Optimization Advanced Assignment

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1 Problem:

Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ to the plane $\mathbf{r}.(2\mathbf{i} + \mathbf{j} + 3\mathbf{k} - 26 = 0)$. Also find the image of P in the plane.

2 Solution:

2.1 Theory:

Given the position vector of the point P as $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$.

The plane eqn is:

$$\mathbf{n^T x} = c \tag{1}$$

$$\mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \tag{2}$$

$$c = 26 \tag{3}$$

Let O be the foot of perpendicular, PO is perpendicular to the given plane. Hence, the directional vectors will be a scalar multiple of plane's directional vectors. The line equation is given as,

$$\mathbf{a} + \lambda \mathbf{m} = \mathbf{O} \tag{4}$$

$$\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \mathbf{O} \tag{5}$$

2.2 Mathematical Calculation:

As the point O lies on the plane, we can substitute the above coordinates in the plane equation,

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}^T \mathbf{O} = 26 \tag{6}$$

$$\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \mathbf{O} \tag{7}$$

The point O should be solved S.T $||\mathbf{O} - \mathbf{P}||$ is minimum. Let it be V.

$$V = \min_{O,P} \ ||\mathbf{O} - \mathbf{P}|| \tag{8}$$

$$S.T \quad \mathbf{O} = \mathbf{P} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}^T \mathbf{O} = 26 \tag{10}$$

$$\tag{11}$$

From the above equations using quadform, we get O and $||\mathbf{O} - \mathbf{P}||$ as,

$$\mathbf{O} = \begin{pmatrix} 3 \\ 3.5 \\ 5.5 \end{pmatrix} \tag{12}$$

$$V = \min_{O,P} \ ||\mathbf{O} - \mathbf{P}|| = 1.87 \tag{13}$$

$$\tag{14}$$

As the Point O is the midpoint of P and its image Q,

$$(\mathbf{P} + \mathbf{Q})/\mathbf{2} = \mathbf{O} \tag{15}$$

$$\mathbf{Q} = \mathbf{2O} - \mathbf{P} \tag{16}$$

$$\mathbf{Q} = \begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix} \tag{17}$$

3 Conclusion:

Therefore, quadform is used for finding the minimum length of P to the plane(perpendicular distance) and the foot of perpendicular. O is $\begin{pmatrix} 3 \\ 3.5 \\ 5.5 \end{pmatrix}$ and image of P is Q $\begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix}$ respectively.