



**NORTH CAROLINA STATE UNIVERSITY**  
**DEPARTMENT OF ELECTRICAL ENGINEERING**

**MAXIMALLY FLAT RESPONSE FILTER DESIGN IN AWR USING**  
**STUBS.**

**ECE 549 FINAL PROJECT**

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## TABLE OF CONTENT

1. INTRODUCTION
2. DESIGN PROCESS
3. DESIGN OF 3<sup>RD</sup> ORDER FILTER
4. TUNING OF 3<sup>RD</sup> ORDER FILTER
5. DESIGN OF 4<sup>TH</sup> ORDER FILTER
6. TUNING OF 4<sup>TH</sup> ORDER FILTER
7. LOWPASS FILTER DESIGN
8. HAND CALCULATIONS
9. REFERENCES

## INTRODUCTION

A flat band filter is also called a Butterworth filter and when this is plotted on a logarithmic scale, The passband magnitude is flat and rolls off into the stopband when it starts moving away from the center frequency. This filter suffers from a slow roll-off and hence requires more elements than any other design such as Chebyshev or elliptical filters. The filter roll-off is 6dB per octave for every added stage. The design for this filter is governed by the polynomials which determine the coefficients for design. The problem that occurs in this filter is, at higher orders the stability of the filter is very bad and might also introduce phases and noise. This filter is used in communication systems that require a flat response. Butterworth filters are mostly used in the analog domain. They are mostly coupled with amplifiers to produce a noise-free output.

Relative Permittivity	3dB bandwidth (GHz)	Center frequency (GHz)	Lower 3dB frequency (GHz)	Upper 3dB frequency (GHz)	Lower 20 dB attenuation frequency (GHz)	Upper 20 dB attenuation frequency (GHz)
<b>4.2</b>	<b>2.2</b>	<b>12</b>	<b>10.9</b>	<b>13.1</b>	<b>9.8</b>	<b>14.2</b>

Table 1. Assigned parameters for filter designing.

From the above parameters, the order of the filter can be determined using the formula given below.

$$\underline{|T(s=j\omega)|^2 = T_0^2 / (1 + (\omega/\omega_0)^{2n})}$$

$\omega_0$  is the frequency where the equation tends to  $T_0$ .

The above equation is then solved by substituting the LHS with 17db as that is the 3dB point for the given filter design. Solving the above equation gives the value of 2.8. this is taken by considering  $f/f_0 = 2$ .

This is considered as 3. From this, we can say that the filter is a 3<sup>rd</sup> order filter.

While designing filters, rounding off floating points to the highest decimal allows the filter to work in an optimal range as it ensures that the desired frequencies are passed and the unwanted frequencies are attenuated.

## DESIGN PROCESS

The design process for a bandpass filter starts with the calculations for a low-pass filter and is then scaled into a bandpass filter. The lowpass filter design parameters when scaled to bandpass filter parameters give us the following set of equations.

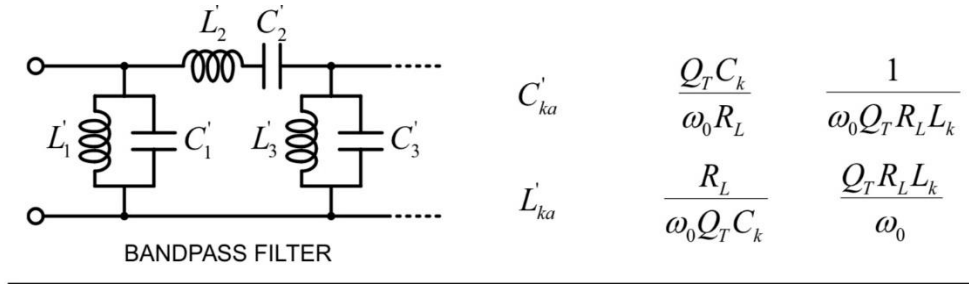


Fig 1 bandpass filter equations

The  $\omega_0$  for the design specifications assigned is 5.45. this can be obtained using the formula below

$$Q_T = \frac{\omega_0}{\omega_2 - \omega_1}$$

Fig 2 . calculation of  $Q_T$

Where  $\omega_0$  = center frequency,  $\omega_1$  = lower frequency,  $\omega_2$  = upper frequency.

For the above set of equations, the following program is scripted in order to automate the calculation process and give us the results. The following program is used to design 3<sup>rd</sup> and 4<sup>th</sup>-order filters.

*The below code allows the user to provide the inputs.*

```
prompt="order of the filter";
n=input(prompt);
prompt2="enter the load resistance value";
Rl=input(prompt2);
prompt3="enter the frequency.";
fc=input(prompt3);
prompt4="enter the Qt value ";
Qt=input(prompt4);
wo=2*3.14*fc;
clc
if (n==3)
*/The below equations are for a 3rd order Butterworth equation./*
    g0=1;
    g1=1;
    g2=2;
    g3=1;
    g4=1;
```

```
*/The below equations give the user L and C values for a 3rd order filter/*
```

```
L1=(R1/(wo*Qt*g1))  
c1=(Qt*g1)/(wo*R1)  
L2=((Qt*R1*g2)/wo)  
C2=1/(wo*Qt*R1*g2)  
L3=(R1/(wo*Qt*g3))  
c3=((Qt*g3)/(wo*R1))
```

```
elseif (n==4)
```

```
*/The below values are the coefficients of a 4th-order polynomial Butterworth equation./*
```

```
g0=1;  
g1=0.7684;  
g2=1.8478;  
g3=1.8678;  
g4=0.7654;  
g5=1;
```

```
clc
```

```
*/The equations below give the user L and C values for a 4th-order filter./*
```

```
L1=(R1/(wo*Qt*g1))  
c1=(Qt*g1)/(wo*R1)  
L2=((Qt*R1*g2)/wo)  
C2=1/(wo*Qt*R1*g2)  
L3=(R1/(wo*Qt*g3))  
c3=((Qt*g3)/(wo*R1))  
L4=((Qt*R1*g4)/wo)  
C4=1/(wo*Qt*R1*g4)
```

```
end
```

```
*/The below equations give us the characteristic impedances required to calculate the  
length and width of the stubs./*
```

```
zo=R1;  
z04=0;  
z01=(zo/((4/3.14)*Qt*g1))  
z02=(zo/((4/3.14)*Qt*g2))  
z03=(zo/((4/3.14)*Qt*g3))  
z04=(zo/((4/3.14)*Qt*g4))
```

The above code gives us the impedances below for both 3<sup>rd</sup> and 4<sup>th</sup>-order filters.

