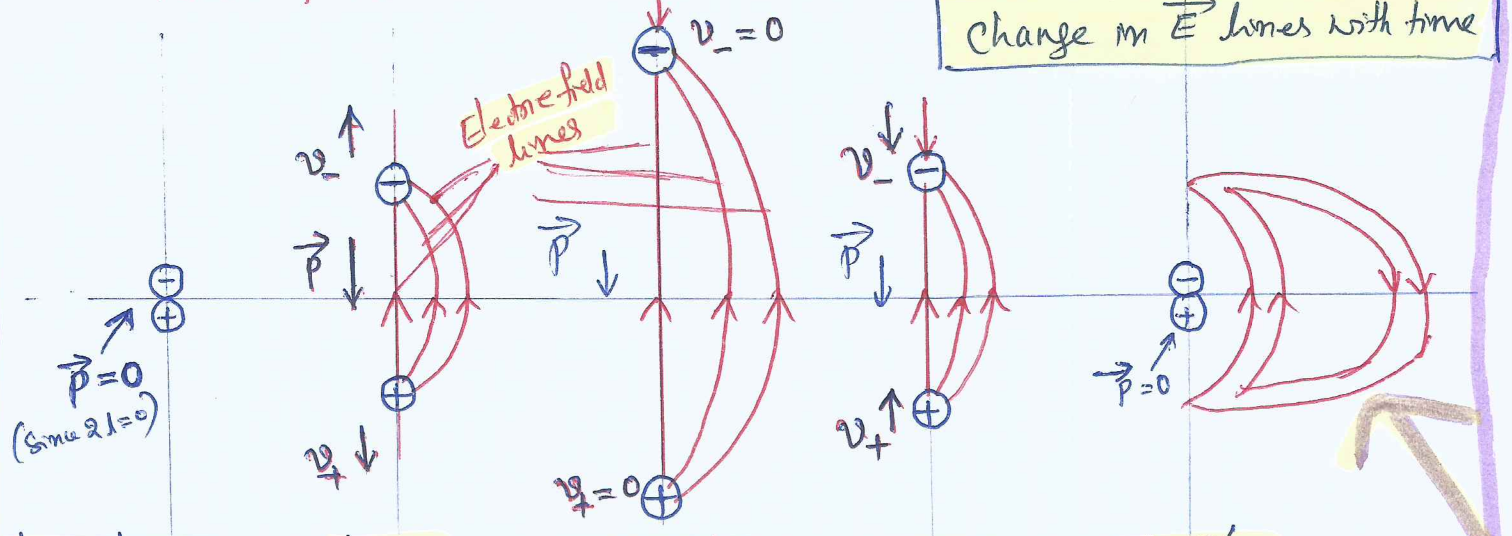
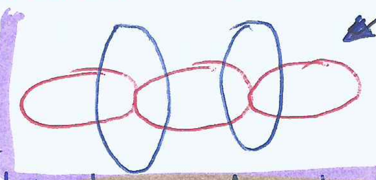
Following figure depicts the electric field ($\vec{E}$) lines at different instants of time



time $t = 0$ (a) | $t > 0$ (b) | $t = T/4$ (c) | $t > T/4$ (d) | $t = T/2$ (e)

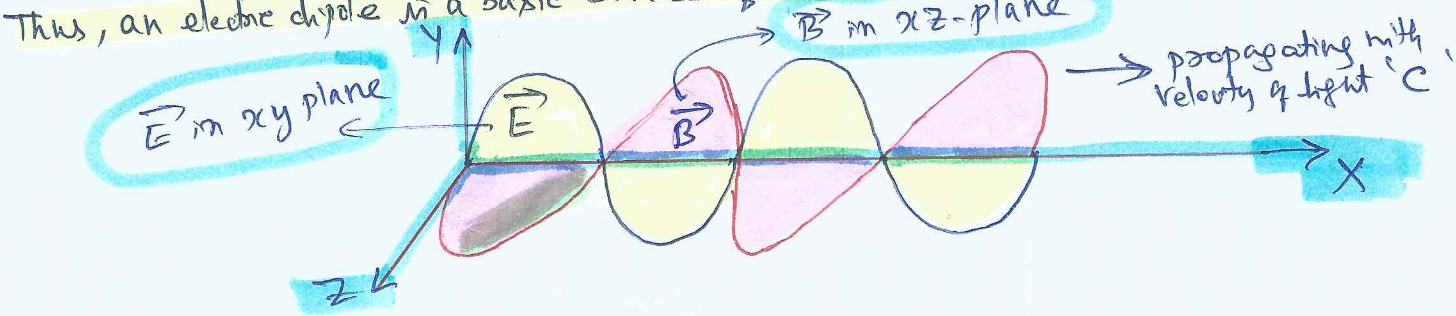
- At $t = 0$, $\vec{p} = 0$ as both +ve and −ve charges coincide ($2l = 0$); $\vec{p} = q \times 2l$
- As t increases, these produces a dipole moment as the separation between +ve and −ve charges increases. So, electric field lines start from +ve charge and end on the −ve charge.
- $\vec{p}$ is maximum at $t = T/4$ (when $2l$ is max and $\vec{p}$ is maximum) $\Longrightarrow$ Also spreading of $\vec{E}$ lines also becomes maximum.
- After $t = T/4$, $\vec{p}$ decreases and at $t = T/2$, $\vec{p} = 0$. At this stage, electric field lines break away from the dipole and keep on propagating in space. These break away electric field lines combine with the electric field lines of the previous half cycle of dipole oscillation and form closed loops (see fig (e) above)

- SO, this "time-varying electric field" becomes a source for "time-varying magnetic field" ($\vec{B}$)
- ~~The~~ $\vec{B}$ at any point oscillates with a frequency = freq. of oscillating current in the antenna.
- The "time-varying $\vec{B}$" inturn produces "time-varying $\vec{E}$" and this cycle repeats. Thus $\vec{E}$ & $\vec{B}$ appear in the form of interwoven chains



- Thus, for example, $\vec{E}$ (varying along Y-axis ⇒ xy plane) and $\vec{B}$ (varying along Z-axis ⇒ xz plane) radiated from an oscillating electric dipole combine together to form an EM wave, which propagates along x-direction.

Thus, an electric dipole is a basic source of EM waves.



- Last few pages describe "production of EM waves" by various methods.

- Next page, let us discuss what is the Nature (characteristics or properties) of this radiated Electromagnetic (EM) waves.

Sec 8.3.2 of NCERT book

Nature of Electromagnetic Waves

(or characteristics or properties)

(1) EM waves donot require any material medium for their propagation. They can propagate in vacuum as well as in a medium.

→ Speed of em waves in free space or vacuum is given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \, m s^{-1}$$

→ speed of em waves in a medium is given by

$$v = \frac{1}{\sqrt{\mu \epsilon}}$$; where μ = permeability of the medium, ϵ = permittivity of the medium.

→ We will see more about "speed of em waves" in subsequent pages.

(2) EM waves are "transverse" in nature ⟹ $\vec{E}$ and $\vec{B}$, which constitute the em waves are mutually ⊥r to each other as well as ⊥r to the direction of the propagation of the wave.

→ We will prove transverse nature of em waves in subsequent pages.

(3) EM waves carry energy as they travel from one point to another point in the space. The average energy densities of an em wave is given by

$$U_{av} = \frac{1}{2} \epsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

- EM waves transport energy. The rate of energy of em wave transported per unit area is represented by a quantity called "Poynting Vector" ($\vec{S}$)

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$; unit of $\vec{S}$ is watt m^{-2}

The direction of $\vec{S}$ at any point gives the direction of energy transported at that point.

→ We will see more about Energy density of EM waves in subsequent pages.

(4) Intensity of an e.m. wave depends on the amplitude of $\vec{E}$ / $\vec{B}$ ∴ $I = \frac{1}{2} \epsilon_0 c E_0^2$

→ We will see more about intensity of EM wave in subsequent pages.

(5) EM waves carry momentum and exert radiation pressure (P) on the surface they fall and is given by

$$\text{pressure } P = \frac{1}{A} \frac{dp}{dt} = \frac{I}{c}$$

→ We will see more about momentum & pressure of EM wave in subsequent pages.

P.T.O. →

more properties of EM waves.

(6) EM waves are self-sustaining electric and magnetic field oscillations in space.

(7) The amplitude of electric field (E_0) and amplitude of mag. field (B_0) are related as $\boxed{c = \frac{E_0}{B_0}}$

(8) An accelerated charged particle is a basic source of EM waves.

(9) $\vec{E}$ is mainly responsible for the "optical effects" of EM waves.
$\therefore$ $\vec{E}$ vector is called "Light vector".

(10) EM waves can be reflected, refracted and diffracted.

(11) EM waves show the phenomenon of Interference.

(12) EM waves can be "polarized". This can be observed by turning radio receiver at certain direction for best reception.

(13) EM waves follow the superposition principle.

(14) EM are not deflected by magnetic and electric fields.

---

problem #1 : In a plane EM wave, $\vec{E}$ oscillates with $f = 2 \times 10^{10}$ Hz and amplitude of $40 \, V m^{-1}$ (i) what is λ of wave (ii) and Energy density due to electric field $\vec{E}$

(i) λ of em wave $\quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{2 \times 10^{10}} = 1.5 \times 10^{-2} m \quad | \quad c = f\lambda$

(ii) Energy Density due to $\vec{E} \rightarrow U_E = \frac{1}{4} \epsilon_0 E_0^2$

$$U_E = \frac{1}{4} \times (8.85 \times 10^{-12}) \times (40)^2 = 3.54 \times 10^{-9} \, J m^{-3}$$

---

problem #2 : The amplitude of oscillating $\vec{E}$ in an em wave is $50 \, V m^{-1}$. what is the amplitude of the oscillating mag. field?

$\rightarrow$ Velocity of EM wave in vacuum is given by

$$c = \frac{E_0}{B_0} \quad \text{or} \quad B_0 = \frac{E_0}{c}$$

Here, $E_0 = 50 \, V m^{-1}$; $c = 3 \times 10^8 \, m s^{-1}$

$$\therefore B_0 = \frac{50 \, V m^{-1}}{3 \times 10^8 \, m s^{-1}} = 1.67 \times 10^{-7} \, V m^{-2} s$$

$$= 1.67 \times 10^{-7} \text{ Tesla}$$

$$= 1.67 \times 10^{-7} \, T$$

Let us discuss in detail about the following nature (properties) of an EM wave.

