Department of Electronic and Telecommunication Engineering

University of Moratuwa

BM4152 - Biosignal Processing

Assignment 3 - Continuous and Discrete Wavelet Transforms

PUSHPAKUMARA H.M.R.M. 200488E

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1 Continuous Wavelet Transform

1.1 Introduction

$$W_x(s,\tau) = \int_{-\infty}^{\infty} x(t)\, \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{a}\right)\, dt \tag{1}$$

where s = *scaling factor*, τ = *translation* and ψ = *wavelet function*

1.2 Wavelet properties

i. Derive the Mexican hat function

Mexican hat function $m(t)$

$$m(t) = -\frac{d^2}{dt^2}\, g(t) \tag{2}$$

Gaussian Function $g(t)$

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \tag{3}$$

where $\mu = 0$ and $\sigma = 1$

So, we can simplify this as,

$$g(t) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{t^2}{2}} \tag{4}$$

The first derivative of the Gaussian function

$$\frac{d}{dt} g(t) = -\frac{t}{\sqrt{2\pi}}\, e^{-\frac{t^2}{2}} \tag{5}$$

Second derivation of the Gaussian function

$$\frac{d^2}{dt^2} g(t) = -\frac{1}{\sqrt{2\pi}}\, \left(e^{-\frac{t^2}{2}}\,(1-t^2)\right) \tag{6}$$

According to the equation 2, the Maxican hat function is,

$$m(t) = \frac{1}{\sqrt{2\pi}}\, \left(t^2-1\right) e^{-\frac{t^2}{2}} \tag{7}$$

ii. Calculate the normalizing factor of $m(t)$

The energy E is defined as:

$$E = \int_{-\infty}^{\infty} m^2(t)\, dt = 1 \tag{8}$$

Given that the Mexican hat function is:

$$m(t) = A\left(t^2 - 1\right)e^{-\frac{t^2}{2}} \tag{9}$$

We need to solve the following integral to determine the value of A:

$$\int_{-\infty}^{\infty} A^2\left(t^2 - 1\right)^2 e^{-t^2}\,dt = 1 \tag{10}$$

To solve the integral

$$I = \int_{-\infty}^{\infty} A^2\left(t^2 - 1\right)^2 e^{-t^2}\,dt = 1,$$

we first expand the integrand:

$$(t^2 - 1)^2 = t^4 - 2t^2 + 1.$$

Thus, we can rewrite I as:

$$I = A^2 \int_{-\infty}^{\infty} (t^4 - 2t^2 + 1)e^{-t^2}\,dt.$$

Next, we separate the integral:

$$I = A^2\left(\int_{-\infty}^{\infty} t^4 e^{-t^2}\,dt - 2\int_{-\infty}^{\infty} t^2 e^{-t^2}\,dt + \int_{-\infty}^{\infty} e^{-t^2}\,dt\right).$$

Now, we calculate each integral:

1. Gaussian Integral:

$$\int_{-\infty}^{\infty} e^{-t^2}\,dt = \sqrt{\pi}.$$

2. Integral of $t^2 e^{-t^2}$:

$$\int_{-\infty}^{\infty} t^2 e^{-t^2}\,dt = \frac{1}{2}\sqrt{\pi}.$$

3. Integral of $t^4 e^{-t^2}$:

$$\int_{-\infty}^{\infty} t^4 e^{-t^2}\,dt = \frac{3}{4}\sqrt{\pi}.$$

Substituting these results back into the expression for I:

$$I = A^2\left(\frac{3}{4}\sqrt{\pi} - 2\cdot\frac{1}{2}\sqrt{\pi} + \sqrt{\pi}\right) = A^2\left(\frac{3}{4}\sqrt{\pi} - \sqrt{\pi} + \sqrt{\pi}\right) = A^2\left(\frac{3}{4}\sqrt{\pi}\right).$$

Setting $I = 1$:

$$A^2 \cdot \frac{3}{4}\sqrt{\pi} = 1 \Rightarrow A^2 = \frac{4}{3\sqrt{\pi}}.$$

We know:

$$A = \frac{Normalizing\_factor}{\sqrt{2\pi}}.$$

Therefore:

$$Normalizing\_factor = A \cdot \sqrt{2\pi}$$

$$Normalizing\_factor = 2 \cdot \sqrt{\frac{2\sqrt{\pi}}{3}}$$

iii. Normalized Mexican hat mother wavelet $\psi(t)$

$$\psi(t) = 2 \cdot \sqrt{\frac{2\sqrt{\pi}}{3}} \cdot \frac{1}{2\pi}(1 - t^2) \cdot e^{-\frac{t^2}{2}}$$

$$\psi(t) = \frac{2}{\sqrt{3\sqrt{\pi}}} \cdot (1 - t^2) \cdot e^{-\frac{t^2}{2}} \tag{11}$$

by including the scaling factor s:

$$\psi_s(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t}{s}\right)$$

$$\psi_s(t) = \frac{2}{\sqrt{3\sqrt{\pi}}} \cdot \frac{1}{\sqrt{s}} \cdot \left(\left(\frac{t}{s}\right)^2 - 1\right) \cdot e^{-\frac{1}{2}\left(\frac{t}{s}\right)^2} \tag{12}$$

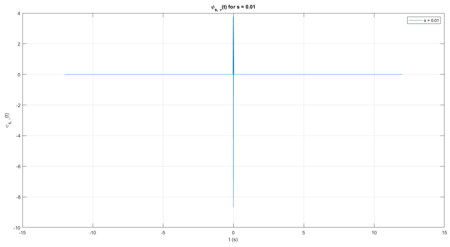
This is the normalized and scaled Mexican hat mother wavelet function.

Further, we can add the translation term as well.
by including translating factor (τ):

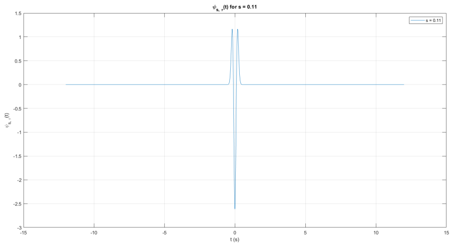
$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - \tau}{s}\right)$$

$$\psi_{s,\tau}(t) = \frac{2}{\sqrt{3\sqrt{\pi}}} \cdot \frac{1}{\sqrt{s}} \cdot \left(\left(\frac{t - \tau}{s}\right)^2 - 1\right) \cdot e^{-\frac{1}{2}\left(\frac{t-\tau}{s}\right)^2} \tag{13}$$

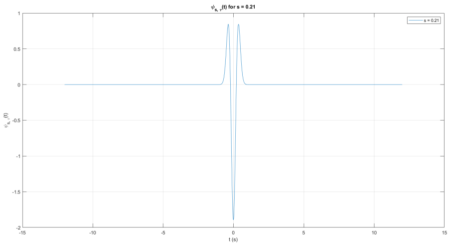
iv. Generate the Mexican hat daughter wavelet for scaling factors of 0.01:0.1:2



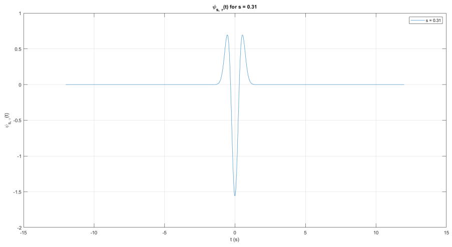
(a) s = 0.01



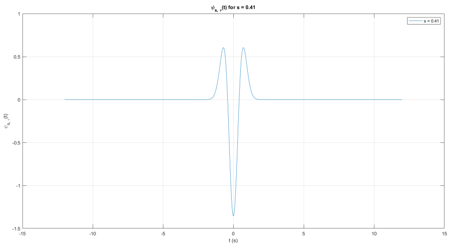
(b) s = 0.11



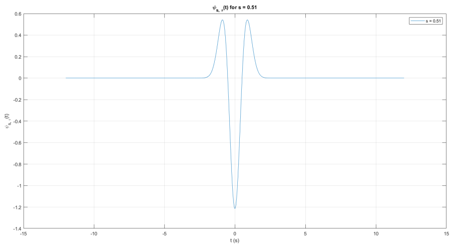
(c) s = 0.21



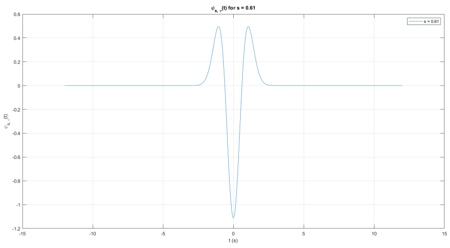
(d) s = 0.31



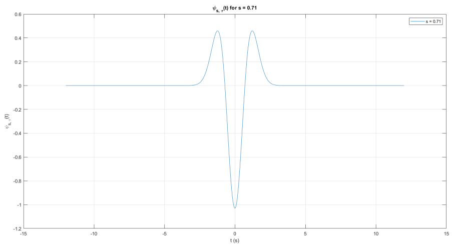
(e) s = 0.41



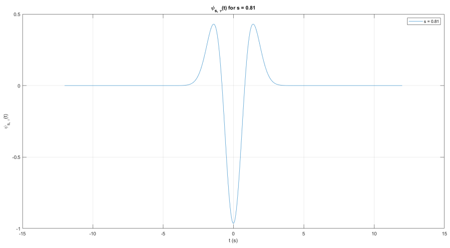
(f) s = 0.51



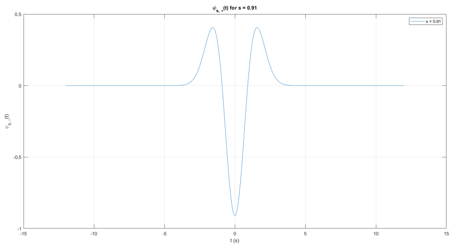
(g) s = 0.61



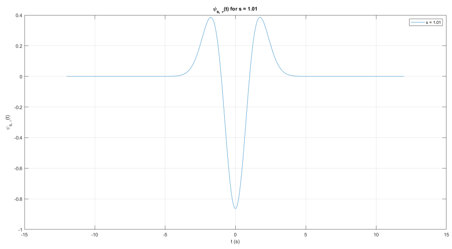
(h) s = 0.71



(i) s = 0.81



(j) s = 0.91



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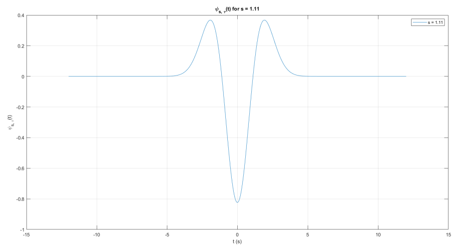

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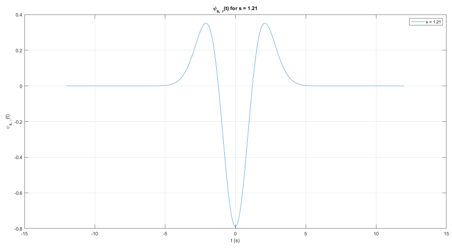

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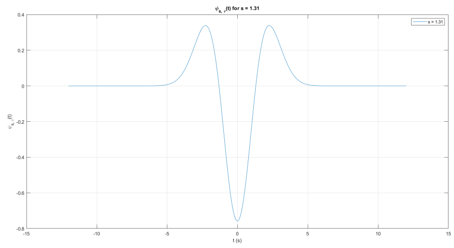

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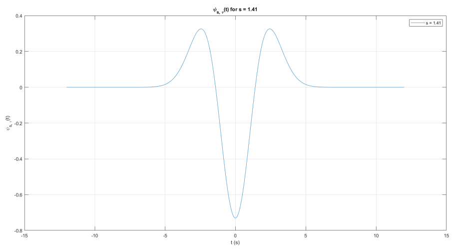
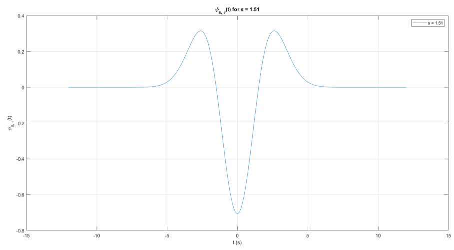


(e) s = 1.41



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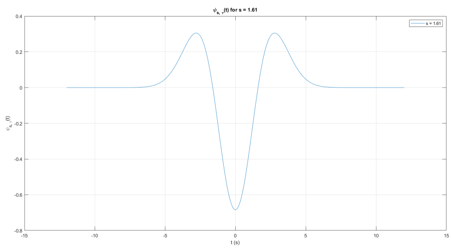
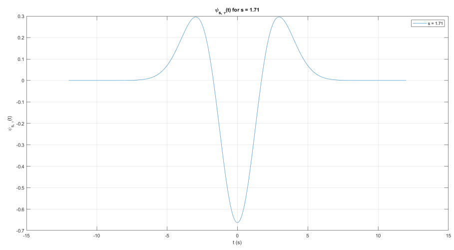
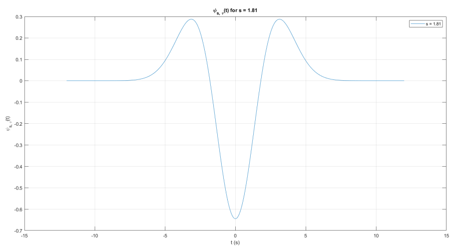


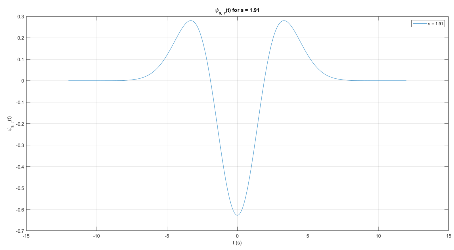
(g) s = 1.61



(h) s = 1.71



(i) s = 1.81



(j) s = 1.91

Figure 1: Generated Mexican hat daughter wavelets.

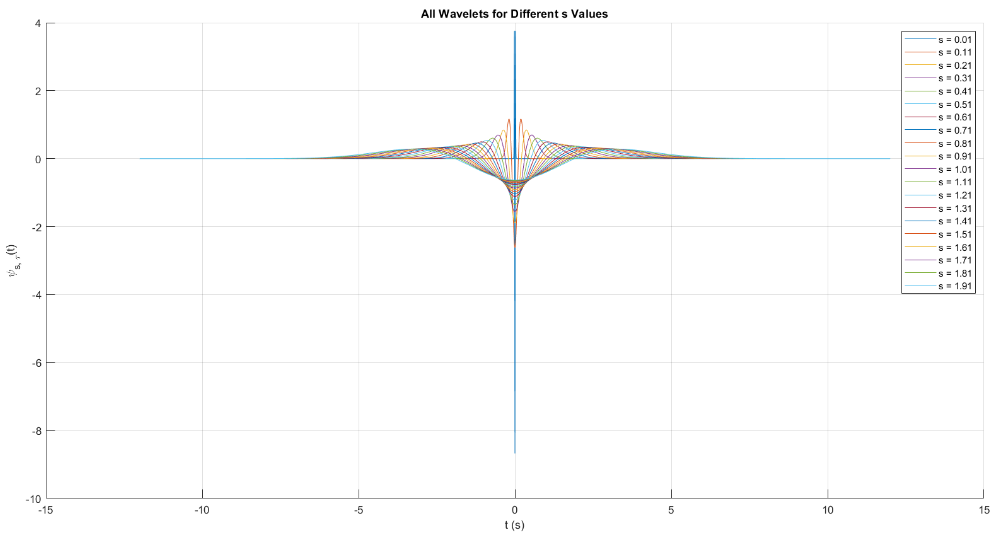


Figure 2: All Mexican hat wavelets in a single plot for comparison.

v. Verify the wavelet properties of zero mean, unity energy, and compact support.

- All the daughter wavelets exhibit symmetry, which implies that each wavelet has a mean of zero.
- Additionally, each daughter wavelet displays a high magnitude within a very short interval, indicating compact support.
- Since all magnitudes are below 1, the unity gain property can be reasonably assumed.

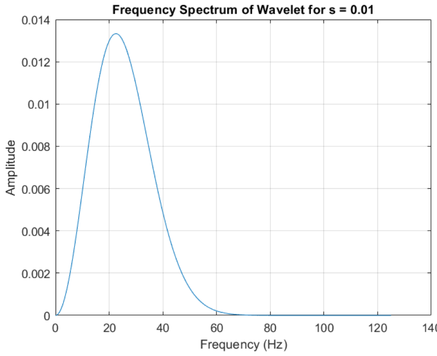
I have calculated each of these properties to validate these observations, as presented below.

Command Window

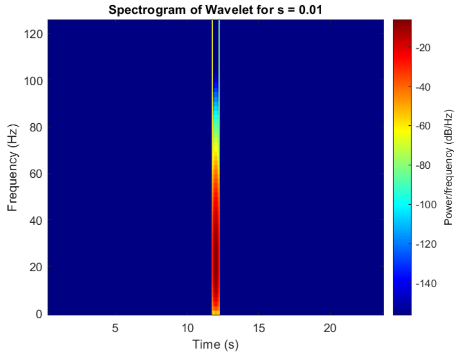
```
Wavelet s = 0.01: Mean = 0.0000, Energy = 1.0000, Compact Support = 0.0520 s (99.78%)
Wavelet s = 0.11: Mean = 0.0000, Energy = 1.0000, Compact Support = 0.5720 s (97.62%)
Wavelet s = 0.21: Mean = 0.0000, Energy = 1.0000, Compact Support = 1.0960 s (95.43%)
Wavelet s = 0.31: Mean = 0.0000, Energy = 1.0000, Compact Support = 1.6160 s (93.27%)
Wavelet s = 0.41: Mean = 0.0000, Energy = 1.0000, Compact Support = 2.1320 s (91.12%)
Wavelet s = 0.51: Mean = 0.0000, Energy = 1.0000, Compact Support = 2.6520 s (88.95%)
Wavelet s = 0.61: Mean = 0.0000, Energy = 1.0000, Compact Support = 3.1760 s (86.77%)
Wavelet s = 0.71: Mean = 0.0000, Energy = 1.0000, Compact Support = 3.6960 s (84.60%)
Wavelet s = 0.81: Mean = 0.0000, Energy = 1.0000, Compact Support = 4.2160 s (82.43%)
Wavelet s = 0.91: Mean = 0.0000, Energy = 1.0000, Compact Support = 4.7360 s (80.27%)
Wavelet s = 1.01: Mean = 0.0000, Energy = 1.0000, Compact Support = 5.2560 s (78.10%)
Wavelet s = 1.11: Mean = 0.0000, Energy = 1.0000, Compact Support = 5.7800 s (75.92%)
Wavelet s = 1.21: Mean = 0.0000, Energy = 1.0000, Compact Support = 6.3000 s (73.75%)
Wavelet s = 1.31: Mean = 0.0000, Energy = 1.0000, Compact Support = 6.8200 s (71.58%)
Wavelet s = 1.41: Mean = 0.0000, Energy = 1.0000, Compact Support = 7.3400 s (69.42%)
Wavelet s = 1.51: Mean = 0.0000, Energy = 1.0000, Compact Support = 7.8600 s (67.25%)
Wavelet s = 1.61: Mean = 0.0000, Energy = 1.0000, Compact Support = 8.3800 s (65.08%)
Wavelet s = 1.71: Mean = 0.0000, Energy = 1.0000, Compact Support = 8.9000 s (62.92%)
Wavelet s = 1.81: Mean = 0.0000, Energy = 1.0000, Compact Support = 9.4200 s (60.75%)
Wavelet s = 1.91: Mean = 0.0000, Energy = 1.0000, Compact Support = 9.9440 s (58.57%)
```

Figure 3: Mean, energy, and the compression percentage of each wavelet.

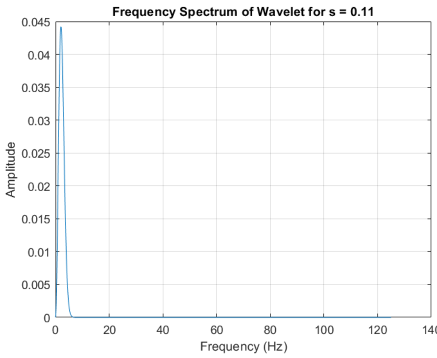
vi. Plot and comment on the spectra of daughter wavelets



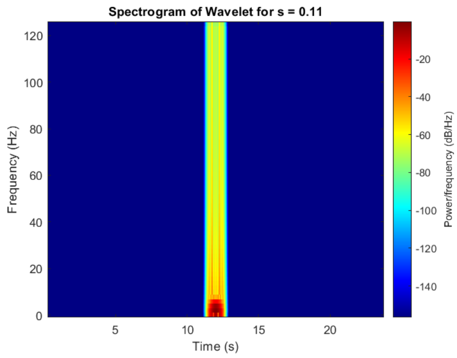
(a) s = 0.01 (spectrum)



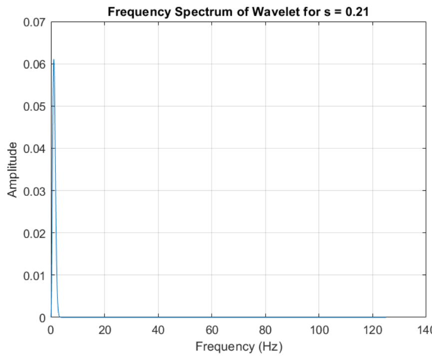
(b) s = 0.01 (spectrogram)



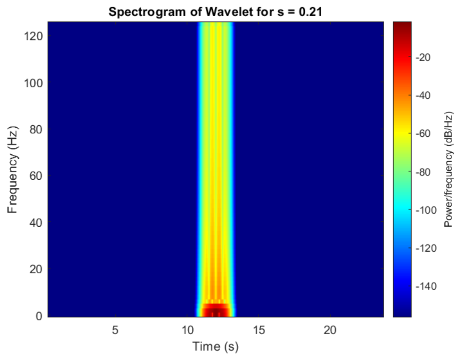
(c) s = 0.11 (spectrum)



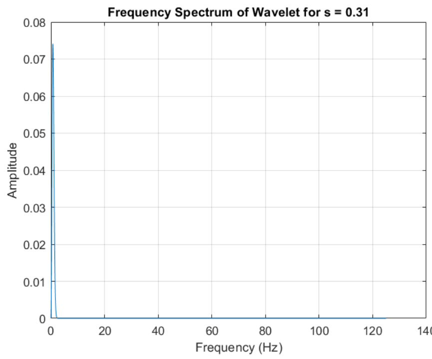
(d) s = 0.11 (spectrogram)



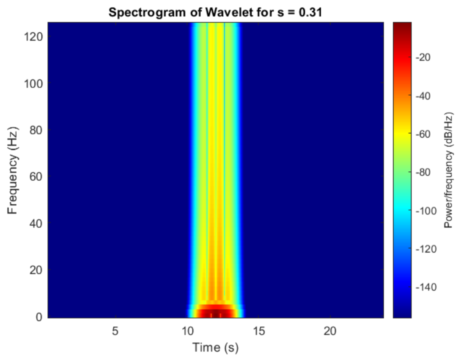
(e) s = 0.21 (spectrum)



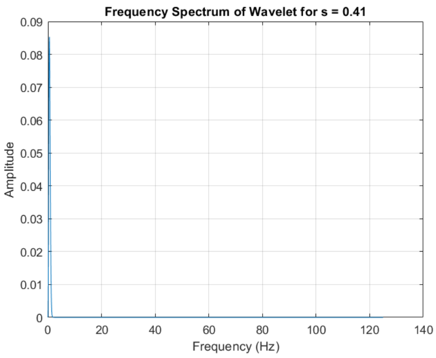
(f) s = 0.21 (spectrogram)



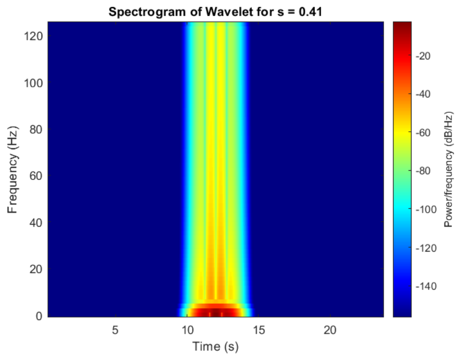
(g) s = 0.31 (spectrum)