	Z1	Z2	Z3	Z4
3 <sup>rd</sup> order	7.2018	3.6009	7.2018	0
4 <sup>th</sup> order	9.325	3.8975	3.8558	9.4092

Table 2. impedances observed using the code above.

The necessary parameters used for calculating the length and width using the above impedances are

- Substrate thickness 100um(4mil)
- Loss tangent 0.0002
- Diameter of the vias between (5-20) chosen depending on the width of the stub.
- Rho of 0.735
- 1/2oz copper metallization
- Normalized relative permittivity 4.2

The designing process should first be simulated using lumped elements as it gives us a better insight into how the filter should perform once converted into stubs.

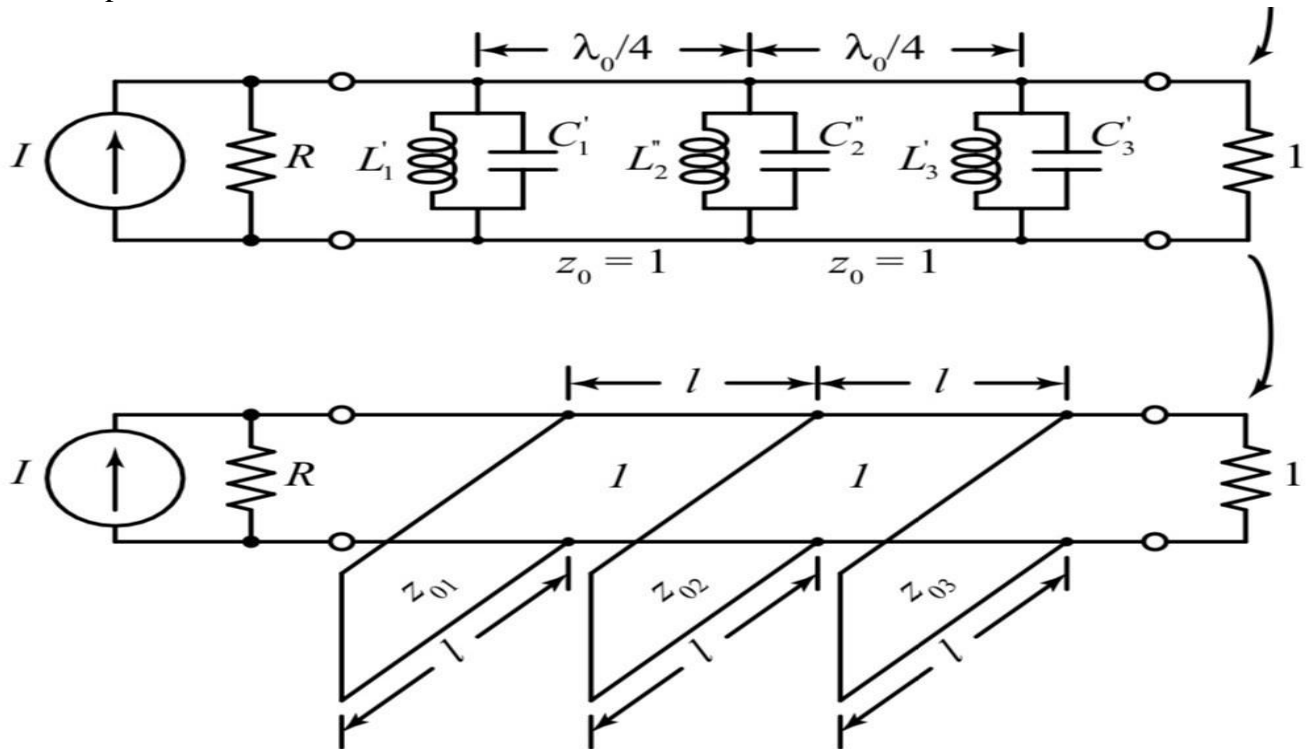


Fig 3. Describes the conversion from lumped elements to stubs.

## DESIGN OF A 3<sup>RD</sup> -ORDER FILTER

The polynomial coefficient for a 3rd-order Butterworth filter is given below.

g0	g1	g2	g3	g4
1	1	2	1	1

*Table 3. 3<sup>RD</sup> order Butterworth polynomial*

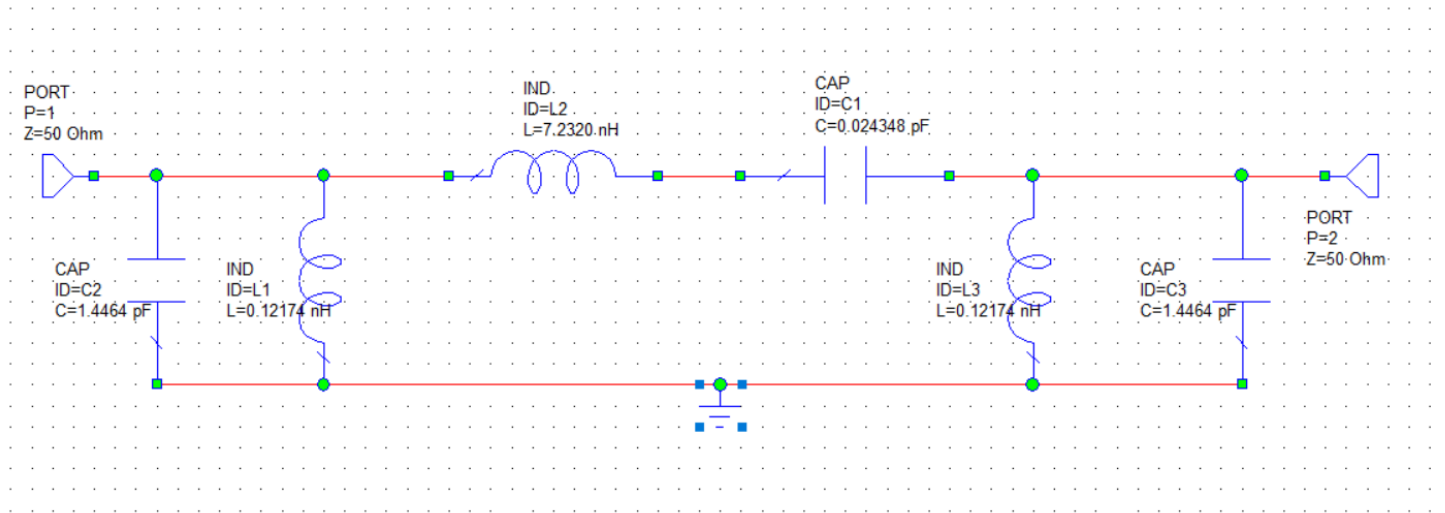
The equations used to design the lumped elements are

