EN3150 Assignment 04: Kernel Methods

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1. $\phi(x) = (1, \sqrt{2}x, x^2)$
$\phi(z) = (1, \sqrt{2}z, z^2)$

$$\langle \phi(x), \phi(z) \rangle = \langle (1, \sqrt{2}x, x^2), (1, \sqrt{2}z, z^2) \rangle$$

$$= 1 + 2xz + x^2z^2$$

$$= (1 + xz)^2$$

$$\therefore \quad k(x, z) = (1 + xz)^2$$

2. If $x = (x_1, x_2)$ and $z = (z_1, z_2)$

$$k(x, z) = (1 + \langle (x_1, x_2), (z_1, z_2) \rangle)^2$$

$$= (1 + x_1z_1 + x_2z_2)^2$$

3. $k(x, z) = (1 + x_1z_1 + x_2z_2)^2$

$$= (1 + x_1z_1 + x_2z_2)(1 + x_1z_1 + x_2z_2)$$

$$= 1 + x_1^2z_1^2 + x_2^2z_2^2 + 2x_1z_1 + 2x_2z_2 + 2x_1z_1x_2z_2$$

4. $k(x, z) = 1 + x_1^2 z_1^2 + x_2^2 z_2^2 + 2x_1 z_1 + 2x_2 z_2 + 2x_1 z_1 x_2 z_2$

$= (1, x_1^2, x_2^2, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_1 x_2)$
$(1, z_1^2, z_2^2, \sqrt{2}z_1, \sqrt{2}z_2, \sqrt{2}z_1 z_2)$

$= (1, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_1 x_2, x_1^2, x_2^2)$
$(1, \sqrt{2}z_1, \sqrt{2}z_2, \sqrt{2}z_1 z_2, z_1^2, z_2^2)$

$= \langle \phi(x_1, x_2), \phi(z_1, z_2) \rangle$

$\therefore \phi(x) = \phi(x_1, x_2) = (1, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_1 x_2, x_1^2, x_2^2)$

4.
$$G_1 = \begin{bmatrix} k(x_1,x_1) & k(x_1,x_2) & k(x_1,x_3) & k(x_1,x_4) \\ k(x_2,x_1) & k(x_2,x_2) & k(x_2,x_3) & k(x_2,x_4) \\ k(x_3,x_1) & k(x_3,x_2) & k(x_3,x_3) & k(x_3,x_4) \\ k(x_4,x_1) & k(x_4,x_2) & k(x_4,x_3) & k(x_4,x_4) \end{bmatrix}$$

$x_1 = (1 \quad 5)$ $x_2 = (3 \quad 4)$ $x_3 = (4 \quad 2)$ $x_4 = (10 \quad 12)$

$$k = (1 + x^T z)^2$$

$$G = \begin{bmatrix} 27^2 & 24^2 & 15^2 & 71^2 \\ 24^2 & 26^2 & 21^2 & 79^2 \\ 15^2 & 21^2 & 21^2 & 65^2 \\ 71^2 & 79^2 & 65^2 & 245^2 \end{bmatrix} = \begin{bmatrix} 729 & 576 & 225 & 5041 \\ 576 & 676 & 441 & 6241 \\ 225 & 441 & 441 & 4225 \\ 5041 & 6241 & 4225 & 60025 \end{bmatrix}$$

↑ Symmetric matrix

Eigenvalues;

$\lambda_1 = 61397.1$
$\lambda_2 = 382.206$
$\lambda_3 = 81.5318$
$\lambda_4 = 10.1186$

