Probability

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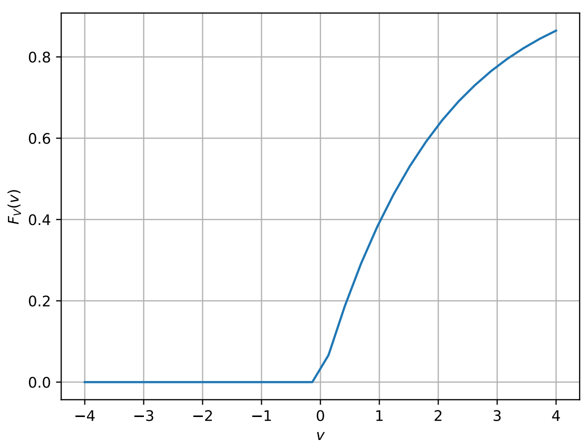

Fig. 1.1. The CDF of X

1 From Uniform to Other

1.1 Generate samples of

$$V = -2\ln(1-U) \tag{1.0.1}$$

and plot its CDF. **Solution:** The python code is at

```
https://github.com/RavishaJain/
    Assigment_3/blob/main/codes/
    UTO.py
```

The CDF is plotted in Figure given below

1.2 Find a theoretical expression for $F_V(x)$. **Solution:** Note that the function

$$v = f(u) = -2\ln(1-u) \tag{1.0.2}$$

is monotonically increasing in [0, 1] and $v \in \mathbb{R}^+$. Hence, it is invertible and that inverse function is given by

$$u = f^{-1}(v) = 1 - \exp\left(-\frac{v}{2}\right) \tag{1.0.3}$$

Therefore, from the monotonicity of v, and using the cdf of uniform variables,

$$F_U(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases} \tag{1.0.4}$$

$$F_V(v) = F_U\left(1 - \exp\left(-\frac{v}{2}\right)\right) \tag{1.0.5}$$

$$\implies F_V(v) = \begin{cases} 0 & v < 0 \\ 1 - \exp\left(-\frac{v}{2}\right) & v \ge 0 \end{cases} \tag{1.0.6}$$