EM215: Numerical Methods

Lab Assignment 1

(1) Approximate the first derivative of

$$f(x) = -0.1x^4 - 0.5x^2 - 0.5x + 1.2$$

at $x = 0.5$ with forward, backward and centered difference formulas with step sizes, $h, \frac{h}{2}, \frac{h}{2^2}, \ldots, \frac{h}{2^n}$ where $h = 1.0$ and $n = 10$. Calculate the true error and the true percentage error of each approximation, given that true value of $f'(0.5) = -1.05$.

(i) Give the approximations as a table. Use 10 significant digits to show the true errors.

(ii) Use these approximations to graphically show that the first derivative approximations by the forward and the backward difference formulas have errors of $O(h)$ and that of the centered difference formula are of $O(h^2)$.

(2) Consider the problem of falling body given in page 2 of your lecture note. Refer to exercises 1.1 (d) and 1.2 (c) in pages 4 – 5 as well. Starting with the following mathematical model for the above problem, derive necessary solutions and answer the given questions.

$$\frac{dv}{dt} = g - \frac{c}{m}v$$

(a) Instead of the velocity at time $t = 0$, if it is assumed that the velocity at some time $t = t_x$ was known as $v = v_x$, derive the analytical solution for the velocity, v.

(b) If $t_x = 10\ s$ and $v_x = 44.87\ m/s$, derive a numerical scheme (similar to the one given in page 3) to calculate velocities of the body from time $t = 0 - 10\ s$.

(c) Using the analytical solution you derived in part (a) above, show graphically, the variation of velocity of the body for $t = 0 - 10\ s$. Use black colour for the graph.

(d) Use computer to solve the numerical scheme in part (b) above to give the velocity of the body for $t = 0 - 10\ s$. Use red colour for the graph, and plot it on the same axes as those of (c).

(e) Discuss the possible reasons for any discrepancies of the two solutions (numerical (c) and analytical (d)).

Submit your solutions as a short report. You may use either Matlab or Python for parts (c), (d). Give your codes as an appendix.