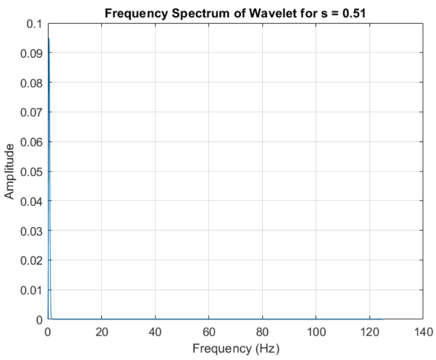
(h) s = 0.31 (spectrogram)



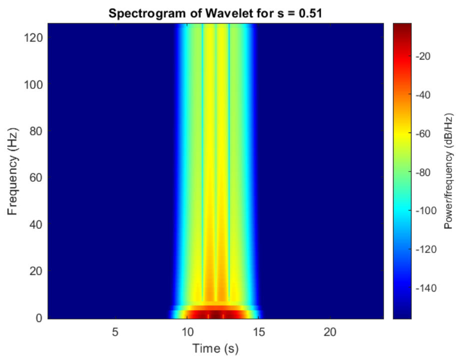
(a) s = 0.41 (spectrum)



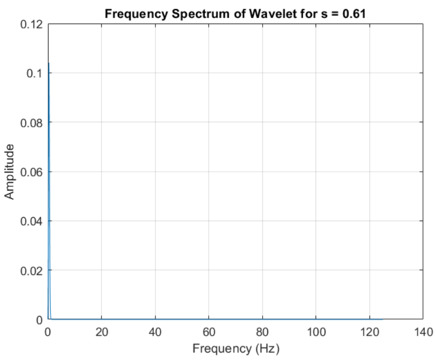
(b) s = 0.41 (spectrogram)



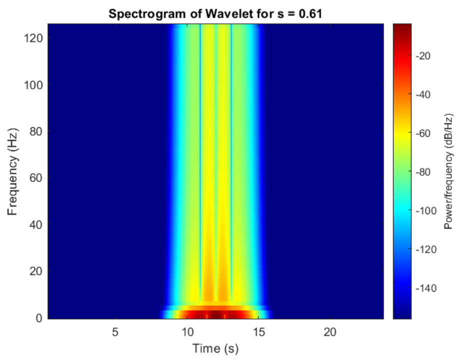
(c) s = 0.51 (spectrum)



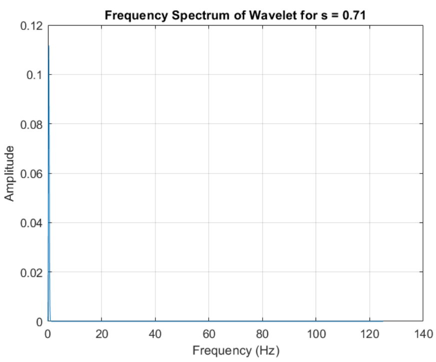
(d) s = 0.51 (spectrogram)



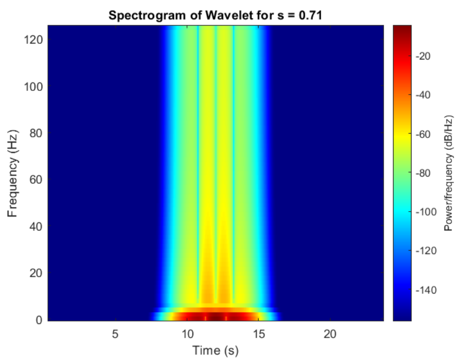
(e) s = 0.61 (spectrum)



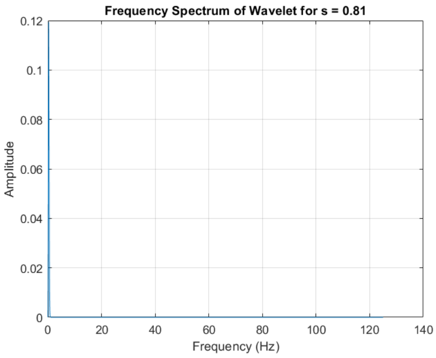
(f) s = 0.61 (spectrogram)



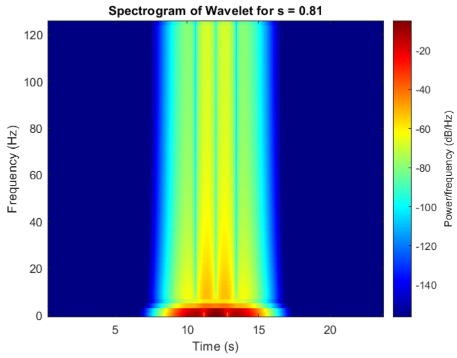
(g) s = 0.71 (spectrum)



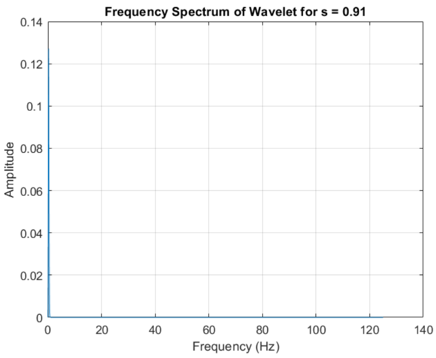
(h) s = 0.71 (spectrogram)



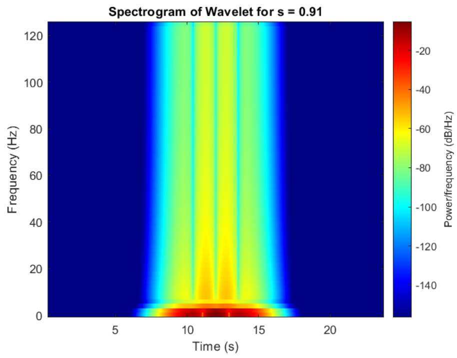
(a) s = 0.81 (spectrum)



(b) s = 0.81 (spectrogram)



(c) s = 0.91 (spectrum)



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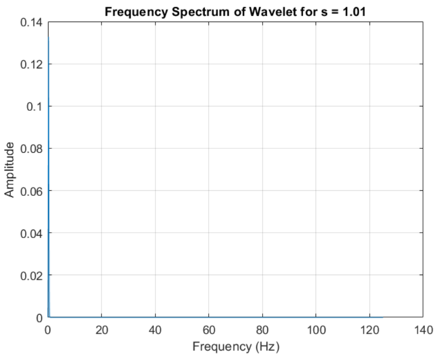

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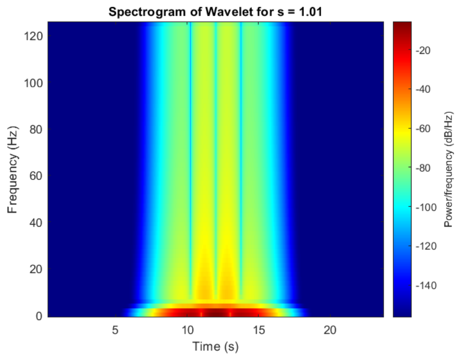

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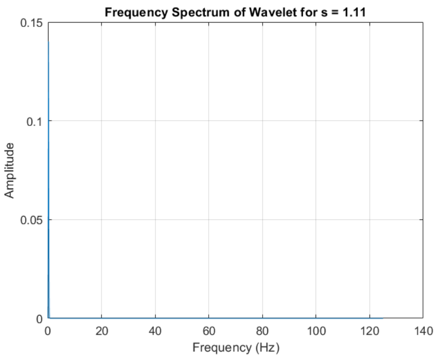
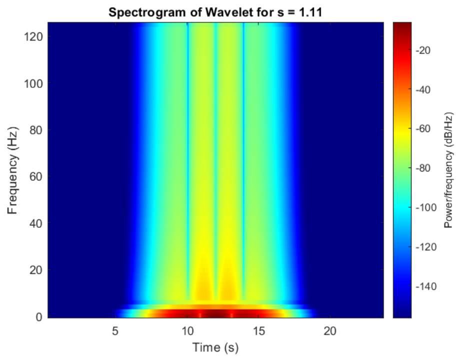
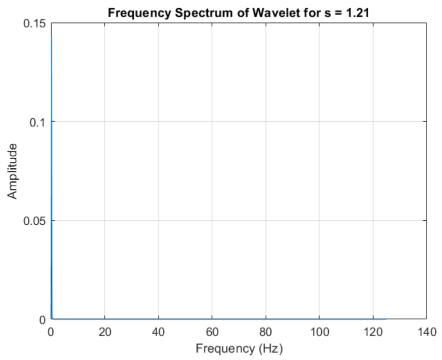


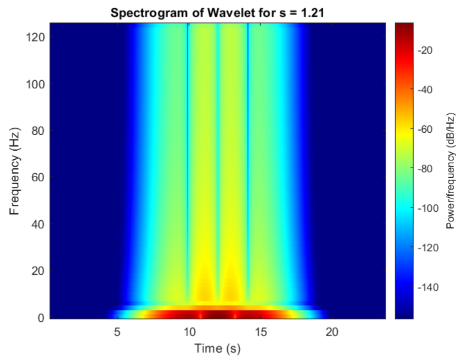
(g) s = 1.11 (spectrum)



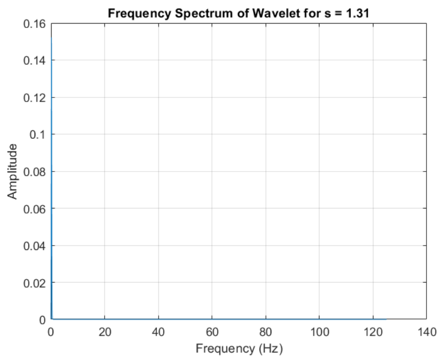
(h) s = 1.11 (spectrogram)



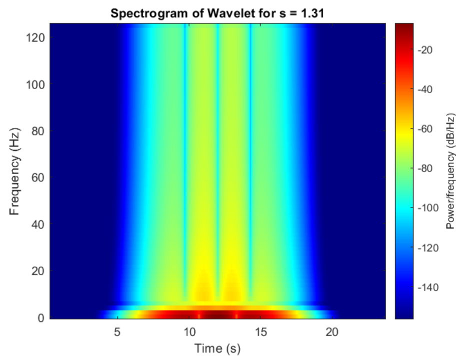
(a) s = 1.21 (spectrum)



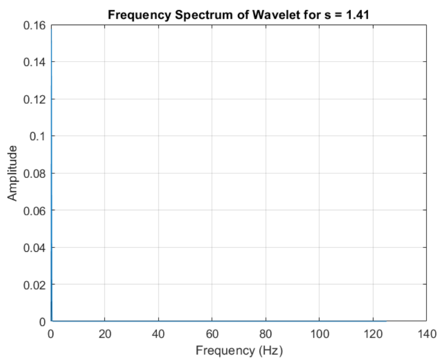
(b) s = 1.21 (spectrogram)



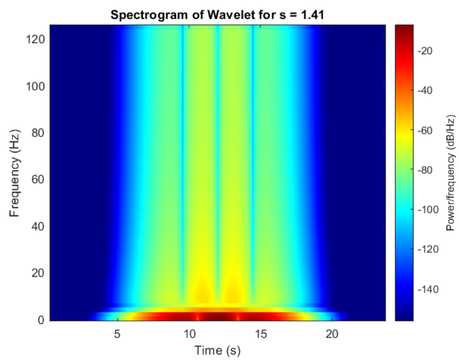
(c) s = 1.31 (spectrum)



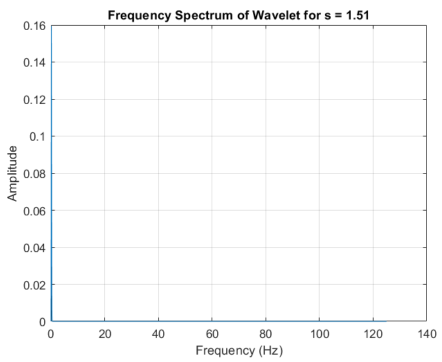
(d) s = 1.31 (spectrogram)



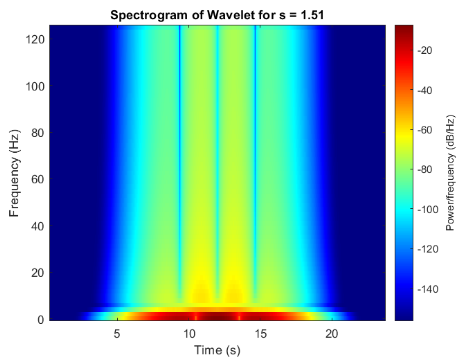
(e) s = 1.41 (spectrum)



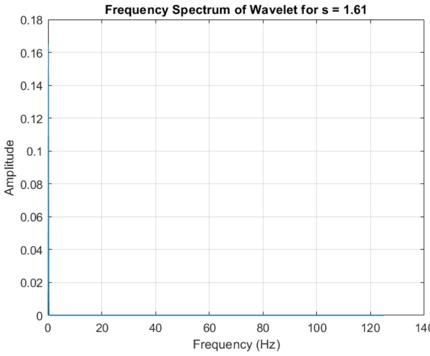
(f) s = 1.41 (spectrogram)



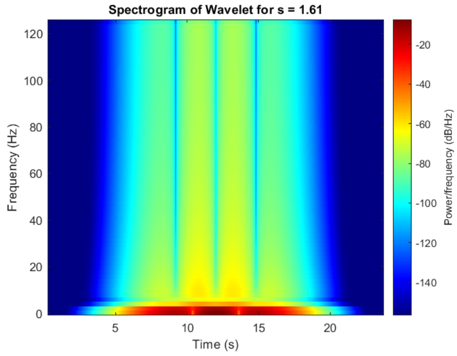
(g) s = 1.51 (spectrum)



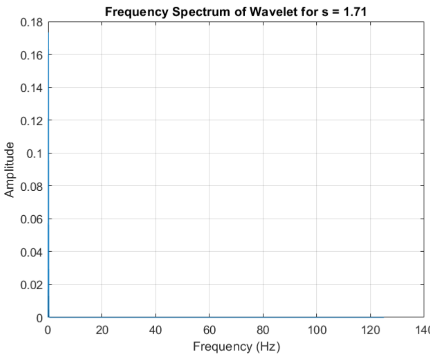
(h) s = 1.51 (spectrogram)



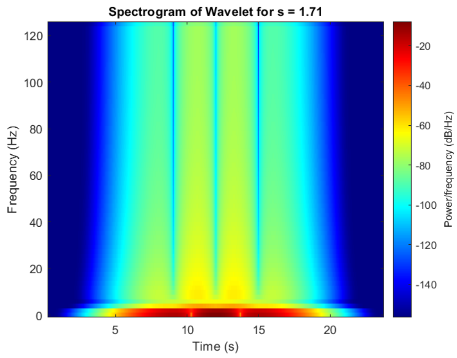
(a) s = 1.61 (spectrum)



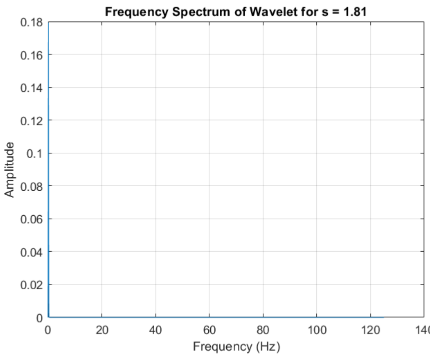
(b) s = 1.61 (spectrogram)



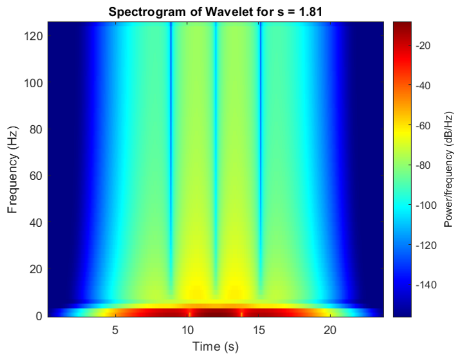
(c) s = 1.71 (spectrum)



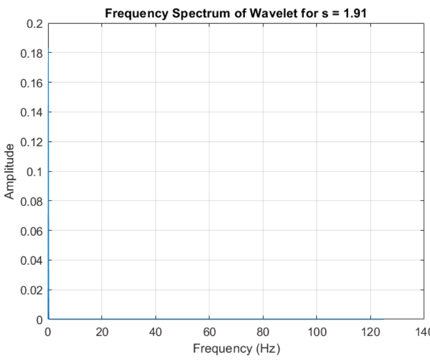
(d) s = 1.71 (spectrogram)



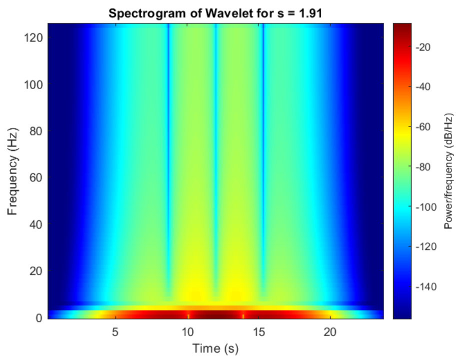
(e) s = 1.81 (spectrum)



(f) s = 1.81 (spectrogram)



(g) s = 1.91 (spectrum)



(h) s = 1.91 (spectrogram)

Figure 8: Spectrum and spectrogram of the generated Mexican hat daughter wavelets.

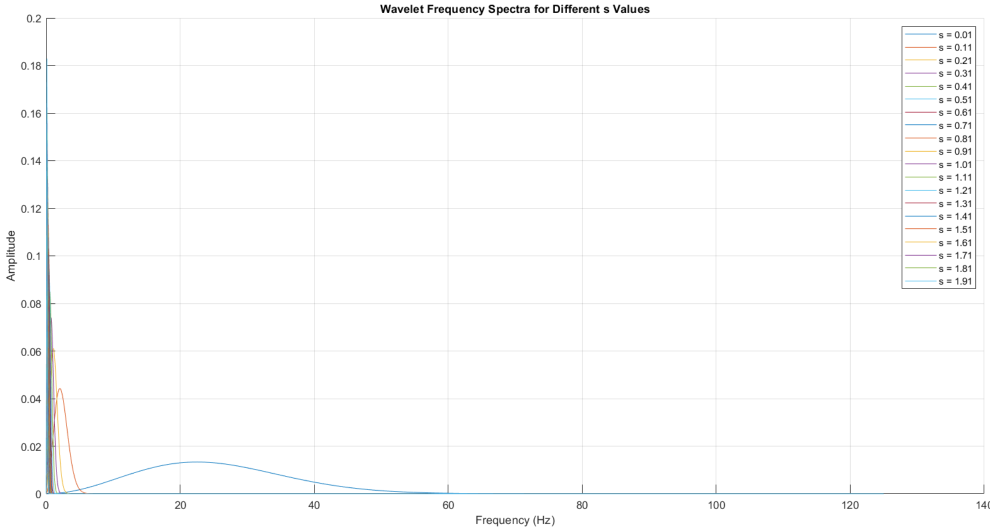


Figure 9: All wavelet spectrums in a single plot for comparison

As the scale increases, the wavelet expands in the time domain, while its spectral profile becomes more concentrated in the frequency domain, emphasizing lower frequencies.

This behavior demonstrates the trade-off between time and frequency resolution, where broader wavelets are more effective in capturing low-frequency components.

1.3 Continuous Wavelet Decomposition

i. Create the waveform on MATLAB

$$x[n] = \begin{cases} \sin(0.5\pi \frac{n}{fs}), & 1 \leq n < \frac{3N}{2} \\ \sin(1.5\pi \frac{n}{fs}), & \frac{3N}{2} \leq n < 3N \end{cases} \tag{14}$$

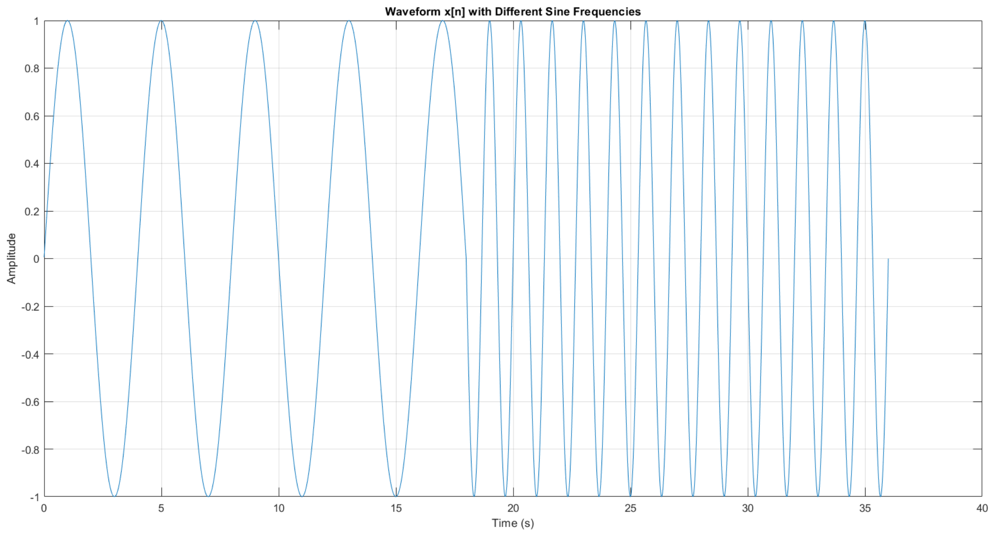
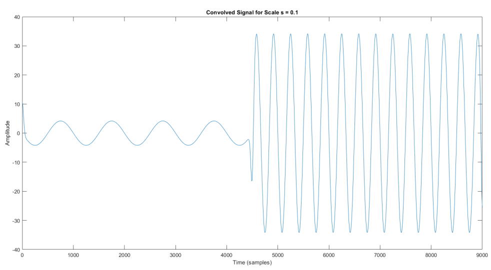
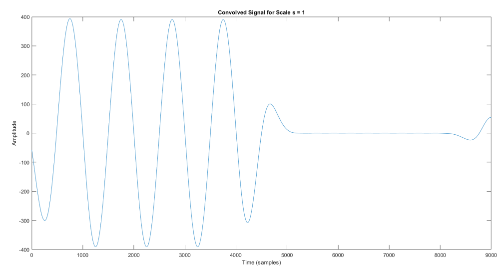


Figure 10: The created waveform

ii. Apply the scaled Mexican hat wavelets to $x(n)$



(a) s = 0.1



(b) s = 1.0

Figure 11: Example convolved signals for two different scales

iii. Visualize the spectrogram

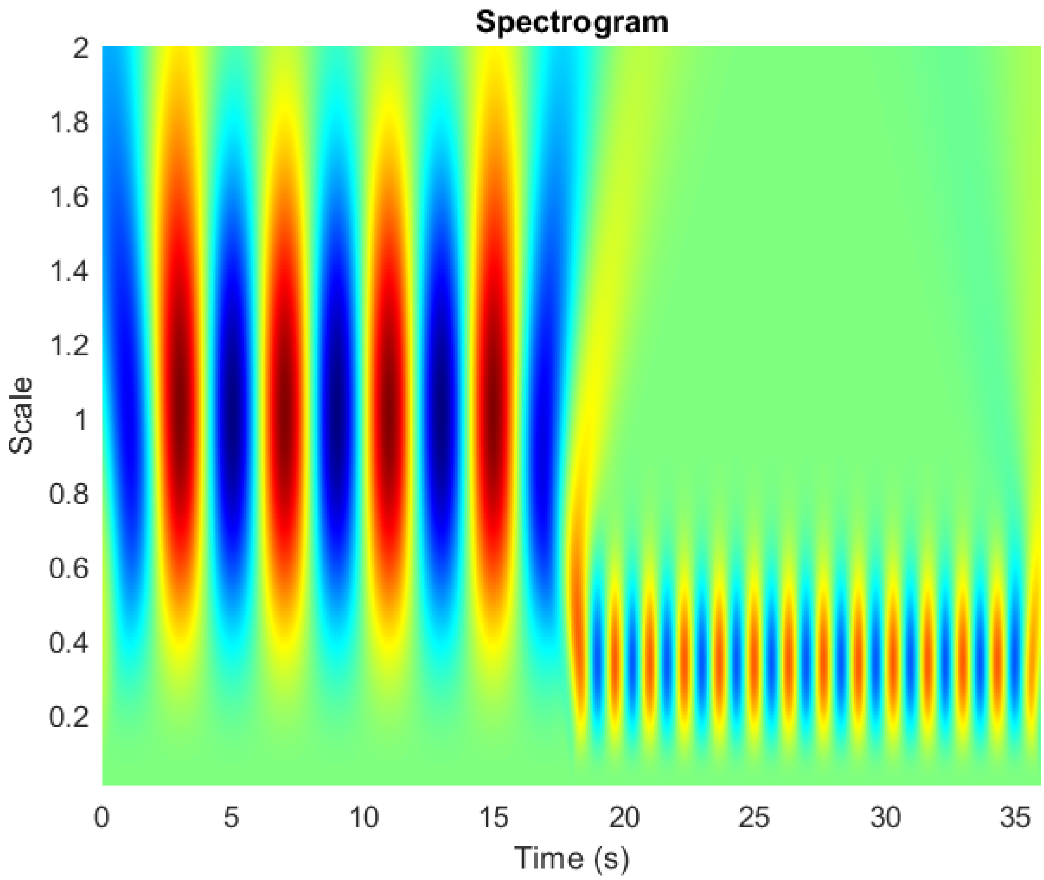


Figure 12: Spectrogram from the CWT for $x(n)$

However, this spectrogram is not exactly similar to the spectrogram provided in the assignment (figure 13).

