

P(h): If height is  $h$ , maximum number of nodes is  $2^h - 1$ .

BS: If  $h=0$ ,  $2^0 - 1 = 1 = \text{max nodes}$ .  $\therefore P(1)$  is true.

IS: Assume  $h \geq 1$  and  $P(k)$  is true for all  $0 \leq k < h$ .

In a tree with  $h$ , left & right subtrees have height less than  $h$ .  
Max height for each is  $h-1$ . By IH, max number of nodes in left & right subtrees is  $2^{h-1} - 1$  and  $2^{h-1} - 1$ .

$\therefore$  Maximum number of nodes in tree is  $1 + (2^{h-1} - 1) + (2^{h-1} - 1) = 2^h - 1$ .