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Conditional Probability & Dependent Events

1) Understanding Conditional Probability

Conditional Probability refers to the probability of an event given that another event has already occured. It is denoted as:

$$P(B|A)$$

where;

$P(B|A)$ - Probability of event B occuring given that A has occured:

Example Drawing Two Aces From a Deck

- Incorrect Approach (assuming independence):

$$p(\text{two aces}) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169} \quad (\text{incorrects})$$

- Correct Approach (considering dependence)

$P(A_1) = 4/52$ (First card is an ace)

$P(A_2 | A_1) = 3/51$ (Second card is an Ace given first was an ace)

Final Probability

$$P(A_1 \text{ and } A_2) = P(A_1) \times P(A_2 | A_1) = \frac{4}{52} * \frac{3}{51} = \frac{1}{221}$$

2) General formula for dependent Events

If events A and B are not independent, then,

$$P(A \text{ and } B) = P(A) * P(B|A)$$

where;

$P(A)$ = Probability of event A occuring

$P(B|A)$ = probability of B occuring given that A has already occurred.

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The Birthday Paradox

The birthday problem asks: what is the probability that at least two people in a room of 25 share a birthday?

At first glance, you might think the probability is around 25/365 ≈ 0.068, but the actual probability is higher!

1 Complementary Approach

Instead of calculating the probability of at least one match directly, it's easier to calculate the probability that no one share a birthday and subtract from 1

$$P(\text{at least one match}) = 1 - P(\text{no matchs})$$

2 Probability of No shared birthdays

Person 1 can have any of the 365 days : $P_1 = 1$

Person 2 can have a different birthday than pevious 1 $P_2 = \frac{364}{365}$

Person 3 can have avoid both previous birthdays $P_3 = 363/365$

Patter continues till Person 25 $P_{25} = 341/365$

The probobability that no two people share a birthday is :

$$P(\text{no match}) = \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \cdots \frac{341}{365}$$

$$P(\text{no match}) \approx 0.431$$

Now we subtract this from 1

$$P(\text{at least one match}) = 1 - 0.431 = 0.569$$

So, there is a 56.9% chance that at leas two people in a room of 25 share a birthday!

Key Takeaways:
Counterintuitive: The probability is much higher than most expect!
Complement Rule: Instead of finding P(match) directly, it's easier to find P(no match) and subtract from 1.

The Gambler's fallacy
The Gambler's fallacy is the mistaken belief that past independent events influence future outcomes in a way that "balances" results

Example Coin flips
A fair coin is flipped five times, landing heads each time. Many people incorrectly assume that tails is now more likely on the sixth flip because "tails are due" to balance out the results

Reality: Each coin flip is independent. previous outcomes have no effect on future flips. The probability of heads or tails on any single flip remains:
$$P(\text{heads}) = P(\text{tails}) = \frac{1}{2}$$

Sample Spaces and Events
Rolling an ordinary six-sided die is a familiar example of a random experiment, an action for which all possible outcomes can be listed, but for which the actual outcome on any given trial of the experiment cannot be predicted with certainity.

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Definition: A random experiment is a mechanism that produces a definite outcome that cannot be predicted with certainity. The sample space associated with a random experiment is the set of all possible outcomes. An event is a subset of the sample space.

What is a sample space?
A sample space is the set of all possible outcomes in a random experiment.

Example
If you flip a coin, there are two possible outcomes: Heads (H) or Tails (T). Sample Space(S) = {H, T}
If you roll a six-sided die, the possible outcomes are 1, 2, 3, 4, 5 and 6. Sample Space (S) = {1, 2, 3, 4, 5, 6}

What is an Event?
→ An event is a subset of the sample space. It consists of one or more outcomes.
Example
Rolling an even number on a six-sided die. The possible outcomes are 2, 4, 6 Event E = {2, 4, 6}
Rolling a number greater than 2. The possible outcomes are 3, 4, 5 or 6 Event T = {3, 4, 5, 6}

Examples of Sample Spaces for different experiments
Flipping Two Coins
If we flip two indistinguishable coins (same type of coin), the outcomes can be :-

Two Heads (HH)
Two Tails (TT)
One Head and One Tail (HT or TH - (Counted as one outcome since the coins are identical

