Name:	*Ravi Chaudhary*
SID:	**10947986**
Title:	**Balancing of rotor in multi-plane.**
Submission Date:	**09/11/2021**

Abstract:

The balancing of rotor experiment was carried out to confirm the possibility of static and dynamic balancing of the system in a multi-plane system. In the experiment, a balancing rig and 4 masses were used where the unknown axial distance of the masses were found through mrl-diagram followed by the measurement of angular position from mr-diagram drawn graphically. The analytical solution of the experiment was also executed to compare with the results obtained graphically and the possible errors were discussed in a percentage. The errors obtained were supposed due to general error in scaling, parallax error, random error and calculation error in scaling. Ultimately, the system concluded to be dynamically balanced applying the graphically calculated and measured angles and distances which also showed the static balance of the system.

Introduction:

Rotors are widely used to manufacture different machinery and mechanical parts which includes automobiles manufacturing, aerospace turbines and other industrial turbomachinery's. The mass revolving around the shaft must coincide with the axis of rotation of the shaft otherwise, it rises unbalanced centrifugal force, leads to the unwanted vibration in the machines **(Harrop, Montague, & Nguyen, 2012)**. The centrifugal force is directly proportional with the speed and increases with the square of the speed. Therefore, the critical speed should be considered because sometimes the unbalance of rotor cannot be observed at lower speed but it increases with the increasement of speed **(Fedák, Záskalický, & Gelvanič, September 8th 2014)**.The precise balancing minimizes vibration reducing the risk of failure, unnecessary noises, increase the lifespan and quality of the machines.

Suppose, an object is moving with an angular velocity (ω) in distance 'r' from the axis of rotation, the outward force experienced on a mass gives centrifugal force, i.e.,

$$Fc=m*r*\omega^2.$$

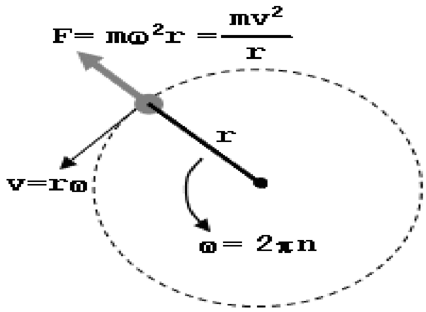


Fig. 1: Centrifugal Force **(Engineers Edge, n.d.)**

There are two types of balancing i.e., static balancing and dynamic balancing. The centrifugal force plays an important role to balance any system statically and dynamically.

- To create a static balance on the system, axis of rotation and axis of centre gravity passing through the centre of mass must lie on the same line and,
- Sum of all the centrifugal forces 'Fc' in the rotating system must be equal to zero **(Varde, 2020).**

i.e., Σ Fc=0

For the system to be in dynamic balance, resultant centrifugal force 'Fc' produced and resultant couple due to the dynamic forces must be equal to zero **(Varde, 2020)**.

Fig. 2: Static and dynamic balancing **(allbiz, n.d.)**

The experiment was carried out for the balancing of overall system in multiplane where two unbalanced masses were lying on the axis of rotation and it was balanced using two extra masses calculating the proper distance on the shaft and angles in different planes on the axis of rotation. These values and angles were calculated either graphically through the sketch of force polygon and couple polygon and also analytically using certain formulas and were compared as percentage error.

Apparatus required:

To perform the experiment theoretically a **graph paper, protector, ruler and pencil** was required. During the experiment, a **rig** consisting **safety cove**r was used. The rotating shaft was mounted horizontally on the nearly frictionless bearings at the end. The operator included **four blocks**, two unbalanced masses with known angular position and axial distance and other two masses was to be added in different plane on the shaft to balance the system calculating their axial distance and angular position. Each block consisted different circular inserts allowing to create 4 blocks of different mass and moments. The protector and the scale were setup to measure the angular position and axial distance of the masses mounted on the shaft respectively. A pulley belt was connected with the DC motor which contributed in the rotation of the mechanism. The system was made statically balanced and the pully-belt was reconnected with motor. The entire system was covered with safety cover, motor turned on and the dynamic balance of the system was confirmed.

