Design and Analysis of Algorithms

Lecture – 19

Single Source Shortest Path

Success is always inevitable with Hard Work and Perseverance

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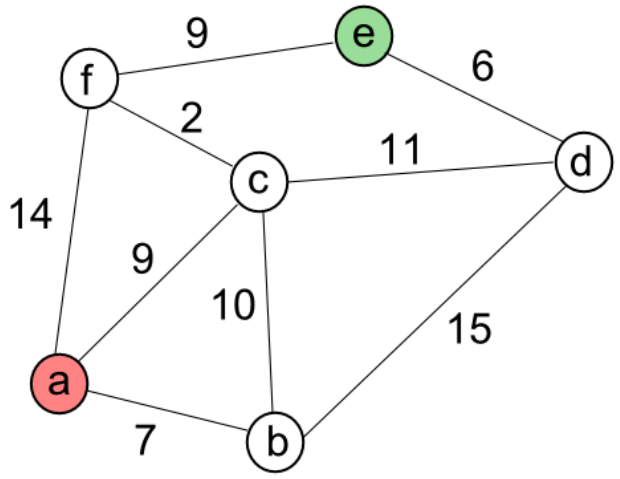
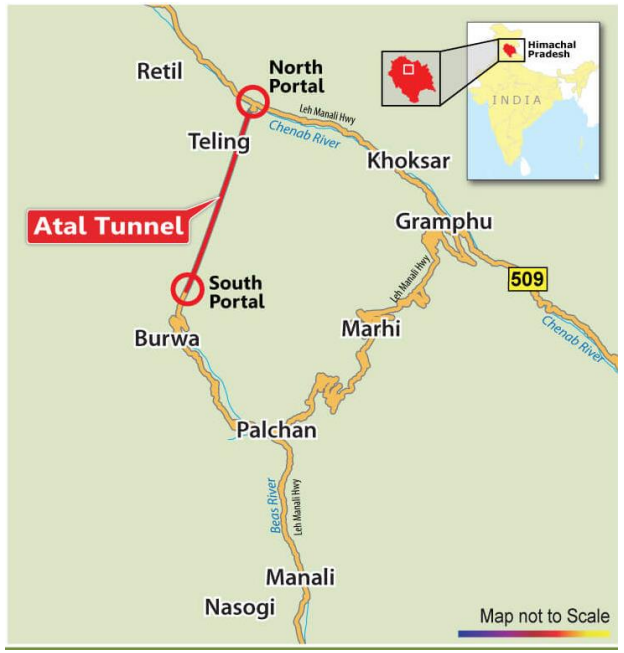
Learning Objective

- Learn the greedy strategy for a classical Problem used in transportation and packet transfer problem

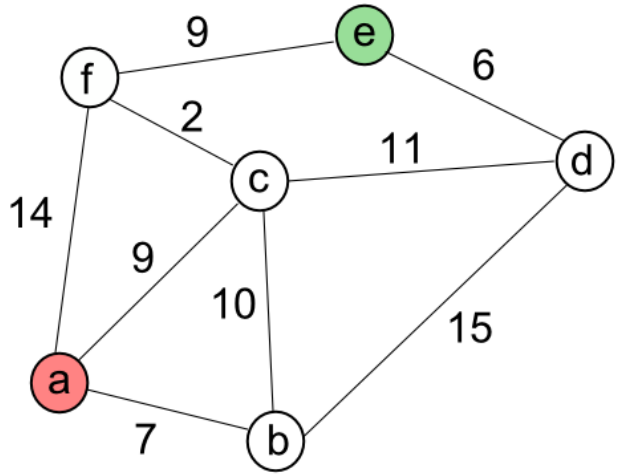
Shortest path

Given a directed / undirected graph of nodes (locations/systems) and a source node. Find the shortest path from the source node to another node in the graph. [Only Positive weights in the edges]





Shortest path



- Min cost path from a to e

  Only vertices f and d will help you to connect with

  Minimum (Min cost path from a to f + cost[f,e] ,

  Min cost path from a to d + cost[d,e])

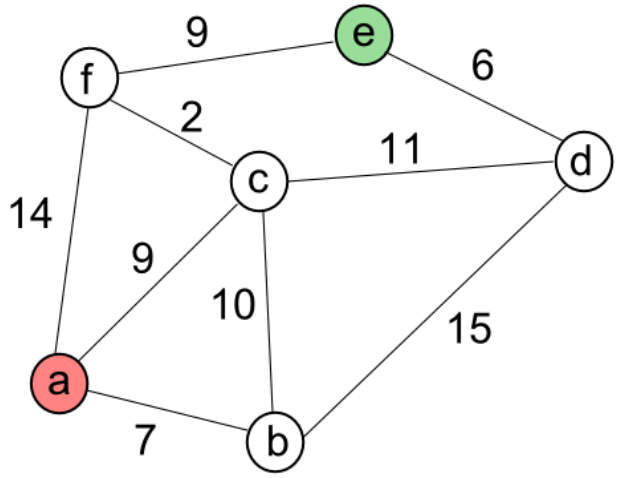
- Let us consider the Minimum cost path from a to d

