# Aryabhatiya

*An Astronomical Treatise*

*of*

## Acharya Aryabhat

*c. 476-550*

*by*


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*for*

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## Historical Background

*"Access to the Vedas is the greatest privilege this century may claim over all previous centuries."* *- Julius Robert Oppenheimer(1904-1967)*

Ved are the collection of memories from old Indo-European civilisations, compiled in Sanskrit, orally passed from generation to generation as a cultural heritage. The word वेद (*Ved*) derived from the Sanskrit root *vid* - "to know", cognate from Proto-Indo-European *u̯(e)id-*[1] "know", which is associated with Greek (ϝ)οἶδα *(w)oida* - "I know"; Latin *videō* - "I see", German *wissen* - "to know", English *wit* - "intelligence".

By the end of *dwapar-yug* (prior to 3,000 BC), Krishna Dvaipāyana, author of the epic *Mahabharata*, categorized these ancient hymns of Ved, called as *Ṛk* (ऋक्), into three attributes of text *(in Saraswati script)* - called *samhita*. These *samhita* named as Rigved samhita, Yajurved samhita, and Saamved samhita - on the basis of the course of discipline. In the second millinuim BC, the Ved recategorized into four *samhita*: Rig, Yajur, Sama and Atharv; with an auxiliary science of Vedang. The Vedang consist of six branches of studies, viz. i) Siksha (to learn proper pronounciation of letters), ii) Kalp (for the procedure of recitation), iii) Vyakarn[2] (grammar for particular *samhita*), iv) Nirukt (etymology and abbreviation), v) Jyotish (Astronomy and time keeping), vi) Chhand[3] (to account the number syllables in hymns). To study Ved, one must has to acquire the knowledge of Vedang.

Ved contains accounts of wars and migrations, astronomy and geometry, administration and agriculture practices, music and much more from distant past. For instance, a stanza from Yajur Ved is given in footnote[4] which enumerates the numerical series as 1, 10, 100, upto $10^{12}$. This numeration is futher extended in the work of Aryabhat, perhaps, geometric progression (with ratio of 10) starting from $10^7$ as *koti* upto $10^{53}$ as *tallaksana* is mention in *Lalitavistara* (biography of Gautama Buddha) - written in 1st century AD.

In Vedang, Asthadhyayi[2] gives the notion of *lopah* (लोपः - deletion) which brought up the concept of nothingness from Vedantic philosophy to mathematical logic. It is termed as *Shunya* शून्य and denoted by a dot in Chhandshastra[3] of Pingala. Later, Bhramagupta formulated the arithmatic rules of *Shunya* (zero) with positive and negative numbers in his *Brahmasputha Siddhanta*. In this essay, we will find, what level of knowledge was available to Brahmagupta before he give the rules of zero in the year of 628 AD.
*Shunya* → Arabic *sifr* → Latinized *zephyrus* → Venetian *zefiro* → French *zéro* → **Zero** [5]

[1] Julius Pokorny's book, Indogermanisches etymologisches Wörterbuch Index: 1125-27

[2] Panini's Asthadhyayi (∼ 4th century BC) is standard and the most concise work on Sanskrit Grammar

[3] Pingala's Chhandshastra (∼3rd century BC) contains combanitorics, fabonacci series, pascal triangle

[4] YJ 17.2: इमा मेऽअग्नऽइष्टका धेनवः सन्त्वेका च दश च दशु च शतं च शतं च सुहस्रं च सहस्रं चायुतं चायुतं च नियुतं च नियुतं च प्रयुतं चार्बुदं च न्यर्बुदं च समुद्रश्च मध्यं चान्तश्च परार्द्धश्चैता मेऽअग्नऽइष्टका धेनवः सन्त्वमुत्रामुष्मिँल्लोके॥॥

[5] Concept of Sunya in Indian antiquity by Dr. Parthassarathi Mukhopadhyay (pg. 21)

# Aryabhatiya

*An Astronomical Treatise*

*"Time follows the process of induction, except the beginning phase."* - c.2021

## Introduction

A dispute about the birthplace of Aryabhat आर्यभट is still exisiting among the scholars, however it is widely accepted that he belonged to the Aśmaka province of West where the river Godavari and Narmada flows.[1] Form his writing, it is clear that he lived in the Gupta era[2] and was present at Pataliputra[3] (present day Bihar state of India) while he complied the treatise *Aryabhatiya*.

