UNIVERSITY OF PERADENIYA
Department of Engineering Mathematics

EM215 Numerical Methods (E/20 Semesters 4)

Tutorial 1

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(1) (a) Evaluate the polynomial, $y = x^3 - 7x^2 + 8x - 0.35$ at $x = 1.37$. Use 3 – digit arithmetic with chopping. Evaluate the percent relative error.

(b) Repeat (a) by expressing y as, $y = \big((x-7)x+8\big)x - 0.35$.
Evaluate the error and compare with part (a).

(2) Evaluate e^{-5} using two approaches: $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \ \ldots$
and $e^{-x} = \frac{1}{e^x} = \frac{1}{1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots}$.
Adding term by term estimate e^{-5} with each approach, and compute the true and the approximate relative errors. Take $e^{-5} = 6.737946999085 \times 10^{-3}$ to calculate errors.

(3) Consider the following function.

$$f(x) = \ln x$$

(a) Approximate $f(2)$ with the Taylor series expansions about $x = 1$ up to,
(i) First order, (ii) 3rd order.

(b) Find the maximum possible truncation errors in the above two estimates.

(c) Use the forward and the centered difference formulas to estimate the first derivative of $f(x)$ at $x = 2$ using a step sizes of $h = 0.25$.

(d) Given that $f'(2) = 0.5$, for each estimate in part (c) calculate the absolute error and compare it with the corresponding maximum possible truncation error.

(4) The Stefan-Boltzmann law can be employed to estimate the rate of radiation of energy H from a surface, as in $H = Ae\sigma T^4$ where H is in watts, $A =$ the surface area (m^2), $e =$ the emissivity that characterizes the emitting properties of the surface (dimensionless), $\sigma =$ a universal constant called the Stefan-Boltzmann constant (= 5.67 × 10−8 W m−2 K−4), and $T =$ absolute temperature (K). Determine the error of H for a copper sphere with radius = 0.15 ± 0.01 m, $e = 0.90 \pm 0.05$, and $T = 550 \pm 20$.

(5) Evaluate and interpret the condition numbers for

(a) $f(x) = \frac{e^{-x}-1}{x}$ for $x = 0.001$

(b) $f(x) = \frac{\sin x}{1+\cos x}$ for $x = 1.0001\pi$

(6) Find the root of $f(x) = e^{-x}(3.2\sin x - 0.5\cos x) = 0$ on the interval [3, 4] using the bisection method. Use the absolute error $E_a = 0.05$ and the function value $|f(x_r)| = 0.01$ as the stopping criteria.

(7) Consider finding a root of $x^4 - x - 10 = 0$.

(a) Write three fixed point iteration schemes to solve the problem.

(b) Using an initial guess of 1.0 identify at least one iterative formulation that converges to a root.

(8) (a) Find the root of $5x + \ln x = 10000$, correct to 4 decimal places using the Newton Raphson Method.

(b) Graphically show that $e^{2x} = x + 6$ has two roots. Calculate the roots correct to 4 decimal places using the Newton Raphson Method.

(9) The Babylonian method (around 1500 BC) for finding square roots by hand is $x_{n+1} = (x_n + S/x_n)/2$, for $S > 0$.

(a) Show that this is a fixed point iteration scheme for solving $x^2 - S = 0$.

(b) Use the method to calculate $\sqrt{3}$ accurate to 4 decimal places.

(10) Determine the real root of $f(x) = 4x^3 - 6x^2 + 7x - 2.3 = 0$,

(a) using bisection method. Employ initial guesses of $x_l = 0$ and $x_u = 1.0$ and a percent relative error of 5% as the stopping criteria.

(b) using fixed point iteration method. Write five different fixed point iteration schemes to find the root. Using an initial guess of $x_0 = 0$ and a percent relative error of 5% as the stopping criteria.

(c) using Newton Raphson method. Use the initial guess $x_0 = -0.5$, and a stopping criteria of 1% percent relative error.

(11) The velocity v of a falling parachutist is given by

$$v = \frac{mg}{c}\left(1 - e^{-\frac{c}{m}t}\right)$$

where $g = 9.8\, m/s^2$.

(a) Use the bisection method with $\varepsilon_a < 5\ \%$ to determine the mass m for a parachutist so that the velocity is $v = 35\, m/s$ at $t = 9\, s$. with drag coefficient $c = 15\, kg/s$. Determine suitable x_l, x_u by trial and error.

(b) Using the Newton Raphson method, find the drag coefficient needed so that an 80-kg parachutist has a velocity of $v = 36 m/s$ at $t = 14\, s$. Use a suitable initial guess.

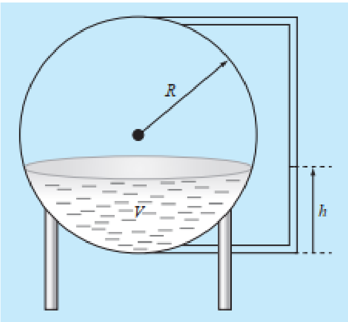
(12) The Manning equation can be written for a rectangular open channel as

$$Q = \left(\sqrt{S}(BH)^{5/3}\right) / \left(n(B + 2H)^{2/3}\right)$$

where $Q =$ flow [m3/s], $S =$ slope [m/m], $H =$ depth [m], and $n =$ the Manning roughness coefficient.

Develop a fixed-point iteration scheme to solve this equation for H given $Q = 5$, $S = 0.0002$, $B = 20$, and $n = 0.03$. Prove that your scheme converges for all initial guesses greater than or equal to zero.

(13)



You are designing a spherical tank to hold water. The volume of liquid it can hold can be computed as, $V = \pi h^2 (3R - h)/3$ where $V =$ volume (m^3), $h =$ depth of water in tank (m), and $R =$ the tank radius (m). If $R = 3$ m, what depth must the tank be filled so that it holds 30 m^3? Use three iterations of the Newton-Raphson method to determine your answer. Determine the approximate relative error after each iteration.