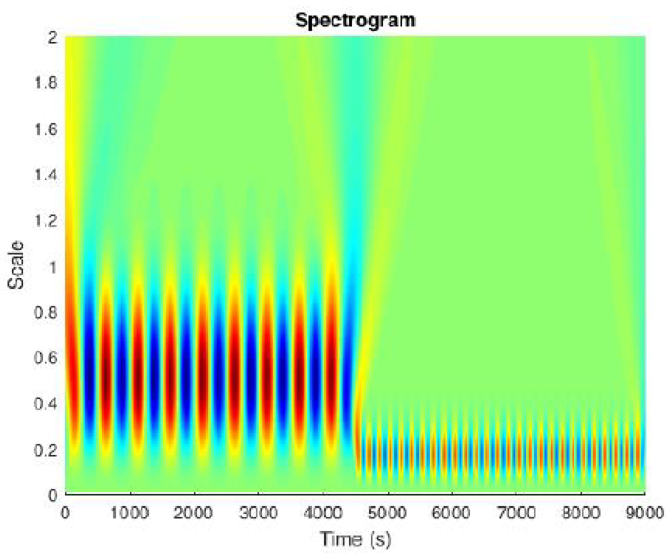


Figure 13: Spectrogram provided in the assignment

I observed that we can obtain this spectrogram by doubling the frequencies in $x(n)$. Then, the updated $x(n)$ as follows:

$$x[n] = \begin{cases} \sin(\pi \frac{n}{fs}), & 1 \leq n < \frac{3N}{2} \\ \sin(3\pi \frac{n}{fs}), & \frac{3N}{2} \leq n < 3N \end{cases} \tag{15}$$

After applying the scaled Mexican hat wavelets to this frequency-doubled signal, we can obtain the following spectrogram, exactly similar to the one provided in the assignment.

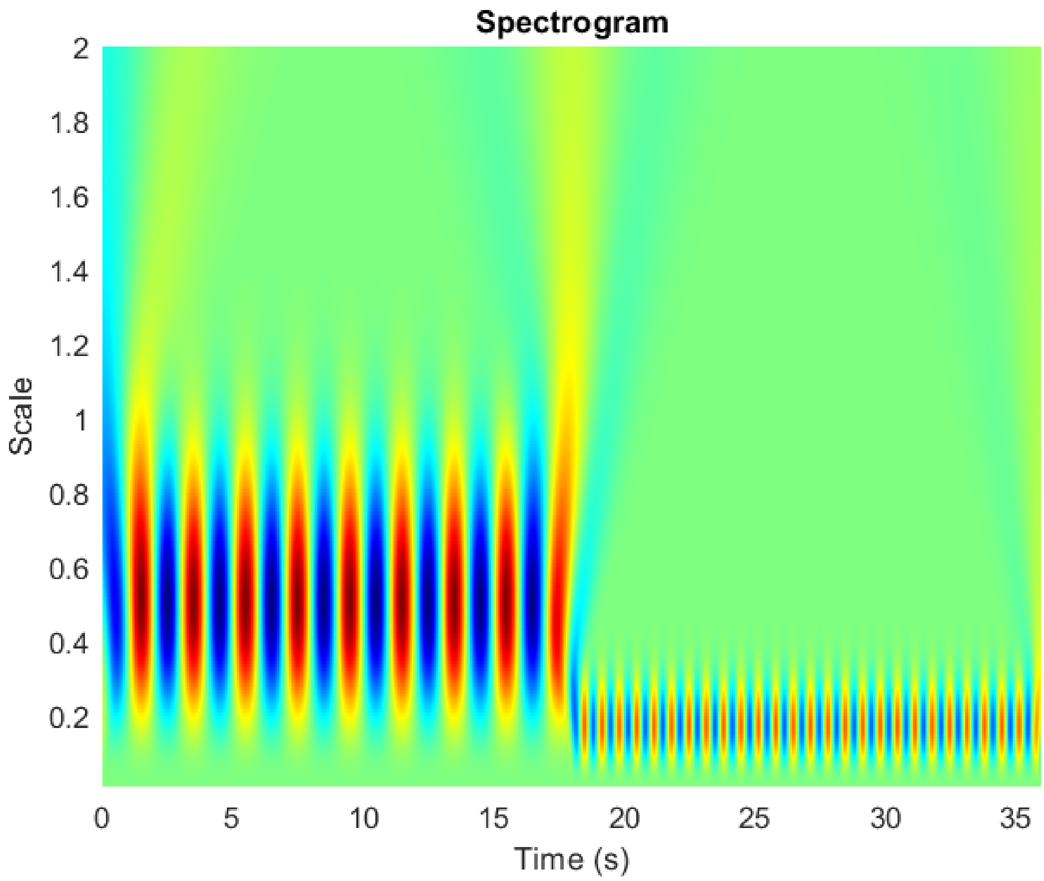
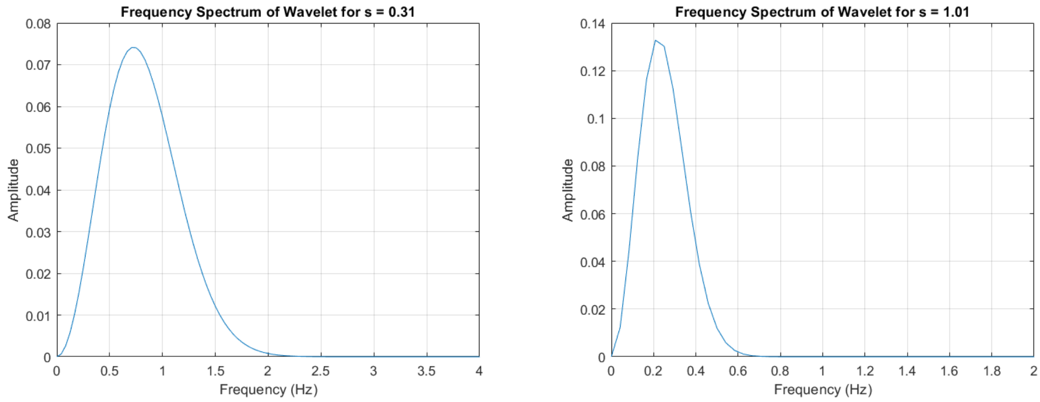


Figure 14: Spectrogram for frequencies-doubled signal

iv. Comment on the plot and how the continuous wavelet coefficients represent the frequency content of $x(n)$

Here, when we observe the spectrogram in figure 12, we can see that the signal contains 2 different frequency components around $s = 1$ and $s = 0.3$. The frequency component related to $s = 1$ can be observed about $17s$ and after that, the frequency component related to $s = 0.3$ has appeared.

Since we have scale values $s = 1.01$ and $s = 0.31$, we can have more details about these frequency components by observing them.



(a) s = 0.31 (b) s = 1.01

Figure 15: Zoomed spectrum for $s = 0.31$ and $s = 1.01$

By observing these spectra, we can find the corresponding two frequencies. Here, $s = 0.31$ gives a frequency around $0.75Hz$, and $s = 1.01$ gives a frequency around $0.25Hz$ which closely aligns with the frequencies of the generated signal.

2 Discrete Wavelet Transform

2.1 Introduction

$$\psi_{m,n}(t) = \frac{1}{\sqrt{s_0^m}}\psi\left(\frac{t - n\tau_0 s_0^m}{s_0^m}\right)$$

where s_0 = scaling step size, τ_0 = translation step size

2.2 Applying DWT with the Wavelet Toolbox in MATLAB

i. Create the following waveforms in MATLAB

$$x_1[n] = \begin{cases} 2\sin(20\pi n) + \sin(80\pi n), & 0 \leq n < 512, \\ 0.5\sin(40\pi n) + \sin(60\pi n), & 512 \leq n < 1024. \end{cases} \tag{16}$$

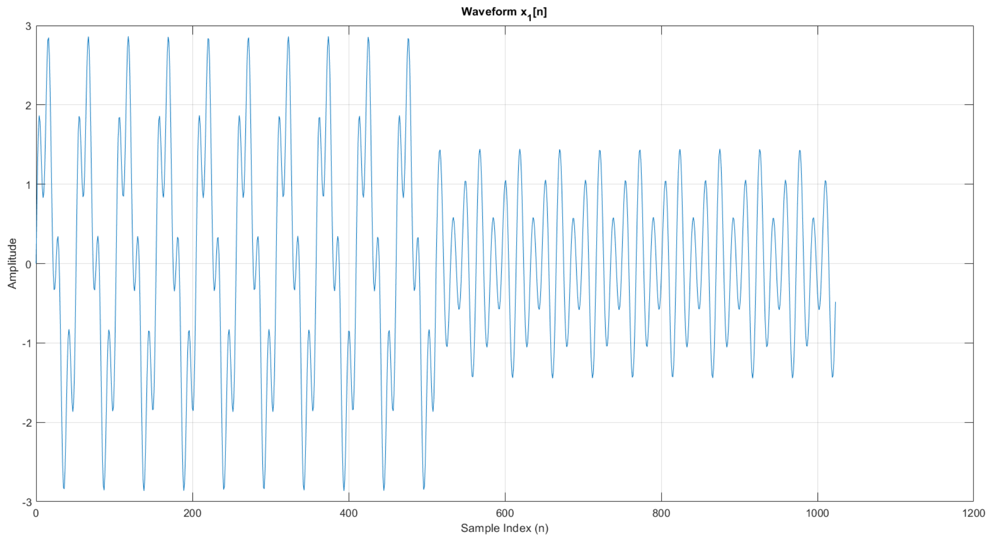


Figure 16: Created $x_1[n]$

$$x_2[n] = \begin{cases} 1, & 0 \leq n < 64, \\ 2, & 64 \leq n < 128, \\ -1, & 128 \leq n < 512, \\ 3, & 512 \leq n < 704, \\ 1, & 704 \leq n < 960, \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

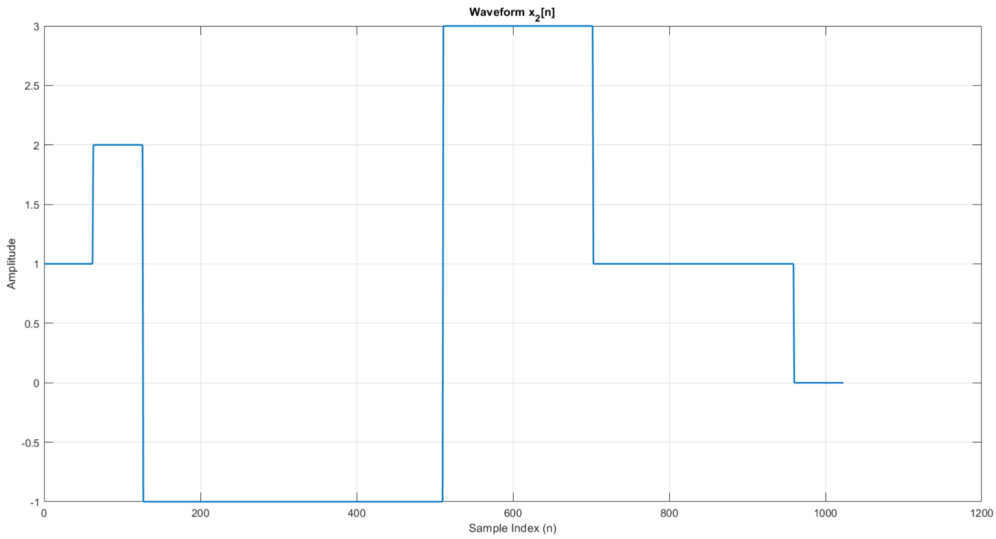


Figure 17: Created $x_2[n]$

Now, corrupt these signals with Additive White Gaussian Noise (AWGN) of 10dB signal-to-noise ratio.

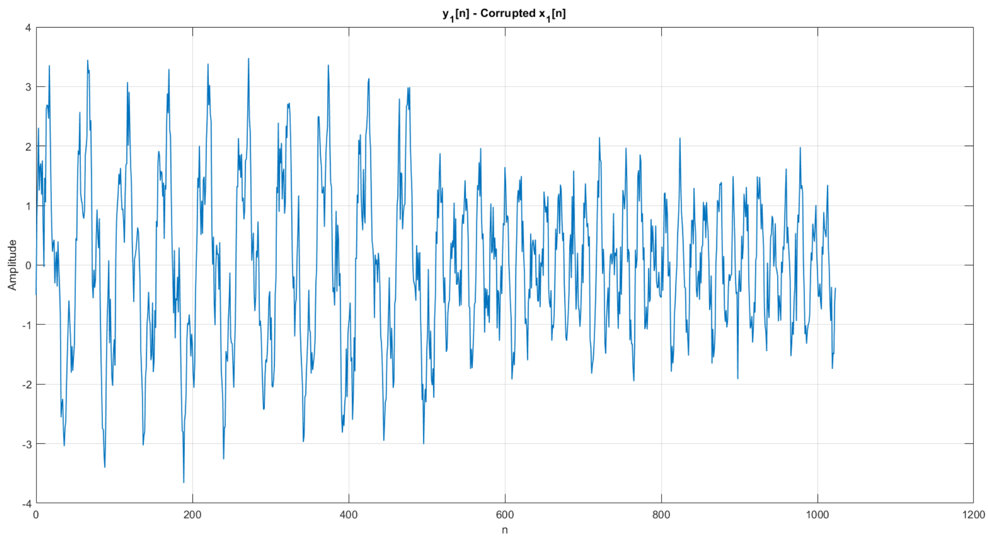


Figure 18: Corrupted $x_1[n]$ signal ($y_1[n]$)

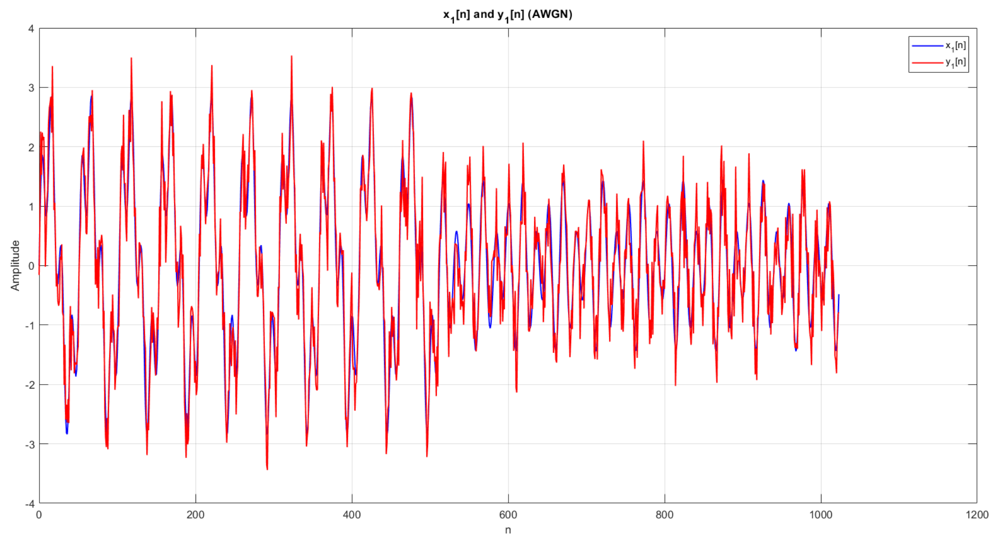


Figure 19: $y_1[n]$ and $x_1[n]$ in a same figure

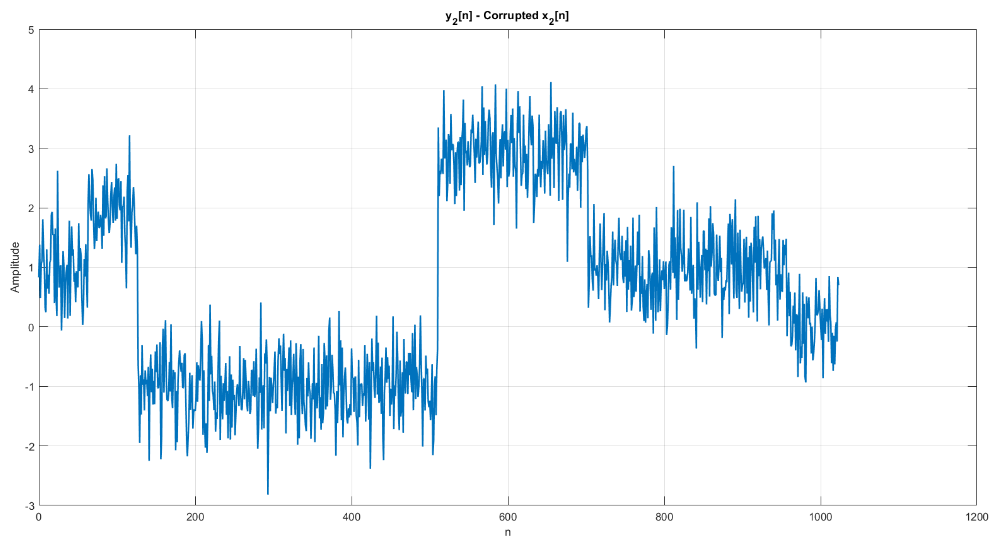


Figure 20: Corrupted $x_2[n]$ signal ($y_2[n]$)

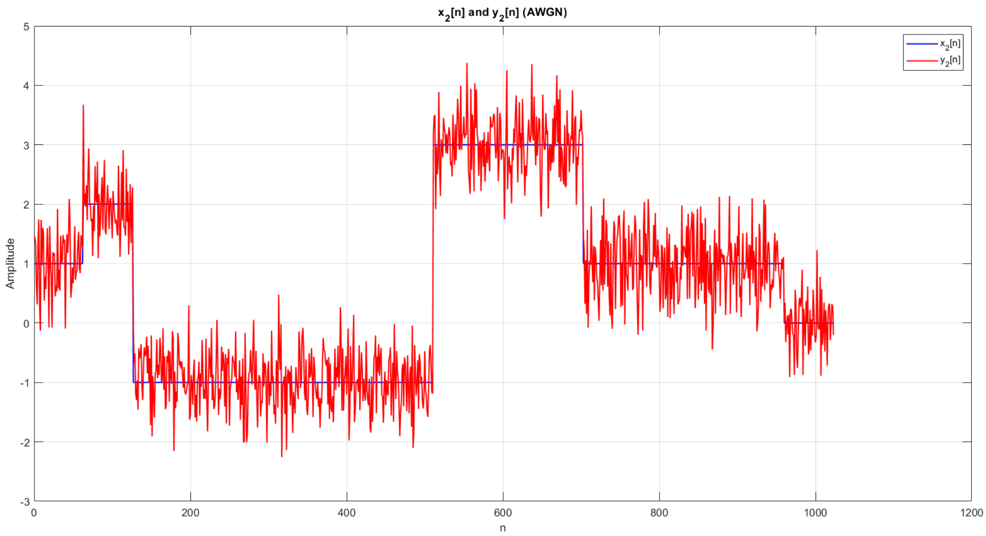


Figure 21: $y_2[n]$ and $x_2[n]$ in a same figure

ii. Observe the morphology of the wavelet and scaling functions of Haar and Daubechies tap 9 using $wavefun()$ command and the $waveletAnalyzer$ GUI

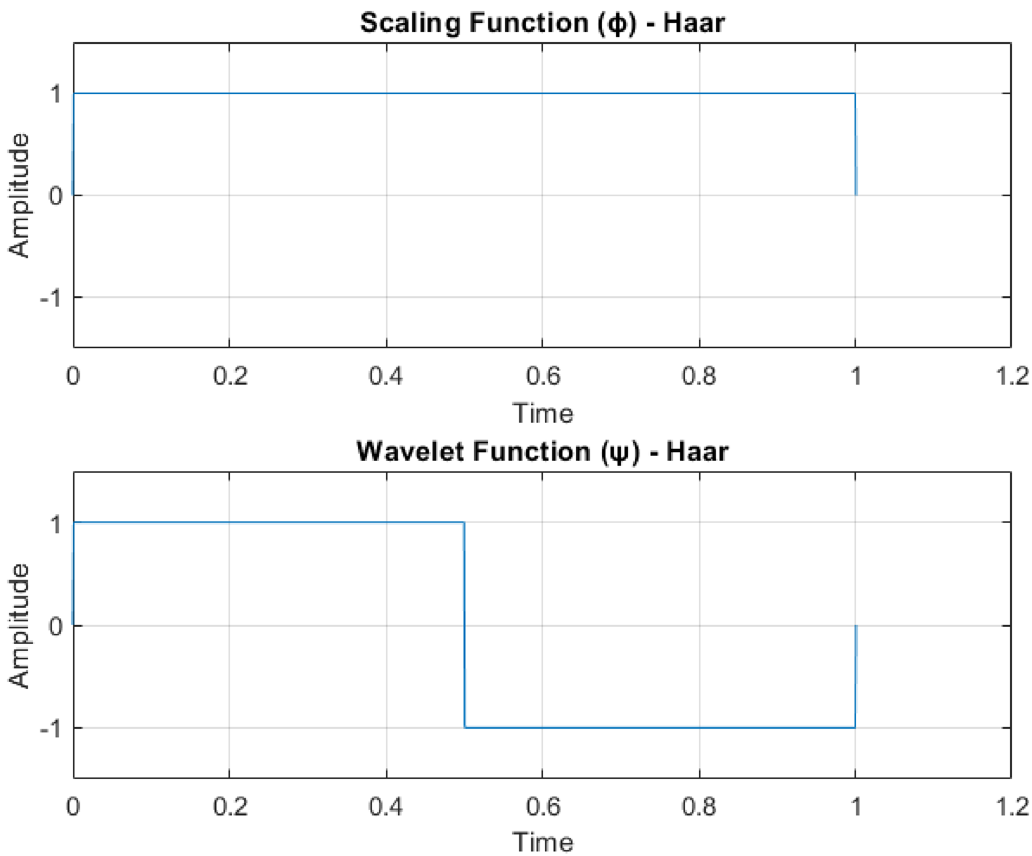


Figure 22: Scaling function (ϕ) and the wavelet function (ψ) of Haar wavelet

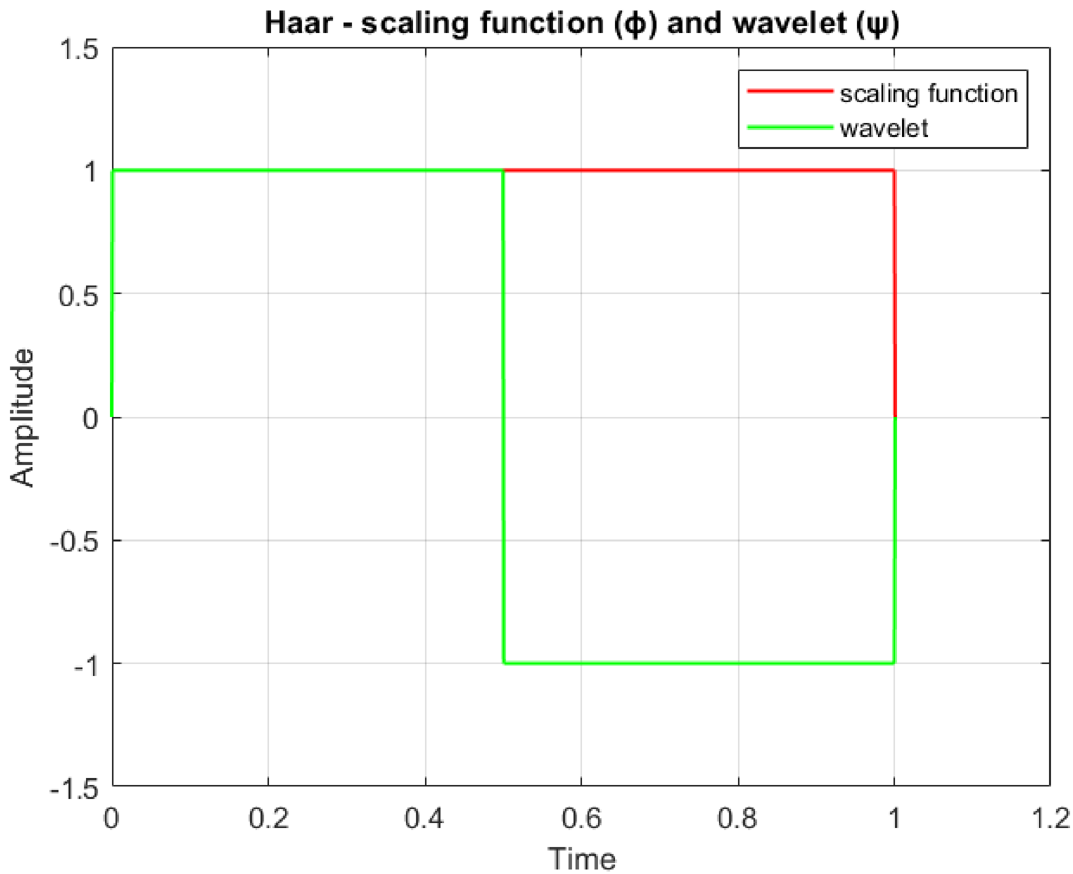


Figure 23: Scaling function (ϕ) and the wavelet function (ψ) of Haar wavelet in a same figure