Experimental procedure:

- At first, the given value of 4 centrifugal forces (mr1=73, mr2=92, mr3=87 and mr4=65) and the angular distance of mr1 (θ=0°) and mr2 (θ=200°) were considered and 1st and 2nd masses with known axial and angular distance were placed accordingly on the rotating shaft in the correct position.
- The angular positions of 3rd and 4th masses were calculated geometrically in a graph paper and to do so, mr1 (from origin) and mr2 were produced according to the given data taking y-axis as a reference. The angular positions of mr3 and mr4 were measured by creating an arc from the end of the mr1 and mr2 respectively with compass at certain given distance (mr3=87 and mr4=65), the bisection produced the angular position of the blocks.
- The produced angles were measured in anticlockwise direction taking 'y' as a reference axis and the values were recorded in a table.
- Now the 'mrl' graph was drawn using the given (mrl1=0mm and mrl2=2760mm) values and the given and calculated angular positions. The mrl3 and mrl4 lines were produced and were measured using the scale. The axial distance of these vectors was then recorded on the table.
- The blocks 3 and 4 with certain masses were fitted on the shaft according to the calculated axial distance and angular position with the help of the scale and the protector attached on the system respectively.
- The static balance of the system was checked by removing the drive belt. Now the dynamic balance of the system was then tested covering the system with the safety cover, connecting the drive belt with the motor switching it on.

Results:

Graphical method:

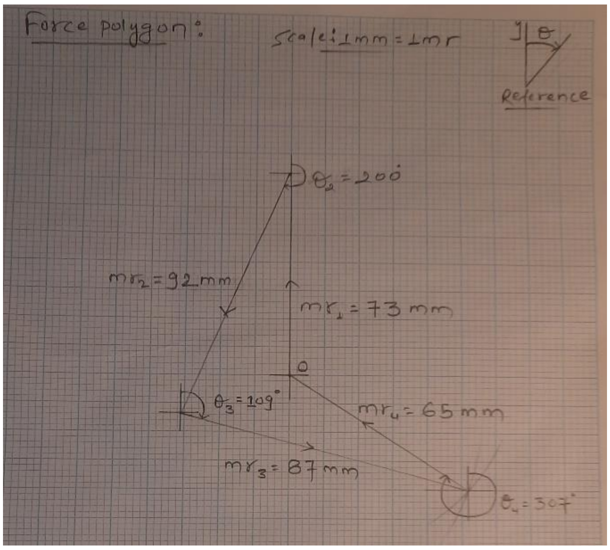


Fig. 3: Force polygon hand-drawn graphically.

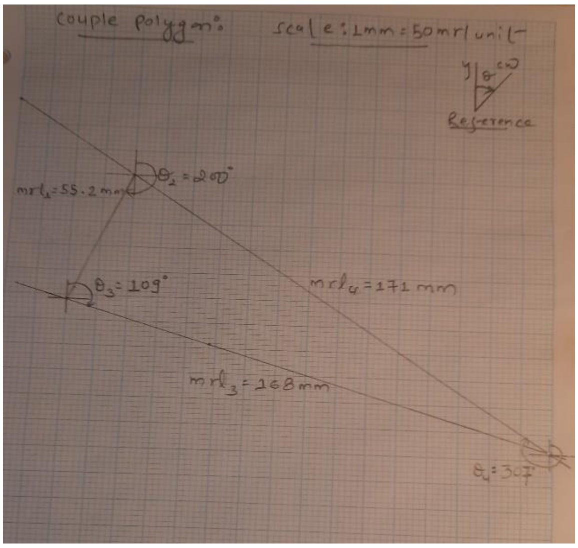


Fig. 4: Couple polygon hand-drawn graphically.

The mr-polygon was drawn with the propose to find (***ϴ3***) and (***ϴ4***). With the concept of static balancing, all the masses were lied on the same plane and the force polygon was produced through which the unknown angular positions i.e., (***ϴ3*** and ***ϴ4***) were derived. The angular values were measured and found to be *109*° and *307*° respectively as shown in figure 3.

