Standardized Test statistics for small samples

I When σ is known

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Date / /
Page No.

2 When σ is unknown: $T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

- The T-statistic follows Student's t-distribution with $(n-1)$ degrees of freedom.
- The population must be normally distributed.
- Critical values for rejection are taken from the t-distribution table

Using the P-value approach
- When using a table, exact p-values are difficult to compute
- Instead, we approximate p-values using a range from the t-table
- The critical value approach is often preferred

Example: Tennis Racket Price comparison
Hypothesis setup
Null Hypothesis: $H_0: \mu = 179$ (No difference)
Alternative Hypothesis: $H_a: \mu < 179$ (online price is lower)
Significance level: $\alpha = 0.05$

Calculations
sample [155, 179, 175, 175, 161]
sample mean: $\bar{x} = 169$
Sample standard deviation $s = 10.39$
Degrees of freedom: $5 - 1 = 4$
Test statistic $T = \frac{169 - 179}{10.39/\sqrt{5}} = -2.152$

Critical value from t-table $= -2.132$
Rejection Region $(-\alpha, -2.132)$

Date / /
Page No.

Conclusion

- Since $T = -2.152$ falls in the rejection region, we reject H_0
- At the 5% significance level, there is enough evidence to conclude that online prices are lower than retail.

p-value Approach

- Since $T = -2.152$ falls between t-values 2.132 and 2.776, the p-value is between 0.025 and 0.050
- Since $p < 0.05$, we reject H_0

Large sample tests for a population proportion
Key concepts

1 Hypotheses : The null hypotheses (H_0) states that the population proportion equals a specific value (p_0). The alternative hypothesis (H_a) can take three forms
   $p < p_0$ (left - tailed test)
   $p > p_0$ (right - tailed test)
   $p \neq p_0$ (two - tailed test)

2 Test-Statistic: The standardized test statistic follows a normal distribution and is ~~collected~~ calculated as

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

where
$\hat{p}$ = sample proportion
p_0 = hypothesized population proportion
$q_0 = 1 - p_0$
n = Sample size

Date / /
Page No.

3 Sample size condition: The test is valid if the interval $[\hat{p} - 3\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + 3\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}]$ falls within $[0, 1]$

Example: Soft Drink Preference (Critical Value Approach)
A soft drink company claims that more than half of adults prefer its beverage over a competitor's. A sample of 500 people was tested, and 270 (54%) preferred the company's brand.

1 Set Hypotheses

$H_0: p = 0.50$ (even preference)

$H_a: p > 0.50$ (majority prefer the company's brand)

Significance Level: $\alpha = 0.05$

2 Calculate Test statistic

$$z = \frac{0.54 - 0.50}{\sqrt{\frac{(0.50)(0.50)}{500}}} = 1.789$$

3 Determine Rejection Region

Right tailed test; critical value = 1.645

since $1.789 > 1.645$, reject H_0

4 Conclusion: At a 5% significance level, there is sufficient evidence that a majority prefer the company's beverage

Testing the Difference Between Two means (Independent Groups)
This section explains how to test for differences between means from two separate groups. Researchers often compare group means rather than focus on individual values

Date / /
Page No.

Assumptions for Testing the Difference between Two means

1. Homogeneity of variance: The two populations have equal variance.
2. Normality: The populations are normally distributed.
3. Independence: Each sample value is independent, each subject contributes only one score.

Test calculation steps:

We use the general t-test formula

$$t = \frac{\text{Statistic} - \text{hypothesized value}}{\text{estimated standard error}}$$

For testing the difference between means:

Statistic: The difference between sample means.

Hypothesized value: 0 (assuming no difference in population means.)

estimate standard error: calculated using pooled variance.

Example: Animal Research study

Students rated how wrong they think animal research is using a 7-point scale. The sample statistics for males and females:

Group	n	Mean	variance
Females	17	5.353	2.743
Males	17	3.882	2.985

1. Compute the difference between means

$$M_1 - M_2 = 5.353 - 3.882 = 1.4705$$

2 Estimate variance

$$MSE = \frac{s_1^2 + s_2^2}{2} = \frac{2.743 + 2.985}{2} = 2.864$$

3 compute the standard error of the difference

$$s_{M1-M2} = \sqrt{\frac{2 \times 2.864}{17}} = 0.5805$$

4 Compute t statistic :

$$t = \frac{1.4705}{0.5805} = 2.533$$

5 find p-value (Two-Tailed Test, df = 32):

Two-tailed p-value : 0.0164

one tailed p-value : 0.0082

Conclusion

Since the two tailed p value (0.0164) is below 0.05, we reject the null hypothesis. This means there is a statistically significant difference between male and female opinion on animal research.

This method allows researchers to determine if differences between sample means reflect real population differences or occur due to chance.