

CSE - 3rd Semester
Digital and Analog Communication

(1)

DAC

Section-A

Fourier Series: → Fourier Series is a tool

used to analyze any periodic signal. After the analysis we obtain the following information about the signal

1. what all frequency component are present in the signal?
2. Their Amplitude
3. The relative phase difference between these frequency components.

Types of Fourier Series: →

- (1) Trigonometric or quadrature Fourier Series
- (2) Polar Fourier Series
- (3) Exponential F. S.

here we are going to discuss only trigonometric Fourier Series

A periodic signal $x(t)$ with a period of T_0 is given the trigonometric Fourier Series as.

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t) \quad (1)$$

where a_0 , a_n and b_n are known as Fourier Coefficients and $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$

The coefficients are as follows:-

1. The value of $a_0 = \frac{1}{T_0} \int_t^{t+T_0} x(t) dt$ (ii)

It is called d.c Component of $x(t)$

2. The value of $a_n = \frac{2}{T_0} \int_t^{t+T_0} x(t) \cdot \cos(n\omega_0 t) dt$ (iii)

3. $b_n = \frac{2}{T_0} \int_t^{t+T_0} x(t) \cdot \sin(n\omega_0 t) dt$ (iv)

now eq (1) becomes.

$$x(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \dots + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \dots$$

↓ ↓ ↓ ↓ ↓ ↓

D.C value fundamental second harmonic

(V)

Conclusion : → The eq (v) is suitable to
Plot the line spectrum.

(3)

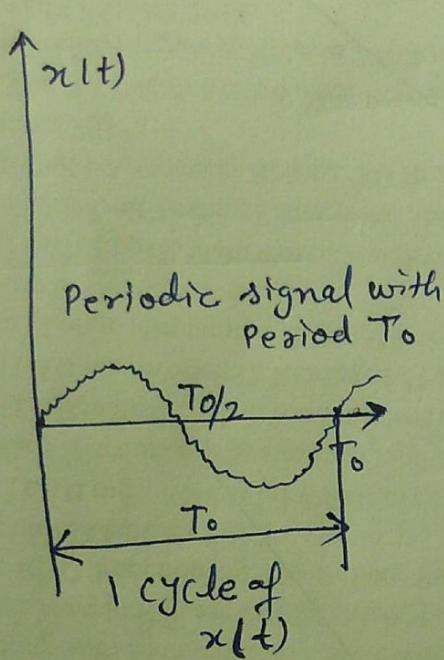


fig (a). Time domain representation of $x(t)$

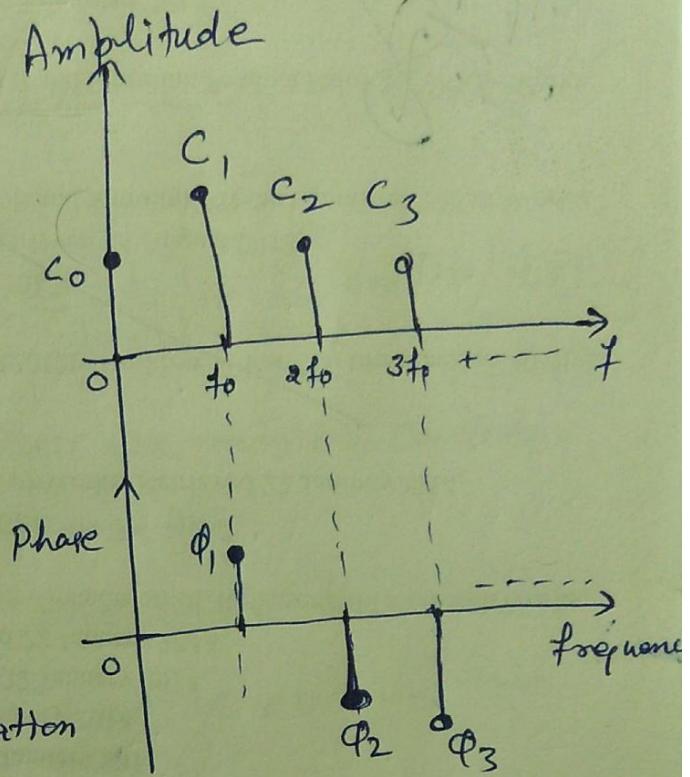
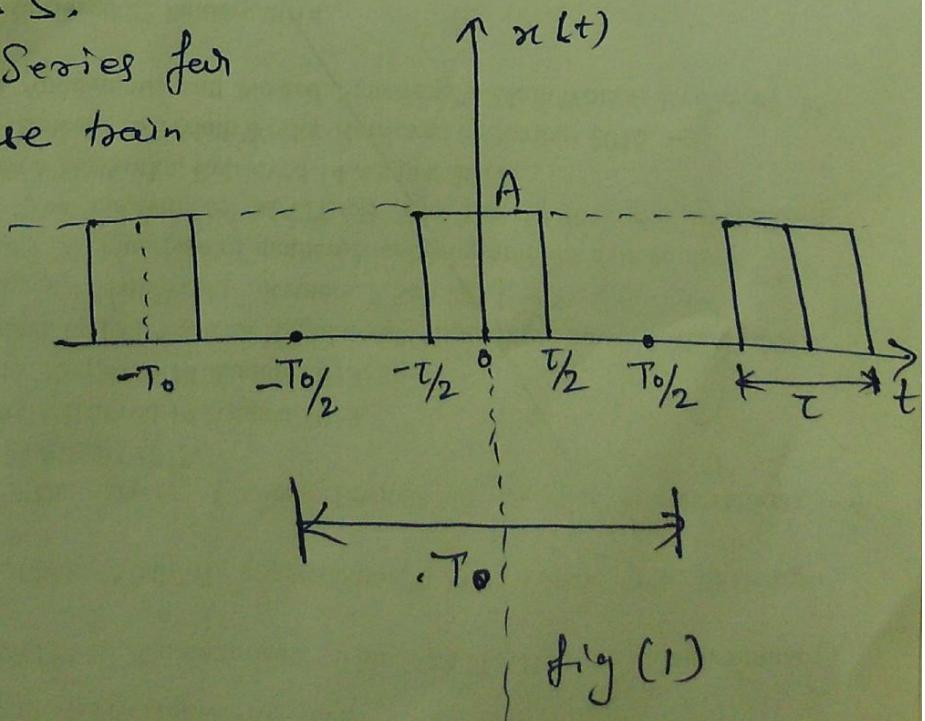


fig (b). Line spectrum of $x(t)$ after solving the Fourier Series

Example 1. Numerical Problem based on F. S.

Q1 Obtain Fourier Series for the rectangular pulse train shown in figure (i)



Solution The Fourier Series is given by (4)

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t) \quad (1)$$

substituting $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$ in eq (1) we get

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left[\frac{2\pi n t}{T_0}\right] + \sum_{n=1}^{\infty} b_n \sin\left[\frac{2\pi n t}{T_0}\right] \quad (2)$$

To find Fourier coefficients, we must consider one cycle of $x(t)$ for integration.