Now the concept of dynamic balancing was applied to find the mrl values of mass 3 and 4 where the couple forces were produced in multiplane. The couple polygon was graphed using the angles **ϴ3** and **ϴ4** obtained from the force polygon and the values were found to be ***mrl3=8400*** kg*mm^2 and ***mrl4=8550*** kg*mm^2.

The couple polygon was drawn to a **scale of 1mm=5mrl**. So, the axial distance of $m * r * l_3$ and $m * r * l_4$ was calculated using formula:

$$\mathbf{l_n} = \frac{\mathbf{(m*r*l_n)*scale}}{\mathbf{m*r_n}}$$

- $l_3 = \frac{(m*r*l_3)*scale}{m*r_3} = \frac{168*50}{87} = 96.6mm$

  ⇨ $m*r*l_3 = 87*96.6 = 8404.2$*(Unit)*mm*^{2}
- $l_4 = \frac{(m*r*l_4)*scale}{m*r_4} = \frac{171*50}{65} = 131.5mm$

  ⇨ $m*r*l_4 = 65*131.5 = 8547.5$*(Unit)*mm*^{2}

Table No. 1: The graphically obtained data showing angular position, axial position and couple.

	No. of mass	Centrifugal force =(mass*radius)	Angular Position	Axial Position	Couple
Notations		*m*r*	*ϑ*	*l*	*(m*r*l)*
Unit		*(Unit)*mm*	*Degrees* (°)	*mm*	*(Unit)*mm^2
	1	73	0	0	0
	2	92	200	30	2760
	3	87	**109**	**96.6**	**8404.2**
	4	65	**307**	**131.5**	**8547.5**

Analytical method calculation:

Analytical calculation was done taking all the couples into their horizontal and vertical components. The angles ***Θ3*** and ***Θ4*** were calculated using MATLAB (**script and calculations shown in abstract**).

Table No. 2: The analytically obtained data showing angular position, axial position and couple.

	No. of mass	Centrifugal force =(mass*radius)	Angular Position	Axial Position	Couple
Notations		*m*r*	*ϑ*	*l*	*(m*r*l)*
Unit		*(Unit)*mm*	*Degrees* (°)	*mm*	*(Unit)*mm^2
	3	87	**107.541**	**88.09**	**7664.1**
	4	65	**307.615**	**123.6**	**8033.8**

The percentage error was calculated between graphical and analytical method using the formula below and the calculation can be found in **appendix**:

- percentage error $= \frac{Analytical\ Value - Experimental\ Value}{Analytical\ Value} * 100$

Table No. 3: The calculated percentage error (**mass 3 and 4**) between the analytical and graphical method was shown below.

No. of mass	Graphical Method			Analytical Method			Percentage Error (%)		
	ϑ Degrees (°)	*l (mm)*	*(m*r*l) (Unit* mm^2)*	*ϑ Degrees (°)*	*l (mm)*	*(m*r*l) (Unit* mm^2)*	*ϑ Degrees (°)*	*l (mm)*	*(m*r*l) (Unit* mm^2)*
3.	109	96.6	8404.2	107.541	88.09	7664.1	**1.37**	**9.66**	**9.66**
4.	307	131.5	8547.5	307.615	123.6	8033.8	**0.20**	**6.39**	**6.39**

Discussion:

The analytical method of calculation was performed and compared with the values obtained graphically. It differences in between those values was below 10% which was acceptable if applied to real life unbalancing and vibration problems in the rotor. The highest percentage error found to be approximately **9.66** which could be due to the scaling factor, 1mm or 1° inaccuracy in measurement might lead to 50kg*mm^2 difference in the mrl value (Reference: ***Fig 4***). Other possible errors might be human error in exact measurement, the parallax error- due to viewing ruler or protector at an oblique angle with respect to the axis **(Wallulis, 2018)**. In spite of the theoretical and experimental error analysis, errors causing unbalance in the real-world scenario also needs to be considered.

- *Tooling error*: Error due to inaccurately machined diameter, face location and threated section for damping **(Norfield, 2007)**.
- *Eccentricity*: When the geometric and rotating centre line don't coincide with each other **(Industrial Research and Development, n.d.)**.
- *Distortion*: caused due to distribution of weights and so on.