1. Transverse Nature of EM wave
2. Speed of EM wave
3. Energy density of EM waves
4. Intensity of EM waves
5. Momentum and pressure of EM wave

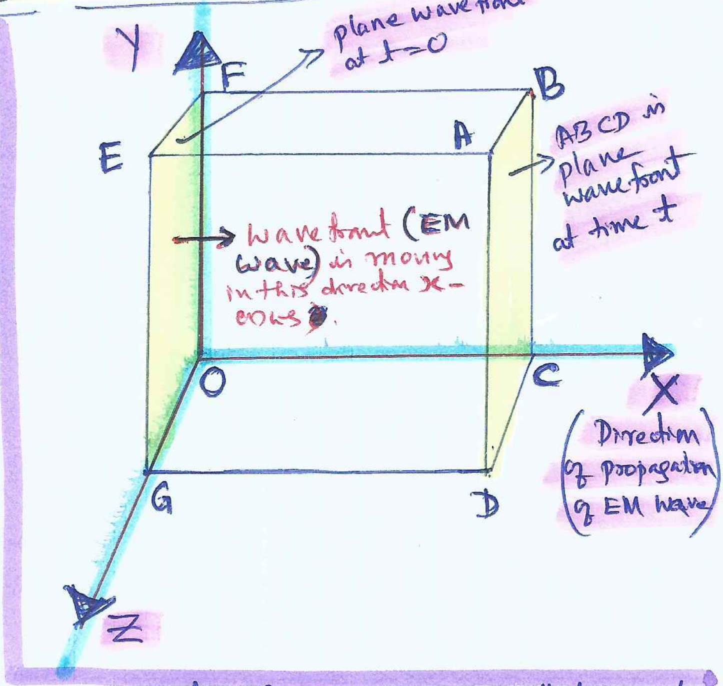
1 prove that EM waves are transverse in nature:

Defn: → A changing electric field ($\vec{E}$) produces a changing mag. field ($\vec{B}$) and vice versa which give rise to a transverse wave known as EM wave. The time-varying $\vec{E}$ and $\vec{B}$ are mutually $\perp^r$ to each other and also $\perp^r$ to the direction of propagation of this wave.

- Maxwell showed that a time-varying $\vec{E}$ produces a time-varying $\vec{B}$ and vice versa. This alternate production of time-varying $\vec{E}$ and $\vec{B}$ gives rise to the propagation of e-m waves. The variation of $\vec{E}$ and $\vec{B}$ are mutually $\perp^r$ to each other as well as to the direction of the propagation of the wave ⟹ the EM waves are "transverse" in nature.

proof: → (proof for Transverse Nature of EM waves)

- Suppose a plane EM wave travelling along X-direction with its "wave front" in the YZ plane.
- ABCD is the position of wavefront at time t.
- The values of $\vec{E}$ and $\vec{B}$ to the left of ABCD will depend on x and t (not on y and z as we have considered EM wave travelling in x-direction).



- According to Gauss' law, the total electric flux across the parallelopiped ABCD GEFO is zero since it does not enclose any charge.

ie $\oint \vec{E} \cdot \vec{ds} = 0$ (or) considering all 6 sides of parallelopiped,

$$\oint_{ABCD} \vec{E} \cdot \vec{ds} + \oint_{OFEG} \vec{E} \cdot \vec{ds} + \oint_{ADGE} \vec{E} \cdot \vec{ds} + \oint_{BCOF} \vec{E} \cdot \vec{ds} + \oint_{OCDG} \vec{E} \cdot \vec{ds} + \oint_{ABFE} \vec{E} \cdot \vec{ds} = 0 \quad \longrightarrow (1)$$

.... Contd from pre. page.

- Since $\vec{E}$ does not depend on y and z, so the contribution to the electric flux coming from the faces normal to y and z axes cancel out in pairs.

i.e. $\oint_{OCDG} \vec{E}\cdot\vec{ds} + \oint_{ABFE} \vec{E}\cdot\vec{ds} = 0 \longrightarrow (2)$ and

$\oint_{ADGE} \vec{E}\cdot\vec{ds} + \oint_{BCOF} \vec{E}\cdot\vec{ds} = 0 \longrightarrow (3)$

$\therefore$ eqn (1) becomes $\oint_{ABCD} \vec{E}\cdot\vec{ds} + \int_{OFEG} \vec{E}\cdot\vec{ds} = 0 \longrightarrow (4)$

Now, $\int_{ABCD} \vec{E}\cdot\vec{ds} = \int_{ABCD} E_x\, ds \cos 0 = E_x \int_{ABCD} ds$ $\quad (\because \vec{E_x}$ is $\parallel$ to $\vec{ds})$

$= E_x \times$ area of face ABCD $= E_x S \longrightarrow (5)$

and $\int_{OGEF} \vec{E}\cdot\vec{ds} = \int_{OGEF} E_x'\, ds \cos 180 = -E_x \int_{OGEF} ds$ $\quad (\because \vec{E_x'}$ is antiparallel to $\vec{ds})$

$= -E_x \times$ area of face "OGEF" $= -E_x S \longrightarrow (6)$

Where E_x and E_x' are the x-components of $\vec{E}$ on the faces ABCD & OGEF respectively.

$\therefore$ Eqn (4) becomes $E_x S - E_x' S = 0$ or $S(E_x - E_x') = 0$

Since $S \neq 0$, $(E_x - E_x')$ must be 0

$\therefore E_x - E_x' = 0$ or $\boxed{E_x = E_x'} \longrightarrow (7)$

Eq (7) $E_x = E_x'$ shows that the value of the x-component of electric field $\vec{E}$ does not change with time. $\Rightarrow \vec{E}$ along x-axis is static. Since, the static $\vec{E}$ cannot produce and propagate wave, hence the $\vec{E}$ parallel to the direction of EM wave is zero $\Rightarrow E_x' = E_x = 0$

$\Rightarrow \vec{E}$ is $\perp^r$ to the direction of propagation of EM wave.

III^{ly}, it can be proved, $\vec{B}$ is $\perp^r$ ———— do ————

$\therefore$ Since both $\vec{E}$ and $\vec{B}$ are $\perp^r$ to the direction of the propagation of EM wave, so EM wave is "transverse in nature"

※ Speed of EM waves ※

※ Obtain a relation for Velocity of Light (c) in terms of
(i) E_0 and B_0 → where E_0 / B_0 are the maximum values of Electric field & Mag. field respectively.
(ii) μ_0 and ϵ_0 → μ_0 → permeability of the free space (vacuum)
ϵ_0 → permittivity of the free space (vacuum)

(1) To prove that $c = E_0 / B_0$; c → velocity of light (speed of EM wave)

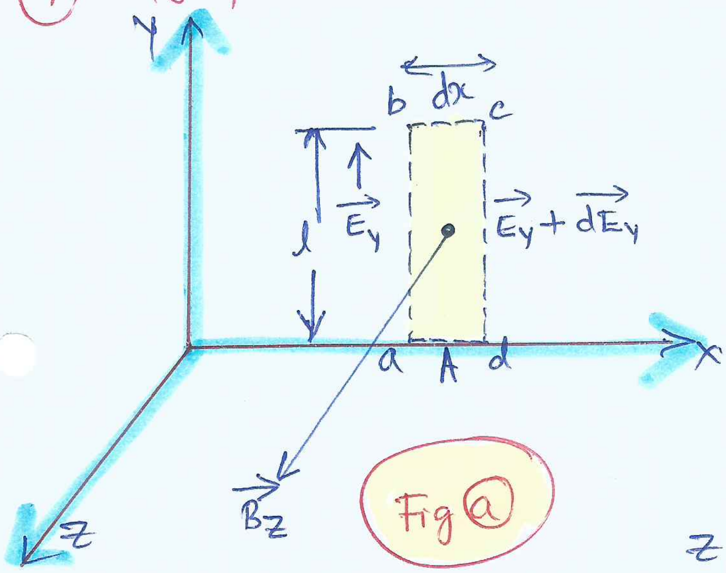


Fig (a)

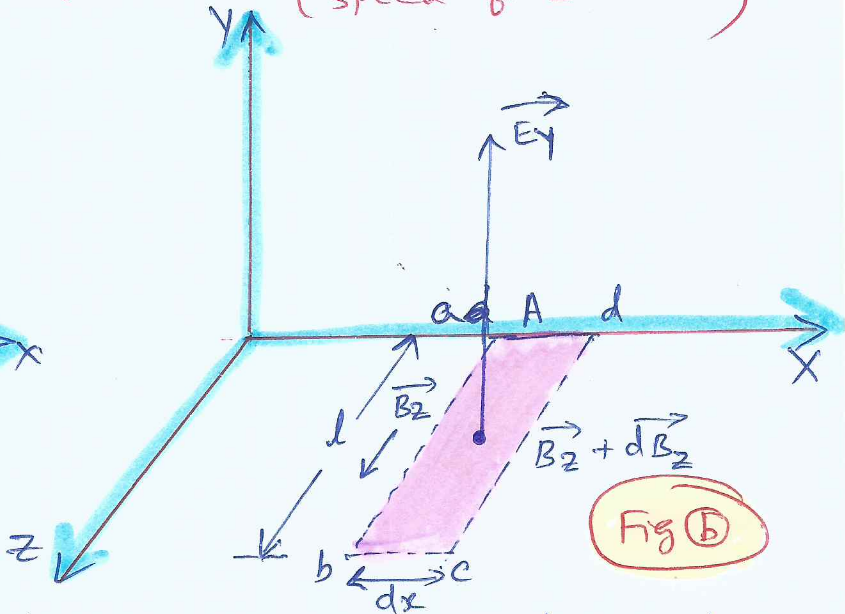


Fig (b)

- Consider a point A on the x-axis. Around this point, consider a rectangle abcd of length l and width dx (See fig (a))
- when the EM wave crosses the point A, the mag. flux (ϕ_B) through the rectangle will change. According to Faraday's law of electromagnetic induction, induced electric fields appear along the sides of the rectangle.
- As per Faraday's law of e-m induction $\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt} \longrightarrow (1)$

Now, $\oint \vec{E} \cdot \vec{dl} = \int_{ad} \vec{E} \cdot \vec{dl} + \int_{dc} \vec{E} \cdot \vec{dl} + \int_{cb} \vec{E} \cdot \vec{dl} + \int_{ba} \vec{E} \cdot \vec{dl}$

