Problems and Search

Chapter 2

Outline

- State space search

- Search strategies

- Problem characteristics

- Design of search programs

State Space Search

Problem solving = Searching for a goal state

State Space Search: Playing Chess

- Each position can be described by an 8-by-8 array.

- Initial position is the game opening position.

- Goal position is any position in which the opponent does not have a legal move and his or her king is under attack.

- Legal moves can be described by a set of rules:
  - Left sides are matched against the current state.
 - Right sides describe the new resulting state.

State Space Search: Playing Chess

- State space is a set of legal positions.

- Starting at the initial state.

- Using the set of rules to move from one state to another.

- Attempting to end up in a goal state.

State Space Search: Water Jug Problem

"You are given two jugs, a 4-litre one and a 3-litre one.Neither has any measuring markers on it. There is a pump that can be used to fill the jugs with water. How can you get exactly 2 litres of water into 4-litre jug."

3gl

4gl
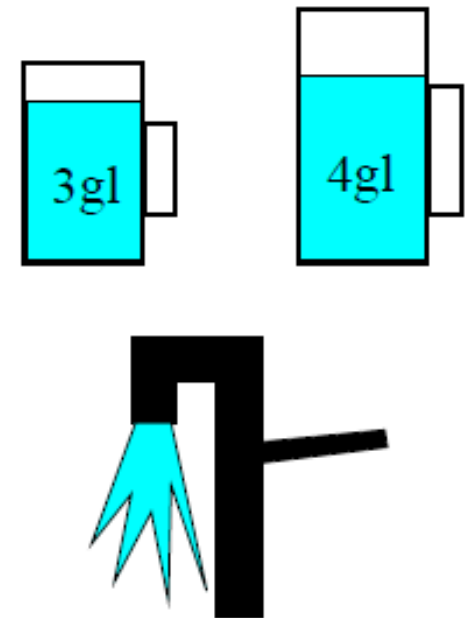
State Space Search: Water Jug Problem

- State: (x, y)

    x = 0, 1, 2, 3, or 4   y = 0, 1, 2, 3

- Start state: (0, 0).

- Goal state: (2, n) for any n.

- Attempting to end up in a goal state.

State Space Search: Water Jug Problem

1. $(x, y) \rightarrow (4, y)$
   if $x < 4$

2. $(x, y) \rightarrow (x, 3)$
   if $y < 3$

3. $(x, y) \rightarrow (x - d, y)$
   if $x > 0$

4. $(x, y) \rightarrow (x, y - d)$
   if $y > 0$

State Space Search: Water Jug Problem

5. $(x, y) \rightarrow (0, y)$
    if $x > 0$

6. $(x, y) \rightarrow (x, 0)$
    if $y > 0$

7. $(x, y) \rightarrow (4, y - (4 - x))$
    if $x + y \geq 4$, $y > 0$

8. $(x, y) \rightarrow (x - (3 - y), 3)$
    if $x + y \geq 3$, $x > 0$

State Space Search: Water Jug Problem

9. $(x, y) \rightarrow (x + y, 0)$
    if $x + y \leq 4$, $y > 0$

10. $(x, y) \rightarrow (0, x + y)$
    if $x + y \leq 3$, $x > 0$

11. $(0, 2) \rightarrow (2, 0)$

12. $(2, y) \rightarrow (0, y)$

State Space Search: Water Jug Problem

1. current state = (0, 0)

2. Loop until reaching the goal state (2, 0)
   – Apply a rule whose left side matches the current state
   – Set the new current state to be the resulting state

   (0, 0)
   (0, 3)
   (3, 0)
   (3, 3)
   (4, 2)
   (0, 2)
   (2, 0)

State Space Search: Water Jug Problem

The role of the condition in the left side of a rule
$\Rightarrow$ restrict the application of the rule
$\Rightarrow$ more efficient

1. $(x, y) \qquad \rightarrow (4, y)$
    if $x < 4$

2. $(x, y) \qquad \rightarrow (x, 3)$
    if $y < 3$

State Space Search: Water Jug Problem

Special-purpose rules to capture special-case knowledge that can be used at some stage in solving a problem

11. $(0, 2)$ $\rightarrow (2, 0)$

12. $(2, y)$ $\rightarrow (0, y)$

State Space Search: Summary

1. Define a state space that contains all the possible configurations of the relevant objects.

2. Specify the initial states.

3. Specify the goal states.

4. Specify a set of rules:
   - What are unstated assumptions?
   - How general should the rules be?
   - How much knowledge for solutions should be in the rules?