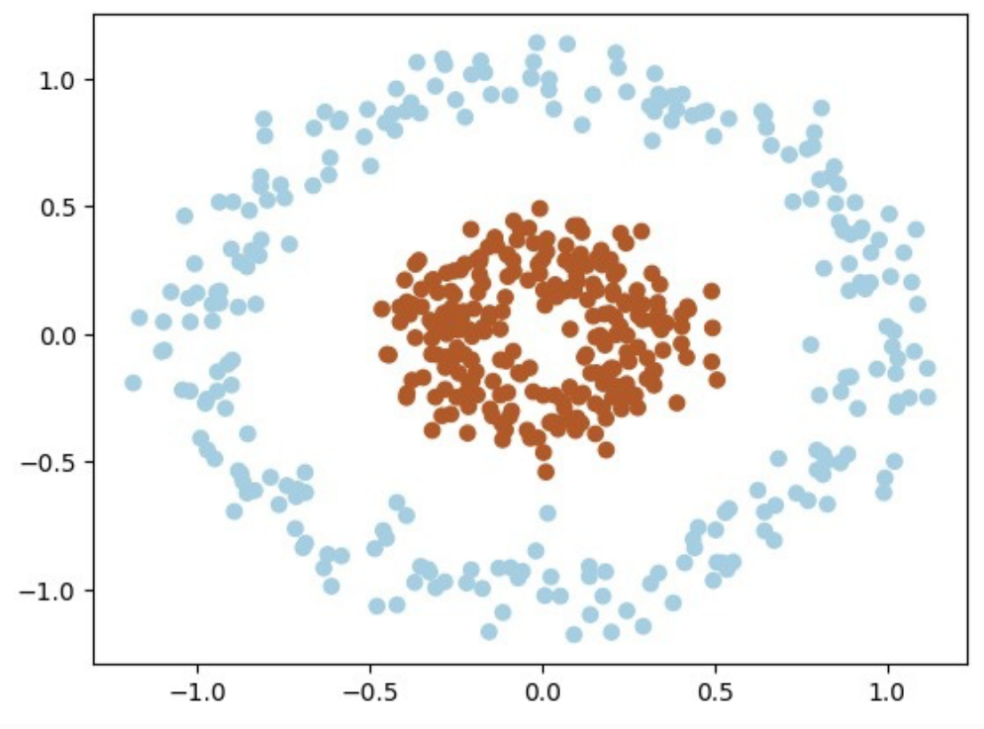
} All eigen values are non-negative

The kernel matrix is a symmetric matrix and all the eigenvalues are non-negative.

∴ The kernel matrix is a positive semi definite matrix.

∴ This is a valid kernel

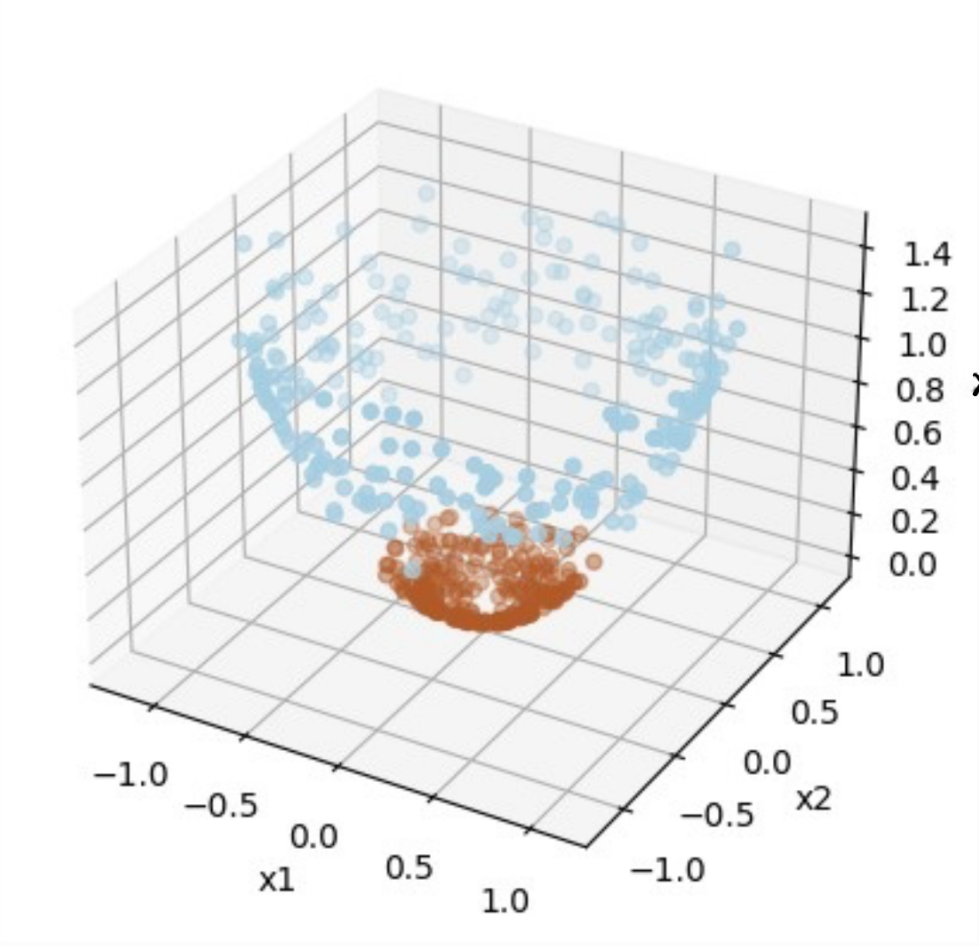
5. a)