Fig a →
$$= \int_{ad} E\,dl \cos 90^\circ + \int_{dc} E\,dl \cos 0^\circ + \int_{cb} E\,dl \cos 90^\circ + \int_{ba} E\,dl \cos 180$$
$$= 0 + \int_{dc} (E_y + dE_y)\,dl + 0 - \int_{ba} E_y\,dl$$
$$= (E_y + dE_y) \int_{dc} dl - E_y \int_{ba} dl$$
$$= (E_y + dE_y)\,l - E_y\,l \quad = \cancel{E_y l} + dE_y \cdot l - \cancel{E_y l}$$
$$= dE_y \times l \longrightarrow (2)$$

Now, $\phi_B = B_Z \times$ area of rectangle abcd $= B_Z \times l\,dx \longrightarrow (3)$

P.T.O →

using eqns ② and ③ in eqn ①, we get

$$dE_y \times \cancel{l} = -\frac{d}{dt}(B_z\, l\, dx) = -\cancel{l}\, dx \frac{dB_z}{dt}$$

$$\therefore \boxed{\frac{dE_y}{dx} = -\frac{dB_z}{dt}} \longrightarrow ④$$

Since both E_y and B_z depend upon the two variables x and t, therefore eqn ④ can be written in the form of partial derivatives

$$\Rightarrow \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \longrightarrow ⑤$$

The magnitude of time varying electric field and magnetic field constituting a plane EM wave are given by

$$E = E_0 \sin(kx - \omega t) \longrightarrow ⑥$$

$$B = B_0 \sin(kx - \omega t) \longrightarrow ⑦$$

where, E_0 & B_0 are the maximum values of electric field & mag. field respectively.

and $\boxed{k = \frac{2\pi}{\lambda}}$ is the propagation vector or wave vector

Differentiating ⑥ wrt x, we get $\frac{\partial E_y}{\partial x} = E_0 k \cos(kx - \omega t) \rightarrow ⑧$

Differentiating ⑦ wrt t, we get $\frac{\partial B_z}{\partial t} = -B_0 \omega \cos(kx - \omega t) \rightarrow ⑨$

plug values of eqn ⑧ and ⑨ in eqn ⑤, we get

$$E_0 k \cancel{\cos(kx - \omega t)} = B_0 \omega \cancel{\cos(kx - \omega t)}$$

$$E_0 k = B_0 \omega$$

$$\text{or } E_0 = \frac{\omega}{k} B_0$$

Since $\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$ → Since $c = f\lambda$; $\frac{\omega}{k} = c$

$\therefore E_0 = c B_0$; where c is the speed of EM wave

$$\therefore \boxed{c = \frac{E_0}{B_0}} \longrightarrow ⑩$$

(2) To prove that $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ (See fig (b) (previous page))

- As the EM wave crosses the point A on the x-axis, the changing electric flux through the rectangle abcd of length l and width dx in XZ plane induces a mag. field along the rectangle.
- According to Ampere - Maxwell eqn, we have

$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0 I_c + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

Since, there is no conduction current $I_c = 0$

$$\therefore \oint_C \vec{B}\cdot\vec{dl} = \mu_0 \epsilon_0 \frac{d\phi_E}{dt} \longrightarrow (1)$$

Now $\oint_C \vec{B}\cdot\vec{dl} = \int_{ab} \vec{B}\cdot\vec{dl} + \int_{bc} \vec{B}\cdot\vec{dl} + \int_{cd} \vec{B}\cdot\vec{dl} + \int_{da} \vec{B}\cdot\vec{dl}$

$$= \int_{ab} B_z\, dl \cos 0^\circ + \int_{bc} B\, dl \cos 90^\circ + \int_{cd} (B_z + dB_z)\, dl \cos 180^\circ + \int_{da} B\, dl \cos 90^\circ$$

$$= \int_{ab} B_z\, dl - \int_{cd} (B_z + dB_z)\, dl$$

$$= B_z \int_{ab} dl - (B_z + dB_z) \int_{cd} dl$$

$$= B_z l - (B_z + dB_z) l = \cancel{B_z l} - \cancel{B_z l} - dB_z \times l$$

$$\therefore \oint_C \vec{B}\cdot\vec{dl} = \cancel{dB_z} - dB_z \times l \longrightarrow (2)$$

Also $\phi_E = E_y \times$ area of rectangle abcd

$$\phi_E = E_y \times l\, dx \longrightarrow (3)$$

plug (2) and (3) in (1)

$$-dB_z \times l = \mu_0 \epsilon_0 \frac{d}{dt}(E_y \times l\, dx)$$

$$-dB_z \times \cancel{l} = \mu_0 \epsilon_0 \cancel{l}\, dx \frac{dE_y}{dt}$$

$$\therefore -\frac{dB_z}{dx} = \mu_0 \epsilon_0 \frac{dE_y}{dt} \longrightarrow (4)$$

(4) Can be written as partial derivatives as both B & E depend upon x and t.

$$\therefore -\frac{\partial B_z}{\partial x} = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t} \longrightarrow (5)$$

Since $E_y = E_0 \sin(kx - \omega t)$ and $B_z = B_0 \sin(kx - \omega t)$

P.T.O →

$$\therefore -\frac{\partial}{\partial x}\left[B_0 \sin(kx-\omega t)\right] = \mu_0 \epsilon_0 \frac{\partial}{\partial t}\left[E_0 \sin(kx-\omega t)\right]$$

or $$-B_0 k \cancel{\cos(kx-\omega t)} = -\mu_0 \epsilon_0 E_0 \omega \cancel{\cos(kx-\omega t)}$$

$$\therefore B_0 k = \mu_0 \epsilon_0 E_0 \omega$$

$$\therefore B_0 = \mu_0 \epsilon_0 E_0 \left(\frac{\omega}{k}\right) \qquad \text{Since } \frac{\omega}{k} = c$$

$$B_0 = \mu_0 \epsilon_0 E_0 c$$

$$\boxed{\frac{E_0}{B_0} = \frac{1}{\mu_0 \epsilon_0 c}} \longrightarrow (6) \qquad \text{Since } \frac{E_0}{B_0} = c$$

$$\therefore c = \frac{1}{\mu_0 \epsilon_0 c}$$

or $$c^2 = \frac{1}{\mu_0 \epsilon_0}$$

or $$\boxed{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}} \longrightarrow (7)$$

which is the expression for the speed of EM wave in Vacuum.

For Vacuum, $\mu_0 = 4\pi \times 10^{-7}\ T\,m\,A^{-1}$
$\epsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

$$c = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = 3 \times 10^{8}\ ms^{-1} \rightarrow (8)$$

Thus speed of an e-m wave in vacuum = Velocity of light in vacuum.

$\Rightarrow$ Hence, an EM wave moves with speed of light in Vacuum.

$\rightarrow$ In a Material medium, the speed of EM wave is given by $v = \frac{1}{\sqrt{\mu \epsilon}}$

where μ = permeability of the medium
ϵ = permittivity of the medium

$\rightarrow$ We ~~Know that~~ Know that Refractive index (n) of the medium is given by $n = \frac{c}{v} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \cdot \sqrt{\mu \epsilon} = \sqrt{\frac{\mu \epsilon}{\mu_0 \epsilon_0}}$

$$n = \sqrt{\left(\frac{\mu}{\mu_0}\right)\left(\frac{\epsilon}{\epsilon_0}\right)} = \sqrt{\mu_r \epsilon_r}, \quad \text{where}$$

$\mu_r = \frac{\mu}{\mu_0}$ $\rightarrow$ is the relative permeability of the medium
$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ $\rightarrow$ is the relative permittivity of the medium

Energy density of EM waves

- Energy density of EM wave is defined as the energy per unit volume of the space in which it travels.
- EM waves carry energy as they travel through the space.
- This energy is shared equally by the electric and mag. field.
- We know that, in a free space, energy density of a "static" electric field $\vec{E}$ is

$$U_E = \frac{1}{2}\epsilon_0 E^2 \longrightarrow (1)$$

- In a free space, energy density of a mag. field B is given by

$$U_B = \frac{B^2}{2\mu_0} \longrightarrow (2)$$

- ∴ Total energy density of electric & mag. fields in free space,

$$U = U_E + U_B = \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0} \longrightarrow (3)$$

- However, in EM wave, both $\vec{E}$ and $\vec{B}$ are time varying ⇒ they both vary sinusoidally in space and time.

∴ Average energy density of EM wave can be obtained by replacing E & B in eq^n (3) with the r.m.s. values of E and B giving average energy density of EM wave as,

$$U_{av} = \frac{1}{2}\epsilon_0 E_{rms}^2 + \frac{B_{rms}^2}{2\mu_0} \longrightarrow (4)$$

- We know that $E_{rms} = \frac{E_0}{\sqrt{2}}$ and $B_{rms} = \frac{B_0}{\sqrt{2}}$, where

→ E_0 and B_0 are the peak or max. values of electric & mag. fields respectively. ∴ Eq. (4) becomes

$$U_{av} = \frac{\epsilon_0 E_0^2}{4} + \frac{B_0^2}{4\mu_0} \longrightarrow (5)$$

Since $B_0 = \frac{E_0}{c} \longrightarrow (6)$

Also $c^2 = \frac{1}{\mu_0 \epsilon_0}$

$$U_{av} = \frac{\epsilon_0 E_0^2}{4} + \frac{E_0^2}{4\mu_0 c^2}$$

$$\therefore U_{av} = \frac{\epsilon_0 E_0^2}{4} + \frac{E_0^2 \cancel{\mu_0}\epsilon_0}{4\cancel{\mu_0}}$$

$$\therefore U_{av} = \frac{\epsilon_0 E_0^2}{4} + \frac{\epsilon_0 E_0^2}{4} = \frac{\epsilon_0 E_0^2}{2} = \epsilon_0\left(\frac{E_0}{\sqrt{2}}\right)\left(\frac{E_0}{\sqrt{2}}\right)$$

$$\therefore \boxed{U_{av} = \epsilon_0 E_{rms}^2} \longrightarrow (7)$$

P.T.O →

Eq (5) in $U_{av} = \dfrac{\epsilon_0 E_0^2}{4} + \dfrac{B_0^2}{4\mu_0}$, this can be written as

(Since $E_0 = cB_0$)

$$U_{av} = \frac{\epsilon_0 c^2 B_0^2}{4} + \frac{B_0^2}{4\mu_0}$$

(Since $c^2 = \dfrac{1}{\mu_0 \epsilon_0}$, $\epsilon_0 c^2 = \dfrac{1}{\mu_0}$)

$$= \frac{B_0^2}{4\mu_0} + \frac{B_0^2}{4\mu_0}$$

$$U_{av} = \frac{B_0^2}{2\mu_0} = \frac{1}{\mu_0}\left(\frac{B_0}{\sqrt{2}}\right)\left(\frac{B_0}{\sqrt{2}}\right) = \frac{B_{rms}^2}{\mu_0}$$

$$\therefore \boxed{U_{av} = \frac{B_{rms}^2}{\mu_0}} \longrightarrow (8)$$

Also $U_E = \dfrac{1}{2}\epsilon_0 E_0^2 = \dfrac{1}{2}\epsilon_0 (cB_0)^2 = \dfrac{c^2 \epsilon_0 B_0^2}{2} = \dfrac{B_0^2}{2\mu_0}$

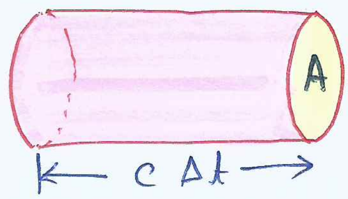
$$\therefore U_E = \frac{B_0^2}{2\mu_0} = U_B \quad (\text{as per eqn } (2))$$

Thus. $\boxed{U_E = U_B}$ $\Rightarrow$ Energy density of static electric field E = that of ~~a~~ magnetic field.

