Since $n = \frac{N}{l}$, where N = total no of turns of solenoid

$$\therefore \boxed{B = \frac{\mu_0 N I}{l}} \longrightarrow (5)$$

→ Magnetic field inside a solenoid can be increased by inserting a magnetic material like iron rod of permeability μ. In that case

$$\boxed{B' = \frac{\mu N I}{l} = \frac{\mu_r \mu_0 N I}{l} = \mu_r B}$$

Example (solenoid): A closely wound solenoid 80 cm long has 5 layers of winding of 400 turns each. The diameter of solenoid is 1.8 cm. If $I = 8$ A, find magnitude of $\vec{B}$ inside the solenoid near its centre.

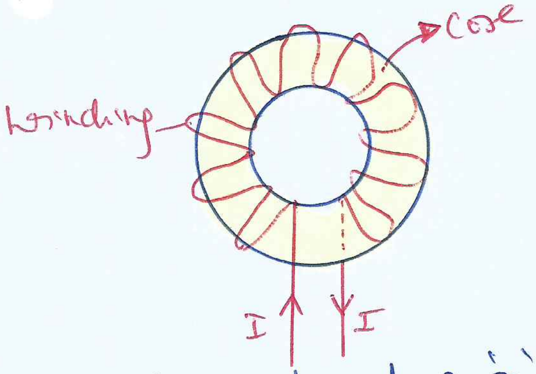
Ans $\vec{B}$ at a point inside the solenoid is

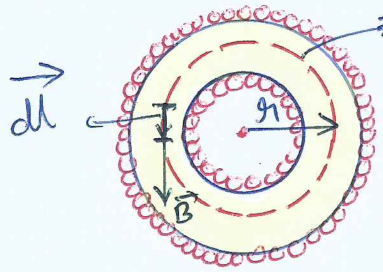
$$B = \frac{\mu_0 N I}{l} = \frac{4\pi \times 10^{-7} \times (400 \times 5) \times 8}{(80 \times 10^{-2})}$$

$$= 8\pi \times 10^{-3}\,T \approx 2.5 \times 10^{-2}\,T$$

The Toroid

A toroid is hollow olar ring of finite thickness on which a large number of turns of an insulated wire are closely wound. A toroid can be considered as a ring ~~type~~ shaped closed solenoid.





- Consider a toroid of 'n' turns/unit length and I be the current thro' toroid. The mag. field lines mainly remain in the "core of toroid" and are in the form of concentric circles. Consider such a circle of radius 'r'. (See fig)
- By symmetry, $\vec{B}$ in the core is constant and is along the tangent to path of concentric circle $\vec{dl}$. $\vec{dl}$ is $\parallel$ to $\vec{B}$ $\Rightarrow \theta = 0$, $(\cos\theta = 1$.

$$\therefore \oint \vec{B}\cdot\vec{dl} = \oint B\,dl\cos\theta = B\int dl \quad (\because B \text{ is uniform})$$

P.T.O →

$$\oint \vec{B}\cdot\vec{dl} = B \times \text{Circumference of the circle of radius } r$$

$$\oint \vec{B}\cdot\vec{dl} = B \times 2\pi r \longrightarrow (1)$$

As per Ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 \times \text{net current enclosed by the circle of radius 'r'}$$

$$= \mu_0 \times \text{total no of turns} \times I$$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 (n \times 2\pi r)\, I \longrightarrow (2)$$

Comparing (1) and (2), $B \times \cancel{2\pi r} = \mu_0 (n \times \cancel{2\pi r}) I$

$$\therefore \boxed{B = \mu_0 n I}$$ → which is same as mag. field due to a current carrying solenoid

→ Also if N is the total no of turns of a toroid, then

$N = n \times 2\pi r$ or $n = \dfrac{N}{2\pi r}$

$$\therefore \boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

→ * Imp: (1) A point inside the core (hollow space) → the field $\vec{B} = 0$ since there is no current enclosed by the circle thro' Q.