Data generation

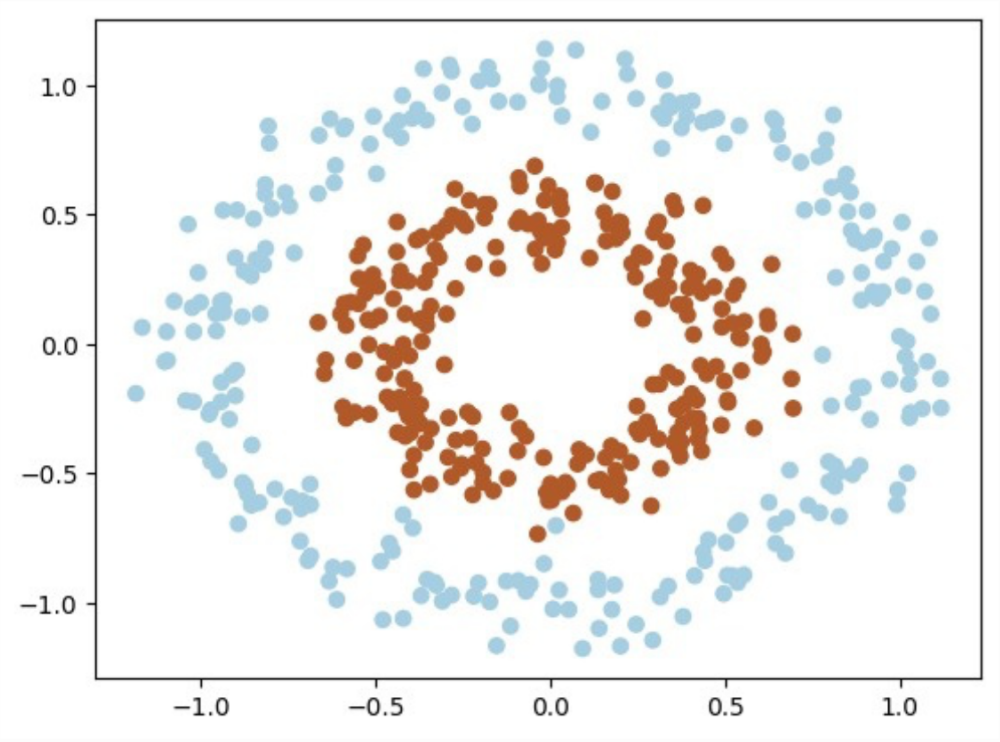
b) Data mapping: 2D space ⟶ 3D space

ϕ: $x = (x_1, x_2) \longrightarrow \phi(x) = (x_1, x_2, x_1^2 + x_2^2) \in \mathbb{R}^3$