Conclusion:

To conclude, the main objective of the experiment was to achieve static and dynamic balance in the multiplane system. The system setup balanced dynamically which proved the fact "The dynamically balanced system is balanced statically itself" **(DerekNorfield, 2007)**. Practically, it can be concluded that the perfect balancing i.e., zero vibration in the system is nearly impossible because of tolerances, random and epistemic errors. In the experiment all the 4-masses were added in the system according to the calculations done graphically and analytical calculation was also carried out. It can be concluded that percentage error below **10%** is negligible in balancing the system as it has no adverse effect regarding vibration. For the further improvement in the result, common errors related to scales and other systematic and epistemic errors should be observed deeply, high precision devices and measurements plays vital role to obtain accuracy.

References:

allbiz. (n.d.). Retrieved from Static & dynamic balancing apparatus ls-12020: https://my.all.biz/static-dynamic-balancing-apparatus-ls-12020-g55895

DerekNorfield. (2007). The balancing process. In DerekNorfield, *Practical Balancing of Rotating Machinery* (pp. 99-181). Elsevier.

Engineers Edge. (n.d.). Retrieved from Centrifugal Force Equations and Calculator: https://www.engineersedge.com/physics/centrifugal_force.htm

Fedák, V., Záskalický, P., & Gelvanič, Z. (September 8th 2014). *Analysis of Balancing of Unbalanced Rotors and Long Shafts using GUI MATLAB.* London: IntechOpen. Retrieved from https://www.intechopen.com/chapters/46862

Harrop, C., Montague, E., & Nguyen, V. (2012). *Self-balancing in Rotating Machinery.* Worcester: Mechanical Engineering Department. Retrieved from https://web.wpi.edu/Pubs/E-project/Available/E-project-042612-075233/unrestricted/MQP_Final_Report.pdf

Industrial Research and Development. (n.d.). Retrieved from Causes of an Unbalanced RotorIRD: https://irdproducts.com/phone/causes-of-an-unbalanced-rotor.html

Norfield, D. (2007). Tooling and production. In D. Norfield, *Practical Balancing of Rotating Machinery* (pp. 183-211). Elsevier Ltd.

Varde, G. (2020, April 15). *Gautam Varde*. Retrieved from Static and Dynamic Balancing (DOM/TOM)_01: https://youtu.be/Nnaeg1xKSog

Wallulis, K. (2018, April 30). *Sciencing*. Retrieved from How to Prevent Parallax Error: https://sciencing.com/to-simple-homemade-sextant-7448600.html

<u>Appendix</u>:

Analytical Calculation:

The analytical calculation solved using MATLAB:

let a=Θ_3 and b=Θ_4

- syms a b

>> eqn1 = 73*cosd(0) + 92*cosd(200) + 87*cosd(a) + 65*cosd(b) == 0;

>> eqn2 = 73*sind(0) + 92*sind(200) + 87*sind(a) + 65*sind(b) == 0;

>> sol = solve([eqn1, eqn2], [a, b]);

>> asol = sol.a

asol =

(360*atan((13*85403789126173195056934905259670212585526538414004209215^(1/2))/212172034776804222801675115885 +
169444851951920877829311430656/212172034776804222801675115885))/pi

-

(360*atan((13*85403789126173195056934905259670212585526538414004209215^(1/2))/212172034776804222801675115885 - 169444851951920877829311430656

/212172034776804222801675115885))/pi

>> (360*atan((13*854037891261731950569349052596702125855265384140042092 15^(1/2))/2121720347768042228016751158 85 + 16944485195192087782931143065 6/2121720347768042228016751158 85))/pi

ans =

107.5410

>> bsol = sol.b

bsol =

(360*atan(854037891261731950569349052596702125855265384140042092 15^(1/2)/1009938951550348524254389535 - 194764197645886066470472908 8/2019877903100697048508779 07))/pi

-

(360*atan(854037891261731950569349052596702125855265384140042092 15^(1/2)/1009938951550348524254389535 + 194764197645886066470472908 8/2019877903100697048508779 07))/pi

>> (360*atan(854037891261731950569349052596702125855265384140042092 15^(1/2)/1009938951550348524254389535 - 194764197645886066470472908 8/2019877903100697048508779 07))/pi