Searching

Photo by Steven E. Newton <snewton@oac.hsc.uth.tmc.edu>

Search Strategies

Requirements of a good search strategy:

1. It causes motion

    Otherwise, it will never lead to a solution.

2. It is systematic

    Otherwise, it may use more steps than necessary.

3. It is efficient

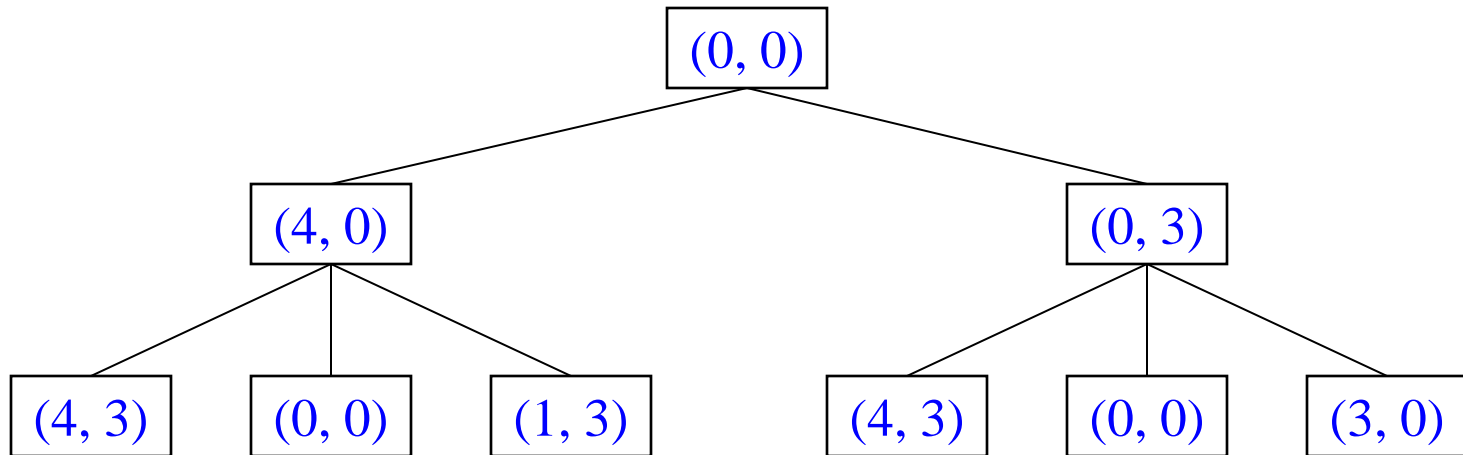   Find a good, but not necessarily the best, answer.

Search Strategies

1. **Uninformed search** (blind search)

   Having no information about the number of steps from the current state to the goal.

2. **Informed search** (heuristic search)
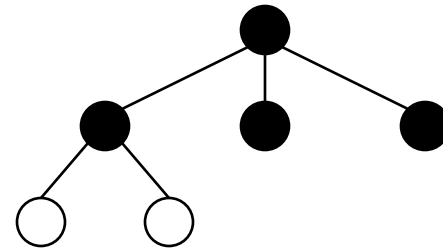
   More efficient than uninformed search.
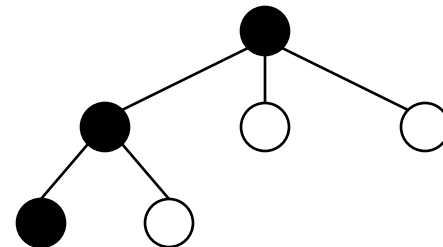
Search Strategies