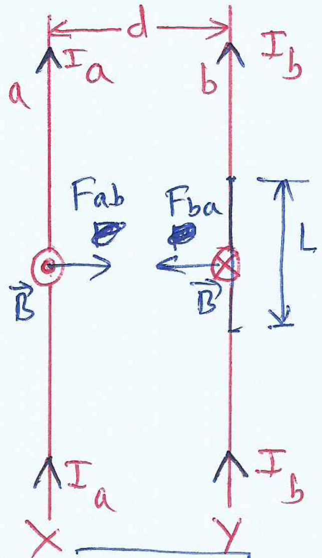
(2) A point 'R' outside the core → $\vec{B}$ is again zero, since each turn of winding passes twice thro' the area enclosed by the circle thro' point R, carrying equal currents in opposite directions, so that the net current enclosed by this circle (of radius R) is zero.

IMP (3) The mag. field of a Toroid is thus zero at all points except within the core.

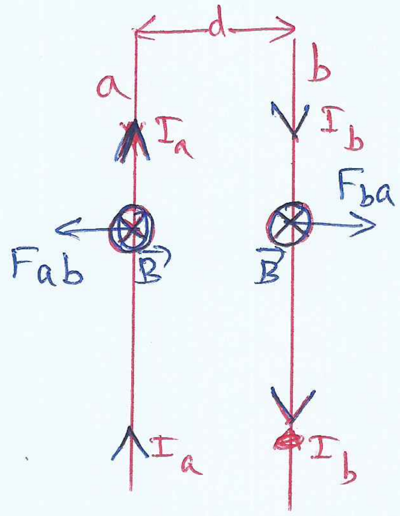
NCERT book
Sec 4.9

Force between two parallel Currents : Defn of Ampere.

- Any conductor carrying current has a mag. field around it. If we have two current carrying conductors, each will have its own mag. field and we might expect these to interact each other.



Fig(a)



Fig(b)

- Fig. shows two long parallel conductors 'a' and 'b' separated by a distance 'd' and carrying the currents I_a and I_b in the same direction [Fig(a)]

- Conductor 'a' produces same mag. field $\vec{B}$ at all points on conductor 'b'. The right hand rule (palm rule #1) says the direction of $\vec{B}$ is downwards ⊗ [when conductors a and b are ~~at~~ on the plane of this page]. The magnitude of this mag. field due to 'a' is B_a

$$\boxed{B_a = \frac{\mu_0 I_a}{2\pi d}} \quad \text{(as per Ampere's Circuital law)} \longrightarrow (1)$$

- So, what is the force on a segment of conductor 'b' due to this B_a. ~~The cond~~ As per FLHR (palm rule #2), conductor 'b' will experience a force towards conductor 'a'. [See fig (a)]. We label this force as F_{ba} [Force on 'b' due to 'a']

- Magnitude of this force $F_{ba} = I_b L B_a$ $\left(\text{But } B_a = \frac{\mu_0 I_a}{2\pi d}\right)$

$$\boxed{F_{ba} = \frac{\mu_0 I_a I_b}{2\pi d} L} \longrightarrow (2)$$

- Similarly "Force on 'a' due to 'b'" $\Rightarrow F_{ab} = \frac{\mu_0 I_a I_b}{2\pi d} L$ and direction is towards conductor 'b'.

$\therefore$ ~~$F_{ab} = -F_{ba}$~~ $\therefore$ $\boxed{F_{ba} = -F_{ab}} \longrightarrow (3)$

- Note that eqn (3) is consistent with Newton's III law. Thus, at least for parallel conductors and steady currents, we have shown that the B-S law and Lorentz force yield results in accordance with Newton's III law.

- Fig (b) → shows oppositely directed parallel currents repel each other. Thus parallel currents attract, and antiparallel currents repel.

P.T.O →

From pre. page

$\therefore$ parallel Currents attract, and antiparallel currents repel.