✗ Intensity of EM wave ✗.

- Intensity of EM wave is defined as the energy crossing per unit area per unit ~~time~~ time $\perp^r$ to the direction of propagation of the wave.

$\therefore$ Intensity $= \dfrac{\text{Energy}}{\text{Area} \times \text{time}}$



- Consider an EM wave travelling along X-direction (axis). The distance travelled by the wave in time $\Delta t = c \times \Delta t$, where c = speed of EM wave.
- Now consider an imaginary cylinder of length $(c \times \Delta t)$ and area of cross-section A (see fig). The energy contained in the cylinder is

U = Average energy density $\times$ Volume of the cylinder

$= U_{av} \times (c\,\Delta t)\,A = \dfrac{1}{2}\epsilon_0 E_0^2 \times (c\,\Delta t)\,A$

Intensity of e-m wave $\boxed{I = \dfrac{U}{A \times \Delta t} = \dfrac{1}{2}\epsilon_0 c E_0^2}$ Thus intensity of em wave is directly $\propto$ to the square of amplitude of electric field.

Since $E_0 = \sqrt{2}\,E_{rms}$ $\therefore$ $\boxed{I = \epsilon_0 c E_{rms}^2}$

※ Momentum and pressure of EM waves ※

- The Momentum carried by an EM wave is given by

$$p = \frac{\text{Energy of the wave}}{\text{Speed of the wave}} = \frac{U}{c} \longrightarrow (1)$$

- If this EM wave falls on a surface and is completely absorbed by the surface, then no wave is reflected from the surface. Hence, change in momentum of the EM wave after falling on the surface, $dp = p - 0 = p$. If dt is the time during which the wave is completely absorbed, then the force exerted on the surface is given by

$$F = \frac{dp}{dt} = \frac{p}{dt} \quad \text{(Newton's II law of motion)}$$

- The force exerted per unit area of the surface is known as "radiation pressure" (P)

$$\Rightarrow P = \frac{F}{A} = \frac{1}{A}\cdot\frac{dp}{dt} = \frac{1}{A}\times\frac{U}{c\times dt}$$

$$\therefore P = \left(\frac{U}{A\times dt}\right)\cdot\frac{1}{c} \qquad \text{Since Intensity } I = \frac{\text{Energy}}{\text{Area}\times\text{time}}$$

$$P = \frac{I}{c}$$

$\therefore$ pressure exerted by EM wave, $\boxed{P = \frac{I}{c}}$

- Radiation pressure of visible light is $7\times10^{-6}\ \mathrm{N\,m^{-2}}$
- If the surface is a perfect reflector, then the wave falling on this surface is completely reflected. It means the initial momentum of the wave $= p$ and the final momentum $= -p$. So, the change in momentum of the wave after falling on the surface, $dp = p-(-p) = 2p$

$\therefore$ Hence, force exerted on the surface is $\rightarrow F = \frac{dp}{dt} = \frac{2p}{dt}$

(S) **Imp:** Eqn (1) above is Momentum = energy (work) / speed

how? We know that momentum $p = F\times t$

also work $W = F\times S$

$\therefore F = W/S$

$$\therefore p = \frac{W}{S}t = \frac{W}{(S/t)} = \frac{W}{c}$$

Imp

$$\therefore p = \frac{W}{c} = \frac{\text{energy}}{\text{speed}}$$

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* When the Sun shines on your hand, you feel energy being absorbed from the EM waves (your hands get warm).
* EM waves also transfer momentum to your hand but since c is very large, the amount of momentum transferred is extremely small and you do not feel the pressure.
* In 1903, the American scientists Nicols and Hull succeeded in measuring "radiation pressure" of visible light and verified eq^n $p = \frac{U}{c}$. It was found to be the order of $7 \times 10^{-6}\ N\ m^{-2}$
* Thus, on a surface area of $10\ cm^2$, the force due to radiation is ONLY about 7×10^{-9} N.

---

* The great technological importance of EM waves stems from their capability to carry energy from one place to another. The radio and TV signals from broadcasting stations carry energy. Light carries energy from the Sun to the earth, thus making life possible on the earth.

Problem: A source of light has energy flux of 10 watt/m². The light is falling ⊥^{r} to the surface of area 5 cm². If the surface completely absorbs the incident light, then find (i) the momentum delivered to the surface and (ii) force exerted on the surface in 10 minutes.

Given Energy flux = $10\ W m^{-2}$; Area = $5\ cm^2 = 5 \times 10^{-4} m^2$

Time t = 10 min = 10×60 = 600 s

Total energy incident on the surface

U = Energy flux × Area × time

$= 10\ W \cancel{m^{-2}} \times 5 \times 10^{-4}\ \cancel{m^2} \times 600\ s = 3\ J$

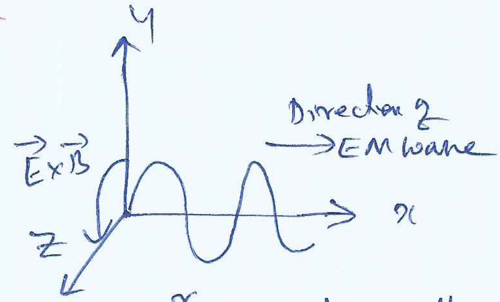
(i) Momentum delivered to the surface $p = \frac{U}{c} = \frac{3\ J}{3 \times 10^8\ m s^{-1}} = 1 \times 10^{-8}\ kg\ m s^{-1}$

(ii) Force exerted on the surface for 10 minutes $F = \frac{p}{t} = \frac{1 \times 10^{-8}\ kg\ m s^{-1}}{600\ s}$

$(p = F \times t)$

$F = 1.6 \times 10^{-11}$ N

Problem: A plane EM wave of $f = 25$ MHz travels in space along x-direction. At a particular point in space and time, $\vec{E} = 6.3\,\hat{j}$ V/m. What is B at this point



→ We know that $B = \frac{E}{c}$

$$\therefore B = \frac{\overset{2.1}{\cancel{6.3}}\ V m^{-1}}{\cancel{3}\times 10^{8}\ m s^{-1}} = 2.1\times 10^{-8}\ T$$

As per right hand thumb rule since $\vec{E}\times\vec{B}$ is along $+x$ direction, then ~~and hence~~ B is along $+\hat{z}$ direction

$$\therefore B = 2.1\times 10^{-8}\,\hat{k}\ T$$

Problem: The mag. field in a plane EM wave is given by

$$B_y = 2\times 10^{-7}\sin(0.5\times 10^{3}x + 1.5\times 10^{11}t)\ \text{Tesla}$$

(a) what is the λ and f of the wave

(b) write an expression for the $\vec{E}$?

We know that $B_y = B_0 \sin(kx - \omega t)$

(a) Given $B_y = 2\times 10^{-7}\sin(0.5\times 10^{3}x + 1.5\times 10^{11}t)$

→ EM wave travelling in $-x$ direction.
and $k = \frac{2\pi}{\lambda}$, $\omega = \frac{2\pi}{T}$
if '+', wave in $-x$ direction
if '−', wave in $+x$ direction

$$\therefore k = 0.5\times 10^{3} = \frac{2\pi}{\lambda}$$

$$\therefore \frac{\lambda}{2\pi} = \frac{1}{0.5\times 10^{3}} \qquad \therefore \lambda = \frac{2\pi}{0.5\times 10^{3}}$$

$$\lambda = 4\pi\times 10^{-3}\ m = 12.6\times 10^{-3}\ m = 12.6\times 10^{-3}\times 10^{2}\ cm = 12.6\times 10^{-1}\ cm$$

$$\boxed{\lambda = 1.26\ cm}$$

And $\omega = 1.5\times 10^{11} = 2\pi f$

$$\therefore f = \frac{1.5\times 10^{11}}{2\pi} = 0.239\times 10^{11}\ Hz = \boxed{23.9\ GHz}$$

(b) Given $B_0 = 2\times 10^{-7}$; we know that $E_0 = cB_0 = 3\times 10^{8}\times 2\times 10^{-7} = 60$

$$\therefore E_0 = 60\ V m^{-1}$$

General eqn for electric field is

$$E_z = E_0 \sin(kx - \omega t)$$

$$E_z = 60\sin(0.5\times 10^{3}x + 1.5\times 10^{11}t)\ V m^{-1}$$

$\vec{E}$ is along $+z$ direction

problem: Light with an energy flux of $18\, W/cm^2$ falls on a non-reflecting surface at normal incidence. If the surface has an area of $20\, cm^2$, find the average force exerted on the surface during a 30 minute time span.

