HW 5

Thursday, February 16, 2023 10:21 PM

1

a)

	Penny	Nickel	Dime	Quater
1	H	H	H	H
2	H	H	H	T
	H	H	T	H
	H	T	H	H
	T	H	H	H
3	H	H	T	T
	H	T	T	H
	T	T	H	H
	T	H	H	T
		H	T	
		T	H	
		H	T	
		T	H	
4	H	T	T	T
	T	T	T	H
	T	T	H	T
	T	H	T	T
5	T	T	T	T

*Similiar to Table 2.1

$\Rightarrow 2^4 = 16$ microstates w/ 5 macrostates : 0H, 1H, 2H, 3H, & 4H gives 5 macrostates

b) 1 has 4 head, 2 has 3 heads, 3 has 2 heads, 4 has 1 head, & 5 has 0 heads $\therefore$ The Macrostates + their Probabilities:

0H: $\Omega(0) = 1$ + probality $= \boxed{1/16}$
1H: $\Omega(1) = 4$ + probability $= 4/16 = \boxed{1/4}$
2H: $\Omega(2) = 6$ + Probability $= 6/16 = \boxed{3/8}$
3H: $\Omega(3) = 4$ + Probability $= 4/16 = \boxed{1/4}$
4H: $\Omega(4) = 1$ + Probability $= \boxed{1/16}$

Brute force from the table in a)

c) $\text{Prob} = \frac{\Omega(n)}{\Omega(all)}$, from eq 2.1 + $\Omega(N,n) = \frac{N!}{n!(N-n)!}$, where $N = 4$ coins, from eq 2.6, where $\Omega(n)$ is the Multiplicity

0H: $\text{Prob}_0 = \frac{\Omega(0)}{\Omega(all)} = \frac{4!}{0!\cdot(4-0)!}\cdot\Omega(all)^{-1}$, $\Omega(all) = 2^4 = 16$, $= \frac{4}{4}16^{-1} = \boxed{\frac{1}{16}}$

1H: $\text{Prob}_1 = \frac{\Omega(1)}{16} = \frac{4!}{16\cdot1!(4-1!)} = \frac{4\cdot6}{16\cdot6} = \frac{4}{16} = \boxed{\frac{1}{4}}$

2H: $\text{Prob}_2 = \frac{\Omega(2)}{16} = \frac{4!}{16\cdot2!(4-2!)} = \frac{4\cdot3\cdot2}{16\cdot2\cdot2} = \frac{6}{16} = \boxed{\frac{3}{8}}$

3H: $\text{Prob}_3 = \frac{\Omega(3)}{16} = \frac{4!}{16\cdot3!(4-3)!} = \frac{4\cdot3\cdot2}{16\cdot3\cdot2\cdot1} = \frac{4}{16} = \boxed{\frac{1}{4}}$

4H: $\text{Prob}_4 = \frac{\Omega(4)}{16} = \frac{4!}{16\cdot4!(4-4)!} = \boxed{\frac{1}{16}}$

2 20 coins $\therefore N = 20$

a) Microstates $= 2^N = 2^{20} = \underline{1{,}048{,}576 \text{ microstates} = \Omega(all)}$

b) b/c there are so many microstates + using fair coins, each sequence is equally probable to $\sim \frac{1}{2^{20}}$

c) $12H = n \therefore \Omega(n) = \Omega(12) = \frac{20!}{12!(20-12)!} = \frac{20\cdot19\cdot18\cdot17\cdot16\cdot15\cdot14\cdot13\cdot12!}{12!\ 8!} = \frac{20\cdot19\cdot18\cdot17\cdot16\cdot15\cdot14\cdot13}{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2} = \frac{4\cdot19\cdot3\cdot17\cdot2\cdot5\cdot2\cdot13}{4\cdot2} = 19\cdot3\cdot17\cdot10\cdot13 = \boxed{125{,}970}$

$\therefore \text{Prob} = \frac{\Omega(n)}{\Omega(all)} = \frac{125{,}978}{1{,}048{,}576} = .12 = \boxed{12\%}$

Can also write $\Rightarrow \binom{20}{12} = \binom{20}{8} = 12\%$

3 50 coins $\therefore N = 50$

a) # of Microstates $= \Omega(all) = 2^N = 2^{50} = \boxed{1.1259\times10^{15}}$

b) Diff ways for 25H + 25T? $\rightarrow$ Multiplicity of 25H $= \Omega(25) = \binom{50}{25} = \boxed{1.264\times10^{14}}$

c) Probability of 25H? $\rightarrow P(25) = \frac{\Omega(25)}{\Omega(all)} = \binom{50}{25}/1.1259\times10^{15} = 0.1123 = \boxed{11.23\%}$