Here we will consider one cycle from

$t = -T_0/2$ to $t = T_0/2$. Let us obtain the Fourier coefficients now

1. To obtain the value of a_0 :

$$a_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) dt \quad \text{--- As } x(t) \text{ exists from } -T_0/2 \text{ to } T_0/2 \text{ only}$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A \cdot dt$$

As $x(t) = A$, for $-T_0/2 \leq t \leq T_0/2$

$$\therefore a_0 = \boxed{\frac{A}{T_0} T}$$

(3)

Q. To obtain the value of a_n : (5)

$$\begin{aligned}
 a_n &= \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \cdot \cos\left[\frac{2\pi n t}{T_0}\right] dt = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} A \cdot \cos\left[\frac{2\pi n t}{T_0}\right] dt \\
 &= \frac{2A}{T_0} \cdot \frac{1}{\frac{2\pi n}{T_0}} \left[\sin\left(\frac{2\pi n t}{T_0}\right) \right]_{-T_0/2}^{T_0/2} \\
 &= \frac{A}{n\pi} \left[\sin\left(\frac{2\pi n T}{2T_0}\right) - \sin\left(-\frac{2\pi n T}{2T_0}\right) \right] \\
 &= \frac{A}{n\pi} \left[\sin\left(\frac{\pi n T}{T_0}\right) + \sin\left(\frac{\pi n T}{T_0}\right) \right]
 \end{aligned}$$

$$a_n = \frac{2A}{n\pi} \sin\left[\frac{n\pi T}{T_0}\right]$$

(4)

3. To obtain the value of b_n :

$$\begin{aligned}
 b_n &= \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \cdot \sin\left[\frac{2\pi n t}{T_0}\right] dt \\
 &= \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} A \cdot \sin\left[\frac{2\pi n t}{T_0}\right] dt
 \end{aligned}$$

$$b_n = -\frac{2A}{T_0} \cdot \frac{1}{\frac{2\pi n}{T_0}} \left[\cos\left(\frac{2\pi n t}{T_0}\right) \right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \quad (6)$$

$$= -\frac{A}{n\pi} \left[\cos\left(\frac{n\pi t}{T_0}\right) - \cos\left(-\frac{n\pi t}{T_0}\right) \right]$$

$$= -\frac{A}{n\pi} \left[\cancel{\cos\left(\frac{n\pi t}{T_0}\right)} - \cancel{\cos\left(\frac{n\pi t}{T_0}\right)} \right]$$

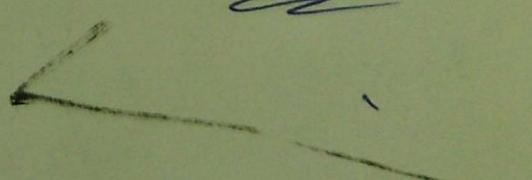
$b_n = 0$

(5)

now Putting the value of a_0 , a_n and b_n in of (2) we get the trigonometric Fourier Series for $x(t)$ as,

$$x(t) = \frac{AT_0}{T_0} + \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\left[\frac{n\pi t}{T_0}\right] \cos\left[\frac{2\pi n t}{T_0}\right]$$

Ans



Fourier Transform:

7.

The Fourier transform of a signal $x(t)$ is defined as

$$X(w) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad \text{I}$$

or

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \quad \text{II}$$

Inverse Fourier Transform:

The signal $x(t)$ can be obtained back from Fourier transform $X(f)$ by using the inverse Fourier transform. It is defined as

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(w) \cdot e^{j\omega t} dw \quad \text{III}$$

OR

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \quad \text{IV}$$

Conditions for the existence of F.T:

For a Periodic signals the integration is obtained over one period however for the non-periodic signals, it will be obtained over a range $-\infty$ to ∞ .

The signal $x(t)$ will have to satisfy the following conditions so that its F.T can be obtained.

1. The function $x(t)$ should be single valued in any finite interval T .
2. $x(t)$ should have a finite number of discontinuities in any finite interval T .
3. The function $x(t)$ should have a finite number of maxima and minima in any finite interval of time T .
4. The function $x(t)$ should be an absolutely integral function i.e $\int_{-\infty}^{\infty} |x(t)| dt < \infty$

The conditions stated above are sufficient conditions, but they are not the necessary conditions.

Merits of F.T:

- (1) It is possible to uniquely recover the original time function $x(t)$.
- (2) We can evaluate the convolutional integrals using F.T
- (3) F.T is very useful in comm. Systems

Limitations of F.T: → The most important limitation of F.T is that there are many time functions for which the F.T does not exist, because such functions are not absolutely integrable.

Properties of F.T: →

(i) Linearity or superposition: →

if $x_1(t) \xleftrightarrow{F} X_1(f)$ and $x_2(t) \xleftrightarrow{F} X_2(f)$

then all constants such as a_1 and a_2 we can write

$$[a_1 x_1(t) + a_2 x_2(t)] \xleftrightarrow{F} [a_1 X_1(f) + a_2 X_2(f)]$$

That means the linear combination of inputs gets transformed into linear combination of their Fourier transform. ①

(2) Time scaling: → Let $x(t) \xleftrightarrow{F} X(f)$ and let α be a constant. Then the time scaling Property states that.

$$x(\alpha t) \xleftrightarrow{F} \frac{1}{|\alpha|} X(f/\alpha) \quad \text{②}$$

(i) For $\alpha < 1$, $x(\alpha t)$ will be a compressed signal but $X(f/\alpha)$ will be an expanded version of $X(f)$.

(2) For $\alpha > 1$, $x(\alpha t)$ will be expanded signal in the time domain. But the F.T $X(f/\alpha)$ represents a compressed version of $X(f)$.

(3) Duality or Symmetry Property : \rightarrow

10.

It states that

if $x(t) \xleftrightarrow{F} X(f)$ then.

$$\boxed{x(t) \xleftrightarrow{F} x(-f)} \rightarrow \textcircled{3}$$

i.e. t and f can be interchanged.

The duality theorem tell us that if $x(t) \xleftrightarrow{F} X(-f)$ then the shape of the signal in time domain and the shape of the spectrum can be interchanged.

(4) Time shifting : \rightarrow It states that if $x(t)$ and $X(f)$ form a fourier transform pair then

$$\boxed{x(t-t_d) \xleftrightarrow{F} e^{-j2\pi f t_d} X(f)} \rightarrow \textcircled{4}$$

here the signal $x(t-t_d)$ is a time shifted signal.
It is the same signal $x(t)$ only shifted in time.

(5) Frequency Shifting : \rightarrow The frequency shifting characteristics states that if $x(t)$ and $X(f)$ form a Fourier transform pair then

$$\boxed{e^{j2\pi f_c t} x(t) \xleftrightarrow{F} X(f-f_c)} \rightarrow \textcircled{5}$$

Here f_c is a real constant.

(6) Differentiation in time domain : \rightarrow Some processing techniques involves differentiation and integration of the signal $x(t)$. This property is applicable if and only if the derivative of $x(t)$ is Fourier transformable.

Let $x(t) \xleftarrow{F} X(f)$ and let the derivative of $x(t)$ be Fourier transformable. Then,

$$\boxed{\frac{d}{dt} x(t) \xleftarrow{F} j2\pi f X(f)} \quad (6)$$

That means differentiating the signal in time domain is equivalent to multiplying its F.T by $(j2\pi f)$.