$$\begin{aligned}
 L1 &= (Rl / (wo * Qt * g1)); & C1 &= (Qt * g1) / (wo * Rl); \\
 L2 &= ((Qt * Rl * g2) / wo); & C2 &= 1 / (wo * Qt * Rl * g2); \\
 L3 &= (Rl / (wo * Qt * g3)); & C3 &= ((Qt * g3) / (wo * Rl));
 \end{aligned}$$

The above equations give the following values for inductors and capacitors

$$\begin{aligned}
 L1 &= 12.174 \mu\text{H} & C1 &= 1.4464 \text{pF} \\
 L2 &= 7.2320 \text{nH} & C2 &= 24.348 \text{fF} \\
 L3 &= 12.174 \mu\text{H} & C3 &= 1.4464 \text{pF}
 \end{aligned}$$

When we use the above equations in AWR and design using lumped elements, we get the following response



*Fig 4. Lumped elements design of 3<sup>rd</sup> order Butterworth filter*

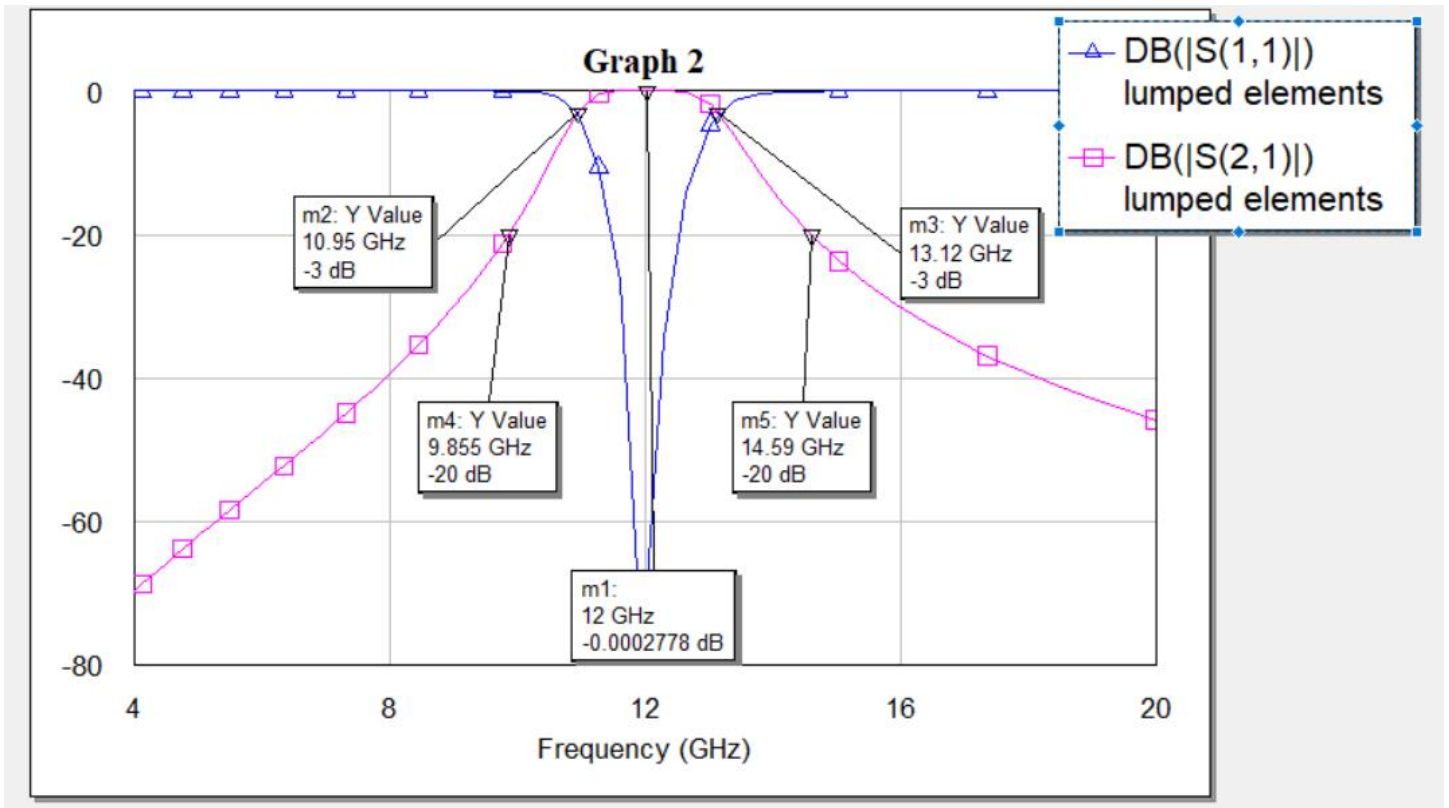


Fig 5. 3<sup>rd</sup> order Butterworth filter response

From the above response, we can estimate the 3dB point and 20dB point.