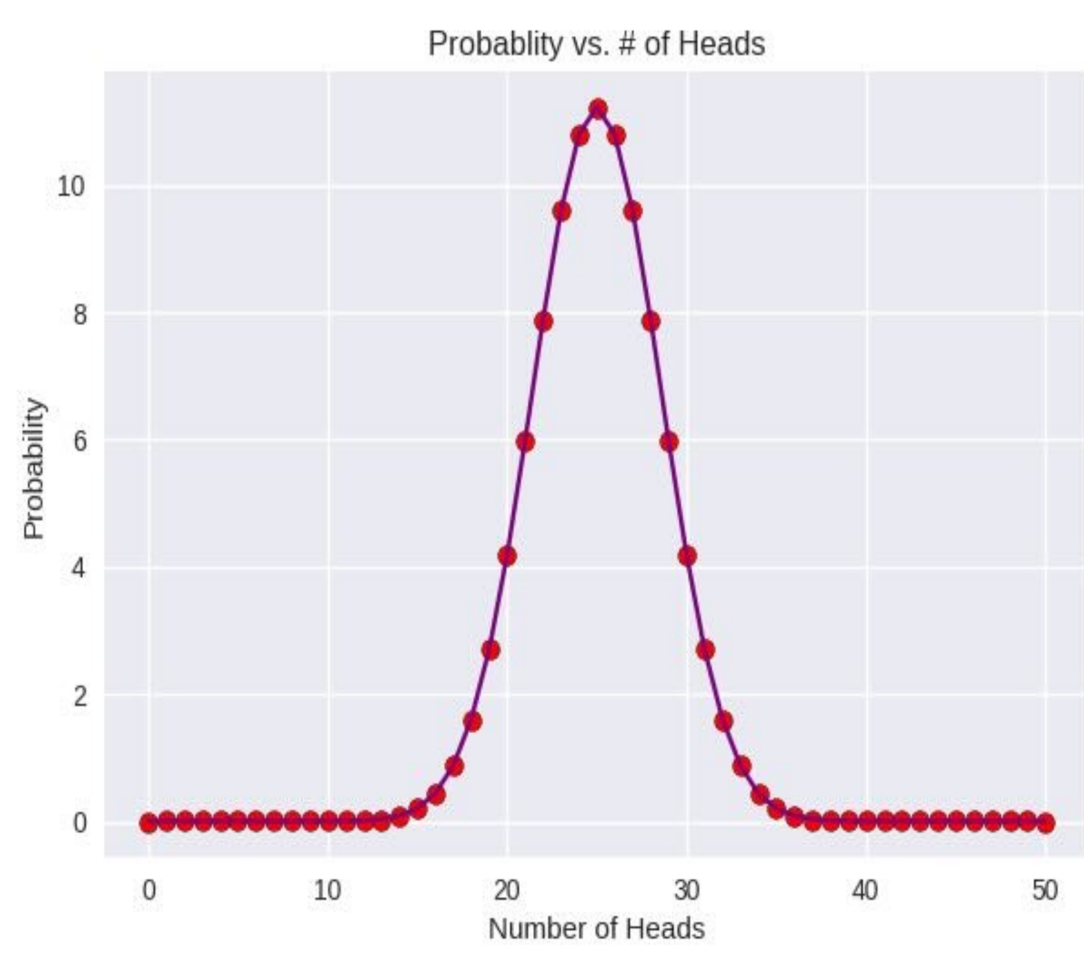
d) $P(30)H = P(20)T = \binom{50}{30}/1.1259\times10^{15} = .0419 = \boxed{4.19\%}$

e) $P(40)H = P(10)T = \binom{50}{10}/1.1259\times10^{15} = 9.124\times10^{-6} = \boxed{9.124\times10^{-4}\%}$

f) $P(50)H = P(0)T = \binom{50}{50}/1.1259\times10^{15} = 8.88\times10^{-16} = \boxed{8.88\times10^{-14}\%}$

$\therefore$ Highest Probability is if even amount of H & T at 25H & 25T

g) Making a dataset including Probality, Multiplicities, & # of Heads to plot the curve in Python:



Probability	Num of Heads	Multiplicity	N	Mult of All
8.88178419700125E-14	0	1	50	1125899906842624
4.44089209850062E-12	1	50		
1.08801856413265E-10	2	1225		
1.74082970261224E-09	3	19600		
2.04547490056939E-08	4	230300		
1.88183690852384E-07	5	2118760		
1.41137768139288E-06	6	15890700		
8.87151685446952E-06	7	99884400		
4.76844030927736E-05	8	536878650		
0.000222527214432944	9	2505433700		
0.000912361579175069	10	10272278170		
0.00331767846972752	11	37353738800		
0.0107824550266145	12	121399651100		
0.0315179454624115	13	354860518600		
0.083297427293516	14	937845656300		
0.199913825504439	15	2250829575120		
0.437311493290959	16	4923689695575		
0.874622986581919	17	9847379391150		
1.60347547540018	18	18053528883775		
2.7005902743582	19	30405943383200		
4.18591492525522	20	47129212243960		
5.97987846465031	21	67327446062800		
7.88256706703905	22	88749815264600		
9.59616860335188	23	108043253365600		
10.7956896787709	24	121548660036300		
11.2275172659217	25	126410606437752		
10.7956896787709	26	121548660036300		
9.59616860335188	27	108043253365600		
7.88256706703905	28	88749815264600		
5.97987846465031	29	67327446062800		
4.18591492525522	30	47129212243960		
2.7005902743582	31	30405943383200		
1.60347547540018	32	18053528883775		
0.874622986581919	33	9847379391150		
0.437311493290959	34	4923689695575		
0.199913825504439	35	2250829575120		
0.083297427293516	36	937845656300		
0.0315179454624115	37	354860518600		
0.0107824550266145	38	121399651100		
0.00331767846972752	39	37353738800		
0.000912361579175069	40	10272278170		
0.000222527214432944	41	2505433700		
4.76844030927736E-05	42	536878650		
8.87151685446952E-06	43	99884400		
1.41137768139288E-06	44	15890700		
1.88183690852384E-07	45	2118760		
2.04547490056939E-08	46	230300		
1.74082970261224E-09	47	19600		
1.08801856413265E-10	48	1225		
4.44089209850062E-12	49	50		
8.88178419700125E-14	50	1		

