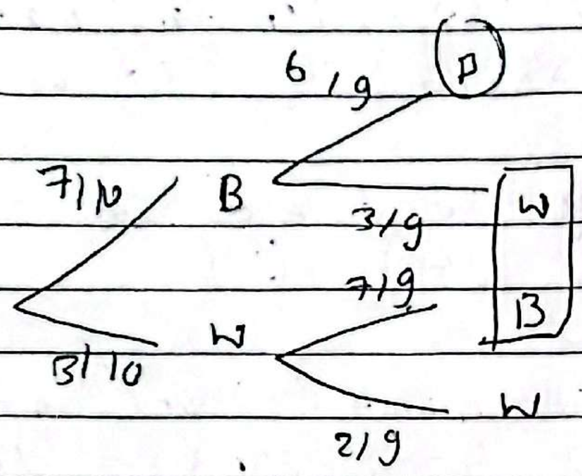
Tree diagrams for probability

Tree diagram principles :

- The probability of an event at any node is the product of probabilities along the path from the starting node.
- for events that correspond to several final nodes, the ~~pro~~ total probability is the sum of the probability at those nodes.

Example

Drawing marbles from Jar (7 black, white)



a. Probability both are black $P(B_1 \cap B_2) = 0.47$

b. Probability exactly one is black $P(BW) + P(WB) = 0.46$

c. Probability at least one is black P(at least one black)

$1 - 0.07 = 0.93$ (using complement rule)

Permutation and Combination

Key Concepts

1) Possible orders:

When arranging n items, the number of different orders is calculated using the factorial of n (denoted as n!)

formula: Number of orders : n!

Example: for three pieces of candy (green, yellow, red), the number of possible orders is

$3! = 3 \times 2 = 6$

2) Multiplication Rule:

When choosing from multiple categories, the total number of combinations is the product of the number of choices in each category.

Example: In a restaurant with 3 soups, 6 entree's and 4 deserts, the total number of possible meals is

$$3 \times 6 \times 4 = 72$$

3) Permutations:

Permutations are used when the order of selection matters for example selecting two pieces of candy from four different colors

Formula : $$^nP_r = \frac{n!}{(n-r)!}$$

Example: For picking 2 pieces out of 4 :

$$^4P_2 = \frac{4!}{(4-2)!} = 12$$

In this case, order matters (eg red and yellow is different from yellow and red).

4) Combinations

Combinations are used when the order of selection does not matter. for example selecting 2 pieces of candy from four different colors.

Formula

$$^nC_r = \frac{n!}{(n-r)! \cdot r!}$$

Example for choosing 2 pieces out of 4

$$^4C_2 = \frac{4!}{(4-2)! \cdot 2!} = 6$$

Here, the order does not matter. (Choosing red and ye[llow] is the same as choosing yellow and red)

Summary
- Permutations (order matters) and Combinations (order does not matters) are essential for calculating different outcomes in probability and statistics.

Random Variables and Distribution

Common Discrete Random variables

Definition of a Random Variable
A random variable is a numerical quantity that results from a random experiment. It is usually denoted by a capital letter (eg X or Z), while specific values it can take are represented by lowercase letters eg (x or z)

Examples of Random Variables

Experiment	Random Variable X	Possible Values of X
Rolling two fair dice	Sum of the dots on top faces.	2, 3, 4, . . . , 12

Types of Random Variables
1. Discrete Random variable
   - Has a finite or countable set of possible values.
   - Typically arises from counting processes

Examples :- Number of heads in 10 coin flips.
- Number of customers arriving at a store in an hour

2 Continuous Random Variable
· can take any value within an interval
· Typically arises from measurement processes
Examples! - A person's height
The time a light bulb last before burning out

Probability Distribution of Discrete Random Variables
A probability Distribution assigns probabilities to each possible values of a discrete random variable X.