↳ This rule is opposite to electrostatics → like ~~poles~~ charges repel, unlike ~~poles atta~~ charges attract.

Let f_{ba} represent the magnitude of the force "F_{ba} per unit length". Then eqn ② becomes

$$\boxed{f_{ba} = \frac{F_{ba}}{L} = \frac{\mu_0 I_a I_b}{2\pi d}} \quad (4)$$

IMP: Eqn ④ is used to define "Ampere" which is one of the seven SI base units.

Defn of Ampere (adopted in 1946):

→ "The "Ampere" is the value of that steady current which, when maintained in each of the two very long, straight, parallel conductors of negligible cross-section, and placed one metre apart in vacuum (μ_0), would produce on each of these conductors a force equal to 2×10^{-7} newtons per metre of length"

↳ how force $= 2\times10^{-7}\ N m^{-1}$

→ we know that $\frac{\mu_0}{4\pi} = 10^{-7}$ $\quad \therefore \boxed{\frac{\mu_0}{2\pi} = 2\times10^{-7}}$

$$\boxed{f_{ba} = \left(\frac{\mu_0}{2\pi}\right)\frac{I_a I_b}{d}}$$

Defn of Coulomb:

"When a steady current of 1 A is set-up in a conductor, the quantity of charge that flows through its cross-section in 1 s is one coulomb (1 C)"

* We have seen that $F_{ba} = -F_{ab}$ in previous pages consistent with Newton's III law. It turns out that when we have time-dependent currents / and / or / charges in motion, Newton's III law may not hold for forces betn charges and / or conductors. An essential consequence of Newton's III law in Mechanics is conservation of "Momentum" of an isolated system. This, however, holds even for the case of time-dependent situations with e-m fields, provided the momentum carried by fields is also taken into account.

Another View on explanation of "Force between two parallel currents" (Info only).

We have seen that parallel currents attract and antiparallel currents repel;

→ Fig (a) → parallel currents ⟹ currents in conductors PQ and RS are in same direction.
⟹ they attract

→ Fig (b) → Anti-parallel currents
⟹ currents in conductors PQ and RS are in opposite direction ⟹ they repel.

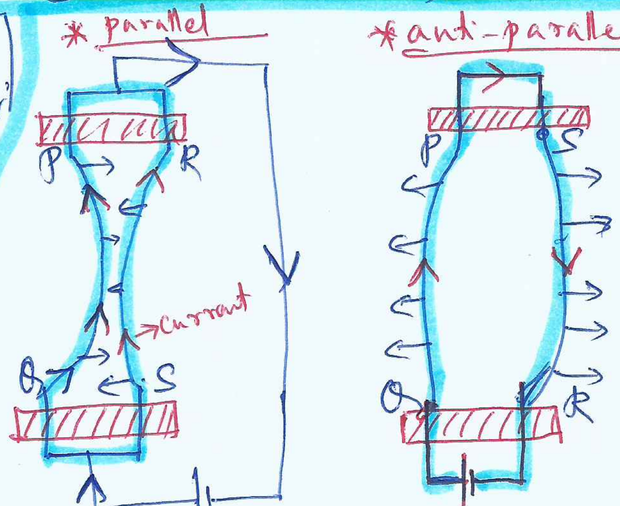


Fig (a)