4 If we have 52 cards, $N = 52$, + 5 cards per hand, $\Omega(5)$:

$\Omega(all) = 2^{52} = 4.5\times10^{15}$ + $\Omega(5) = \binom{52}{5} = 2{,}598{,}960$

$\therefore$ b/c only 4 hands are Royal Flushes, the Probability of getting a Royal Flush is: $P(RF) = \frac{4}{\Omega(5)} = 1.539\times10^{-6} = \boxed{1.539\times10^{-4}\%}$, which is a little better than 1 in a million but still slim

5 a) $N=3, q=4$:

400 310 301 220 211
040 031 130 202 121
004 103 013 022 112

$3\times5 = \underline{15 \text{ microstates}}$: $\Omega(3,4)\binom{4+3-1}{4} = \frac{(4+3-1)!}{4!(3-1)!} = \frac{6!}{4!\,2!} = \frac{6\cdot5\cdot4\cdot6}{4\cdot6\cdot2} = \frac{30}{2} = \boxed{15}$

b) $N=3, q=5$:

500 410 014 320 023 311 212
050 401 041 302 032 131 122
005 104 140 203 230 113 221

$3\times7 = \underline{21 \text{ microstates}}$: $\Omega(3,5) = \binom{5+3-1}{5} = \frac{7!}{5!(3-1)!} = \frac{7\cdot6\cdot5\cdot4\cdot6}{5\cdot4\cdot6\cdot2} = \frac{7\cdot6}{2} = 7\cdot3 = \boxed{21}$

c) $N=3, q=6$:

600 510 015 420 204 330 321 213 222 114
060 501 105 024 240 033 312 123 411
006 150 051 402 042 303 231 132 141

$3\times9+1 = 27+1 = \underline{28 \text{ microstates}}$: $\Omega(3,6) = \binom{6+3-1}{6} = \frac{8!}{6!(3-1)!} = \frac{8\cdot7\cdot6!}{6!\,2!} = \frac{8\cdot7}{2} = \boxed{28}$

d) $N=4, q=2$:

2000 1100 0101
0200 0110 1001
0020 0011
0002 1010

$4\times2+2 = 8+2 = \underline{10 \text{ microstates}}$: $\Omega(4,2) = \binom{2+4-1}{2} = \frac{5!}{2!(4-1)!} = \frac{5\cdot4\cdot3\cdot2}{2\cdot3\cdot2} = \frac{20}{2} = \boxed{10}$

e) $N=4, q=3$:

3000 2100 2010 0120 1110
0300 0210 1020 1002 0111
0030 0021 2001 0102 1011
0003 1200 0201 0012 1101

$4\times5 = \underline{20 \text{ microstates}}$: $\Omega(4,3) = \binom{3+4-1}{3} = \frac{6!}{3!\,3!} = \frac{6\cdot5\cdot4\cdot3!}{3!\,3!} = \frac{6\cdot5\cdot4}{6} = \boxed{20}$

f) If $N=1$, all the energy will belong to that oscillator, meaning 1 microstate denoted by q :

$\Omega(1,q) = \binom{q+1-1}{q} = \frac{q!}{q!} = \boxed{1}$

g) Since $q=1$, there are an infinite amount of microstates, denoted by N:

$\Omega(N,1) = \binom{1+N-1}{1} = \frac{N!}{(N-1)!} = \boxed{N}$

6 If $N = 30$ & $q = 30$, the # of microstates:

$\Omega(30,30) = \binom{30+30-1}{30} = \frac{59!}{30!\,29!} = \boxed{5.91\times10^{16}}$

7 If $N = 4$, # of lines is $N-1 = 3$ & if $q=2$, 2 dots :

••||| |||••
•|•|| ||•|•
•||•| ||••|
•|||• |•||•
|••|| |•|•|

$5\times2 = 10$ microstates, equivalent from 2.5 d), just diff notation