Search Strategies: Blind Search

- ## Breadth-first search
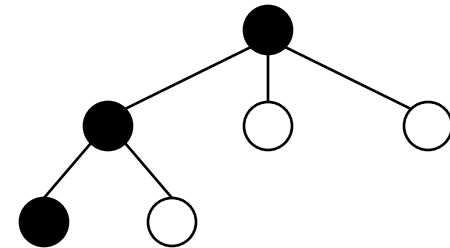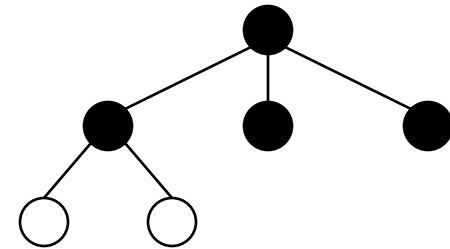 Expand all the nodes of one level first.

- ## Depth-first search
  Expand one of the nodes at the deepest level.

Search Strategies: Blind Search

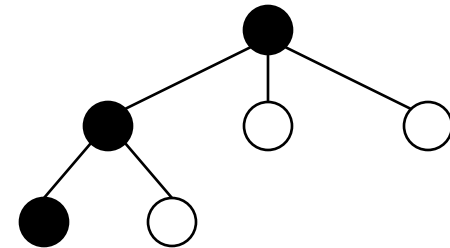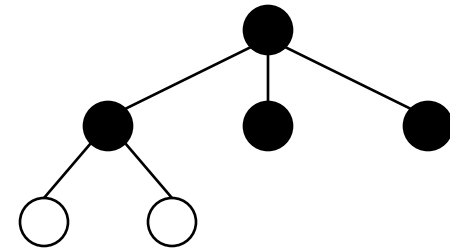| Criterion	Breadth-First	Depth-First
Time		
Space		
Optimal?		
Complete?		

b: branching factor   d: solution depth   m: maximum depth

Search Strategies: Blind Search

Criterion	Breadth-First	Depth-First
Time	b^d	b^m
Space	b^d	bm
Optimal?	Yes	No
Complete?	Yes	No

b: branching factor   d: solution depth   m: maximum depth

Search Strategies: Heuristic Search

- Heuristic: involving or serving as an aid to learning, discovery, or problem-solving by experimental and especially trial-and-error methods. (Merriam-Webster's dictionary)

- Heuristic technique improves the efficiency of a search process, possibly by sacrificing claims of completeness or optimality.
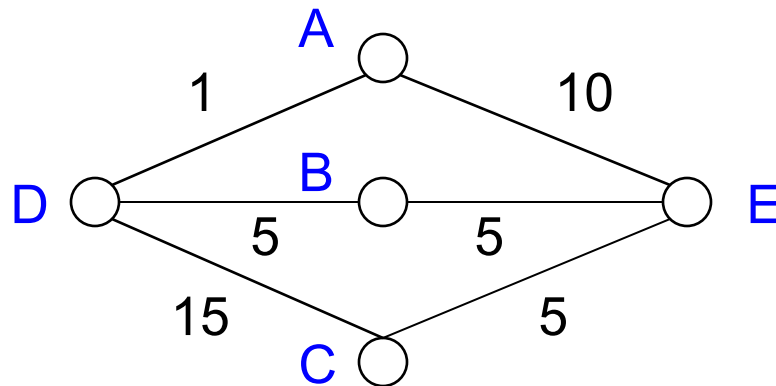
Search Strategies: Heuristic Search

- Heuristic is for combinatorial explosion.

- Optimal solutions are rarely needed.

Search Strategies: Heuristic Search

The Travelling Salesman Problem

"A salesman has a list of cities, each of which he must visit exactly once. There are direct roads between each pair of cities on the list. Find the route the salesman should follow for the shortest possible round trip that both starts and finishes at any one of the cities."