(7) Integration in time domain : \rightarrow Integration in time domain is equivalent to dividing the Fourier transform by $(j2\pi f)$.

i.e if $x(t) \xleftarrow{F} X(f)$ and provided that $X(0) = 0$ then

$$\boxed{\int_{-\infty}^t x(\lambda) d\lambda \xleftarrow{F} \frac{1}{j2\pi f} X(f)} \quad (7)$$

⑧ Multiplication in Time Domain (Multiplication theorem) 12

The multiplication theorem states that : if $x_1(t) \xrightarrow{F} X_1(f)$ and $x_2(t) \xrightarrow{F} X_2(f)$ are the two fourier transform pairs then :

$$\boxed{x_1(t) \cdot x_2(t) \xrightarrow{F} \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(f-\tau) d\tau} \quad \textcircled{1}$$

This means that the multiplication of two signals $x_1(t)$ and $x_2(t)$ in the time domain gets transformed into convolution of their frequency transforms in the frequency domain.

$$\therefore \boxed{x_1(t) \cdot x_2(t) \xrightarrow{F} X_1(f) * X_2(f)} \quad \textcircled{2}$$

⑨ Convolution in the Time Domain (Convolution Theorem)

This property states that the convolution of signals in the time domain will be transformed into the multiplication of their fourier transforms in the freq. domain.

i.e $\boxed{[x_1(t) * x_2(t)] \xrightarrow{F} X_1(f) \cdot X_2(f)} \quad \textcircled{3}$

Spectral Density functions:

13.

The spectral density of a signal, defines the distribution of energy or power per unit bandwidth as a function of frequency.

The spectral density of energy signals is called energy spectral density (ESD) while that of the Power signal is called as Power spectral density (PSD).

(1) Energy spectral density (ESD):

According to Rayleigh's energy theorem, the total energy of a signal is given by

$$E = \int_{-\infty}^{\infty} |X(f)|^2 df$$

This means that the energy of a signal $x(t)$ is a result of energies contributed by all the spectral components of the signal $x(t)$. Thus it is possible to obtain the energy of the signal by calculating the area under the square of its amplitude spectrum $|X(f)|$. The Rayleigh energy theorem not just provide a useful method of evaluating the total energy but it also tell us that $|X(f)|^2$ can provide us the distribution of energy of signal $x(t)$ in the freq. domain.

Power Spectral Density (PSD): \rightarrow Let $x(t)$ be a real valued power signal. Then the average normalized power of this signal is defined as 14

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt \quad \text{--- (1)}$$

If the signal $x(t)$ is periodic with a period T_0 then according to the Parseval's Power theorem, the average signal power is given by

$$P = \sum_{n=-\infty}^{\infty} |C_n|^2 \quad \text{--- (2)}$$

For a periodic signal C_n is expressed as follows

$$\boxed{C_n = \frac{1}{T_0} \times (x(nT_0))} \quad \text{--- (3)}$$

Putting eq (3) into eq (1) we get

$$P = \sum_{n=-\infty}^{\infty} \frac{1}{T_0^2} |x(nT_0)|^2$$

$$\boxed{P = \frac{1}{T_0^2} \sum_{n=-\infty}^{\infty} |x(nT_0)|^2} \quad \text{--- (4)}$$

But the F.T of a periodic signal is not continuous. It is discrete and present only at freq. $\pm f_0, \pm 2f_0, \pm 3f_0, \dots$

so now we can define the power spectral density of a periodic signal $x(t)$ as follows.

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$$\boxed{\text{PSD : } S(f) = \frac{1}{T_0^2} \sum_{n=-\infty}^{\infty} |x(nT_0)|^2 \delta(f - nT_0)} \quad (5)$$

The most important property of PSD is that the area under PSD is equal to the average power of the signal $x(t)$

$$\boxed{P = \int_{-\infty}^{\infty} S(f) df} \quad (6)$$

Rayleigh's Energy theorem : \rightarrow The Rayleigh's energy theorem is analogous to the Parseval's Power theorem. It states that the total energy of the signal $x(t)$ is equal to the sum of energies of the individual spectral components in the frequency domain. The total normalized energy of a signal $x(t)$ is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad (7)$$

According to Rayleigh's energy theorem

$$\text{Total } ^{\text{normalized}} \text{energy} : E = \int_{-\infty}^{\infty} |X(f)|^2 df \quad (8)$$

Thus the total energy is equal to the area under the signal corresponding to the square of the amplitude spectrum $|X(f)|$ of the signal.

Data Transmission System : →Physical Layer Connections : →

Modulation : Before the modulation we have to know the following things:

① Baseband and Bandpass signals : →

(a) Baseband signal : → The information or input signal to a comm. system can be analog i.e sound, picture or it can be digital i.e the computer data. The electrical equivalent of this original information signal is known as the baseband signal.

In some systems, called the baseband transmission system, the baseband signals are directly transmitted (without modulation).

The frequency spectrum of a baseband signal is shown in fig (i). It occupies the frequency spectrum right from 0 Hz

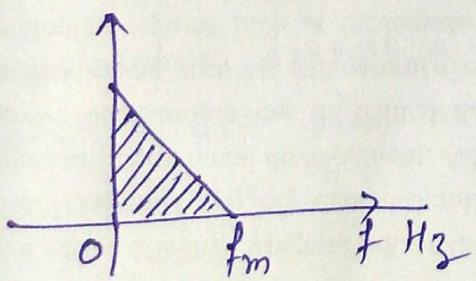


fig (i). Spectrum of a baseband signal

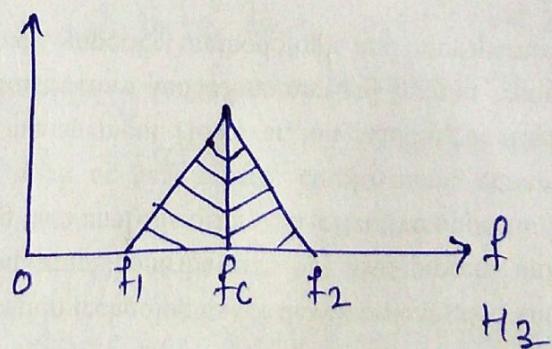


fig (2) Spectrum of a bandpass signal.

(b) Bandpass signal : \rightarrow It can be defined as a signal which has a non zero lowest frequency in its spectrum. That means the frequency spectrum of a bandpass signal extends from f_1 to f_2 Hz.

The modulated signal is called as the bandpass signal. It is obtained by shifting the baseband signal in frequency domain. The spectrum is shown in fig(2).

Examples of bandpass signals are the ultrasound waves, visible light, radio waves etc.

Modulation : \rightarrow In the modulation process, two signals are used namely the modulating signal and the carrier. In modulation process some parameter of the carrier wave (such as amplitude, freq, or phase) is varied in accordance with the modulating signal. This modulated signal is then transmitted by the transmitter.

The receiver will "demodulate" the received modulated signal and get the original information signal back. This demodulation is exactly opposite to modulation.