The treatise consists of four chapters (*pad*) which are categorized into two sections: i) Dasagitika sutra - 13 verses of which 10 are aphorisms stating the parameters required for calculations of Astronomy; ii) Aryastast - 108 verses spread over three chapters in a set of 33, 25 and 50 verses respectively.

1. Gitika-pad (*Aphorisms*)
2. Ganita-pad (*Mathematics*)
3. Kaalkriya-pad (*Act of time*)
4. Gola-pad (*Spherical geometry*)

In the first chapter, he put forward some definitions, e.g. larger unit of time (Kalp, Manu, Yug), circular units (degree, min, sine), linear units (yojana, hasta, angula); he gave number of revolution of planets in a Yug; diameter and inclination of planets, and table of sine-difference.

In Ganita-pad, he gave rule to find square root and cube root, volume of pyramid, sphere, summation of Arithmatic progression, $\Sigma n^2$ $\Sigma n^3$, compound interest, and some other computations.

Kaalkriya-pad devoted to time cycles, number of revolution in 43,20,000 years (yug) of other planets, metrics for larger time cycle than yug, motion of planets through eccentric circles as well as epicyles, correction on superior and inferior planets, distance and speed of planets and so on.

The last chapter, Gola pad, is the longest one. Gola refers to celestial sphere and deals with spherical Astronomy. It start with the position of eliptic with respect to zodiac signs, Right Ascensions of zodiac, visibility of planets, longitute of observer, Gonom of the Sun, parallax of Sun and Moon, Occurence of eclipse and so on.

[1] K. V. Sarma (2001). "Āryabhaṭa: His name, time and provenance

[2] See: Kaalkriya pad 3.10 of Aryabhatiya gives his age 23 in c. 499

[3] See: Gitika pad 1.1 of Aryabhatiya

## Numerical Notation

To begin his work, Aryabhat gave a notation of the number system, using Sanskrit letters in Devnagari script, which counts upto $10^{18}$. In the second stanza of the first chapter, he wrote:

वर्गाक्षराणि वर्गेऽवर्गेऽवर्गाक्षराणि कात् ङ्यौ यः ।
खद्विनवके स्वरा नव वर्गेऽवर्गेऽवर्गाक्षराणि नवान्त्यवर्गे वा ॥ १.२ ॥
Classified consonants are the numbers from 1 to 25,
un-classified consonants acquire tenth place, begin from 3.
Nine vowels define the place-value of the digit,
Classified and Unclassified are two labels which vowel determines. (1.2)

Explaination: Only 42 letters of Classical Sanskrit had been used to formulate the number system, of which 9 vowels are assigned for the place-value notation to fill upto 16 digits. Rest of 33 consonants are divided into two sets: of which 25 letters are from Classified set, denoting numbers from 1 to 25; remaining 8 letters of Unclassified set stand for 30, 40, 50 to 100. The division of Classified and Unclassified consonants are based on the sound - work of previous writters like Pingala.

**Numerical Notation by Aryabhata**
**Devnagri Script**

० १ २ ३ ४ ५ ६ ७ ८ ९

**Vowels:** Place Value

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| अ = $10^0$ | इ = $10^2$ | उ = $10^4$ | ऋ = $10^6$ | ऌ = $10^8$ | ए = $10^{10}$ | ऐ = $10^{12}$ | ओ = $10^{14}$ | औ = $10^{16}$ |

**Consonants:** Face Value

**Vargiya** (classified)

| | | | | | |
|---|---|---|---|---|---|
| Velar | क (१) | ख (२) | ग (३) | घ (४) | ङ (५) |
| Palatal | च (६) | छ (७) | ज (८) | झ (९) | ञ (१०) |
| Retroflex | ट (११) | ठ (१२) | ड (१३) | ढ (१४) | ण (१५) |
| Dental | त (१६) | थ (१७) | द (१८) | ध (१९) | न (२०) |
| Labial | प (२१) | फ (२२) | ब (२३) | भ (२४) | म (२५) |

**Avargiya** (un-classified)

| | | | | |
|---|---|---|---|---|
| Semi-Vowel | य (३०) | र (४०) | ल (५०) | व (६०) |
| Sibilant | श (७०) | ष (८०) | स (९०) | |
| Aspirate | ह (१००) | | | |

The number 57753336 in expanded form:
$6 + 30 + 3.10^2 + 30.10^2 + 5.10^4 + 70.10^4 + 7.10^6 + 50.10^6$
च+ य+ ग.इ+ य.इ+ ङ.उ+ श.उ+ छ.ऋ+ ल.ऋ
चयगियिङुशुछ्लृ is the numerical notation of **57753336** in Sanskrit.

