Homework 11
due Jan 26, 2022

In homeworks 9 and 10 we made the preparations required for the project. The goal is to simulate a three-dimensional scalar ϕ^4 theory for a two-component real scalar field $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$. Equivalently ϕ_1 and ϕ_2 can be thought of the real and imaginary parts of a complex field Φ. We use a hypercubic lattice of size $L \times L \times L$, the lattice volume is denoted by $V = L^3$. The action of the system is

$$S = \sum_x \left\{ -2\kappa \sum_\mu \phi_x^T \phi_{x+\hat{\mu}} + \phi_x^T \phi_x + \lambda \left(\phi_x^T \phi_x - 1 \right)^2 \right\} , \tag{1}$$

in terms of two parameters κ and λ.

- Implement a Hybrid Overrelaxation algorithm as explained in the lecture, cf. [1].
- Implement routines to measure $\frac{1}{V} \sum_x \phi_{x,i},\ i = 1, 2$, from which one can measure the magnetization

  $$m = \frac{1}{2V} \left(\left| \sum_x \phi_{x,1} \right| + \left| \sum_x \phi_{x,2} \right| \right) , \tag{2}$$

  the value of $\phi_x^T \phi_x$ and the susceptibility of the magnetization, which is defined by the expectation value

  $$\chi_m = V \left\langle (m - \langle m \rangle)^2 \right\rangle . \tag{3}$$

- If your code is correct, you should be able to measure values consistent with the following:

λ	κ	L	$\langle m \rangle$	$\langle \phi^T \phi \rangle$
0.2	0.1	8	0.0308(1)	0.82927(3)
0.2	0.2	8	0.0590(3)	0.86591(5)
0.2	0.3	8	0.4489(11)	1.05154(9)
0.2	0.1	16	0.01092(2)	0.82927(1)
0.2	0.2	16	0.0210(1)	0.86588(2)
0.2	0.3	16	0.435(3)	1.05093(3)

- We set $\lambda = 2$. At a critical value κ_c of κ a second order phase transition in the universality class of the three-dimensional XY model occurs. On a finite lattice of size L, $\kappa_c(L)$ can be estimated by the position of the peak of the magnetic susceptibility $\kappa_c = \max_\kappa(\chi_m)$. Simulate a series of lattice volumes $L = 16, 32$ using 2×8 cores. For each volume do a scan in κ, measure χ_m and estimate the position of the maximum $\kappa_c(L)$. Compare them to the value of $\kappa_c = 0.25495\ldots$ quoted for $L = 48$ in [2].
  *Remark:*Keep in mind that, in order to obtain a statistically significant sample, the number of updates should be $\mathcal{O}\left(10^6\right)$.

- For the error analysis download the MATLAB program `autocorr.m` from the moodle page. What do you observe as the phase transition is approached?

(30 points)

References

[1] B. Bunk,
Monte Carlo methods and results for the electro-weak phase transition,
Nucl. Phys. Proc. Suppl. **42** (1995) 566.

[2] M. Hasenbusch and T. Török,
High precision Monte Carlo study of the 3-D XY universality class,
`http://arxiv.org/abs/cond-mat/9904408`.