Probability

RAVISHA JAIN
CC22MTECH14001
CLIMATE CHANGE DEPARTMENT

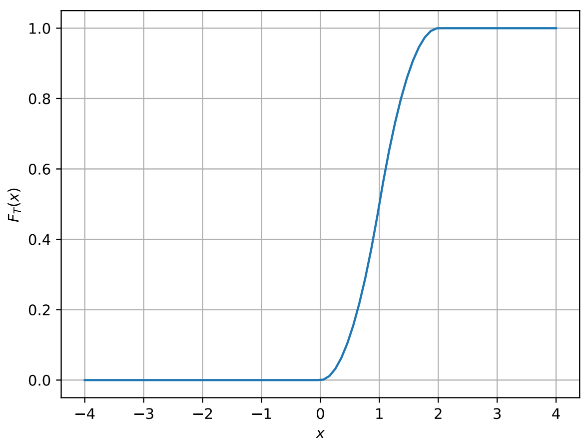

Fig. 1.2. The CDF of T

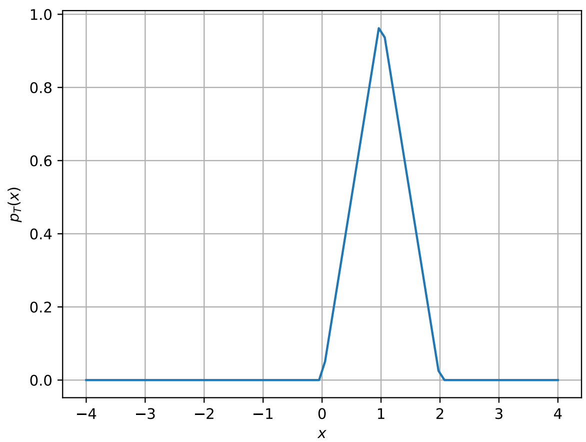

Fig. 1.3. The PDF of T

1 Triangular Distribution

1.1 Generate

$$T = U_1 + U_2 \tag{1.1.1}$$

Solution: The value of T is generated in tri.dat file

```
https://github.com/RavishaJain/
    Assignment_4/blob/main/Codes
    /exrand.c
https://github.com/RavishaJain/
    Assignment_4/blob/main/Codes
    /coeffs.h
```

1.2 Find the CDF of T.
Solution: The Python code for the cdf is

```
https://github.com/RavishaJain/
    Assignment_4/blob/main/Codes
    /TRIcdf.py
```

1.3 Find the PDF of T.
Solution: The Python code for the pdf is

```
https://github.com/RavishaJain/
    Assignment_4/blob/main/Codes
    /TRIpdf.py
```

1.4 Find the theoretical expressions for the PDF and CDF of T.
Solution: We know,

$$F_T(t) = \Pr(U_1 + U_2 \le t) \tag{1.4.1}$$
$$= \Pr(U_1 \le t - U_2) \tag{1.4.2}$$

Here U_1 and U_2 are independent variables on [0,1],hence we can use convolution formula

$$f_T(t) = \int_0^1 F_{U_1}(t-x)F_{U_2}(x)dx \tag{1.4.3}$$
$$F_T(t) = \int_0^1 f_Z(u)dx \tag{1.4.4}$$

Now , we use the fact that U_1 and U_2 are independent $Unif(0,1)$ random variables. We have following cases:

a) $t < 0$: Using ,

$$F_T(t) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases} \tag{1.4.5}$$

Therefore, $F_T(t) = 0$.

b) We will
$0 \leq t < 1$: We have,$0 \leq t < 1$
Therefore,

$$f_T(t) = \int_0^t 1.1 dx = t \tag{1.4.6}$$

$$f_T(t) = \int_0^t t dt = \left(\frac{t^2}{2}\right)_0^t \tag{1.4.7}$$

$$= \frac{t^2}{2} \tag{1.4.8}$$

c) $1 \leq t < 2$: Here, we need to be careful about range of U_1, U_2

$$f_T(t) = \int_{t-1}^1 1.1 dx = 2 - t \tag{1.4.9}$$

$$F_T(t) = \int_0^1 t dt + \int_1^t (2 - t) dt \tag{1.4.10}$$

$$= \left(\frac{t^2}{2}\right)_0^1 + 2 \left(t\right)_1^t - \left(\frac{t^2}{2}\right)_0^t \tag{1.4.11}$$

$$= 2t - \frac{t^2}{2} - 1 \tag{1.4.12}$$

d) $t \geq 2$: Here, $F_T(t) = 1$.

Therefore,

$$F_T(t) = \begin{cases} 0 & t < 0 \\ \frac{t^2}{2} & 0 \leq t < 1 \\ 2t - \frac{t^2}{2} - 1 & 1 \leq t < 2 \end{cases} \tag{1.4.13}$$

and,

$$f_T(t) = \begin{cases} t & 0 \leq t < 1 \\ 2 - t & 1 \leq t < 2 \\ 0 & \text{otherwise} \end{cases} \tag{1.4.14}$$

1.5 Verify your results through a plot.
Solution: The plots for the results are in Fig 1.2 and Fig 1.3.