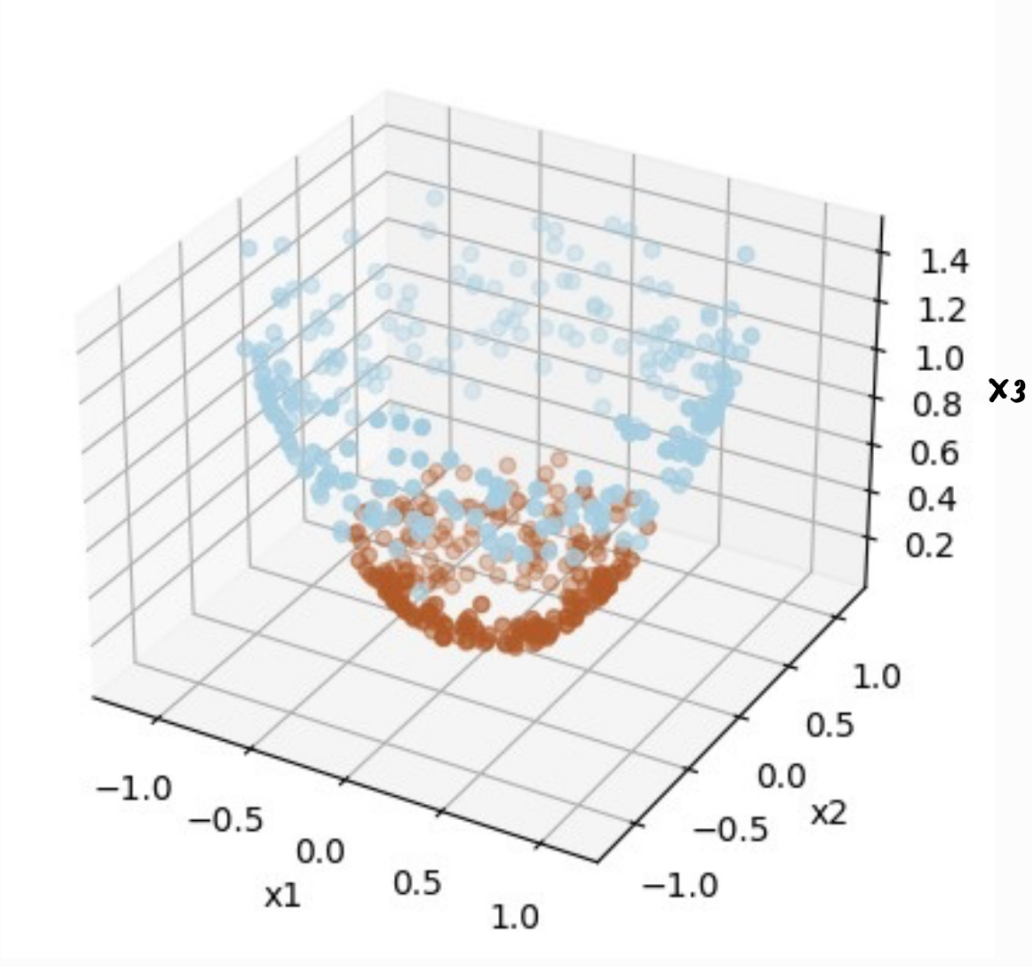


We can see here, the data is linearly separable in 3D space.

c) Change factor to 0.5



2D - space



3D - space

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X, y = make_circle(n_samples = 500, factor = 0.5, noise = 0.1)
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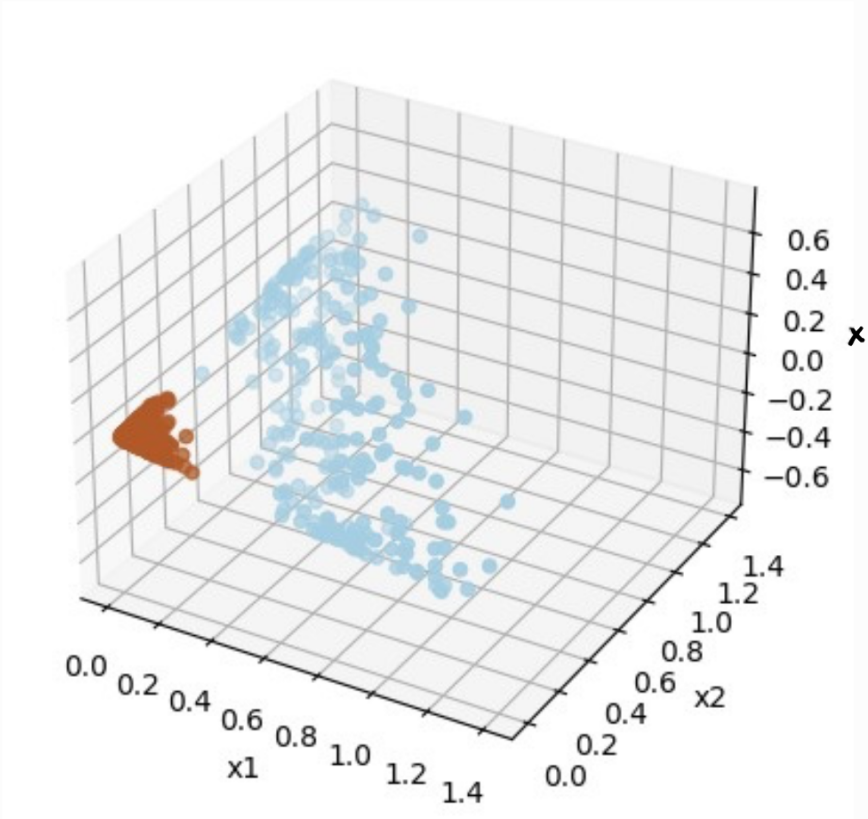
In this line in the data generation code, the "factor" parameter controls the relative size of the inner circle compared to the outer circle.