Search Strategies: Heuristic Search

Nearest neighbour heuristic:

1. Select a starting city.

2. Select the one closest to the current city.

3. Repeat step 2 until all cities have been visited.

Search Strategies: Heuristic Search

Nearest neighbour heuristic:

1. Select a starting city.

2. Select the one closest to the current city.

3. Repeat step 2 until all cities have been visited.

$$O(n^2) \text{ vs. } O(n!)$$

Search Strategies: Heuristic Search

- Heuristic function:

    state descriptions $\rightarrow$ measures of desirability
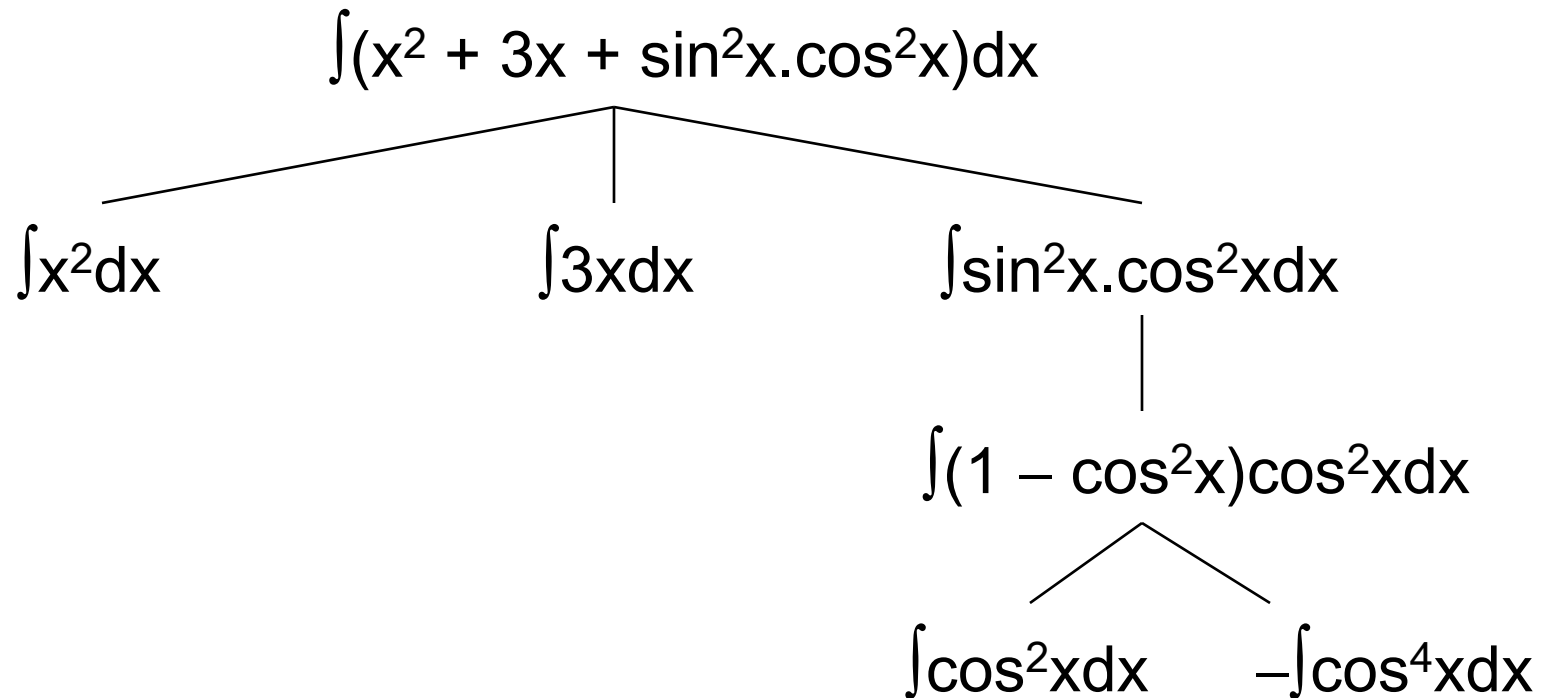
Problem Characteristics

To choose an appropriate method for a particular problem:

- Is the problem decomposable?
- Can solution steps be ignored or undone?
- Is the universe predictable?
- Is a good solution absolute or relative?
- Is the solution a state or a path?
- What is the role of knowledge?
- Does the task require human-interaction?