→ Given energy flux $= 18\, W/cm^2$

Area $= 20\, cm^2$

time $= 30\, m = 30 \times 60\, s$

∴ Total energy falling on the surface

$= $ energy flux × Area × time

$$= \left(\frac{18\, W}{\cancel{cm^2}}\right) \times (20\, \cancel{cm^2}) \times (30 \times 60\, s)$$

$$= 6.48 \times 10^5\, \frac{J}{\cancel{s}}\, \cancel{s} = \underline{6.48 \times 10^5\, J}$$

∴ total momentum delivered (for complete absorption) is

$$p = \frac{U}{c} = \frac{\cancel{6.48}^{\,2.16} \times 10^5\, J}{\cancel{3} \times 10^8\, m s^{-1}} = 2.16 \times 10^{-3}\, kg\, m/s$$

∴ Average force exerted on the surface is

$$F = \frac{\text{Momentum}}{\text{time}} = \frac{p}{t} = \frac{2.16 \times 10^{-3}}{\cancel{3 \times 10^8}\, 30 \times 60} = \underline{1.2 \times 10^{-6}\, N}$$

extra

→ If the surface is a perfect reflector, the change in momentum will be $= p - (-p) = 2p = 2 \times 2.16 \times 10^{-3}\, kg\, ms^{-1}$

Now ave. force $F = \dfrac{2 \times 2.16 \times 10^{-3}}{30 \times 60} = \underline{2.4 \times 10^{-6}\, N}$

problem: Calculate the electric and magnetic fields produced by the radiation coming from a 100 W bulb at a dist. of 3 m. Assume that the efficiency of bulb is 2.5% and it is a point source. (34)

→ The bulb, as a point source, radiates light in all directions uniformly. At a distance of 3 m, the surface area of the surrounding sphere is $A = 4\pi r^2 = 4\pi (3)^2 = 113\ m^2$

→ The intensity (I) at this distance $= \frac{\text{Power}}{\text{Area}} = \frac{100\ W \times 2.5\%}{113\ m^2}$

$$I = 0.022\ W m^{-2}$$

→ Half of this intensity is provided by $\vec{E}$ and half by $\vec{B}$.

$$\frac{1}{2} I = \frac{1}{2} \epsilon_0 E_{rms}^2 c = \frac{1}{2} \times 0.022$$

$$\therefore E_{rms} = \sqrt{\frac{0.022}{(8.85 \times 10^{-12})(3 \times 10^8)}}\ V m^{-1}$$

$$E_{rms} = 2.9\ V m^{-1}$$

$$\therefore E_0 = \sqrt{2} E_{rms} = \sqrt{2} \times 2.9 = 4.07\ V m^{-1}$$

and $B_0 = ?$ Since $B_0 = \frac{E_0}{c} = \frac{4.07}{3 \times 10^8} = 1.36 \times 10^{-8}$

$$B_0 = 1.36 \times 10^{-8}\ T.$$

→ Thus, you see that electric field strength of the light that you use for reading is fairly large (for electric field strength of TV or FM waves is few $\mu V/m$

→ Since $B_0 = 1.36 \times 10^{-8}\ T$ → Note that although the energy in the mag. field = energy in the electric field, the mag. field strength is evidently very weak.

problem: A beam is travelling along x-axis is described by the mag. field as $B_z = 5 \times 10^{-9} \sin \omega (t - x/c)$ => $B_z = 5 \times 10^{-9} \sin (\omega t - kx)$
Calculate the max. electric and magnetic forces on a charge, i.e. alpha particle moving along y-axis with a speed of $3 \times 10^7\ m s^{-1}$, charge on electron $= 1.6 \times 10^{-19} C$

→ $B_0 = 5 \times 10^{-9}\ T$

Charge on alpha particle, $q = +2e = 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19} C$

$$v = 3 \times 10^7\ m s^{-1}$$

$$\therefore E_0 = B_0 c = 5 \times 10^{-9} \times 3 \times 10^8 = 1.5\ V m^{-1}$$

→ Max. force on alpha particle due to $\vec{E} = q E_0 = (3.2 \times 10^{-19}) \times 1.5$
$= 4.8 \times 10^{-19}\ N$

→ Max. Force on alpha particle due to $\vec{B} = q v B_0 = (3.2 \times 10^{-19})(3 \times 10^7)(5 \times 10^{-9})$
$= 48 \times 10^{-21}\ N$
$= 4.8 \times 10^{-20}\ N$

Important Info

Electromagnetic (EM) waves (Important Info)

Electromagnetic (EM) waves are changing electric and magnetic fields, transporting energy and momentum through space. EM waves are solutions of Maxwell's equations, which are the fundamental equations of electrodynamics. EM waves require no medium, they can travel through empty space. Sinusoidal plane waves are one type of electromagnetic waves. Not all EM waves are sinusoidal plane waves, but all electromagnetic waves can be viewed as a linear superposition of sinusoidal plane waves traveling in arbitrary directions. A plane EM wave traveling in the x-direction is of the form

$\mathbf{E}(x,t) = \mathbf{E}_{max}\cos(kx - \omega t + \phi)$, $\mathbf{B}(x,t) = \mathbf{B}_{max}\cos(kx - \omega t + \phi)$.

E is the electric field vector, and **B** is the magnetic field vector of the EM wave. For electromagnetic waves **E** and **B** are always perpendicular to each other and perpendicular to the direction of propagation.

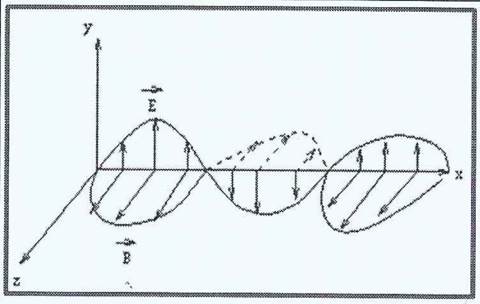
The direction of propagation of EM wave is the direction of E x B.

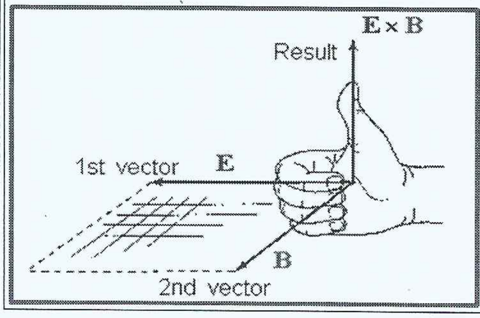
- Let the fingers of your right hand point in the direction of **E**.
- Orient the palm of your hand so that, as you curl your fingers, you can sweep them over to point in the direction of **B**.
- Your thumb points in the direction of **E** x **B**.

****

- Let **i** denote the x-direction, **j** the y-direction and **k** the z-direction.
- If for a wave traveling in the x-direction, then
  - **E** = E **j**
  - **B** = B **k**
  - **j** x **k** = **i**.

Electromagnetic waves are transverse waves.





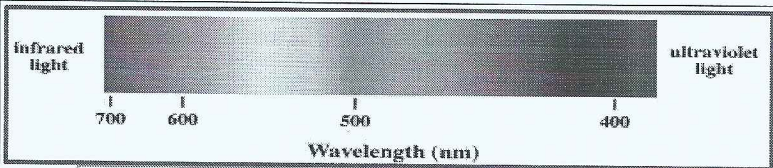
The wave number is $k = 2\pi/\lambda$, where λ is the wavelength of the wave. The frequency f of the wave is $f = \omega/2\pi$, ω is the angular frequency. The **speed** v of any periodic wave is the product of its λ and f. **$v = f\lambda$**

The speed of any electromagnetic waves **in free space** is the **speed of light** $c = 3*10^8$ m/s. Electromagnetic waves can have any wavelength λ or frequency f as long as $\lambda f = c$.

When electromagnetic waves travel through a medium, the speed of the waves in the medium is $v = c/n$, where n is the index of refraction of the medium. When an EM wave travels from one medium with index of refraction n_1 into another medium with a different index of refraction n_2, then its frequency remains the same, but its speed and wavelength change. For air n is nearly equal to 1.

The electromagnetic spectrum

Electromagnetic waves are categorized according to their frequency f or, equivalently, according to their wavelength $\lambda = c/f$. Visible light has a wavelength range from ~400 nm to ~700 nm. Violet light has a wavelength of ~400 nm, and a frequency of ~$7.5*10^{14}$ Hz. Red light has a wavelength of ~700 nm, and a frequency of ~$4.3*10^{14}$ Hz. Visible light makes up just a small part of the full electromagnetic spectrum. Electromagnetic waves with shorter wavelengths and higher frequencies include ultraviolet light, X-rays, and gamma rays. Electromagnetic waves with longer wavelengths and lower frequencies include infrared light, microwaves, and radio and television waves.

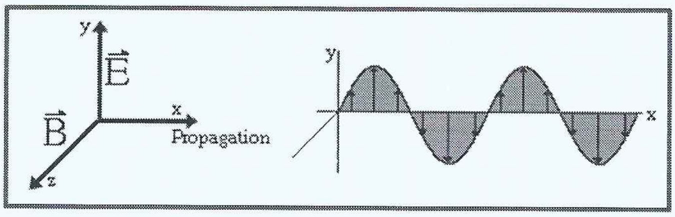


Type of Radiation	Frequency Range (Hz)	Wavelength Range
gamma-rays	$10^{20} - 10^{24}$	$< 10^{-12}$ m
x-rays	$10^{17} - 10^{20}$	1 nm - 1 pm
ultraviolet	$10^{15} - 10^{17}$	400 nm - 1 nm
visible	$4 - 7.5*10^{14}$	750 nm - 400 nm
near-infrared	$1*10^{14} - 4*10^{14}$	2.5 µm - 750 nm
infrared	$10^{13} - 10^{14}$	25 µm - 2.5 µm
microwaves	$3*10^{11} - 10^{13}$	1 mm - 25 µm
radio waves	$< 3*10^{11}$	> 1 mm

Polarization

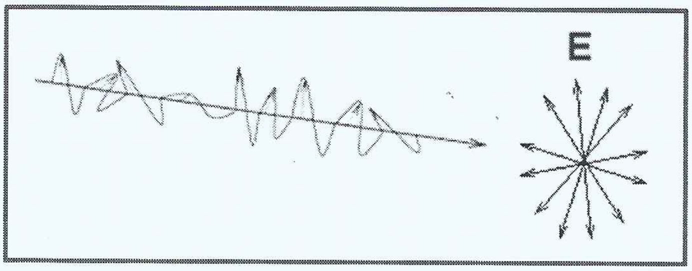
Polarization is a phenomenon peculiar to transverse waves. Longitudinal waves such as sound cannot be polarized. Light and other electromagnetic waves are transverse waves made up of mutually perpendicular, fluctuating electric and magnetic fields. In the diagram on the right, an EM wave is propagating in the x-direction, the electric field oscillates in the xy-plane, and the magnetic field oscillates in the xz-plane. A line traces out the electric field vector as the wave propagates.

For a linearly polarized electromagnetic wave traveling in the x-direction, the angle the electric field makes with the y-axis is unique.