**The lumped elements are then converted into stubs using the below process.**

1. The design using lumped elements is converted to stub impedances using the following impedance formulas.

$$z_{01} = (z_0 / ((4/3.14) * Q_t * g_1))$$

$$z_{02} = (z_0 / ((4/3.14) * Q_t * g_2))$$

$$z_{03} = (z_0 / ((4/3.14) * Q_t * g_3))$$

the formulas give us the following results

$$Z_{01} = 7.2018\Omega \quad Z_{02} = 3.6009\Omega \quad Z_{03} = 7.2018\Omega$$

2. The following impedances are then plugged into the microstrip length and width calculation equation
3. Rearranging the above equations gives us the length and width requirements for a given microstrip stub. In this case, the length and width of the microstrip stubs are as follows,
4. 3<sup>rd</sup> order filter design using stubs is shown below. The design also includes vias which are used to connect the stubs to the ground. The diameters of the vias are selected by determining the length of the stub to be approximately the diameter of the stub. This practice ensures that the stub is grounded correctly and no reflections occur due to imperfect via matching.



## TUNING OF 3<sup>RD</sup>-ORDER FILTER DESIGN

The tuning of the 3rd-order filter consists of the following steps.

1. Design the stubs with the length and widths calculated.
2. Slowly add and subtract the values of W and L to match the design.
3. This adding and subtracting of values shifts the graph depending on the values provided to reach element.

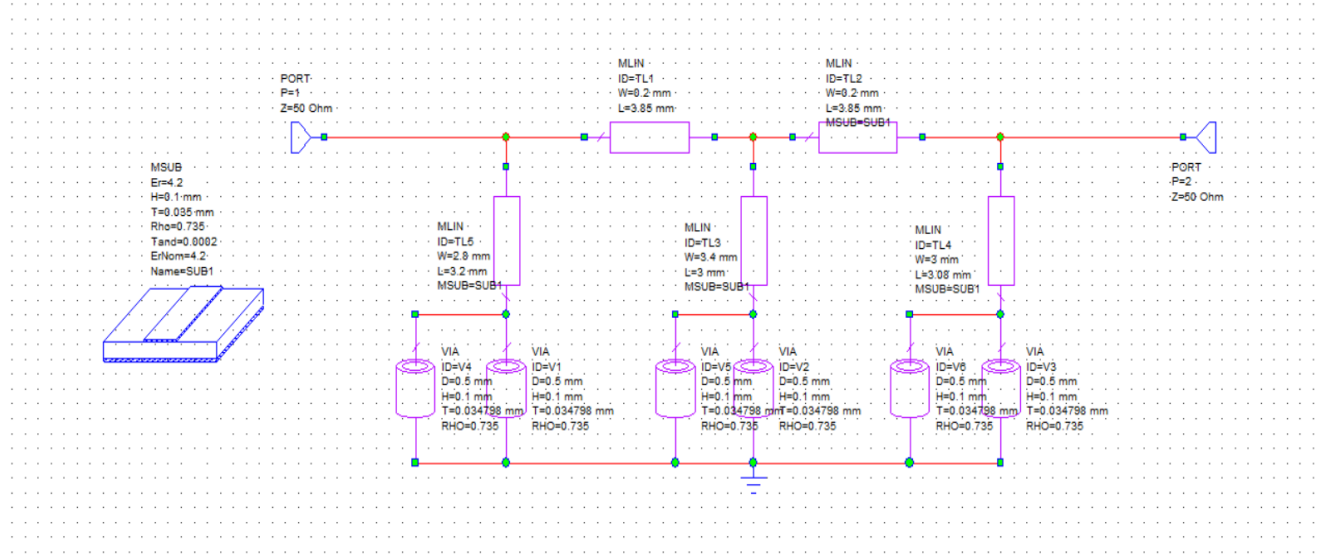


Fig 6. 3<sup>rd</sup> order Butterworth filter implemented in stubs.

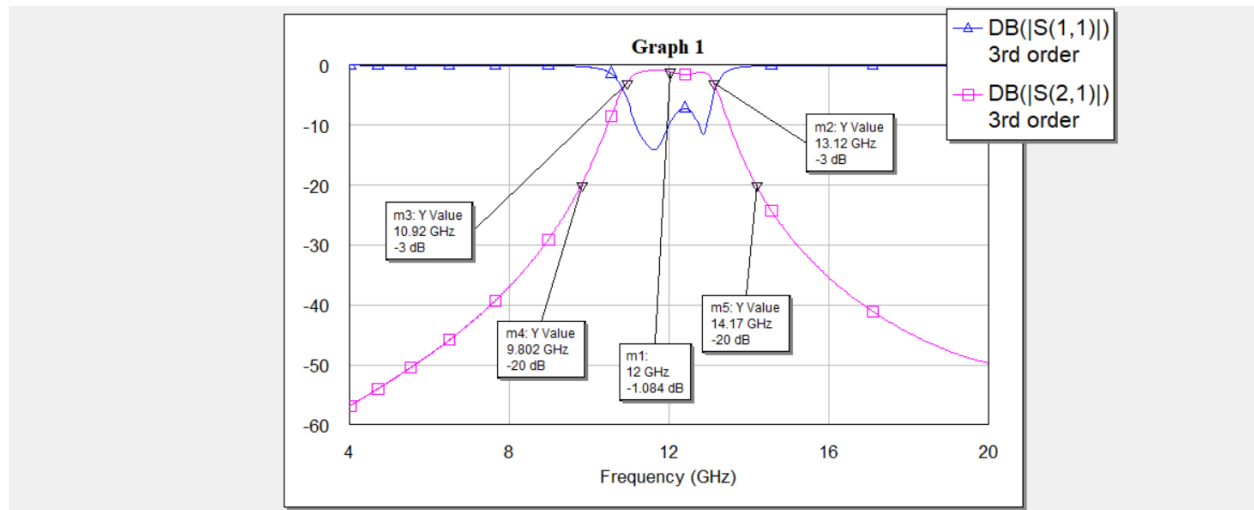


Fig 7. 3<sup>rd</sup> order Butterworth filter graph representing the design parameters.