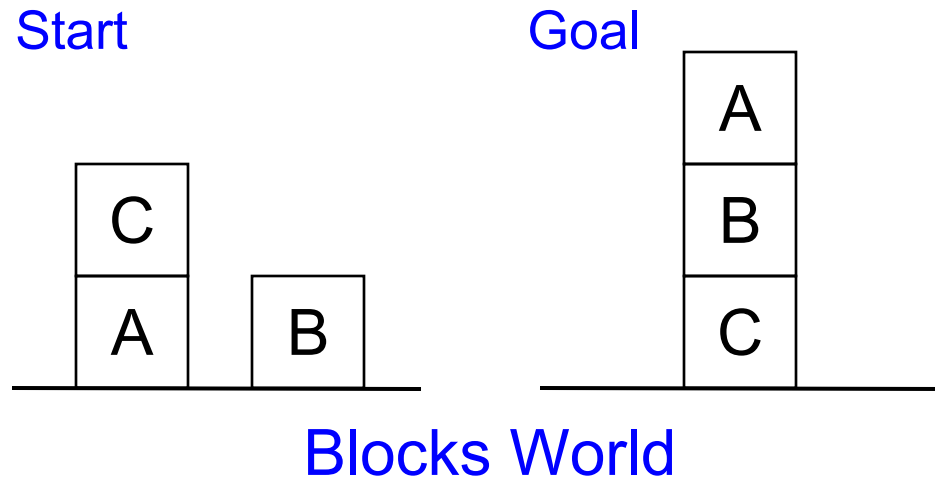# Is the problem decomposable?

- Can the problem be broken down to smaller problems to be solved independently?

- Decomposable problem can be solved easily.

Is the problem decomposable?

$$\int(x^2 + 3x + \sin^2x.\cos^2x)dx$$

$\int x^2dx$ $\int 3xdx$ $\int \sin^2x.\cos^2xdx$

$$\int(1 - \cos^2x)\cos^2xdx$$

$\int\cos^2xdx$ $-\int\cos^4xdx$

Is the problem decomposable?

Start   Goal

Blocks World

CLEAR(x) $\rightarrow$ ON(x, Table)

CLEAR(x) and CLEAR(y) $\rightarrow$ ON(x, y)