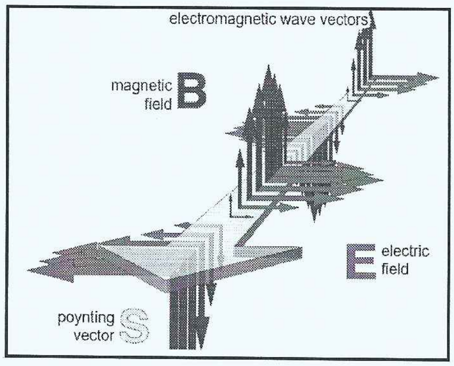
An unpolarized electromagnetic wave traveling in the x-direction is a superposition of many waves. For each of these waves the electric field vector is perpendicular to the x-axis, but the angle it makes with the y-axis is different for different waves. For unpolarized light traveling in the x-direction E_y and E_z are randomly varying on a timescale that is much shorter than that needed for observation.

The diagram on the right depicts unpolarized light. Natural light is, in general, unpolarized.



Electromagnetic waves transport energy through space. In free space this energy is transported by the wave with speed c. The magnitude of the energy flux S is the amount of energy that crosses a unit area perpendicular to the direction of propagation of the wave per unit time. It is given by $S = EB/(\mu_0) = E^2/(\mu_0 c)$, since for electromagnetic waves $B = E/c$. The units of S are $J/(m^2s)$. μ_0 is a constant called the permeability of free space, $\mu_0 = 4\pi*10^{-7}\ N/A^2$. Note: The energy transported by an EM wave is proportional to the square of the amplitude, E^2, of the wave.

Electromagnetic waves transport energy. **EM wave also transport momentum**. The magnitude of the momentum flux S/c is the amount of momentum that crosses a unit area perpendicular to the direction of propagation of the wave per unit time. If an electromagnetic wave is absorbed, momentum conservation requires that the object acquires momentum. The radiation exerts **radiation pressure** on the object. If the radiation is reflected instead of absorbed, then its momentum changes direction. The radiation pressure on an object that reflects the radiation is therefore twice the radiation pressure on an object that absorbs the radiation.



6. The figure shows the direction of propagation, direction of the electric field, and/or the direction of the magnetic field for four electromagnetic waves.

Direction of Electric field	Direction of Magnetic field	Direction of propagation	Answer (Direction?)
+y	?	+x	+z
?	+y	+x	-z
+z	?	-y	-x
+z	+y	?	-x

For each of the waves use the Right-Hand Rule to determine the missing information. Point the fingers of your right hand in the direction of the electric field, bend them in the direction of the magnetic field, and then your thumb will point in the direction of the wave propagation. If the electric field or magnetic field is unknown, guess the direction to see if your thumb points in the correct direction of propagation.

1. (a) The magnetic field points in the $\boxed{+z}$ direction.

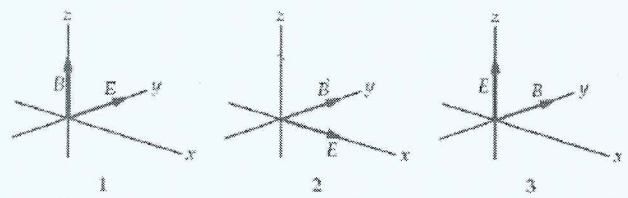
2. (b) The electric field points in the $\boxed{-z}$ direction.

3. (c) The magnetic field points in the $\boxed{-x}$ direction.

4. (d) The wave propagates in the $\boxed{-x}$ direction.

9. The figure at the right shows three electromagnetic waves with various orientations.

The propagation direction is the same as the direction of $\vec{E} \times \vec{B}$. Use the Right-Hand Rule to determine the directions of propagation.



To find the direction of propagation of an E&M wave, point the fingers of the right hand in the direction of the electric field, curl them toward the direction of the magnetic field, and your thumb will point in the direction of propagation. Applying this rule, we find the following directions of propagation: case 1, positive *x* direction; case 2, positive *z* direction; case 3, negative *x* direction.

Direction of Electric field	Direction of Magnetic field	Direction of wave propagation	Answer (Direction?)	Remarks
+y	?	+x	+z	The Magnetic field points in the +z direction
?	+y	+x	-z	The Electric field points in the -z direction
+z	?	-y	-x	The Magnetic field points in the -x direction
+z	+y	?	-x	The wave propagates in the -x direction

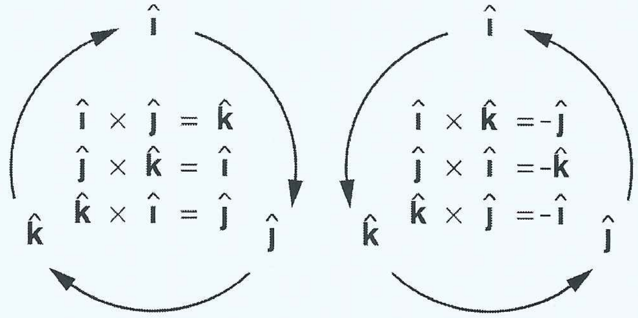
The direction of propagation of EM wave is the direction of E x B (Cross Product of E and B vectors)

- Let the fingers of your right hand point in the direction of **E**.
- Orient the palm of your hand so that, as you curl your fingers, you can sweep them over to point in the direction of **B**.
- Your thumb points in the direction of **E x B**.

OR we can also use Right Hand Palm Rule #1 :

If we stretch our right-hand palm such that the **"thumb"** points in the **"direction of wave propagation"** and the **"fingers"** pointing in the direction of **"electric field"**, then the **"perpendicular drawn from the palm"** will give the direction of **"magnetic field"**

Direction of Electric field	Direction of Magnetic field	Direction of wave propagation	Direction of Electric field	Direction of Magnetic field	Direction of wave propagation	Direction of Electric field	Direction of Magnetic field	Direction of wave propagation
+x	+y	+z	+y	+x	-z	+z	+y	-x
	-y	-z		-x	+z		-y	+x
	+z	-y		+z	+x		+x	+y
	-z	+y		-z	-x		-x	-y
-x	+y	-z	-y	+x	+z	-z	+y	+x
	-y	+z		-x	-z		-y	-x
	+z	+y		+z	-x		+x	-y
	-z	-y		-z	+x		-x	+y



The cross product of a cyclic pair of unit vectors is positive.

The cross product of an anticyclic pair of unit vectors is negative.

Cross product of unit vectors:

- $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$
- $\hat{\imath} \times \hat{\jmath} = \hat{k}$; $\hat{\jmath} \times \hat{k} = \hat{\imath}$; $\hat{k} \times \hat{\imath} = \hat{\jmath}$
- $\hat{\jmath} \times \hat{\imath} = -\hat{k}$; $\hat{k} \times \hat{\jmath} = -\hat{\imath}$; $\hat{\imath} \times \hat{k} = -\hat{\jmath}$
- a x b ≠ b x a ; a x b = -(b x a)
- a x (b+c) = (a x b) + (a x c)
- $\hat{\imath} \times \hat{\jmath} = \hat{k}$; $\hat{\jmath} \times \hat{\imath} = -\hat{k}$; $\hat{\imath} \times -\hat{\jmath} = \hat{\jmath} \times \hat{\imath} = -\hat{k}$ (see table below)

i x j = k	-i x j = j x i = -k	j x i = -k	-j x i = i x j = k	k x j = -i	-k x j = j x k = i
i x -j = j x i = -k	-i x -j = i x j = k	j x -i = i x j = k	-j x -i = j x i = -k	k x -j = j x k = i	-k x -j = k x j = -i
i x k = -j	-i x k = k x i = j	j x k = i	-j x k = k x j = -i	k x i = j	-k x i = i x k = -j
i x -k = k x i = j	-i x -k = i x k = j	j x -k = k x j = -i	-j x -k = j x k = i	k x -i = i x k = -j	-k x -i = k x i = j

From the previous page, we have this following table....

Direction of Electric field	Direction of Magnetic field	Direction of wave propagation	Direction of Electric field	Direction of Magnetic field	Direction of wave propagation	Direction of Electric field	Direction of Magnetic field	Direction of wave propagation
+x	+y	+z	+y	+x	-z	+z	+y	-x
	-y	-z		-x	+z		-y	+x
	+z	-y		+z	+x		+x	+y
	-z	+y		-z	-x		-x	-y
-x	+y	-z	-y	+x	+z	-z	+y	+x
	-y	+z		-x	-z		-y	-x
	+z	+y		+z	-x		+x	-y
	-z	-y		-z	+x		-x	+y

In terms of i, j & k unit vectors, we can rewrite the above table as follows → satisfies cross product rule.

Direction of Electric field	Direction of Magnetic field	Direction of wave propagation	Direction of Electric field	Direction of Magnetic field	Direction of wave propagation	Direction of Electric field	Direction of Magnetic field	Direction of wave propagation
+i	+j	+k	+j	+i	-k	+k	+j	-i
	-j	-k		-i	+k		-j	+i
	+k	-j		+k	+i		+i	+j
	-k	+j		-k	-i		-i	-j
-i	+j	-k	-j	+i	+k	-k	+j	+i
	-j	+k		-i	-k		-j	-i
	+k	+j		+k	-i		+i	-j
	-k	-j		-k	+i		-i	+j

i x j = k	-i x j = j x i = -k	j x i = -k	-j x i = i x j = k	k x j = -i	-k x j = j x k = i
i x -j = j x i = -k	-i x -j = i x j = k	j x -i = i x j = k	-j x -i = j x i = -k	k x -j = j x k = i	-k x -j = k x j = -i
i x k = -j	-i x k = k x i = j	j x k = i	-j x k = k x j = -i	k x i = j	-k x i = i x k = -j
i x -k = k x i = j	-i x -k = i x k = j	j x -k = k x j = -i	-j x -k = j x k = i	k x -i = i x k = -j	-k x -i = k x i = j