Multiplexing : \rightarrow It is the process of combining several message signals together and send them over the same communication channel. Three multiplexing techniques

- (1) Frequency Division Multi - (FDM)
- (2) Time division Multi - (TDM)
- (3) Code division Multi - (CDM)

Advantages of Modulation :

(18)

- (1) Reduction in the height of antenna
- (2) Avoids mixing of signals
- (3) Increase the range of communication
- (4) Multiplexing is possible
- (5) Improve quality of reception.

i) Reduction of Antenna height : → For radio signals transmission, the antenna height must be a multiple of $(\lambda/4)$, $\lambda \rightarrow$ wavelength $\lambda = \frac{C}{f}$

so for example for a signal $f = 10 \text{ KHz}$, the antenna height is

$$\text{Minimum antenna height} = \lambda/4 = \frac{C}{4f} = \frac{3 \times 10^8}{4 \times 10 \times 10^3} \\ = 7500 \text{ m} \quad \text{i.e. } 7.5 \text{ Km}$$

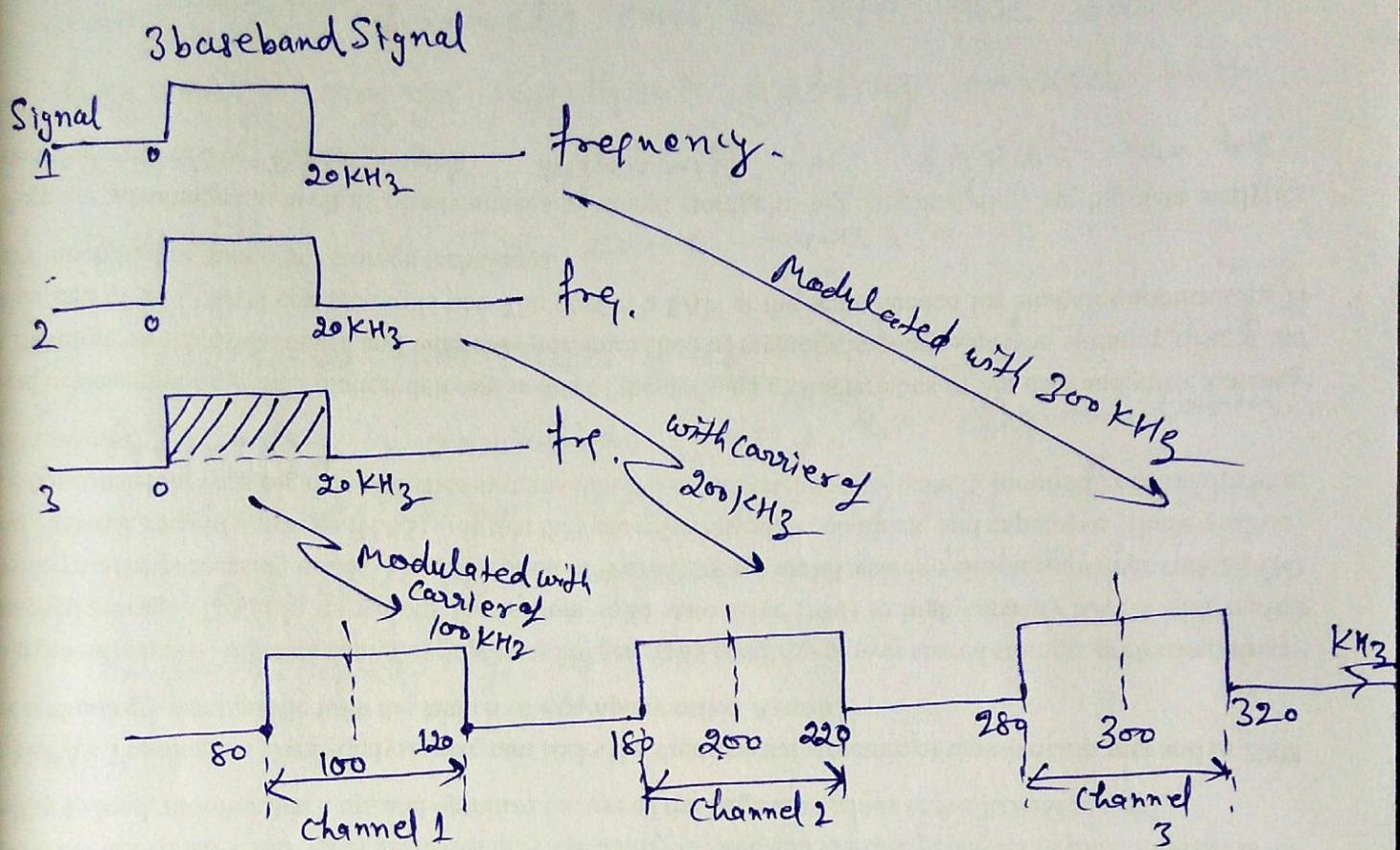
now consider the modulated signal at $f = 1 \text{ MHz}$, the Antenna height is

$$\text{MAH} = \lambda/4 = \frac{C}{4f} = \frac{3 \times 10^8}{4 \times 10 \times 10^6} = 75 \text{ m}$$

so it reduces the antenna height.

(2) Avoids mixing of signals : if the baseband signals are transmitted without using the modulation by more than one transmitter, then all the signals will be in the same freq range i.e. $0 \text{ to } 20 \text{ KHz}$ then receiver is not able to separate them from each other.

so if each baseband sound signal is used to modulate a different carrier then they will occupy different slots in the frequency domain. 19



fig(1). Modulation avoids mixing of signals

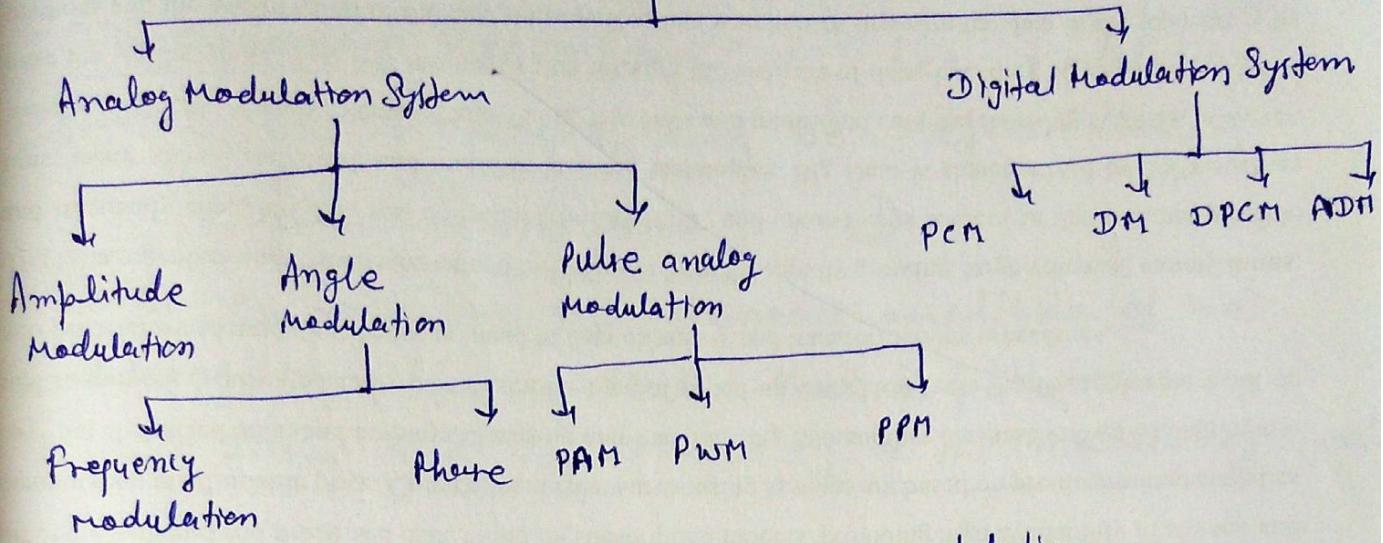
- ③ Increases the range of Comm: → The low frequency signal can not travel a long distance when they are transmitted. They get heavily attenuated (suppressed). But the modulation process increases the freq. of the signal to be transmitted. Hence it increases the range of communication.