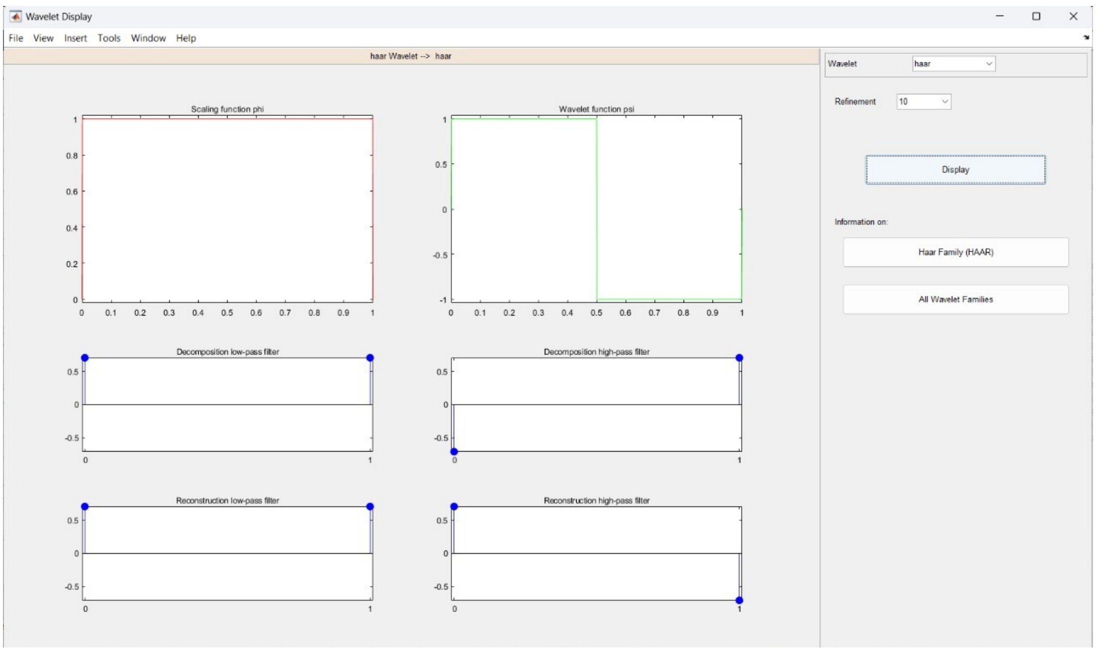


Figure 24: Haar wavelet using waveletAnalyzer GUI

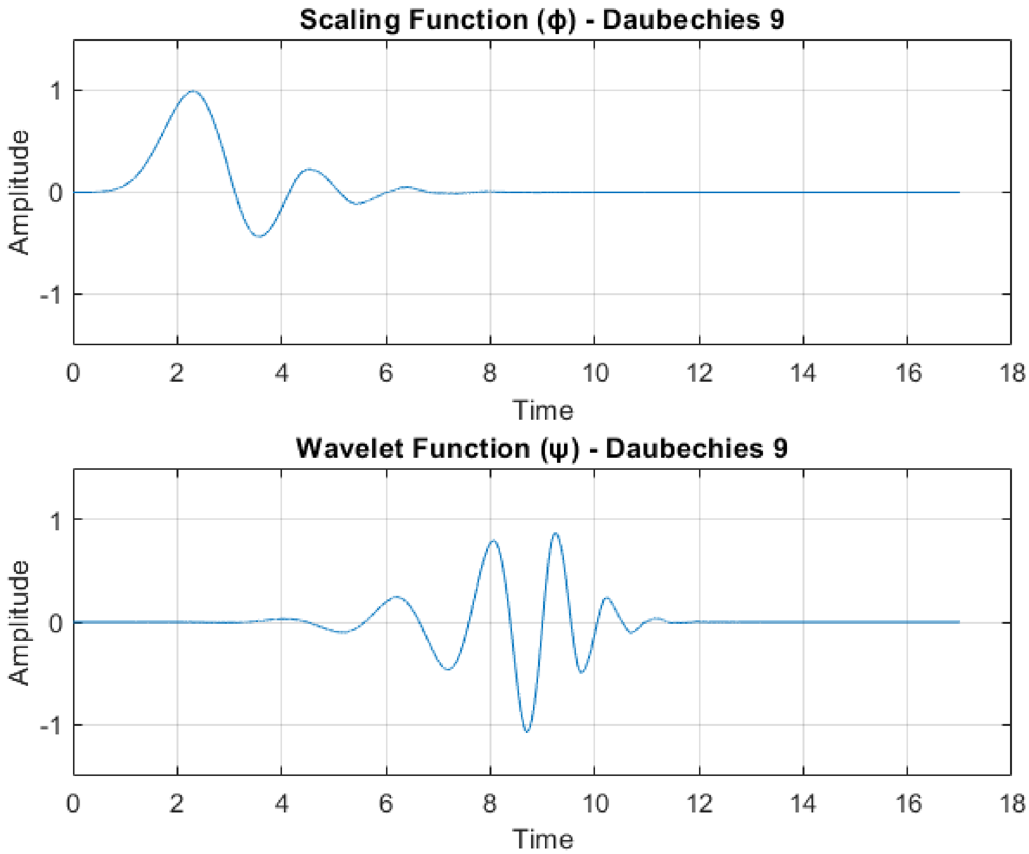


Figure 25: Scaling function (ϕ) and the wavelet function (ψ) of db9 wavelet

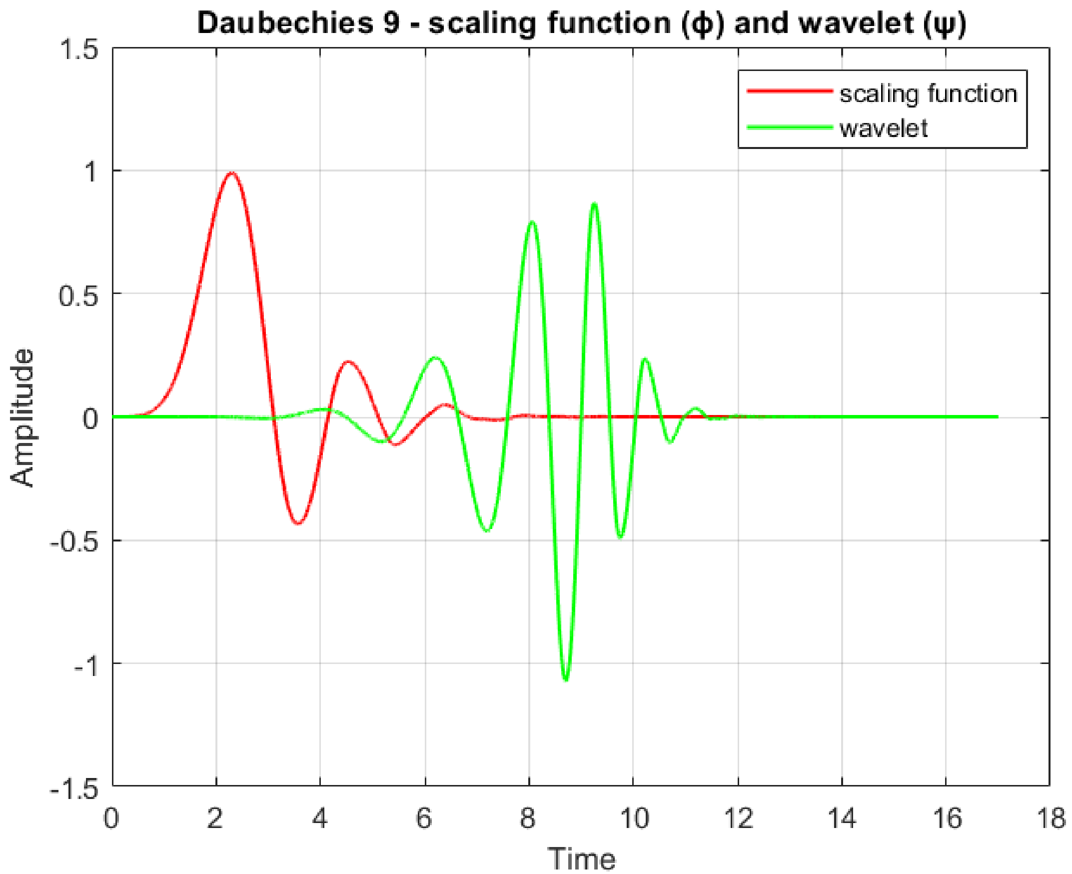


Figure 26: Scaling function (ϕ) and the wavelet function (ψ) of db9 wavelet in a same figure

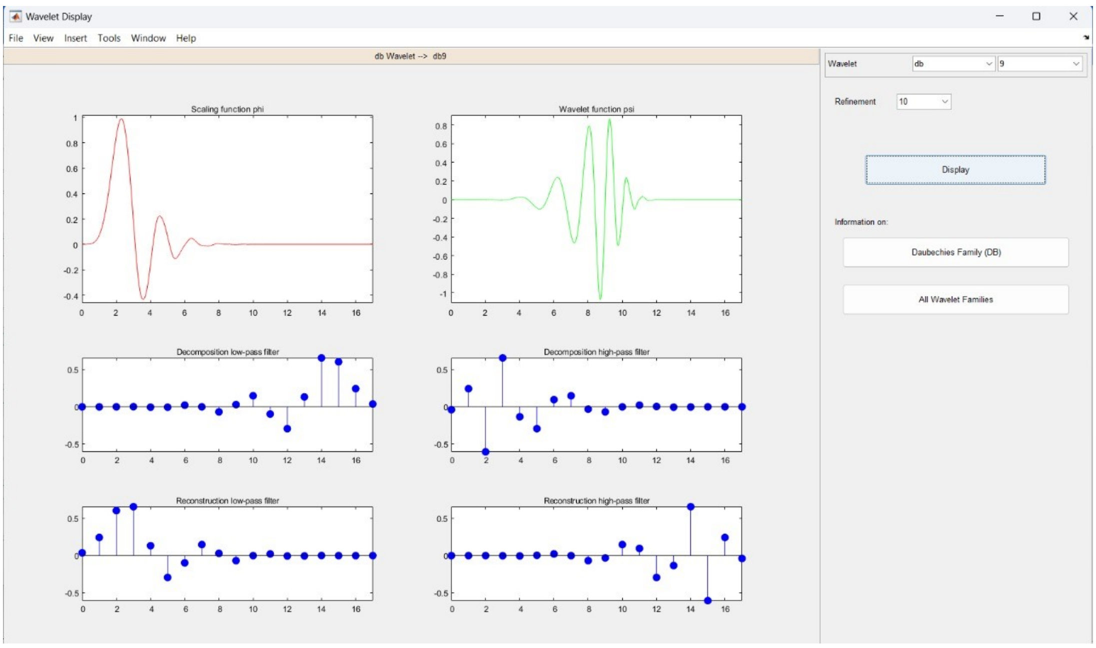


Figure 27: db9 wavelet using waveletAnalyzer GUI

iii. Calculate the 10-level wavelet decomposition of the signal using wavelet ‘db9’ and ‘haar’. Use the command *wavedec*()

```
% Decomposition using Daubechies 9 wavelet (y1)
level = 10;
[C_db9_y1, L_db9_y1] = wavedec(y1, level, 'db9');
```

Listing 1: Decomposition y1 using Daubechies 9 wavelet

```
% Decomposition using Daubechies 9 wavelet (y2)
level = 10;
[C_db9_y2, L_db9_y2] = wavedec(y2, level, 'db9');
```

Listing 2: Decomposition y2 using Daubechies 9 wavelet

```
% Decomposition using Haar wavelet (y1)
level = 10;
[C_haar_y1, L_haar_y1] = wavedec(y1, level, 'haar');
```

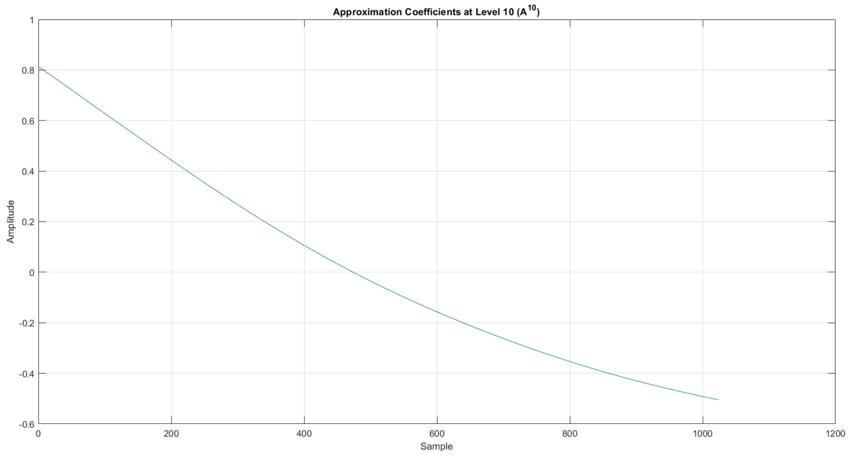
Listing 3: Decomposition y1 using Haar wavelet

```
% Decomposition using haar wavelet (y2)
level = 10;
[C_haar_y2, L_haar_y2] = wavedec(y2, level, 'haar');
```

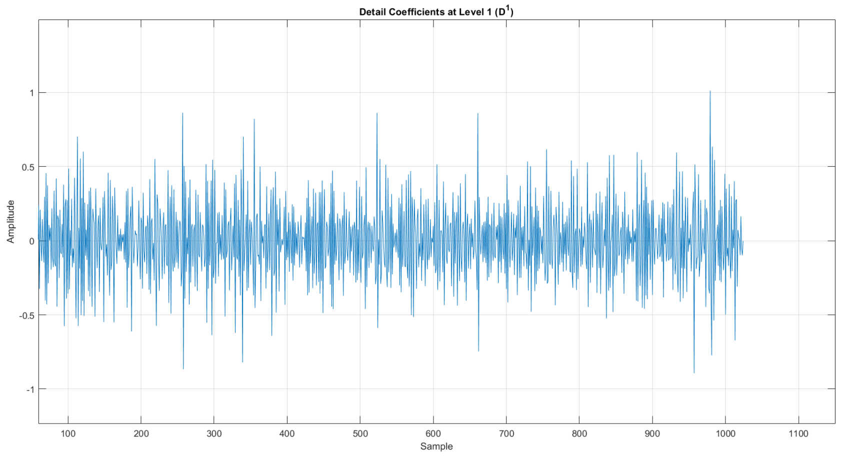
Listing 4: Decomposition y2 using haar wavelet

iv. Use the inverse DWT to reconstruct $A^{10}, D^{10}, D^{9}, \ldots, D^{2}, D^{1}$

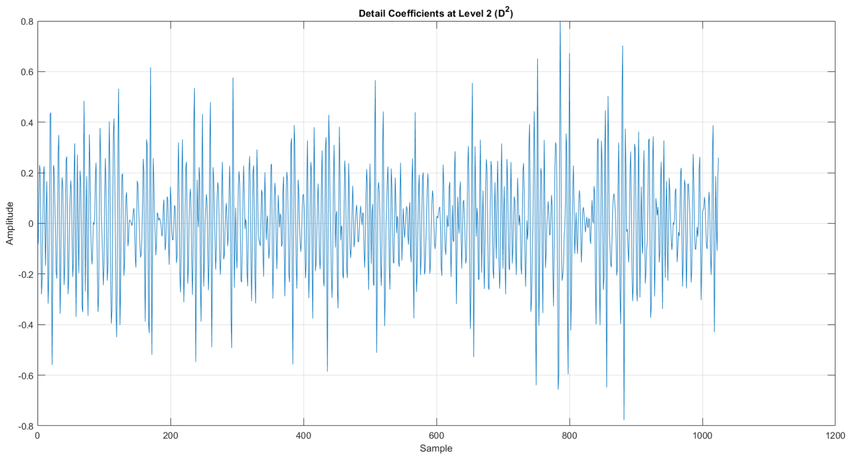
1. Reconstructed $A^{10}, D^{10}, D^{9}, ..., D^{2}, D^{1}$ for signal $y1$ with $db9$ wavelet



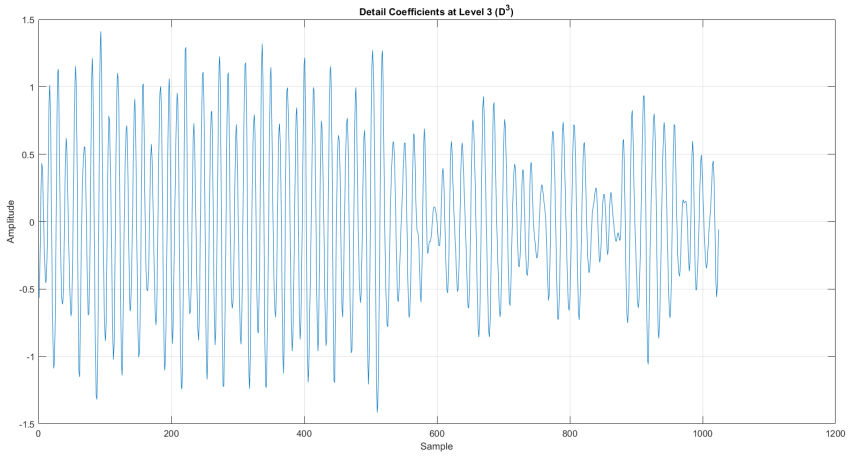
(a) Approximation at level 10 (A^{10})



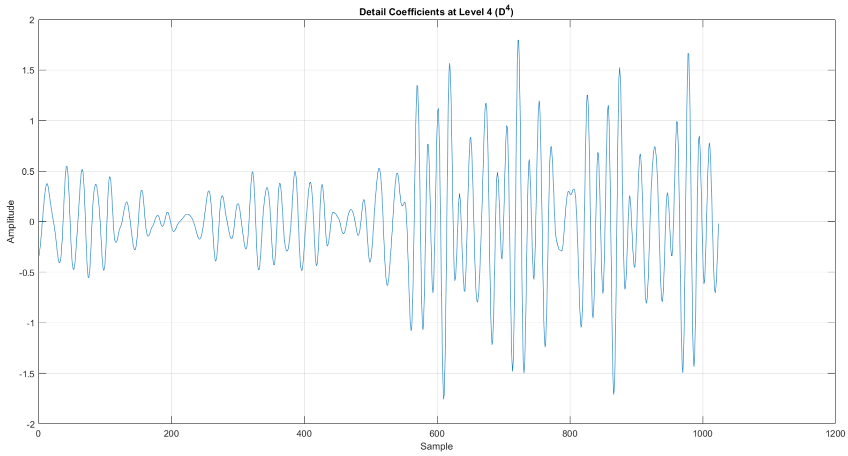
(b) Detail at level 1 (D^{1})



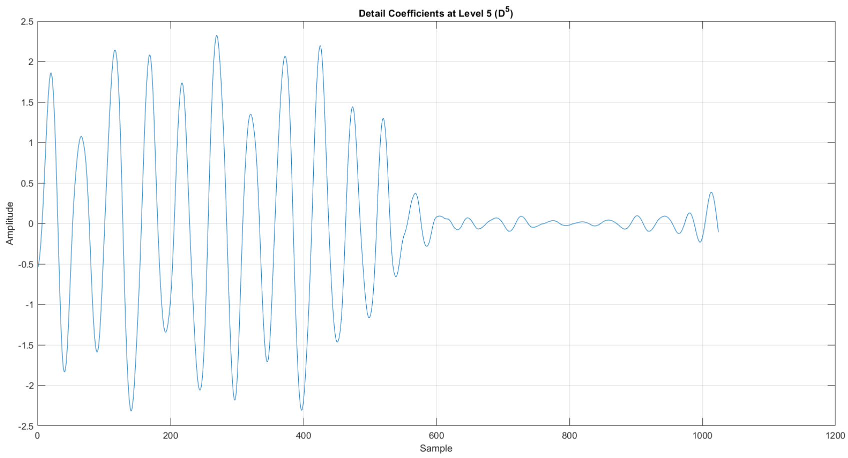
(c) Detail at level 2 (D^{2})



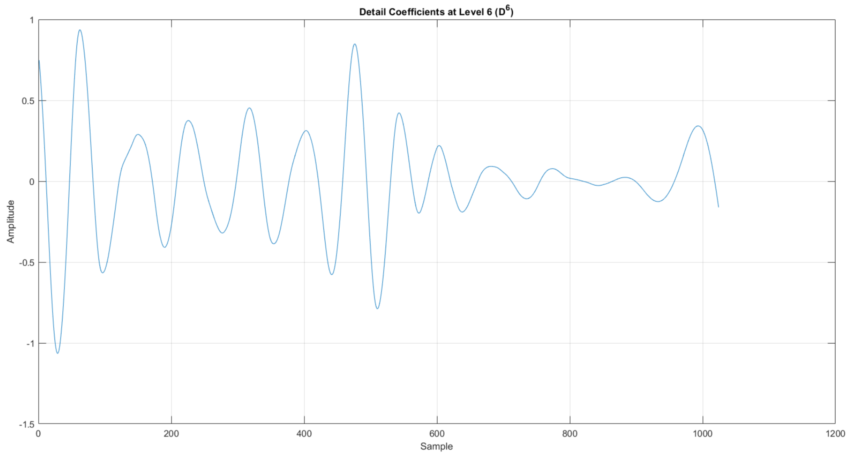
(d) Detail at level 3 (D^3)