Info

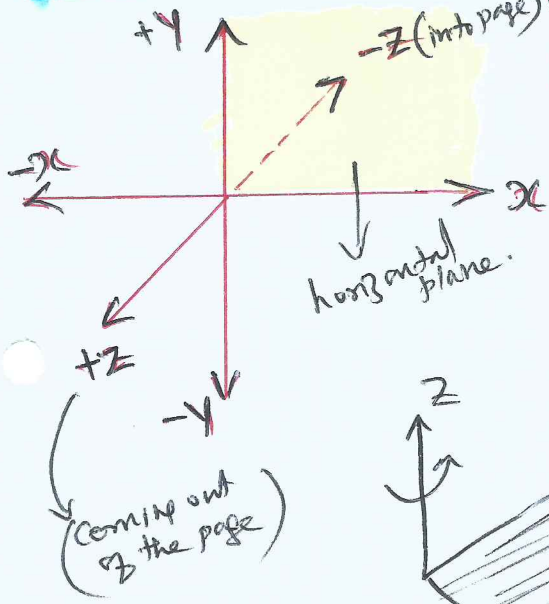
Cartesian Co-ordinate Systems (3-dimensions)

Two possible co-ordinate system

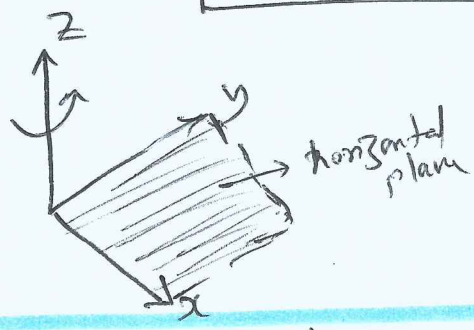
(We use this)

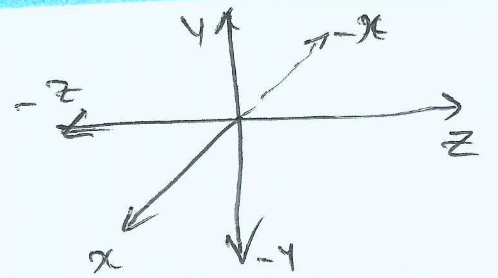
Right-handed (or positive) Cartesian co-ordinate system

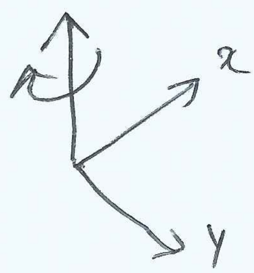
Left-handed (or negative) Cartesian co-ordinate system.



If xy plane is horizontal on the page, Z-axis points upwards from page, -Z axis points downwards from page.





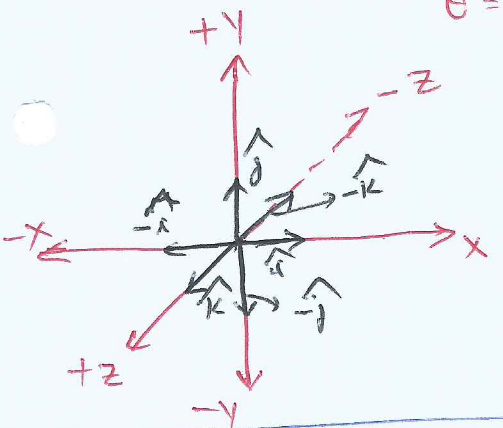


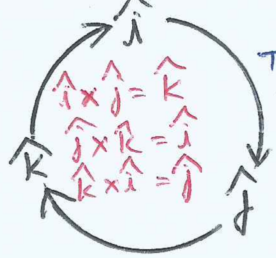
Right-handed cartesian co-ordinate system.

Cross product

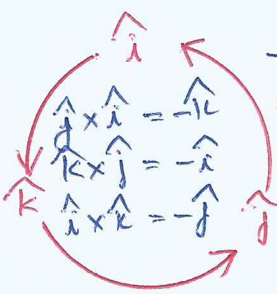
$A \times B = AB \sin\theta$

θ = is the smaller angle from A to B.





The cross product of a cyclic pair of unit vectors is POSITIVE



The cross product of an anti-cyclic pair of unit vectors is NEGATIVE

and $i \times i = j \times j = \hat{k} \times \hat{k} = 0$

Since $\sin 0° = 0$

dot product of unit vectors

$A \cdot B = AB\cos\theta$

$\theta \to$ Le betw A and B (smaller angle)

Unit vectors:

$i \cdot i = j \cdot j = k \cdot k = 1$

(Since $i \cdot i = 1 \cdot 1 \cos 0 = 1$)

and

$i \cdot j = j \cdot k = k \cdot i = 0$

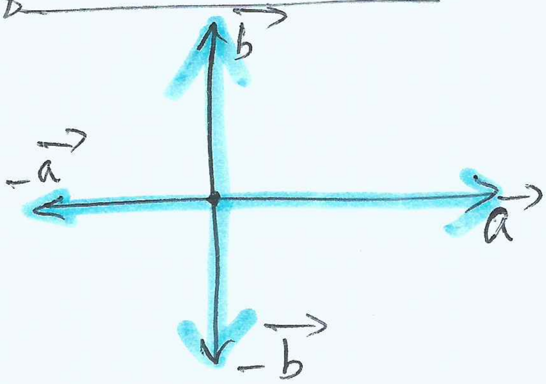
(Since i, j, k are mutually $\perp^r$, $\cos 90° = 0$)

$a \times b = \|a\| \, \|b\| \sin(\theta) \, \hat{n}$; where θ is the angle betw a and b in the plane containing them (hence, it is betw 0° and 180°), $\|a\|$ & $\|b\|$ are the magnitudes of vectors $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is the unit vector $\perp^r$ to the plane containing $\vec{a}$ and $\vec{b}$ by the right hand rule. If the vectors $\vec{a}$ and $\vec{b}$ are parallel (or anti-parallel) (θ is 0° or 180° $\to \sin 0 = \sin 180 = 0$), then $a \times b = 0$ (zero vector)

Imp Info : Cross product of two vectors.

- We know that $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

$$\therefore \boxed{\begin{aligned} \vec{a} \times \vec{b} &= -(\vec{b} \times \vec{a}) \quad \text{or} \\ \vec{a} \times \vec{b} &= -\vec{b} \times \vec{a} \quad \text{or} \\ \vec{a} \times \vec{b} &= \vec{b} \times -\vec{a} \end{aligned}}$$



- ~~However~~ This implies

$$-(\vec{b} \times \vec{a}) = \vec{a} \times \vec{b}$$
$$-\vec{b} \times \vec{a} = \vec{a} \times \vec{b}$$
$$\vec{b} \times -\vec{a} = \vec{a} \times \vec{b}$$

using Right hand rule (as explained in page 37)

$$\boxed{\text{But} \quad -\vec{b} \times -\vec{a} = \vec{b} \times \vec{a}}$$

- If $\vec{a}$ and $\vec{b}$ are on the ~~same~~ horizontal plane (plane of the page), then

$\vec{a} \times \vec{b} = \odot$ $\perp^r$ to the page and pointing upwards

$-(\vec{b} \times \vec{a}) = \odot$ —— do ——

$-\vec{b} \times \vec{a} = \odot$ —— do ——

$\vec{b} \times -\vec{a} = \odot$ —— do ——

$\vec{b} \times \vec{a} = \otimes$ $\perp^r$ to the page and pointing downwards.

$-\vec{b} \times -\vec{a} = \otimes$ —— do ——

problem :→ If an EM wave enters a medium, what is its wavelength if speed of the wave decreases by 25%.

→ We know that $c = f\lambda$ and also we know that when an em wave enters a medium, the frequency of the wave remains constant, but the λ decreases

so, from eqn $c = f\lambda \rightarrow \lambda = \frac{c}{f}$ $\therefore$ $\lambda \propto$ speed of the wave

If speed decreases by 25%, λ also decreases by 25%.

$$\frac{\lambda'}{\lambda} = \frac{\left(\frac{3}{4}\right)\cancel{c/f}}{\cancel{c/f}} = \frac{3}{4} \qquad \therefore \boxed{\lambda' = 0.75\lambda}$$

ELECTROMAGNETIC (EM) SPECTRUM (Sec 8.4 NCERT book)

At the time Maxwell predicted the existence of electromagnetic waves, the only familiar electromagnetic (EM) waves were the visible light waves. The existence of ultraviolet and infrared waves was barely established. By the end of the 19th century, X-rays and gamma rays had also been discovered. We now know that, EM waves include visible light waves, X-rays, gamma rays, radio waves, microwaves, ultraviolet and infrared waves. The classification of EM waves according to frequency is the electromagnetic spectrum (See next page). There is no sharp division between one kind of wave and the next. **The classification is based roughly on how the waves are produced and/or detected.**

We briefly describe these different types of EM waves, in order of decreasing wavelengths.

- **Radio waves**
  - Radio waves are produced by the accelerated motion of charges in conducting wires (by oscillating electric circuits)
  - They are generally in the frequency range from 500 kHz to about 1000 MHz.
  - They are used in radio and television communication systems.
    - The AM band is from 530 kHz to 1710 kHz.
    - Higher frequencies up to 54 MHz are used for *short wave* bands.
    - TV waves range from 54 MHz to 890 MHz.
    - The FM radio band extends from 88 MHz to 108 MHz.
    - Cellular phones use radio waves to transmit voice communication in the UHF band.
  - Properties: Reflection, diffraction.

- **Microwaves**
  - Microwaves (short-wavelength radio waves), with frequencies in the GHz range, and are produced by special vacuum tubes (called klystrons, magnetrons and Gunn diodes).
  - Uses :
    - Due to their short wavelengths, they are suitable for the radar systems used in aircraft navigation. Radar also provides the basis for the speed guns used to time fast balls, tennis-serves, and automobiles.
    - Long distance wireless communication via satellites.
    - Microwave ovens are an interesting domestic application of these waves. In such ovens, the frequency of the microwaves is selected to match the resonant frequency of water molecules so that energy from the waves is transferred efficiently to the kinetic energy of the molecules. This raises the temperature of any food containing water.
  - Properties: Reflection, polarization.
  - **MICROWAVE OVEN (for information)**
    - The spectrum of *electromagnetic radiation* contains a part known as *microwaves*. These waves have frequency and energy smaller than visible light and wavelength larger than it. What is the principle of a microwave oven and how does it work? Our objective is to cook food or warm it up. All food items such as fruit, vegetables, meat, cereals, etc., contain water as a constituent. Now, what does it mean when we say that a certain object has become warmer? When the temperature of a body rises, the energy of the random motion of atoms and molecules increases and the molecules travel or vibrate or rotate with higher energies. The frequency of rotation of water molecules is about 300 crore hertz, which is 3 gigahertz (GHz). If water receives microwaves of this frequency, its molecules absorb this radiation, which is equivalent to heating up water. These molecules share this energy with neighbouring food molecules, heating up the food.
    - One should use porcelain vessels and not metal containers in a microwave oven because of the danger of getting a shock from accumulated electric charges. Metals may also melt from heating. The porcelain container remains unaffected and cool, because its large molecules vibrate and rotate with much smaller frequencies, and thus cannot absorb microwaves. Hence, they do not get heated up.
    - Thus, the basic principle of a microwave oven is to generate microwave radiation of appropriate frequency in the working space of the oven where we keep food. This way energy is not wasted in heating up the vessel. In the conventional heating method, the vessel on the burner gets heated first, and then the food inside gets heated because of transfer of energy from the vessel. In the microwave oven, on the other hand, energy is directly delivered to water molecules which is shared by the entire food.