Fig b

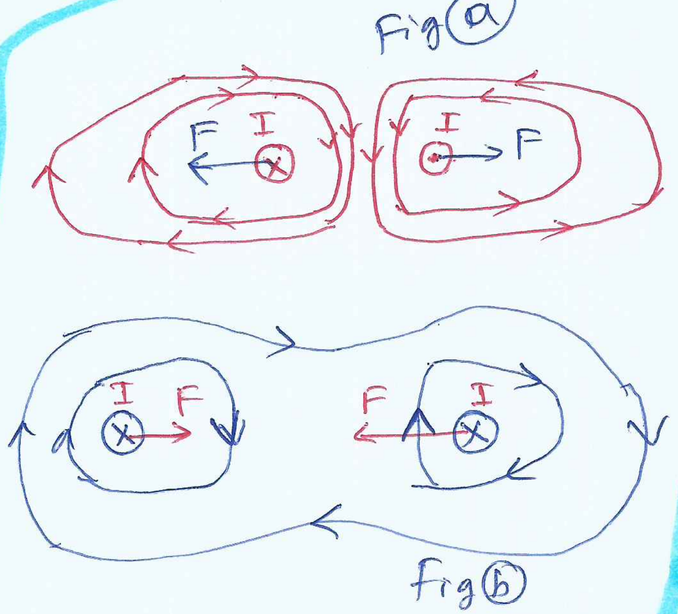
Contd - - - -

Info | -50- | ~~-12-~~

Force between Currents

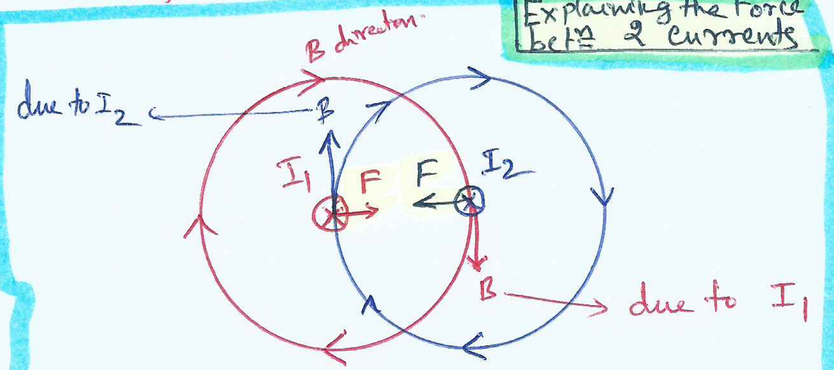
Any electric current has a mag. field around it. If we have two currents, each will have its own mag. field, and we might expect these to interact.

→ Fig ⓐ shows anti-parallel currents, one flowing in to the page and the other flowing out of the page. Direction of $\vec{B}$ is as per RH Thumb rule and in fig ⓐ, there is an extra strong field in the space between the wires. ⟹ repulsive force on the two wires.



→ Fig ⓑ shows the same idea, but for two parallel currents in the same direction and in the space left wires, the mag. fields cancel out. The wires are pushed together
⟹ Attractive force on the two wires.

↓ we can explain above forces based on motor effect (Fleming LHR).



- Consider I_1 → its mag. field lines at conductor I_2 is tangential as shown. So, as per FLHR, force on I_2 conductor due to I_1 is towards left.
- Similarly, consider I_2 → its mag. field lines at conductor I_1 is tangential as shown in figure. So, as per FLHR, force on I_1 conductor due to I_2 is towards Right
⟹ Attractive force ~~due~~ on the two wires.

⟹ These are an example of an action and reaction pair, as described by Newton's third law of motion.

* parallel currents attract and anti-parallel currents repel

over

Not in Syllabus

※ Comparison betⁿ Electric and Magnetic forces:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \longrightarrow (1)$$

Mag. force betⁿ parallel current elements $I_1 dl_1$ and $I_2 dl_2$ separated by a distance d is given by $F_m = \frac{\mu_0}{4\pi}\left(\frac{I_1 I_2}{r^2}\right) dl_1 \cdot dl_2 \longrightarrow (2)$

But $I_1 dl_1 = \frac{q_1}{t} \cdot dl_1 = q_1 v_1 \longrightarrow$ Similarly $I_2 dl_2 = q_2 v_2$

$\rightarrow$ v_1 and v_2 are drift velocities of electrons in 2 current elements respectively,

