Line Assignment

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1 Problem:

Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus

2 Solution:

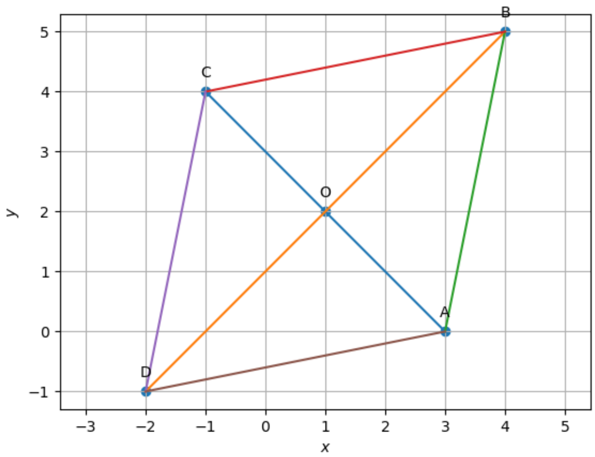

Figure 1: Rhombus

2.1 Theory:

Let us assume two vectors $\mathbf{a}$ and $\mathbf{b}$ for sides BA and CB. The diagonals AC, BD are the addition and subtraction of the two vectors:

$$
\begin{align}
(\mathbf{B}-\mathbf{A}) &= \mathbf{a} \tag{1}\\
(\mathbf{C}-\mathbf{B}) &= \mathbf{b} \tag{2}\\
(\mathbf{C}-\mathbf{A}) &= (\mathbf{C}-\mathbf{B})+(\mathbf{B}-\mathbf{A}) \tag{3}\\
(\mathbf{C}-\mathbf{A}) &= \mathbf{b}+\mathbf{a} \tag{4}\\
(\mathbf{D}-\mathbf{B}) &= \mathbf{b}-\mathbf{a}] \tag{5}\\
& \tag{6}
\end{align}
$$

2.2 Mathematical Calculation:

Let the two diagonals be $\mathbf{a}+\mathbf{b}$, $\mathbf{b}-\mathbf{a}$. Since the diagonals are at right angle to each other,

$$
\begin{align}
0 &= (\mathbf{a}+\mathbf{b})^{\mathbf{T}}.(\mathbf{b}-\mathbf{a}) \tag{7}\\
||\mathbf{b}||^2 - ||\mathbf{a}||^2 &= 0 \tag{8}\\
||\mathbf{b}|| &= ||\mathbf{a}|| \tag{9}\\
& \tag{10}
\end{align}
$$

Hence, the two sides of the quadrilateral are equal. We need to prove the third side is also equal. Now, In triangle BOA and AOD;

$$
\begin{align}
\mathbf{O}-\mathbf{B} &= \mathbf{p} \tag{11}\\
\mathbf{O}-\mathbf{D} &= -\mathbf{p} \tag{12}\\
\mathbf{O}-\mathbf{A} &= \mathbf{r} \tag{13}\\
& \tag{14}
\end{align}
$$

$$
\begin{align}
\mathbf{a} &= (\mathbf{O}-\mathbf{B})-(\mathbf{O}-\mathbf{A}) \tag{15}\\
\mathbf{d} &= (\mathbf{O}-\mathbf{D})-(\mathbf{O}-\mathbf{A}) \tag{16}\\
||\mathbf{a}||^2 &= ||\mathbf{p}||^2 + ||\mathbf{r}||^2 - 2\mathbf{p}^{\mathbf{T}}\mathbf{r} \tag{17}\\
||\mathbf{d}||^2 &= ||{-\mathbf{p}}||^2 + ||\mathbf{r}||^2 + 2\mathbf{p}^{\mathbf{T}}\mathbf{r} \tag{18}\\
& \tag{19}
\end{align}
$$

The terms $\mathbf{p}^{\mathbf{T}}\mathbf{r}$ is equal to zero as they is perpendicular.Therefore,

$$
\begin{align}
||\mathbf{a}||^2 &= ||\mathbf{p}||^2 + ||\mathbf{r}||^2 \tag{20}\\
||\mathbf{d}||^2 &= ||\mathbf{p}||^2 + ||\mathbf{r}||^2 \tag{21}\\
Clearly, ||\mathbf{a}|| &= ||\mathbf{d}|| \tag{22}\\
& \tag{23}
\end{align}
$$

Hence, all three sides are equal and it's a rhombus.

3 Construction:

Consider any three vertices of the rhombus. Using the vertices, find the midpoint of the diagonals, then find the fourth point using the midpoint and remaining vertex.

variable	length/point	Description
A	[3,0]	Vertex A
B	[4,5]	Vertex B
C	[-1,4]	Vertex C
D	[D-x,D-y]	Vertex D
M	(A+B)/2	midpoint
(D-x,D-y)	(2*M[0]-B[0],2*M[1]-B[1])	vertex of D