(zettaHz = 10^{21} Hz) ; (yottaHz = 10^{24} Hz)

Electromagnetic spectrum (The electromagnetic spectrum is the range of frequencies (the spectrum) of electromagnetic radiation and their respective wavelengths and photon energies. The electromagnetic spectrum covers electromagnetic waves with frequencies ranging from below one hertz to above 10^{25} hertz)

Very Imp

Class		
Ionizing radiation	γ	Gamma rays
	HX	Hard X-rays
	SX	Soft X-rays
700nm (Red) to 400nm (Violet)	EUV	Extreme ultraviolet
	NUV	Near ultraviolet
Visible	NIR	Near infrared
1.6 eV to 3.4 eV: the photon energy of visible light	MIR	Mid infrared
	FIR	Far infrared
Micro-waves and radio waves	EHF	Extremely high frequency
	SHF	Super high frequency
	UHF	Ultra high frequency
	VHF	Very high frequency
	HF	High frequency
	MF	Medium frequency
	LF	Low frequency
	VLF	Very low frequency
	ULF	Ultra low frequency
	SLF	Super low frequency
	ELF	Extremely low frequency

Frequency	λ	(1) Photon Energy = hf = hc/λ
300 EHz (10^{18}) (E=Exa)	1 pm (10^{-12})	1.24 MeV (M= 10^6)
30 EHz (10^{18})	10 pm (10^{-12})	124 keV
3 EHz (10^{18})	100 pm (10^{-12})	12.4 keV
300 PHz (10^{15})	1 nm (10^{-9})	1.24 keV
30 PHz (10^{15})	10 nm (10^{-9})	124 eV
3 PHz (10^{15}) (P= peta)	100 nm (10^{-9})	12.4 eV
300 THz (10^{12})	**1 μm (10^{-6})**	**1.24 eV**
30 THz (10^{12})	10 μm (10^{-6})	124 meV
3 THz (10^{12})	100 μm (10^{-6})	12.4 meV
300 GHz (10^9)	1 mm (10^{-3})	1.24 meV (m = 10^{-3})
30 GHz (10^9)	1 cm (10^{-2})	124 μeV
3 GHz (10^9)	1 dm (10^{-1})	12.4 μeV
300 MHz (10^6)	1 m	1.24 μeV (μ = 10^{-6})
30 MHz (10^6)	10 m	124 neV
3 MHz (10^6)	100 m	12.4 neV
300 kHz (10^3)	1 km (10^3)	1.24 neV (n = 10^{-9})
30 kHz (10^3)	10 km (10^3)	124 peV
3 kHz (10^3)	100 km (10^3)	12.4 peV
300 Hz	1 Mm (10^6)	1.24 peV (pico = 10^{-12})
30 Hz	10 Mm (10^6)	124 feV
3 Hz	100 Mm (10^6)	12.4 feV (femto = 10^{-15})

Photon energy is the energy carried by a single photon. The amount of energy is directly proportional to the photon's electromagnetic frequency and inversely proportional to the wavelength. The higher the photon's frequency, the higher its energy. Equivalently, the longer the photon's wavelength, the lower its energy.

Photon energy is solely a function of the photon's wavelength. Other factors, such as the intensity of the radiation, do not affect photon energy. In other words, two photons of light with the same colour and therefore, same frequency, will have the same photon energy, even if one was emitted from a wax candle and the other from the Sun.

If photons are in fact massless, photon energy would not be related to mass through equivalence $E = mc^2$. The only two observed kinds of so-called massless energetic particles are photons and gluons. Photons are said to have relativistic mass. Moreover, some hypotheses propose that all mass or "rest mass" might itself actually be composed of stacked relativistic mass, secondary to motion, since no material body can be truly at "rest" relative to all fields. In this hypothesis, as motion becomes zero, mass also becomes zero. On the other hand, photons have motion and varying energy depending on the frequency and wavelength, suggesting that various forms of the photon each have different mass equivalence. Thus, "$E = mc^2$" would show that mass and motion are inextricably linked and fundamentally interchangeable concepts for all matter.

****

Photon energy $E = hf = hc/\lambda$; since c & h are constants ; $\boxed{c = 3 \times 10^8 ms^{-1} ; \quad h = 6.63 \times 10^{-34}\ Js ; \quad 1eV = 1.6 \times 10^{-19}\ J}$

Substituting c, h in the above equation and convert Joule to eV, we get

$\boxed{E = 1.2398 \times 10^{-6} / \lambda (m)\ eV, \lambda \text{ in meters} \approx 1.24 \times 10^{-6} / \lambda (m)\ eV, \lambda \text{ in meters}}$; $E = 1.24 / \lambda (\mu m)\ eV$, where λ in μm

Eg : Therefore, ONE PHOTON energy at 1 μm wavelength (*λ of near Infrared radiation*) is ≈ 1.24 eV

- **Infrared waves**
  - Infrared waves are produced by hot bodies and by rotational and vibrational transitions in molecules. This band lies adjacent to the low-frequency or long-wavelength end of the visible spectrum.
  - Infrared waves are sometimes referred to as *heat waves*. This is because water molecules present in most materials readily absorb infrared waves (many other molecules, for example, CO2, NH3, also absorb infrared waves). After absorption, their thermal motion increases, that is, they heat up and heat their surroundings.
  - Uses :
    - Infrared lamps are used in physical therapy.
    - Infrared radiation also plays an important role in maintaining the earth's warmth or average temperature through the greenhouse effect. Incoming visible light (which passes relatively easily through the atmosphere) is absorbed by the earth's surface and reradiated as infrared (longer wavelength) radiations. This radiation is trapped by greenhouse gases such as carbon dioxide and water vapour.
    - Infrared detectors are used in Earth satellites, both for military purposes and to observe growth of crops.
    - Electronic devices (for example semiconductor light emitting diodes) also emit infrared and are widely used in the remote switches of household electronic systems such as TV sets, video recorders and hi-fi systems.
  - Properties: Heating effect on thermopile and bolometer, reflection, refraction, diffraction, penetration through fog.

- **Visible rays**
  - Production : Radiated by excited atoms in gases and incandescent bodies
  - It is the most familiar form of electromagnetic waves. It is the part of the spectrum that is detected by the human eye. It runs from about 4×10^{14} Hz to about 7×10^{14} Hz or a wavelength range of about 700 – 400 nm.
  - Visible light emitted or reflected from objects around us provides us information about the world. Our eyes are sensitive to this range of wavelengths.
  - Different animals are sensitive to different range of wavelengths. For example, snakes can detect infrared waves, and the 'visible' range of many insects extends well into the utraviolet.
  - Uses:
    - Reveals the structure of molecules and arrangement of electrons in external shells of atoms.
  - Properties: Reflection, refraction, interference, diffraction, polarization, photoelectric effect, photographic action and sensation of sight.

- **Ultraviolet rays**
  - Production: By Sun, arc, vacuum spark and ionized gases
    - The sun is an important source of ultraviolet light. But fortunately, most of it is absorbed in the ozone layer in the atmosphere at an altitude of about 40 – 50 km. UV light in large quantities has harmful effects on humans. Exposure to UV radiation induces the production of more melanin, causing tanning of the skin. UV radiation is absorbed by ordinary glass. Hence, one cannot get tanned or sunburn through glass windows. Welders wear special glass goggles or face masks with glass windows to protect their eyes from large amount of UV produced by welding arcs.
  - It covers wavelengths ranging from about 4×10^{-7} m (400 nm) down to 6×10^{-10}m (0.6 nm).
  - Uses:
    - Due to its shorter wavelengths, UV radiations can be focussed into very narrow beams for high precision applications such as LASIK (*Laserassisted in situ keratomileusis*) eye surgery.
    - UV lamps are used to kill germs in water purifiers.
    - In detection of invisible writing, forged documents, finger prints and to preserve food stuffs, to destroy bacteria and for sterilizing the surgical instruments, in burglars' alarm, in studying molecular structure.
  - Properties: All properties of gamma rays, but less penetrating, produce photoelectric effect, absorbed by atmospheric ozone, harmful to human body.

X-rays

- One common way to generate X-rays is to bombard a metal target by high energy electrons.
- It covers wavelengths from about 10^{-8} m (10 nm) down to 10^{-13} m (10^{-4} nm).
- Uses:
  - In Science, for studying structures of inner atomic electron shells and crystals.
  - In surgery, for detection of fractures, diseased organs, formation of bones and stones, observing the progress of healing bones.
  - In engineering, for detecting faults, cracks, flaws and holes in finished metal products
  - In radio therapy, to cure untraceable skin diseases, in cancer like diseases.
- Properties: All properties of gamma rays, but less penetrating.

Gamma rays

- This high frequency radiation is produced in nuclear reactions and also emitted by radioactive nuclei (by transitions of atomic nuclei and decay of certain elementary particles)
- They lie in the upper frequency range of the electromagnetic spectrum and have wavelengths of from about 10^{-10}m to less than 10^{-14}m.
- Uses: They are used in medicine to destroy cancer cells. It also provides information about structure of atomic nuclei.
- Properties: Chemical reaction on photographic plates, fluorescence, ionisation, diffraction, highly-penetrating, chargeless, harmful to human body.

Below table summarises different types of electromagnetic waves, their production and detections. As mentioned earlier, the demarcation between different regions is not sharp and there are over laps.

TABLE 8.1 DIFFERENT TYPES OF ELECTROMAGNETIC WAVES

Type	Wavelength range	Production	Detection
Radio	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1m to 1 mm	Klystron valve or magnetron valve	Point contact diodes
Infra-red	1mm to 700 nm	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eye Photocells Photographic film
Ultraviolet	400 nm to 1nm	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	1nm to 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic film Geiger tubes Ionisation chamber
Gamma rays	<10^{-3} nm	Radioactive decay of the nucleus	-do-