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Summary: Probability of Complements

Key Concepts
- The complement of an Event A (denoted A^c) consists of all outcomes not in A

  Probability Rule $P(A^c) = 1 - P(A)$

- This rule is especially useful when calculation P(A) directly is difficult

Example: Coin Toss (At least One Heads in 5 tosses)
- Total possible outcomes in 5 tosses : $2^5 = 32$
- Only 1 outcome has all tails = TTTTT
- $P(\text{All Tails}) = \frac{1}{32}$ so

$P(\text{At Least One Heads}) = 1 - \frac{1}{32} = \frac{31}{32} \approx 97\%$

Summary: Intersection and Mutually Exclusive Events

(1) Intersection of Events ($A \cap B$)

The intersection of two events A and B, denoted $A \cap B$, consists of all outcomes that belong to both A and B

Example Rolling a Die

Let E = "rolling an even number" → $E = \{2, 4, 6\}$

Let T = "rolling a number greater than two" → $T = \{3, 4, 5, 6\}$

Intersection: $E \cap T = \{4, 6\}$ (numbers that are both even and greater than 2)

(2) Mutually exclusive Events

Events A and B are mutually exclusive if they cannot occur together, meaning their intersection is empty

$P(A \cap B) = 0$

Key point : A and its complement A^c are always mutually exclusive

Example : Mutually Exclusive Events of Rolling a Die

Let E = "rolling an even number" → $E = \{2, 4, 6\}$

To find mutually exclusive events, pick an Event A that shares no outcomes with E

$A = \{1, 3, 5\}$ (odds number complement of E)

These choice work because they do not contain any numbers from E, making $A \cap E \neq \emptyset$ and $P(A \cap E) = 0$

Summary : Union of Events and Additive Rule of Probability

1) Union of Events ($A \cup B$)

The union of two events A and B, denote $A \cup B$, consists of all outcomes that belong to either, A B, or Both.

Example : Two Child family

Sample Space $S = \{bb, bg, gb, gg\}$ (first letter = first born, second letter = second born)

Events : - B = at least one boy → B $\{bb, bg, gb\}$
- D = different genders → $D = \{bg, gb\}$
- M = same gender → $M = \{bb, gg\}$

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· Unions

$B \cup D = B = \{bb, bg, gb\}$ (all already included in B)

$B \cup M = S$ (entire sample space)

2) Additive Rule of Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example Tutoring service

Given Probabilities:

· Need Math help $P(M) = 0.63$

Need English help $P(E) = 0.34$

Need both $P(M \cap E) = 0.27$

Find Probability of needing either math or English help

$$P(M \cup E) = P(M) + P(E) - P(M \cap E)$$
$$= 0.63 + 0.34 - 0.27 = 0.70$$

Why subtract? Without it we'd overcount students needing help in both subjects.

Summary Conditional Probability & Independence of Events

Conditional Probability

The conditional probability of an event A given that event B has occurred is denoted as $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example (Rolling a Die)

a) Probability of Rolling a 5 given that the number is odd

Given $O = \{1,3,5\}$ (odd numbers) $F = \{5\}$

$$P(F|O) = \frac{P(F \cap O)}{P(O)} = \frac{1/6}{3/6} = \frac{1}{3}$$

b) Probability that the number is odd given that it is 5

Since Rolling a 5 garantees an Odd number, $P(O|F) = 1$

Example (Marriage age and Gender Analysis)

A classification table provides data on 902 individuals who were married before age 40, categorized by gender and age at First marriage.

	E	W	H	Total
M	43	293	114	450
F	82	299	71	452
Total	125	592	185	902

M: male

F: female

E: a teenager when first married

W: in one's twenties when first married

H: in one's thirties when first married

(a) Probability that a randomly selected person was a teenager at first married

$P(E) = 125/902 \approx 0.139$ or (14%)

(b) Probability that a male was a teenager at first married

since it is known, that the person selected is male, all the females may be removed from consideration

From the male only data $P(E|M) = 43/450 \approx 0.096$ or (10%)

Example (Overweight and Hypertension)
A contingency table summarizes the population distrib based on overweight status and hypertension

	O	O^c
H	0.09	0.11
H^c	0.02	0.78

$H \Rightarrow$ The person selected suffers Hyper
$O \Rightarrow$ The person selected is overweight

ⓐ Probability that a person has hypertension given they are overweight

$$P(H|O) = \frac{P(H \cap O)}{P(O)} = \frac{0.09}{0.09+0.02} = 0.8182$$

ⓑ Probability that a person has hypertension given they are not overw

$$P(H|O^c) = \frac{0.11}{0.11+0.78} = 0.186$$

ⓒ Since $P(H|O) = 0.8182$ is over six times larger than $P(H|O^c) = 0.1236$, overweight individuals are significantly more likely to have hypertension.

Independence of Events
1 Definition of Independence
Two events A and B are independent if

$$P(A \cap B) = P(A) \cdot P(B)$$

If this condition does not hold, A and B are dependent

2 Practical Uses of Independence
Checking independence. If you can compute $P(A)$, $P(B)$ and $P(A \cap B)$.

- If $P(A \cap B) = P(A) \cdot P(B)$, then A and B are independent
- If $P(A \cap B) \neq P(A) \cdot P(B)$, then A and B are not independent

Calculating Joint Probability: If A and B are independent calculate $P(A \cap B)$ as $P(A) \cdot P(B)$

Example Rolling a fair die with $A = \{3\}$ and $B = \{1, 3, 5\}$

$P(A) = 1/6$, $P(B) = 1/2$, $P(A \cap B) = 1/6$

Since $P(A \cap B) = \frac{1}{6}$ and $P(A) \cdot P(B) = \frac{1}{12}$. A and B are not independent

Example Analyzing marital status and gender among adults under 40:

$P(F) = 452/902$, $P(E) = 125/902$, $P(F \cap E) = 83/902$

$P(F \cap E) \neq P(F) \cdot P(E)$ the events F and E are not independent.