Probability

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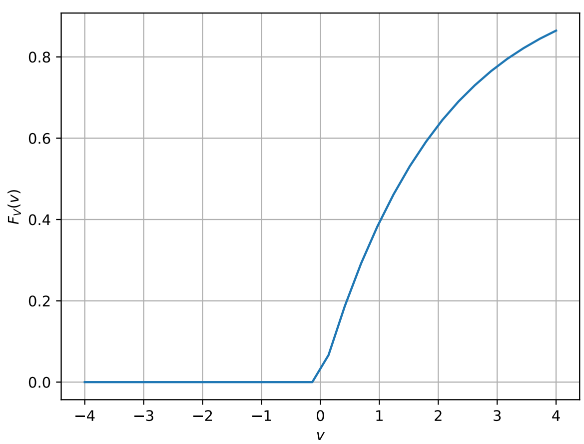

Fig. 1.1. The CDF of X

1 From Uniform to Other

1.1 Generate samples of

$$V = -2\ln(1-U) \tag{1.0.1}$$

and plot its CDF. **Solution:** The python code is at

```
$ wget https://raw.
    githubusercontent.com/goats
    -9/ai1110-assignments/master
    /manual/codes/3_1.py
```

The CDF is plotted in Figure given below

1.2 Find a theoretical expression for $F_V(x)$. **Solution:** We know that from uniform random variable pdf:

$$f(x) = \begin{cases} 0 & x \le 0 \\ x & 0 \le x \le 1 \\ 1 & x > 1 \end{cases} \tag{1.0.2}$$

Therefore,

$$\begin{aligned} & 0 \le U \le 1 \\ & = 1-0 \ge 1-U \ge 1-1 \\ & = 1 \ge 1-U \ge 0 \\ & = 0 \le 1-U \le 1 \\ & = -\infty \le \ln(1-U) \le 0 \\ & = -\infty \; leq 2\ln(1-U) \le 0 \\ & = \infty \ge -2\ln(1-U) \ge 0 \\ & = 0 \ge 2\ln(1-U) \ge \infty \end{aligned}$$

For $u \in U, v \in V$

$$\begin{aligned} V &= -2\ln(1-U) \\ &= \frac{-V}{2} = \ln(1-U) \\ &= e^(\frac{-V}{2}) = 1-U \\ &= U = 1-e^(\frac{-v}{2}) \end{aligned}$$

$$F_V(v) = F_V(1-e^(\frac{-v}{2}) \tag{1.0.3}$$

$$F_V(x) = \begin{cases} 1-e^{(\frac{-x}{2})} & x \ge 0 \\ 0 & x < 0 \end{cases} \tag{1.0.4}$$