Design and Analysis of Algorithms

Lecture – 17

Greedy Algorithms

Success is always inevitable with Hard Work and Perseverance

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Learning Objective

- Understand the Combinatorial Optimization Problems
- General Strategy of Greedy Algorithm
- Well-Known Examples

Combinatorial Optimization Problems

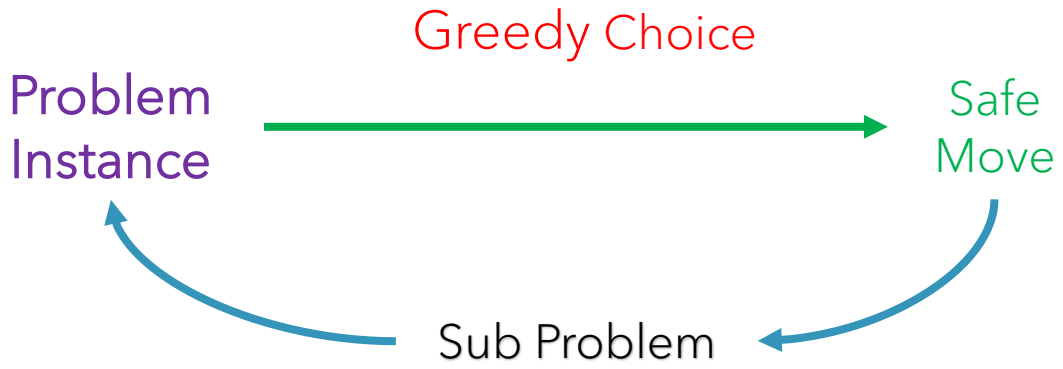
- There can be many solutions to a problem instance
- Optimal Solution
  - Maximize (or) Minimize an Objective Criterion

Find the largest number containing the following set of digits {3, 9, 2, 1,9, 6}

- Problem Instance : {3, 9, 2, 1, 9, 6}
- Feasible solution : 391296, 961293, 993126 **(6!)**
- Optimal Solution : 993126

Greedy Strategy

- Solution is built in stages



1. Make a greedy choice
2. Reduce the problem
3. Iterate until the problem can directly be solved

A safe move is a move which is consistent with some optimal solution

Pause & Think

Find the largest number containing the following set of digits {3, 9, 2, 1,9, 6}

Which Greedy Choice offers a safe move?

Possible Greedy Choice

1. Select the largest digit from the set
2. Select the smallest digit from the set
3. Select some random digit from the set

Function Largest Number(A, n)

```
Sort(A, n) # Sort elements in decreasing order
num = A[0]
for(int i=1; i<n; i++){
        num = num * 10 + A[i]
}
```

Time Complexity

Input Size : n

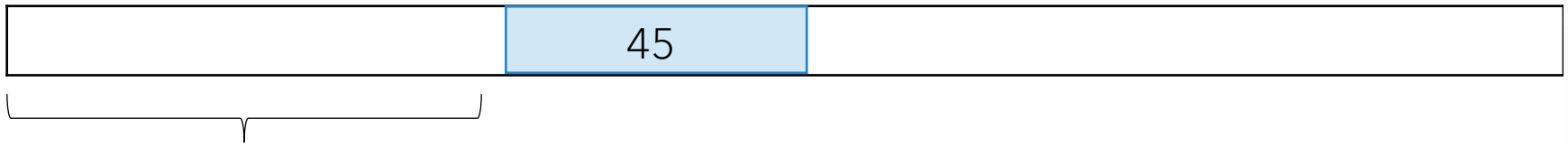
Basic Operation : Assignment (Counting sort) , Addition

$T(n) = Sort\ O(n) + Addition\ O(n) = \ O(n)$

Optimal Storage on Tapes

There are n programs to be stored on a computer tape of length l (Sequential Access). Each of these program i has a length l_i. Find a permutation of the programs such that mean retrieval time is minimized.