## Mathematical Tools

### Circumference

Calculation of the precise value of $\pi$ is a mathematical quest standing still in front of the modern age. Aryabhat was the first among the indian mathematician who wrote about the $\pi$ and give the ratio of *circumferece to diameter* as 3.1416; while mentioning the ratio as an aproximation.

चतुरधिकं शतमष्टगुणां द्वाषष्टिस्तथा सहस्त्राणाम् ।
अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाह: ॥
Add 4 to 100, then multiply by 8 and add to 62,000;
Circle of diameter 20,000 has approximately the same circumference. (2.10)

### Trignometry

In the verse 4 of chapter 2, a method is explained to calculate the square root of a given number. Using that square root, in verse 2.11, directions to calculate the length of a chord of the circle is given as:

समवृत्तपरिधिपादं छिन्द्यात् त्रिभुजाच्चतुभुजाच्चैव ।
समचापज्यार्धानि तु विष्कम्मार्धे यथेष्टानि ॥ ३.११॥
Evenly divide the circumference of the circle into desire parts.
From right-angled triangle and quadilateral, one can get required Rsine.(3.11)

Explanation: Aryabhata didn't wrote any algorithm to calculate the length of the chord, however, Bhaskara I (c.600 - 680) describe the whole process in his commentory on Arybhatiya[4]. Perhaps, Aryabhat gave the numerical values of Rsine($R.sin\theta$) as an aphorism in introductory chapter.

In the twelveth verse of the first chapter, Aryabhat wrote a series of numbers using his notation to express $sin\theta$ every 225 minutes (3°45'). In a compact form, he wrote:

मखि भखि फखि धखि णखि ञखि ङखि हस्झ स्ककि किष्ग श्घकि किघ्व ।
घ्लकि किग्र हक्य धकि किच स्ग श्झ ङ्व क्ल ष्त फ छ कलार्धज्या: ॥ १.१२ ॥
225 224 222 219 215 210 205 199 191 183 174
164 154 143 131 119 106 93 79 65 51 37 22 7 half-chord-difference. (1.12)

Explaination: See the next page.

[4]See Aryabhatiya of Aryabhata (Kripa Shankar Sukla) pg (45 - 51)

| Term | Unit | Expansion | Arabic | R*sin | c.f/R | Angle |
|---|---|---|---|---|---|---|
| 1 | मखि | म + ख.इ | 25 + 2x$10^2$ | 225 | 225/3438 | sin(03°45') |
| 2 | भखि | भ + ख.इ | 24 +2x$10^2$ | 224 | 449/3438 | sin(07°30') |
| 3 | फखि | फ + ख.इ | 22 + 2x$10^2$ | 222 | 671/3438 | sin(11°15') |
| 4 | धखि | ध + ख.इ | 19 + 2x$10^2$ | 219 | 890/3438 | sin(15°00') |
| 5 | णखि | ण + ख.इ | 15 + 2x$10^2$ | 215 | 1105/3438 | sin(18°45') |
| 6 | ञखि | ञ + ख.इ | 10 + 2x$10^2$ | 210 | 1315/3438 | sin(22°30') |
| 7 | ङखि | ङ + ख.इ | 5 + 2x$10^2$ | 205 | 1520/3438 | sin(26°15') |
| 8 | हस्झ | ह + स् + झ | 100 + 90 + 9 | 199 | 1719/3438 | sin(30°00') |
| 9 | स्ककि | स्+ क + क.इ | 90 + 1 + 1x$10^2$ | 191 | 1910/3438 | sin(33°45') |
| 10 | किष्ग | क.इ + ष् + ग | 1x$10^2$ + 80 + 3 | 183 | 2093/3438 | sin(37°30') |
| 11 | श्घकि | श् + घ+ क.इ | 70 + 4 + 1x$10^2$ | 174 | 2267/3438 | sin(41°15') |
| 12 | किघ्व | क.इ + घ् + व | 1x$10^2$ + 4 + 60 | 164 | 2431/3438 | sin(45°00') |
| 13 | घ्लकि | घ् + ल+ क.इ | 4 + 50 + 1x$10^2$ | 154 | 2585/3438 | sin(48°45') |
| 14 | किग्र | क.इ + ग् + र | 1x$10^2$ + 3 + 40 | 143 | 2728/3438 | sin(52°30') |
| 15 | हक्य | ह + क् + य | 100 + 1 + 30 | 131 | 2859/3438 | sin(56°15') |
| 16 | धकि | ध + क.इ | 19 + 1x$10^2$ | 119 | 2978/3438 | sin(60°00') |
| 17 | किच | क.इ + च | 1x$10^2$ + 6 | 106 | 3084/3438 | sin(63°45') |
| 18 | स्ग | स् + ग | 90 + 3 | 93 | 3177/3438 | sin(67°30') |
| 19 | श्झ | श् + झ | 70 + 9 | 79 | 3256/3438 | sin(71°15') |
| 20 | ङ्व | ङ् + व | 5 + 60 | 65 | 3321/3438 | sin(75°00') |
| 21 | क्ल | क् + ल | 1 + 50 | 51 | 3372/3438 | sin(78°45') |
| 22 | प्त | प् + त | 21 + 16 | 37 | 3409/3438 | sin(82°30') |
| 23 | फ | फ | 22 | 22 | 3431/3438 | sin(86°15') |
| 24 | छ | छ | 7 | 7 | 3438/3438 | sin(90°00') |