  You can reach d either from b (or) c

  Which ever vertex has a shortest path from a will be used to reach d

Single source shortest path

- Shortest path from source vertex to every other vertex is to be known
- Greedy method
  - Find shortest path to one vertex
  - Reduce the problem

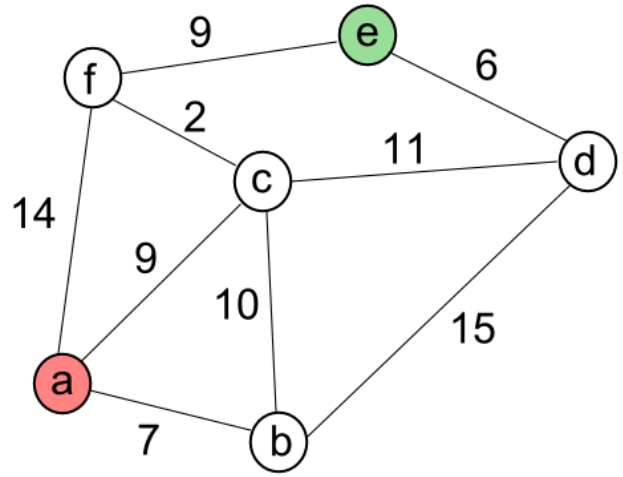


- From a , b can be reached at a distance of 7.
- Can you reduce this cost to b through alternate paths?

a

b from a at a distance of 7

c	from a	at a distance of 9
d	from a	∞
e	from a	∞
f	from a	at a distance of 14



Vertex C
Can it be reached using the shortest path found?

What will be the cost

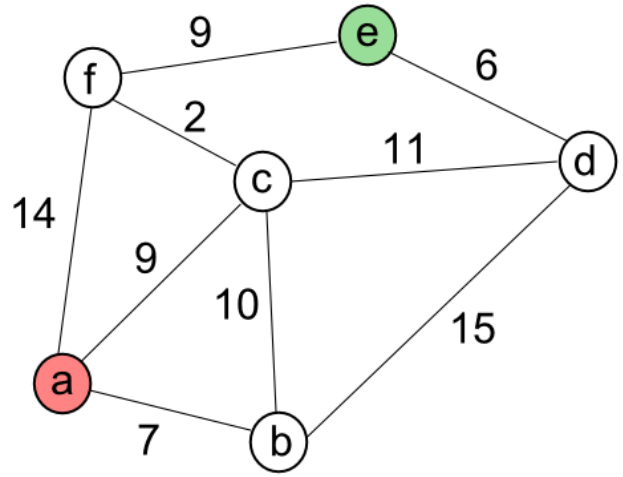
A----------B----------C ✓

Cost of path (A,B) + Cost of edge [B,C]
Should be less

a

b from a at a distance of 7

c	from a	at a distance of 9
d	from b	at a distance of 22
e	from a	∞
f	from a	at a distance of 14



Vertex D
Can it be reached using the shortest path found?

What will be the cost

A----------B----------D ✓

Cost of path (A,B) + Cost of edge [B,D]
Should be less

Procedure

Let S indicate the vertices for which shortest path is found

Initialize the distance matrix with cost of edges from source vertex

1. Find the vertex $u^*$with min cost in distance matrix
2. Include the vertex u^* to set S
3. Update the distance matrix for remaining vertices v^* which are not part of S

    Update only if there is an edge from u^* to v^*

    and existing dist value $\mathbf{dist}[v^*] > dist[u^*] + cost[u^*, v^*]$

a	b	c	d	e	f
-	7(a)	9(a)	∞	∞	14(a)

a	b	c	d	e	f
-	7(a)	9(a)	22(b)	∞	14(a)

a-> b = 7

a	b	c	d	e	f
-	7(a)	9(a)	20(c)	∞	11(c)

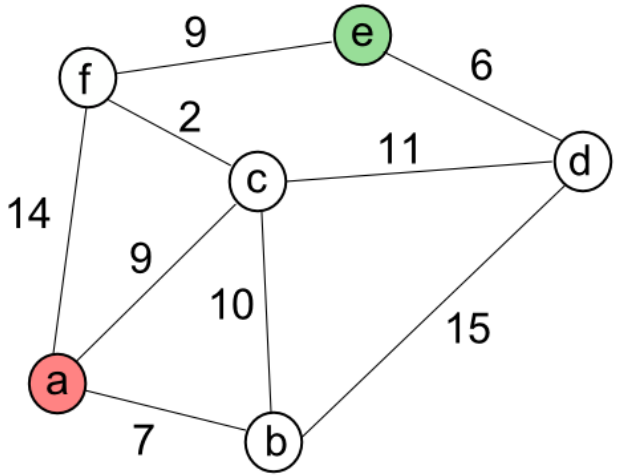
a-> c = 9

a	b	c	d	e	f
-	7(a)	9(a)	20(c)	20(f)	11(c)

a-> c ->f = 9

a	b	c	d	e	f
-	7(a)	9(a)	20(c)	20(f)	11(c)

a-> c ->d= 20



a	b	c	d	e	f
-	7(a)	9(a)	20(c)	20(f)	11(c)

a-> c ->f->e= 20

Summary

- Discussed about greedy strategy for identifying shortest path between nodes in a graph

Thank You

Happy Learning

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