Eq (2) becomes $F_m = \frac{\mu_0}{4\pi}\frac{q_1 q_2}{r^2} v_1 v_2 \longrightarrow (3)$

$$\therefore \frac{F_e}{F_m} = \frac{\frac{1}{\cancel{4\pi}\epsilon_0}\cancel{\frac{q_1 q_2}{r^2}}}{\frac{\mu_0}{\cancel{4\pi}}\cancel{\frac{q_1 q_2}{r^2}} v_1 v_2} = \frac{1}{(\mu_0\epsilon_0) v_1 v_2}$$

Now, $C = \frac{1}{\sqrt{\mu_0\epsilon_0}}$ or $\boxed{\mu_0\epsilon_0 = \frac{1}{C^2}}$ $\quad C = 3\times10^8 \, ms^{-1}$

$$\therefore \boxed{\frac{F_e}{F_m} = \frac{C^2}{v_1 v_2}}$$

Drift Velocity of an electron is $10^{-5}\, ms^{-1}$

$$\therefore v_1 = v_2 = 10^{-5}\, m s^{-1}$$

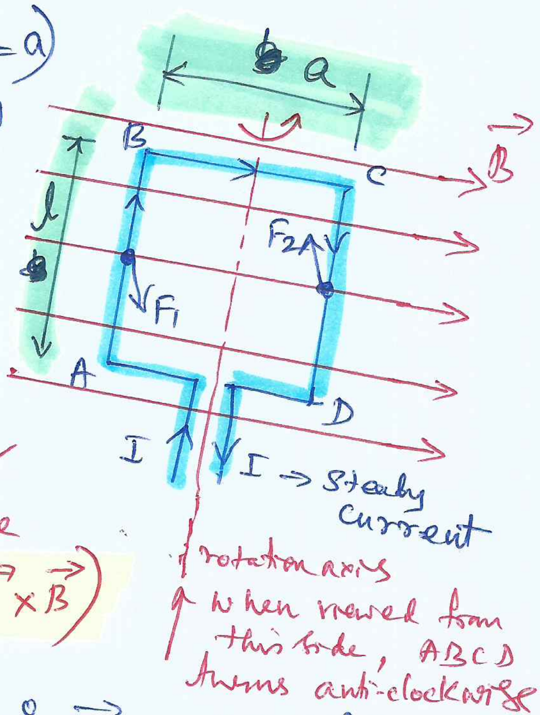
$$\frac{F_e}{F_m} = \frac{(3\times10^8)^2}{(10^{-5})^2} = \frac{9\times10^{16}}{10^{-10}} = 9\times10^{26}$$

$$\frac{F_e}{F_m} \approx 10^{27} \qquad \therefore \boxed{F_e \approx 10^{27} F_m}$$

This shows electric force >>>> mag. force.

Over

Torque on a Rectangular Current loop in a uniform $\vec{B}$



- Consider ▭ar current loop ABCD (length = l, breadth = a) placed in uniform $\vec{B}$ (See fig). I is steady current in the loop ABCD.
- Consider simple case: Loop ABCD and $\vec{B}$ are in the same plane.
  → Sides AB and CD are cutting across flux lines @ 90^0.
  → Sides BC and AD are parallel to $\vec{B}$.
- We know that force acting on a conductor of length l carrying current I in the mag. field is given by $\vec{F} = I(\vec{l} \times \vec{B})$

$$F = IlB\sin\theta \longrightarrow (1)$$

- Since BC & AD are parallel to $\vec{B}$, $\theta = 0^0$, $\vec{F} = 0$; Arms BC and AD experiences no force.
- Since AB and DC are $\perp^r$ to $\vec{B}$, $\theta = 90^0$, $\vec{F} = IlB$ (maximum); Arms AB and DC experiences force.
- As per FLHR (or palm rule #2), F_1 is into the page and F_2 is out of page and their magnitude is equal. Therefore, only two forces F_1 & F_2 act on the loop and they are equal in magnitude and opposite in direction. Therefore, the net force on the loop is zero. However, $\vec{F_1}$ and $\vec{F_2}$ form a couple and as per figure, the torque on the loop tends to rotate it anti-clockwise. Torque = one of the forces × separation

Max. Torque: $\tau = IlB \times a = I(la)B = IAB$

In Vector form: $\vec{\tau} = I(\vec{A} \times \vec{B}) = IAB\sin\theta$

If loop has 'N' turns $\vec{\tau} = N\tau = NIAB\sin\theta \longrightarrow (2)$

where A = area of the rectangle.