④ Multiplexing is Possible \rightarrow This is possible $\textcircled{25}$
only with modulation. The multiplexing allows the same channel to be used by many signals. So many TV channels can ~~be~~ use the same frequency range without getting mixed with each other. OR different freq. signals can be transmitted at the same time.

⑤ Improves the quality of reception \rightarrow With frequency modulation (FM) ^{and}, the digital Comm. techniques like PCM, the effect of noise is reduced to a great extent. This improve quality of reception.

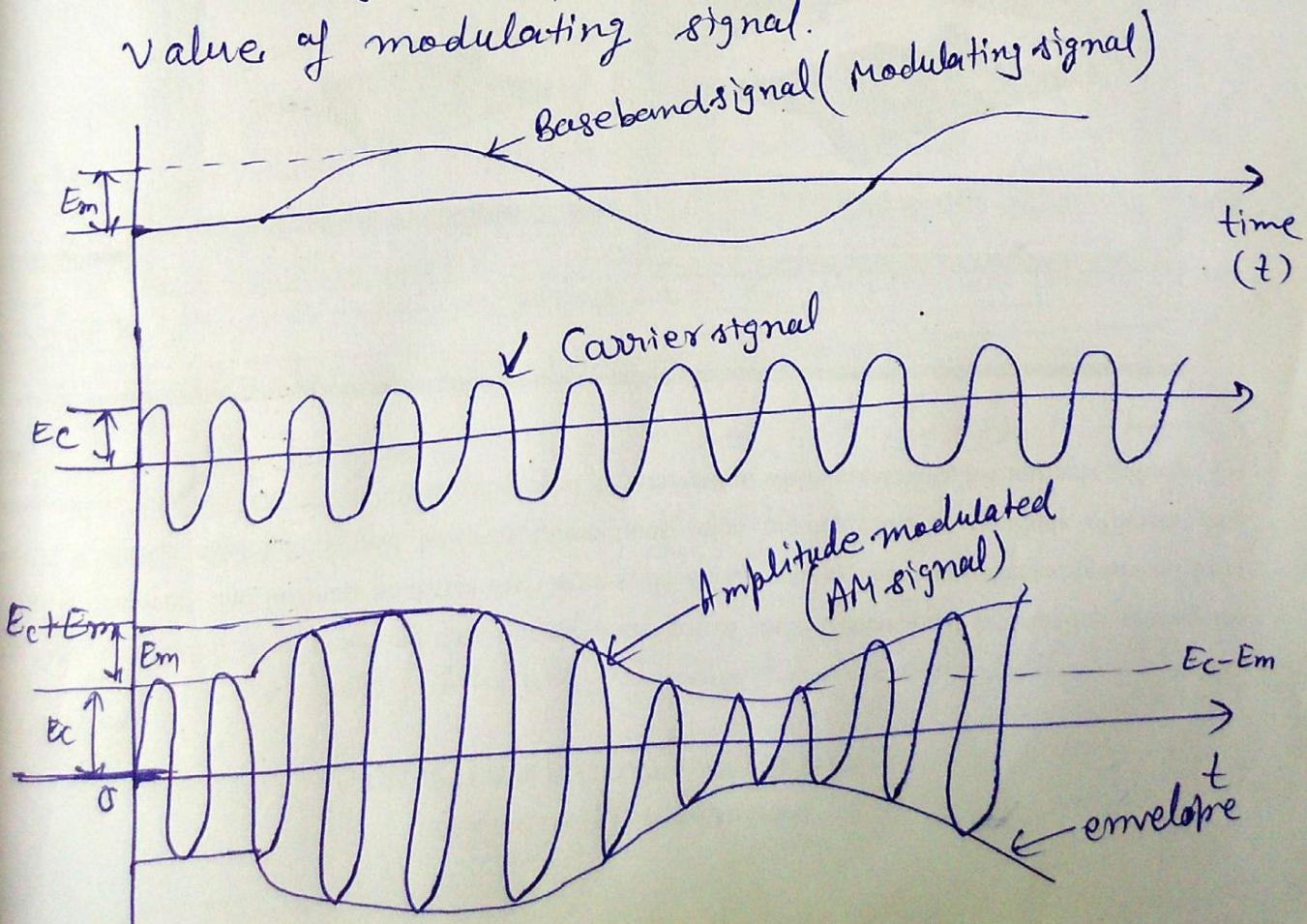
Modulation Systems

(21)



Classification of Modulation

(a) Amplitude Modulation : \rightarrow It is the process of changing the amplitude of a high frequency carrier signal in proportion with the instantaneous value of modulating signal.



Mathematical Representation of an AM wave : \rightarrow (22)

Time domain Description : \rightarrow let modulating signal be sinusoidal and be represented as

$$e_m = E_m \cos \omega_m t \quad \text{--- (1)}$$

where $e_m \rightarrow$ is the instantaneous amplitude of the modulating signal, $E_m \rightarrow$ is the peak amplitude,

$\omega_m = 2\pi f_m$ and $f_m \rightarrow$ frequency of the modulating signal.

let the carrier signal also be sinusoidal at a much higher freq. The instantaneous carrier signal e_c is given by

$$e_c = E_c \cos \omega_c t \quad \text{--- (2)}$$

The AM wave is expressed by the expression

$$c_{AM} = A \cos (2\pi f_c t) \quad \text{--- (3)}$$

value of envelope, the modulating signal either get subtracted or added from the peak carrier amplitude E_c as shown in fig hence we can represent the instantaneous value of envelope as

$$A = E_c + E_m = E_c + E_m \cos (\omega_m t) \quad \text{--- (4)}$$

Hence the AM wave is given by

$$c_{AM} = A \cos (2\pi f_c t) = [E_c + E_m \cos (\omega_m t)] \cos (\omega_c t)$$

(23)

$$\therefore e_{AM} = E_c \left[1 + \frac{E_m}{E_c} \cos(2\pi f_m t) \right] \cos(2\pi f_c t)$$

let $\frac{E_m}{E_c}$ = modulation index.

$e_{AM} = E_c [1 + m \cos(2\pi f_m t)] \cos(2\pi f_c t)$

(5)

It is time domain representation of an AM signal.

Modulation Index : \rightarrow

$$m = \frac{E_m}{E_c}$$

when $E_m \leq E_c$, the modulation index ' m ' has values between 0 and 1 and no distortion is introduced in the AM wave. But if $E_m > E_c$ then m is greater than 1. This will distort the shape of AM signal. The distortion is called as "over modulation".