Table 1: The aphorism consists of 25 terms, of which the first 24 are the numerical values as deciphered above. This means, the first quadrant is divided into 24 intervals of 3°45'. The last word of the aphorism represents the trignometric function as 'half-chord difference' (कलार्धज्या: *klardhjya*). Here, *Jya* (ज्या) refers to the chord of a circle, which is double of the sinθ in an analogy to right-angled triangle. So, the *ardh-jya* (अर्धज्या:) refers to the 'half-chord' i.e.*perpendicular*(P) - product of *Radius* (R) and sinθ for any circle. Also, *kl* (कल) translated as 'difference', means the terms represent the difference in two consecutive half-chords. By adding all the 24 terms (differences), the radius of the circle can be obtained as 3438 units.

## Uranology

According to ancient Vedic calendar, time is eternal and relative to the subject, hence, it need to be measured in two distinct scales as per the context: Human scale or Cosmic scale. In Human scale, one revolution of Earth around the sun counts for one year, and in Cosmic scale, one revolution of Sun around a hypothetical point counts for a Kalp (a day of Brahma). To relate these two scales of time measurement, Aryabhat wrote in the third chapter, for refering the year of his writing:

षष्ट्यब्दानां षष्टिर्यदा व्यतीतास्त्रयश्च युगपादा: ।
त्र्यधिका विंशतिरब्दास्तदेह मम जन्मनोऽतीता:॥ ३.१० ॥
When sixty times sixty years and three yugas has elapsed.
Twenty-three years had then passed since my birth. (3.10)

He refered the year 499 AD from the beginning of the $28^{th}$ mahayug[5], of which three out of four yugas (i.e. *Krita* Yug, *Treta* Yug, and *Dwapr* Yug), and 3,600 years of *Kali Yuga* has passed away when his age was 23 years old.

To reach at this result, first he defined the length of a Yug in terms of Human years and gave a relation of it to the Kalp in the first chapter. He wrote:

युगरविभगणाः ख्युघृ शशि चयगियिङुःशुछ्लृ कु ङिशिबुण्लृष्खृ प्राक् ।
शनि डुङ्विघ्व गुरु ख्रिच्युभ कुज भद्लिङ्नुखृ भृगुबुधसौराः॥ १.३ ॥
In a yuga, Sun revolves 43,20,000 times; Moon 5,77,53,336; Earth 1,58,22,37,500.
Saturn 1,46,564; Jupiter 3,64,224; Mars 22,96,824; Venus Mercury Sun-alike. (1.3)

In the above stanza, Yug (unit of time) is defined to be 43,20,000 years long, which is equals to 1,58,22,37,500 days or 5,77,53,336 revolution of moon around the earth or 1,46,564 revolution of Saturn around the sun and so on. To express these ten digit long numerals, he devised a very unique mathematical notation which is known as place-value system in present days. This elegant notation in *Devnagari* देवनागरी script is briefly explained in the Numerical Notation section of this essay.