Since in this simple case $\theta = 90^0$, $\boxed{\tau = NIAB} \longrightarrow (3)$

- Consider an angle θ betn $\vec{B}$ and normal to plane of coil, the torque has decreased compared to previous case and is given by $\tau = IAB\sin\theta$
- When $\theta = 0$, torque acting on the loop $\tau = NIAB\sin 0 = 0$. Thus, loop will be in a position of equilibrium.

P.T.O →

Thus, the net force on a current loop due to uniform $\vec{B}$ is zero but the torque on current loop may or may not be zero.

$\tau = IAB \sin\theta$ → This can be expressed as vector product of the "magnetic moment of the coil" and the "mag. field". We define the "mag. moment of the current loop" as

$m = IA$

$$\boxed{\tau = m \times B} \quad (4)$$

This is analogous to electrostatic case (electric dipole of dipole moment P_e in an electric field E)

$\vec{\tau} = \vec{P_e} \times \vec{E}$

→ Dimension: $[A][L^2]$ and unit is Am^2

If loop has N closely wound turns $\boxed{m = NIA}$.

Example (1): A current-carrying circular loop lies on a smooth horizontal plane. Can a uniform $\vec{B}$ be set-up in such a manner that the loop turns around itself (i.e. turns about vertical axis).

→ No, we know that the torque on a current-loop due to a mag. field $\vec{B}$ is given by $\tau = \vec{m} \times \vec{B} = I(\vec{A} \times \vec{B})$, where $\vec{A}$ is the area vector of the loop. The loop can turn around itself only when τ is along vertical. Since $\vec{A}$ is vertical (loop is placed horizontally), τ cannot be vertical.

Example 2: A current-carrying cirlar loop is located in a uniform external $\vec{B}$. If the loop is free to turn, what is the orientation of stable equilibrium? Show that in this orientation, the flux of total field (external field + field produced by the loop) is maximum.

→ The torque on the loop (area A) is $\tau = IAB \sin\theta$, where θ is the angle between the normal to the plane of loop and the field $\vec{B}$. In stable equilibrium, the loop will have its plane ⊥r to the field $\vec{B}$ ($\theta = 0$ and so $\tau = 0$). In this orientation, the mag. field produced by the loop is in the same direction as external field, both being normal to the plane of the loop. This will result in maximum flux of the total field

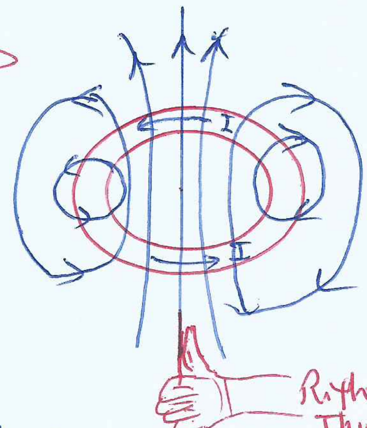
P.T.O

Example 3: A loop of irregular shape carrying current is located in an external mag. field. If the wire is flexible, why does it change to a circular shape?

→ It assumes circular shape with its plane normal to the field to maximize flux, since for a given perimeter, a circle encloses greater area than any other shape.

