Simulation of a three-dimensional scalar ϕ^4 theory

Lab II project

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9.1 Lattice setup - Geometry

User inputs the parameters:

- 1. $NPROC0$ and $NROC1$: number of processes in $\mu = 0, 1$ direction
- 2. L_0, L_1 and L_2: global three-dimensional Euclidean lattice size (even)

Create a two-dimensional grid of processes.

- $cpr[2]$: coordinates
- $npr[4]$: rank of neighboring process

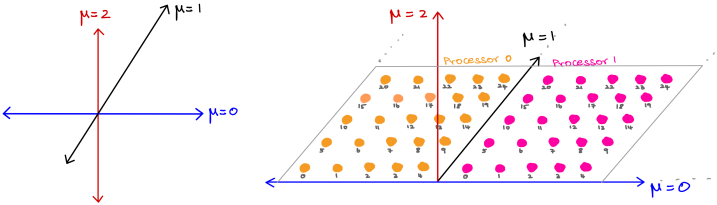

Figure: Dimensions

9.1 Lattice setup - Geometry

Count the boundary points of the local lattice.

- $FACE0$: number of boundary points in 0 direction
- $FACE1$: number of boundary points in 1 direction
- $BNDRY$: total number of boundary points

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User input nPROC0, nPROC1:  4,  4
and Global lattice size:L0 = 20, L1 = 20, L2 = 20
face0 = 100
face1 = 100
bndry = 400
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Figure: User input

9.1 Lattice setup - Geometry

Figure: Dimensions

9.1 Lattice setup - Geometry

Determine follwing arrays:

int *ipt*[*VOLUME*]

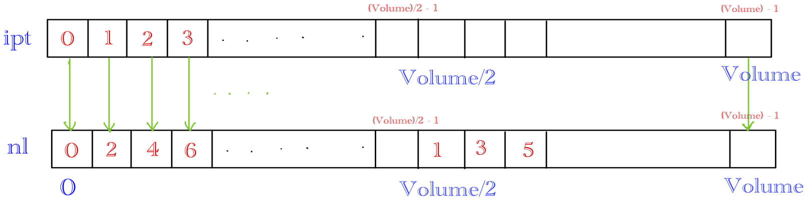


Figure: ipt

9.1 Lattice setup - Geometry

Determine follwing arrays:

iup[VOLUME][2] idn[VOLUME][2] map[BNDRY]

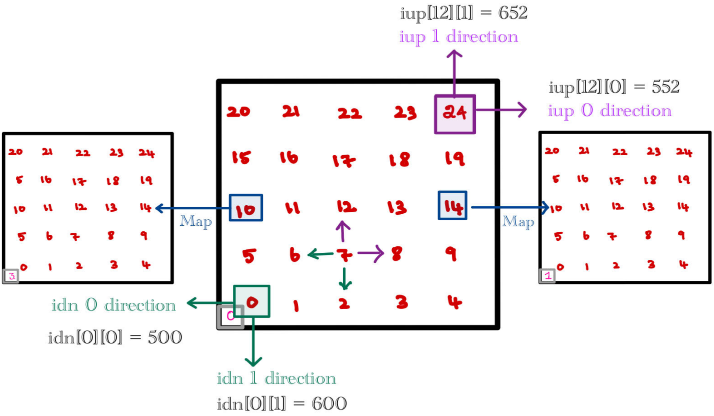


Figure: iup, idn, map

9.2 Random Walk

Define global coordinates:

$$\mathbf{x} = (x_0, x_1, x_2) = (cpr[0]L_0 + x_0, cpr[1]L_1 + x_1, x_2)$$

- for $1 \le i \le N_{meas} = 10^4$
  - choose starting point $\mathbf{x}^{(0)}$ at random
  - for $1 \le t \le T = 100$
    - move in a random direction $\mathbf{x}^{(t)} = \mathbf{x}^{(t-1)} \pm \hat{\mu}$
    - measure distance $d_i(t) = \sqrt{\sum_\mu min[(\mathbf{x}_\mu^{(0)} - \mathbf{x}_\mu^{(t)})^2, (NPROC_\mu L_\mu - (\mathbf{x}_\mu^{(0)} - \mathbf{x}_\mu^{(t)}))^2]}$
  - write values $d_i(t), t = 1, 2, ..., T$ into a file