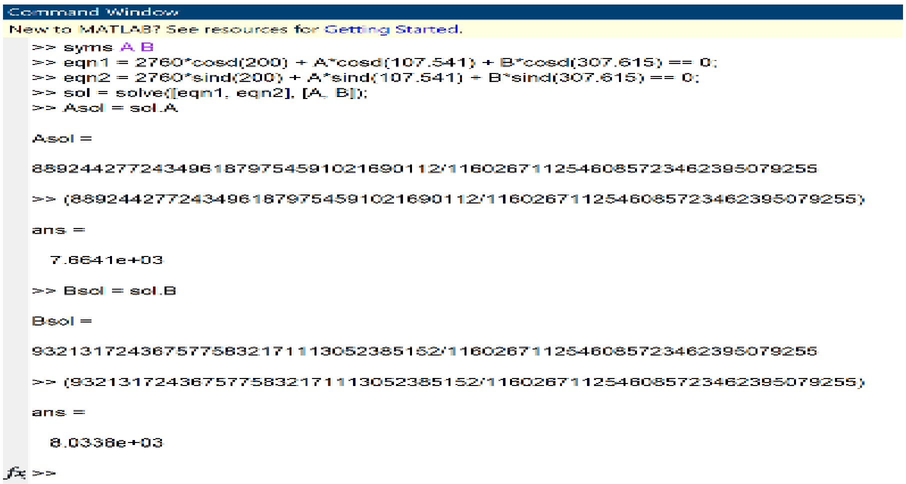
ans =

-52.3855

>> (360-52.3855)

ans =

307.6145



```
Command Window
New to MATLAB? See resources for Getting Started.
>> syms A B
>> eqn1 = 2760*cosd(200) + A*cosd(107.541) + B*cosd(307.615) == 0;
>> eqn2 = 2760*sind(200) + A*sind(107.541) + B*sind(307.615) == 0;
>> sol = solve([eqn1, eqn2], [A, B]);
>> Asol = sol.A

Asol =

8892442772434961879754591021690112/1160267112546085723462395079255

>> (8892442772434961879754591021690112/1160267112546085723462395079255)

ans =

  7.6641e+03

>> Bsol = sol.B

Bsol =

9321317243675775832171113052385152/1160267112546085723462395079255

>> (9321317243675775832171113052385152/1160267112546085723462395079255)

ans =

  8.0338e+03

fx >>
```

***Fig.5**: MATLAB script for angles.*

Let A=$(m*r*l)_3$ and B=$(m*r*l)_4$

- syms A B

>> eqn1 = 2760*cosd(200) + A*cosd(107.541) + B*cosd(307.615) == 0;

>> eqn2 = 2760*sind(200) + A*sind(107.541) + B*sind(307.615) == 0;

\>> sol = solve([eqn1, eqn2], [A, B]);

\>> Asol = sol.A

Asol =

8892442772434961879754591021690112/1160267112546085723462395079255

\>> (8892442772434961879754591021690112/1160267112546085723462395079255)

ans =

7.6641e+03

\>> Bsol = sol.B

Bsol =

9321317243675775832171113052385152/1160267112546085723462395079255

\>> (9321317243675775832171113052385152/1160267112546085723462395079255)

ans =

8.0338e+03

```
Command Window
New to MATLAB? See resources for Getting Started.
>> syms a b
>> eqn1 = 73*cosd(0) + 92*cosd(200) + 87*cosd(a) + 65*cosd(b) == 0;
>> eqn2 = 73*sind(0) + 92*sind(200) + 87*sind(a) + 65*sind(b) == 0;
>> sol = solve([eqn1, eqn2], [a, b]);
>> asol = sol.a

asol =

 (360*atan((13*8540378912617319505693490525967021258552653841400420921 5^(1/2))/2121720347768042228016751 15885 + 1694448519519208778293114 30656/2121720347768042228O
-(360*atan((13*8540378912617319505693490525967021258552653841400420921 5^(1/2))/2121720347768042228016751 15885 - 1694448519519208778293114 30656/2121720347768042228O

>> (360*atan((13*8540378912617319505693490525967021258552653841400420921 5^(1/2))/2121720347768042228016751 15885 + 1694448519519208778293114 30656/212172034776804222

ans =

 107.5410

>> bsol = sol.b

bsol =

 (360*atan(85403789126173195056934905259670212585526538414004209215^(1/2)/100993895155034852425438953 5 - 1947641976458860664704729088/2019877903100697048508779 07))
-(360*atan(85403789126173195056934905259670212585526538414004209215^(1/2)/100993895155034852425438953 5 + 1947641976458860664704729088/2019877903100697048508779 07))

>> (360*atan(85403789126173195056934905259670212585526538414004209215^(1/2)/100993895155034852425438953 5 - 1947641976458860664704729088/2019877903100697048508779 07

ans =

 -52.3855

>> (360 - 52.3855)

ans =

 307.6145

fx >>
```