Conditions
1 Each probability $P(x)$ must be between 0 and 1:
$$0 \le P(x) \le 1$$
2 The sum of all probabilities must be 1:
$$\sum P(x) = 1$$

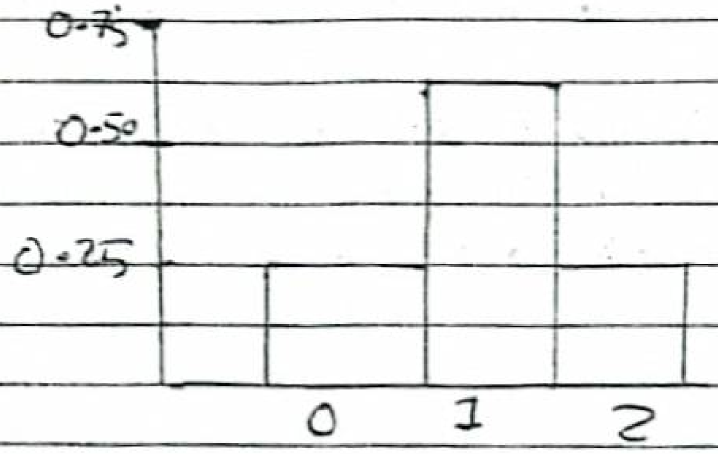
Example 1: Tossing a fair coin Twice
let x be the number of heads observed.

Step 1: construct the probability Distribution
The sample space for tossing two fair coins:
$$S = \{hh, ht, th, tt\}$$
Possible values of X : 0, 1, 2
$X = 0$ (no heads) : $\{tt\} \Rightarrow P(0) = 1/4 = 0.25$
$X = 1$ (one heads) : $\{ht, th\} \Rightarrow P(1) = 2/4 = 0.5$
$x = 2$ (two heads) $= \{hh\} \Rightarrow P(2) = 2/4 = 0.5$

x	0	1	2
$P(x)$	0.25	0.50	0.25

step2 find $P(x \geq 1) = P(1) + P(2)$
$= 0.75$

Probability Distribution for Tossing a fair coin twice



Mean, Variance and Standard deviation of a Discrete Random variable

1) Mean (expected Value)
The mean of a discrete random variable X is given by
$$\mu = E(X) = \Sigma x P(x_i)$$

Example: Computing the mean
given the probability distribution

x	-2	1	2	3.5
$P(x)$	0.21	0.34	0.24	0.21

Appropriate formula
$\mu = (-2)(0.21) + 1(0.34) + 2(0.24) + 1.5(0.21)$
$= 1.135$
so the expected value is 1.135

2) Variance and Standard Deviation
The variance of a discrete random variable is
$$\sigma^2 = \Sigma (x - \mu)^2 P(x)$$
$$\sigma^2 = [\Sigma x^2 P(x)] - \mu^2$$

The standard deviation is the square root of the variance
$$\sigma = \sqrt{\sigma^2}$$

Example Raffle Ticket Problem
Each ticket costs \$1. The prize and net gains are

Outcome	Prize	Net gain (X)	Probability P(X)
1st Prize	\$300	\$299	0.001
2nd Prize	\$200	\$199	0.001
3rd Prize	\$100	\$99	0.001
No prize	\$0	\$-1	0.997

step 1 Compute P(wining)
$P(w) = P(299) + P(199) + P(99) = 0.003$

step 2: Compute Expected value E(X)
$E(X) = 299(0.001) + 199(0.001) + 99(0.001) - 1(0.997)$
$= -0.4$

Interpretation: The negative expected value means that, on average a player loses 40 cent per ticket over many trials

Compute Mean, Variance and Standard Deviation

Given Probability Distribution

x	-1	0	1	4
$P(x)$	0.2	0.5	$a(0.2)$	0.1

Step1 Compute a (Missing Probability)

Since probabilities must sum to 1

$$a = 1 - (0.2 + 0.5 + 0.1) = 0.2$$

Step2 compute Mean μ

$$\mu = -1(0.2) + 0(0.5) + 1(0.2) + 4(0.1)$$
$$= 0.4$$

Step3 Compute variance σ^2

using

$$\sigma^2 = \Sigma (x - \mu)^2 P(X)$$

$$\sigma^2 = (-1 - 0.4)^2 (0.2) + (0 - 0.4)^2 (0.5) + (1 - 0.4)^2 (0.2) + (4 - 0.4)^2 (0.1)$$

$$\therefore \sigma^2 = 1.84$$

Step4 Compute Standard deviation σ

$$\sigma = \sqrt{1.84} = 1.36$$