- Pointer in the tape will always be positioned in the front
- Access to an item depends on its position



Retrieval time for a program $R_i = \sum_{j=1}^{k} l_j$

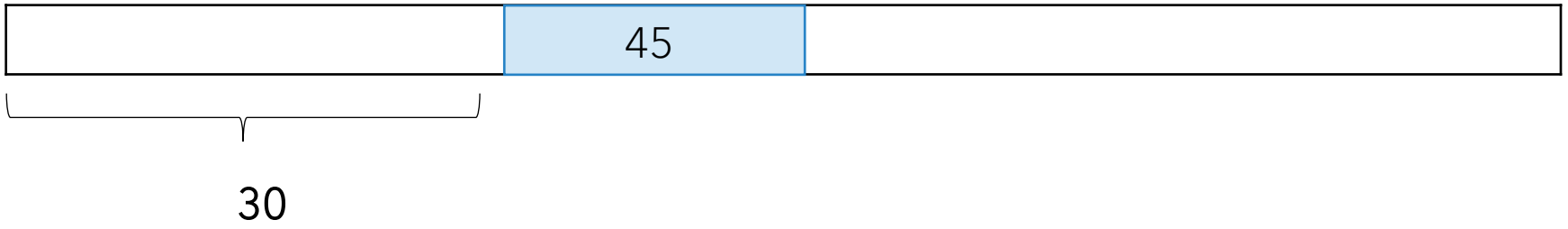
(sum of length of all programs stored before i^{th} program)

Mean Retrieval time = Average retrieval time of all the programs

$$= 1/n \sum_{i=1}^{n} R_i$$

Pause & Think

- What will be the retrieval time for the program with length 45?



- Given 3 programs with length (8, 11, 13) , Find the mean retrieval time for the ordering shown below

Problem Instance

n = 3 $(l_1, l_2, l_3) = (5, 10, 3)$

Possible Orderings =

Orderings	MRT
1, 2, 3	1/3 * $\big((\mathbf{5}) + (\mathbf{5+10}) + (\mathbf{5+10+3})\big) = \mathbf{12.67}$
1, 3, 2	1/3 * $\big((\mathbf{5}) + (\mathbf{5+3}) + (\mathbf{5+3+10})\big) = \mathbf{10.33}$
2, 1, 3	1/3 * $\big((\mathbf{10}) + (\mathbf{10+5}) + (\mathbf{10+5+3})\big)$ = 14.33
2, 3, 1	1/3 * $\big((\mathbf{10}) + (\mathbf{10+3}) + (\mathbf{10+3+5})\big)$ = 13.67
3, 1, 2	1/3 * $\big((\mathbf{3}) + (\mathbf{3+5}) + (\mathbf{3+5+10})\big)$ = 9.67
3, 2, 1	1/3 * $\big((\mathbf{3}) + (\mathbf{3+10}) + (\mathbf{3+10+5})\big)$ = 11.33

Optimal Solution = 3, 1, 2

Function Storage(A, n)

Sort(A, n) # Sort elements in increasing order
return index [of elements after sorting]

Time Complexity

Input Size : n

Basic Operation : Comparison (sort)

$T(n) = O(n \log n)$

Optimal Storage on Multiple Tapes

Given multiple tapes of infinite length, Find the optimal storage of programs onto the tapes such that MRT is minimized

- We always want the programs to be present at the start of tape
  - If there are equal number of programs and tapes
  - Every program is stored onto a different tape
- Optimal Ordering in a single tape (Increasing length)

Example

if there are three programs 5 , 8, 12 to be stored onto two tapes

On storing P_3 onto T_1 , retrieval time will be increased

Hence, optimal strategy will be to store the program onto T_0

Approach

Let there be m tapes and n programs

Optimal Solution :

Sort the programs based on increasing order of length

Place the first m programs onto tapes $T_0, T_1, T_2 \ldots T_{m-1}$

Again, Place the next m programs onto tapes $T_0, T_1, T_2 \ldots T_{m-1}$

i^{th} program will be stored onto tape $T_{i \bmod m}$

Summary

- Discussed general strategy of greedy algorithm

Thank You

Happy Learning

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