Fig.6: MATLAB script solved for $(m * r * l)_3$ and B=$(m * r * l)_4$

Percentage Error Calculation:

Error in angles

1. %error (θ_3) =$\frac{107.541-109}{107.541}$*100 =1.37%

2. %error (θ_4) = $\frac{307.615-307}{307.615}$*100 =0.20%

Error in axial distance

1. %error (l_3) = $\frac{88.09-96.6}{88.09}$*100 =9.66%
2. %error (l_4) = $\frac{123.6-131.5}{123.6}$*100 =6.39%

Error in axial distance

1. %error (mrl_3) = $\frac{7664.1-8404.2}{7664.1}$*100 =9.66%
2. %error (mrl_4) = $\frac{8033.8-8547.5}{8033.8}$*100 =6.39%

Result:

The force polygon and couple polygon could be drawn using Catia to obtain more accurate angles and mrl values.

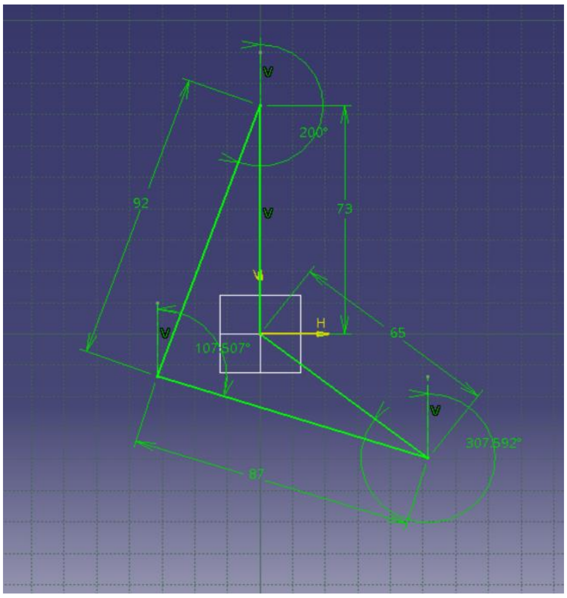


Fig.7: Force Polygon drawn using Catia V5

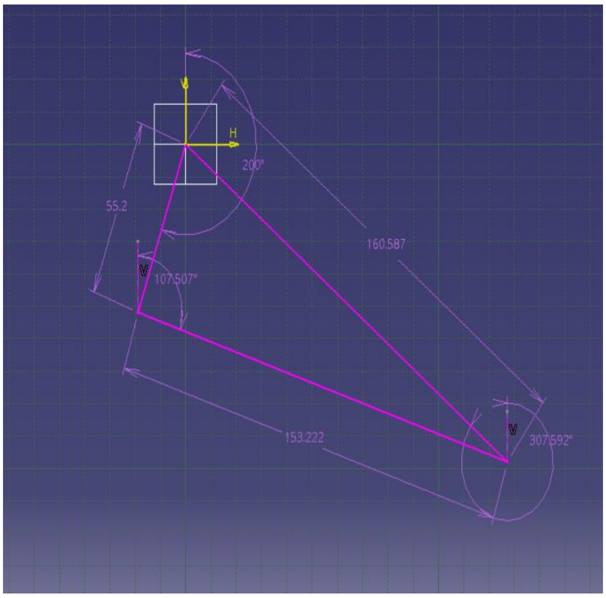


Fig.8: Couple Polygon drawn using Catia V5