To deduce a relation in between Human year, Yug, and Kalp, few other units of time need to be introduced. Aryabhat related all these parameters as follow:

काहो मनयो ढ मनुयुगा: श्ख गतास्ते च मनुयुगा: छ्ना च ।
कल्पादेर्युगपादा ग च गुरुदिवसाच्च भारतात् पूर्वम् ॥ १.५ ॥
Kalp equals 14 Manu, Manu equals 72 mahayug. Present is 7th Manu, 28th mahayug.
On Thursday, Kalp began, 6 Manu 27 mahayug 3 yug had elapsed prior to Bharat. (1.5)

[5]of *Vaivasvata Manu* 27 mahayug is passed away, present is 28th. Matsya Purana (290.3-12)

Here, 'prior to Bharat' refer to - *before the Battle of Bharat*[6]. After a century of that battle, a rare conjunction of Saturn, Jupiter, Mars, Venus, Mercury, Sun and - Southern lunar node (केतु *Ketu*) near the fixed star ζ-Piscium (रेवती *Revati*) on the night of 17 to 18 Feb, 3102 BC (Julian Calendar) marks the beginning of the Dark age (कलयुग *Kalyug*)[7][8]. As of 18 Feb 2021, Kalyug has been completing its 5123 years in the span of 43,20,000 years.

The knowledge of time keeping is as ancient as Ved, which Aryabhat must had aquired in his childhood, traditionally. In his calculation, he used the unit Kalp as 72 mahayug, which is a new correction introduce by him. From the original sources [9], these parameters are described as: 'A Kalp is a period of fourteen Manvantar, each spans of roughly 71 mahayug. One Mahayug stands for 4,32,00,000 years, so one Manvantar is a period of 306,720,000 years and one Kalp is 4,320,000,000 years.'

In the above stanza, he emphasis that, from the present manvantar, *Vaivasvata manvantar*, 27 mahayuga has passed away and of the current 28th mahayug, Kreta (1,728,000 years), Treta (1,296,000 years) and Dwapr (864,000 years) have passed away.

The word मन्वन्तर manvantar is made up of मनु+ अन्तर manu + antar, man-v-antar i.e. the lifespan of a manu. Manu मनु means the progenitor of humanity, derived word Manav – 'of manu' – means human in the English language. And the English word 'man' from Proto-Germanic – 'manwaz'.

[6]Mahabharat of Krishna Dvaipāyana, Aryabhatiya of Aryabhta Vol 1 K-V- Sharma (pg 10)
[7]सुर्य सिध्दान्त Surya Siddhant - Solar Principle
[8]Godwin 2011, p. 301
[9]Manusmriti Ch. 1, Slokas 67, 71-72, 79-80

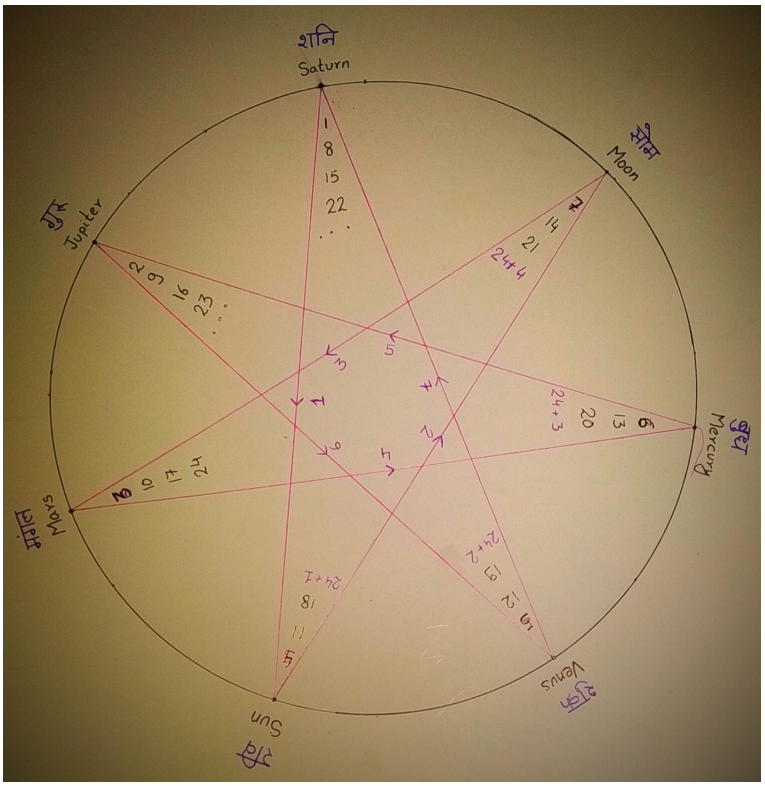
## 7 days of Week

The following verse explains the one day cycle of 24 periods and how it is related to the planetary system.

