Line Assignment

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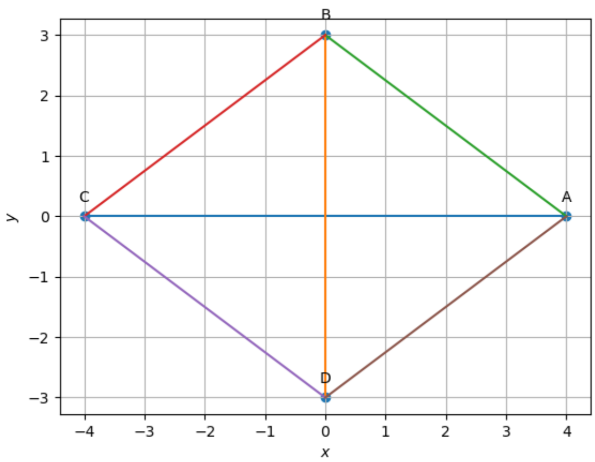

Figure 1: Rhombus

1 Problem:

Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus

2 Solution:

2.1 Theory:

Given the diagonals bisect at right angles, we need to prove that all sides are equal, using vector algebra. Let us assume two vectors **A** and **B**. The diagonals are the addition and subtraction of the two vectors:

$$(\boldsymbol{A}+\boldsymbol{B}),(\boldsymbol{B}-\boldsymbol{A})$$

2.2 Mathematical Calculation:

Let the two diagonals be $\boldsymbol{X}$, $\boldsymbol{Y}$. Since the diagonals are at right angle to each other,

$$\boldsymbol{X}=\boldsymbol{A}+\boldsymbol{B}$$
$$\boldsymbol{Y}=\boldsymbol{B}-\boldsymbol{A}$$
$$cos\theta=\frac{(\boldsymbol{A}+\boldsymbol{B}).(\boldsymbol{B}-\boldsymbol{A})}{||\boldsymbol{A}+\boldsymbol{B}||.||\boldsymbol{B}-\boldsymbol{A}||}$$
$$cos90^\circ=0=(\boldsymbol{A}+\boldsymbol{B}).(\boldsymbol{B}-\boldsymbol{A})$$
$$||\boldsymbol{B}||^2-||\boldsymbol{A}||^2=0$$
$$||\boldsymbol{B}||=||\boldsymbol{A}||$$

Hence, the two sides of the quadrilateral are equal. We need to prove the third side is also equal. In triangle BOA and BOC;

$$\boldsymbol{A}=\boldsymbol{X}-\boldsymbol{Y}$$
$$\boldsymbol{D}=-\boldsymbol{X}-\boldsymbol{Y}$$

Now,

$$||\boldsymbol{A}||^2=||\boldsymbol{X}-\boldsymbol{Y}||^2$$
$$||\boldsymbol{D}||^2=||-\boldsymbol{X}-\boldsymbol{Y}||^2$$
$$||\boldsymbol{A}||^2=||\boldsymbol{X}||^2-2(\boldsymbol{X^T}.\boldsymbol{Y})+||\boldsymbol{Y}||^2$$
$$||\boldsymbol{D}||^2=||\boldsymbol{X}||^2+2(\boldsymbol{X^T}.\boldsymbol{Y})+||\boldsymbol{Y}||^2$$

As the diagonals are perpendicular to each other, the terms $+2(\boldsymbol{X^T}.\boldsymbol{Y})$ and $-2(\boldsymbol{X^T}.\boldsymbol{Y})$ will be equal to zero.

$$||\boldsymbol{A}||^2=||\boldsymbol{X}||^2+||\boldsymbol{Y}||^2$$
$$||\boldsymbol{D}||^2=||\boldsymbol{X}||^2+||\boldsymbol{Y}||^2$$
$$||\boldsymbol{A}||=||\boldsymbol{D}=||\boldsymbol{B}||$$

Hence, we have proved that the given quadrilateral is a parallelogram. A parallelogram with its diagonals as perpendicular bisectors is known as Rhombus.

3 Construction:

The construction of rhombus can be done using only two diagonals, taken as d1 and d2.

variable	**length/point**	**Description**
d1	8	length of diagonal 1
d2	6	length of diagonal 2
A	(d1/2,0)	point A
B	(0,d2/0)	point B
C	(-d1/2,0)	point C
D	(0,-d2/2)	point D