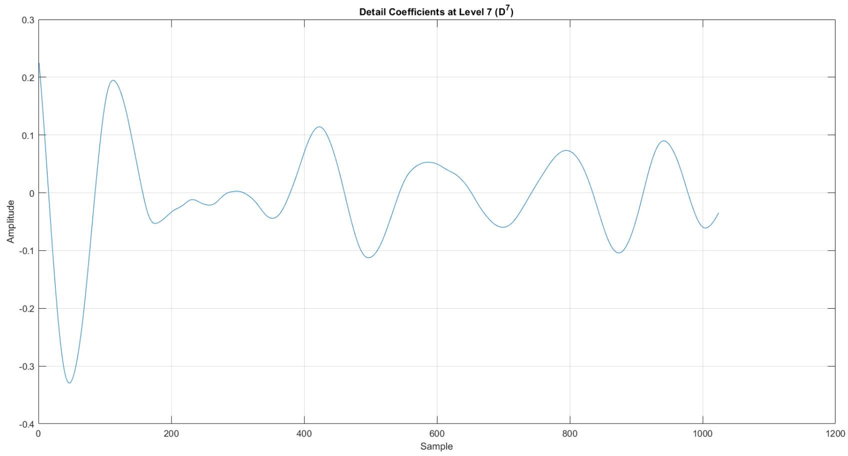
(e) Detail at level 4 (D^4)



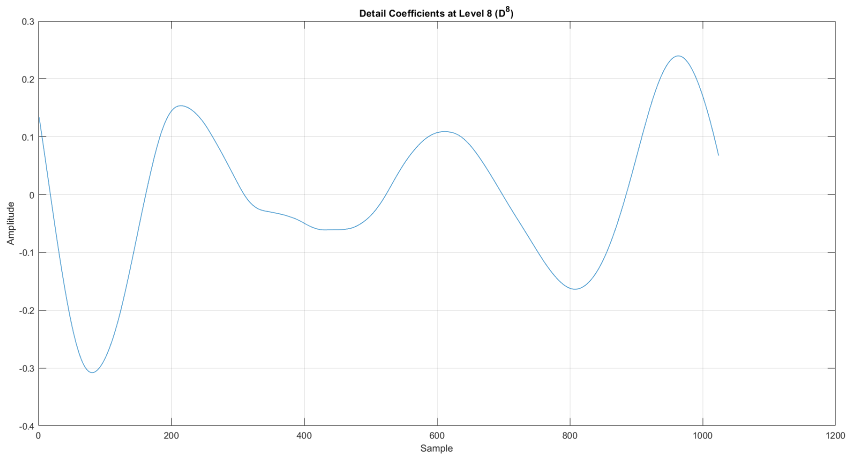
(f) Detail at level 5 (D^5)



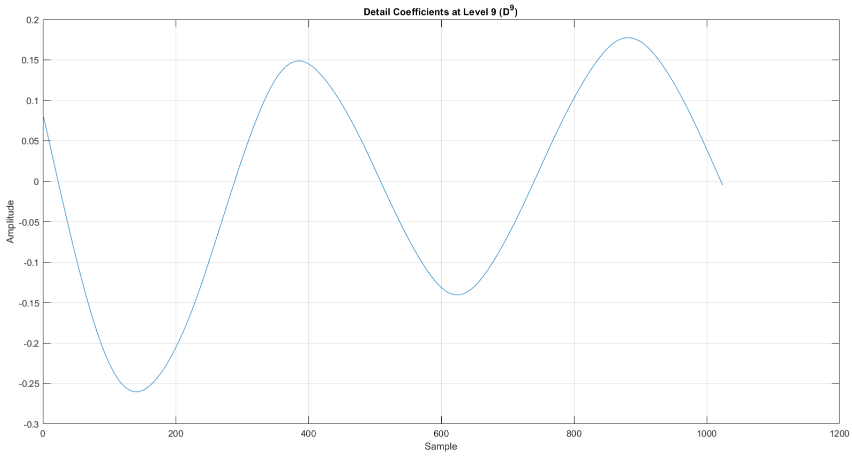
(g) Detail at level 6 (D^6)



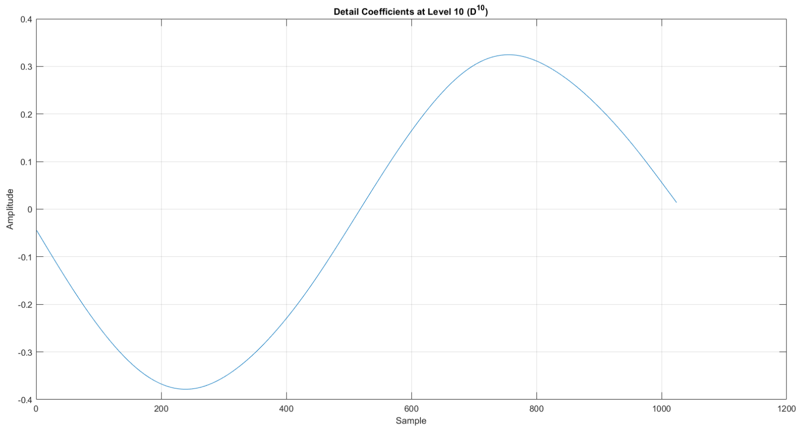
(h) Detail at level 7 (D^7)



(i) Detail at level 8 (D^8)



(j) Detail at level 9 (D^9)



(k) Detail at level 10 (D^{10})

Figure 27: Reconstructed wavelet sub-bands for signal $y1$ using db9 wavelet

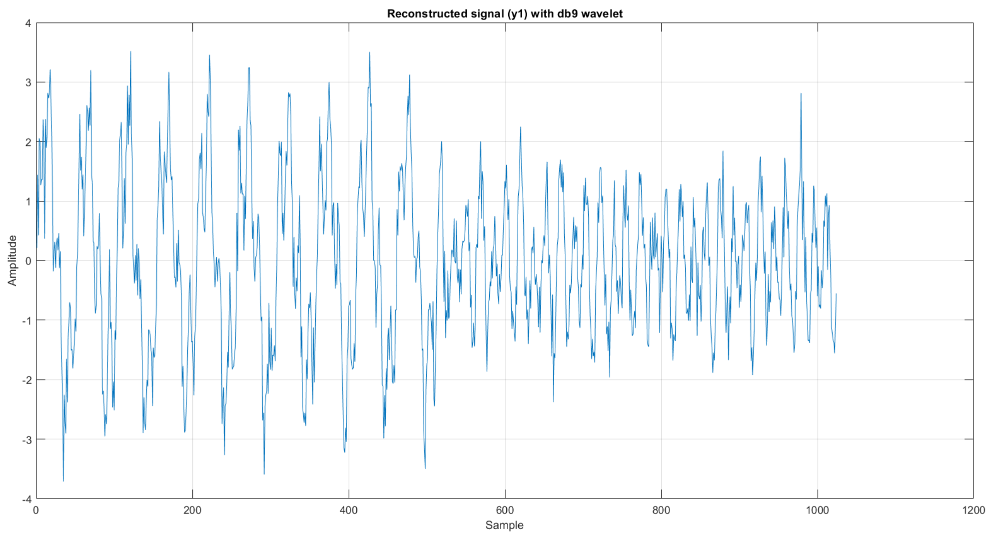
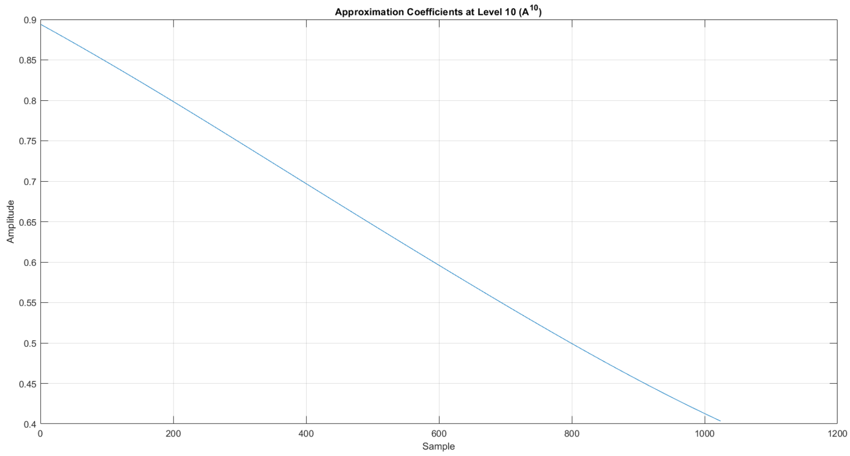
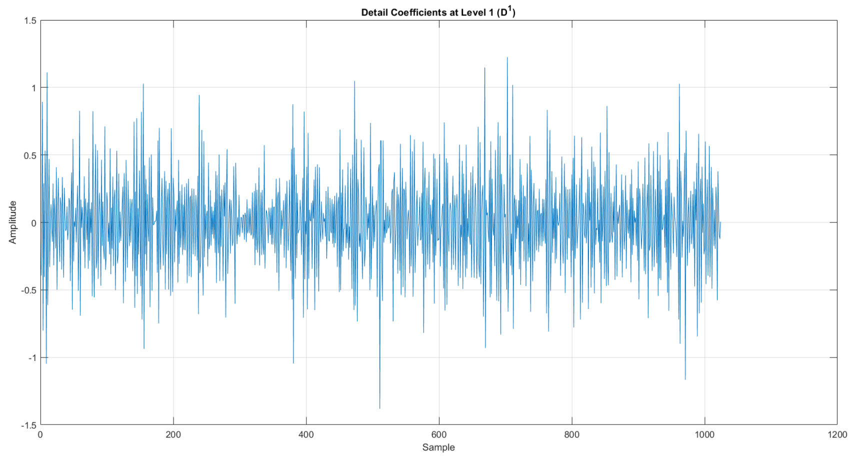


Figure 28: Reconstructed signal $y1$ using wavelet coefficients

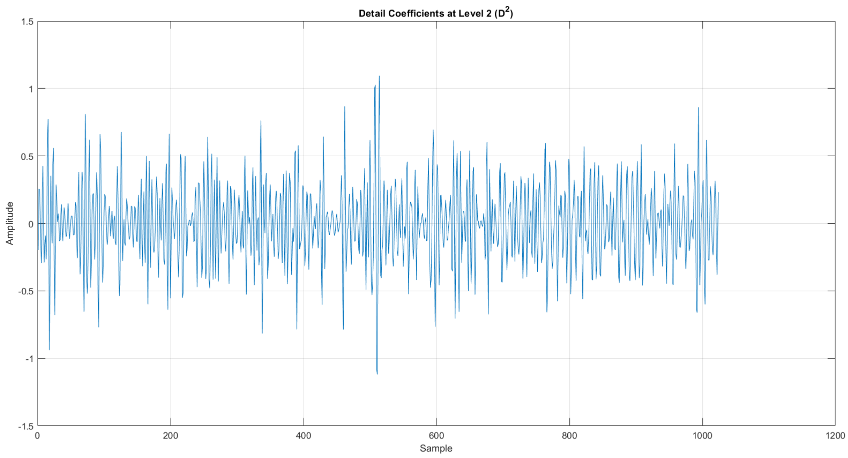
2. Reconstructed $A^{10}, D^{10}, D^9, ..., D^2, D^1$ for signal $y2$ with $db9$ wavelet



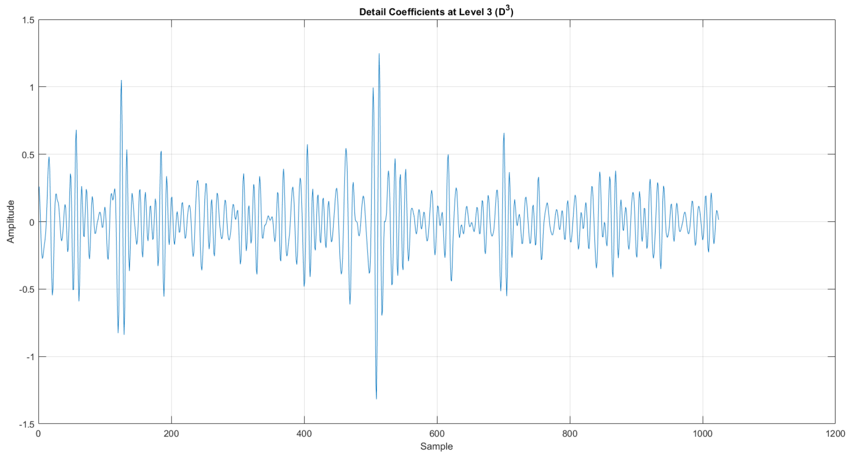
(a) Approximation at level 10 (A^{10})



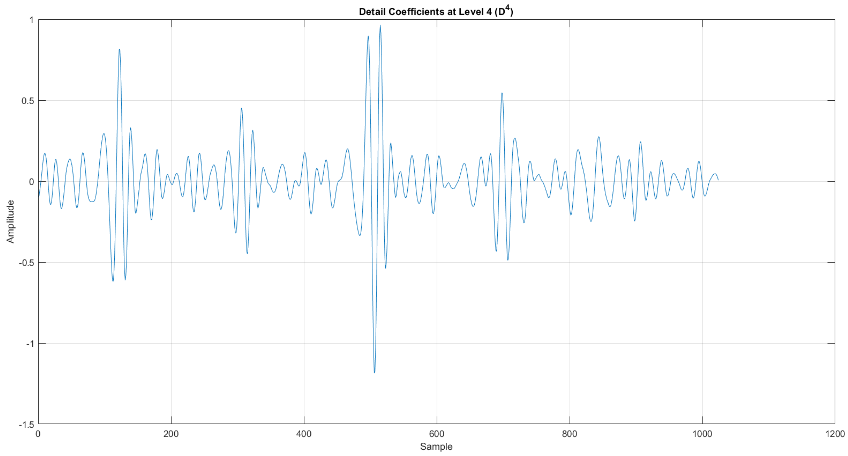
(b) Detail at level 1 (D^1)



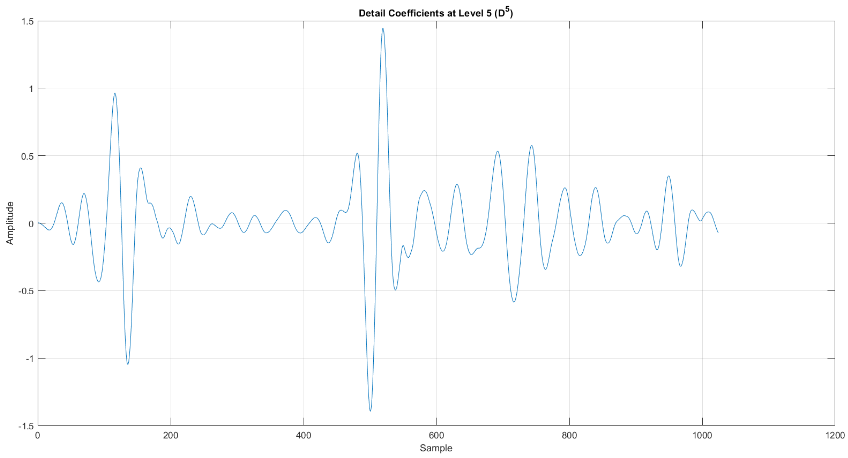
(c) Detail at level 2 (D^2)



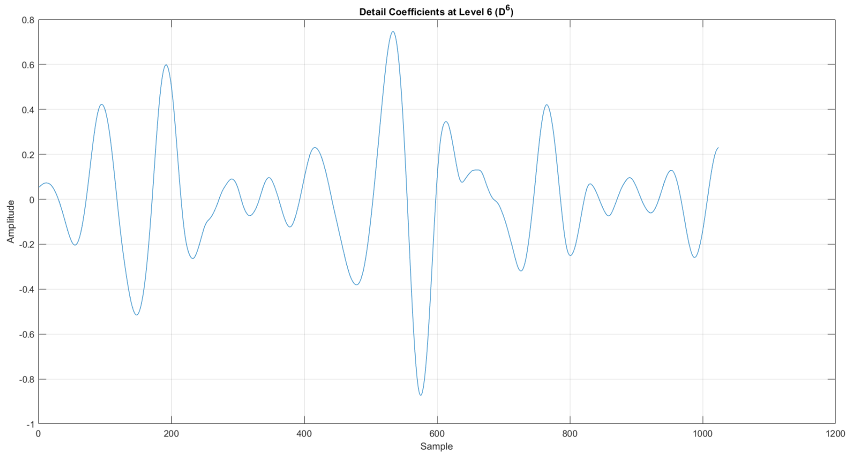
(d) Detail at level 3 (D^3)



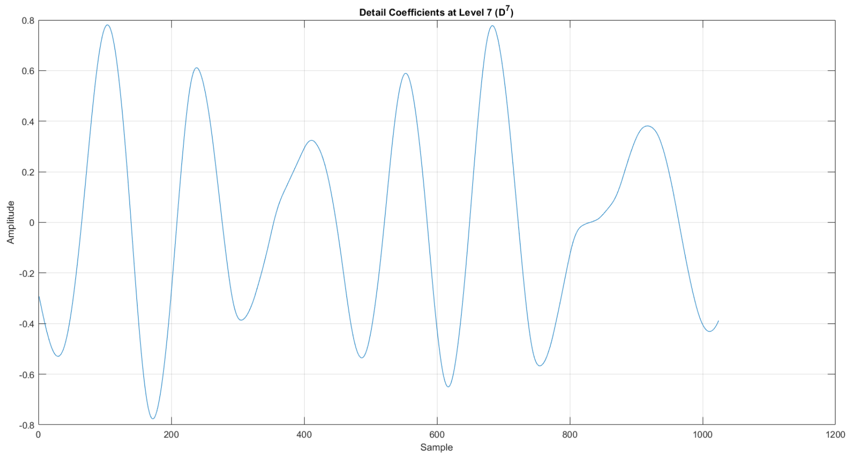
(e) Detail at level 4 (D^4)



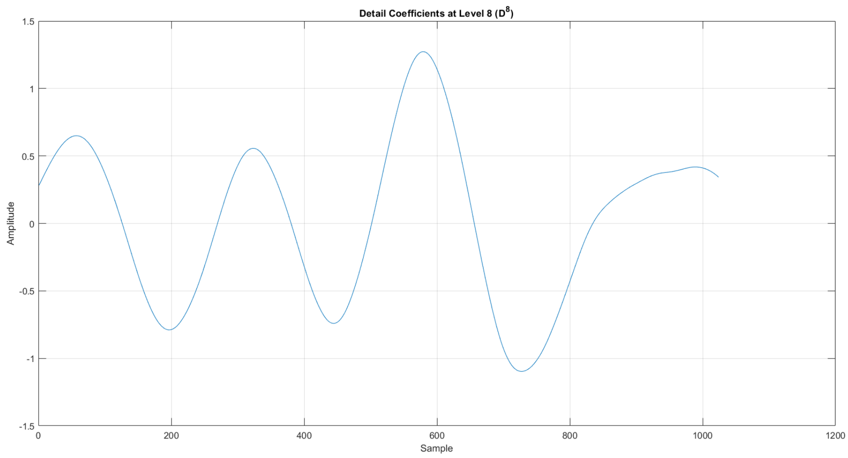
(f) Detail at level 5 (D^5)



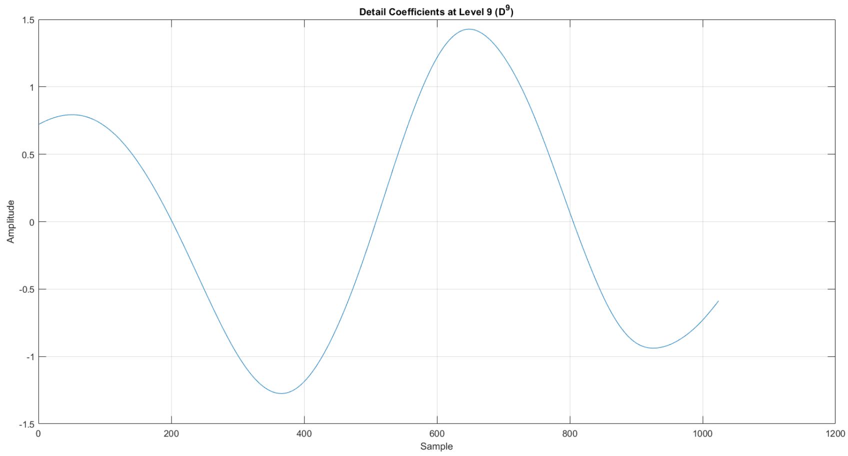
(g) Detail at level 6 (D^6)



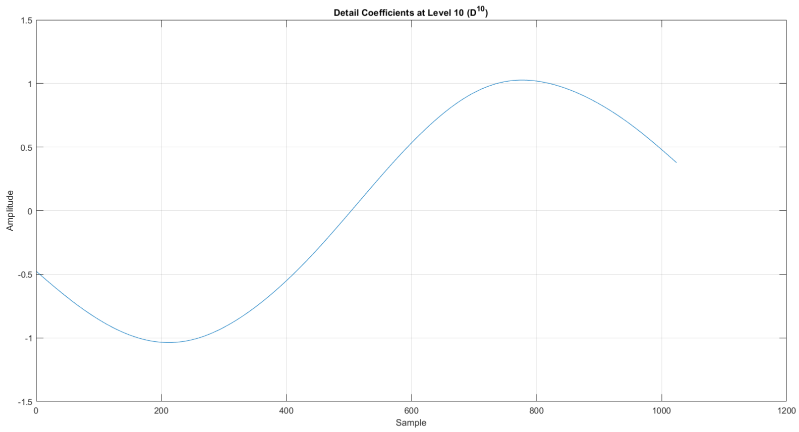
(h) Detail at level 7 (D^7)



(i) Detail at level 8 (D^8)



(j) Detail at level 9 (D^9)



(k) Detail at level 10 (D^{10})