Is the problem decomposable?
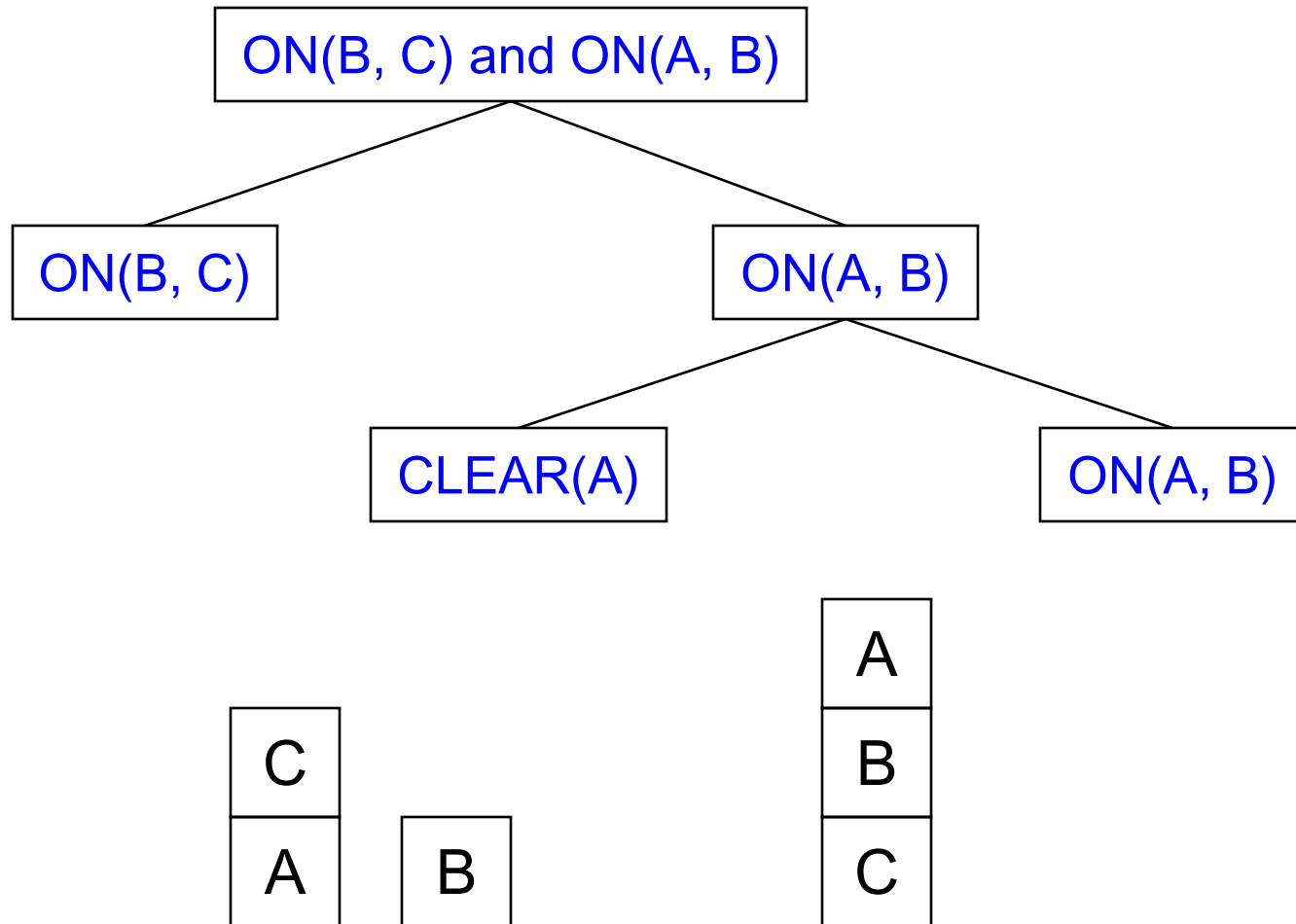
Can solution steps be ignored or undone?

Theorem Proving

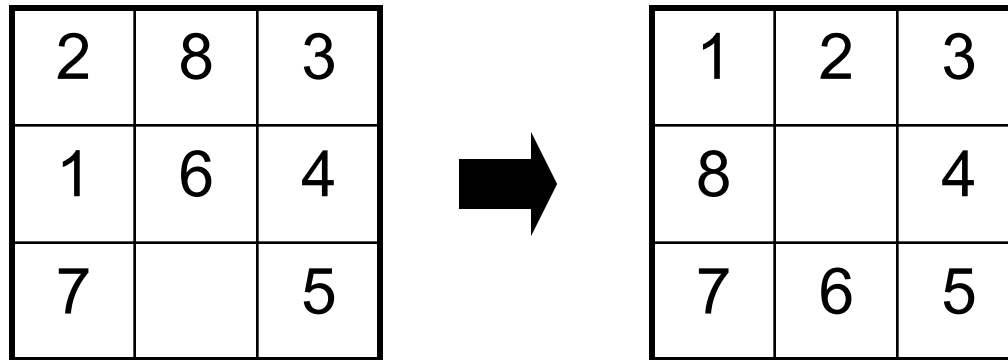A lemma that has been proved can be ignored for next steps.

