ASSIGNMENT-10

Challa·Ravi teja.
192325065 (CSA0670)

1. Big Omega Notation : Prove that $g(n)=n^3+2n^2+4n$ is $\Omega(n^3)$

A: $g(n) \geq Cn^3$.

$g(n)=n^3+2n^2+4n$

for finding constants C and n_0

$n^3+2n^2+4n \geq Cn^3$.

Divide both sides with n^3.

$1+\frac{2n^2}{n^3}+\frac{4n}{n^3} \geq C$

$1+\frac{2}{n}+\frac{4}{n^2} \geq C$

Here $2/n$ and $4/n^2$ approches 0

$1+2/n+4/n^2 \approx 1$

Example $C=1/2$

$1+2/n+4/n^2 \geq 1/k$ $\quad (1 \geq 1/2, n \geq 1)$

$1+2/n+4/n^2 \geq 1$ $\quad (n \geq 1, n_0=1)$

$1+2/n+4/n^2 \geq 1/2$

Thus, $g(n)=n^3+2n^2+4n$ is indeeded $\Omega(n^3)$

2. Big theta Notation : Determine a shether $h(n)=4n^2+3n$ is $\Theta(N^2)$ or Not.

A: $C_1n^2 \leq h(n) \leq C.2n^2$

In upper bound $h(n)$ is $O(n^2)$

In Lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(O(n^2))$:

$h(n)=4n^2+3n$

$h(n) \leq C.2n^2$

$4n^2+3n \leq C.2n^2$

$4n^2+3n \leq 5n^2$

let's $C_2=5$

Divide both sides by n^2

$4 + 3/n \leq 5$

$h(n) = 4n^2+3n$ is $O(n^2)$ $(c_2 = 5, n_0 = 1)$

lower bound: $h(n) = 4n^2 + 3n$

$h(n) \geq 4n^2$

$4n^2 + 3n \geq c_1 \cdot n^2$

let's $c_1 = 4 \Rightarrow 4n^2 + 3n \geq 4n^2$

Divide both sides by n^2

$4 + 3/n \geq 4$

$h(n) = 4n^2 + 3n$ $(c_1 = 4, n_0 = 1)$

$h(n) = 4n^2 + 3n$ is $\Theta(n^2)$

3. let $F(n) = n^3 - 2n^2 + n$ and $g(n) = n^2$ show whether $F(n) = \Omega(g(n))$ is true or False and Justify your answer.

A: $F(n) \geq c \cdot g(n)$

Substituting $F(n)$ and $g(n)$ into this inequality we get

$n^3 \cdot 2n^2 + n \geq c \cdot (-n^2)$

Find c and n_0 holds $n \geq n_0$

$n^3 \cdot 2n^2 + n \geq \cdot cn^2$

$n^3 \cdot 2n^2 + n + c \cdot n^2 \geq 0$

$n^3 + (c-2)n^2 + n \geq 0$

$n^3 + (c-2)n^2 + n \geq 0 \qquad n^3 > 0$

$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0 \quad (c = 2)$

$F(n): n^3 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

Therefore the statement $F(n) = \Omega(g(n))$ is True.

4. Determine whether $h(n) = n\log n + n$ is in $\Theta(n\log n)$. prove a rigorous proof for your conclusion.

A: $C_1 . n\log n \le h(n) \le C_2 . n\log n$.

Upper bound:

$h(n) \le C_2 . n\log n$.

$h(n) = n\log n + n$

$n\log n + n \le C_2 . n\log n$.

Divide both sides by $n \log n$.

$1 + n/n\log n \le 2$.

$1 + 1/\log n \le C_2$ (simplify)

$1 + 1/\log n \le 2$ $(C_2 = 2)$

Then $h(n)$ is $O(n\log n)$ $(C_2 = 2, n_0 = 2)$

lower bound: $h(n) \ge C_1 . n\log n$.

$h(n) = n\log n + n$.

$n\log n + n \ge C_1 . n\log n$.

Divide both sides by $n \log n$

$1 + n/n\log n \ge C_1$

$1 + 1/\log n \ge C_1$ (simplify)

$1 + 1/\log n \ge C_1$ $C_1 = 1$

$1/\log n \ge 0$ For all $n > 1$

$h(n)$ is $\Omega(n\log n)$ $(C_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $\Theta(n\log n)$

5. Solve the following Reccurence Relations and find the order of growth for solution $T(n) = 4T(n/2) + n^2, T(1) = 1$.

A:

$T(n) = 4T(n/2) + n^2, \; T(1) = 1$

$T(n) = aT(n/2) + F(n)$

$a = 4, \; b = 2, \; F(n) = n^2.$

Applying Master theorem.

$T(n) = aT(n/b) + F(n)$

$F(n) = O(n^{\log_b a - 1})$ $\left(\begin{matrix} \epsilon > 0 \\ T(n) = \Theta(n^{\log_b a}) \end{matrix} \right)$

$F(n) = \Theta(n^{\log_b a})$, Then $T(n) = \Theta(n^{\log_b a} \log n)$

$F(n) = \Omega(n^{\log_b a + 1})$, Then $T(n) = F(n)$

Calculating $\log_b a$:

$\log_b a = \log_2 4 = 2.$

$F(n) = n^2 = \Theta(n^2)$ (Comparing $F(n)$ with $n^{\log_b a}$)

$F(n) = \Theta(n^2) = \Theta(n^{\log_b a})$. — Case 2

$T(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b a} \cdot \log n)$

$= \Theta(n^2 \log n)$

Order of growth.

$T(n) = 4T(n/2) + n^2$ with

$T(1) = 1$ is $\Theta(n^2 \log n)$