Figure 28: Reconstructed wavelet sub-bands for signal y2 using db9 wavelet

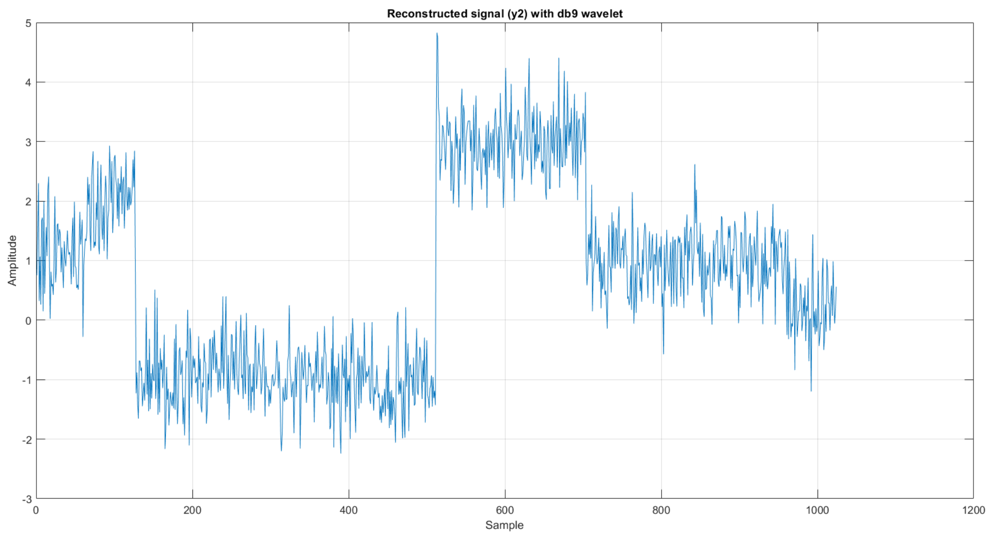
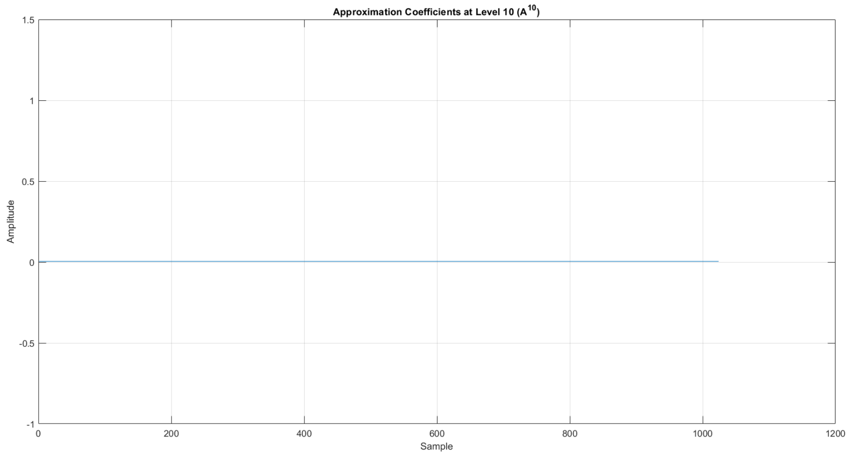
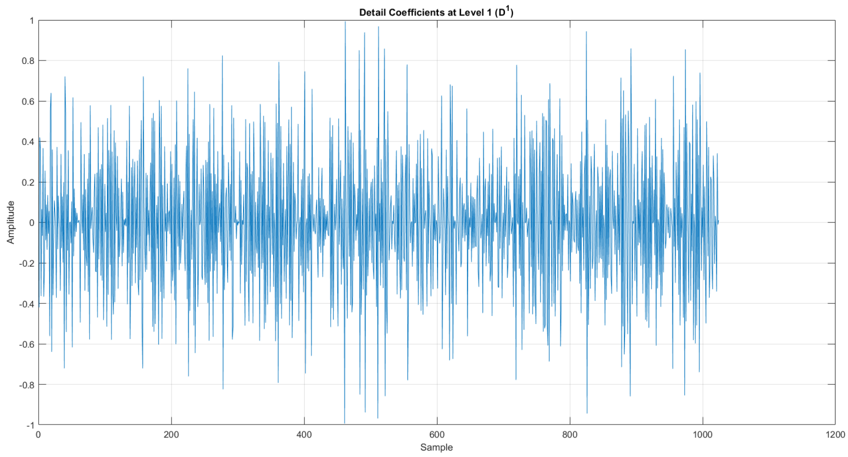


Figure 29: Reconstructed signal y2 using wavelet coefficients

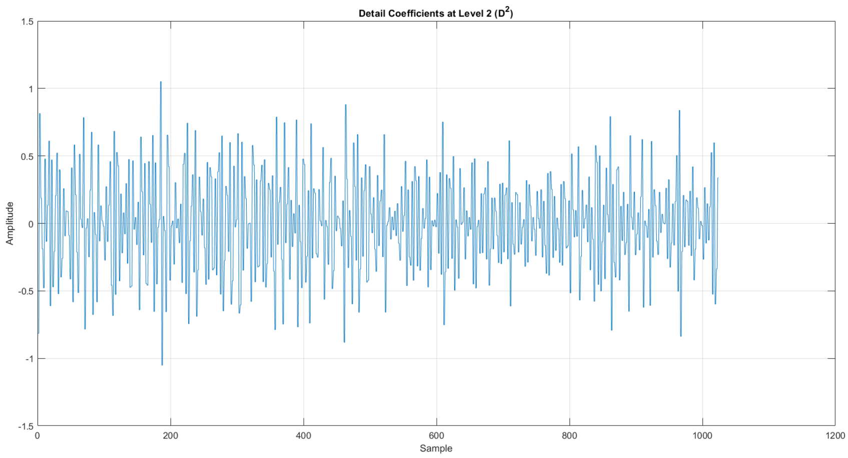
3. Reconstructed $A^{10}, D^{10}, D^9, ..., D^2, D^1$ for signal $y1$ with *haar* wavelet



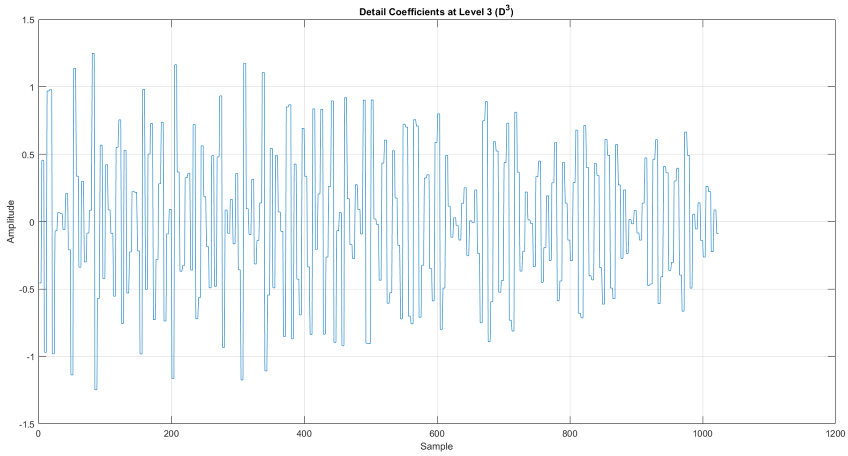
(a) Approximation at level 10 (A^{10})



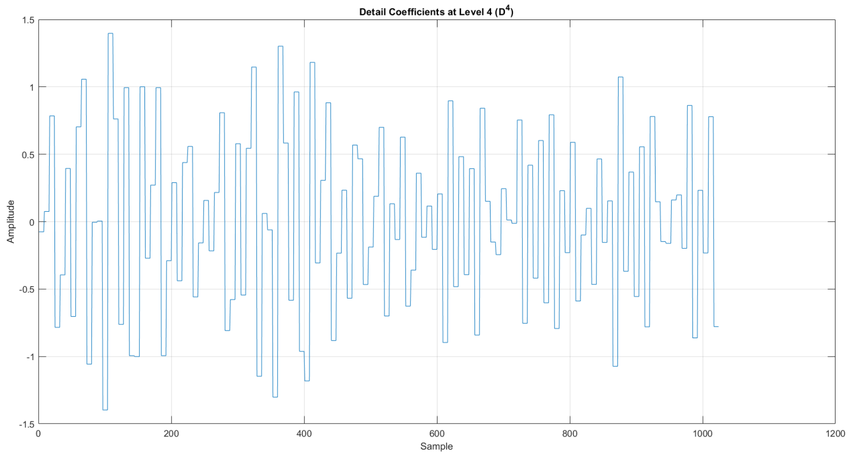
(b) Detail at level 1 (D^1)



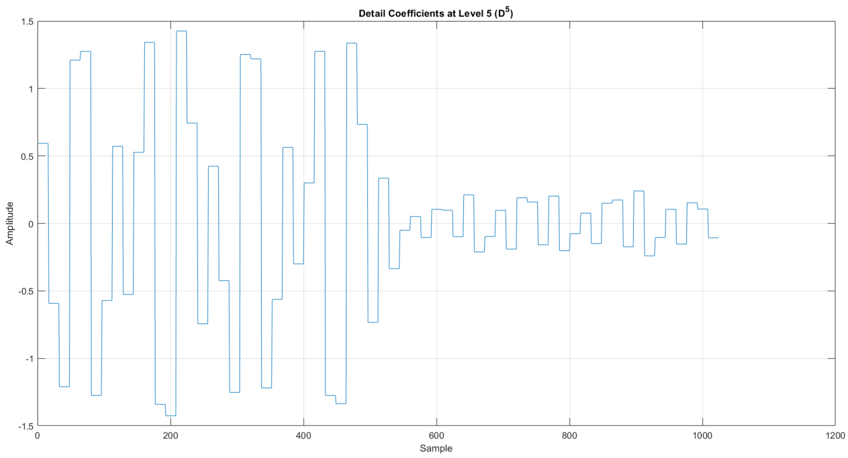
(c) Detail at level 2 (D^2)



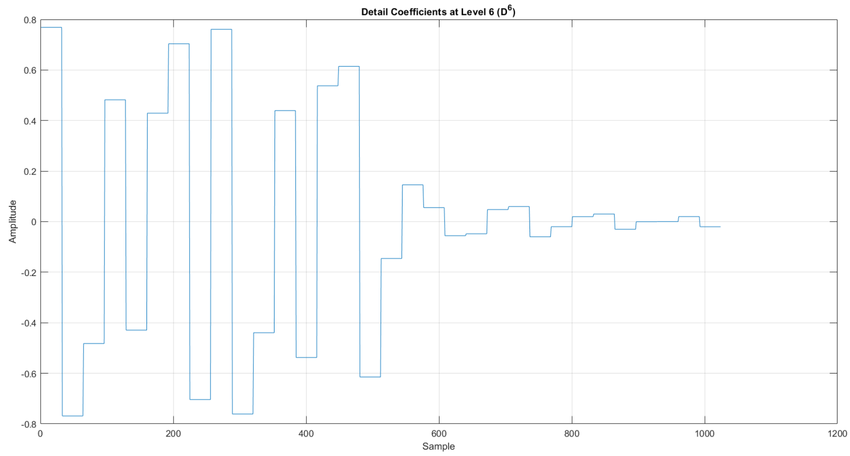
(d) Detail at level 3 (D^3)



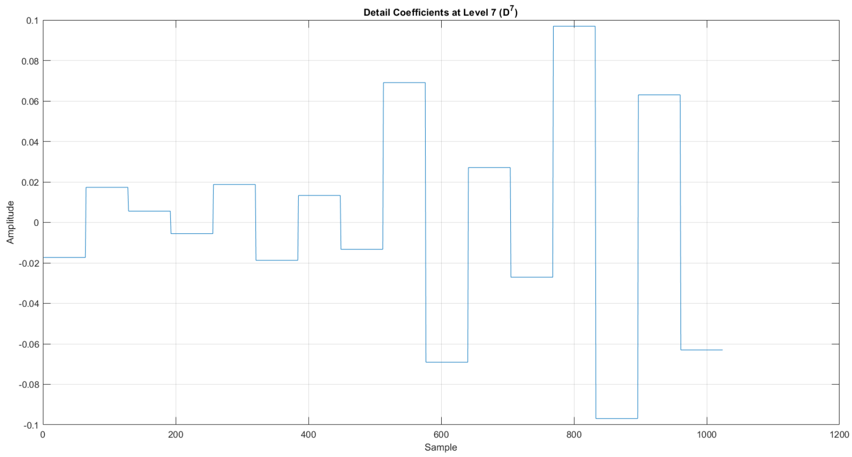
(e) Detail at level 4 (D^4)



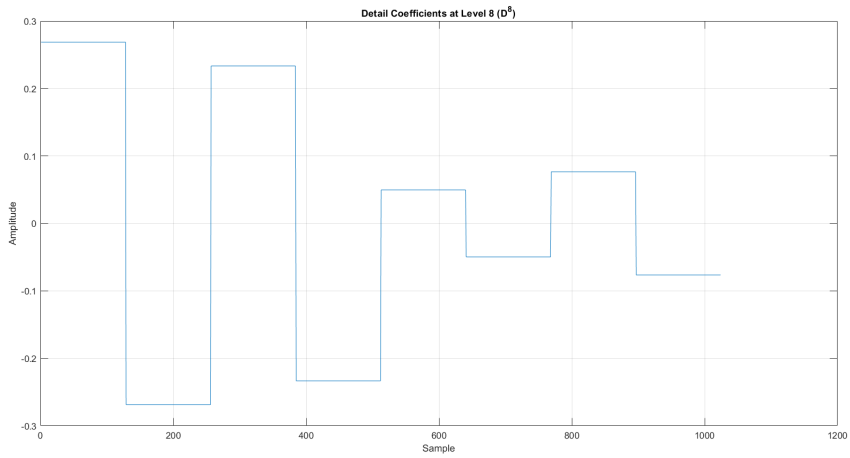
(f) Detail at level 5 (D^5)



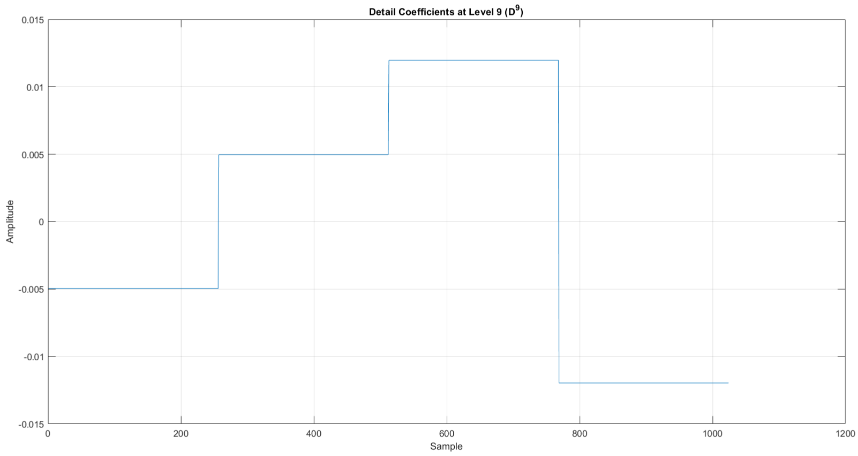
(g) Detail at level 6 (D^6)



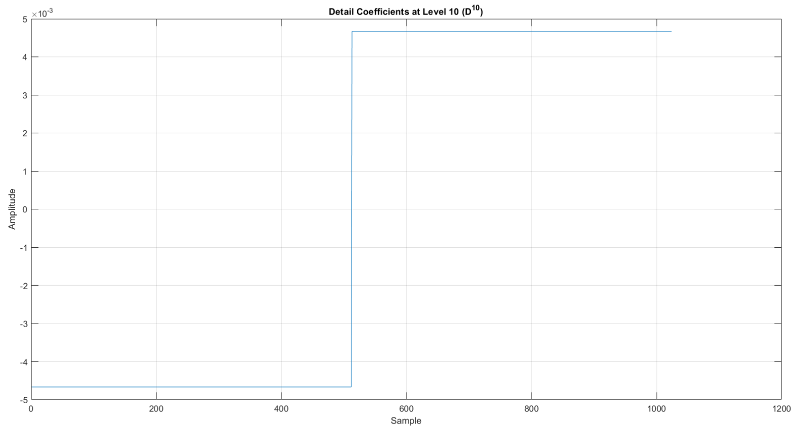
(h) Detail at level 7 (D^7)



(i) Detail at level 8 (D^8)



(j) Detail at level 9 (D^9)



(k) Detail at level 10 (D^{10})

Figure 29: Reconstructed wavelet sub-bands for signal $y1$ using haar wavelet

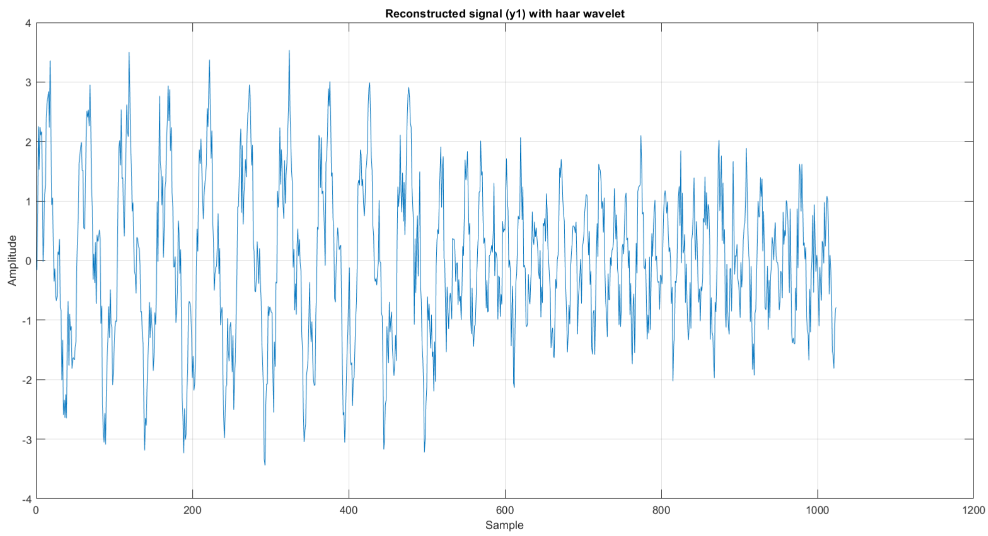
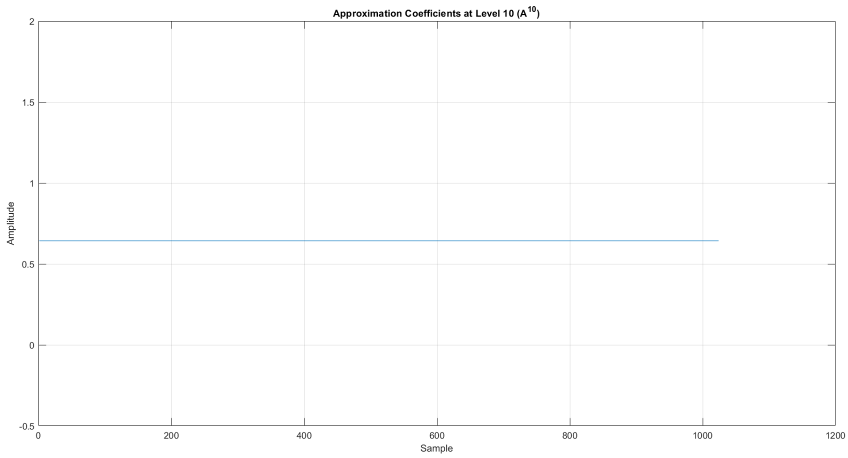
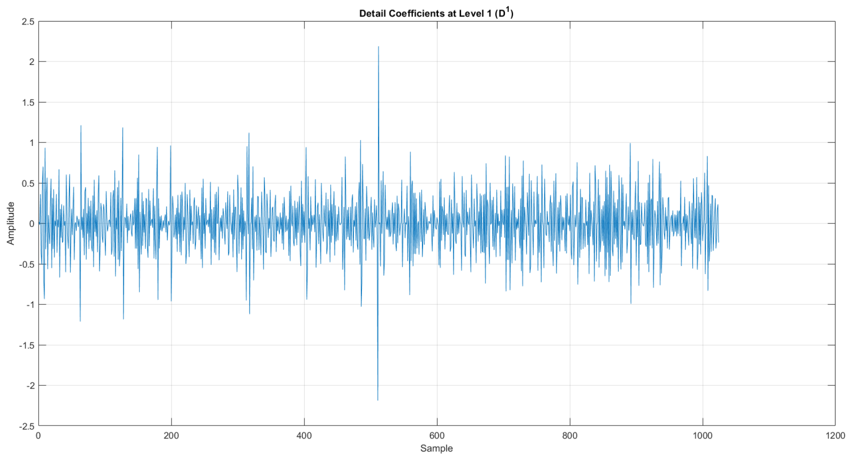


Figure 30: Reconstructed signal $y1$ using wavelet coefficients

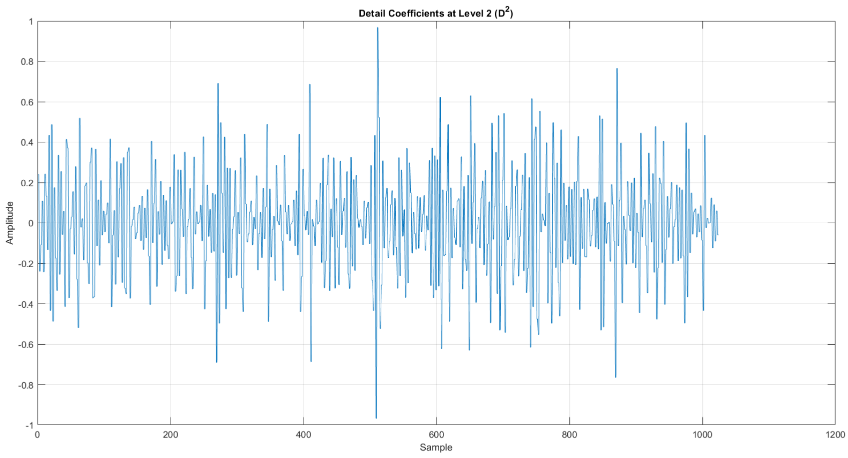
4. Reconstructed $A^{10}, D^{10}, D^{9}, ..., D^{2}, D^{1}$ for signal $y2$ with *haar* wavelet



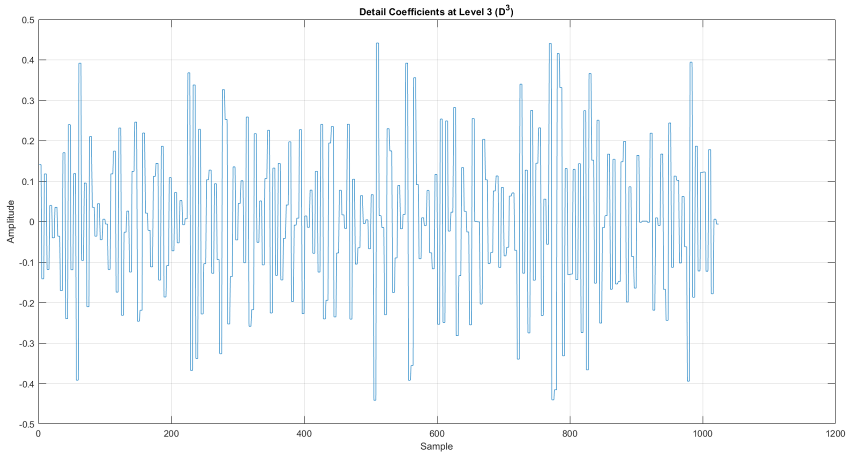
(a) Approximation at level 10 (A^{10})



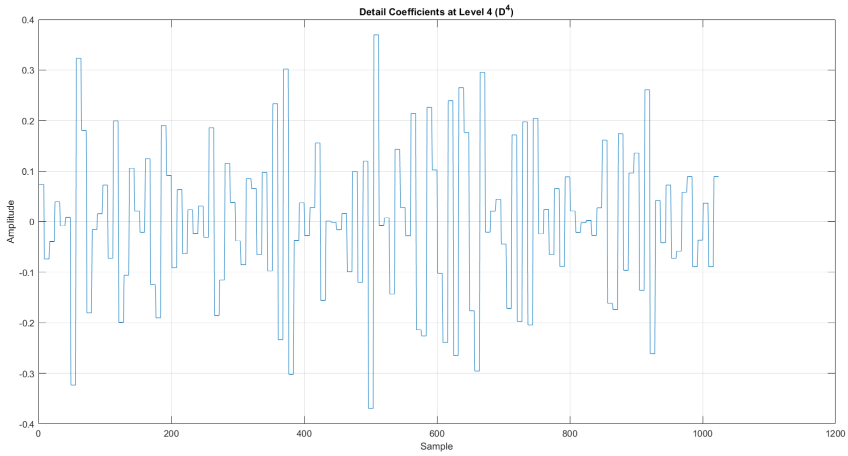
(b) Detail at level 1 (D^{1})



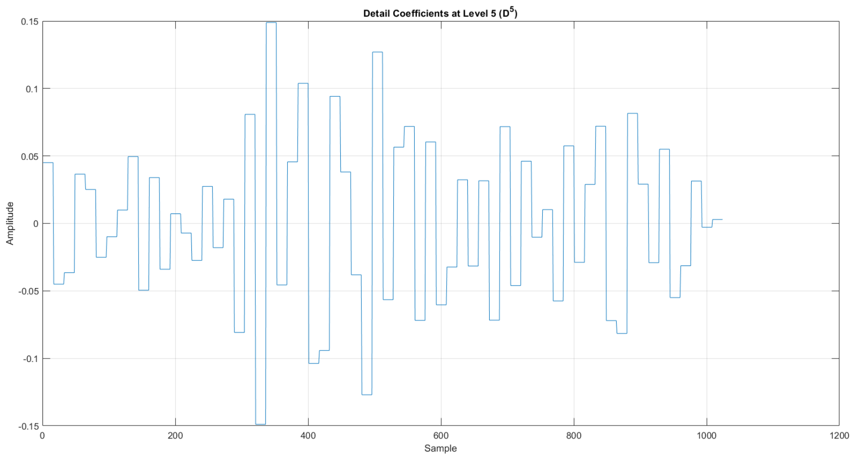
(c) Detail at level 2 (D^{2})