सप्तैते होरेशाः शनैश्चराध्या यथाकरमं शीघ्राः ।
शीघ्रक्रमाच्चतुर्था भवन्ति सूर्योदयाद् दिनपाः ॥ ३.१६ ॥

Seven planet beginning with Saturn, arranged in increasing velocity, are the lords of the successive hours.
Fourth planet occurring in the order is the lord of successive day, which is reckoned from sunrise.(3.16)

According to above stanza, seven planets for 24 hours will be arranged as:
**Saturn**, Jupiter, Mars, **Sun**, Venus, Mercury, **Moon**, Saturn, Jupiter, **Mars**, Sun, Venus, **Mercury**, Moon, Saturn, **Jupiter**, Mars, Sun, **Venus**, Mercury, Moon, **Saturn**, Jupiter and Mars are the respective lord of every hour for Saturday. Since, Mars was the last, successive hour will be Sun, so the Sunday is the lord of next day.
The lord of the day is the lord of the first hour of that day and every fourth lord will be the lord of successive day. Hence, the lords of the seven days are: Saturn (Saturni), Sun(Solis), Moon(Lunae), Mars(Martis), Mercury(Mercurii), Jupiter(Iovis) and Venus(Veneris) respectively.



## Shortcoming in Aryabhatiya

Being a great source of learning for centuries, merely, the treatise Aryabhatiya is not free from the flaws. While living in 21th century, critizing a work of 5th century is not an act of honour, however, refining the shortcoming in the deeds of predecessors and revise it, is the first duty of any intellect.

In practice, different levels of omission can be made in the work of Aryabhat, it can be based on mathematical, observational, or orthodoxial practices. For example, in the last chapter - *Gola-pad*, while explaining the phenomenon of eclipse, he wrote:

चन्द्रो जलमर्कोऽग्निः मृदभूश्च्छायापि या तमस्तद्धि ।
धादयति शशी सूर्यं शशिनं महती च भूच्छाया ॥

Moon corresponds to water, Sun is to fire, Earth is to soil and Shadow is darkness.
Moon eclipses the Sun and the great Shadow of the Earth eclipses the Moon.(4.37)

Here, it can be underlined that Moon is not the water. Similarly, table of Rsine often revised by later mathematician to get more precise values. e.g Aryabhat II (c. 950) updated 4th and 16th Rsine to 889' and 2977' respectively.

## Biography of Aryabhat

Aryabhat born on c. 13.Apr.499[10] and become the head of Nalanda University where he complied *Aryabhatiya*and *Aryabhat sidhhanta* (translated by Abu Hasan Ahwazi[1] in Arabic).

He was the earliest astronomer of India who calculated the sidereal rotation of the Earth as $23^h56^m4.1^s$, as precise as the modern value ($23^h56^m4.0905^s$). His student Latadeva known for commentary on *Romoka-sidhhant* (Byzantine (Rome) Astronomy)[11] and *Paulisa Sidhhanta*(Hellenistic astronomy).

[10] Aniversay celebrated by Bihar Research Society
[11] McEvilley, (2001), p385

## Future Work

Brahmagupta (c. 628) Haridatta (c. 683) Sankaranarayan (c. 869) from Kerela Aryabhata (c. 950) Udayadivakara (c. 1073) Bhaskara II(c. 1150) Suryadeva (c. 1191) Paramesvara (c. 1431) Yallaya (c. 1480) Nilkantha (c. 1500) Raghunatha-raja (c. 1597) Jyeshtadev (c. 1603) Visvanath (c. 1629)

भद्राम्बुद्धिसिद्धजन्मगणितश्रद्धा स्म यद् भूपगी:

भ र ब ध स ध ज म ग ण त र ध म य भ प ग

4 2 3 9 7 9 8 5 3 5 6 2 9 5 1 4 1 3

Mādhava follows Āryabhata and develops the power series for sine, cosine and arctangent using recursion.