	Before tuning	After tuning
Stub 1	$W:2.32897172mm$ $L:3.164941854mm$	$W:2.8mm$ $L:3.2$
Stub 2	$W:4.928392226mm$ $L:3.108469526mm$	$W:3.4mm$ $L:3mm$
Stub 3	$W:2.32897172mm$ $L:3.164941854mm$	$W:3.0mm$ $L:3.08mm$
Transmission line stub	$W:0.175584866mm$ $L:3.609673756mm$	$W:0.2mm$ $L:3.85mm$

Table 4. before and after tuning 3<sup>rd</sup> order filter

## DESIGN OF 4<sup>TH</sup> ORDER BUTTERWORTH FILTER

Designing for the 4<sup>th</sup>-order filter requires the fourth-order polynomial equations which are specified in the table below

G <sub>0</sub>	G <sub>1</sub>	G <sub>2</sub>	G <sub>3</sub>	G <sub>4</sub>	G <sub>5</sub>
50	0.7654	1.8478	1.8478	0.7654	50

Table 5. 4<sup>th</sup> order Butterworth Polynomial

By using the equations given below, the capacitors and inductors values can be obtained.

$$\begin{aligned}
 L1 &= (Rl/(wo * Qt * g1)) \\
 c1 &= (Qt * g1)/(wo * Rl) \\
 L2 &= ((Qt * Rl * g2)/wo) \\
 C2 &= 1/(wo * Qt * Rl * g2) \\
 L3 &= (Rl/(wo * Qt * g3)) \\
 c3 &= ((Qt * g3)/(wo * Rl)) \\
 L4 &= ((Qt * Rl * g4)/wo) \\
 C4 &= 1/(wo * Qt * Rl * g4)
 \end{aligned}$$

Using the above equations gives us the following values for 4<sup>th</sup> order Butterworth filter gives us the following lumped elements circuit and its corresponding graph.

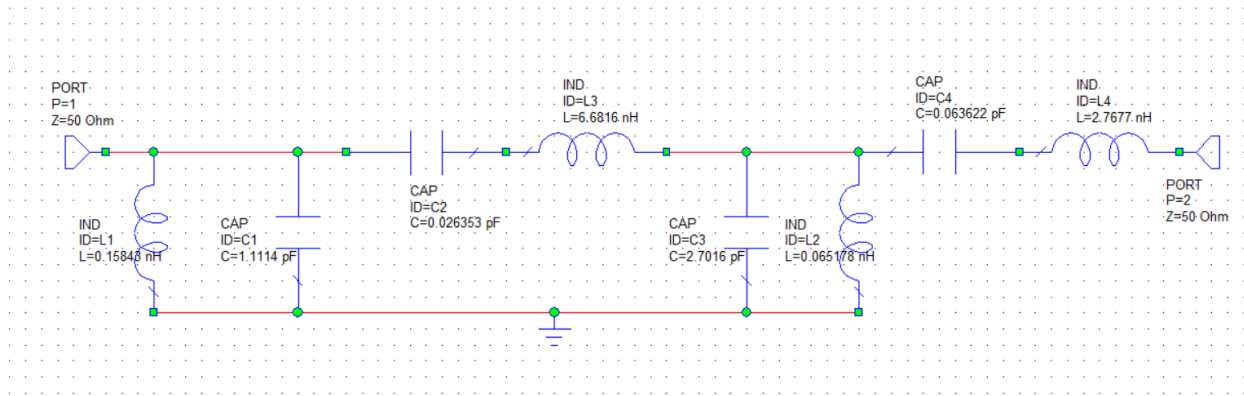


Fig 8. 4<sup>th</sup> order Butterworth filter design using lumped elements.

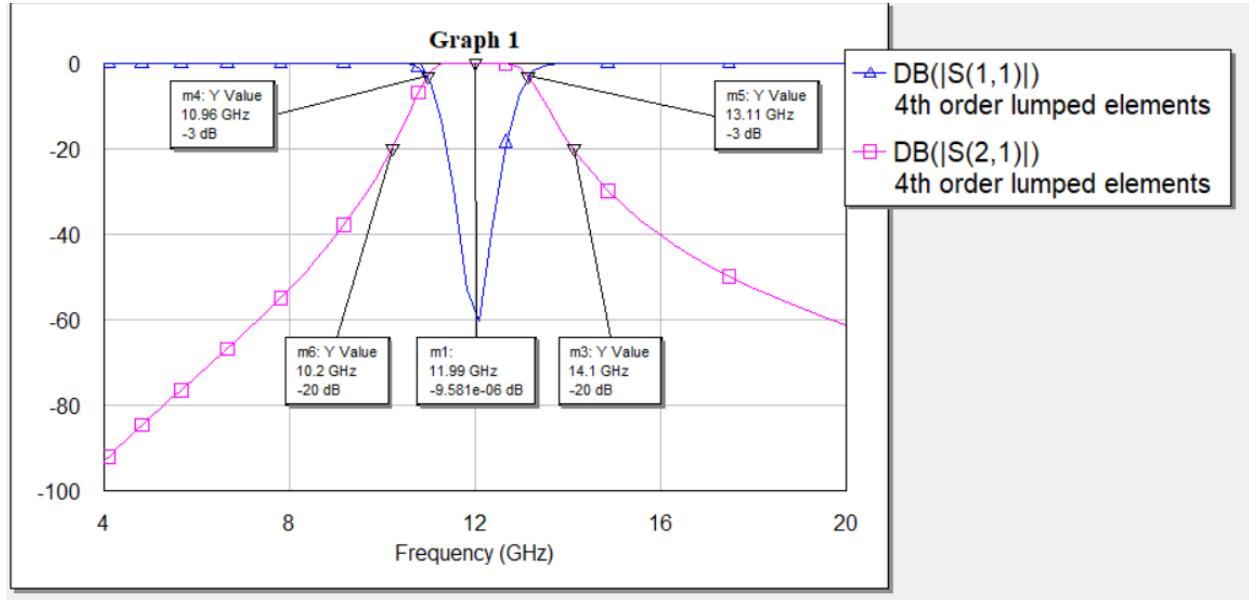


Fig 9. S-parameters graph for 4<sup>th</sup> order Lumped elements

Using the similar equations that are used for the 3<sup>rd</sup> order filters, we can convert the capacitors and inductors into stubs. This is done using the steps below.