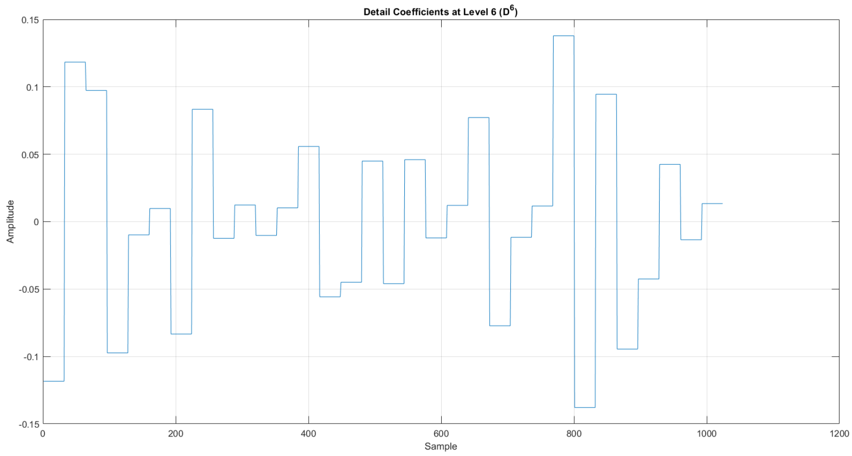
(d) Detail at level 3 (D^3)



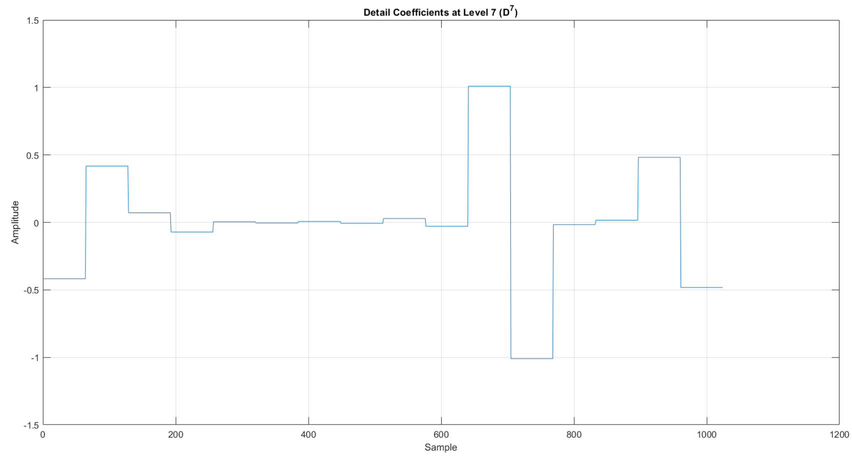
(e) Detail at level 4 (D^4)



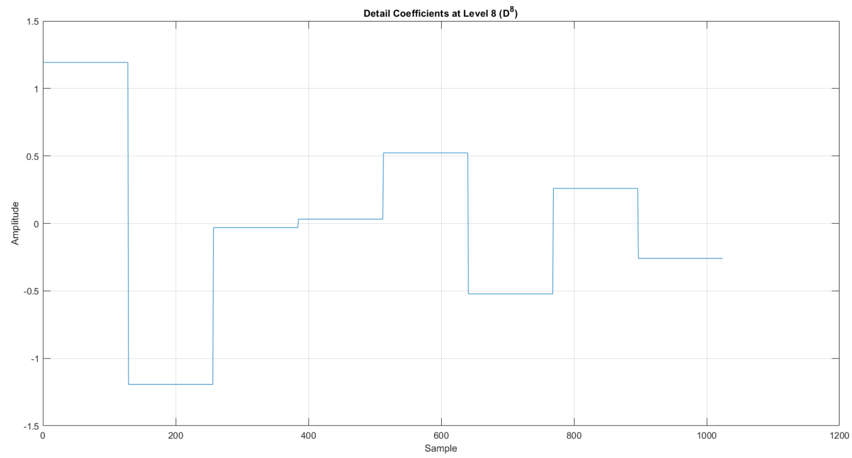
(f) Detail at level 5 (D^5)



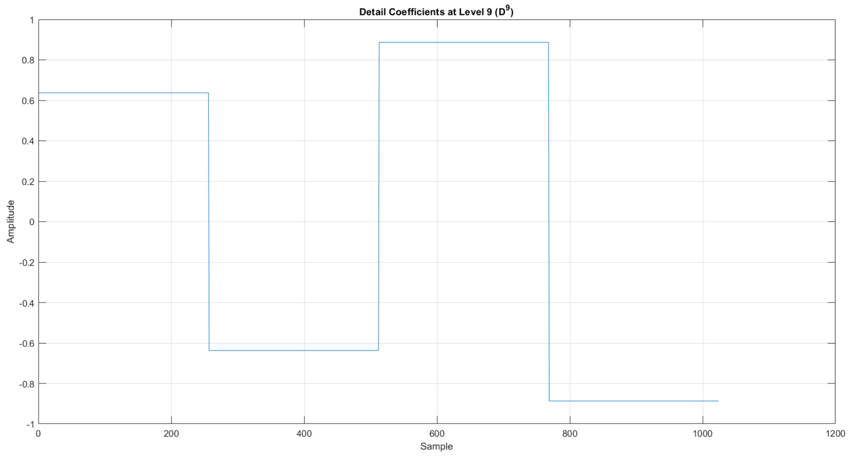
(g) Detail at level 6 (D^6)



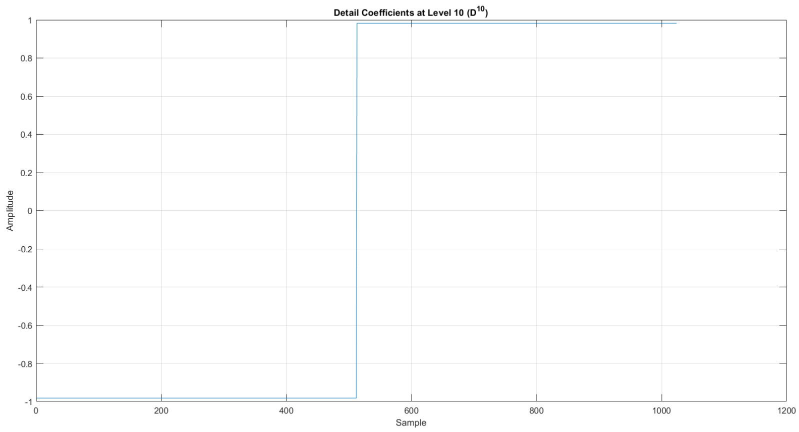
(h) Detail at level 7 (D^7)



(i) Detail at level 8 (D^8)



(j) Detail at level 9 (D^9)



(k) Detail at level 10 (D^{10})

Figure 30: Reconstructed wavelet sub-bands for signal y2 using haar wavelet

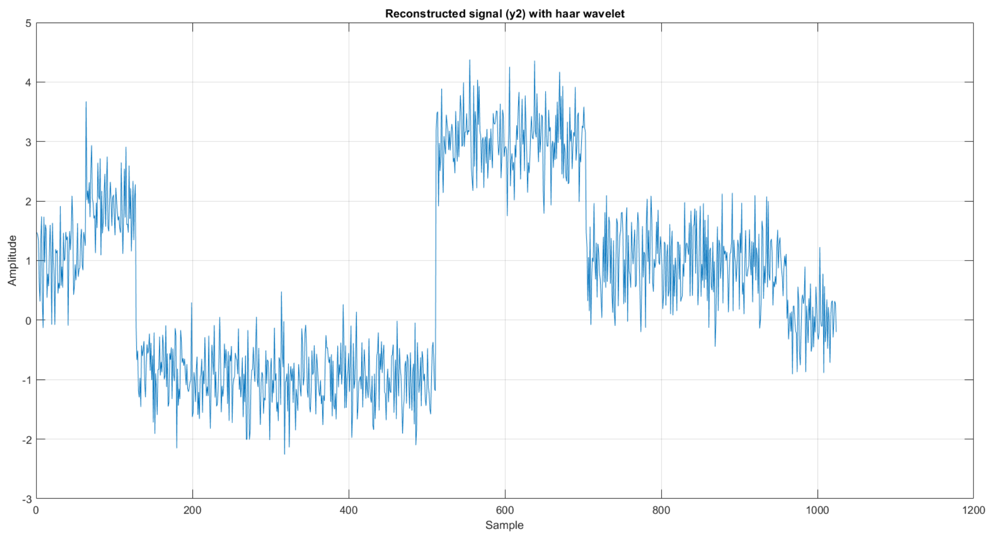


Figure 31: Reconstructed signal y2 using wavelet coefficients

In the above work, I reconstructed the signals by using wavelet coefficients. Those reconstructions are as follows.

- Figure 28 : Reconstructed signal $y_1[n]$ with '$db9$' wavelet
- Figure 29 : Reconstructed signal $y_2[n]$ with '$db9$' wavelet
- Figure 30 : Reconstructed signal $y_1[n]$ with '$haar$' wavelet
- Figure 31 : Reconstructed signal $y_2[n]$ with '$haar$' wavelet

All of these signals are obtained by using the following equation.

$$y = \Sigma D^i + A \tag{18}$$

To verify this equation, I calculated the energy difference between the original signal and the reconstructed signal. The calculated energy differences are as below.

Table 1: Energy difference between original and reconstructed signal

Signal	**Wavelet**	**Energy Difference**
y1	db9	2.9665e-07
	haar	1.5916e-12
y2	db9	4.9803e-07
	haar	5.9117e-12

With these results, we can verify the equation 18 since the energy difference between the original signal and the reconstructed signal by using equation 18 is very low.

Also, we can observe that the 'haar' wavelet has performed well when reconstructing the signals.

2.3 Signal Denoising with DWT

i. Plot the magnitude of wavelet coefficients (stem plot) of the above signal in descending order

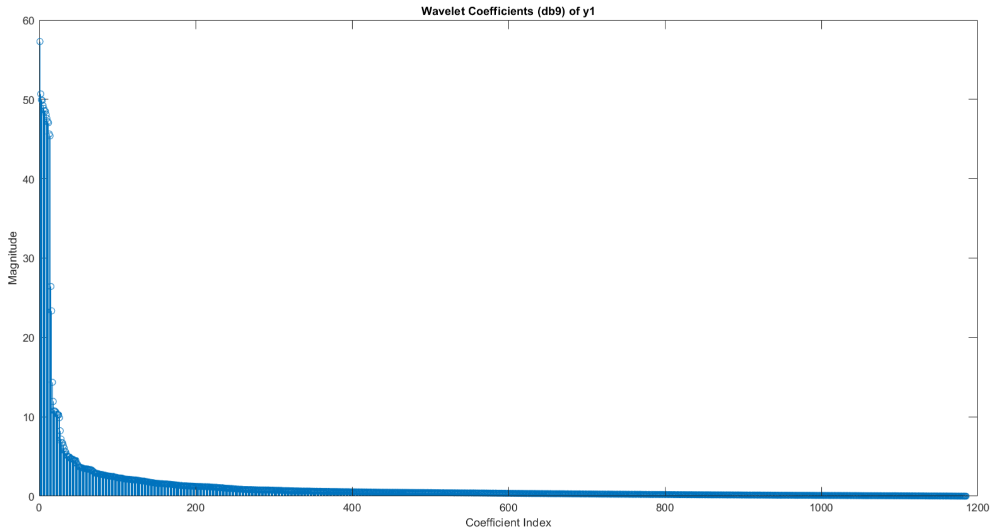


Figure 32: Magnitudes of wavelet coefficients of y1 (with 'db9') in descending order

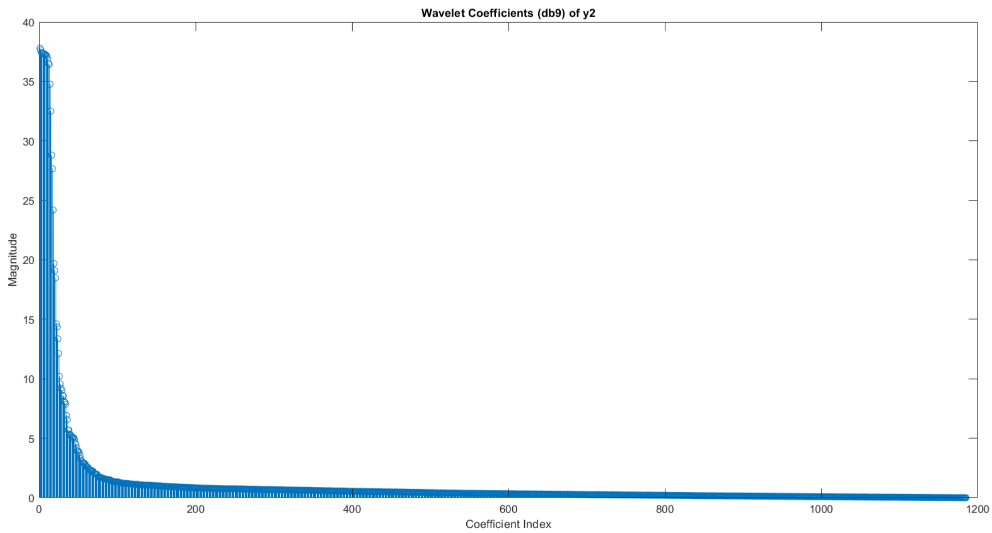


Figure 33: Magnitudes of wavelet coefficients of y2 (with 'db9') in descending order

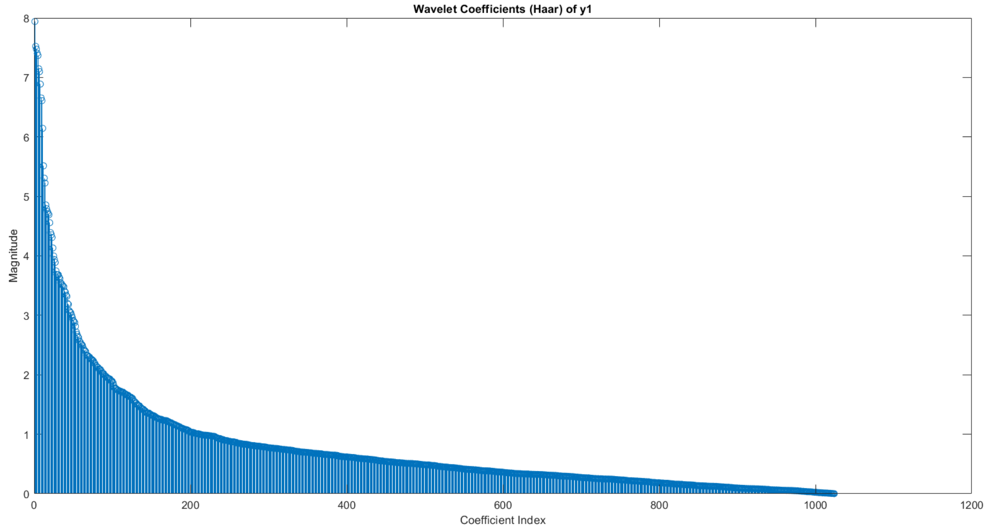


Figure 34: Magnitudes of wavelet coefficients of y1 (with 'haar') in descending order

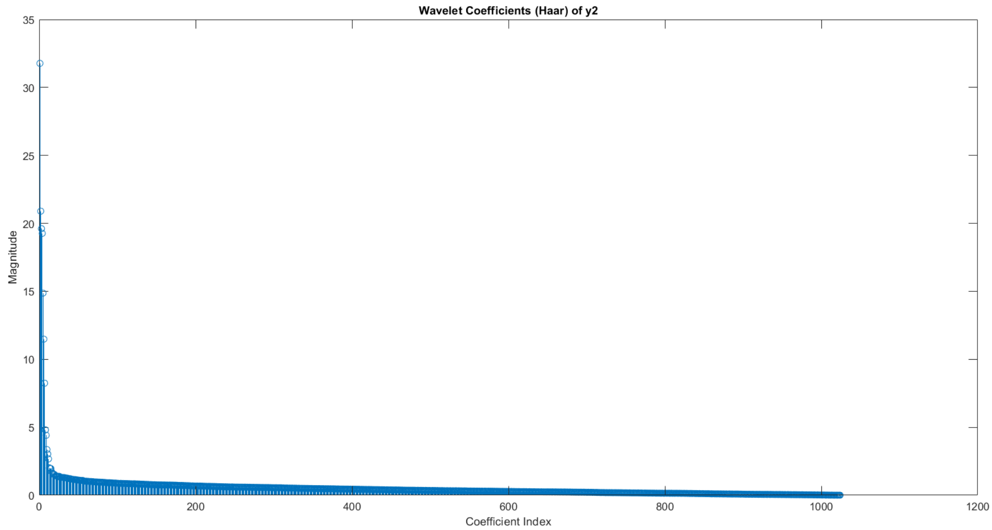


Figure 35: Magnitudes of wavelet coefficients of y2 (with 'haar') in descending order

ii. Select a threshold by observation assuming low magnitude coefficients contain noise. Reconstruct the signal with suppressed coefficients.

```
% threshold for y1 with db9
thresh_y1_db9 = 0.9;
```

Listing 5: Selected threshold for y1 (with ‘db9’)

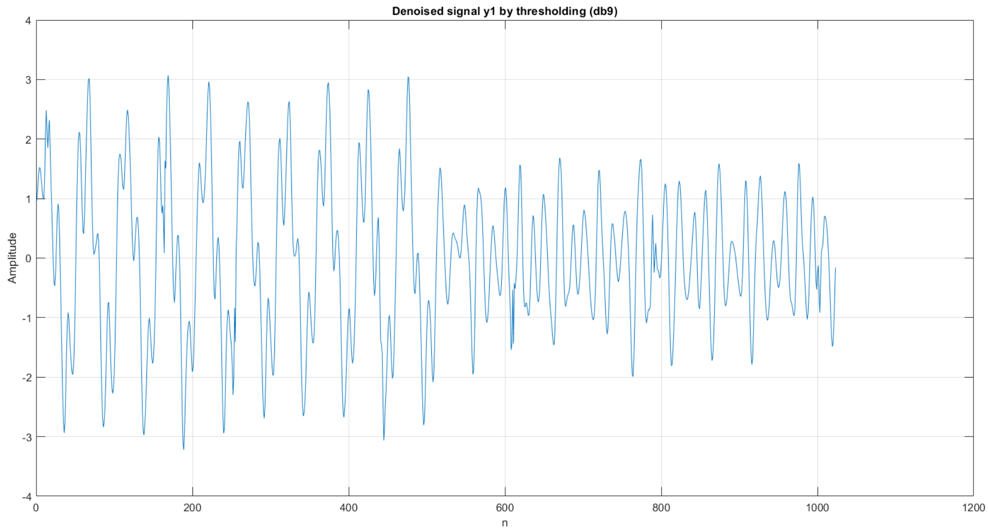


Figure 36: Denoised signal y1 (with ‘db9’)

```
% threshold for y2 with db9
thresh_y2_db9 = 1.5;
```

Listing 6: Selected threshold for y2 (with ‘db9’)

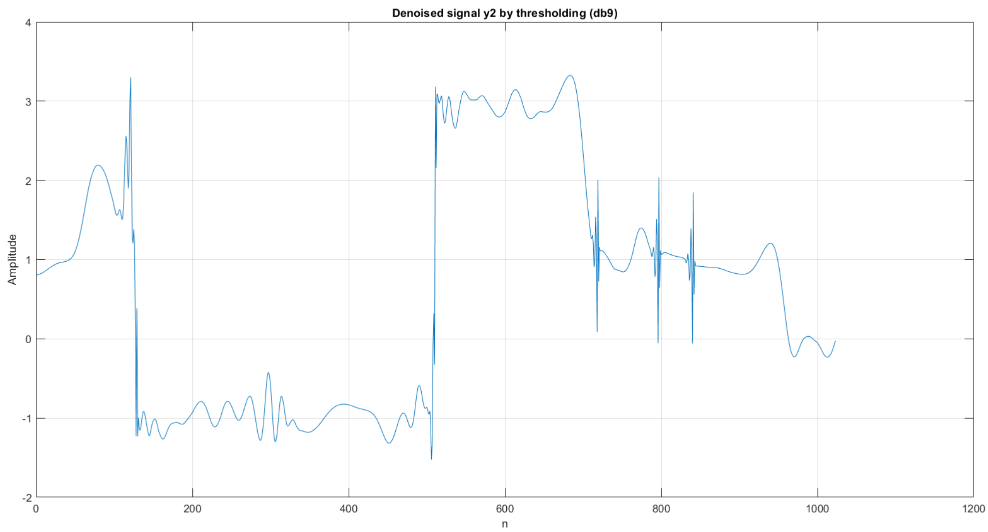


Figure 37: Denoised signal y2 (with ‘db9’)

```
% threshold for y1 with haar
thresh_y1_haar = 0.25;
```

Listing 7: Selected threshold for y1 (with 'haar')

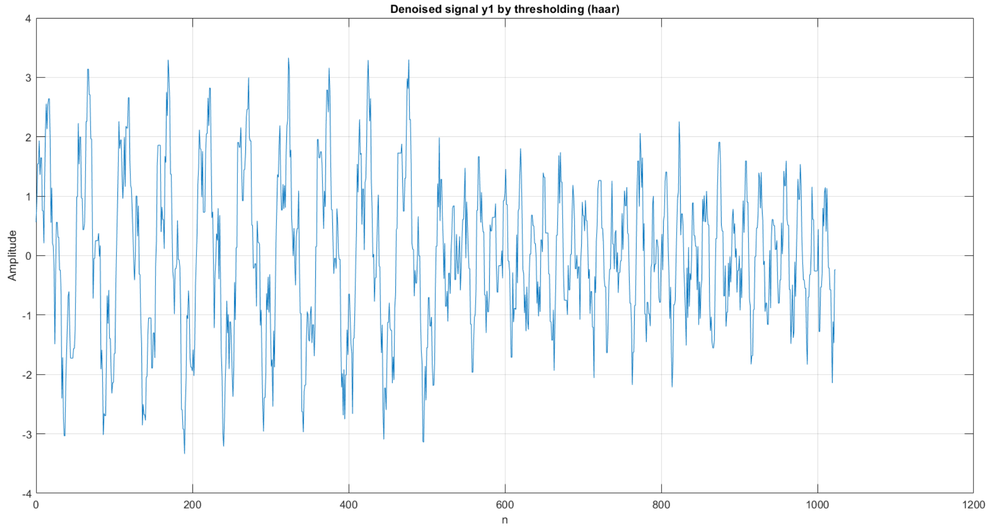


Figure 38: Denoised signal y1 (with 'haar')

```
% threshold for y1 with haar
thresh_y2_haar = 2;
```

Listing 8: Selected threshold for y1 (with 'haar')

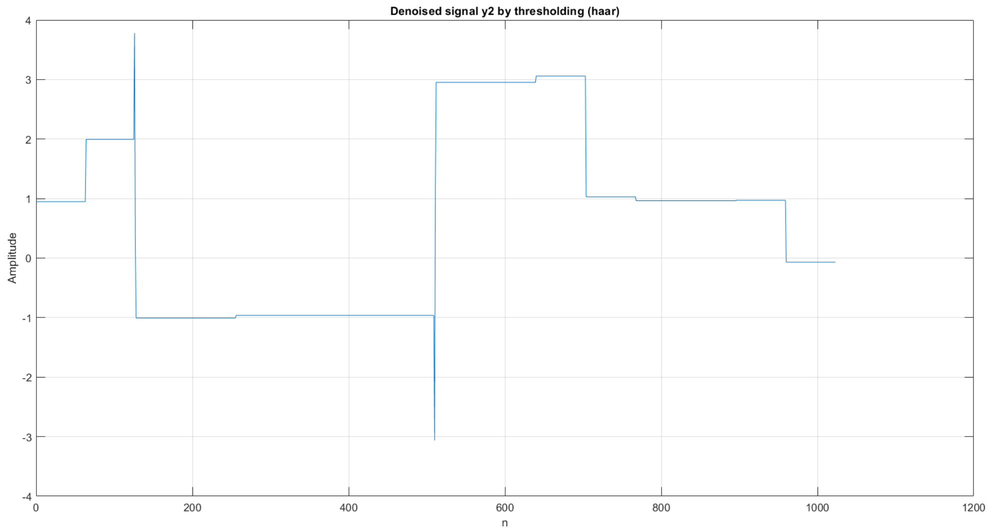


Figure 39: Denoised signal y2 (with 'haar')

iii. Calculate the root mean square error (RMSE) between the original and denoised signal.

