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- Use a two-tailed test when:
We want to detect any significant difference (higher or lower)
We want to avoid missing unexpected effects

Caution
- A one-tailed test cannot detect an effect in the opposite direction
- Deciding whether to use a one-tailed or two-tailed test must be done before analyzing the data.

Summary: Understanding the Observed Significance (p-value) in Hypothesis Testing

1 What is the observed significance (p-value)?
The observed significance (p-value) measures how rare our sample result would be if the null hypothesis (H_0) were true
- It is the probability of obtaining a test statistic as extreme as (or more extreme than) the observed value assuming H_0 is true
- A small p-value suggests strong evidence against H_0, supporting the alternative hypothesis (H_a)
- A large p-value means the sample result is not unusual so there is not enough evidence to reject H_0.

2 How to interpret the p-value?
- If $p \leq a$ (eg 0.05), we reject H_0 → The result is significant
- If $p > a$, we fail to reject H_0 → The result is not significant (but we do not "accept" H_0)

3 Example: Left-Tailed Test (One-Tailed Tests)

Pain Relievers Study (Left-Tailed test)

- Test statistic: $Z = -1.89$
- P-value $P(Z \le -1.89) = 0.0294$
- Interpretation: If H_0 were true, only 2.94% of samples would produce results this extreme or more extreme.

If $a = 0.05$, then $p = 0.0294 < 0.05$, so we reject H_0 (evidence for H_a)

- If $a = 0.01$, then $p = 0.0294 > 0.01$, so we fail to reject H_0 (not enough evidence)

4) Example: Two-tailed test

Example: large sample test

Test statistic: $Z = 2.490$

Right-Tail Area: 0.0064

since it is a two-tailed test, we double the tail area →

$p = 2 \times 0.0064 = 0.0128$

Interpretation: If H_0 were true, only 1.28% of samples would give results this extreme

Summary: The P-value Approach to Hypothesis Testing

The p-value approach provides a systematic way to determine whether to reject the null Hypothesis (H_0). Instead of computing rejection regions using critical values, we directly compare the p-value to the significance level (a)

Steps for Hypothesis testing using the p-value Approach

1 State the Hypotheses (H_0 and H_a)

2 Identify the test statistic and its distribution.

3 Compute the test statistic using sample data

4) Find the p-value from the test statistic.
5) compare p-value to a and make a decision:
   if $p \leq a$, reject H_0 (significantly result)
   if $p > a$, do not reject H_0 (not significant)
6 Interpret the conclusion in context

Testing the Null Hypothesis
The null Hypothesis states that the population mean (μ) is equal to be a hypothesized value. A significance test assesses how likely it is to obtain a sample mean that differs from this hypothesized value.

Example 1: Subliminal Message Experiment
Hypothesis: Does subliminal messaging influence picture choice?
Sample: 9 subjects, each making 100 choices.
Null Hypothesis (H_0) = $\mu = 50$ (no influence).
Sample Mean (M): 51
The probability of obtaining a mean of 51 or greater is computed assuming a normal distribution given:
$\sigma = 5$
$N = 9$, so Standard Error (SE) = $\sigma / \sqrt{N} = 5 / \sqrt{9} = 1.667$

Using a Normal distribution calculator:
- One tailed probability ($M \geq 51$): 0.274 → H_0 is not rejected (not statistically significant)
- Two-tailed probability ($M \leq 49$ or $M \geq 51$): 0.548

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Z-Score Approach

Before modern calculators, probabilities were computed using the z-score formula $z = \frac{M - \mu}{\sigma_M}$

For this example $z = \frac{51 - 50}{1.667} = 0.60$

$\therefore$ The probability remains 0.274, confirming previous calculation

Summary: Large Sample Tests for a Population Mean

Large sample hypothesis test for a population mean are based on the Central limit Theorem (CLT), which states that the sampling distribution of the sample mean $\bar{x}$ is approximately normal when the sample size $n \geq 30$, with mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Test statistic formulation:

If population standard deviation σ is known: $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

If σ is unknown (typically the case), replace it sample standard deviations

$$z = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

Since the sample is large, z follows the standard normal distribution

Example: Testing a New Pain Reliever

Objective: Test if a new pain reliever reduces pain faster than a standard one, which has a mean relief time of 3.5 minutes.

- Sample Data

$n = 50$, $\bar{x} = 3.1$, $s = 1.5$

Hypothesis: $H_0 : \mu = 3.5$, $H_a : \mu < 3.5$

Compute Test Statistic

$$z = \frac{3.1 - 35}{1.5/\sqrt{50}} = -1.886$$

Critical value (5% significance, left-tailed test): $-z_{0.05} = -1.6$

Decision: Since $Z = -1.886$ falls in the rejection region, reject H_0

Conclusion: The new Pain reliever is significantly faster at reducing pain than the standard one at at the 5% level.

Summary of small sample Tests for a Population mean

Key Concepts

- In large samples, the Central Limit Theorem ensures statistical validity
- For small samples, we assume the population follows a normal distribution to ensure validity
- If population standard deviation (σ) is known, the test statistic follows the standard normal distribution (Z- distribution).
- If σ is unknown, the test statistic follow student is t- distribution with $(n-1)$ degrees of freedom.