1. The equations below are used to calculate the characteristic impedance of the stubs.

$$z_0 = R_1;$$

$$z_{01} = (z_0 / ((4/3.14) * Q_t * g_1))$$

$$z_{02} = (z_0 / ((4/3.14) * Q_t * g_2))$$

$$z_{03} = (z_0 / ((4/3.14) * Q_t * g_3))$$

$$z_{04} = (z_0 / ((4/3.14) * Q_t * g_4))$$

the above equations give us the following stub impedance values

$$z_{01} = 9.3725 \Omega \quad z_{02} = 3.8975 \Omega$$

$$z_{03} = 3.8558 \Omega \quad z_{04} = 9.4092 \Omega$$

using the below equation, we can calculate the length and width of the equations.

2. Using the length and width when plugged into AWR, it can be observed that the 3dB points and 20dB attenuation points are easy to obtain.
3. This filter provides us with better cutoff compared to 3<sup>rd</sup> order filter and also gives us less length and width measurements. Minimal tuning is required to obtain the responses.

## TUNING OF 4<sup>TH</sup> ORDER FILTER

The process for tuning the circuit is as follows.

1. Design the filter for the required frequency.
2. Plot the graph by using S11 and S12 parameters.
3. Mark the 3dB points for the filter.
4. Tune the filter by changing the W and L values to get the desired response.
5. It can be observed that tuning the L values shifts the graph in terms of frequency and attenuation. Tuning the W values widens and shrinks the graphs response.

### THE CIRCUIT AND GRAPH AFTER TUNING THE 4<sup>TH</sup> ORDER FILTER.

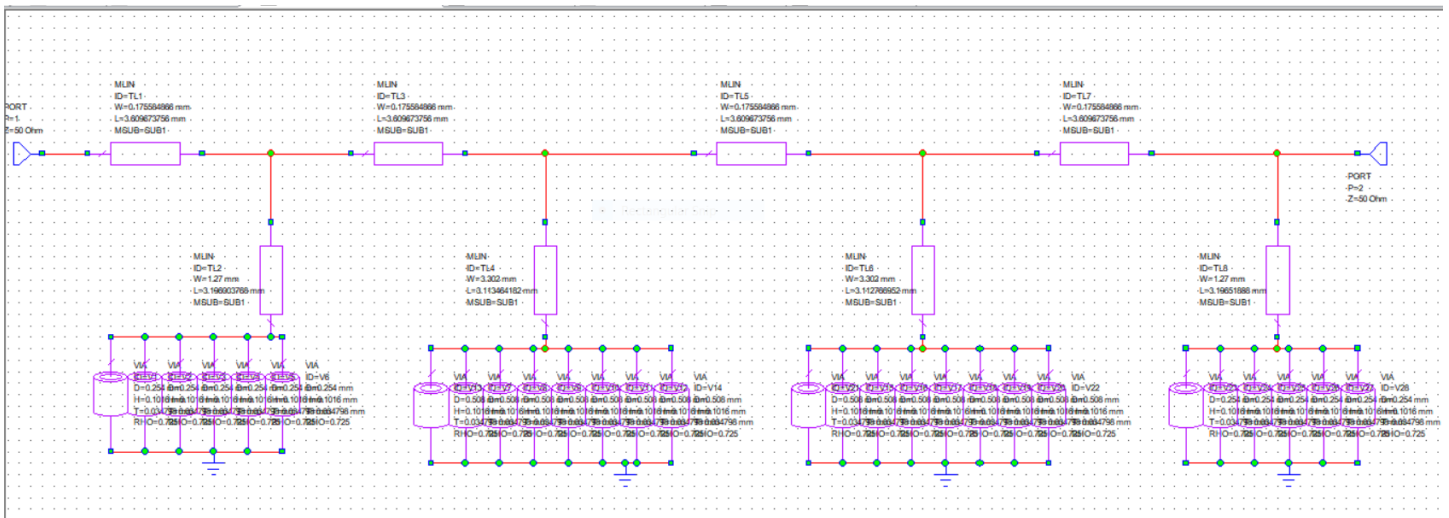


Fig 10. 4<sup>th</sup> order Butterworth filter circuit using microstrip elements.

Multiple Vias are used in the above filter design to reduce the number of reflections because of improper coupling. The vias help the stubs to be match perfectly with the ground plane.