Table 2: Root mean square error (RMSE) between original and denoised signals

Wavelet	**Signals**	**Root Mean Square Error (RMSE)**
db9	denoised y1 and x1	0.292020
	denoised y2 and x2	0.269244
haar	denoised y1 and x1	0.389568
	denoised y2 and x2	0.152065

Plot the two signals on the same plot and interpret the results

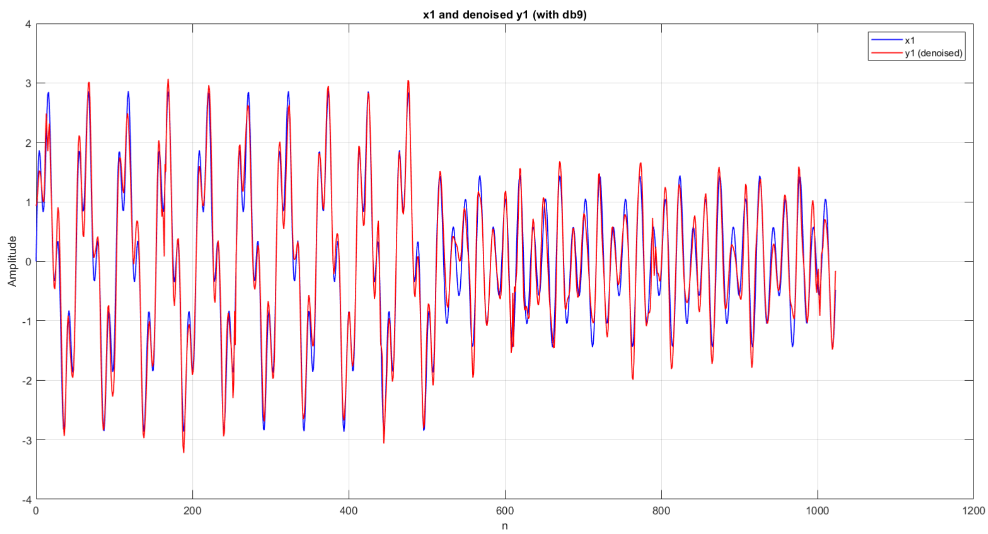


Figure 40: x1 and denoised y1 (with 'db9') on the same figure

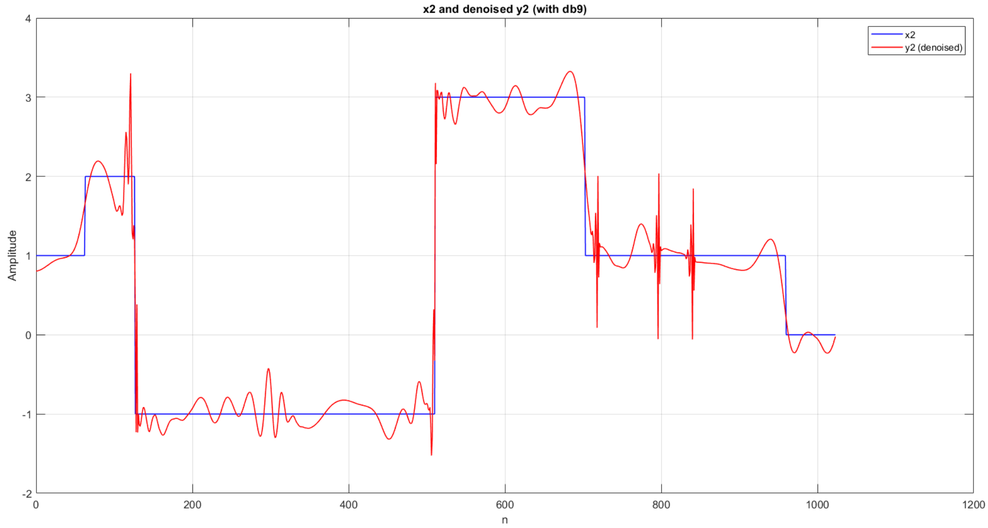


Figure 41: x2 and denoised y2 (with 'db9') on the same figure

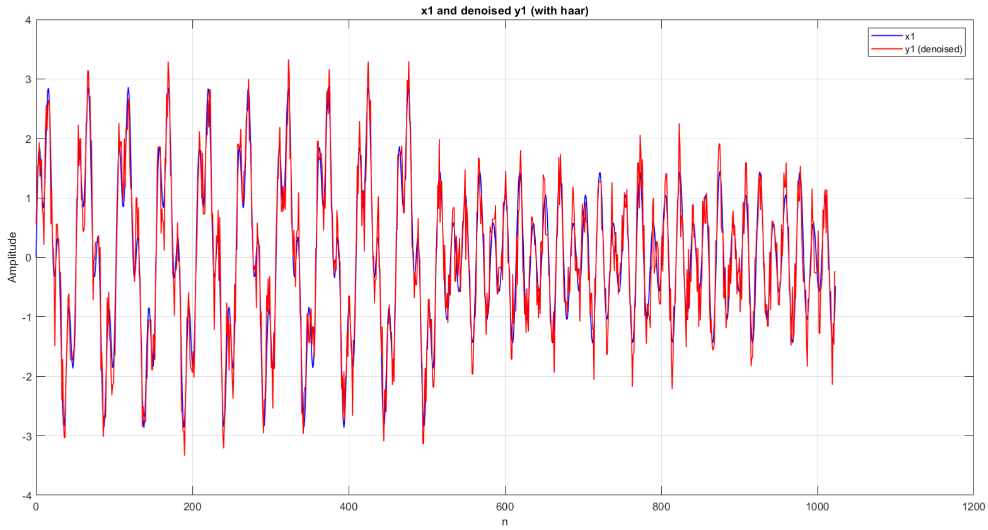


Figure 42: x1 and denoised y1 (with 'haar') on the same figure

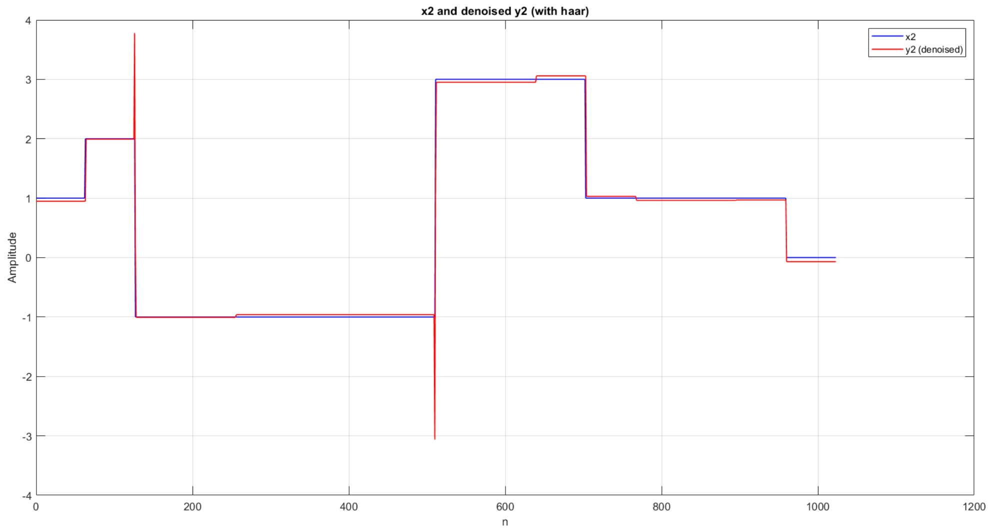


Figure 43: x2 and denoised y2 (with 'haar') on the same figure

iv. Repeat the same procedure with 'haar' wavelet

I have done the same procedure above with the 'haar' wavelet as well.

v. Compare the RMSE and the reconstructed wave morphology of the two waveforms with the two wavelets and comment on the suitability of the wavelets used.

Table 3: Table for comparing the stability of the wavelets used (Based on RMSE)

		Denoisiong	
		y1	y2
Wavelet	db9	**0.292020**	0.269244
	haar	0.389568	**0.152065**

The **'db9'** wavelet shows the better performance when denoising the $y_1[n]$ signal and the **'haar'** wavelet shows better performance when denoising the $y_2[n]$.

This is because the signal $y_1[n]$ is a sinusoidal shape signal (actually sin combination) and the **'db9'** wavelet is more compatible with it. Because of that, 'db9' shows more stability better performance in $y_1[n]$.

When it comes to $y_2[n]$, it is a rectangular signal and the **'haar'** wavelet, that has rectangular shape is more compatible with $y_2[n]$. Because of that, 'haar' shows more stability and better performance in $y_2[n]$.

2.4 Signal Compression with DWT

i. You are given the aV_R lead of an ECG sampled at 257 Hz. Obtain the discrete wavelet coefficients of the signal (use 'db9' and 'haar' wavelets).

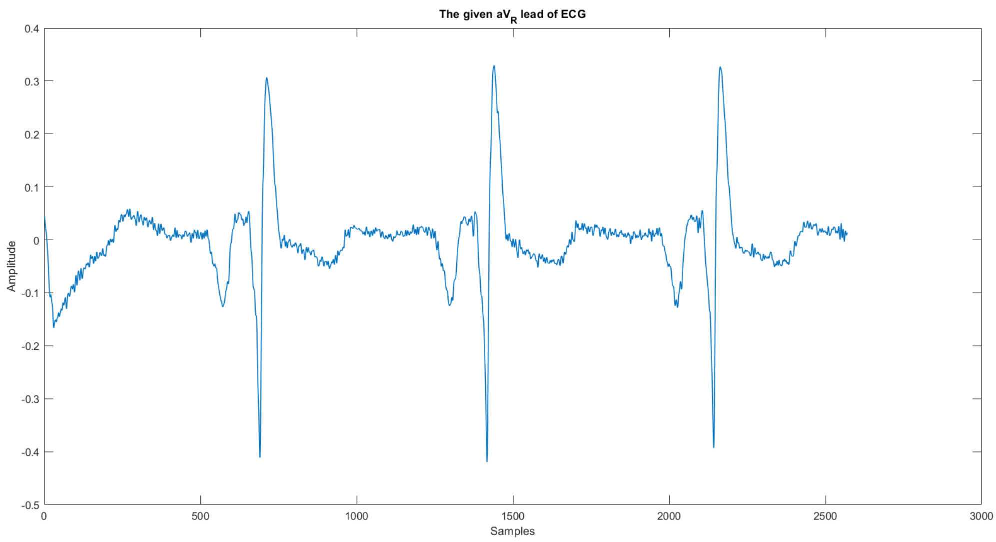


Figure 44: The given $aV_R leadofanECG$

ii. Arrange the coefficients in descending order and find the number of coefficients that are required to represent 99% of the energy of the signal.

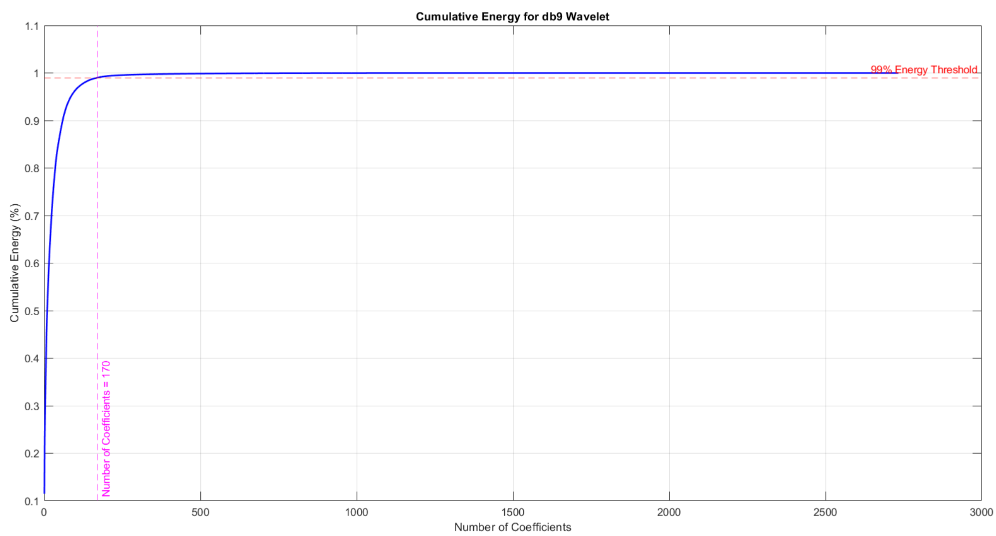


Figure 45: Cumulative energy of 'db9' wavelet coefficients

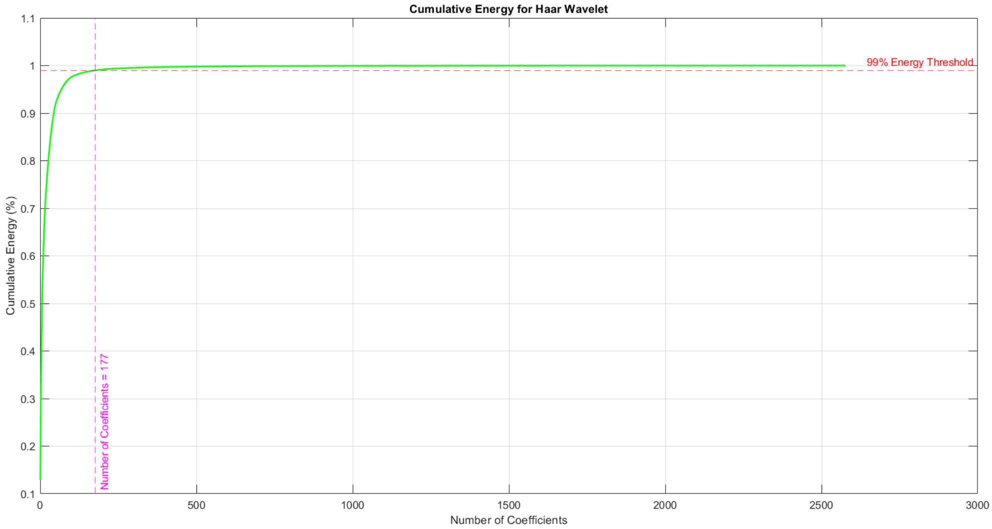


Figure 46: Cumulative energy of 'haar' wavelet coefficients

Table 4: Energy and coefficient details

Wavelet	Number of Coefficients	Total Energy	99% Energy	Required Coefficients
db9	2733	17.9618	16.16562	**170**
haar	2577	15.7709	14.19381	**177**

iii. Compress the signal and find the compression ratio.

When compressing the signal with the 'db9' wavelet, the threshold is set to the absolute value of the amplitude of 170 coefficient of the sorted wavelet coefficient, and all the coefficients that have less amplitude than this threshold are set to zero.

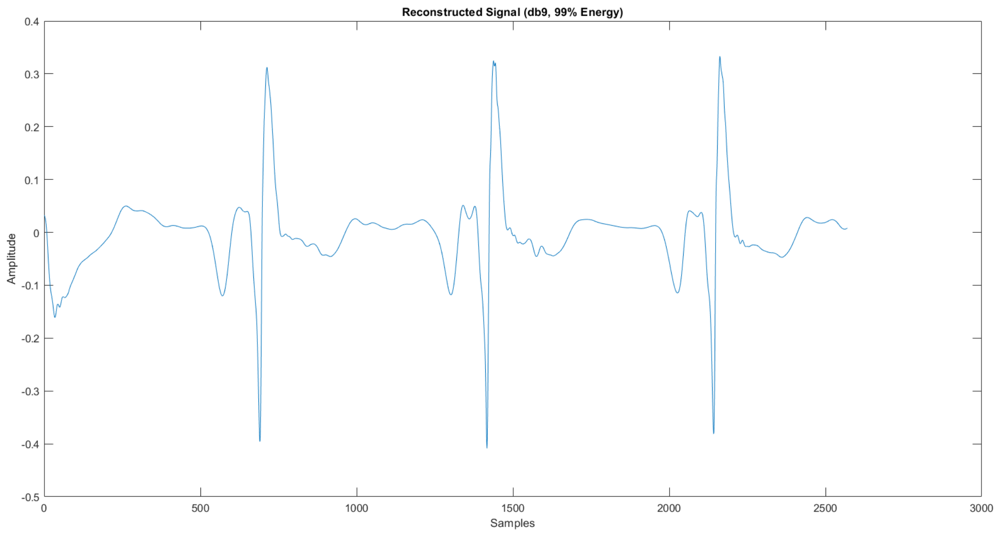


Figure 47: Reconstructed signal using compressed 'db9' coefficients

When compressing the signal with the 'haar' wavelet, the threshold is set to the absolute value of the amplitude of 177 coefficient of the sorted wavelet coefficient, and all the coefficients that have less amplitude than this threshold are set to zero.

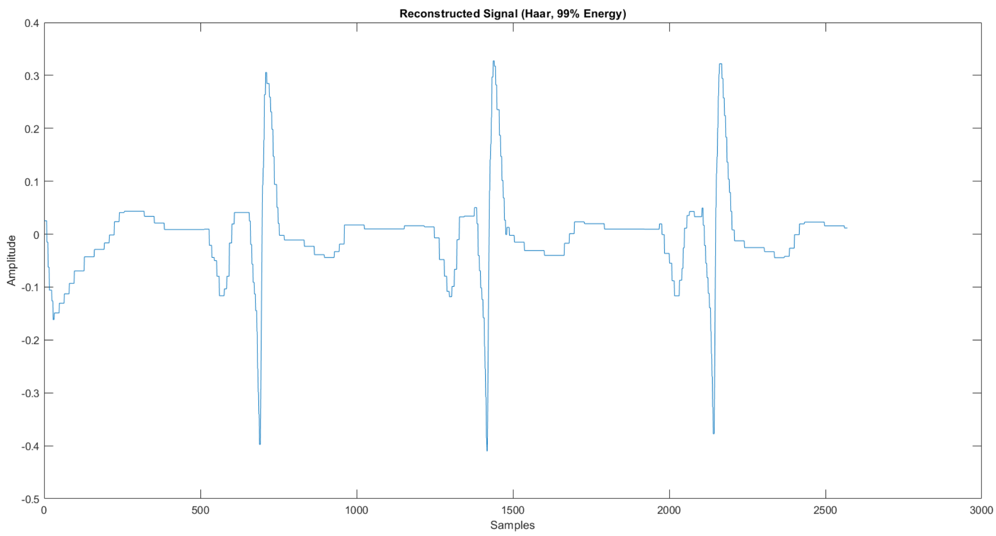


Figure 48: Reconstructed signal using compressed 'haar' coefficients

The compression ratio can be defined as;

$$\text{Compression Ratio (CR)} = \frac{\text{Total number of wavelet coefficients}}{\text{Number of wavelet coefficients after compressed}}$$

Table 5: Compression Ratio

Wavelet	Number of Coefficients	Number of Coefficients After Compressed	Compression Ratio
db9	2733	170	**16.08**
haar	2577	177	**14.56**

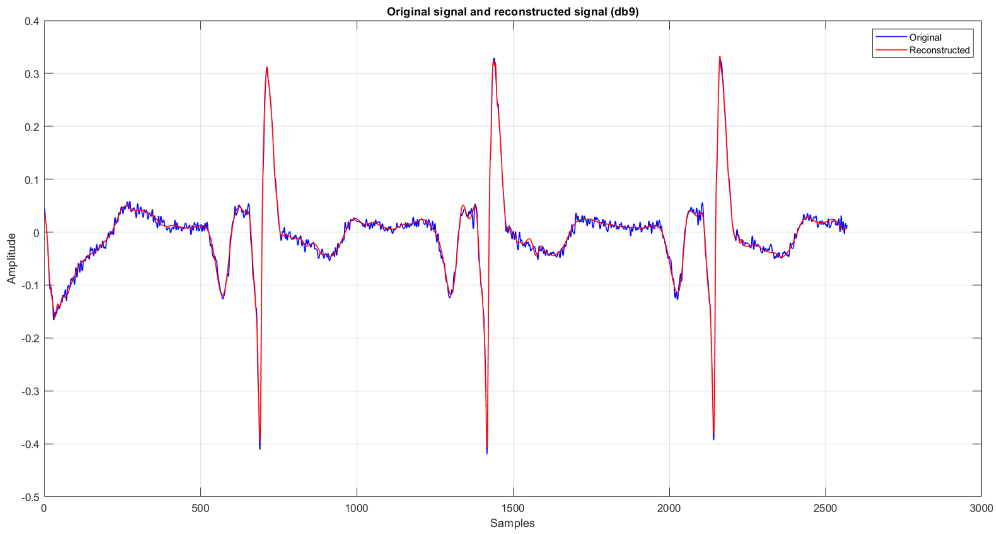


Figure 49: Original aV_R lead signal and reconstructed signal ('db9')

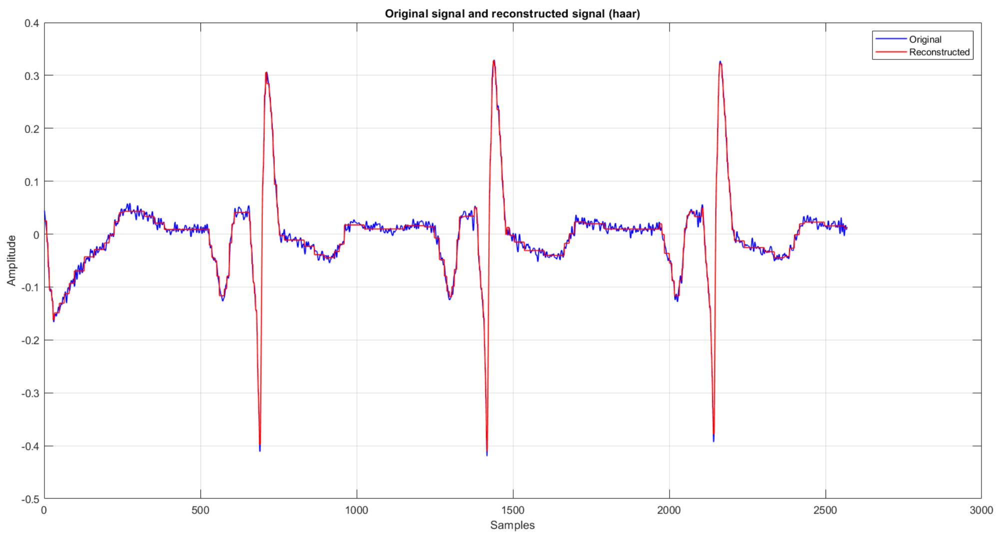


Figure 50: Original aV_R lead signal and reconstructed signal ('haar')

Examining the morphology of the reconstructed signal against the original ECG signal reveals that the reconstruction effectively retains the essential information, preserving the overall shape and key characteristics of the ECG waveform.

Both the ‘db9’ and ‘haar’ wavelets maintain the primary features of the signal; however, the ‘haar’ wavelet compression introduces a noticeable rectangular pulse shape in the reconstructed signal, leading to a loss of finer details. While the fundamental aspects of the waveform remain intact, the use of the Haar wavelet results in a more pronounced distortion, which may affect the accuracy of specific measurements derived from the ECG.

In contrast, the ‘db9’ wavelet compression showcases a more refined reconstruction, closely resembling the original signal while minimizing the loss of critical morphological details.