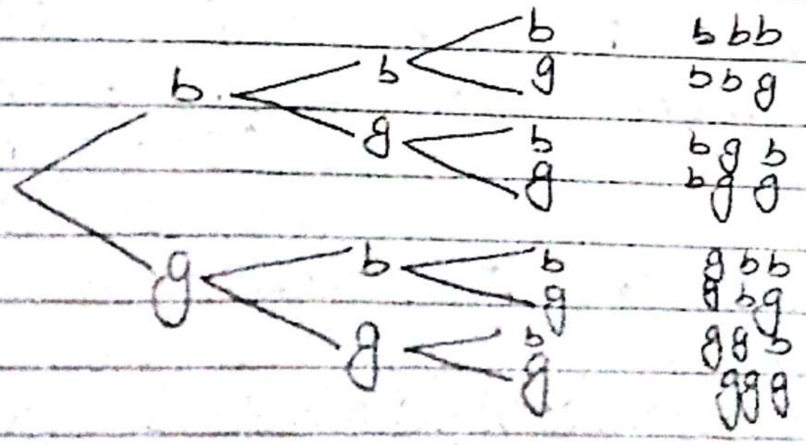
$\therefore$ Sample space is $S = \{HH, TT, (\text{Difference}) D\}$

If the coins are distinguishable (eg penny and a nickel), the sample space includes : $S = \{HH, HT, TH, TT\}$, since now we can tell which coin landed on Heads or tails

~~Three~~ Child Family (Tree diagram Representation)

A tree diagram is helpful in identifying all possible outcomes of a random experiment, particularly one that can be viewed as proceeding in stages.

Solution Two of the outcomes are "two boys then a girl" (bbg) and "a girl then two boys" (gbb). Clearly, there are many outcomes. Listing all of them systematically can be difficult without a structured approach.



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A tree diagram is constructed as follows
There are two possiblities for the first child boy or girl;
For each possibility, there are two option for the second child: boy or girl.
This process is repeated for the third child.

Each segment of the tree represents a decision, and each final node represents an outcome. Reading from top to bottom of the final nodes in the tree: the sample space is:

$S = \{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg\}$

Simplified Exploration of Probability Concepts with Examples
What is Probability?
Probability is a number between 0 and 1 that represent the likelihood of an event happening.
0 means impossible (eg rolling a 7 on six-sided die)
1 mean certain (eg rolling a number between 1 and 6 on a six-sided die)

In real life, we often express probability as a percentage. for example, if the chance of a rain is 70% the probability is written as 0.70.

- Probability of an Outcome
Each outcome in a sample space has a probability assigned to it. The sum of all probabilities must equal 1.

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Example flipping a fair coin
Sample Space $S = \{H, T\}$
Each outcome (heads or tails) has an equal probabil
$P(H) = \frac{1}{2}$, $P(t) = \frac{1}{2}$

2 Probability of an Event
An event is a subset of sample space. The probabi
of an event is the sum of the probabilities of all the individual outcomes in that Event.

Example Rolling a Number greater than 2
Event : $T = \{3, 4, 5, 6\}$
$P(T) = P(3) + P(4) + P(5) + P(6)$
$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$
$\approx 66.67\%$.

3 Probability in Real life : Selecting a Random Student
Suppose a high school has students of different ethnic backgrounds :

Outcome	white	black	hispanic	asian	other
Probability	0.51	0.27	0.11	0.6	0.5

Example: Probability of Selecting Black Student
$P(B) = 0.27 = 27\%$.

Probability of selecting someone who is not black
$P(N) = P(W) + P(H) + P(A) + P(O) = 0.73 = 73\%$

4 Probability in Groups (Two-way Table example)
If we break the student down further by gender and race, we can use two way table to find probabilities

Example Probability of selecting a black student
There are 12% black males and 15% black females
$P(B) = P(BM) + P(BF) = 0.12 + 0.15$
$= 0.27$
$= 27\%$

Key Takeaways on Probability
- Sample Space : The set of all possible outcomes on a random experiment.
- Event : A subset of the sample space, representing specific outcomes of interest.
- Probability of an Outcome : Always a number between 0 and 1
  0 = impossible event
  1 = Certain event
- Total probability Rule : The sum of probabilities of all outcomes in the sample space equals 1.
- Probability of an Event : The sum of the probabilities of all the individual outcomes the make up that event.