which was firstly translated by William Jones in 1777.

| Count | $10^0$ = अ | $10^2$ = इ (ि) | $10^4$ = उ (ु) | $10^6$ = ऋ (ृ) | $10^8$ = ऌ (ॢ) | $10^{10}$ = ए | $10^{12}$ = ऐ | $10^{14}$ = ओ | $10^{16}$ = औ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | क १ | कि | कु | कृ | कॢ | के | कै | को | कौ |
| 2 | ख २ | खि | खु | खृ | खॢ | खे | खै | खो | खौ |
| 3 | ग | गि ३०० | गु | गृ | गॢ | गे | गै | गो | गौ |
| 4 | घ | घि ४०० | घु | घृ | घॢ | घे | घै | घो | घौ |
| 5 | ङ | ङि | ङु ५०,००० | ङृ | ङॢ | ङे | ङै | ङो | ङौ |
| 6 | च | चि | चु ६०,००० | चृ | चॢ | चे | चै | चो | चौ |
| 7 | छ ७ | छि | छु | छृ ७०,००,००० | छॢ | छे | छै | छो | छौ |
| 8 | ज ८ | जि | जु | जृ ८०,००,००० | जॢ | जे | जै | जो | जौ |
| 9 | झ | झि ९०० | झु | झृ | झॢ ९०,००,००,००० | झे | झै | झो | झौ |
| 10 | ञ | ञि १,००० | ञु | ञृ | ञॢ १,००,००,००,००० | ञे | ञै | ञो | ञौ |
| 11 | ट | टि | टु १,१०,००० | टृ | टॢ | टे | टै | टो | टौ |
| 12 | ठ | ठि | ठु १,२०,००० | ठृ | ठॢ | ठे | ठै | ठो | ठौ |
| 13 | ड १३ | डि | डु | डृ १,३०,००,००० | डॢ | डे | डै | डो | डौ |
| 14 | ढ १४ | ढि | ढु | ढृ १,४०,००,००० | ढॢ | ढे | ढै | ढो | ढौ |
| 15 | ण | णि १,५०० | णु | णृ | णॢ १,५०,००,००,००० | णे | णै | णो | णौ |
| 16 | त | ति १,६०० | तु | तृ | तॢ १,६०,००,००,००० | ते | तै | तो | तौ |
| 17 | थ | थि | थु १,७०,००० | थृ | थॢ | थे | थै | थो | थौ |
| 18 | द | दि | दु १,८०,००० | दृ | दॢ | दे | दै | दो | दौ |
| 19 | ध १९ | धि | धु | धृ १,९०,००,००० | धॢ | धे | धै | धो | धौ |
| 20 | न २० | नि | नु | नृ | नॢ २,००,००,००,००० | ने | नै | नो | नौ |
| 21 | प | पि २,१०० | पु | पृ | पॢ २,१०,००,००,००० | पे | पै | पो | पौ |
| 22 | फ | फि २,२०० | फु | फृ | फॢ | फे | फै | फो | फौं |
| 23 | ब | बि | बु २,३०,००० | बृ | बॢ | बे | बै | बो | बौं |
| 24 | भ | भि | भु २,४०,००० | भृ | भॢ | भे | भै | भो | भौं |
| 25 | म २५ | मि | मु | मृ २,५०,००,००० | मॢ | मे | मै | मो | मौं |
| 30 | य ३० | यि | यु | यृ ३,००,००,००० | यॢ | ये | यै | यो | यौं |
| 40 | र | रि ४,००० | रु | रृ | रॢ ४,००,००,००,००० | रे | रै | रो | रौं |
| 50 | ल | लि ५,००० | लु | लृ | लॢ ५,००,००,००,००० | ले | लै | लो | लौं |
| 60 | व | वि | वु ६,००,००० | वृ | वॢ | वे | वै | वो | वौं |
| 70 | श | शि | शु ७,००,००० | शृ | शॢ | शे | शै | शो | शौं |
| 80 | ष ८० | षि | षु | षृ ८,००,००,००० | षॢ | षे | षै | षो | षौं |
| 90 | स ९० | सि | सु | सृ ९,००,००,००० | सॢ | से | सै | सो | सौं |
| 100 | ह | हि १०,००० | हु | हृ | हॢ १०,००,००,००,००० | हे | है | हो | हौं |

Appendix - 1