$\therefore \% \text{ Modulation} = \frac{E_m}{E_c} \times 100$

(6)

Frequency spectrum of the AM wave (Freq. Domain) : \rightarrow

We know that

$$\therefore e_{AM} = E_c (1 + m \cos \omega_m t) \cos \omega_c t \quad \rightarrow (1)$$

simplifying we get

$$e_{AM} = E_c \cos \omega_c t + m E_c \cos \omega_m t \cos \omega_c t \quad \rightarrow (2)$$

we know that

(24)

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

\therefore eq ② becomes.

$$e_{AM} = \underbrace{E_c \cos \omega_c t}_{\text{Carrier}} + \underbrace{\frac{m E_c}{2} \cos(\omega_c + \omega_m)t + \frac{m E_c}{2} \cos(\omega_c - \omega_m)t}_{\text{Upper sideband}} + \underbrace{\frac{m E_c}{2} \cos(\omega_c - \omega_m)t}_{\text{Lower sideband}}$$

(3)

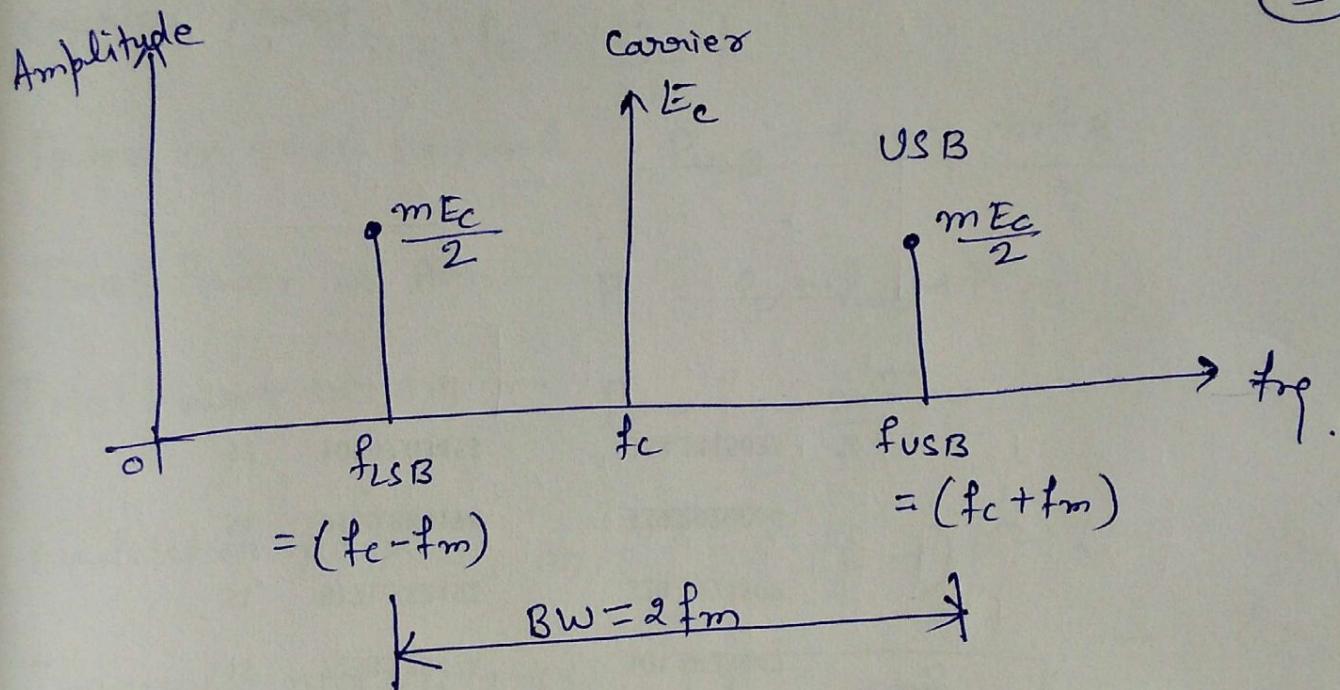


fig (i) Single sided frequency spectrum of AM wave.

Bandwidth of AM wave: \rightarrow

$$BW = f_{USB} - f_{LSB} = (f_c + f_m) - (f_c - f_m)$$

$$\boxed{BW = 2f_m} \rightarrow (4)$$

Important Formulae :

Equation for an AM wave : $E_{AM} = E_C (1 + m \cos \omega_m t) \cos \omega_c t$

$$\text{Modulation index } m = \frac{E_m}{E_C}$$

$$\text{Amplitude of each sideband} = \frac{m E_C}{2}$$

$$\text{frequency of sidebands } f_{USB} = f_c + f_m, f_{LSB} = f_c - f_m$$

$$\text{Bandwidth of AM wave} \quad BW = 2f_m$$

$$\text{Carrier Power} \quad P_C = \frac{E_C^2}{2R}$$

$$\text{Power in each sideband} \quad P_{USB} = P_{LSB} = \frac{m^2 P_C}{4}$$

$$\text{Total Power in AM} \quad P_t = P_C + P_{USB} + P_{LSB}$$

$$\text{Total Power in AM} : P_t = \left[1 + \frac{m^2}{2} \right] P_C$$

$$\text{modulation index} \quad m = \left[2 \left(\frac{P_t}{P_C} - 1 \right) \right]^{\frac{1}{2}}$$

$$\text{Transmission efficiency} \quad \boxed{\eta = \frac{m^2}{2 + m^2}}$$

Advantages, Disadvantages and Application of AM

(i) disadvantage ! \rightarrow (i) Power wastage takes place

(2) AM needs large bandwidth

(3) AM wave gets affected due to noise.