Ignorable!

Can solution steps be ignored or undone?

The 8-Puzzle

2	8	3
1	6	4
7		5

➡

1	2	3
8		4
7	6	5

Moves can be undone and backtracked.

Recoverable!

Can solution steps be ignored or undone?

Playing Chess

Moves cannot be retracted.

Irrecoverable!

Can solution steps be ignored or undone?

- **Ignorable problems** can be solved using a simple control structure that never backtracks.

- **Recoverable problems** can be solved using backtracking.

- **Irrecoverable problems** can be solved by recoverable style methods via **planning**.

Is the universe predictable?

The 8-Puzzle

Every time we make a move, we know exactly what will happen.

Certain outcome!

Is the universe predictable?

Playing Bridge

We cannot know exactly where all the cards are or what the other players will do on their turns.

Uncertain outcome!

Is the universe predictable?

- For certain-outcome problems, planning can used to generate a sequence of operators that is guaranteed to lead to a solution.

- For uncertain-outcome problems, a sequence of generated operators can only have a good probability of leading to a solution.

  Plan revision is made as the plan is carried out and the necessary feedback is provided.

Is a good solution absolute or relative?

1. Marcus was a man.

2. Marcus was a Pompeian.

3. Marcus was born in 40 A.D.

4. All men are mortal.

5. All Pompeians died when the volcano erupted in 79 A.D.

6. No mortal lives longer than 150 years.

7. It is now 2004 A.D.

Is a good solution absolute or relative?

1. Marcus was a man.
2. Marcus was a Pompeian.
3. Marcus was born in 40 A.D.
4. All men are mortal.
5. All Pompeians died when the volcano erupted in 79 A.D.
6. No mortal lives longer than 150 years.
7. It is now 2004 A.D.

Is Marcus alive?

Is a good solution absolute or relative?

1. Marcus was a man.
2. Marcus was a Pompeian.
3. Marcus was born in 40 A.D.
4. All men are mortal.
5. All Pompeians died when the volcano erupted in 79 A.D.
6. No mortal lives longer than 150 years.
7. It is now 2004 A.D.

Is Marcus alive?

Different reasoning paths lead to the answer. It does not matter which path we follow.

Is a good solution absolute or relative?

The Travelling Salesman Problem

We have to try all paths to find the shortest one.

Is a good solution absolute or relative?

- Any-path problems can be solved using heuristics that suggest good paths to explore.

- For best-path problems, much more exhaustive search will be performed.

Is the solution a state or a path?

Finding a consistent intepretation

"The bank president ate a dish of pasta salad with the fork".

- – "bank" refers to a financial situation or to a side of a river?
- – "dish" or "pasta salad" was eaten?
- – Does "pasta salad" contain pasta, as "dog food" does not contain "dog"?
- – Which part of the sentence does "with the fork" modify? What if "with vegetables" is there?

No record of the processing is necessary.

Is the solution a state or a path?

The Water Jug Problem

The path that leads to the goal must be reported.

Is the solution a state or a path?

- A path-solution problem can be reformulated as a state-solution problem by describing a state as a partial path to a solution.

- The question is whether that is natural or not.

What is the role of knowledge

Playing Chess
Knowledge is important only to constrain the search for a solution.

Reading Newspaper
Knowledge is required even to be able to recognize a solution.

Does the task require human-interaction?

- Solitary problem, in which there is no intermediate communication and no demand for an explanation of the reasoning process.

- Conversational problem, in which intermediate communication is to provide either additional assistance to the computer or additional information to the user.

Problem Classification

- There is a variety of problem-solving methods, but there is no one single way of solving all problems.