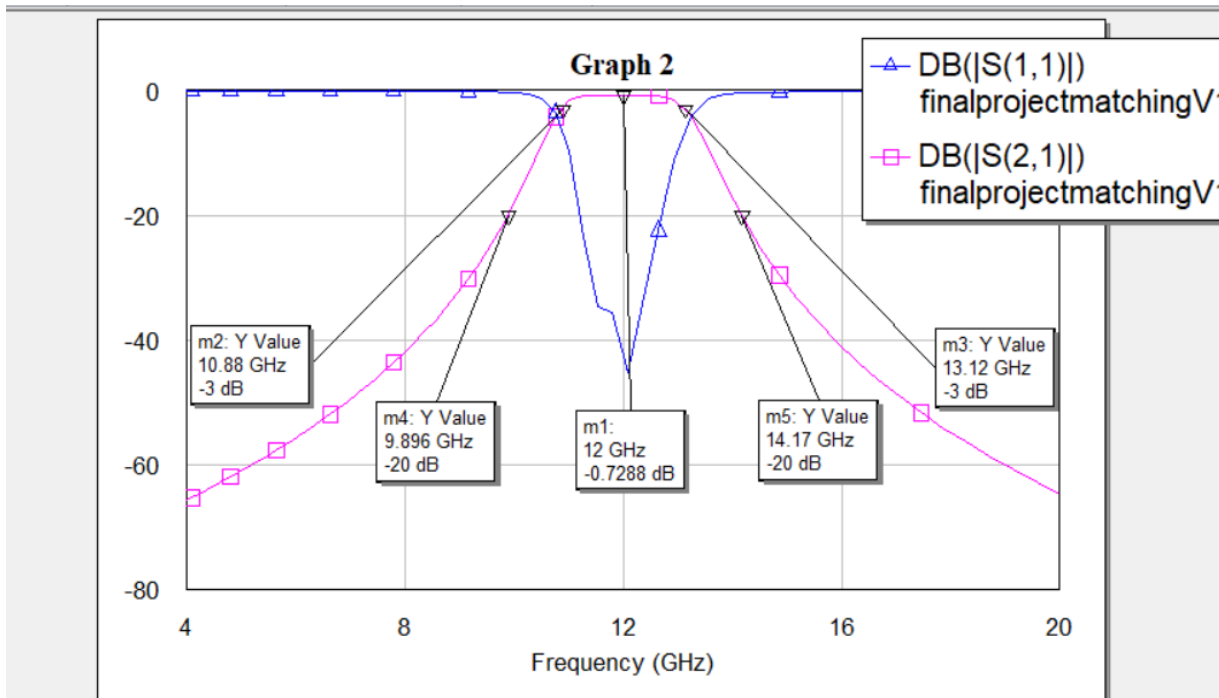


Fig 11. S-parameters depicting the output specs for the given filter design.

	Before Tuning	After Tuning
Transmission Lines	W: 0.175584866mm L: 3.609673756mm	W: 0.175584866mm L: 3.609673756mm
Stub 1	W: 1.7324451mm L: 3.196003768mm	W: 1.27mm L: 3.196003768mm
Stub 2	W: 4.531694264mm L: 3.113464182mm	W: 3.302mm L: 3.113464182mm
Stub 3	W: 4.583782044mm L: 3.112766952mm	W: 3.302mm L: 3.112766952mm
Stub 4	W: 1.724748138mm L: 3.19651888mm	W: 1.27mm L: 3.19661888mm

Table 6. before and after tuning 4<sup>th</sup> order filter.

## CONCLUSION

The design specification given are met using both the filter designs but it can be seen that the 4<sup>th</sup> order filter performs slightly better compared to 3<sup>rd</sup> order filters. It is also seen that 4<sup>th</sup> order filter does not resonate as compared to the 3<sup>rd</sup>-order filter. It can be observed that the filter presents us with lower attenuation compared to 3<sup>rd</sup> order filter at 12GHz. 4<sup>th</sup> order also provides a steeper roll off compared to 3<sup>rd</sup> order filter this added advantage of 2 extra elements gives the designer the ease to tune the filter much more effectively compared to 3<sup>rd</sup> order.

The width and length ratios are a matter of concern as they do not meet the design specification. This can be resolved if we change use much thinner substrates at a cost of introducing distortions and added manufacturing difficulty or by including TEEs in the filter design. Considering other filter design methods can also prove to be useful in this case coupled line filters.

4<sup>th</sup> order filters require less tuning to achieve the design specifications compared to 3<sup>rd</sup> order filters. This can be seen from the above comparison tables.

## APPENDIX

Given -

Relative permittivity = 4.2

3dB bandwidth (GHz) = 2.2

Center frequency (GHz) = 12

Lower 3dB frequency (GHz) = 10.9

Upper 3dB frequency (GHz) = 13.1

Lower 20dB attenuation frequency (GHz) = 9.8

Upper 20dB attenuation frequency (GHz) = 14.2

$$Q_T = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{12}{13.1 - 10.9} \Rightarrow 5.45$$

We know that the butterworth coefficients for a 3<sup>rd</sup> order filter are  $g_0 = 1, g_1 = 1, g_2 = 1, g_3 = 1, g_4 = 1$ .

$$C'_{ka} \Rightarrow \left( \frac{Q_T \times C_k}{\omega_0} \right) \frac{1}{R_L}$$

$\therefore$  for odd values of elements the above value should be used.

~~$C'_{ka} \Rightarrow \left( \frac{Q_T \times C_k}{\omega_0} \right) \frac{1}{R_L}$~~

$$L'_{ka} = \frac{R_L}{\omega_0 Q_T C_k}$$

For odd values of elements, the conversion above is used.

$$\therefore C'_1 = \frac{5.45 (1)}{(2\pi \times 12 \times 10^9)(50)} = 1.445 \text{ pF}$$

$$L'_1 = \frac{50}{(2\pi \times 12 \times 10^9)(5.45)(1)} = 121.6 \text{ pH}$$

For even values,

$$L'_k = \frac{B_T R_L L_2}{\omega_0}$$

$$C'_k = \frac{1}{\omega_0 B_T R_L L_2}$$

$$L'_2 = \frac{(5.45)(50)(2)}{\omega_0}$$

$$C'_2 = \frac{1}{(2\pi \times 12 \times 10^9)(50)(2)}$$

$$= 7.22 \times 10^{-9} \text{H}$$

$$C'_2 = 24.52 \text{fF}$$

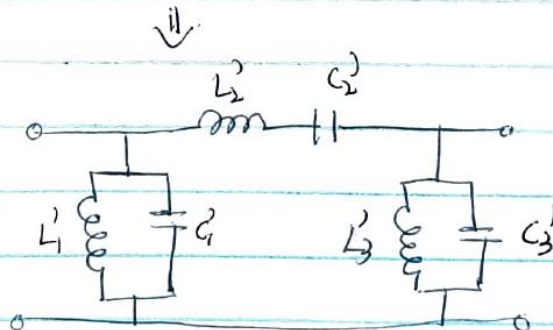
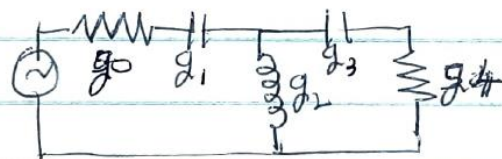
$$= 7.22 \text{nH}$$

As 3<sup>rd</sup> order filter is an odd value filter,  
 $C_3$  &  $L_3$  values are similar to  $L_1$  &  $C_1$  values.