Advantages : (i) AM transmitters are less complex

(2) AM receivers are simple, detection is easy

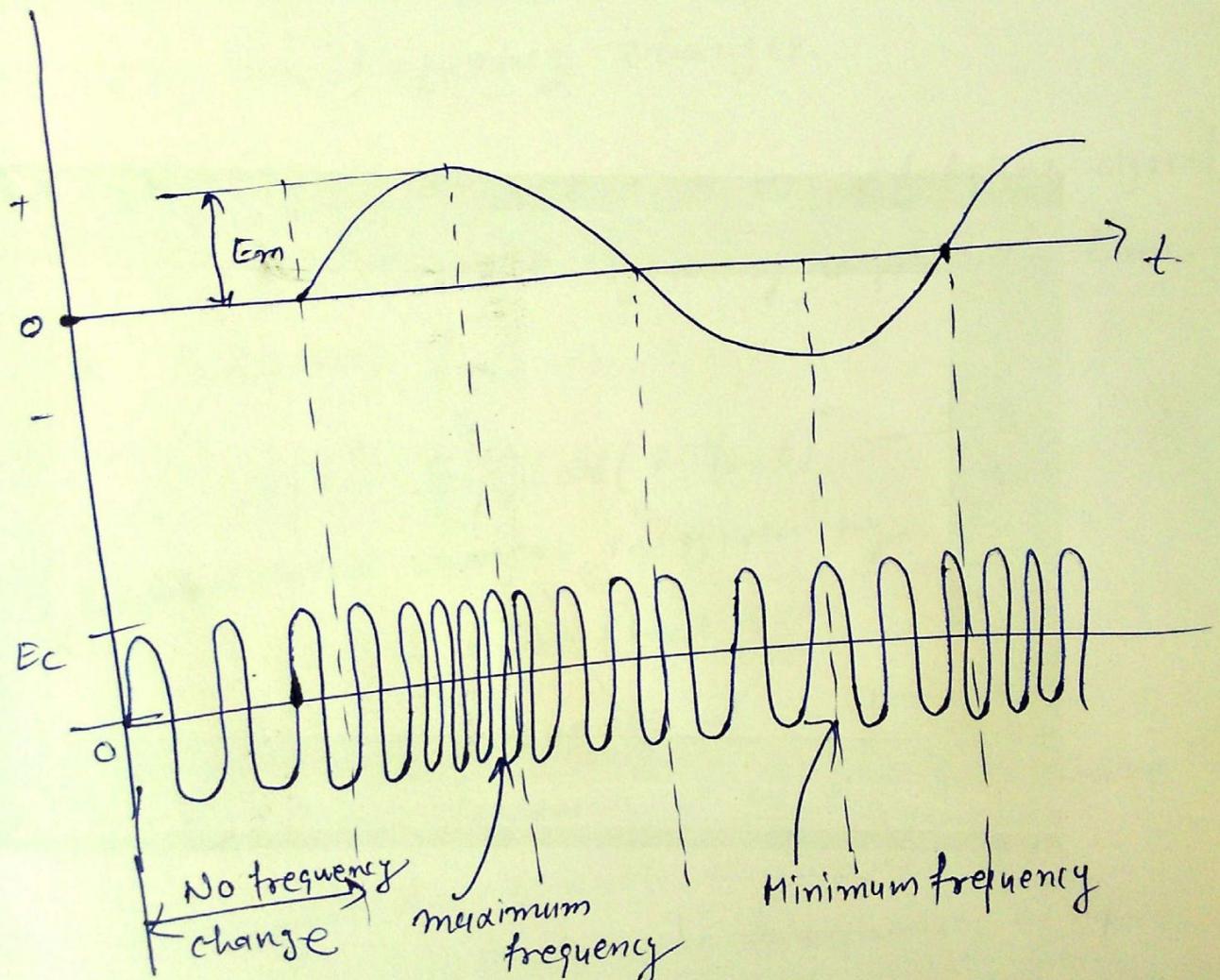
(3) AM receivers are cost efficient. Hence even a common person can afford to buy it.

(4) AM wave can travel a longer distance

(5) Low bandwidth.

- Applications : (i) Radio broadcasting. (26)
(2) Picture transmission in a TV System.

(2) Frequency modulation (FM) : \rightarrow



Time domain display of FM wave

FM is a system of modulation in which the (27)
 instantaneous frequency of the carrier is varied in
 proportion with the amplitude of the modulating
 signal. The amplitude of the carrier signal
 remains constant. Thus the information is
 conveyed via frequency changes.

For the single tone FM, the modulating signal
 $x(t)$ be a sinusoidal signal of amplitude E_m
 and frequency f_m

$$\therefore x(t) = E_m \cos(2\pi f_m t) \quad \textcircled{1}$$

The unmodulated carrier is given by

$$e_c = A \sin(\omega c t + \phi)$$

The instantaneous frequency of an FM wave:

In FM, the frequency of the FM wave
 varies in accordance with the modulating
 voltage. The instantaneous frequency of the
 FM wave is denoted by $f_i(t)$ and is given

$$\begin{aligned} \therefore f_i(t) &= f_c + k_f x(t) = f_c + k_f \cdot E_m \cos(\omega_m t) \\ &= f_c + \delta \cos(\omega_m t) \end{aligned}$$

where $\delta = k_f E_m$ and it is called as frequency
 deviation. where k_f is a constant.

Mathematical formulae :-

$$e_{FM} = E_c \sin \left[\omega_c t + \frac{\delta}{f_m} \sin \omega_m t \right] \quad \text{--- (1)}$$

But $\frac{\delta}{f_m} = m_f$ i.e. the modulation index of FM wave.

Hence the equation for FM wave is given as

$$e_{FM} = E_c \sin \left[\omega_c t + m_f \sin \omega_m t \right] \quad \text{--- (2)}$$

Frequency of FM wave varies according to the modulating signal.

FM wave is a sine wave

Peak amplitude of FM wave is constant and equal to the peak amplitude of the carrier.

Advantages of FM :-

- (1) Improved noise immunity.
- (2) Low Power is required to be transmitted to obtain the same quality of received signal at the receiver.
- (3) Covers a large area with the same amount of transmitted Power.
- (4) Transmitted Power remains constant.
- (5) All the transmitted Power is useful.

Diseadvantages : (1) Very large bandwidth is required.

- (2) Since the space wave propagation is used, the radius of transmission is limited by the line of sight.
- (3) FM transmission and reception equipments are complex.

Applications of FM :

- (1) Radio broadcasting
- (2) Sound broadcasting in TV
- (3) Satellite communication
- (4) Police wireless
- (5) Point to Point Communication.

Frequency spectrum of FM wave: \rightarrow It can only

be solve by using the Bessel functions.

so equation of FM is given by

$$\begin{aligned}
 c_{FM} = s(t) = & E_c \left\{ J_0(m_f) \sin \omega_c t \right. \\
 & + J_1(m_f) \left[\sin(\omega_c + \omega_m)t - \sin(\omega_c - \omega_m)t \right] \\
 & + J_2(m_f) \left[\sin(\omega_c + 2\omega_m)t + \sin(\omega_c - 2\omega_m)t \right] \\
 & + J_3(m_f) \left[\sin(\omega_c + 3\omega_m)t - \sin(\omega_c - 3\omega_m)t \right] \\
 & \left. + \dots \right\}
 \end{aligned}$$

e_{FM} = Carrier + Infinite number of sidebands. (30)

∴

$$e_{FM} = \underbrace{J_0(m_f) E_c \sin \omega_c t}_{\text{Carrier}} + \underbrace{J_1(m_f) E_c [\sin(\omega_c \pm \omega_m)t] + \dots}_{\text{Pair of first sidebands.}}$$

