Probability

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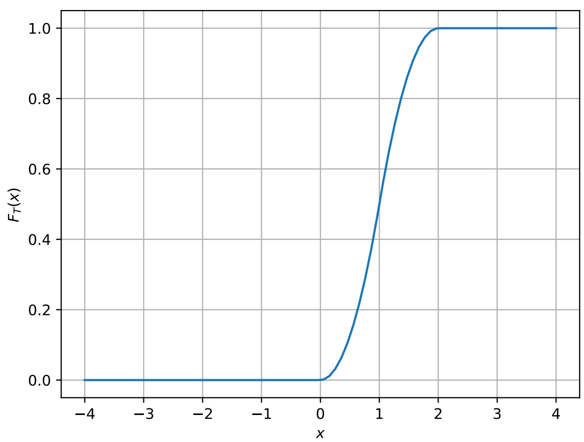

Fig. 1.2. The CDF of T

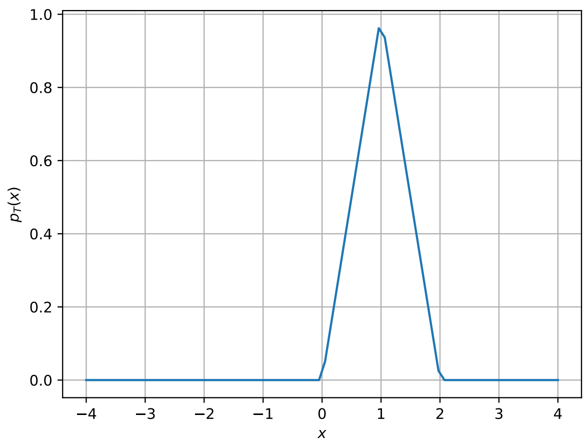

Fig. 1.3. The PDF of T

1 Triangular Distribution

1.1 Generate

$$T = U_1 + U_2 \tag{1.1.1}$$

Solution: The value of T is generated in tri.dat file

```
https://github.com/RavishaJain/
    Assignment_4/blob/main/Codes
    /exrand.c
https://github.com/RavishaJain/
    Assignment_4/blob/main/Codes
    /coeffs.h
```

1.2 Find the CDF of T.
Solution: The Python code for the cdf is

```
https://github.com/RavishaJain/
    Assignment_4/blob/main/Codes
    /TRIcdf.py
```

1.3 Find the PDF of T.
Solution: The Python code for the pdf is

```
https://github.com/RavishaJain/
    Assignment_4/blob/main/Codes
    /TRIpdf.py
```

1.4 Find the theoretical expressions for the PDF and CDF of T.
solution

1.5 Find the theoretical expressions for the PDF and CDF of T.

Solution: We know that $T = U_1 + U_2$. Accordingly,

$$
\begin{aligned}
F_T(t) &= PrU_1 + U_2 \le t \\
&= \int_0^1 \Pr\left(U_1 + U_2 \le t \mid U_1 = x\right) f_{U_1}(x)dx \\
&= \int_0^1 \Pr\left(x + U_2 \le t \mid U_1 = x\right) f_{U_1}(x)dx \\
&= \int_0^1 \Pr\left(U_2 \le t - x \mid U_1 = x\right) f_{U_1}(x)dx \\
&= \int_0^1 \Pr\left(U_2 \le t - x\right) f_{U_1}(x)dx \\
&= \int_0^1 F_{U_2}(t-x) f_{U_1}(x)dx
\end{aligned}
$$

Now we can take the derivative (with respect to t) of the CDF to get the density ($F_U 2$ becomes $f_U 2$):

$$
\begin{aligned}
f_T(t) &= \frac{d}{dt} F_T(t) \\
&f_{U_1}(x) f_{U_2}(t-x)dx
\end{aligned}
$$

where U_1 and U_2 are uniform random variables in [0, 1]. Clearly, $0 \le U_1 + U_2 \le 2$.

Again we know that

$$
f_{U_1}(x) = \begin{cases} 0 & \text{for x } < 0 \\ 1 & \text{for x } \ge 0 \end{cases}
$$

$$
F_{U_1}(x) = \Pr\left(U_1 \le x\right) = \begin{cases} 0 & \text{for x } < 0 \\ x & \text{for } x \in [0,1] \\ 1 & \text{for x } > 1 \end{cases}
$$

and

$$
f_{U_2}(x) = \begin{cases} 0 & \text{for x } < 0 \\ 1 & \text{for x } \ge 0] \end{cases}
$$

$$
F_{U_2}(x) = \Pr\left(U_2 \le x\right) = \begin{cases} 0 & \text{for x } < 0 \\ x & \text{for } x \in [0,1] \\ 1 & \text{for x } > 1 \end{cases}
$$

Now to deal with the interval from 0 to 2, it is useful to break this down into four cases:

(i) $t < 0$: In this case, it is clear that $F_T(t) = 0$.

(ii) $0 \le t < 1$: We need $t - x \ge 0$ and thus $x \le t$ and therefore, we integrate from $x = 0$ and $x = t$.

$$
\begin{aligned}
F_T(t) &= \int_0^t F_{U_2}(t-x) F_{U_1}(x)dx \\
&= \int_0^t (t-x)dx \\
&= \left[tx - \frac{x^2}{2}\right]_0^t \\
&= \frac{t^2}{2}
\end{aligned}
$$

(iii) $1 \le t < 2$: Here, we need $t - x \le 1$ and thus $x \le t - 1$ and therefore, we integrate from 0 to $x = t - 1$ and $x = t - 1$ to $x = 1$.

$$
\begin{aligned}
F_T(t) &= \int_0^{t-1} F_{U_2}(t-x) F_{U_1} \\
&\int_{t-1}^1 F_{U_2}(t-x) F_{U_1}(x)dx \\
&= \int_0^{t-1} dx + \int_{t-1}^1 (t-x)dx \\
&= t - 1 + t(2-t) - \frac{1-(t-1)^2}{2} \\
&= -\frac{t^2}{2} + 2t - 1
\end{aligned}
$$

(iv) $t \ge 2$: Also $F_T(t) = 1$.

So, the CDF for T is given by

$$
F_T(t) = \Pr\left(T \le t\right) = \begin{cases} 0 & x < 0 \\ \frac{t^2}{2} & 0 \le t < 1 \\ 2t - \frac{t^2}{2} - 1 & 1 \le t < 2 \\ 1 & x \ge 2 \end{cases} \tag{1.5.1}
$$