9.2 Random Walk

Plot the average: $\langle d^2(t) \rangle = \frac{1}{N_{meas}} \sum_{i=1}^{N_{meas}} d_i^2(t)$

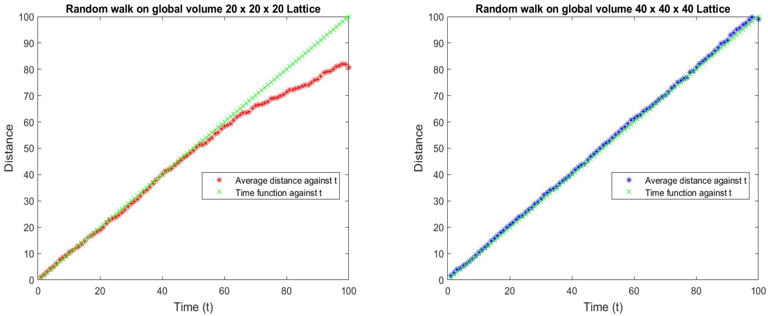
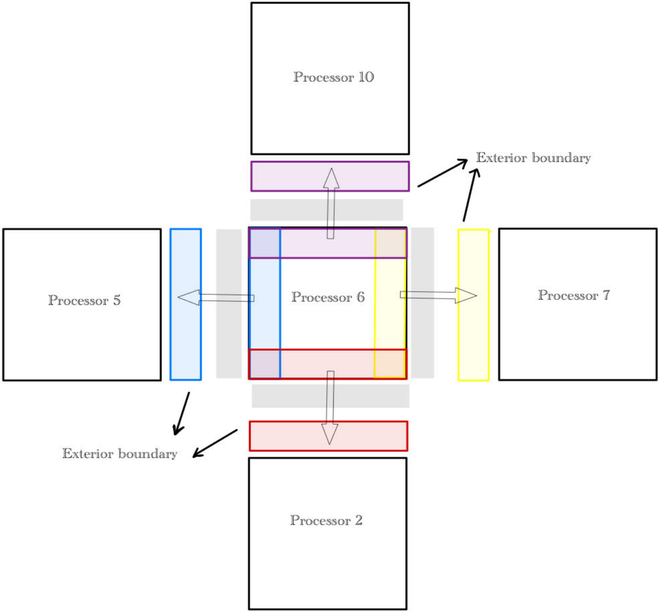


Figure: Results

10 Lattice setup - Communication

- Define a structure 'field' to represent the scalar field $\phi = (\phi_1, \phi_2)^T$, where each elemet of ϕ is real
- The field is represented by an array $fi[(VOLUME + BNDRY)]$
- The boundary values are stored with oredering: first even then odd points with dirctions $-0, +0, -1$ and $+1$ accordingly
- Communication between processors via nearest neighbor interactions

10 Lattice setup- Communication



11 Simulate a three-dimensional scalar ^{4} theory

- Lattice of size $L \times L \times L$ with volume $V = L^3$
- The action of the system is
  $S = \sum_x \{-2\kappa \sum_\mu \phi_x^T \phi_{x+\hat{\mu}} + \phi_x^T \phi_x + \lambda(\phi_x^T \phi_x - 1)^2\}$
- measure magnetization
  $m = \frac{1}{2V}(|\sum_x \phi_{x,1}| + |\sum_x \phi_{x,2}|)$
- susceptibility of the magnetization $\chi_m = V\langle(m - \langle m\rangle)^2\rangle$

End

Thank you for your attention.