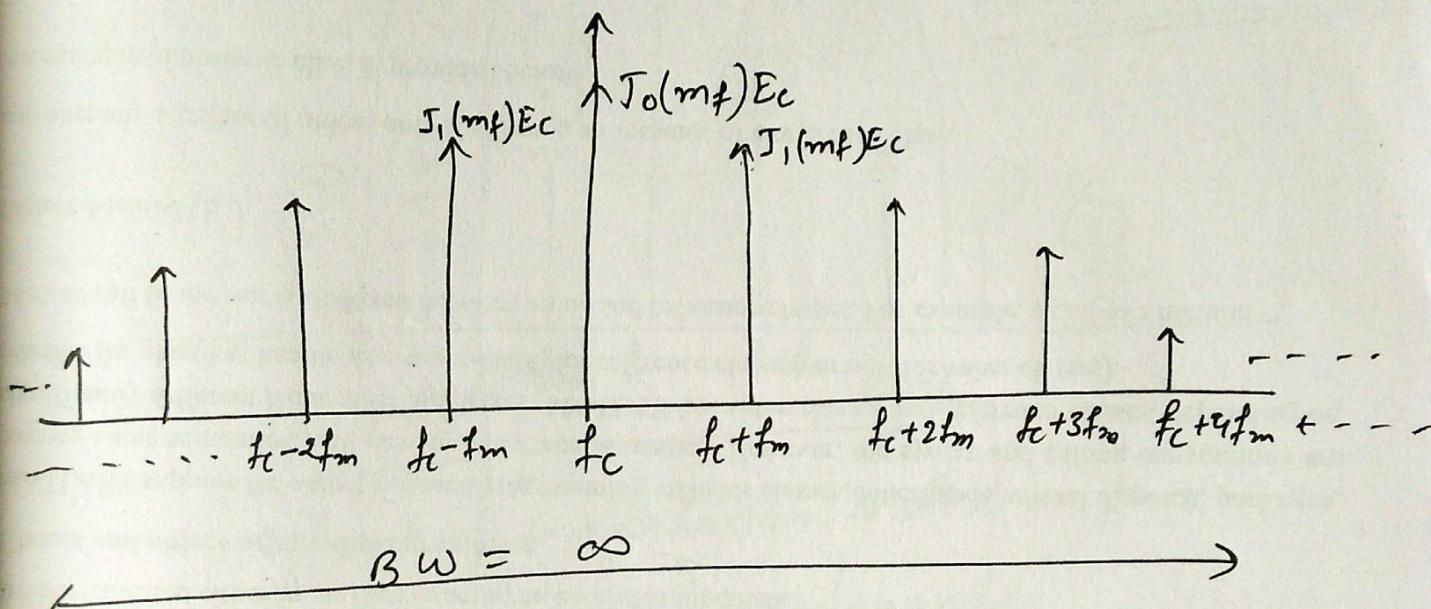
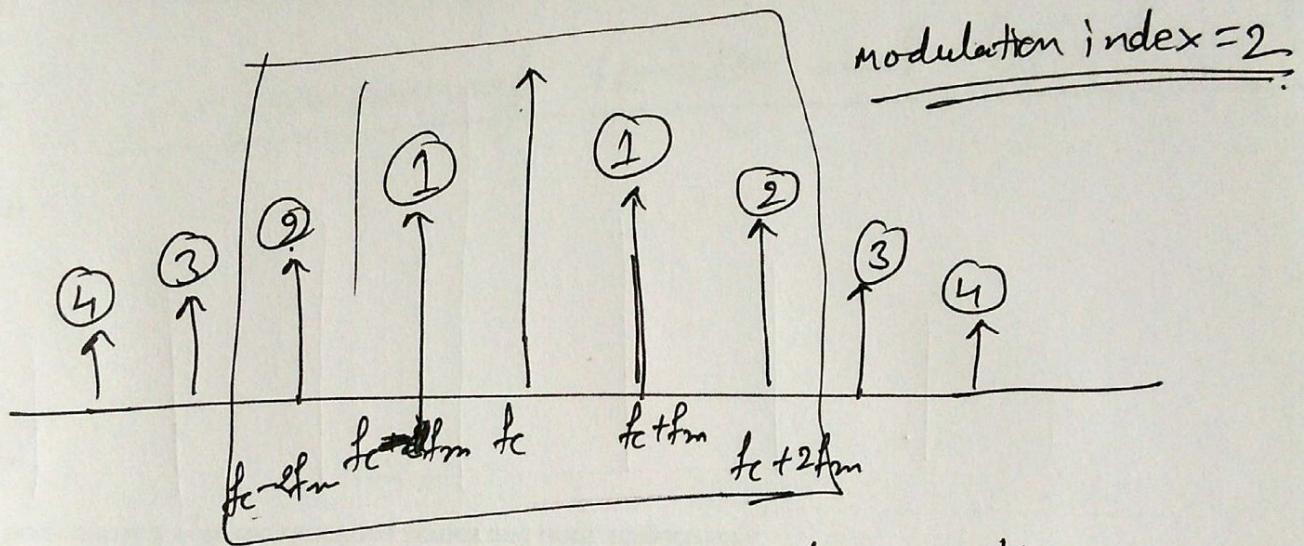
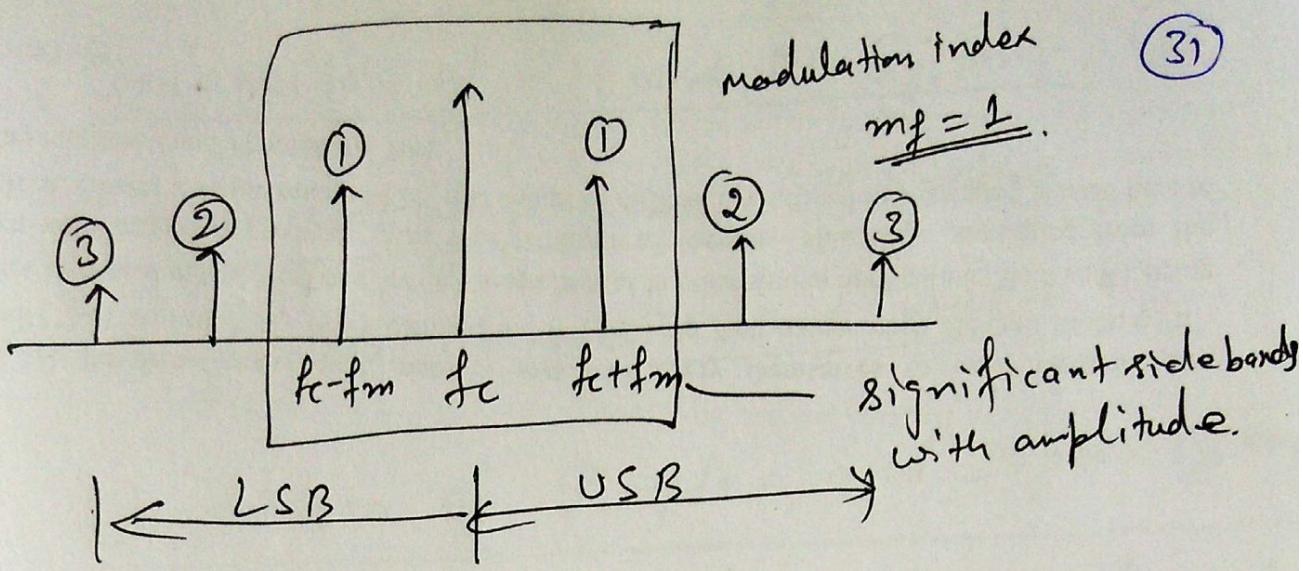


fig (i) Ideal frequency spectrum of FM wave

Note: → The No of sidebands depends upon the modulation index. because the values of J-coefficient are dependent on the modulation index. (m_f).

m_f determines how many sidebands components have significant amplitude.



effect of modulation index on the significant number of sidebands.

$$BW = 2fm \times \text{Number of significant sidebands}$$

- Types of FM : \rightarrow
- (1) Narrow band FM (NBFM)
 - (2) Wide band FM (WBFM)

Q1

Comparison of FM and PM System

(32)

Q2

Comparison of FM and AM System

Q3.

Comparison of NBFM and WBFM

Q4

Comparison of Simplex and Duplex system

Q5.

Comparison of Analog and digital signal