

How to Compute Pearson's r (correlation coefficient)

Steps

1. Compute the mean of X and Y
 \bar{X}, \bar{Y}

2. Calculation Deviation scores (x and y);
subtract the mean from each value of $X \Rightarrow x = X - \bar{X}$
subtract the mean from each value of $Y \Rightarrow y = Y - \bar{Y}$
These deviation scores represent how far each value is from the mean

3. Create the product of Deviation scores (xy):
Multiply corresponding deviation scores x and y

4. Sum of xy, x^2 and y^2

5. Use Pearson's r formula:

$$r = \frac{\sum (x \times y)}{\sqrt{\sum (x^2) \times \sum (y^2)}}$$

Let's calculate Pearson's correlation for the data in Table

X	Y	x (deviation)	y (deviation)	x^2	y^2	xy
1	4	-3	-5	9	25	15
3	6	-1	-3	1	9	3
5	10	1	1	1	1	1
5	12	1	3	1	9	3
6	13	2	4	4	16	8

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$$X = (2+3+5+5+6) / 5 = 4$$

$$Y = (4+6+10+12+13) / 5 = 9$$

$$r = \frac{\sum xy}{\sqrt{\sum x^2} \sqrt{\sum y^2}}$$

$$= \frac{30}{\sqrt{(16) \times (60)}} \approx 0.968$$

What is the correlation between the two variables X and Y listed below?

SN	X	Y	X deviation	Y deviation	xy	x ²	y ²
1	8	10	-1.42	0.08	-0.1136	2.064	0.0064
2	10	9	0.58	-0.92	-0.5336	0.3364	0.8464
3	10	11	0.58	1.08	0.62464	0.3364	1.1664
4	11	11	1.58	1.08	1.7064	2.4964	1.1664
5	12	8	2.58	-1.92	-4.9536	6.6564	3.6864
6	12	10	2.58	0.08	0.2064	6.6564	0.0064
7	15	14	5.58	4.08	22.7664	31.1364	16.6464
8	5	8	-4.42	-1.92	8.4864	19.5364	3.6864
9	11	11	1.58	1.08	1.7064	2.4964	1.1664
10	9	9	-0.42	-0.92	0.3864	0.1764	0.8464
11	11	12	1.58	2.08	3.2864	2.4964	4.3264
12	10	13	0.58	3.08	1.7864	0.3364	9.4864
13	7	12	-2.42	2.08	-5.0336	5.8564	0.8464
14	8	7	-1.42	-2.92	4.1464	0.2064	8.5264
15	6	9	-3.42	-0.92	3.1464	11.6964	0.8464
16	15	12	5.58	2.08	11.6604	31.1364	4.3264
17	9	10	-0.42	0.08	-0.0336	0.1764	0.0064
18	10	11	1.58	1.08	1.7864	6.6564	1.1664
19	10	12	2.42	2.08	2.6236	5.8564	1.1664
20	8	5	-1.42	-4.92	6.9864	0.2064	24.2064
21	8	7	-2.42	-2.92	4.1464	2.0264	8.5264
22	8	10	-1.42	0.08	-0.1136	2.0264	0.0064
23	6	9	-3.42	-0.92	3.1464	11.6964	0.8464
24	10	9	0.58	-0.92	-0.5336	0.3364	0.8464

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}}$$

$$= 0.4868$$

Computing the correlation of the variables $(r) = 0.4868$

Introduction to Probability Standard

Probability is a branch of mathematics that deals with the likelihood of events occurring. It quantifies uncertainty ranging from 0 (impossible) to 1 (certain). Key concepts include experiments, outcomes, events, and probability rules. It is widely used in statistics, finance, science and every day decision making.

Summary of Probability Approaches

① Symmetrical Outcomes

- Probability is based on equally likely outcomes
- Example: A fair coin has a 50% chance of landing on heads or tails. A six sided die has a $1/6$ chance for each number

② Frequentist Approach

- Probability is based on long term relative frequency
- Example: If it rained 62% of the past 100,000 days in Seattle, the probability of rain tomorrow is estimated at 0.62. However, this approach becomes tricky when considering changing weather conditions.

③ Subjective Probability

Probability is based on personal belief or opinion.

Example: predicting an election outcome (eg. "Ms Garcia has a 70% chance of winning") reflects opinion rather than an objective probability.

Basic Probability Concepts

1 Probability of a single Event

When all outcomes are equally likely, the probability of an event occurring is

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total Number of possible outcomes}}$$

Example: Rolling a die

- A six sided die has six possible outcomes i.e. (1, 2, 3, 4, 5, 6)

- Probability of rolling a 1: $\frac{1}{6}$

- Probability of rolling a 1 or 6: $\frac{2}{6} = \frac{1}{3}$

2 Probability of Two Independent Events Both Occurring

Example: Rolling Two dice and getting a sum of 6

There are 36 possible outcomes when rolling two dice

Favourable outcomes for a sum of 6

(1, 5), (2, 4), (3, 3), (4, 2), (5, 1) \rightarrow 5 cases

Probability $\frac{5}{36}$

3 Probability of an Event Not Occurring

- If $P(A)$ is the probability of an event occurring, the probability of it not occurring is $1 - P(A)$

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Example: Rolling two dice and Not getting a sum of 6
Probability of rolling a 6 is $\frac{5}{36}$

Probability of not rolling a 6: $1 - \frac{5}{36}$
 $= \frac{31}{36}$

Probability of Independent Events

1 Definition of Independent Events

Two events, A and B are independent if the probability of B occurring remains the same regardless of whether A occurs.

* Example: Tossing a fair coin twice

- Probability of heads on the second toss remains $\frac{1}{2}$, whether the first toss was heads or tails.
- Independent Events
 - 1 First toss is heads
 - 2 Second toss is heads

* Example of Non Independent Events

- Rain in Houston vs Rain in Galveston
- If it rains in Houston, it is more likely to rain in nearby Galveston

2 Probability of Both Events Occurring (A and B)

If A and B are independent, the probability of both occurring is

$$P(A \text{ and } B) = P(A) \times P(B)$$

Example coin flip & Rolling a six sided die (heads and getting 1)

$$P(\text{Heads}) = \frac{1}{2} \quad P(\text{rolling a 1}) = \frac{1}{6}$$

$$P(\text{Heads and Rolling a 1}) = \frac{1}{2} \times \frac{1}{6}$$

$$= \frac{1}{12}$$

3 Probability of Either Event Occurring (A or B)

For independent events, the probability that either A or B happens is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

example: Rolling a die & Flipping a coin (rolling a 6 or Heads)

$$P(6) = \frac{1}{6} \quad P(\text{Heads}) = \frac{1}{2} \quad P(6 \text{ and Heads}) = \frac{1}{12}$$

$$P(6 \text{ or Heads}) = P(6) + P(\text{Heads}) - P(6 \text{ and Heads})$$

$$= \frac{1}{6} + \frac{1}{2} - \frac{1}{12}$$

$$= \frac{7}{12}$$

4 Alternative Approach using "Not" Probability

Sometimes, it's easier to calculate the probability of an event not occurring and subtract from 1

Example: Rolling a die three times (at least one roll is a 1)

probability of not rolling a 1 in a single throw

$$P(\text{not 1}) = 1 - \frac{1}{6} = \frac{5}{6}$$

probability of not rolling a 1 in three throws

$$\left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \right) = \frac{125}{216}$$

probability of at least one roll being a 1

$$1 - \frac{125}{216} = \frac{91}{216}$$