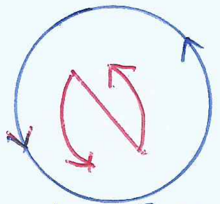
Circular Current loop as a magnetic dipole (Sec 4·10·2 in NCERT)

- A "magnetic dipole" consists of 2 unlike poles of equal strengths by a small distance. A bar magnet or a compass needle are magnetic dipoles (N and S poles). Mag. poles always exist in pair.
- A current-carrying solenoid, or a current-loop, behaves like a bar magnet. A bar magnet having 'N' and 'S' poles at its ends is a magnetic dipole, and so a "current-loop" is also a magnetic dipole. Let us compute the mag. moment of a current-loop –
- When current flows in a current-~~loop~~ circular loop, $\vec{B}$ is set up around the current loop.
  The circular current-loop behaves ~~like~~ as a mag. dipole. One face of the loop as N-pole and the other face as S-pole.

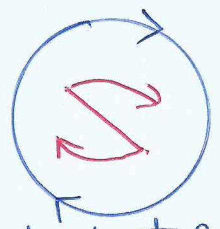


Fig①

Right hand Thumb rule to find $\vec{B}$ direction.



Look at a Face where I is in anti-clockwise direction, that face is N-pole

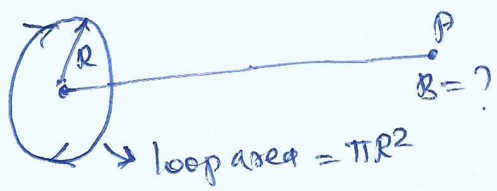


Look at a face where I is in clockwise direction, that face is S-pole.

- We know the eqn for the following diagram

$$\boxed{B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}}$$

If $x >>> R$



→ loop area $= \pi R^2$

$$B = \frac{\mu_0 R^2 I}{2x^3} \longrightarrow (i)$$

$$\therefore B = \frac{\mu_0 I}{2x^3}\left(\frac{1}{\pi}\right)\pi R^2 \qquad \therefore B = \frac{\mu_0 I A}{2\pi x^3}$$

We define "mag. moment" $\vec{m}$ to have a magnitude $= IA$; $|\vec{m}| = IA$.

Hence $\vec{B} \simeq \frac{\mu_0 \vec{m}}{2\pi x^3}$ $\quad \therefore \boxed{\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{2\vec{m}}{x^3}} \longrightarrow (2)$

PTO

The eq^n ~~...~~ $\cancel{\mu_0 = \mu}$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{x^3} \rightarrow (2)$$

is very similar to an expression obtained for electric ~~dipole of~~ field of a dipole. The similarity may be seen if we substitute

$\mu_0 \rightarrow 1/\epsilon_0$

$\vec{m} \rightarrow \vec{p}_e$ (electrostatic dipole)

$\vec{B} \rightarrow \vec{E}$ (electrostatic field)

We obtain $E = \dfrac{2p_e}{4\pi\epsilon_0 x^3}$ $(r >>> a)$ → which is precisely the field for an electric dipole at a point on its axis.

→ This is at point on the "dipole axis".

(i) At a point on the equatorial axis [⇒ $\vec{E}$ on ⊥ bisector of the dipole $\vec{E} = \dfrac{p_e}{4\pi\epsilon_0 x^3}$ $(r >> a)$; where x is the distance from the dipole.

↳ Replace $\vec{p} \rightarrow \vec{m}$ and $\mu_0 \rightarrow 1/\epsilon_0$, then $\vec{B}$ for a point on the plane of the loop at a distance x from the ~~ce~~ centre, For $x >>> R$,

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{\vec{m}}{x^3} \quad ; \quad x >> R} \rightarrow (3)$$

This is applicable to any planar ~~loop~~ current loop.

Difference is : Electric dipole is built up of 2 elementary "charges" (or electric mono-poles → they can exist independently).

: In magnetism, mag. dipole (or a current-loop) is the most elementary element → there is no 2 separate elementary ~~...~~ units ⇒ Mag. monopoles, are ~~not~~ known to exist.

Conclusion : In this section, we have shown that a current-loop

(i) produces a mag. field (See fig (1) in page 54) and behaves like a magnetic dipole at large distances

(ii) Current-loop is ~~is~~ subjected to ~~a~~ torque ~~like~~ like a mag. needle.