$$\therefore C_3 = 1.447 \text{PF}$$

$$L_3 = 121.6 \text{pH}$$

$\therefore$  From butterworth design,



For stub conversion:-

$$\frac{Z_0}{Z_{01}} = \frac{L}{\pi} B_T C_k$$

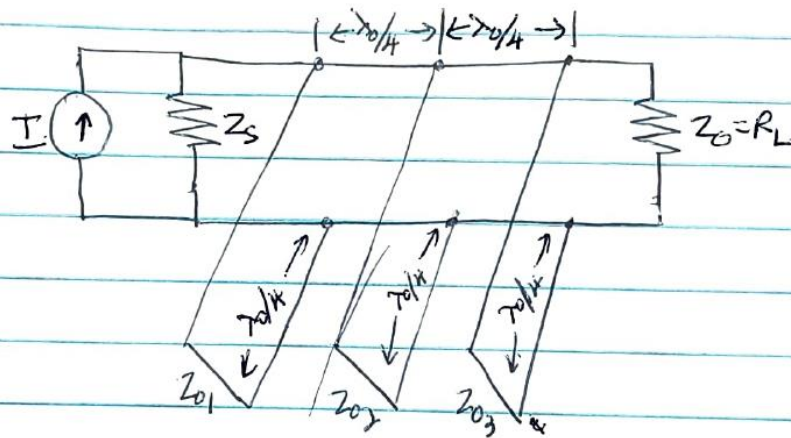
$$Z_{01} = \frac{Z_0}{\frac{L}{\pi} B_T C_k} \Rightarrow \frac{50}{\frac{4}{\pi} (5.45)(1)} = 7.20 \Omega$$



$$Z_{02} = \frac{Z_0}{\frac{4}{\pi} B_T L_K} = \frac{50}{\frac{4}{\pi} (5.45)(2)} = 3.60 \Omega$$

$$Z_{03} = \frac{Z_0}{\frac{4}{\pi} B_T C_K} = \frac{50}{\frac{4}{\pi} (5.45)(1)} = 7.20 \Omega$$

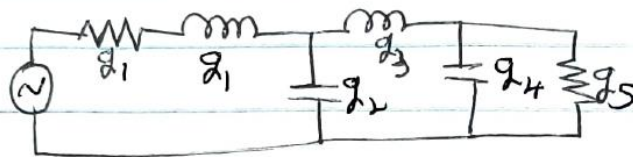
$\therefore$  final design looks something like below:-



$\therefore$  using formulas from Steer's textbook volume "2",  
The following lengths and widths are observed.

Similar to 3<sup>rd</sup> order filter, using 4<sup>th</sup> order filter coefficients,

$$g_0 = 1 \Omega, g_1 = 0.7654, g_2 = 1.8478, g_3 = 1.8478, \\ g_4 = 0.7654, g_5 = 1$$



using the same formula used in 3<sup>rd</sup> order filter, gives the values.

$$L_a^1 = \frac{R_L}{\omega_0 g_1} = 0.15 \text{ nH}$$

$$C_a^1 = \frac{g_5}{\omega_0 R_L} = 1.1 \text{ pF}$$

$$L_a^2 = \frac{g_2 R_L L_2}{\omega_0} = 6.67 \times 10^{-9} \text{ H}$$

$$C_a^2 = 0.026 \times 10^{-12} \text{ F}$$

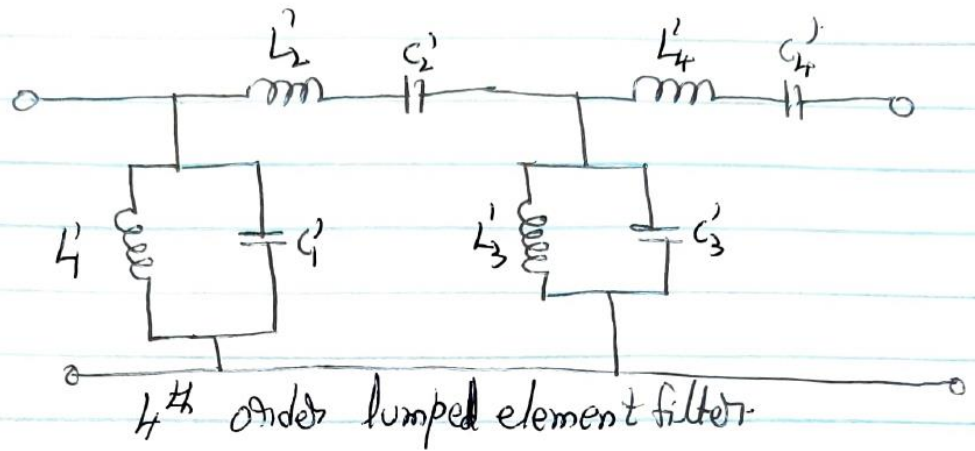
$$L_a^3 = 65.8 \times 10^{-12} \text{ H}$$

$$C_a^3 = 2.67 \times 10^{-12} \text{ F}$$

$$L_a^4 = 2.76 \times 10^{-9} \text{ H}$$

$$C_a^4 = 6.35 \times 10^{-14} \text{ F}$$

$$= 0.0635 \times 10^{-12} \text{ F}$$



we know  $z_0 = 44 - \epsilon_p$

$$\Rightarrow 44 - 4.2 = 39.8 \Omega$$

which the given parameters  
when we plug them into the below equation  
gives us the  $\frac{w}{h}$  of the line.

$$\frac{z_0}{z_{0k}} = \frac{4}{\pi} \cot L_k \Rightarrow z_{0k} = \frac{z_0}{\frac{4}{\pi} \cot L_k}$$

$$= 9.4140 \Omega$$

$$\frac{z_0}{z_{02}} = \frac{4}{\pi} \cot L_k \Rightarrow z_{02} = \frac{z_0}{\frac{4}{\pi} \cot L_k}$$

$$= 3.8975 \Omega$$

$$z_{03} = 3.8558 \Omega$$

$$z_{04} = 9.4092 \Omega$$

using the below formulas,

$$H' = \frac{Z_0 \sqrt{2(\epsilon_r + 1)}}{119.9} + \frac{1}{2} \left( \frac{\epsilon_r - 1}{\epsilon_r + 1} \right) \left( \ln \frac{\pi}{2} + \frac{1}{\epsilon_r} \ln \frac{4}{\pi} \right)$$

$$\frac{\omega}{h} = \left( \frac{\exp H'}{8} - \frac{1}{4 \exp H'} \right)^{-1}.$$

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