Probability

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1 Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution:

```
https://github.com/RavishaJain/Assignment_1/blob/main/Codes/exrand.c
https://github.com/RavishaJain/Assignment_1/blob/main/Codes/coeffs.h
```

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \tag{1.2.1}$$

Solution: The following code plots Fig. 1.2

```
https://github.com/RavishaJain/Assignment_1/blob/main/Codes/cdf_plot.py
```

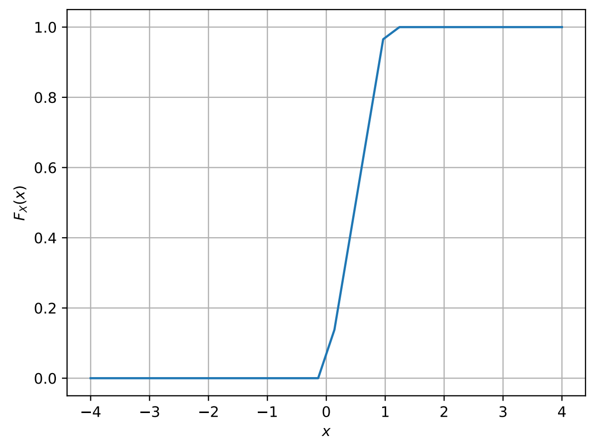

Fig. 1.2. The CDF of U

1.3 Find a theoretical expression for $F_U(x)$.
Solution: The CDF for probability distribution is

$$f(x) = \begin{cases} \frac{1}{1-0} & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

The CDF of a random variable X is defined as

$$F_u(x) = P(U \leq x) = \int_{-\infty}^{\infty} f(x)\,dx$$

, for $x \in \mathbb{R}$.
So we will split into three intervals i.e.

$$F_u(x) = P(U \leq x) = \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx$$

According to the given we will slove them separately,
Case I:

$$\int_{-\infty}^{0} f(x)dx$$

As we can see x=0,therefore f(x)=0 ,where $\lim_{x\to 0},\ f(x) = 0$
Case II:

$$\int_{0}^{1} f(x)dx$$

where $0 \leq x \leq 1$ then,

$$\int_{0}^{1} f(x)dx = \frac{1}{1-0}\int_{0}^{1} dx$$
$$= \left[\frac{1}{1-0}\right]_0^1$$
$$= 1$$

Case III:

$$\int_{1}^{\infty} f(x)dx$$

As we can see, $\lim_{x\to 1},\ f(x) = 1$

Thus, CDF for random variable is as follows:

$$F_u(x) = P(0 \le x \le 1) =$$

$$f(x) = \begin{cases} 0 & x \le 0 \\ \frac{1}{1-0} & 0 \le x \le 1 \\ 1 & x \le 1 \end{cases} \quad (1.3.1)$$

1.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \quad (1.4.1)$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \quad (1.4.2)$$

Write a C program to find the mean and variance of U.

Solution: The mean is 0.50 and the variance is 0.083

```
https://github.com/RavishaJain/
    Assignment_1/blob/main/Codes
    /Mean.c
https://github.com/RavishaJain/
    Assignment_1/blob/main/Codes
    /Variance.c
https://github.com/RavishaJain/
    Assignment_1/blob/main/Codes
    /coeffs.h
```

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \quad (1.5.1)$$

Solution: We know that ,

$$\text{var}[U] = E[U - E[U]]^2$$
$$E\left[U^2 - 2UE[U] + (E[U]^2)\right]$$
$$E\left[U^2\right] - 2E[U]E[U] + E\left[E\left[U^2\right]\right]$$
$$E\left[U^2\right] - 2E[U] + E\left[U^2\right]$$
$$E\left[U^2\right] - E[U]^2$$

So we get,

$$E\left[U^2\right] - E[U]^2 \quad (1.5.2)$$

Now for $\mathrm{E}\left[U^2\right]$ and $\mathrm{E}[U]$;
As we know mean is given by,

$$E[U] = \int_0^1 dF_U(x) \quad (1.5.3)$$

According to the equation given above,

$$E[U] = \int_0^1 dF_U(x)$$
$$= \int_0^1 x dx$$
$$= \left[\frac{x}{2}\right]_0^1$$
$$= \frac{1}{2}$$
$$= 0.50$$

And Variance is given by,

$$E\left[U^2\right] = \int_0^1 x^2 dF_U(x) \quad (1.5.4)$$

Therefore, accordingly we see that,

$$E\left[U^2\right] = \int_0^1 x^2 dF_U(x)$$
$$= \int_0^1 x dx$$
$$= \left[\frac{x^3}{3}\right]_0^1$$
$$= \frac{1}{3}$$
$$= 0.33$$

Now substituting the values of $\mathrm{E}[U]$ and $\mathrm{E}\left[U^2\right]$ in equation (1.5.2) we get,

$$\text{var}[U] = \frac{1}{3} - (\frac{1}{2})^2$$
$$= \frac{1}{3} - \frac{1}{4}$$
$$= \frac{1}{12}$$
$$= 0.083$$

Hence Proved