→ This led Ampere to suggest that all magnetism is due to circulating currents. This seems to be partly true and no mag. monopoles have been seen so far.

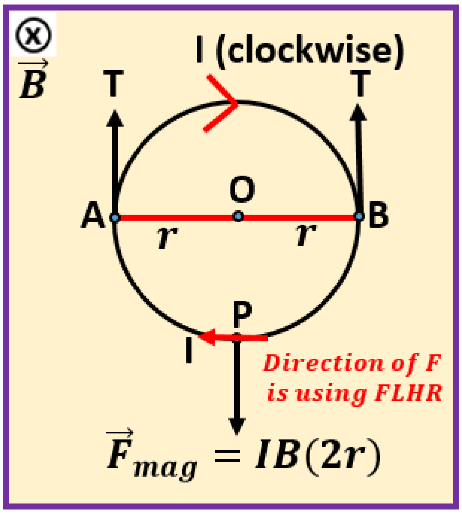
→ However, elementary particles such as an electron or a proton also carry an intrinsic magnetic moment, not accounted by circulating currents.

-- 55a --

Problem : **A small circular flexible loop of wire of radius r carrying a current I is placed in a uniform magnetic field B. The tension in the loop will be doubled if**

(a) I is doubled	(b) B is halved	(c) r is doubled	(d) Both B and I are doubled

Solution #1: Let the flexible circular current loop is on the plane of the paper and the direction of current is clockwise. Let $\vec{B}$ *be* perpendicular to the plane of the paper and pointing into the paper (as shown in the figure). ∴ The angle θ between the current loop plane and $\vec{B}$ is 90^0. Given that the loop is flexible so that it can be slightly extended or contracted.



- Note that the flexible circular loop is already carrying a current I and is then placed in uniform B, hence the current loop experiences a force. The direction of the force is always outward normal to each and every point on the circular current loop. The force as per FLHR is on the plane of the paper and outwards.
- The arc APB is trying to experience a downward force (at point P) and the loop is trying to move down on the plane of the paper; however, the tension at A and B will not allow the loop to move down.
- To find the tension in the loop, let A and B are the diametrically opposite points, therefore the net tension on half of the wire loop = 2T. This tension will balance the force on the current loop due to $\vec{B}$. We need to find the force
- Net force F on half of the circular loop = $I(\vec{l}\,x\,\vec{B}) = IlB\,sin\theta = Ilb\,sin90^0 = IlB$, where $l\ is\ the\ shortest\ length\ between\ A\ and\ B = 2r\ ;\ \therefore l = 2r$
- $\therefore F = I\,x\,2r\,x\,B = 2IBr.\ The\ tension\ (2T)\ balances\ this\ force\ F\ ,hence\ \ 2T = F$
- $Hence, 2T = 2IBr\ ;\ \ \therefore\ \ \boldsymbol{T = IBr\ is\ the\ tension\ in\ the\ whole\ loop}$
- As T is directly proportional to $I, B\ and\ r$, in the given problem (a) and (c) are the correct answers.

****

- Extra:
  - Note that if $both\ B\ \&\ I\ are\ doubled$, the tension will increase by 4 times (tension will be quadrupled)
  - The end result is the closed current loop will not experience any net force and hence the current loop remains stationary in the uniform magnetic field.
  - An easier way of looking into the net force being zero in a **CLOSED** current loop is as follows → Force on the conducting loop is given by $F = I(\vec{l}\,x\,\vec{B}), where\ l\ is\ the\ displacement\ vector$. In a closed loop, the initial and final points are same, therefore displacement vector $\vec{l} = 0\ and\ hence\ \vec{F} = 0$. **Therefore, the force acting on a closed current loop in a uniform magnetic field is always ZERO.** If the magnetic field is not uniform or the loop is open ended, there will be a net force on the current carrying wire.