Increasing the "factor" value makes the inner circle more prominent and larger in size relative to the outer circle. This change could effect the difficulty of the classification task, as the circles become more overlapping and less distinct.

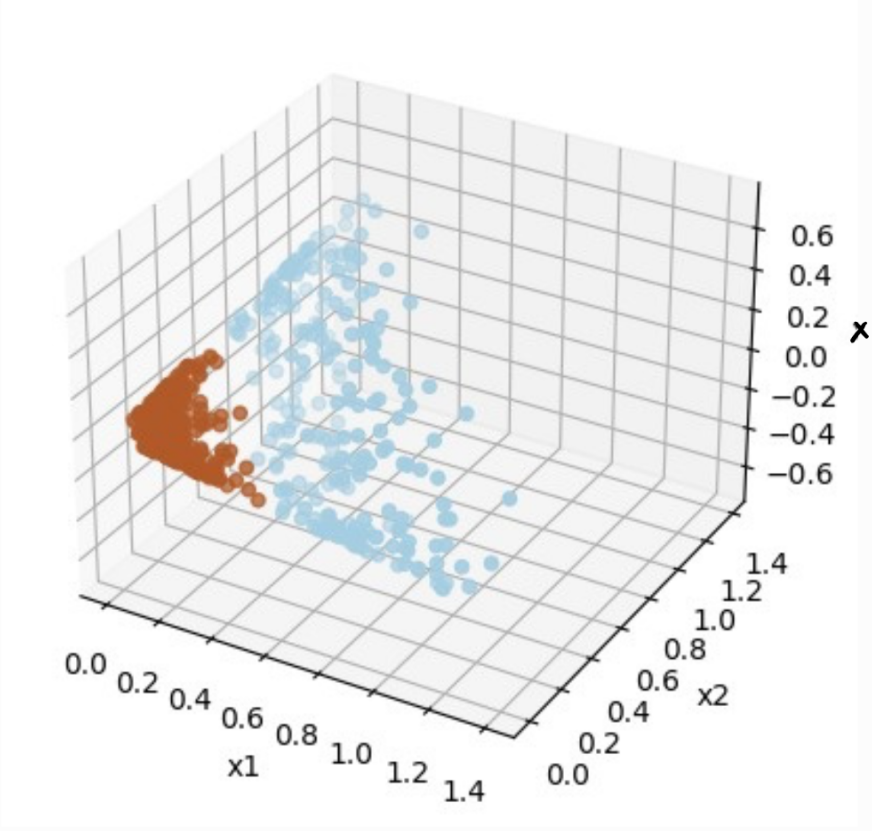
With "factor = 0.5", the inner circle have a larger radius compared to the "factor = 0.3" and we can observe overlapping datas in 2D projection as well as 3D projection.

New data mapping function

$$\phi : x = (x_1, x_2) \longrightarrow (x_1^2, x_2^2, x_1 x_2)$$



factor = 0.3



factor = 0.5

Compared to the first mapping function, we can see a very clear boundery between two classes in this mapping functions when factor = 0.3

When factor = 0.5 also, first mapping function have more overlapping points compared to the new mapping function.

d) Run linear SVC on data

Data	factor value	Accuracy	Precision	Recall	F1 score
Without mapping	0.3	0.65	0.59	0.96	0.73
	0.5	0.51	0.51	0.66	0.57
With first mapping function	0.3	0.99	0.99	1.00	0.99
	0.5	0.99	0.99	1.00	0.99
With second mapping function	0.3	0.99	0.99	1.00	0.99
	0.5	0.99	0.97	1.00	0.99

Note:

All codes are attached in the .zip file as well as the GitHub repository.

GitHub: https://github.com/RavinduMPK/EN3150-Pattern-Recognition/tree/main/Assignment%2004