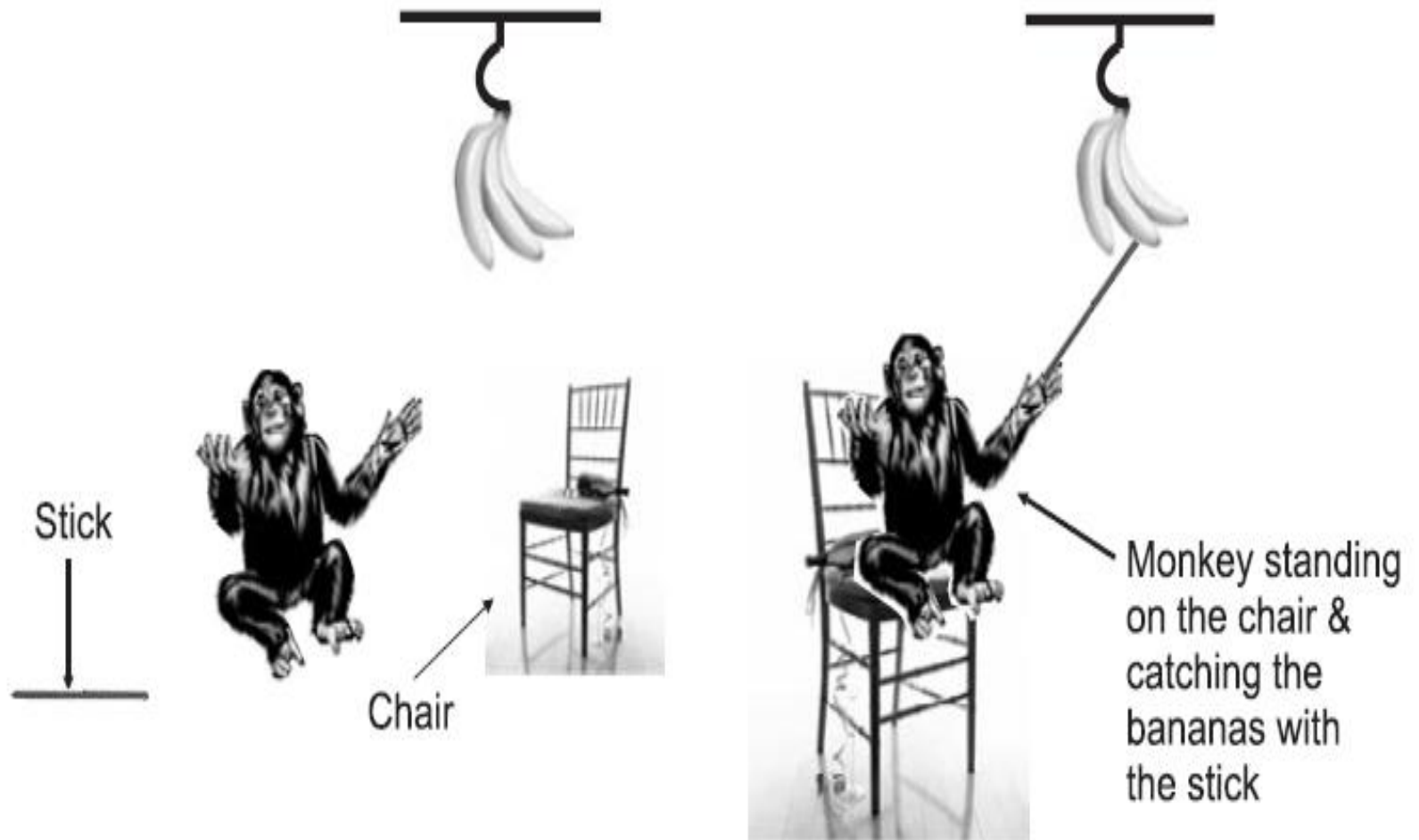- Not all new problems should be considered as totally new. Solutions of similar problems can be exploited.

Fig 2.12: Monkey and banana problem

Homework

1) 1-7 (Chapter 2 – AI Rich & Knight)

2) 5,3 litre jug (want 4 litre in 5 litre jug).

3) 11,4 litre jug (want exactly 1 litre of water in 4 litre jug)

4) 7,3 litre obtain exactly 5 litre in 7 litre jug